Using Mathematical Maturity to Shape Our Teaching, Our Careers, and Our Departments

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Since 2006 I've been the director of the Williams College Project for Effective Teaching (PET). While PET does a number of things, its main goal is to help mentor new faculty in all disciplines, helping them to thrive at Williams as teachers and as scholars. It is a great job, as I spend a lot of time talking to ambitious, smart young people; I've also learned an amazing amount about other departments. One thing that has truly surprised me is that no other discipline has a concept analogous to our mathematical maturity. I am sure that a professor of history, say, is more "historically mature" than a first-year student, but it is not the rhetoric that they use. In fact, it takes some effort to give people even an idea of what we mean by mathematical maturity. But it is a vital notion.

First, though, what exactly is "mathematical maturity"? Most of us probably think of it as primarily being about understanding proofs. We see huge leaps in understanding in our students when they finally get it. At Williams, this most often happens in the first course in real analysis or in abstract algebra. A student is struggling to just pass and then is suddenly doing A-level work.

For me, it happened in the first month of college. Bruce Palka was setting up a new honors program in math at the University of Texas. Our text was Spivak's *Calculus*. I found the course hard and had at first no real clue about what was going on. How I got through the first homework problem set, I still don't know. On the second, though not deliberately, I basically copied off of Michael Lacey (who was far more mathematically mature than I; in fact, I suspect he still is). He didn't bother to show up for a meeting for the third homework assignment (I don't blame him), leaving me with my equally ignorant colleagues. Then it happened. It was a Thursday, David Bowie's "Diamond Dogs" was echoing in my head, and I got it. Suddenly problems that seemed impenetrable became intriguing. I could work the problems. This was one of the greatest days of my life.

But there are kinds of mathematical maturity that come even before doing proofs. One of the stumbling blocks for many college students is their lack of fluency in high school algebra. Those who speak this language can get through almost all of the first few years of college math. Those

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who don't, struggle with calculus. Speaking this language is a type of mathematical maturity. Maybe most of you found your first exposure to basic algebra easy. I didn't. We had an ambitious seventh-grade teacher who, in the middle of the year, tried to introduce some algebra to us. I suspect it was just solving a single linear equation in one variable. I just didn't get it. All I remember is that there was an equation with an equal sign, then on the next line a new equation, still with an equal sign. I had no clue as to what was going on. We soon returned to the normal stuff, coming back to this algebra at the end of the year, at which time it all made perfect sense. I had become high school algebraically mature.

This story of mathematical growth can be extended and fleshed out, so that all of us can foster our own personal mathematical maturities, tracing our own mathematical paths from nursery school to the retirement home, from "the cradle to the grave". Who among us would not like to end our days, at age 102, in a hospital bed, surrounded by a loving spouse, children, and grandchildren (maybe with one great-great-grandchild, an infant in arms), and have as our last thought our last moment of insight into mathematics?

How can we use the idea of mathematical maturity to influence our teaching, our careers, and our departments? For teaching, this is fairly clear. We can and should use the idea of mathematical maturity to help shape our classes. Too often our classes are reduced to the mere teaching of technique (which most students think of as all of mathematics). We should always bring in the big picture and keep emphasizing what the goal is. For me, in each lecture there should be a clearly stated punchline, which in turn should be linked to the goals of the unit, the goals of the semester, and to mathematics overall, all at the age-appropriate level of mathematical maturity. This will not only prevent the mindless recitation of facts and techniques but also help to determine what is important and what is mere detail. This is also where research can influence our teaching. Back in the mid-1980s, Ken Hoffman became the first math lobbyist, after a successful career as a mathematician at MIT. Back then I heard him speak once at a colloquium about trying to get an increase in government math funding. He said that all congressmen, and for that matter everyone else in government, thought that math was important. Of that, none had any doubts. What they didn't know was that math was still going on. Those of us in the audience had at that moment a certain smug self-satisfaction, looking down on the philistines in Washington. But then Hoffman asked who was to blame, pointing an accusing finger at the audience. After all, how many students are in math classes every day? How could they possibly know that math is still going on if not told by their teachers? Yes, we are to blame. Mention research. Not in abstract terms but in terms of how it enters into the classroom.

For example, in the late 1980s, as a young postdoc I cared very much about algorithms for factoring multivariable polynomials over the complex numbers. This led me in my classes to emphasize factoring as a key type of problem. Did I go on and on, with all the details of then current algorithms? No. Instead, whenever factoring came up, I just mentioned that if you bump up the number of dimensions, then you would be doing current work that was important. This can be done no matter what type of research you are doing.

Mathematical maturity should also be used to help guide each of us in our careers. Most readers of the *Notices* have Ph.D.s. A large percentage work at schools with heavy teaching loads. These are folks who look sheepish if you ask them about their research, despite the fact that if they are teaching four or more courses a semester, it almost certainly precludes time for research. There has to be another way for these talented people to continue to foster their mathematical maturity and not get stuck explaining precalculus over and over again for the next fifty years. Here is one idea. A group of us (most at schools with far heavier teaching loads than Williams) are writing an introductory problem book for algebraic geometry, creating an informal network of collaborators. We would like for this to become a model for other mathematicians.

What about our departments? We all know of departments in which people stay in their offices, leaving only to teach their classes. Such departments are only as strong as each individual member and are particularly toxic to junior folks. Certainly there are some straightforward methods, such as a weekly department colloquium or daily lunch crowd. There are more radical ideas, such as Williams's DADA seminars and January orgies (not what you think). For a description of these, see some of the past entries on the Williams Math/Stat department blog.

Thus mathematical maturity should not be thought of as a single thing that you either have or don't have. Instead, it is more of a process, one that each of us can develop over our lifetimes. Further, the rhetoric of types of intellectual maturity should be spread to our colleagues in other departments. Eventually, maybe how we teach math can be used as a model even for those in the humanities.