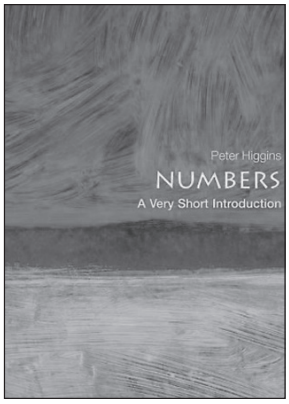
Book Review



Numbers: A Very Short Introduction

Reviewed by Rafe Jones

Numbers: A Very Short Introduction
Peter M. Higgins
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Numbers: A Very Short Introduction, by Peter M. Higgins, is a little book with big aims. Its purpose, set forth in the first sentence, is "to explain, in language that will be familiar to everyone, what are the various kinds of numbers that arise and how they behave." While total fulfillment of this goal lies outside the reach of even much longer works, I found this to be an admirable effort that succeeds to a surprising degree in its 144 undersized pages.

Let me treat first the book's goal of explaining the kinds and behavior of numbers that arise in mathematics. The first five chapters describe the integers and their properties, mostly those involving prime numbers. In an opening chapter with the catchy title "How not to think about numbers", Higgins gives a leisurely account of basic properties of decimal representation and divisibility. He then proceeds to discuss primes and their properties in greater depth, describing unique factorization, the infinitude of the primes, Goldbach's conjecture, and much else besides. Then comes a tour of the flavors of positive integers: perfect numbers, friendly numbers, binomial coefficients, and Stirling numbers, to name a few. Amidst this tour is a full chapter devoted to detailing the RSA encryption algorithm. In Chapter 6, titled "Below the water line of the number iceberg", Higgins dives into a dissection of the real numbers, including irrationals and their colorful history, transcendentals, and a discussion of the "reality" of real numbers. In the penultimate chapter comes an excellent account of the cardinality of infinite sets, the Cantor middle-third set, and continued fractions. Finally, the complex numbers make up the backbone of Chapter 8. All in all, the book covers quite a bit of ground, saying something interesting about each of the principal kinds of numbers in wide use in mathematics. It achieves this about as gracefully as its length permits. That is to say, it sometimes skips about frenetically, such as in Chapter 5 while enumerating flavors of integers, but generally it follows the natural thread from counting numbers outward to the complex numbers. Even when the pace feels rushed, the interest of each nugget and the almost uniformly good quality of the writing minimize the distraction.

If the choice of subject and organization are merely good, the exposition is outstanding. Higgins's background as both a research mathematician and mathematics writer shows clearly in his prose. He writes with a casual flair that sacrifices nothing in clarity and makes the book engaging at every turn. His explanations nearly always find an elusive middle ground: they are precise enough to contain all the outlines of a rigorous proof, but colloquial enough to tell a friend over coffee. For instance, I appreciated the rigor in his description of how Cantor's diagonal argument that the real numbers are uncountable cannot be circumvented by simply adding the missing number to the original list: no matter how many times this is done, "…Cantor's point remains valid: although we can keep creating lists that contain additional numbers that were previously overlooked, there can never be one specific list that contains every real number" (p. 88). I enjoyed the conversational quality of this passage about transcendental numbers: "It is not at all clear that there are any such numbers. However, they do exist and they form a very secretive society, with those in it not readily divulging their membership of the club" (p. 79). Occasionally Higgins's writing even rises to the poetic, as when he describes the transcendental

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numbers as the dark matter on the real number line (p. 91), or when on p. 23 he remarks how the Riemann hypothesis would imply that "primality, in the realm of the very large, takes on the mantle of randomness, with no additional pattern or structure to emerge."

I believe that the book succeeds in its promise to explain the world of numbers in language that is familiar to all. However, that is not to say that no mathematical background is required to get at all the details given in the book. For example, to follow the proof of uniqueness of prime factorization in Chapter 2, the reader must be comfortable with abstract notation for numbers (e.g., n, p, q) and the idea of proof by contradiction. The primary mystery of the book for me is determining whom, exactly, it is written for. Surely it is for neither the true math neophyte nor anyone with a mathematics degree. I suspect its ideal reader is a relatively educated person, perhaps with a degree in a scientific discipline, or maybe an undergraduate partway through a math major and looking for outside reading.

While this ideal audience may not be the broadest, happily the book is engaging and varied enough to hold some interest for just about any reader. The neophyte will enjoy at least Chapter 1 and selected other passages, such as the description of the Hilbert Hotel in Chapter 7, which I found to be particularly well rendered. At the other extreme, the number theorist will smile at the wry humor (the exclamation mark in a factorial, we learn on p. 54, alerts us to its rather alarming rate of growth) and is likely to find many of the descriptions, if not the objects, to be fresh.

I should also note the altogether pleasant physical dimensions of the book. Its weight and length allow it to be picked up with one hand in a perfectly comfortable way, and the small page size makes it almost beg to be thumbed through. I found myself carrying it around even when secretly I knew I wouldn't have time to read any of it. One minor unpleasantness came in the form of several typos and usage problems, which were jarring given the otherwise good quality of the writing and seemingly could have been eliminated in so short a book. *Numbers* contributes to the dizzying expanse of coverage provided by the "Very Short Introduction" series, with pocket-size titles covering ground from *Advertising* to *Schopenhauer*. (There is also *Mathematics*, by Timothy Gowers, for those who are interested.)

In summary, this well-written little book contains a surprising amount of information and is well worth reading or recommending to others. Any reader, but particularly one with a bit of mathematical background, will likely come away enlightened, amused, and wanting to learn more.