Book Review

What's Luck Got to Do with It?

Reviewed by Andrew I. Dale

The origin of the study of mathematical probability is often, though incorrectly, seen as arising in an exchange of letters between Antoine Gombauld (the Chevalier de Méré), Blaise Pascal, and Pierre Fermat in the mid-seventeenth century. This "origin" was rooted in gambling, yet probability theory itself has had little, if any, effect on gamblers' behavior. In *What's Luck Got to Do with It?*, a book enlivened by numerous literary and personal anecdotes, Mazur explores various facets of gambling and luck in a manner that will appeal not only to the general reader but also to those who relish little-known facts and tidbits.

Divided into three parts, *What's Luck Got to Do with It?* leads the reader through historical, mathematical, and psychological aspects of matters relating to gambling. The reader must draw his own conclusions about the wisdom of indulging in such a pastime, for Mazur does not sermonize. Although he no more preaches against gambling than he advocates it, one gets a distinct sense of the unreasonableness of gambling and of its obsessive and destructive nature.

Authors of earlier centuries were less restrained in their opinions of gamblers. For instance, in 1785 Samuel Johnson, in his usual forthright and inimitable style, defined a *gambler* as "A knave whose practice it is to invite the unwary to game and cheat them," while a *gamester* was "one who is vitiously addicted to play; a merry, frolicksome person" or a prostitute.

The ancient gods and goddesses were thought to be bearers of luck (good or bad), and even in these more enlightened times Dame Fortune is believed to be influenced by things like a rabbit's foot or a stepladder. So much for free will! A belief in one's luck may of course result in a warm inner glow, and while not denigrating the psychological value of such a benefit, Mazur notes that "this book concentrates on the mathematics behind gambling to empower the reader who knew—all along—that the powerful illusion of luck is not some acquired supernatural essence but something that can be cogently explained by rules of probability" [p. xvii].

Mazur begins his first chapter with a most descriptive passage that invites one to picture the brute Neanderthals "reflexively gambling every day against the impending extinction of their race" [p. 3] as preparations are made for a sabre-toothed tiger hunt. This fierce scene is contrasted with the image of the proto-human child, innocently indulging in a gentle game with astragali (huckle bones). Mazur then goes on to consider topics ranging from rock painting to the Tudors in England.

Games and gambling were the subject of tight control in England from early times, though the reasons for such control seem perhaps silly today. Act 33, Henry VIII, c. 9, prohibited unlawful games because they interfered with other activities more useful to the kingdom: for example, the maintenance of archery, which was considered more important than the social evils of things like crime and neglect of divine service. In practice, of course, gambling was allowed (or at any rate winked at) to the rich but forbidden to the poor: Mazur records that Henry VIII (1491–1547) and

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his courtiers indulged themselves in the privacy of the royal quarters whenever they wished. By the end of the nineteenth century various statutes had clearly defined which games were lawful and which were not. The former included backgammon, billiards, bowls, chess, cricket, football, golf, rowing, tennis, whist, and wrestling; among the unlawful were dice (excluding backgammon) and lotteries, while boxing was doubtful.

Coin tossing is discussed in Chapter 2: the naive reader may have his faith in things like "the probability of a *head* when a coin is tossed is 1/2" shaken on reading [3]. Attention is also paid to Pascal's Triangle (I missed a reference to [5] here) and Jacob (Jacques, Jakob, James) Bernoulli's *Ars Conjectandi*. (Incidentally, it is not clear from Mazur's index that James and Jacob were the same person.)

The solution of the *problem of points*— concerned with the division of stakes when a game is called off before the agreed-on concluding stage is reached—unfortunately receives little attention from Mazur (see his p. 29). I would also like to have seen more on the St. Petersburg Paradox.

It is a pity that more is not made here of the gambler's ruin problem. For example, suppose a gambler decides to bet on the *n*th trial only when he has seen many more *heads* than *tails* in the preceding $n - 1$ outcomes. Or, after such a sequence of observations, he may decide to bet on *tails* because of the "maturity of chances" (the Monte Carlo fallacy). Unfortunately all such gambling systems are futile.

Feller provides a precise definition of a gambling system, giving it as "a set of fixed rules which for every trial uniquely determine whether or not the bettor is to bet … the rules must be such as to ensure an indefinite continuation of the game. … The importance of this statement was first recognized by von Mises, who introduced the impossibility of a successful gambling system as a fundamental axiom" [6, p. 199]. Shafer and Vovk [9] also take the hypothesis of the impossibility of a gambling system as a fundamental idea, an assumption that, together with something similar on dynamic hedging, serves as a starting point for their development of probability.

Mazur's reader is next taken across the Atlantic to North America and is first of all treated to a discussion of riverboat gamblers. Figure 4.1 in the book showing such a person sharking a businessman recalls Ambrose Bierce's definition in *The Devil's Dictionary*: "The gambling known as business looks with austere disfavor upon the business known as gambling." Things like shipwrecks, the stock exchange, hedge funds, and insurance are discussed, preparing the way for Chapter 5, in which is detailed the general, almost global, economic collapse that started in 2008.

The purchase of an annuity is perhaps one of the soundest gambles in which one can indulge, yet even here there may be pitfalls. Bierce took note of this in defining insurance as "An ingenious modern game of chance in which the player is permitted to enjoy the comfortable conviction that he is beating the man who keeps the table" (op. cit.).

In the anecdote with which Chapter 5 starts, Mazur talks of greed, risk, and reckless behaviour. He details some of the recent bank and financial scandals, often caused by those supposedly "in the know" gambling that some event or other would or would not occur and then finding that they were wrong. Mazur goes so far as to write, "The banking industry's extensive risks … were reckless ventures goaded by unrestrained greed" [p. 61]: Wall Street and Las Vegas are but sisters under the skin. There is discussion of the reasons for the fall of the world financial equities, a fall that one may perhaps uncharitably say came at exactly the right time to be featured in Mazur's book.

In Chapter 7 Mazur notes that, when properties of a long sequence of events are noticed, one should not believe that these properties necessarily hold for shorter subsequences. Ville [10] in fact showed that there exist collectives—i.e., sequences, in which every sub-sequence selected in advance has the same proportion of "successes", say—taking values in {0,1} that have limiting frequency 1/2 yet are such that in every initial segment the relative frequency of 1's, say, exceeds 1/2. This phenomenon occurs in connection with a long run of reds (say) in roulette: those who would firmly bet on a black on the next throw (because of some belief in a "balancing effect") should bear in mind that there is also a possibility, however unsavory one might find it, that the roulette wheel is unfair.

Mazur provides a reasonable discussion of Galton's quincunx (a device intended to illustrate the tendency of a number of small accidental causes to approximate a normal distribution), leading to an investigation of coin tossing in a "double or nothing" game. Here the matter of *risk* arises, leading in turn to *utility* and *value*.

Consideration is given to Daniel Bernoulli and his 1738 paper on utility [1]. The view expressed here, Lopes has noted, was that "For Bernoulli, utility was a *psychological* construct capturing the common intuition" [8, p. 482], and she illustrates this by citing the following passage: "any increase in wealth, no matter how insignificant, will always result in an increase in utility which is inversely proportionate to the quantity of goods already possessed" [1, p. 25] (Sommer's translation). Lopes also notes, however, that the interpretation of *utility* explored in von Neumann and Morgenstern's

masterly work of 1944 was not Bernoulli's, the former now being only for money under risk.

Starting with the binomial distribution in Chapter 8, Mazur leads on in the usual way to the normal. I suspect that the general reader will be perplexed by the continuity correction that appears in the formula on p. 115. (Incidentally, it is worth remarking that the different descriptions "bell-shaped" and "gend'arme's hat", the latter attributed to "a lively French statistician" by Edgeworth [4, p. 600], merely reflect the change in shape of the Normal distribution under various changes of scale.)

That "Truly Astonishing Result", the weak law of large numbers, receives careful attention and illustration in Chapter 9. It is important in considering this law to bear in mind Jacob Bernoulli's own preamble to his result:

> What cannot be ascertained a priori, may at least be found out a posteriori from the results many times observed in similar situations, since it should be presumed that something can happen or not happen in the future in as many cases as it was observed to happen or not to happen in similar circumstances in the past.

[Sylla's translation, 2006, p. 327]. That is, a probability can be "learned" from observation.

It is a pity that there are a number of slips that detract from the usefulness of Mazur's presentation here. For example, in (1) on p. 120 kp should be Np (perhaps occasioned by too slavish a following of [12]?); (2) the symbols μ and σ appearing in Chebyshev's Inequality on p. 121 are not defined; and (3) at the top of p. 122 "at least 1/4" should be "at most 1/4". Once again Mazur scores, however, in exploring the connection between mathematics and the random winds of fortune.

In the last chapter in his second section, "The Skill/Luck Spectrum", Mazur investigates the essential eight gambling games: roulette, craps, slot machines (almost the worst betting values), lotteries, blackjack, poker, horse racing (pari-mutuel betting in America), and sports. He presents the odds on various hands in poker and considers the "pure luck" gambles of lotteries. Here we find the basic gambling strategy: "maximize expectation while minimizing risk" [p. 136].

There is apparently an old French proverb to the effect that "There are two great pleasures in gambling: that of winning and that of losing," and the investigation of these pleasures is explored in the final section.

Theories of gambling addiction, Mazur suggests, have resulted from "tensions between the demands of conscience and the performances of the ego" [p. 155]. Further, it seems that there

may well be those in whom a hidden gambling tendency can be aroused by any one of a number of environmental factors. "It is still a mystery," writes Mazur, "why some habitual social gamblers can manage their gambling pleasures while others lose all judgment of rational gambling behavior in thrill-seeking flirtation with jeopardy" [p. 179]. To some degree one must ask whether one is investigating how people *ought* rationally to act or how they actually *do* act.

Chapter 11, beginning with personal reminiscences, is concerned with the "house money" effect, observed when a gambler freely risks money he has gained from the house. "That fantasy of controlling chance—the overconfident belief in one's personal luck—is the gambler's illusion. It is the daring that confuses chance with skill" [p. 166].

Knowing when to stop playing is perhaps one of the most difficult aspects of gambling. Is it, for instance, greed that makes contestants on live television shows continue even though their chances of a large win are decreasing? Mazur also notes the importance of the *house effect*, viz. "under some circumstances, an earlier gain can increase a subject's eagerness to gamble and an earlier loss can decrease his or her willingness to take risks" [p. 172], and further, "Behavior toward risk depends not only on how that risk is formulated but also on the risk taker's view of gains and losses. For example, a venture may be presented in terms of a risk-aversion or a risk-seeking experience" [p. 178]. Here it might well be the case that one averse to risk may prefer a game having a lower expected value (utility?) to one in which the expected value is higher if the possible losses in the first instance are smaller than those in the second.

Psychopathological theories of the twentieth-century distinguishing between social and neurotic gamblers are explored in Chapter 13. Mazur differentiates between *pathological* and *problem* gamblers: briefly, in the case of the former there is manifested a preoccupation with gambling, irrational behavior, and continuation of such behavior even in the face of adverse consequences, while in the latter the gambler's behavior has a harmful effect not only on himself but also on his family, friends, etc. Freud's ideas are of course considered, a consequence of which is "that the gambler's true motivation may not be his conscious will to win but an unconscious desire caused by some internal conflict, possibly even an unconscious desire to lose" [p. 183]. Or could belief in luck be more a desire to control than a wish to win? After exploring such things Mazur is reluctantly led to conclude that current thinking "is still all theoretical and inconclusive" [p. 186].

There is also discussion of behavioral theories of psychology, where it is concluded that

"Reinforcement and conditioning … motivate the gambler's decisions" [p. 191]. Early wins may also encourage a player to continue: *luck is on my side*. Other behaviorists believe that gambling is driven by boredom or even the euphoria and the action—perhaps even like that induced by drugs (interestingly, Mazur notes on p. 264 that some casinos in the United States have successfully fought smoking bans, the argument being that smoking and gambling go hand in hand for many people). Other psychologists would see the workings of an irrational mind or even the influence of a mixture of "pure" theories.

Mazur concludes this chapter by considering what makes a person a gambler. Neuroscience has shown heightened levels of dopamine during the gambling process. However, "dopamine transmission does not differentiate the activities of extensive gambling, obsessive drinking, and so forth" [p. 200]. Can one in fact conclude anything more than *quot homines tot sententiæ*?

Chapter 14 describes the "hot hands" phenomenon, in which a gambler is enticed back to the gaming table even though losing. Is it caused by a need for excitement? Does one feel "hot" after a winning streak? Does one argue that luck is on one's side or that one has a certain amount of luck in one's account, so to speak, that may be drawn? If the latter, then perhaps one's chances of winning decrease.

The final chapter is mainly concerned with gambling on slot machines, and Mazur notes the importance of being aware of the machine paybacks, often actually less than naively expected. Drawing a comparison with entropy, Mazur writes that "In the long run the chips will drift uniformly in the direction of the house's baited treasury" [p. 214].

The main text ends as follows:

> I would argue that some—if not most—gambling behavior is primarily connected to an intrinsic desire to manipulate luck in order to validate life, to test the forces of uncertainty under a fantasy of knowing something unknowable or to experiment with the new. … Gambling is confirmation that *someone* is in control; it is as natural as belief in God. [p. 216]

While the five appendices are useful, it is unfortunate that they contain some serious errors. For example, in Appendix C [p. 227] it is stated that in a binomial experiment with k successes in N trials "the expected ratio of successes is k/N". What is in fact required is $E[k/N] = p$. Appendix D shows some confusion between conjunction and disjunction: for instance, we find "the probability of *A or B* happening is the product of the probability of *A* and the probability of *B*" [p. 234].

The reading of *What's Luck Got to Do with It?* has led me to find out more about many of the topics Mazur discusses. Despite some of the shortcomings I have mentioned here, I have no hesitation in recommending it to the interested layman, who will find the treatment and style fascinating and will be grateful to Mazur for having set his feet on a path "to fresh woods and pastures new".

References

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