University students come from many geographic locations and types of secondary and post-secondary schools (including public, private, and preparatory schools) with very different mathematical backgrounds. This results in a diversity of mathematical knowledge, augmented by the fact that what constitutes precalculus varies with source institution and instructor preferences, confounding traditional indicators of student knowledge and maturity such as high school grades. Additionally, students and institutions have disparate expectations of sufficient preparation for higher-educational institutions.

Mathematics placement at the University of Illinois prior to 2007 was an assessment based on ACT math scores. Because of undesirably high proportions of students failing to successfully complete Calculus I, the mathematics department began searching for a more effective placement program. Data from the placement program indicated that the ACT math score was a poor measure of preparedness, while external research indicated that the online software product ALEKS can serve as a preparedness measure for calculus, as higher initial ALEKS scores correlated with higher final grades [3].

ALEKS¹ (Assessment and Learning in Knowledge Spaces) is an artificially intelligent mathematics learning product commercially available to individual users and institutions for assessment and gain of knowledge. ALEKS mathematics courses are available for many grade levels and courses up to precalculus as well as for other subjects.²

In 2007 the mathematics department at the University of Illinois began a new placement program using ALEKS, which was chosen for the ability of students to be assessed remotely via the Internet with immediate reporting and for the ability of ALEKS to provide personalized remediation. The placement exam is an ALEKS assessment—an adaptive series of questions that determines the knowledge state of a student, which is a set of items such as the ability to plot an exponential function or to solve an equation involving rational expressions. The total number of items in the assessment is approximately 200, of which students are asked approximately thirty directly (performance on the remaining items is inferred from these responses). The initial assessment primes the learning module within ALEKS, and the organization of items within the knowledge space allows ALEKS to present to a particular student items that he or she is ready to learn or review. Students are automatically reassessed as they progress.

The remediation component of ALEKS complements the use of an assessment as a placement exam by allowing students to self-remediate before courses begin. At Illinois, the students must

²For ALEKS course products, see http://www.aleks.com/about_aleks/course_products.
achieve the minimum measure of readiness in the four months preceding the start of a course. This approach allows the absolute enforcement of the placement policy, avoiding the difficulties with static and not easily repeatable placement exams (e.g., paper and pencil exams given in large groups on few occasions). Moreover, the responsibility for review of prerequisite material is shifted from the instructors and course to the students. The learning mechanism allows students to refresh forgotten knowledge and distinguish themselves from students who are in a novel learning situation. The placement program and policy were specifically designed to be independent of uncontrollable variables such as instructor variation and textbook selection. The placement exam and syllabi can be modified to respond to variations in the student population and course goals while maintaining an objective standard of readiness.

All students complete an ALEKS assessment and are offered the opportunity to use the mechanisms within ALEKS (the learning module) to remediate if they do not place into the course they wish to take. Roughly 20 percent of the students being assessed independently use the learning module or take another assessment to improve their placement. ALEKS is also used in on-campus summer programs that reach out to certain subsets of the student population to help prepare them for college-level studies. A sufficient ALEKS score is the only access point to enrollment—grades from any prerequisite courses neither allow nor deny access to enrollment under the placement policy. The placement score is the highest cardinality achieved on an assessment within four months of the start of the course.

The underlying hypothesis of the placement program at the University of Illinois is that ALEKS effectively measures the current knowledge of students before beginning a course and that the initial knowledge should be indicative of student performance. Three years of data (over 20,000 assessments) support this hypothesis. In many of the courses and semesters examined, the data shows very high correlations between mean grades over small ALEKS score ranges and range midpoints. This greatly outperforms the former placement policy. Similar correlations for ACT math scores were generally much lower, less consistent year to year, and sometimes negative. (Similar correlations were found for the SAT in previous work by Baron and Norman [1].) ALEKS assessments also report scores for subcategories such as trigonometry and geometry, radical expressions, and exponentials and logarithms. Data analysis indicates that ALEKS scores and some subscores correlate well with final grades and that the ALEKS-based placement program lowered failure and withdrawal rates in nearly all the placement classes in each semester. The placement exam does not require mastery of every subcategory; nevertheless, a sufficient score for calculus indicates broad mathematical knowledge and maturity and near-mastery or mastery of some subcategories. For placement, only the total score was used; the subscore correlations are an interesting consequence of the data provided by an ALEKS assessment.

Using the subscore data in aggregate, Illinois is able to compare the knowledge states of students completing precalculus to those entering directly into calculus. This allows an objective measurement of the effectiveness of precalculus, as taught by the university, as preparation for calculus. If students want to take one of the placement courses in a future semester at Illinois, they must take another assessment so that the knowledge state is current. This allows the progress of some students to be followed through successive courses, enabling the measure of specific knowledge gains and aggregate knowledge gain in several subcategories. Results from these comparisons will be included in upcoming publications.

Departmental changes due to the use of ALEKS also include a reduction in advising staff in the mathematics department and increases in enrollment (perhaps because of the limited temporal validity of the assessments). Lower withdrawal percentages yield more stable rosters, which may allow for more effective course planning and management of instructional resources. The success of the placement program at the University of Illinois has led to several other institutions adopting similar placement mechanisms, including the University of Arizona, Arizona State, the University of Colorado, the University of Missouri, and others.

The authors believe that the success of the placement program is based largely on the accountability of a knowledge-based placement exam that is independent of grades in previous coursework and the active assumption of responsibility of preparation by students. The link between the placement exam and the personalized remediation mechanism strengthens the effectiveness of the placement program, allowing students to demonstrate preparation in a low-risk and high-reward setting. Finally, we believe the success of any placement program hinges on strict enforcement, which is university policy at Illinois. These properties and policies support a successful program that serves the student body and university well.

References
