

### On the Article of Schneiderman

In a recent article, Rob Schneiderman misjudges his music-theoretical competence, making several false statements about music in general and my own work in particular.

First, my orbifolds model inefficient as well as efficient voice leading. What matters is just that voice-leading distance is relevant to the creation or perception of music, a point almost nobody disputes.

Second, I have never asserted that efficient voice leading is more than a norm or “rule of thumb”. My project is to describe the circumstances under which this important norm can be satisfied.

Third, Schneiderman claims, without evidence, that the “rule of thumb” is relevant only for beginners. But this is easily testable: we can ask how often composers like Bach use efficient voice leading. (Very frequently, it turns out.) My book, *A Geometry of Music*, is filled with statistical arguments demonstrating that the norm is robust throughout Western music.

Fourth, Schneiderman seems not to realize that efficient voice leading is used to connect scales in the process of modulation. In *A Geometry of Music*, as well as in several earlier papers, I have shown that this idea helps us understand classical music, twentieth-century music, and jazz.

Fifth, Schneiderman asserts, without evidence, that efficient voice leading is relevant only when an unremarkable (or “benign”) accompaniment is desired. But Part II of my book presents dozens of examples where voice-leading distance, as modeled by geometry, reveals a logic not obvious on the musical surface. There is nothing particularly unremarkable or “benign” about these passages.

Sixth, Schneiderman makes the manifestly implausible claim that “the experienced creator of music certainly hears every voice”. This flies in the face of perceptual studies of “voice denumerability”. Nobody follows all the voices in Ligeti’s *Requiem* or Tallis’s *Spem in Alium*, and

very few can follow all the voices in a complex baroque fugue.

Seventh, Schneiderman contrasts “what sounds good” with the norm of efficient voice leading. But psychological studies suggest that efficient voice leading contributes to the sense of musical coherence.

I want to encourage readers to take a look at *A Geometry of Music*. The book is meant to be accessible to mathematicians and musicians both, and it is written by a practical, committed composer—someone who spends most of his life writing music, and who is constitutionally skeptical of theorizing as an end in itself.

—Dmitri Tymoczko  
Princeton University  
dmitri@princeton.edu

(Received July 25, 2011)

### Response to Tymoczko

Although I would much rather engage in a more philosophical musicomathematical dialogue, Mr. Tymoczko’s letter does present some clear examples of how our viewpoints fundamentally differ. For me, a geometric “model” would relate some mathematical and musical properties in a way that provides some transferable insight. From his work I do not see how our understanding of orbifolds significantly enhances our “understanding” of music (and the converse is certainly not true). As my essay mentions, parameterizations of possible musical states can stimulate experimentation, but the creator of music ultimately chooses what sounds right—yes, I believe that Bach, for instance, heard “every voice” of his beautiful and often complex compositions—and any musical value resulting from these choices does not have a well-defined correspondence to “logic” living in any parameter space.

I also do not share Mr. Tymoczko’s belief that the notion of efficient voice leading is “important”: Any statistical prevalence is explained

by the elementary observation that musical tones tend to move incrementally more frequently than in large jumps; and examples of high prevalence in chorales and left-hand jazz piano accompaniments illustrate my use of the word “benign” to describe efficient voice leading, as the choral music bows to the religious lyric while the pianist’s left hand provides subtle harmonic support for the featured melodic development in the right hand. My unwillingness to accept the importance of efficient voice leading is related to my general dissatisfaction with theories that parade lists of carefully chosen examples while ignoring the existence of multitudes of counterexamples: Sounds that may or may not have any amount of efficient voice leading, also may or may not be good/bad, happy/sad, soulful/trite, coherent/incoherent, interesting/boring,...., etc.

—Rob Schneiderman  
Lehman College  
City University of New York  
robert.schneiderman@lehman.cuny.edu

(Received July 27, 2011)

### On the Article of Schneiderman

In a recent *Notices* article [1], Rob Schneiderman wrote an extremely negative review of several works in the field of mathematics and music. In regard to [2] and [3], of which I was the principal author, he provides some important corrections. My descriptions of percussion scalograms and spectrograms as “objective” representations of musical data was an unfortunate choice of words. In the future, I will try to choose my words more accurately. It is possible that percussion scalograms will not end up providing an effective tool for analyzing musical rhythm. As pointed out in [2] and [3], however, our method is related to research by other workers. Their research, which was not discussed in Schneiderman’s critique, may hold up even if ours does not.

For Gabor transforms, however, the probability is near zero that such a widely applied technique—in audio engineering, computer music, and even singing pedagogy (e.g., [4] and [5])—would prove to be ill-founded. Of course, it will undoubtedly be enhanced by new developments in time-frequency analysis. Some of these new developments were described in [3]. The work of the Feichtinger school, centered at NuHAG, is particularly profound both in applied and pure mathematical approaches and was extensively referenced in [3]. Much of this work is motivated by musical applications (e.g., [6]). The wide-ranging use and theoretical foundations for time-frequency methods was surveyed in [3], with over fifty references to its literature. As to the limited range of musical examples that Schneiderman faults [3] for, all I can say is that the examples were chosen to illustrate the points we wanted to make—the points that were relevant to our goal of briefly surveying over three decades of interdisciplinary work by scientists, engineers, mathematicians, and musicians, with a total of one hundred references. We leave it to *Notices* readers to judge whether Schneiderman's critique of [3] is fair and accurate.

## References

- [1] R. SCHNEIDERMAN, Can one hear the sound of a theorem?, *AMS Notices*, August 2011.
- [2] X. CHENG, J. V. HART, AND J. S. WALKER, Time-frequency analysis of musical rhythm, *AMS Notices*, March 2009.
- [3] G. W. DON, K. K. MUIR, G. B. VOLK, and J. S. WALKER, Music: Broken symmetry, geometry, and complexity, *AMS Notices*, January 2010.
- [4] R. MILLER, *The Art of Singing*, Oxford Univ. Press, 1996.
- [5] S. MCCOY, *Your Voice: An Inside View. Multimedia Voice Science and Pedagogy*, Inside View Press, 2006.
- [6] M. DÖRFLER (P.I.), AUDIOMINER project, available at <http://www.ofai.at/research/impl/projects/audiominer.html>.

—James S. Walker  
University of Wisconsin-Eau Claire  
walkerjs@uwec.edu

(August 10, 2011)

## On Hearing the Sound of a Theorem

Rob Schneiderman's essay "Can one hear the sound of a theorem?" was an article waiting to be written. As a mathematician for many years—now retired—and a composer starting a decade before my retirement, I have always bridled at the common observation that mathematics and music are intimately related. (My answer has always been that they're related in one direction only. Mathematicians are much more likely to enjoy, and even to be talented in, music than musicians are to enjoy mathematics: anyone with talent in both areas is bound to realize at some point that mathematics is likely to afford him a better living than music....)

My musical compositions draw on no mathematics. I doubt very much that anyone hearing them, who doesn't know me, would guess that I spent most of my life as a mathematician. When I compose, I am guided by melodic, harmonic, and rhythmic considerations, and above all by the emotional content of what I write. Intuition plays a key role, of course, as it did in my mathematical research, but this would be true in any creative pursuit, whether in the arts or in the sciences. As Schneiderman's article shows, any attempt to analyze music using mathematical tools is bound to produce only trivial observations, and any attempt to create worthwhile music using mathematical tools in place of musical ones is bound to fail.

—Jacob E. Goodman  
City University of New York,  
Emeritus  
goodman@sci.ccnycuny.edu

(August 15, 2011)

## Mathematical Intimidation

John Ewing's article, "Mathematical Intimidation" [May 2011 *Notices*] is right on the money. Indeed, "value added" is a more slippery notion than even his article suggests. There is no way to know just how much additional proficiency corresponds to raising a test score a given amount, such as ten points, and there is certainly no way to compare the gain in

proficiency of a student whose score increases from, say, 40 to 50 with that of a student whose score increases from 70 to 80. Thus it can be totally misleading to base a comparison of teachers on the average score changes of their classes.

—Bob Stein  
California State University,  
San Bernardino  
bstein@csusb.edu

(August 10, 2011)

## Corrections

The email address given for author James Schwartz, "Gerhard Hochschild (1915–2010)", *Notices*, September 2010 issue, page 1082, was incorrect. The correct address is [jameschwartz1@gmail.com](mailto:jameschwartz1@gmail.com).

The Bôcher Prize citation for Gunther Uhlmann (reprinted in the April 2011 issue of the *Notices*, page 604), states that "The prize also recognizes Uhlmann's incisive work on boundary rigidity with L. Pestov and with P. Stepanov...". The name should be P. Stefanov.

—Sandy Frost

The *Notices* invites readers to submit letters and opinion pieces on topics related to mathematics. Electronic submissions are preferred ([notices-letters@ams.org](mailto:notices-letters@ams.org)); see the masthead for postal mail addresses. Opinion pieces are usually one printed page in length (about 800 words). Letters are normally less than one page long, and shorter letters are preferred.