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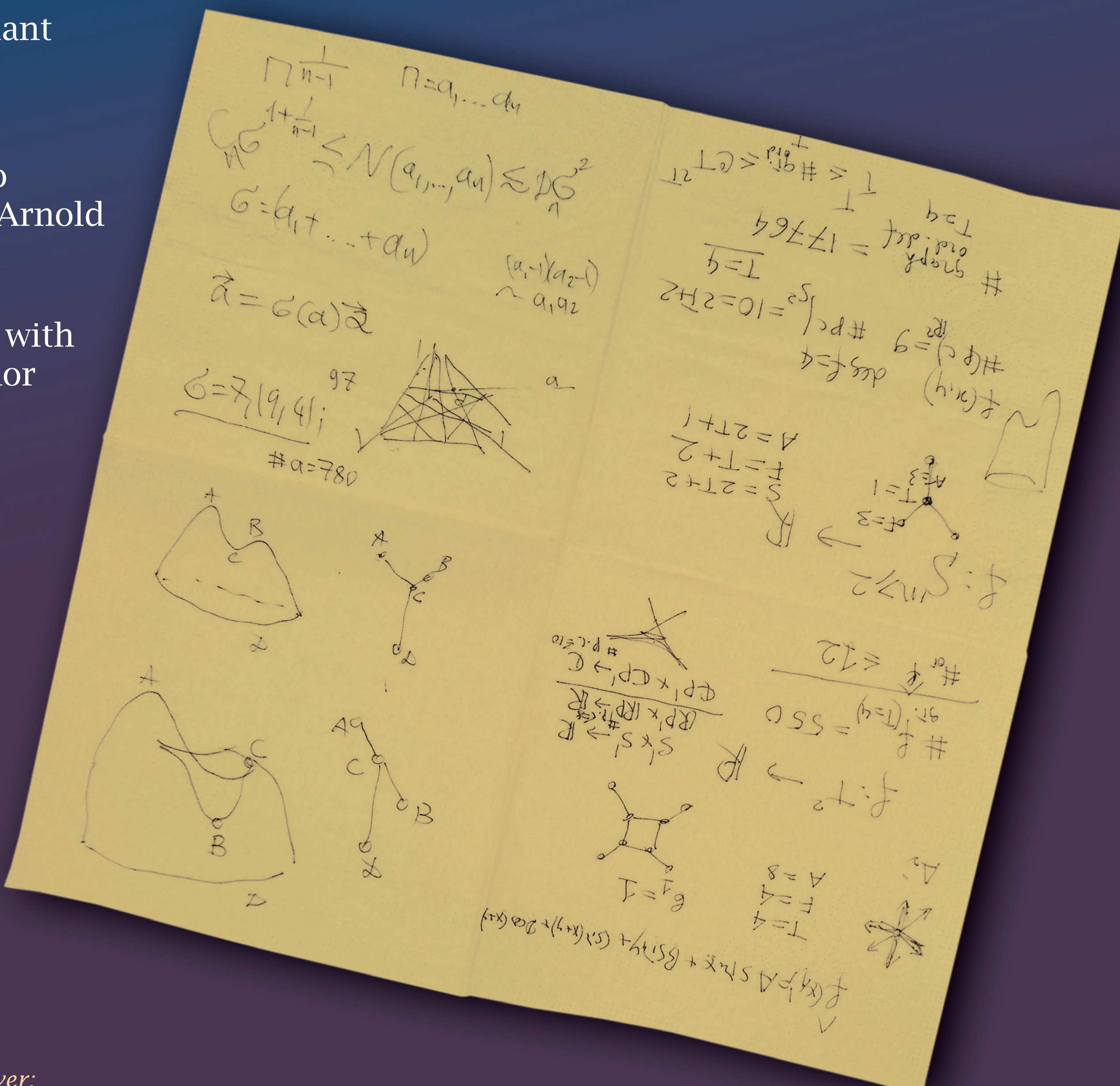

About the Cover: A mathematician's dinner table conversation (see page 399)

The American Mathematical Society presents

The AMS Einstein Public Lecture in Mathematics

Gunther Uhlmann

Professor of Mathematics, University of California, Irvine and University of Washington

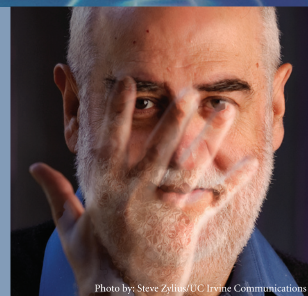

Photo by: Steve Zylius/UC Irvine Communications

Cloaking: Science Meets Science Fiction

Photo by: Steve Zylius/UC Irvine Communications

Saturday, March 17, 2012
5 p.m. with a reception to follow
McCrea Auditorium in Funger Hall (Room 108)
on The George Washington University campus

Can we make objects invisible? This has been a subject of human fascination for millennia in Greek mythology, movies, science fiction, etc., including the legend of Perseus versus Medusa and the more recent *Star Trek* and *Harry Potter*. In the last decade or so there have been several scientific proposals to achieve invisibility. We will introduce some of these in a non-technical fashion concentrating on the so-called "transformation optics" that has received the most attention in the scientific literature.

Photo by: E. SCHREMPP/SPL

Uhlmann's research provides the foundation for recent breakthroughs in "cloaking." He has received many awards, including a Guggenheim Fellowship, a Sloan Fellowship and the Bôcher Memorial Prize. In addition, Uhlmann has been an invited speaker at the International Congress of Mathematicians in Berlin and the International Congress of Industrial and Applied Mathematics in Zurich.

Sponsored by the American Mathematical Society.

Hosted by The George Washington University Department of Mathematics.

This event is part of the AMS 2012 Spring Eastern Sectional Meeting, March 17-18.

ams.org/meetings/sectional/2194_events.html

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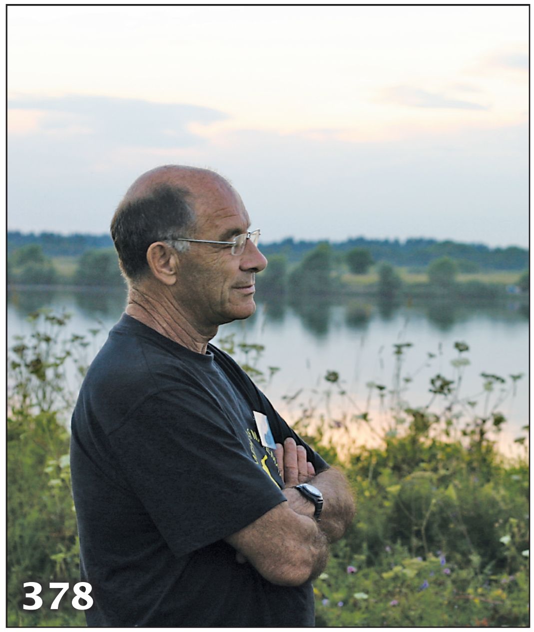

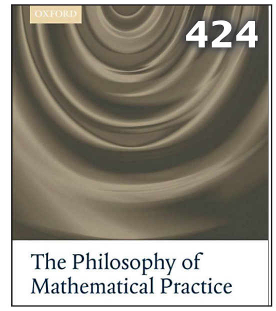

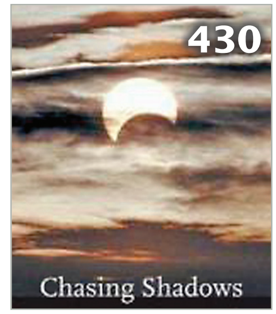

The March issue features a fascinating and broadly based article on circulant matrices. Two other feature articles highlight the distinct personalities of Abel Laureate John Milnor and recently deceased mathematician Vladimir Arnold. There are also interesting contributions concerning author rights and teaching students about ethical issues.

—*Steven G. Krantz, Editor*

Features

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Commentary

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of the American Mathematical Society

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Departments

From the AMS Secretary

Opinion

International Summer School for Students

There are many attractive summer opportunities in mathematics for high school or college students. The AMS site www.ams.org/programs/students/high-school/emp-math-camps lists about forty summer math camps and programs for high school students, and the site www.ams.org/programs/students/undergrad/emp-reu lists eighty-five research experience for undergraduates (REU) summer programs.

We are writing to discuss a new initiative: the International Mathematical Summer School for Students. The first school took place in Bremen in July of 2011; the next one will be in Lyon in August of 2012. The plan is to have the school every year, alternating between Germany and France.

The school differs from the above-mentioned summer camps and REU programs in a couple of ways. First, the summer school participants belong to both groups, high school and college students: specifically, the rising high school seniors, the fresh high school graduates, and the college freshmen and sophomores.

Second, the school is international: there were close to one hundred participants in Bremen, representing twenty-nine countries (the language of the school is English). By regions: Western Europe, forty-one students; Eastern Europe, twenty-four; North America, seven; South America, eight; Asia, six; Africa, six. By age: twenty-two students with two years of university education, twenty-two with one year, twenty-nine who had completed high school but not yet started university, and nineteen still at high school.

The school is inspired by the Russian summer school "Contemporary Mathematics" that has been running for ten years at Dubna near Moscow (www.mccme.ru/dubna/, the site is in Russian); close to two hundred videos of the lectures are posted at www.mathnet.ru/php/conference.phtml?eventID=15&confid=149&option_lang=eng, some of them in English).

The main idea of the school is simple: to invite bright students of mathematics from across the world and to have them learn from—and actively interact with—leading international mathematicians. The instructors were selected by the following criteria: they should present contemporary research in their areas and should be able to communicate in a not-too-technical way; and they should share their excitement and be willing to spend time with the students during the breaks and after classes. To quote from the interview with one of the lecturers, Don Zagier, describing his "random" path to research mathematics early on his career:

> …in college, even, nobody told me what the interesting things are that point to research. It was only in graduate school that I finally found out what modern mathematics is about.

(This interview, along with the videos of the lectures, is available at the website of the school http://math.jacobs-university.de/summerschool/index.php).

Indeed, to many participants, doing advanced mathematics mostly meant being good at solving olympiad problems (many of the participants were members of their national teams for the International Mathematical Olympiad). Judging by the questionnaires filled out at the end of the school, most of the students left with a much wider view of what mathematicians actually do.

Due to space limitations, we cannot give a complete list of the speakers and the topics of their talks. There were nine days of lectures (and one full day devoted to an excursion)—thirty-seven lectures altogether, given by fourteen instructors. Some lectures were plenary, and some ran in two parallel sessions; each speaker gave at least one plenary talk. We cannot help mentioning a few names: J. Conway, J. Hubbard, T. Tokieda (his series of talks "Geometry and physics" was probably the most popular among the students), W. Werner, D. Zagier, G. Ziegler (the authors of this note were among the speakers as well).

What impact did the school have on its participants? We asked them in a questionnaire at the end of the program. A sample of their responses captures their enthusiasm:

> …helped me rediscover the beauty of mathematics and it made me reconsider a career in mathematical research.

> …highlighted how fun mathematics can be and how diverse the mathematical areas are which one can choose from.

> …I am now sure that I want to go into research and more or less which areas I want to go into.

> …inspiring to see such successful mathematicians who are happy and love their jobs.

> …interaction/communication with world-class mathematicians had a great impact on my views, changed a bit my perception of mathematics. I am thinking about becoming a mathematician.

> …I understood better that mathematics is a lot more than mathematical olympiads.

> …now I am really sure that math is all I want to focus on in my future.

> …the school made me think that mathematicians can be happy people… Also, I see mathematics as more unified.

> …I have often felt very lonely in my life, and when I arrived here and got a chance to interact with other participants, that feeling vanished. I can not express how much happiness this has brought to me, and I have all intentions of continuing with mathematics, to be a part of this wonderful community. If I had any doubts before—they're all gone now, this is what I want to do.

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Letters to the Editor

AMS Fellows—Arbeit macht das Leben süss

I completely agree with Richard Laugesen's letter in the December 2011 *Notices*, "AMS Fellows—A Modest Proposal". However, I think that Richard has missed one important point that I would add as a friendly amendment. While it is clear that, as Richard states, any paper submitted by an AMS Fellow should immediately and automatically be accepted by any journal, what is to become of papers submitted by regular folks? Well, certainly we cannot have the mathematical hoi polloi refereeing and reviewing papers written by their peers (or, egads! written by Fellows!). What would happen to quality?

It seems obvious that AMS Fellows should be the only ones who are allowed to referee for journals and to review for MathSciNet. It is only in this way that the vast wisdom of AMS Fellows may be passed down, undiluted, through the generations to the common mathematical folk.

—John Oprea
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(Received November 28, 2011)

Opportunities for Mathematicians in the Startup World

The article by Daniel Krasner in the December 2011 *Notices* was intriguing. I read it while commuting to a startup company in Washington, DC, where I worked from June until December 2011. My path to this position was somewhat different from the author's. I consulted for the firm and then was invited to join full-time, and I'll return to my position as a tenured professor at Georgetown University in August 2012. The author's description of his work is close to my own experience. I was modifying billing spreadsheets, working with heavy-tailed probability distributions, and writing R code for data cleaning, all in the same week.

Does the startup world really offer opportunities for mathematical scientists at the beginning of their career? I believe this is the case, but it surely is not for everybody. Somebody with an undergraduate or Master's degree may be better off in a larger company that has training and mentoring programs. Ph.D. holders may prefer the well-defined career paths that a government agency or a big company can offer. But somebody with a new Ph.D. may also have just the right capabilities for abstraction and problem solving that are essential in the startup world—assuming they retained breadth and flexibility during their training. To succeed, you also have to be willing to use solutions with limited rigorous foundation, to abandon interesting pursuits if they show little potential for business impact, and to pick low-hanging fruit. The goal is to make money, and your and your colleagues' jobs depend on this. Finally, a sense of adventure and a willingness to take risks are important.

Today's Ph.D. programs in the mathematical sciences mostly do not prepare their students for this kind of work. I think that some programs may find interesting opportunities, however. It will enrich our profession if more of our students and colleagues enter that world and succeed in it. Interesting new mathematical questions will be identified, new talent will be attracted to our programs, some of us may end up in the limelight or become rich. Graduate students should not be discouraged from becoming interested in this area. Rather, they should be made aware of these opportunities and be allowed to prepare for them. Consulting opportunities, courses taken outside their field of specialization, and professional contacts with mathematically trained people who are working in the tech world are good first steps.

—Hans Engler
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(Received December 9, 2011)

Response to Quinn

This is a response to the article "A revolution in mathematics? What really happened a century ago and why it matters today", by Frank Quinn, that appeared in the January, 2012 issue of the *Notices*.

My mathematics colleagues almost never think about mathematical logic (see: "The ideal mathematician", Philip J. Davis & Reuben Hersh, `http://people.maths.ox.ac.uk/bui/ideal.pdf`, for what is simultaneously the funniest and most profound description of mathematicians!!). Mathematical logic is almost never taught in mathematics departments—it's taught in computer science departments and philosophy departments—and, when it is, it is taught in a purely technical way with no concern for history or philosophy. Mathematicians still live in Cantor's paradise—or even Eilenberg's paradise—in spite of Russell's paradox; they simply learn not to make certain moves that lead to trouble (as long as the referee doesn't complain, what, me worry?). The various formalizations for avoiding Russell's paradox also prevent one from making certain moves which are usually safe and powerful. So mathematicians work informally and have always done so; there is almost no trace of mathematical logic in most of the history of modern mathematics!! I'm not saying that mathematicians are aware of what I just said; most are totally unaware of these issues and simply working in a successful research tradition.

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(Received December 14, 2011)

The American Mathematical Society

Math in Moscow

Scholarship Program

http://www.ams.org/programs/travel-grants/mimoscow

Study mathematics the Russian way in English

The American Mathematical Society (AMS) invites undergraduate mathematics and computer science majors in the U.S. to apply for a special scholarship to attend a semester in the Math in Moscow program, run by the Independent University of Moscow.

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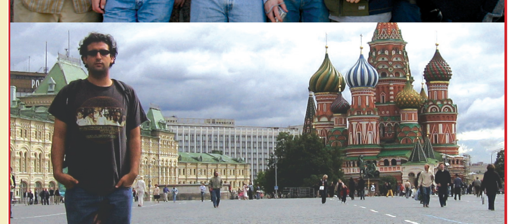
- A 15-week long semester that offers foreign students the chance to spend a semester studying mathematics and computer science in Moscow with internationally recognized research mathematicians.
- Courses are taught in the Russian tradition of teaching mathematics, features of which include: small classes, emphasis on in-depth understanding of carefully selected material, one-on-one interaction with professors.
- Courses are taught in English.
- For more information about the Math in Moscow program and application instructions, visit **http://www.mccme.ru/mathinmoscow/**



On Circulant Matrices

Irwin Kra and Santiago R. Simanca

Some mathematical topics—circulant matrices, in particular—are pure gems that cry out to be admired and studied with different techniques or perspectives in mind.

Our work on this subject was originally motivated by the apparent need of the first author to derive a specific result, in the spirit of Proposition 24, to be applied in his investigation of theta constant identities [9]. Although progress on that front eliminated the need for such a theorem, the search for it continued and was stimulated by enlightening conversations with Yum-Tong Siu during a visit to Vietnam. Upon the first author's return to the U.S., a visit by Paul Fuhrmann brought to his attention a vast literature on the subject, including the monograph [4]. Conversations in the Stony Brook mathematics common room attracted the attention of the second author and that of Sorin Popescu and Daryl Geller* to the subject and made it apparent that circulant matrices are worth studying in their own right, in part because of the rich literature on the subject connecting it to diverse parts of mathematics. These productive interchanges between the participants resulted in [5], the basis for this article. After that version of the paper lay dormant for a number of years, the authors' interest was rekindled by the casual discovery by S. Simanca that these matrices are connected with algebraic geometry over the mythical field of one element.

Circulant matrices are prevalent in many parts of mathematics (see, for example, [8]). We point the reader to the elegant treatment given in [4, §5.2] and to the monograph [1] devoted to the subject. These matrices appear naturally in areas of mathematics where the roots of unity play a role, and some of the reasons for this will unfurl in our presentation. However ubiquitous they are, many facts about these matrices can be proven using *only* basic linear algebra. This makes the area quite accessible to undergraduates looking for research problems or mathematics teachers searching for topics of unique interest to present to their students.

We concentrate on the discussion of necessary and sufficient conditions for circulant matrices to be nonsingular and on various distinct representations they have, goals that allow us to lay out the rich mathematical structure that surrounds them. Our treatment, though, is by no means exhaustive. We expand on their connection to the algebraic geometry over a field with one element, to normal curves, and to Toeplitz's operators. The latter material illustrates the strong presence these matrices have in various parts of modern and classical mathematics. Additional connections to other mathematics may be found in [11].

The paper is organized as follows. In the section "Basic Properties" we introduce the basic definitions and present two models of the space of circulant matrices, including that as a finite-dimensional commutative algebra. Their determinant and eigenvalues, as well as some of

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* *Our colleague Daryl died on February 5, 2011. We dedicate this manuscript to his memory.*

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their other invariants, are computed in the section "Determinants and Eigenvalues". In the section "The Space of Circulant Matrices" we discuss further the space of such matrices and present their third model identifying them with the space of diagonal matrices. In the section "Roots of Polynomials" we discuss their use in the solvability of polynomial equations. All of this material is well known. Not so readily found in the literature is the remaining material, which is also less elementary. In the section "Singular Circulant Matrices" we determine necessary and sufficient conditions for classes of circulant matrices to be nonsingular. The geometry of the affine variety defined by these matrices is discussed in the section "The Geometry of Circ(n)", where we also speculate on some fascinating connections. In the section "The Rational Normal Curves Connection" we establish a relationship between the determinant of a circulant matrix and the rational normal curve in complex projective space and uncover their connection to Hankel matrices. Finally, in the section "Other Connections—Toeplitz Operators" we relate them to the much-studied Toeplitz operators and Toeplitz matrices as we outline their use in an elementary proof of Szegö's theorem.

It is a pleasure for the first author to thank Yum-Tong Siu for outlining another elementary proof of formula (3) and for generating his interest in this topic. He also thanks Paul Fuhrmann for bringing to his attention a number of references on the subject and for the helpful criticism of an earlier draft of this manuscript. It is with equal pleasure that the second author thanks A. Buium for many conversations about the subject of the field with one element and for the long list of related topics that he brought to his attention.

Basic Properties

We fix hereafter a positive integer $n \geq 2$. Our main actors are the n-dimensional complex vector space $\mathbb{C}^n$ and the ring of $n \times n$ complex matrices $\mathbb{M}_n$. We will be studying the multiplication Mv of matrices $M \in \mathbb{M}_n$ by vectors $v \in \mathbb{C}^n$. In this regard, we view v as a column vector. However, at times, it is useful mathematically and more convenient typographically to consider

$$v = (v_0,\ v_1, \dots, v_{n-1}) \in \mathbb{C}_n$$

as a row vector. We define a *shift* operator $T : \mathbb{C}^n \to \mathbb{C}^n$ by

$$T(v_0, v_1, \dots, v_{n-1}) = (v_{n-1}, v_0, \dots, v_{n-2}).$$

We start with the basic and key definition.

Definition 1. The *circulant matrix* $V = \mathrm{circ}\{v\}$ *associated to* the vector $v \in \mathbb{C}^n$ is the $n \times n$ matrix whose rows are given by iterations of the shift operator acting on v; its k^{th} row is $T^{k-1}v$, $k = 1, \dots, n$:

$$V = \begin{bmatrix} v_0 & v_1 & \cdots & v_{n-2} & v_{n-1} \\ v_{n-1} & v_0 & \cdots & v_{n-3} & v_{n-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ v_2 & v_3 & \cdots & v_0 & v_1 \\ v_1 & v_2 & \cdots & v_{n-1} & v_0 \end{bmatrix}.$$

We denote by $\mathrm{Circ}(n) \subset \mathbb{M}_n$ the set of all $n \times n$ complex circulant matrices.

It is obvious that Circ(n) is an n-dimensional complex vector space (the matrix V is identified with its first row) under the usual operations of matrix addition and multiplication of matrices by scalars; hence our first model for circulant matrices is provided by the $\mathbb{C}$-linear isomorphism

(FIRST MODEL) $\qquad \mathcal{I} : \mathrm{Circ}(n) \to \mathbb{C}^n,$

where $\mathcal{I}$ sends a matrix to its first row. Matrices can, of course, be multiplied, and one can easily check that the product of two circulant matrices is again circulant and that for this set of matrices, multiplication is commutative. However, we will shortly see much more and conclude that we are dealing with a mathematical gem. Before that we record some basic facts about complex Euclidean space that we will use.

The ordered n-tuples of complex numbers can be viewed as the elements of the inner product space $\mathbb{C}^n$ with its Euclidean (L^2-norm) and *standard orthonormal basis*

$$e_i = (\delta_{i,0}, \dots, \delta_{i,n-1}),\ i = 0, \dots, n-1,$$

where $\delta_{i,j}$ is the *Kronecker delta* (= 1 for $i = j$ and 0 for $i \neq j$). We will denote this basis by $\mathbf{e}$ and remind the reader that in the usual representation $v = (v_0,\ v_1, \dots, v_{n-1}) = \sum_{i=0}^{n-1} v_i e_i$, the v_i's are the components of v with respect to the basis $\mathbf{e}$.

To explore another basis, we fix once and for all a choice of a primitive nth root of unity

$$\epsilon = e^{\frac{2\pi\imath}{n}},$$

define for $l = 0, 1, \dots, n-1$,

$$x_l = \frac{1}{\sqrt{n}}(1, \epsilon^l, \epsilon^{2l}, \dots, \epsilon^{(n-1)l}) \in \mathbb{C}^n,$$

and introduce a special case of the *Vandermonde matrix*

$$E = \frac{1}{\sqrt{n}} \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ 1 & \epsilon & \cdots & \epsilon^{n-2} & \epsilon^{n-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & \epsilon^{n-2} & \cdots & \epsilon^{(n-2)^2} & \epsilon^{(n-2)(n-1)} \\ 1 & \epsilon^{n-1} & \cdots & \epsilon^{(n-1)(n-2)} & \epsilon^{(n-1)^2} \end{bmatrix}.$$

It is well known and established by a calculation that

$$\text{(1)} \qquad \det E = n^{-\frac{n}{2}} \prod_{0 \leq i < j \leq n-1} (\epsilon^j - \epsilon^i) \neq 0;$$

hence E is nonsingular. In fact, E is a most remarkable matrix: It is *unitary*, $E^{-1} = \overline{E}^t$, and it is *symmetric*, $E^t = E$, and hence $E^{-1} = \overline{E}$; and its columns and rows are the vectors $\{x_l\}$.

We view E as a self-map of $\mathbb{C}^n$ and conclude that $Ee_i = E(e_i) = x_i$. Since E is nonsingular, we see that the $\{x_l\}$ are another orthonormal basis for $\mathbb{C}^n$, to be denoted by $\mathbf{x}$. The $\mathbb{C}$-linear self-map of $\mathbb{C}^n$ defined by the matrix E depends, of course, on the bases for the domain and target; to show this dependence, the map should be denoted as $E_{\mathbf{e},\mathbf{e}}$. Observe that as linear maps, $E_{\mathbf{e},\mathbf{e}} = I_{\mathbf{e},\mathbf{x}}$, where I is the $n \times n$ identity matrix.

To return to circulant matrices, we let

$$W_i = \operatorname{circ}\{e_i\}\,,\ 0 \le i \le n-1.$$

It is obvious that we have a *standard representation* or *form* of circulant matrices:

$$\operatorname{circ}\{(v_0, v_1, \dots, v_{n-1})\} = \sum_{i=0}^{n-1} v_i W_i.$$

It is less obvious, but follows by an easy calculation, that $W_i W_j = W_{i+j}$, where all the indices are interpreted $\bmod n$. Obviously $W_0 = I$, and letting $W = W_1$, we see that $W^i = W_i$.

Remark 2. With respect to the standard basis of $\mathbb{C}^n$, the shift operator T is represented by the transpose W^t of the matrix W. Note that $(W^i)^t = W^{n-i}$.

It is useful to introduce

Definition 3. The (*polynomial in the indeterminate X*) *representer P_V* of the circulant matrix $V = \operatorname{circ}\{(v_0, v_1, \dots, v_{n-1})\}$ is

$$P_V(X) = \sum_{i=0}^{n-1} v_i X^i\,. \tag{2}$$

As usual, we let $\mathbb{C}[X]$ denote the ring of complex polynomials and for $f(X) \in \mathbb{C}[X]$, $(f(X))$, the principal ideal generated by this polynomial. We have established most of the following theorem (the remaining claims are easily verified).

Theorem 4. $\operatorname{Circ}(n)$ *is a commutative algebra that is generated (over $\mathbb{C}$) by the single matrix W. The map that sends W to the indeterminate X extends by linearity and multiplicativity to an isomorphism of $\mathbb{C}$-algebras*

(SECOND MODEL) $\mathcal{J} : \operatorname{Circ}(n) \to \mathbb{C}[X]/(X^n - 1).$

The map that sends a circulant matrix V to its transpose V^t is an involution of $\operatorname{Circ}(n)$ *and corresponds under $\mathcal{J}$ to the automorphism of $\mathbb{C}[X]/(X^n-1)$ induced by $X \mapsto X^{n-1}$.*

Proof. The only nontrivial observation is that multiplication of circulant matrices in standard form corresponds to the multiplication in $\mathbb{C}[X]/(X^n - 1)$. □

Remark 5. The algebra $\mathbb{C}[X]/(X^n - 1)$ can be identified with the space $\mathcal{P}_{n-1}$ of complex polynomials of degree $\le (n-1)$ with appropriate definition of multiplication of its elements. Under this identification, for $V \in \operatorname{Circ}(n)$,

$$\mathcal{J}(V) = P_V(X).$$

Determinants and Eigenvalues

The Basic Theorem

Many results about circulants follow from direct calculations; in particular, the next theorem.

Theorem 6. *If $v = (v_0, v_1, \dots, v_{n-1}) \in \mathbb{C}^n$ and $V = \operatorname{circ}\{v\}$, then*

$$\det V = \prod_{l=0}^{n-1} \left(\sum_{j=0}^{n-1} \epsilon^{jl} v_j \right) = \prod_{l=0}^{n-1} P_V(\epsilon^l). \tag{3}$$

Proof. We view the matrix V as a self-map $V_{\mathbf{e},\mathbf{e}}$ of $\mathbb{C}^n$. For each integer l, $0 \le l \le n-1$, let*

$$\lambda_l = v_0 + \epsilon^l v_1 + \cdots + \epsilon^{(n-1)l} v_{n-1} = P_V(\epsilon^l).$$

A calculation shows that $Vx_l = \lambda_l x_l$. Thus λ_l is an eigenvalue of V with normalized eigenvector x_l. Since, by (1), $\{x_0, x_1, \dots, x_{n-1}\}$ is a linearly independent set of vectors in $\mathbb{C}^n$, the diagonal matrix with the corresponding eigenvalues is conjugate to V, and we conclude that $\det V = \prod_{l=0}^{n-1} \lambda_l$. □

Corollary 7. *All circulant matrices have the same ordered set of orthonormal eigenvectors $\{x_l\}$.*

Corollary 8. *The characteristic polynomial p_V of V is given by*

$$p_V(X) = \det(XI - V) = \prod_{l=0}^{n-1} (X - \lambda_l) = X^n + \sum_{i=n-1}^{0} b_i X^i. \tag{4}$$

(*Here we let the last equality define the b_i's as functions of the λ_l's. They are the elementary symmetric functions of the λ_l's.*)

Corollary 9. *The nullity of $V \in \operatorname{Circ}(n)$ is the number of zero eigenvalues λ_l.*

We use similar symbols for the characteristic polynomial p_V of a circulant matrix V and its representer P_V. They exhibit, however, different relations to V.

If we let $\nu(V)$ denote the nullity of V, the last corollary can be restated as

$$\text{For all } V \in \operatorname{Circ}(n), \nu(V) = \deg\gcd(p_V(X), X^n).$$

Corollary 10. *Let V be a circulant matrix with representer P_V. The following are equivalent:*

(a) *The matrix V is singular.*
(b) *$P_V(\epsilon^l) = 0$ for some $l \in \mathbb{Z}$.*
(c) *The polynomials $P_V(X)$ and $X^n - 1$ are not relatively prime.*

Throughout this paper the symbols λ_l and x_l are reserved for the eigenvalue and eigenvectors we introduce here.

Again, we have a reformulation of part of the last corollary as

For all $V \in \mathrm{Circ}(n)$, $\nu(V) = \deg\gcd(P_V(X), X^n - 1)$.

Remark 11. The shift operator T acts on $\mathrm{Circ}(n)$. If $V \to T\cdot V$ denotes this action, then the traces of $T^k \cdot V$, $k = 0, \ldots, n-1$, uniquely determine V. The representer P_V of a circulant matrix V uniquely determines and is uniquely determined by the matrix. Similarly, the characteristic polynomial and eigenvalues of a circulant matrix uniquely determine each other. From a given set of ordered eigenvalues, we recover the circulant matrix by the next theorem. However, a given set of distinct eigenvalues determines $n!$ circulant matrices.

Determinants of Circulant Matrices

It is easy to see that

$$\begin{aligned}
&\det \mathrm{circ}\{(v_0, v_1, \ldots, v_{n-1})\}\\
&\quad = (-1)^{n-1} \det \mathrm{circ}\{(v_1, v_2, \ldots, v_{n-1}, v_0)\}\\
&\quad = (-1)^{n-1} \det \mathrm{circ}\{(v_{n-1}, v_{n-2}, \ldots, v_1, v_0)\},
\end{aligned}$$

and iterations of these yield that $\det V = (-1)^{k(n-1)} \det T^k \cdot V$ for each integer $0 \le k < n$. However, there is no obvious general relation between

$$\det \mathrm{circ}\{(v_0, v_1, \ldots, v_{n-1})\}$$

and

$$\det \mathrm{circ}\{(v_{\sigma(0)}, v_{\sigma(1)}, \ldots, v_{\sigma(n-1)})\}$$

for $\sigma \in \mathcal{S}_n$, the permutation group on n letters. For example,

$$\det \mathrm{circ}\{(v_0, v_1, v_2)\} = v_0^3 + v_1^3 + v_2^3 - 3v_0v_1v_2, \tag{5}$$

a function that is invariant under the permutation group $\mathcal{S}_3$, while

$$\begin{aligned}
&\det \mathrm{circ}\{(v_0, v_1, v_2, v_3)\}\\
&\quad = (v_0 + v_1 + v_2 + v_3)(v_0 - v_1 + v_2 - v_3)\\
&\qquad \times ((v_0 - v_2)^2 + (v_1 - v_3)^2),
\end{aligned} \tag{6}$$

which, though admitting some symmetries, fails to be invariant under the action of the entire group $\mathcal{S}_4$; for instance, it is not invariant under the transposition that exchanges v_0 and v_1.

The action of $\mathbb{C}^\times$ on $\mathrm{Circ}(n)$ by dilations can be used to understand further the singular circulant matrices. For given $a \in \mathbb{C}^\times$, we have that

$$\begin{aligned}
&\det \mathrm{circ}\{(av_0, av_1, \ldots, av_{n-1})\}\\
&\qquad = a^n \det \mathrm{circ}\{(v_0, v_1, \ldots, v_{n-1})\},
\end{aligned}$$

and we may cast these matrices as the projective variety in $\mathbb{P}^{n-1}(\mathbb{C})$ given by the locus of det on $\mathrm{Circ}(n)$. The decomposition of this variety into its irreducible components yields a geometric interpretation of the zeroes of various multiplicities of this function on $\mathrm{Circ}(n)$.

The Space of Circulant Matrices

To obtain our third model for $\mathrm{Circ}(n)$, we start by defining $\mathbb{D}_n$ to be the space of $n \times n$ diagonal matrices. This space is clearly linearly isomorphic to $\mathbb{C}^n$.

Theorem 12. *All elements of* $\mathrm{Circ}(n)$ *are simultaneously diagonalized by the unitary matrix E; that is, for V in* $\mathrm{Circ}(n)$,

$$E^{-1}VE = D_V \tag{7}$$

is a diagonal matrix and the resulting map

$$\mathcal{D} : \mathrm{Circ}(n) \to \mathbb{D}_n \tag{THIRD MODEL}$$

is a $\mathbb{C}$*-algebra isomorphism.*

Proof. The $n \times n$ matrix E represents the linear automorphism of $\mathbb{C}^n$ that sends the unit vector e_l to the unit vector x_l. If V is a circulant matrix and D_V is the diagonal matrix with diagonal entries given by the ordered eigenvalues of V: $\lambda_0, \lambda_1, \ldots, \lambda_{n-2}, \lambda_{n-1}$, then (7) holds. The map $\mathcal{D}$ is onto, because for all $D \in \mathbb{D}_n$, EDE^{-1} is circulant. □

Corollary 13. *The inverse of an invertible element of* $\mathrm{Circ}(n)$ *also belongs to* $\mathrm{Circ}(n)$.

Proof. If V is a nonsingular circulant matrix, then D_V is invertible and $D_V^{-1} = D_{V^{-1}}$. □

Corollary 14. *The characteristic polynomial of* $V \in \mathrm{Circ}(n)$ *is given by*

$$p_V(X) = \det(XI - V) = \det(XI - D_V).$$

Remark 15. The last corollary encodes several facts that can be established by other methods:

- Let $p \in \mathcal{P}_{n-1}$. If $p(X) = \sum a_i X^i$ and $\lambda_l = p(\epsilon^l)$, then the elementary symmetric functions of the λ_l's belong to the ring generated (over $\mathbb{Z}$) by the a_i's.
- Given an ordered set $\{\lambda_l\}$, the unique polynomial p satisfying $\lambda_l = p(\epsilon^l)$ is $\det(XI - D_V)$.

Roots of Polynomials

Each $n \times n$ circulant matrix V has two polynomials naturally associated to it: its representer P_V and its characteristic polynomial p_V. These are both described explicitly in terms of the eigenvalues λ_l of V. The characteristic polynomial p_V is the unique monic polynomial of degree n that vanishes at each λ_l. The representer P_V is the unique polynomial of degree $\le n-1$ whose value at ϵ^l is λ_l.

The roots of the characteristic polynomial of an arbitrary $n \times n$ matrix V (these are the eigenvalues of the matrix V) are obtained by solving a monic degree n polynomial equation. In the case of circulant matrices, the roots of p_V are easily calculated using the representer polynomial P_V. Thus, if a given polynomial p is known to be the characteristic polynomial of a *known* circulant matrix V, its zeroes can be readily found. This remark is the basis of [8] and of the section "Polynomials of Degree ≤ 4". Every monic polynomial p is the characteristic polynomial of some circulant matrix V, and so a very natural problem ensues: If we are given that $p = p_V$ for a collection of circulant matrices V, can we determine one such V or, equivalently its representer P_V directly from p? If so, then the n roots of p are the values of P_V at the n nth roots of unity.

The General Case

The vector space $\mathcal{P}_{n-1}$ of polynomials of degree $\leq n-1$ is canonically isomorphic to $\mathrm{Circ}(n)$ (both are canonically isomorphic as vector spaces to $\mathbb{C}^n$). Let $\mathcal{M}$ be the affine space of monic polynomials of degree n (again identifiable with $\mathbb{C}^n$). We define a map

$$\Lambda : \mathcal{P}_{n-1} \to \mathcal{M}$$

as follows. For each $p \in \mathcal{P}_{n-1}$, there exists a unique $V \in \mathrm{Circ}(n)$ such that $p = P_V$. Send p to p_V. The map Λ is holomorphic; in fact, it is algebraic. We have already remarked that it is generically $n!$ to 1. We define three subspaces:

(1) $\mathcal{P}^0_{n-1}$ consisting of those

$$\{p \in \mathcal{P}_{n-1} \text{ with } p(\epsilon^i) \neq p(\epsilon^j) \text{ for all integers } 0 \leq i < j \leq n-1\}.$$

(2) $\mathrm{Circ}^0(n)$ consisting of those

$$\{V \in \mathrm{Circ}(n) \text{ with distinct eigenvalues}\}.$$

(3) $\mathcal{M}^0$ consisting of those

$$\{p \in \mathcal{M} \text{ with distinct roots}\}.$$

Each of the subspaces defined is open and dense in its respective ambient spaces. It is quite obvious that

$$\Lambda : \mathcal{P}^0_{n-1} \to \mathcal{M}^0$$

is a complex analytic bijection. An explicit form for the inverse to this map would provide an algorithm for solving equations of all degrees.

Remark 16. We know that

$$\mathcal{P}^0_{n-1} \cong \mathrm{Circ}^0(n) \cong \mathcal{M}^0.$$

Each of these spaces is defined analytically. However, the last one has an alternate algebraic characterization. Let p' denote the derivative of p. The set $\mathcal{M}^0$ can be described as

$$\{p \in \mathcal{M} : \deg \gcd(p, p') = 0\}.$$

Thus, solving general equations can be reduced by an algebraic procedure to solving equations with distinct roots, for the calculation of p' is quite algebraic, and so is the calculation of $d = \gcd(p, p')$ via the division algorithm. The polynomial $\frac{p}{d}$ has no multiple roots.

The problem encountered above is of fundamental importance and quite difficult in general. We now turn our attention to the cases of low degree.

Polynomials of Degree ≤ 4

Circulant matrices provide a unified approach to solving equations of degree ≤ 4; we will illustrate this for degrees 3 and 4. As is quite common, we start with a definition.

Definition 17. Given a monic polynomial p of degree n, a circulant $n \times n$ matrix $V = \mathrm{circ}\{(v_0, \dots, v_{n-1})\}$ is said to *adhere* to p if the characteristic polynomial p_V of V is equal to p.

We learned at a quite early age that to solve the equation

$$p(X) = X^n + \alpha_{n-1}X^{n-1} + \alpha_{n-2}X^{n-2} + \cdots + \alpha_1 X + \alpha_0 = 0,$$

we should use the change of variable $Y = X + \frac{1}{n}\alpha_{n-1}$, which eliminates the monomial of degree $n-1$ in p and leads to the equation

$$q(Y) = Y^n + \gamma_{n-2}Y^{n-2} + \cdots + \gamma_1 Y + \gamma_0 = 0$$

to be solved. If $V = \mathrm{circ}\{(v_0, v_1, \dots, v_{n-1})\}$ adheres to p, then the traceless matrix $V - v_0 I$ adheres to q.

A reasonable program for solving polynomial equations p of degree n can thus consist of changing variables to reduce to an equation q with zero coefficient monomial of degree $n-1$ and then finding a traceless circulant matrix V that adheres to q. The eigenvalues of V are the roots of $p_V = q$ and can be readily computed using the representer P_V of V. In this program we seem to be replacing the difficult problem of solving a monic polynomial equation of degree n by the more difficult problem of solving $n-1$ nonlinear equations in $n-1$ variables. However, because of the symmetries present in the latter set of equations, they may be easier to handle.

Cubics. We illustrate how this works for cubics by finding a circulant matrix $V = \mathrm{circ}\{(0, a, b)\}$ of zero trace that adheres to

$$q(Y) = Y^3 + \alpha Y + \beta.$$

We need to find *any* traceless 3×3 circulant matrix V that adheres to q. Evaluating the representer $P_V(Y) = aY + bY^2$ at $y = e^{j\frac{2\pi\imath}{3}}$, $j = 0, 1, 2$, will then yield the roots of q.

By formula (5) for $\det \mathrm{circ}\{(0, a, b)\}$, we see that

$$p_V(Y) = \det(YI - V) = Y^3 - 3abY - (a^3 + b^3),$$

and so

$$\begin{aligned} 3ab &= -\alpha, \\ a^3 + b^3 &= -\beta. \end{aligned}$$

It then follows that

$$a = \left(\frac{-\beta \pm \sqrt{\beta^2 + \frac{4\alpha^3}{27}}}{2}\right)^{\frac{1}{3}}, \quad b = \left(\frac{-\beta \mp \sqrt{\beta^2 + \frac{4\alpha^3}{27}}}{2}\right)^{\frac{1}{3}}$$

(we are free to choose any consistent set of values since we need only one representer), and the roots of q are given by (the values of P_V at the three cube roots of unity)

$$\begin{aligned} r_1 &= a+b, \\ r_2 &= ae^{\frac{2\pi\imath}{3}} + be^{2\frac{2\pi\imath}{3}}, \\ r_3 &= ae^{2\frac{2\pi\imath}{3}} + be^{\frac{2\pi\imath}{3}}. \end{aligned}$$

Quartics. In order to find the roots of the polynomial

$$q(Y) = Y^4 + \beta Y^2 + \gamma Y + \delta,$$

we search for a $V = \operatorname{circ}\{(0,b,c,d)\}$ such that $p_V(Y) = \det(YI - V) = q(Y)$. By (6),

$$\begin{aligned} p_V(Y) &= Y^4 - (4bd + 2c^2)Y^2 - 4c(b^2+d^2)Y \\ &\quad + (c^4 - b^4 - d^4 - 4bc^2d + 2b^2d^2), \end{aligned}$$

and so

$$\begin{aligned} 4bd + 2c^2 &= -\beta, \\ 4c(b^2+d^2) &= -\gamma, \\ c^4 - b^4 - d^4 - 4bc^2d + 2b^2d^2 &= \delta, \end{aligned}$$

a system in the unknowns a, b, c.

It suffices to say that this system admits solutions. We leave the details of the argument to the reader. We encourage the reader to explore two sets of additional symmetries: the first consisting of solutions of the system with $c = 0$, and the second, solutions with $b = d$.

Singular Circulant Matrices

The eigenvalues of a circulant matrix tell us when it is singular. We develop a number of criteria for singularity relying on this basic fact.

Proposition 18. *If for some k, $|v_k| > \sum_{j\neq k}|v_j|$, then the circulant matrix $V = \operatorname{circ}\{(v_0,\ldots,v_{n-1})\}$ is nonsingular. The result is sharp in the sense that $>$ cannot be replaced by $\geq$.*

Proof. Let P_V be the representer of V. If $P_V(\epsilon^l) = 0$ for some $l \in \mathbb{Z}$, then for $\lambda = \epsilon^l$,

$$v_k\lambda^k = -\sum_{j\neq k} v_j\lambda^j.$$

In particular,

$$|v_k| \leq \sum_{j\neq k}|v_j|,$$

which contradicts the hypothesis. □

Proposition 19. *Let $d \mid n$, $1 \leq d < n$, and assume that the vector $v \in \mathbb{C}^n$ consists of $\frac{n}{d}$ identical blocks (that is, $v_{i+d} = v_i$ for all i, where indices are calculated* mod n*). Then $\lambda_l = 0$ whenever dl is not a multiple of n, and $V = \operatorname{circ}\{v\}$ is singular of nullity $\geq n - d$.*

Proof. Compute for $0 \leq l < n$,

$$\begin{aligned} \lambda_l &= \sum_{i=0}^{n-1}\epsilon^{li}v_i = \sum_{j=0}^{\frac{n}{d}-1}\left(\sum_{i=0}^{d-1}\epsilon^{l(dj+i)}v_{dj+i}\right) \\ &= \sum_{j=0}^{\frac{n}{d}-1}\epsilon^{ldj}\sum_{i=0}^{d-1}\epsilon^{li}v_i \\ &= \frac{1-\epsilon^{nl}}{1-e^{dl}}\sum_{i=0}^{d-1}\epsilon^{li}v_i, \end{aligned}$$

provided dl is not a multiple of n. In particular, $\lambda_l = 0$ for $1 \leq l < \frac{n}{d}$. In general there are $n - d$ integers l such that $0 < l < n$ and dl is not a multiple of n. □

Remark 20. In this case,

$$P_V(X) = \left(\sum_{i=0}^{d-1} v_iX^i\right)\left(\frac{X^n-1}{X^d-1}\right),$$

and the polynomial $\frac{X^n-1}{X^d-1}$ of degree $n-d$ divides both $P_V(X)$ and $X^n - 1$ (see Corollary 10).

Proposition 21. *Let $d \mid n$, $2 \leq d < n$, and assume that the vector $v \in \mathbb{C}^n$ consists of $\frac{n}{d}$ consecutive constant blocks of length d (that is to say, $v_{id+j} = v_{id}$ for $i = 0, 1, \ldots, \frac{n}{d} - 1$ and $j = 0, 1, \ldots, d-1$). Then $\lambda_l = 0$ whenever $l \neq 0$ and $l \equiv 0 \mod \frac{n}{d}$, and V is singular of nullity $\geq d - 1$.*

Proof. In this case

$$\begin{aligned} \lambda_l &= \sum_{i=0}^{n-1}\epsilon^{li}v_i = \sum_{j=0}^{\frac{n}{d}-1}\epsilon^{ldj}v_{dj}\sum_{i=0}^{d-1}\epsilon^{li} \\ &= \frac{1-\epsilon^{ld}}{1-\epsilon^l}\sum_{j=0}^{\frac{n}{d}-1}\epsilon^{ldj}v_{dj}, \end{aligned}$$

provided $l > 0$. In particular, $\lambda_l = 0$ for all $l = k\frac{n}{d}$, with $k = 1, 2, \ldots, d-1$. □

Remark 22. In the above situation,

$$P_V(X) = \left(\sum_{i=0}^{\frac{n}{d}-1} v_iX^{id}\right)\left(\frac{X^d-1}{X-1}\right),$$

and the polynomial $\frac{X^d-1}{X-1}$ of degree $d-1$ divides both $P_V(X)$ and $X^n - 1$ (see Corollary 10).

Proposition 23. *Let $n \in \mathbb{Z}_{>0}$ be a prime. If $V = \operatorname{circ}\{(v_0,\ldots,v_{n-1})\}$ has entries in $\mathbb{Q}$, then $\det V = 0$ if and only if either $\lambda_0 = \sum_{j=0}^{n-1}v_j = 0$ or all the v_j's are equal.*

Proof. If all the v_i's are equal, then all the eigenvalues λ_l of V, except possibly λ_0, are equal to zero. We already know that the vanishing of one λ_l implies that $\det V = 0$. Conversely, assume that $\det V = 0$ and that $\lambda_0 \neq 0$. Then $\lambda_l = 0$ for some positive integer $l < n$. Consider the field extension $\mathbb{Q}[\epsilon]$ and the automorphism A of this field induced by sending $\epsilon \mapsto \epsilon^2$ (A fixes $\mathbb{Q}$, of course). Since n is prime, A generates a cyclic group of automorphisms of $\mathbb{Q}[\epsilon]$ of order $n-1$ that acts transitively on the primitive nth roots of unity: $\{\epsilon, \epsilon^2, \dots, \epsilon^{n-1}\}$. Hence $\lambda_l = 0$ implies that $\lambda_k = 0$ for all integers k with $1 \le k \le n-1$. It remains to show that all the v_i's are equal. Consider the $(n-1) \times n$ matrix

$$\begin{bmatrix} 1 & \epsilon & \epsilon^2 & \cdots & \epsilon^{n-1} \\ 1 & \epsilon^2 & \epsilon^4 & \cdots & \epsilon^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \epsilon^{n-1} & \epsilon^{2(n-1)} & \cdots & \epsilon^{(n-1)^2} \end{bmatrix}$$

(essentially the matrix E in the section "Determinants and Eigenvalues" with the first row deleted). Since it has rank $n-1$, this matrix, when viewed as a linear map from $\mathbb{C}^n$ to $\mathbb{C}^{n-1}$, must have a one-dimensional kernel. This kernel is spanned by the vector $(1, 1, \dots, 1)$. The conclusion follows. □

Proposition 24. *If $\{v_j\}_{0 \le j \le n-1}$ is a weakly monotone sequence (that is, a nondecreasing or nonincreasing sequence) of nonnegative or nonpositive real numbers, then the matrix $V = \mathrm{circ}\{(v_0, v_1, \dots, v_{n-1})\}$ is singular if and only if for some integer $d \mid n$, $d \ge 2$, the vector $v = (v_0, v_1, \dots, v_{n-1})$ consists of $\frac{n}{d}$ consecutive constant blocks of length d. In particular, if the sequence $\{v_j\}_{0 \le j \le n-1}$ is strictly monotone and nonpositive or nonnegative, then V is nonsingular.*

Proof. If the matrix V were singular, then its representer P_V would vanish at an nth root of unity, say λ. It is sufficient to prove the theorem in the case when $\{v_j\}_{0 \le j \le n-1}$ is a nonincreasing sequence of nonnegative real numbers; all other cases reduce to this one by replacing λ with $\frac{1}{\lambda}$ or by appropriately changing the signs of all the v_i's (see also the symmetries discussed at the beginning of the section "Polynomials of Degree ≤ 4"). We may thus assume in the sequel that

$$v_0 \ge v_1 \ge \cdots \ge v_{n-1} \ge 0.$$

Now $P_V(\lambda) = 0$ means that

$$v_0 + v_1\lambda + \cdots + v_{n-1}\lambda^{n-1} = 0,$$

and hence also that

$$v_0\lambda + v_1\lambda^2 + \cdots + v_{n-1}\lambda^n = 0,$$

which yields

$$\begin{aligned} v_0 - v_{n-1} = (v_0 - v_1)\lambda + (v_1 - v_2)\lambda^2 \\ + \cdots + (v_{n-2} - v_{n-1})\lambda^{n-1}. \end{aligned} \tag{8}$$

Observe that if $z_1, \dots, z_m$ are complex numbers such that

$$\sum_{i=1}^m z_i = \left| \sum_{i=1}^m z_i \right| = \sum_{i=1}^m | z_i |, \tag{9}$$

then $z_i \in \mathbb{R}$ and $z_i \ge 0$ for all $i = 1, \dots, m$. Since $|\lambda| = 1$, it follows from (8) that the $z_k = (v_{k-1} - v_k)\lambda^k$, $k = 1, \dots, n-1$, satisfy (9), and thus for each k either $v_{k-1} = v_k$ or $\lambda^k = 1$. The latter holds only if λ is actually a dth root of unity, for some divisor $d \ge 2$ of n, while k is a multiple of d, and the conclusions of the theorem now follow easily, for we may choose the smallest positive integer d such that $\lambda^d = 1$. Then $d \ge 2$, $d \mid n$ and $\lambda^k = 1$ for $1 \le k \le n$ if and only if $k = d, 2d, \dots$ or $n = \frac{n}{d}d$. It follows that $v_k = v_{k-1} = \cdots = v_{k-(d-1)}$. □

The next result deals with circulant matrices whose entries are ± the same nonzero complex number.

Proposition 25. *If $V = \mathrm{circ}\{(v_0, \dots, v_{n-1})\} \in \mathrm{Circ}(n)$ has entries in $\{\pm 1\}$, and $0 < k = |\{j \mid v_j = 1\}| \le n - k$, then*

(a) *$\lambda_0 = 0$ if and only if $k = \frac{n}{2}$.*
(b) *For $0 < l < n$, $\lambda_l = 0$ if and only if*

$$\sum_{\{j \mid v_j = 1\}} e^{lj} = 0.$$

(c) *Assume that $\lambda_0 \ne 0$. V is nonsingular provided that k is not of the form $\sum m_i p_i$, where the p_i run over the distinct positive prime factors of n and the m_i are positive integers. In particular, V is nonsingular if k is less than the smallest positive prime dividing n.*

Proof. If $0 \le l \le n$, the formula for the eigenvalues of V in terms of the representer P_V yields that

$$\lambda_l = \sum_{\{j \mid v_j = 1\}} \epsilon^{lj} - \sum_{\{j \mid v_j = -1\}} \epsilon^{lj}.$$

This establishes part (a). We now observe that

$$\sum_{\{j \mid v_j = 1\}} \epsilon^{lj} + \sum_{\{j \mid v_j = -1\}} \epsilon^{lj} = \sum_{j=0}^{n-1} \epsilon^{lj} = \frac{1 - \epsilon^{ln}}{1 - \epsilon^l} = 0,$$

for $0 < l < n$. Thus part (b) follows. For part (c), we observe that for $0 < l < n$ we have that $\lambda_l \ne 0$ by (b) and the characterization of vanishing sums of n-roots of unity of weight k proven in [10]. □

We end this section (see also [15]) with the following.

Proposition 26. *If*

$$V = \mathrm{circ}\left\{ \left(1, \binom{n}{1} \cdots \binom{n}{n-1} \right) \right\},$$

then the following hold:

(a) $\lambda_l = (1 + \epsilon^l)^n - 1$.
(b) *$\lambda_l = 0$ if and only if $\frac{l}{n} = \frac{1}{3}$ or $\frac{l}{n} = \frac{2}{3}$.*
(c) *V is singular if and only if $n \equiv 0 \bmod 6$, in which case the nullity of V is 2.*

Proof. By Theorem 6, we have that

$$\lambda_l = \sum_{j=0}^{n-1} \binom{n}{j} \epsilon^{lj},$$

and the binomial expansion yields (a). We obtain that $\lambda_l = 0$ if and only if $(1+\epsilon^l)^n = 1$, and so $|1+\epsilon^l| = 1$ if and only if $\cos\frac{2\pi l}{n} = -\frac{1}{2}$, a statement equivalent to the condition $\frac{l}{n} = \frac{1}{3}$ or $\frac{l}{n} = \frac{2}{3}$. This proves (b). Part (c) follows readily since the conditions making $\lambda_l = 0$ are equivalent to n being divisible by 2 and 3, respectively, and λ_l being zero exactly for the two values of l satisfying the condition in (b). □

The Geometry of Circ(n)

Let k be a positive integer. The *affine k-space* over $\mathbb{C}$ is $\mathbb{C}^k$, often denoted by $\mathbb{A}^k_{\mathbb{C}}$. The maximal ideals in the polynomial ring $\mathbb{C}[x_1,\dots,x_k]$ correspond to elements of $\mathbb{C}^k$, with $a = (a_1,\dots,a_k) \in \mathbb{C}^k$ corresponding to the ideal in $\mathbb{C}[x_1,\dots,x_k]$ given by the kernel of the evaluation homomorphism $p \mapsto p(a)$. An *affine variety* $\mathbb{V} \subset \mathbb{C}^k$ is an irreducible component of the zero locus of a collection of polynomials $p_1,\dots,p_l$ in $\mathbb{C}[x_1,\dots,x_k]$. The ideal $I_{\mathbb{V}} = (p_1,\dots,p_l) \subset \mathbb{C}[x_1,\dots,x_k]$ of functions vanishing on $\mathbb{V}$ is prime, and under the above identification the points of V are in one-to-one correspondence with the set of maximal ideals of the ring $\mathcal{O}(\mathbb{V}) = \mathbb{C}[x_1,\dots,x_k]/I_{\mathbb{V}}$, a ring without zero divisors. We say that $\mathbb{V}$ *is cut out by* $p_1,\dots,p_l$, *has ideal* $I_{\mathbb{V}}$, and *ring of global functions* $\mathcal{O}(\mathbb{V})$. Theorem 4 realizes Circ(n) as the ring of global functions of the variety given by the nth roots of unity in $\mathbb{C}$.

Complex projective k-space $\mathbb{P}^k = \mathbb{P}^k_{\mathbb{C}}$ is the set of one-dimensional subspaces of $\mathbb{C}^{k+1}$. A point $x \in \mathbb{P}^k$ is usually written as a homogeneous vector $[x_0 : \dots : x_k]$, by which is meant the complex line spanned by $(x_0,\dots,x_k) \in \mathbb{C}^{k+1} \setminus \{0\}$.

A nonconstant polynomial $f \in \mathbb{C}[x_0,\dots,x_k]$ does not descend to a function on $\mathbb{P}^k$. However, if f is a homogeneous polynomial of degree d, we can talk about the zeroes of f in $\mathbb{P}^k$ because we have the relation $f(\lambda x_0,\dots,\lambda x_k) = \lambda^d f(x_0,\dots,x_k)$, for all $\lambda \in \mathbb{C} \setminus \{0\}$. A *projective variety* $\mathbb{V} \subset \mathbb{P}^k$ is an irreducible component of the zero locus of a finite collection of homogeneous polynomials.

If we replace the role of $\mathbb{C}$ in the above discussion by an arbitrary field $\mathbb{F}$, we obtain the notions of k-dimensional affine space $\mathbb{A}^k_{\mathbb{F}}$ and k-dimensional projective space $\mathbb{P}^k_{\mathbb{F}}$ over $\mathbb{F}$, respectively. Polynomials in $\mathbb{F}[x_1,\dots,x_k]$ define affine varieties in $\mathbb{A}^k_{\mathbb{F}}$, while homogeneous polynomials define projective varieties in $\mathbb{P}^k_{\mathbb{F}}$. These spaces are usually studied for algebraically closed $\mathbb{F}$, but the definitions are valid for more general fields, and we work in this extended context. Let $\mathbb{V}$ be an affine or projective variety over $\mathbb{F}$, the zero locus of a set of polynomials in $\mathbb{F}[x_1,\dots,x_k]$. Given any field extension $\mathbb{E}$ of $\mathbb{F}$, we can talk about the locus of these polynomials in the affine or projective space over the extension field $\mathbb{E}$. These will define the $\mathbb{E}$-points of the variety $\mathbb{V}$, a set which we denote by $\mathbb{V}(\mathbb{E})$. This brings about some additional structure to the $\mathbb{F}$-varieties $\mathbb{V}$, which we can think of as a functor from the category of field extensions of $\mathbb{F}$ and their morphisms to a suitable category of sets and morphisms, with the functor mapping an extension $\mathbb{E}$ of $\mathbb{F}$ to the set $\mathbb{V}(\mathbb{E})$ of $\mathbb{E}$-points of the variety. Using *restrictions* when possible, we may also use this idea in the opposite direction and find the points of a variety with coordinates in a subring of $\mathbb{F}$ when the variety in question is defined by polynomials whose coefficients are elements of the subring. This idea applied to Circ(n) takes us to a rather interesting situation.

Given a variety over $\mathbb{C}$ cut out by polynomials with coefficients in $\mathbb{Z}$, we can use the natural inclusion $\mathbb{Z} \hookrightarrow \mathbb{C}$ to look at the $\mathbb{Z}$-points of the variety and the restricted ring of global functions. In the case of Circ(n), the restricted ring of global functions is $\mathbb{Z}[X]/(X^n - 1)$, and, remarkably, the set of prime ideals, or spectrum, of this latter ring is related to a variety defined over a *field with one element*, a mythical object denoted in the literature by $\mathbb{F}_1$. We elaborate on this connection. It derives from analogies between regular combinatorial arguments and combinatorics over the finite field $\mathbb{F}_q$ with q elements (q a power of a prime).

The number of bases of the $\mathbb{F}_q$-vector space $\mathbb{F}_q^k$ is given by $q^{\frac{k(k-1)}{2}}(q-1)^k[k]_q!$, where $[k]_q = 1 + q + q^2 + \cdots + q^{k-1}$, and where the q-factorial is defined by $[k]_q! = [1]_q \cdot [2]_q \cdots [k]_q$. Similarly, the number of linearly independent j-element subsets is equal to $q^{\frac{j(j-1)}{2}}(q-1)^k[k]_q!/[k-j]_q!$, and for $j \le k$, the number of subspaces of $\mathbb{F}_q^k$ of dimension j is given by

$$\binom{k}{j}_q = \frac{[k]_q!}{[k-j]_q![j]_q},$$

an expression that makes perfect sense when $q = 1$, in which case we obtain the usual binomial. The idea of the mysterious one element field $\mathbb{F}_1$ emerges [12], and we see that the number of $\mathbb{F}_1$-points of projective space—that is to say, the number of 1-dimensional subspaces of $\mathbb{F}_1^n$—must be equal to n. Speculating on this basis, we are led to define a vector space over $\mathbb{F}_1$ simply as a set, a subspace simply as a subset, and the dimensions of these simply as the cardinality of the said sets.

Some relationships between properties of Circ(n) and that of algebraic geometry over $\mathbb{F}_1$ now follow. We think of the group of points of $\mathbb{SL}(n, \mathbb{F}_1)$ as the symmetric group $\mathcal{S}_n$ on n letters and that these n-letters are the $\mathbb{F}_1$-points of the projective space $\mathbb{P}^{n-1}_{\mathbb{F}_1}$. A "variety X over $\mathbb{F}_1$" should have as extension to the scalars $\mathbb{Z}$, a scheme $X_{\mathbb{Z}}$ of finite type over $\mathbb{Z}$, and the points of X should be a finite subset of the set of points in $X_{\mathbb{Z}}$. Going further, in developing algebraic geometry over $\mathbb{F}_1$, some [13] propose replacing the notion played by an ordinary commutative ring by that of a commutative, associative, and unitary monoid M to obtain $\operatorname{Spec}(M \otimes_{\mathbb{F}_1} \mathbb{Z}) = \operatorname{Spec}\mathbb{Z}[M]$. In particular, they define the finite extension $\mathbb{F}_{1^n}$ of degree n as the monoid $\mathbb{Z}/n\mathbb{Z}$, and its spectrum after lifting it to $\mathbb{Z}$ becomes

$$\operatorname{Spec}(\mathbb{F}_{1^n} \otimes_{\mathbb{F}_1} \mathbb{Z}) = \operatorname{Spec}(\mathbb{Z}[X]/(X^n - 1)).$$

Thus, the algebra of circulant matrices with integer coefficients is the ring of global functions of the spectrum of the field extension $\mathbb{F}_{1^n}$ of degree n after lifting it to $\mathbb{Z}$.

The Rational Normal Curves Connection

Theorem 6 has an elaborate proof that is more geometric in nature and longer than the proof by calculation given above. We outline its details.

The *rational normal curve* $C_d \subset \mathbb{P}^d$ of degree d is defined to be the image of the map $\mathbb{P}^1 \to \mathbb{P}^d$ given by

$$\begin{aligned}[z_0 : z_1] &\mapsto [z_0^d : z_0^{d-1}z_1 : \cdots : z_0 z_1^{d-1} : z_1^d] \\ &= [Z_0 : \cdots : Z_d].\end{aligned}$$

It is the common zero locus of the polynomials $p_{ij} = Z_i Z_j - Z_{i-1}Z_{j+1}$ for $1 \le i \le j \le d-1$. The ideal of C_d, $I(C_d) = \{f \in \mathbb{C}[Z_0, \ldots, Z_d] \mid f \equiv 0 \text{ on } C_d\}$ is generated by this set of polynomials.

We view $\{v_0, \ldots, v_{n-1}, \ldots, v_{2n-2}\}$ as a set of $2n-1$ independent variables and consider the matrix with constant antidiagonals given by

$$M = \begin{bmatrix} v_0 & v_1 & \cdots & v_{n-2} & v_{n-1} \\ v_1 & v_2 & \cdots & v_{n-1} & v_n \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ v_{n-2} & v_{n-1} & \cdots & v_{2n-4} & v_{2n-3} \\ v_{n-1} & v_n & \cdots & v_{2n-3} & v_{2n-2} \end{bmatrix}$$

as an $n \times n$ *catalecticant* or *Hankel* matrix. Its 2×2-minors define the ideal of the rational normal curve $C = C_{2n-2} \subset \mathbb{P}^{2n-2}$ of degree $2n-2$.

The other ideals of minors of M also have geometric significance. Since the sum of m matrices of rank one has rank at most m, the ideal I_k of $k \times k$-minors of M, $k \in \{2, \ldots, n\}$, vanishes on the union of the $(k-1)$-secant $(k-2)$-planes to the rational normal curve $C \subset \mathbb{P}^{2n-2}$. The ideal I_k defines the (reduced) locus of these $(k-1)$-secant $(k-2)$-planes to C [14] (see [2] for a modern proof).

The restriction of M to the $(n-1)$-dimensional linear subspace

$$\begin{aligned}\Lambda = \{v_n - v_0 = v_{n+1} - v_1 = \cdots = v_{2n-2} - v_{n-2} = 0\} \\ \subset \mathbb{P}^{2n-2}\end{aligned}$$

coincides, up to row permutations, with the arbitrary circulant matrix

$$V = \operatorname{circ}\{(v_0, v_1, \ldots, v_{n-1})\}.$$

The intersection $\Lambda \cap C$ is the image in $\mathbb{P}^{2n-2}$ of the points whose coordinates $[z_0 : z_1] \in \mathbb{P}^1$ satisfy the equations $(z_0^{n-2}, z_0^{n-3}z_1, \ldots, z_1^{n-2}) \cdot (z_0^n - z_1^n) = 0$ or, equivalently, $z_0^n - z_1^n = 0$. The point $[1 : \epsilon^i] \in \mathbb{P}^1$ gets mapped to the point

$$p_i = [1 : \epsilon^i : \epsilon^{2i} : \cdots : \epsilon^{(n-1)i}], \ 0 \le i \le n-1,$$

and so the restriction of I_k to Λ vanishes on

$$\bigcup_{i_1, i_2, \ldots, i_{k-1} \in \{0, \ldots, n-1\}} \operatorname{span}(p_{i_1}, p_{i_2}, \ldots, p_{i_{k-1}}).$$

In particular, the determinant of the circulant matrix V vanishes on the union of the n distinct hyperplanes

$$\bigcup_{i \in \{0, \ldots, n-1\}} \operatorname{span}(p_0, p_1, \ldots, \hat{p}_i, \ldots, p_{n-1}),$$

where the symbol $\hat{p}_i$ indicates that p_i does not appear. The union of these n hyperplanes in $\mathbb{P}^{2n-2}$ is a degree n subvariety of codimension 1, and thus any degree n polynomial vanishing on it must be its defining equation, up to a scalar factor (because for any hypersurface, its defining ideal is generated by one element and the degree of the hypersurface is the degree of this element). We deduce that $\det(V)$ factors as in the statement of Theorem 6.

Similarly, though the argument is slightly more involved, we can show also that for all $k \in \{2, \ldots, n\}$, the ideal of $k \times k$-minors of the generic circulant matrix V defines the (reduced) union of $(k-2)$-planes

$$\bigcup_{i_1, i_2, \ldots, i_{k-1} \in \{0, \ldots, n-1\}} \operatorname{span}(p_{i_1}, p_{i_2}, \ldots, p_{i_{k-1}})$$

(in contrast with the case of the generic catalecticant matrix, where all ideals of minors are prime).

Other Connections—Toeplitz Operators

We end by discussing briefly a relation between circulant and Toeplitz matrices. The interested reader may consult [6] for more information about the connection.

Let $\{t_{-n+1}, \ldots, t_0, \ldots, t_{n-1}\}$ be a collection of $2n-1$ complex numbers. An $n \times n$ matrix $T = [t_{kj}]$ is said to be *Toeplitz* if $t_{kj} = t_{k-j}$. Thus, a Toeplitz matrix T is a square matrix of the form

$$T = T_n = \begin{bmatrix} t_0 & t_{-1} & \cdots & t_{-(n-2)} & t_{-(n-1)} \\ t_1 & t_0 & \cdots & t_{-(n-3)} & t_{-(n-2)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ t_{n-2} & t_{n-3} & \cdots & t_0 & t_{-1} \\ t_{n-1} & t_{n-2} & \cdots & t_1 & t_0 \end{bmatrix}.$$

These matrices have a rich theory, and they relate naturally to the circulant ones we study here. If we have $t_k = t_{-(n-k)} = t_{k-n}$, then as a special case

T_n is circulant. We use both classes of matrices in a proof of a celebrated spectral theorem to show the depth of their interconnection.

Let φ be a smooth real-valued function on the unit circle with Fourier coefficients $\hat{\varphi}_j = \int_0^{2\pi} e^{-\imath j\theta}\varphi(\theta)d\theta$, and consider the Toeplitz matrix $T_n(\varphi) = (\hat{\varphi}_{i-j})$, $0 \le i, j \le n-1$. The renowned Szegö theorem [7] asserts that if f is a continuous function on $\mathbb{C}$, then

$$\lim_{n\to\infty} \frac{1}{n} \sum_{\lambda \in \operatorname{spec} T_n(\varphi)} f(\lambda) = \frac{1}{2\pi} \int_{\mathbb{S}^1} f(\varphi(e^{i\theta}))d\theta. \tag{10}$$

We sketch a classical argument leading to its proof.

Given a double sequence $\{t_k\}_{k=-\infty}^{+\infty} \subset \mathbb{C}$ in l^1 (and hence also in l^2), let φ be the L^1-function whose Fourier coefficients are the t_j's. We form the sequence of Toeplitz matrices $\{T_n(\varphi) = T_n\}_{n=1}^{+\infty}$, where T_n is defined, as above, by $\{t_{-n+1}, \dots, t_{n-1}\}$, and denote by $\tau_l^{(n)}$, $l = 0, 1, \dots, n-1$ its eigenvalues. The T_n's are Hermitian if and only if φ is real-valued. We study the *asymptotic distribution*

$$\lim_{n\to+\infty} \frac{1}{n} \sum_{l=0}^{n-1} \tau_l^{(n)}, \tag{11}$$

the case of $f(x) = x$ in Szegö's identity (10).

We introduce the circulant matrix $V_n(\varphi) = \operatorname{circ}\{(v_0^{(n)}, \dots, v_{n-1}^{(n)})\}$, where

$$v_k^{(n)} = \frac{1}{n} \sum_{j=0}^{n-1} \varphi\left(\frac{2\pi j}{n}\right) e^{\frac{2\pi jk}{n} i}. \tag{12}$$

For fixed k, this is the truncated Riemann sum approximation to the integral yielding t_{-k}, and since $\varphi \in L^1$, we have $v_k^{(n)} \to t_{-k}$. By Theorem 6, the ordered eigenvalues of $V_n(\varphi)$ are $\lambda_l^{(n)} = \varphi\left(2\pi \frac{l}{n}\right)$, $l = 0, \dots, n-1$, and so, using Riemann sums to approximate the integral of the mth power of φ, $m \in \mathbb{N}$, we conclude that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{l=0}^{n-1} (\lambda_l^{(n)})^m = \frac{1}{2\pi} \int_0^{1\pi} \varphi(\theta)^m d\theta\,. \tag{13}$$

This relates the average of φ to the asymptotic distributions of the eigenvalues of V_n. The special case of Szegö's theorem above is now within reach.

If we can prove that the two sequences of $n \times n$ matrices $\{T_n\}$ and $\{V_n\}$ are *asymptotically equivalent* in the sense that $\lim_{n\to+\infty} ||T_n - V_n|| = 0$, where $\|V\|$ is the Hilbert-Schmidt norm of the operator V, then their eigenvalues are asymptotically equivalent in the sense that

$$\lim_{n\to+\infty} \frac{1}{n} \sum_{l=0}^{n-1} (\tau_l^{(n)} - \lambda_l^{(n)}) = 0,$$

and so (11) equals (13) for $m = 1$. It is convenient to do this by introducing the auxiliary circulant matrix $V_n(\pi_n\varphi) = \operatorname{circ}\{(\tilde{v}_0^{(n)}, \dots, \tilde{v}_{n-1}^{(n)})\}$ of the truncated Fourier series $\pi_n\varphi = \sum_{j=-n+1}^{n+1} t_j e^{ij\theta}$, where $\tilde{v}_k^{(n)}$ is given by (12) with the role of φ played by $\pi_n\varphi$. The matrix $V_n(\pi_n\varphi)$ is also Toeplitz, and its Toeplitz's coefficients are determined solely by $\{t_{-n+1}, \dots, t_{n-1}\}$. The matrices $V_n(\varphi)$ and $V_n(\pi_n\varphi)$ are asymptotically equivalent, and a simple L^2-argument of Fourier series shows that the latter is asymptotically equivalent to $T_n(\varphi)$, and so also the former.

Arbitrary powers of T_n and V_n have asymptotically equivalent eigenvalues, and the general Szegö's theorem follows by applying Weierstrass's polynomial approximation to f.

It is of practical significance that $V_n(\pi_n\varphi)$ encodes finite-dimensional information of the Fourier expansion of φ and spectral information on the zeroth order pseudodifferential operator $\pi_n M_\varphi \pi_n$, where M_φ is the multiplication by φ operator.

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Tribute to Vladimir Arnold

Boris Khesin and Serge Tabachnikov, Coordinating Editors

Vladimir Arnold, an eminent mathematician of our time, passed away on June 3, 2010, nine days before his seventy-third birthday. This article, along with one in the next issue of the *Notices*, touches on his outstanding personality and his great contribution to mathematics.

A word about spelling: we use "Arnold", as opposed to "Arnol'd"; the latter is closer to the Russian pronunciation, but Vladimir Arnold preferred the former (it is used in numerous translations of his books into English), and we use it throughout.

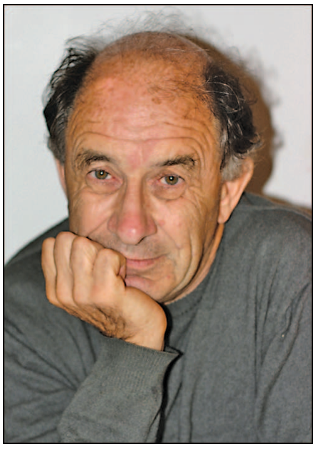
Arnold in His Own Words

In 1990 the second author interviewed V. Arnold for a Russian magazine *Kvant* (Quantum). The readership of this monthly magazine for physics and mathematics consisted mostly of high school students, high school teachers, and undergraduate students; the magazine had a circulation of about 200,000. As far as we know, the interview was never translated into English. We translate excerpts from this interview;[1] the footnotes are ours.

Q: *How did you become a mathematician? What was the role played by your family, school, mathematical circles, Olympiads? Please tell us about your teachers.*

A: I always hated learning by rote. For that reason, my elementary school teacher told my parents that a moron, like myself, would never manage to master the multiplication table.

My first mathematical revelation was when I met my first real teacher of mathematics, Ivan Vassilievich Morozkin. I remember the problem about two old ladies who started simultaneously from two towns toward each other, met at noon, and who reached the opposite towns at 4 p.m. and 9 p.m., respectively. The question was when they started their trip.



Vladimir Igorevich Arnold

We didn't have algebra yet. I invented an "arithmetic" solution (based on a scaling—or similarity—argument) and experienced a joy of discovery; the desire to experience this joy again was what made me a mathematician. A. A. Lyapunov organized at his home "Children Learned Society". The curriculum included mathematics and physics, along with chemistry and biology, including genetics that was just recently banned[2] (a son of one of our best geneticists was my classmate; in a questionnaire, he wrote: "my mother is a stay-at-home mom; my father is a stay-at-home dad").

Q: *You have been actively working in mathematics for over thirty years. Has the attitude of society towards mathematics and mathematicians changed?*

A: The attitude of society (not only in the USSR) to fundamental science in general, and to mathematics in particular, is well described by I. A. Krylov

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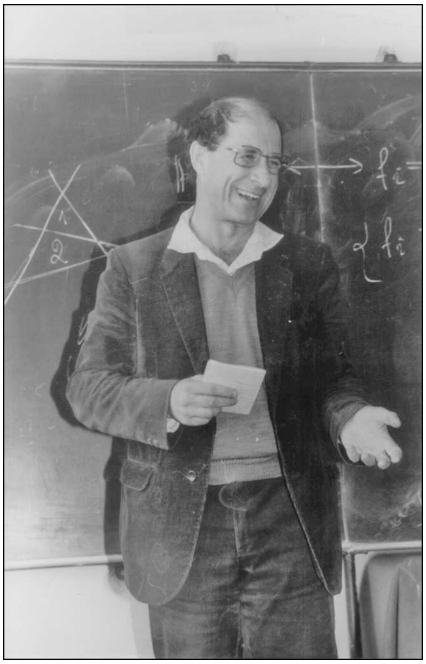
[1] *Full text is available in Russian on the website of* Kvant *magazine (July 1990),* `http://kvant.mirror1.mccme.ru/`.

DOI: http://dx.doi.org/10.1090/noti810

[2] *In 1948 genetics was officially declared "a bourgeois pseudoscience" in the former Soviet Union.*



Vladimir Arnold, circa 1985.

in the fable "The hog under the oak".[3] In the 1930s and 1940s, mathematics suffered in this country less than other sciences. It is well known that Viète was a cryptographer in the service of Henry IV of France. Since then, certain areas of mathematics have been supported by all governments, and even Beria[4] cared about preservation of mathematical culture in this country.

In the last thirty years the prestige of mathematics has declined in all countries. I think that mathematicians are partially to be blamed as well (foremost, Hilbert and Bourbaki), particularly the ones who proclaimed that the goal of their science was investigation of all corollaries of arbitrary systems of axioms.

Q: *Does the concept of fashion apply to mathematics?*

A: Development of mathematics resembles a fast revolution of a wheel: sprinkles of water are flying in all directions. Fashion—it is the stream that leaves the main trajectory in the tangential direction. These streams of epigone works attract the most attention, and they constitute the main mass, but they inevitably disappear after a while because they parted with the wheel. To remain on the wheel, one must apply the effort in the direction perpendicular to the main stream.

A mathematician finds it hard to agree that the introduction of a new term not supported by new theorems constitutes substantial progress. However, the success of "cybernetics", "fractals", "synergetics", "catastrophe theory", and "strange attractors" illustrates the fruitfulness of word creation as a scientific method.

Q: *Mathematics is a very old and important part of human culture. What is your opinion about the place of mathematics in cultural heritage?*

A: The word "mathematics" means science about truth. It seems to me that modern science (i.e., theoretical physics along with mathematics) is a new religion, a cult of truth, founded by Newton three hundred years ago.

Q: *When you prove a theorem, do you "create" or "discover" it?*

A: I certainly have a feeling that I am discovering something that existed before me.

Q: *You spend much time popularizing mathematics. What is your opinion about popularization? Please name merits and demerits of this hard genre.*

A: One of the very first popularizers, M. Faraday, arrived at the conclusion that "Lectures which really teach will never be popular; lectures which are popular will never teach." This Faraday effect is easy to explain: according to N. Bohr, clearness and truth are in a quantum complementarity relation.

Q: *Many readers of* Kvant *aspire to become mathematicians. Are there "indications" and "contraindications" to becoming a mathematician, or can anyone interested in the subject become one? Is it necessary for a mathematician-to-be to successfully participate in mathematical Olympiads?*

A: When 90-year-old Hadamard was telling A. N. Kolmogorov about his participation in Concours Général (roughly corresponding to our Olympiads), he was still very excited: Hadamard won only the second prize, while the student who had won the first prize also became a mathematician, but a much weaker one!

Some Olympiad winners later achieve nothing, and many outstanding mathematicians had no success in Olympiads at all.

Mathematicians differ dramatically by their time scale: some are very good tackling 15-minute problems, some are good with the problems that require an hour, a day, a week, the problems that take a month, a year, decades of thinking. A. N. Kolmogorov considered his "ceiling" to be two weeks of concentrated thinking.

Success in an Olympiad largely depends on one's sprinter qualities, whereas serious mathematical research requires long distance endurance (B. N. Delaunay used to say, "A good theorem takes not 5 hours, as in an Olympiad, but 5,000 hours").

There are contraindications to becoming a research mathematician. The main one is lack of love of mathematics.

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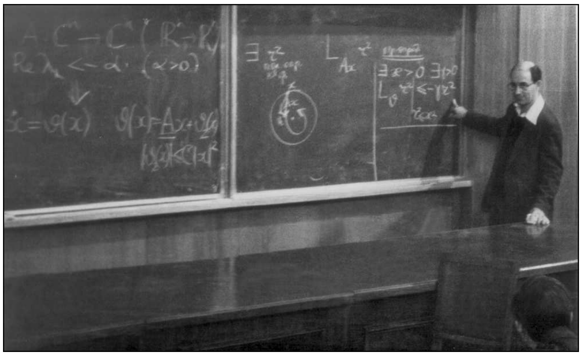
[3] *See A. Givental and E. Wilson-Egolf's (slightly modernized) translation of this early nineteenth-century Russian fable at the end of this interview.*

[4] *The monstrous chief of Stalin's secret police.*



Teaching at Moscow State University, 1983.

But mathematical talents can be very diverse: geometrical and intuitive, algebraic and computational, logical and deductive, natural scientific and inductive. And all kinds are useful. It seems to me that one's difficulties with the multiplication table or a formal definition of half-plane should not obstruct one's way to mathematics. An extremely important condition for serious mathematical research is good health.

Q: *Tell us about the role of sport in your life.*

A: When a problem resists a solution, I jump on my cross-country skis. After forty kilometers a solution (or at least an idea for a solution) always comes. Under scrutiny, an error is often found. But this is a new difficulty that is overcome in the same way.

The Hog Under the Oak

A Hog under a mighty Oak
Had glutted tons of tasty acorns, then, supine,
Napped in its shade; but when awoke,
He, with persistence and the snoot of real swine,
The giant's roots began to undermine.
"The tree is hurt when they're exposed."
A Raven on a branch arose.
"It may dry up and perish—don't you care?"
"Not in the least!" The Hog raised up its head.
"Why would the prospect make me scared?
The tree is useless; be it dead
Two hundred fifty years, I won't regret a second.
Nutritious acorns—only that's what's reckoned!"—
"Ungrateful pig!" the tree exclaimed with scorn.
"Had you been fit to turn your mug around
You'd have a chance to figure out
Where your beloved fruit is born."

Likewise, an ignoramus in defiance
Is scolding scientists and science,
And all preprints at lanl_dot_gov,
Oblivious of his partaking fruit thereof.

Arnold's Doctoral Students

The list below includes those who defended their Ph.D. theses under Arnold's guidance. We have to admit that it was difficult to compile. Along with straightforward cases when Arnold supervised the thesis and was listed as the person's Ph.D. advisor, there were many other situations. For example, in Moscow State University before perestroika, a Ph.D. advisor for a foreigner had to be a member of the Communist Party, so in such cases there was a different nominal Ph.D. advisor while Arnold was supervising the student's work. In other cases there were two co-advisors or there was a different advisor of the Ph.D. thesis, while the person defended the Doctor of Science degree (the second scientific degree in Russia) under Arnold's supervision. In these "difficult cases" the inclusion in the list below is based on "self-definition" as an Arnold student rather than on a formality. We tried to make the list as complete and precise as possible, but we apologize in advance for possible omissions: there were many more people whose work Arnold influenced greatly and who might feel they belong to Arnold's school.

Names are listed chronologically according to the defense years, which are given in parentheses. Many former Arnold's students defended the second degree, the Doctor of Science or Habilitation, but we marked it only in the cases where the first degree was not under Arnold's supervision.

Edward G. Belaga (1965)
Andrei M. Leontovich (1967)
Yulij S. Ilyashenko (1969) (1994, DSci)
Anatoly G. Kushnirenko (1970)
Askold G. Khovanskii (1973)
Nikolai N. Nekhoroshev (1973)
Alexander S. Pyartli (1974)
Alexander N. Varchenko (1974)
Sabir M. Gusein-Zade (1975)
Alexander N. Shoshitaishvili (1975)
Rifkat I. Bogdanov (1976)
Lyudmila N. Bryzgalova (1977)
Vladimir M. Zakalyukin (1977)
Emil Horozov (1978)
Oleg V. Lyashko (1980)
Olga A. Platonova (1981)
Victor V. Goryunov (1982)
Vladimir N. Karpushkin (1982)
Vyacheslav D. Sedykh (1982)
Victor A. Vassiliev (1982)
Aleksey A. Davydov (1983)
Elena E. Landis (1983)
Vadim I. Matov (1983)
Sergei K. Lando (1986)
Inna G. Scherbak (1986)
Oleg P. Scherbak (1986)
Victor I. Bakhtin (1987)
Alexander B. Givental (1987)
Mikhail B. Sevryuk (1988)
Anatoly I. Neishtadt (1976) (1989, DSci)
Ilya A. Bogaevsky (1990)
Boris A. Khesin (1990)
Vladimir P. Kostov (1990)

Boris Z. Shapiro (1990)
Maxim E. Kazarian (1991)
Ernesto Rosales-Gonzalez (1991)
Oleg G. Galkin (1992)
Michael Z. Shapiro (1992)
Alexander Kh. Rakhimov (1995)
Francesca Aicardi (1996)
Yuri V. Chekanov (1997)
Emmanuel Ferrand (1997)
Petr E. Pushkar (1998)
Jacques-Olivier Moussafir (2000)
Mauricio Garay (2001)
Fabien Napolitano (2001)
Ricardo Uribe-Vargas (2001)
Mikhail B. Mishustin (2002)
Adriana Ortiz-Rodriguez (2002)
Gianmarco Capitanio (2004)
Oleg N. Karpenkov (2005)
Alexander M. Lukatsky (1975) (2006, DSci)

At Dubna, 2006.

Alexander Givental

To Whom It May Concern

Но есть и Божий суд ...
М. Ю. Лермонтов, "Смерть Поэта"[6]

Posthumous memoirs seem to have the unintended effect of reducing the person's life to a collection of stories. For most of us it would probably be a just and welcome outcome, but for Vladimir Arnold, I think, it would not. He tried and managed to tell us many different things about mathematics, education, and beyond, and in many cases we've been rather slow listening or thinking, so I believe we will be returning again and again not only to our memories of him but to his own words as well. What is found below is not a memoir, but a recommendation letter, albeit a weak one, for he did *not* get the prize, and yet hopefully useful as an interim attempt to overview his mathematical heritage.

January 25, 2005

Dear Members of [the name of the committee],

You have requested my commentaries on the work of Vladimir Arnold. Writing them is an honorable and pleasurable task for me.

In the essence the task is easy:

Yes, Vladimir Arnold fully merits [the name of the prize] *since his achievements are of extraordinary depth and influence.*

His work indeed resolves fundamental problems, *and* introduces unifying principles, *and* opens up major new areas, *and* (at least in some of these areas) it introduces powerful new techniques too.

On the other hand, writing this letter is not easy, mainly because the ways Arnold's work contributes to our knowledge are numerous and go far beyond my personal comprehension. As Arnold's student, I am quite familiar with those aspects of his work which inspired my own research. Outside these areas, hopefully, I will be able to convey the conventional wisdom about Arnold's most famous achievements. Yet this leaves out the ocean of numerous, possibly less famous but extremely influential, contributions, of which I have only partial knowledge and understanding. So, I will have to be selective here and will mention just a handful of examples which I am better familiar with and which for this reason may look chosen randomly.

Perhaps the most legendary, so to speak, of Arnold's contributions is his work on **small denominators**,[7] followed by the discovery of *Arnold's diffusion*,[8] and known now as part of the Kolmogorov–Arnold–Moser theory. Among other things, this work contains an explanation (or, depending on the attitude, a proof, and a highly technical one) of stability of the solar planetary system. Even more importantly, the KAM theory provides a very deep insight into the real-world dynamics (perhaps one of the few such insights so far, one more being stability of Anosov's systems) and

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[6] *Yet, there is God's Court, too...*, M. Yu. Lermontov, "Death of Poet".

[7] Small denominators III. Problems of stability of motion in classical and celestial mechanics, *Uspekhi Mat. Nauk **18** (1963), no. 6, 91–192, following* Small denominators I. Mappings of a circle onto itself, *Izvestia AN SSSR, Ser. Mat. **25** (1961), 21–86,* Small denominators II. Proof of a theorem of A. N. Kolmogorov on the preservation of conditionally periodic motions under a small perturbation of the Hamiltonian, *Uspekhi Mat. Nauk **18** (1963), no. 5, 13–40, and a series of announcements in DAN SSSR.*

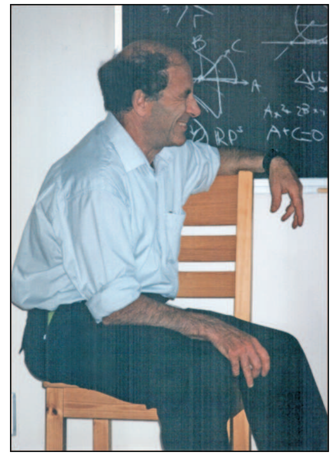
[8] Instability of dynamical systems with many degrees of freedom, *DAN SSSR **156** (1964), 9–12.*

is widely regarded as one of the major discoveries of twentieth-century mathematical physics.

Symplectic geometry has established itself as a universal geometric language of Hamiltonian mechanics, calculus of variations, quantization, representation theory and microlocal analysis of differential equations. One of the first mathematicians who understood the unifying nature of symplectic geometry was Vladimir Arnold, and his work played a key role in establishing this status of symplectic geometry. In particular, his monograph *Mathematical Methods of Classical Mechanics*[9] has become a standard textbook, but thirty years ago it indicated a paradigm shift in a favorite subject of physicists and engineers. The traditional "analytical" or "theoretical" mechanics got suddenly transformed into an active region of modern mathematics populated with Riemannian metrics, Lie algebras, differential forms, fundamental groups, and symplectic manifolds.

Just as much as symplectic geometry is merely a language, **symplectic topology** is a profound problem. Many of the best results of such powerful mathematicians as Conley, Zehnder, Gromov, Floer, Hofer, Eliashberg, Polterovich, McDuff, Salamon, Fukaya, Seidel, and a number of others belong to this area. It would not be too much of an overstatement to say that symplectic topology has developed from attempts to solve a single problem: to prove the Arnold conjecture about Hamiltonian fixed points and Lagrangian intersections.[10] While the conjecture has been essentially proved[11] and many new problems and ramifications discovered, the theory in a sense continues to explore various facets of that same topological rigidity property of phase spaces of Hamiltonian mechanics that goes back to Poincaré and Birkhoff and whose symplectic nature was first recognized by Arnold in his 1965 notes in *Comptes Rendus*.

Arnold's work in Riemannian geometry of infinite-dimensional Lie groups had almost as much of a revolutionizing effect on hydrodynamics as his work in small denominators produced in classical mechanics. In particular, Arnold's seminal paper in *Annales de L'Institut Fourier*[12] draws on his observation that flows of incompressible fluids can be interpreted as geodesics of right-invariant metrics on the groups of volume-preserving diffeomorphisms. Technically speaking, the aim of the paper is to show that most of the sectional curvatures of the area-preserving diffeomorphism group of the standard 2-torus are negative and thus the geodesics on the group typically diverge exponentially. From time to time this result makes the news as a "mathematical proof of impossibility of long-term weather forecasts". More importantly, the work had set Euler's equations on coadjoint orbits as a blueprint and redirected the attention in many models of continuum mechanics toward symmetries, conservation laws, relative equilibria, symplectic reduction, topological methods (in works of Marsden, Ratiu, Weinstein, Moffat, and Freedman among many others).[13]



Between the lectures at Arnoldfest, 1997.

Due to the ideas of Thom and Pham and fundamental results of Mather and Malgrange, **singularity theory** became one of the most active fields of the seventies and eighties, apparently with two leading centers: Brieskorn's seminar in Bonn and Arnold's seminar in Moscow. The theory of critical points of functions and its applications to classification of singularities of caustics, wave fronts and short-wave asymptotics in geometrical optics as well as their relationship with the ADE-classification are perhaps the most famous (among uncountably many other) results

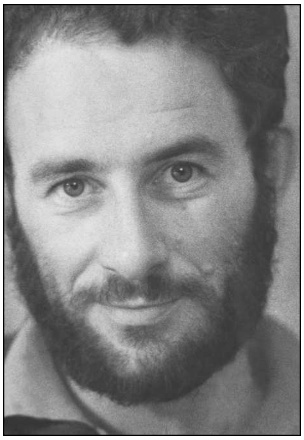
[9] *Nauka, Moscow, 1974.*

[10] *First stated in* Sur une propriété topologique des applications globalement canoniques de la mécanique classique, *C. R. Acad. Sci. Paris* ***261*** *(1965), 3719–3722, and reiterated in a few places, including an appendix to* Math Methods….

[11] *By Hofer (1986) for Lagrangian intersections and by Fukaya–Ono (1996) for Hamiltonian diffeomorphisms, while "essentially" refers to the fact that the conjectures the way Arnold phrased them in terms of critical point of functions rather than (co)homology, and especially in the case of* possibly degenerate *fixed or intersection points, still remain open (and correct just as likely as not, but with no counterexamples in view).*

[12] Sur la géométrie différentielle des groupes de Lie de dimension infinie et ses applications à l'hydrodynamique des fluides parfaits, *Ann. Inst. Fourier* ***16****:1 (1966), 319–361, based on a series of earlier announcements in C. R. Acad. Sci. Paris.*

[13] *As summarized in the monograph* Topological Methods in Hydrodynamics, *Springer-Verlag, 1998, by Arnold and Khesin.*



V. Arnold, 1968.

of Arnold in singularity theory.[14] Arnold's role in this area went, however, far beyond his own papers.

Imagine a seminar of about thirty participants: undergraduates writing their first research papers, graduate students working on their dissertation problems, postgraduates employed elsewhere as software engineers but unwilling to give up their dream of pursuing mathematics even if only as a hobby, several experts—Fuchs, Dolgachev, Gabrielov, Gusein-Zade, Khovansky, Kushnirenko, Tyurin, Varchenko, Vassiliev—and the leader, Arnold—beginning each semester by formulating a bunch of new problems, giving talks or listening to talks, generating and generously sharing new ideas and conjectures, editing his students' papers, and ultimately remaining the only person in his seminar who would keep in mind everyone else's works-in-progress and understand their relationships. Obviously, a lion's share of his students' achievements (and among the quite famous ones are the theory of Newton polyhedra by Khovansky and Kushnirenko or Varchenko's results on asymptotical mixed Hodge structures and semicontinuity of Steenbrink spectra) is due to his help, typically in the form of working conjectures, but every so often through his direct participation (for, with the exception of surveys and obituaries, Arnold would refuse to publish joint papers—we will learn later why).

Moreover, under Arnold's influence, the elite branch of topology and algebraic geometry studying singular real and complex hypersurfaces was transformed into a powerful tool of applied mathematics dealing with degenerations of all kinds of mathematical objects (metamorphoses of wave fronts and caustics, evolutes, evolvents and envelopes of plane curves, phase diagrams in thermodynamics and convex hulls, accessibility regions in control theory, differential forms and Pfaff equations, symplectic and contact structures, solutions of Hamilton-Jacobi equations, the Hamilton-Jacobi equations themselves, the boundaries between various domains in functional spaces of all such equations, etc.) and merging with the **theory of bifurcations** (of equilibria, limit cycles, or more complicated attractors in ODEs and dynamical systems). Arnold had developed a unique intuition and expertise in the subject, so that when physicists and engineers would come to him asking what kind of **catastrophes** they should expect in their favorite problems, he would be able to *guess* the answers in small dimensions right on the spot. In this regard, the situation would resemble experimental physics or chemistry, where personal expertise is often more important than formally registered knowledge.

Having described several (frankly, quite obvious) broad areas of mathematics reshaped by Arnold's seminal contributions, I would like to turn now to some more specific classical problems which attracted his attention over a long time span.

The affirmative solution of the **13th Hilbert problem** (understood as a question about superpositions of *continuous* functions) given by Arnold in his early (essentially undergraduate) work[15] was the beginning of his interest in the "genuine" (and still open) Hilbert's problem: Can the root of the general degree 7 polynomial considered as an algebraic function of its coefficients be written as a superposition of *algebraic* functions of 2 variables? The negative[16] solution to the more general question about polynomials of degree n was given by Arnold in 1970 for $n = 2^r$.[17] The result was generalized by V. Lin. Furthermore, Arnold's approach, based on his previous study of cohomology of braid groups, later gave rise to Smale's concept of topological complexity of algorithms and Vassiliev's results on this subject. Even more importantly, Arnold's study of braid groups via topology of configuration spaces[18] was generalized by Brieskorn to E. Artin's braid groups associated with reflection groups. The latter inspired Orlik–Solomon's theory of hyperplane arrangements,

[14]Normal forms of functions near degenerate critical points, the Weyl groups A_k, D_k, E_k and Lagrangian singularities, *Funct. Anal. Appl.* ***6**, no. 4 (1972), 3–25; see also Arnold's inspiring paper in Proceedings of the ICM-74 Vancouver and the textbooks* Singularities of Differential Maps, *Vols. I and II, by Arnold, Gusein-Zade and Varchenko.*

[15]On the representations of functions of several variables as a superposition of functions of a smaller number of variables, *Mat. Prosveshchenie (1958), 41–46.*

[16]*That is, positive in Hilbert's sense.*

[17]Topological invariants of algebraic functions. II, *Funct. Anal. Appl.* ***4** (1970), no. 2, 1–9.*

[18]The cohomology ring of the group of dyed braids, *Mat. Zametki* ***5** (1969), 227–231.*

K. Saito–Terao's study of free divisors, Gelfand's approach to hypergeometric functions, Aomoto's work on Yang-Baxter equations, and Varchenko–Schekhtman's hypergeometric "Bethe ansatz" for solutions of Knizhnik–Zamolodchikov equations in conformal field theory.

Arnold's result[19] on the **16th Hilbert problem, Part I,** about disposition of ovals of real plane algebraic curves, was immediately improved by Rokhlin (who applied Arnold's method but used more powerful tools from the topology of 4-manifolds). This led Rokhlin to his proof of a famous conjecture of Gudkov (who corrected Hilbert's expectations in the problem), inspired many new developments (due to Viro and Kharlamov among others), and is considered a crucial breakthrough in the history of real algebraic geometry.

Among other things, the paper of Arnold outlines an explicit diffeomorphism between S^4 and the quotient of $\mathbf{C}P^2$ by complex conjugation.[20] The fact was rediscovered by Kuiper in 1974 and is known as *Kuiper's theorem* [31]. Arnold's argument, based on hyperbolicity of the discriminant in the space of Hermitian forms, was recently revived in a far-reaching paper by M. Atiyah and J. Berndt [19].

Another work of Arnold in the same field[21] unified the Petrovsky-Oleinik inequalities concerning topology of real hypersurfaces (or their complements) and brought mixed Hodge structures (just introduced by Steenbrink into complex singularity theory) into real algebraic geometry.

Arnold's interest in the **16th Hilbert problem, Part II**, on the number of limit cycles of polynomial ODE systems on the plane has been an open-ended search for simplifying formulations. One such formulation[22] (about the maximal number of limit cycles born under a nonconservative perturbation of a Hamiltonian system and equivalent to the problem about the number of zeroes of Abelian integrals over a family of real algebraic ovals) generated extensive research. The results here include the general deep finiteness theorems of Khovansky and Varchenko, Arnold's conjecture about nonoscillatory behavior of the Abelian integrals, his geometrization of higher-dimensional Sturm theory of (non)oscillations in linear Hamiltonian systems,[23] various attempts to prove this conjecture (including a series of papers by Petrov-Tan'kin on Abelian integrals over elliptic curves, my own application of Sturm's theory to nonoscillation of hyperelliptic integrals, and more recent estimates of Grigoriev, Novikov–Yakovenko), and further work by Horozov, Khovansky, Ilyashenko and others. Yet another modification of the problem (a discrete one-dimensional analogue) suggested by Arnold led to a beautiful and nontrivial theorem of Yakobson in the theory of dynamical systems [39].

V. Arnold and J. Moser at the Euler Institute, St. Petersburg, 1991.

The classical problem in the theory of Diophantine approximations of inventing the **higher-dimensional analogue of continued fractions** has been approached by many authors, with a paradoxical outcome: there are many relatively straightforward and relatively successful generalizations, but none as unique and satisfactory as the elementary continued fraction theory. Arnold's approach to this problem[24] is based on his discovery of a relationship between graded algebras and Klein's *sails* (i.e., convex hulls of integer points inside simplicial convex cones in Euclidean spaces). Arnold's problems and conjectures on the subject have led to the results of E. Korkina and G. Lauchaud generalizing Lagrange's theorem (which identifies quadratic irrationalities with eventually periodic continued fractions) and to the work of Kontsevich–Sukhov generalizing Gauss's dynamical system and its ergodic properties. Thus the Klein-Arnold generalization, while not straightforward, appears to

[19]The situation of ovals of real algebraic plane curves, the involutions of four-dimensional smooth manifolds, and the arithmetic of integral quadratic forms, *Funct. Anal. Appl.* **5** *(1971), no. 3, 1–9.*

[20]*Details were published much later in* The branched covering $CP^2 \to S^4$, hyperbolicity and projective topology, *Sibirsk. Mat. Zh.* **29** *(1988), no. 5, 36–47.*

[21]The index of a singular point of a vector field, the Petrovsky-Oleinik inequalities, and mixed Hodge structures, *Funct. Anal. Appl.* **12** *(1978), no. 1, 1–14.*

[22]*V. I. Arnold, O. A. Oleinik,* Topology of real algebraic varieties, *Vestnik Moskov. Univ. Ser. I Mat. Mekh. (1979), no. 6, 7–17.*

[23]Sturm theorems and symplectic geometry, *Funct. Anal. Appl.* **19** *(1985), no. 4, 1–10.*

[24]A-graded algebras and continued fractions, *Commun. Pure Appl. Math.* **49** *(1989), 993–1000;* Higher-dimensional continued fractions, *Regul. Chaotic Dyn. 3 (1998), 10–17; and going back to* Statistics of integral convex polyhedra, *Funct. Anal. Appl.* **14** *(1980), no. 1, 1–3; and to the theory of Newton polyhedra.*

Lecturing in Toronto, 1997.

be just as unique and satisfactory as its classical prototype.

The above examples show how Arnold's interest in specific problems helped to transform them into central areas of modern research. There are other classical results which, according to Arnold's intuition, are scheduled to generate such new areas, but to my understanding have not yet achieved the status of important mathematical theories in spite of interesting work done by Arnold himself and some others. But who knows? To mention one: the Four-Vertex Theorem, according to Arnold, is the seed of a new (yet unknown) branch of topology (in the same sense as the Last Poincaré Theorem was the seed of symplectic topology). Another example: a field-theoretic analogue of Sturm theory, broadly understood as a study of topology of zero levels (and their complements) of eigenfunctions of selfadjoint linear partial differential operators.

Perhaps with the notable exceptions of KAM-theory and singularity theory, where Arnold's contributions are marked not only by fresh ideas but also by technical breakthroughs (e.g., a heavy-duty tool in singularity theory—his spectral sequence),[25] a more typical path for Arnold would be to invent a bold new problem, attack its first nontrivial cases with his bare hands, and then leave developing an advanced machinery to his followers. I have already mentioned how the theory of hyperplane arrangements emerged in this fashion. Here are some other examples of this sort where Arnold's work starts a new area.

In 1980 Arnold invented the concepts of **Lagrangian and Legendrian cobordisms** and studied them for curves using his theory of bifurcations of wave fronts and caustics.[26] The general homotopy theory formulation was then given by Ya. Eliashberg, and the corresponding "Thom rings" computed in an award-winning treatise by M. Audin [20]. A geometric realization of Lagrange and Legendre characteristic numbers as the enumerative theory of singularities of global caustics and wave fronts was given by V. Vassiliev [38]. The method developed for this task, namely associating a spectral sequence to a stratification of functional spaces of maps according to types of singularities, was later applied by Vassiliev several more times, of which his work on *Vassiliev invariants of knots* is the most famous one.

Arnold's definition[27] of the **asymptotic Hopf invariant** as the average self-linking number of trajectories of a volume-preserving flow on a simply connected 3-fold and his "ergodic" theorem about coincidence of the invariant with Moffatt's *helicity* gave the start to many improvements, generalizations, and applications of topological methods in hydro- and magneto-dynamics due to M. H. Freedman et al., É. Ghys, B. Khesin, K. Moffatt, and many others.[28]

As one can find out, say, on MathSciNet, Arnold is one of the most prolific mathematicians of our time. His high productivity is partly due to his fearless curiosity and enormous appetite for new problems.[29] Paired with his taste and intuition, these qualities often bring unexpected fruit, sometimes in the areas quite remote from the domain of his direct expertise. Here are some examples.

Arnold's observation[30] on the pairs of triples of numbers computed by I. Dolgachev and A. Gabrielov and characterizing respectively uniformization and monodromy of *14 exceptional unimodal singularities of surfaces* (in Arnold's classification) is known now under the name **Arnold's Strange Duality**. In 1977, due to Pinkham and Dolgachev–Nikulin, the phenomenon received a beautiful explanation in terms of geometry of K3-surfaces. As became clear in the early nineties, Arnold's Strange Duality was the first, and highly nontrivial, manifestation of **Mirror Symmetry**: a profound conjecture discovered by string theorists and claiming a sort of equivalence between symplectic topology and complex geometry (or singularity theory).

[25]A spectral sequence for the reduction of functions to normal forms, *Funct. Anal. Appl.* ***9*** *(1975), 81–82.*

[26]Lagrange and Legendre cobordisms. I, II, *Funct. Anal. Appl.* ***14*** *(1980), no. 3, 1–13; no. 4, 8–17.*

[27]*See* The asymptotic Hopf invariant and its applications, *Selected translations. Selecta Math. Soviet.* ***5*** *(1986), no. 4, 327–345, which is the translation of a 1973 paper and one of Arnold's most frequently quoted works.*

[28]*See a review in Chapter III of V. I. Arnold, B. A. Khesin,* Topological methods in hydrodynamics, *Applied Math. Sciences, vol. 125, Springer-Verlag, NY, 1998.*

[29]*See the unusual book* Arnold's Problems, *Springer-Verlag, Berlin; PHASIS, Moscow, 2004.*

[30]*See* Critical points of smooth functions, *Proceedings of the International Congress of Mathematicians (Vancouver, B. C., 1974), Vol. 1, pp. 19–39.*

Arnold's work in pseudo-periodic geometry[31] encouraged A. Zorich to begin a systematic study of dynamics on Riemann surfaces defined by levels of closed 1-forms, which led to a number of remarkable results of Kontsevich–Zorich [29] and others related to ergodic theory on Teichmüller spaces and conformal field theory, and of Eskin–Okounkov [24] in the Hurwitz problem of counting ramified covers over elliptic curves.

Arnold seems to be the first to suggest[32] that monodromy (say of Milnor fibers or of flag varieties) can be realized by symplectomorphisms. The idea, picked up by M. Kontsevich and S. Donaldson, was upgraded to the monodromy action on the Fukaya category (consisting of all Lagrangian submanifolds in the fibers and of their Floer complexes). This construction is now an important ingredient of the Mirror Symmetry philosophy and gave rise to the remarkable results of M. Khovanov and P. Seidel about faithfulness of such Hamiltonian representations of braid groups [27].

The celebrated Witten's conjecture proved by M. Kontsevich in 1991 characterizes intersection theory on Deligne–Mumford moduli spaces of Riemann surfaces in terms of KdV-hierarchy of integrable systems. A refreshingly new proof of this result was recently given by Okounkov–Pandharipande. A key ingredient in their argument is an elementary construction of Arnold from his work on enumerative geometry of trigonometric polynomials.[33]

Among many concepts owing Arnold their existence, let me mention two of general mathematical stature which do not carry his name.

One is the **Maslov index**, which proved to be important in geometry, calculus of variations, numbers theory, representation theory, quantization, index theory of differential operators, and whose topological origin was explained by Arnold.[34]

The other one is the geometric notion of **integrability in Hamiltonian systems**. There is a lot of controversy over which of the known integrability mechanisms is most fundamental, but there is a consensus that integrability means a complete set of Poisson-commuting first integrals.

This definition and "Liouville's Theorem" on geometric consequences of the integrability property (namely, foliation of the phase space by Lagrangian tori) are in fact Arnold's original inventions.

Similar to the case with integrable systems, there are other examples of important developments which have become so common knowledge that Arnold's seminal role eventually became invisible. Let me round up these comments with a peculiar example of this sort.

The joint 1962 paper of Arnold and Sinai[35] proves structural stability of hyperbolic linear diffeomorphisms of the 2-torus. Their idea, picked up by Anosov, was extended to his famous general stability theory of *Anosov systems* [2]. Yet, according to Arnold, the paper is rarely quoted, for the proof contained a mistake (although each author's contribution was correct, so that neither one could alone be held responsible). By the way, Arnold cites this episode as the reason why he refrained from writing joint research papers.

To reiterate what I said at the beginning, Vladimir Arnold has made outstanding contributions to many areas of pure mathematics and its applications, including those I described above and those I missed: classical and celestial mechanics, cosmology and hydrodynamics, dynamical systems and bifurcation theory, ordinary and partial differential equations, algebraic and geometric topology, number theory and combinatorics, real and complex algebraic geometry, symplectic and contact geometry and topology, and perhaps some others. I can think of few mathematicians whose work and personality would influence the scientific community at a comparable scale. And beyond this community, Arnold is a highly visible (and possibly controversial) figure, the subject of several interviews, of a recent documentary movie, and even of the night sky show, where one can watch an asteroid, *Vladarnolda*, named after him.

I am sure there are other mathematicians who also deserve [the name of the prize], but awarding it to Vladimir Arnold will hardly be perceived by anyone as a mistake.

[31]Topological and ergodic properties of closed 1-forms with incommensurable periods, *Funct. Anal. Appl.* ***25*** *(1991), no. 2, 1–12.*

[32]*See* Some remarks on symplectic monodromy of Milnor fibrations, *The Floer memorial volume, 99–103, Progr. Math., 133, Birkhäuser, Basel, 1995.*

[33]Topological classification of complex trigonometric polynomials and the combinatorics of graphs with an identical number of vertices and edges, *Funct. Anal. Appl.* ***30*** *(1996), no. 1, 1–17.*

[34]*In his paper* On a characteristic class entering into conditions of quantization, *Funct. Anal. Appl.* ***1*** *(1967), 1–14.*

[35]*Arnold, V. I., Sinai, Ya. G.,* On small perturbations of the automorphisms of a torus, *Dokl. Akad. Nauk SSSR* ***144*** *(1962), 695–698.*

Ya. Sinai and V. Arnold, photo by J. Moser, 1963.

Yakov Sinai

Remembering Vladimir Arnold: Early Years

De mortuis veritas[37]

My grandparents and Arnold's grandparents were very close friends since the beginning of the twentieth century. Both families lived in Odessa, which was a big city in the southern part of Russia and now is a part of Ukraine. At that time, Odessa was a center of Jewish intellectual life, which produced many scientists, musicians, writers, and other significant figures.

My maternal grandfather, V. F. Kagan, was a well-known geometer who worked on the foundations of geometry. During World War I, he gave the very first lecture course in Russia on the special relativity theory. At various times his lectures were attended by future famous physicists L. I. Mandelshtam, I. E. Tamm, and N. D. Papaleksi. In the 1920s all these people moved to Moscow.

L. I. Mandelshtam was a brother of Arnold's maternal grandmother. He was the founder and the leader of a major school of theoretical physics that included A. A. Andronov, G. S. Landsberg, and M. A. Leontovich, among others. A. A. Andronov is known to the mathematical community for his famous paper "Robust systems", coauthored with L. S. Pontryagin, which laid the foundations of the theory of structural stability of dynamical systems. A. A. Andronov was the leader of a group of physicists and mathematicians working in Nizhny Novgorod, formerly Gorky, on nonlinear oscillations. M. A. Leontovich was one of the leading physicists in the Soviet Union. In the 1930s he coauthored with A. N. Kolmogorov the well-known paper on the Wiener sausage. I. E. Tamm was a Nobel Prize winner in physics in the fifties. N. D. Papaleksi was a great expert on nonlinear optics.

V. I. Arnold was born in Odessa, where his mother had come for a brief visit with her family. She returned to Moscow soon after her son's birth. When Arnold was growing up, the news that his family had a young prodigy soon became widely known. In those days, when we were both in high school, we did not really know each other. On one occasion, Arnold visited my grandfather to borrow a mathematics book, but I was not there at the time. We met for the first time when we were both students at the mathematics department of the Moscow State University; he was walking by with Professor A. G. Vitushkin, who ran a freshman seminar on real analysis, and Arnold was one of the most active participants. When Arnold was a third-year undergraduate student, he was inspired by A. N. Kolmogorov to work on superposition of functions of several variables and the related Hilbert's thirteenth problem. Eventually this work became Arnold's Ph.D. thesis. When I visited the University of Cambridge recently, I was very pleased to learn that one of the main lecture courses there was dedicated to Arnold's and Kolmogorov's work on Hilbert's thirteenth problem.

Arnold had two younger siblings: a brother, Dmitry, and a sister, Katya, who was the youngest. The family lived in a small apartment in the center of Moscow. During one of my visits, I was shown a tent in the backyard of the building where Arnold used to spend his nights, even in cold weather. It seems likely that Arnold's excellent knowledge of history and geography of Moscow, which many of his friends remember with admiration, originated at that time.

Like me, Arnold loved nature and the outdoors. We did hiking and mountain climbing together. Since I knew Arnold so closely, I often observed that his ideas both in science and in life came to him as revelations. I remember one particular occasion, when we were climbing in the Caucasus Mountains and spent a night with some shepherds in their tent. In the morning we discovered that the shepherds were gone and had left us alone with their dogs. Caucasian dogs are very big, strong, and dangerous, for they are bred and trained to fight wolves. We were surrounded by fiercely barking dogs, and we did not know what to do. Then, all of a sudden, Arnold had an idea. He started shouting very loudly at the dogs, using all the obscenities he could think of. I never heard him use such language either before or after this incident, nor did anybody else. It was a brilliant idea, for it worked! The dogs did not touch Arnold and barely

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[37]*About those who have died, only the truth.*

A hiking expedition, 1960s.

touched me. The shepherds returned shortly afterwards, and we were rescued.

On another occasion, roughly at the same time, as Anosov, Arnold, and I walked from the main Moscow University building to a subway station, which usually took about fifteen minutes, Arnold told us that he recently came up with the Galois theory entirely on his own and explained his approach to us. The next day, Arnold told us that he found a similar approach in the book by Felix Klein on the mathematics of the nineteenth century. Arnold was always very fond of this book, and he often recommended it to his students.

Other examples of Arnold's revelations include his discovery of the Arnold-Maslov cocycle in the theory of semi-classical approximations and Arnold inequalities for the number of ovals in real algebraic curves. Many other people who knew Arnold personally could provide more examples of this kind.

Arnold became a graduate student at the Moscow State University in 1959. Naturally it was A. N. Kolmogorov who became his advisor. In 1957 Kolmogorov gave his famous lecture course on dynamical systems, which played a pivotal role in the subsequent development of the theory. The course was given three years after Kolmogorov's famous talk at the Amsterdam Congress of Mathematics.

Kolmogorov began his lectures with the exposition of the von Neumann theory of dynamical systems with pure point spectrum. Everything was done in a pure probabilistic way. Later Kolmogorov found a similar approach in the book by Fortet and Blank–Lapierre on random processes, intended for engineers.

This part of Kolmogorov's lectures had a profound effect on researchers working on the measure-theoretic isomorphism in dynamical systems, a long-standing problem that goes back to von Neumann. It was shown that when the spectrum is a pure point one, it is the only isomorphism invariant of a dynamical system and that two systems with the same pure point spectrum are isomorphic. The excitement around these results was so profound that people began to believe that the isomorphism theory of systems with continuous spectrum would be just a straightforward generalization of the theory of systems with pure point spectrum. However, this was refuted by Kolmogorov himself. He proposed the notion of entropy as a new isomorphism invariant for systems with continuous spectrum. Since the entropy is zero for systems with pure point spectrum, it does not distinguish between such systems, but systems with continuous spectrum might have positive entropy that must be preserved by isomorphisms. This was a path-breaking discovery, which had a tremendous impact on the subsequent development of the theory.

The second part of Kolmogorov's lectures was centered around his papers on the preservation of invariant tori in small perturbations of integrable Hamiltonian systems, which were published in the *Doklady* of the Soviet Academy of Sciences. Unfortunately there were no written notes of these lectures. V. M. Tikhomirov, one of Kolmogorov's students, hoped for many years to locate such notes, but he did not succeed. Arnold used to claim in his correspondence with many people that good mathematics students of Moscow University could reconstruct Kolmogorov's proof from the text of his papers in the *Doklady*. However, this was an exaggeration. Recently two Italian mathematicians, A. Giorgilli and L. Chierchia, produced a proof of Kolmogorov's theorem, which was complete and close to Kolmogorov's original proof, as they claimed.

Apparently Kolmogorov himself never wrote a detailed proof of his result. There might be several explanations. At some point, he had plans to work on applications of his technique to the famous three-body problem. He gave a talk on this topic at a meeting of the Moscow Mathematical Society. However, he did not prepare a written version of his talk. Another reason could be that Kolmogorov started to work on a different topic and did not want to be distracted. There might be a third reason, although some people would disagree with it. It is possible that Kolmogorov underestimated the significance of his papers. For example, some graduate exams on classical mechanics included the proof of Kolmogorov's theorem, so it was easy to assume that the proof was already known. The theory of entropy, introduced by Kolmogorov roughly at the same time, seemed a hotter and more exciting area. He might have felt compelled to turn his mind to this new topic.

A. Kirillov, I. Petrovskii, V. Arnold.

Arnold immediately started to work on all the problems raised in Kolmogorov's lectures. In 1963 the Moscow Mathematical Society celebrated Kolmogorov's sixtieth birthday. The main meeting took place in the Ceremony Hall of the Moscow State University, with about one thousand people attending. The opening lecture was given by Arnold on what was later called KAM theory, where KAM stands for Kolmogorov, Arnold, Moser. For that occasion, Arnold prepared the first complete exposition of the Kolmogorov theorem. I asked Arnold why he did that, since Kolmogorov presented his proof in his lectures. Arnold replied that the proof of the fact that invariant tori constitute a set of positive measure was not complete. When Arnold asked Kolmogorov about some details of his proof, Kolmogorov replied that he was too busy at that time with other problems and that Arnold should provide the details by himself. This was exactly what Arnold did. I believe that when Kolmogorov prepared his papers for publication in the *Doklady*, he did have complete proofs, but later he might have forgotten some details. Perhaps it can be expressed better by saying that it required from him an effort that he was not prepared to make at that time.

In the following years, Kolmogorov ran a seminar on dynamical systems, with the participation of many mathematicians and physicists. At some point, two leading physicists, L. A. Arzimovich and M. A. Leontovich, gave a talk at the seminar on the existence of magnetic surfaces. Subsequently this problem was completely solved by Arnold, who submitted his paper to the main physics journal in the Soviet Union, called JETP. After some time, the paper was rejected. According to Arnold, the referee report said that the referee did not understand anything in that paper and hence nobody else would understand it. M. A. Leontovich helped Arnold to rewrite his paper in the form accessible to physicists, and it was published eventually. According to Arnold, this turned out to be one of his most quoted papers.

Arnold's first paper related to the KAM theory was about smooth diffeomorphisms of the circle that were close to rotations. Using the methods of the KAM theory, Arnold proved that such diffeomorphisms can be reduced to rotations by applying smooth changes of variables. The problem in the general case was called the Arnold problem. It was completely solved by M. Herman and J.-C. Yoccoz.

A. N. Kolmogorov proved his theorem in the KAM theory for the so-called nondegenerate perturbations of integrable Hamiltonian systems. Arnold extended this theorem to degenerate perturbations, which arise in many applications of the KAM theory.

Arnold proposed an example of the Hamiltonian system which exhibits a new kind of instability and which was later called the Arnold diffusion. The Arnold diffusion appears in many physical problems. New mathematical results on the Arnold diffusion were recently proved by J. Mather. V. Kaloshin found many applications of the Arnold diffusion to problems of celestial mechanics.

In later years Arnold returned to the theory of dynamical systems only occasionally. One can mention his results in fluid mechanics (see his joint book with B. Khesin [16]) and a series of papers on singularities in the distribution of masses in the universe, motivated by Y. B. Zeldovich. But all this was done in later years.

Steve Smale

Vladimir I. Arnold

My first meeting with V. I. Arnold took place in Moscow in September 1961 (certainly I had been very aware of him through Moser). After a conference in Kiev, where I had gotten to know Anosov, I visited Moscow, where Anosov introduced me to Arnold, Novikov, and Sinai. As I wrote later [35], I was extraordinarily impressed by such a powerful group of four young mathematicians and that there was nothing like that in the West. At my next visit to Moscow for the world mathematics congress in 1966 [36], I again saw much of Dima Arnold. At that meeting he introduced me to Kolmogorov.

Perhaps the last time I met Dima was in June 2003 at the one-hundred-year memorial conference for Kolmogorov, again in Moscow. In the intervening years we saw each other on a number of occasions in Moscow, in the West, and even in Asia.

Arnold was visiting Hong Kong at the invitation of Volodya Vladimirov for the duration of the

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fall semester of 1995, while we had just moved to Hong Kong. Dima and I often were together on the fantastic day hikes in the Hong Kong countryside parks. His physical stamina was quite impressive. At that time we two were also the focus of a well-attended panel on contemporary issues of mathematics at the Hong Kong University of Science and Technology. Dima expressed himself in his usual provocative way! I recall that we found ourselves on the same side in most of the controversies, and catastrophe theory in particular.

Dima Arnold was a great mathematician, and here I will just touch on his mathematical contributions which affected me the most.

While I never worked directly in the area of KAM, nevertheless those results had a great impact in my scientific work. For one thing they directed me away from trying to analyze the global orbit structures of Hamiltonian ordinary differential equations, in contrast to what I was doing for (unconstrained) equations. Thus KAM contributed to my motivation to study mechanics in 1970 from the point of view of topology, symmetry, and relative equilibria rather than its dynamical properties. The work of Arnold had already affected those subjects via his big paper on fluid mechanics and symmetry in 1966. See Jerry Marsden's account of how our two works are related [32]. I note that Jerry died even more recently than Dima.

KAM shattered the chain of hypotheses, ergodic, quasi-ergodic, and metric transitivity going from Boltzmann to Birkhoff. That suggested to me some kind of non-Hamiltonian substitute in these hypotheses in order to obtain foundations for thermodynamics [37].

I read Arnold's paper on braids and the cohomology of swallowtails. It was helpful in my work on topology and algorithms, which Victor Vassiliev drastically sharpened.

Dima could express important ideas simply and in such a way that these ideas could transcend a single discipline. His work was instrumental in transforming Kolmogorov's early sketches into a revolutionary recasting of Hamiltonian dynamics with sets of invariant curves, tori of positive measure, and Arnold diffusion.

It was my good fortune to have been a part of Dima Arnold's life and his mathematics.

Mikhail Sevryuk

Some Recollections of Vladimir Igorevich

A very large part of my life is connected with Vladimir Igorevich Arnold. I became his student in the beginning of 1980 when I was still a freshman at the Department of Mechanics and Mathematics of Moscow State University. Under his supervision, I wrote my term papers, master's thesis, and doctoral thesis. At the end of my first year in graduate school, Arnold suggested that I write a monograph on reversible dynamical systems for Springer's Lecture Notes in Mathematics series, and working on this book was one of the cornerstones of my mathematical biography. For the last time, I met Vladimir Igorevich (V. I., for short) on November 3, 2009, at his seminar at Moscow State.

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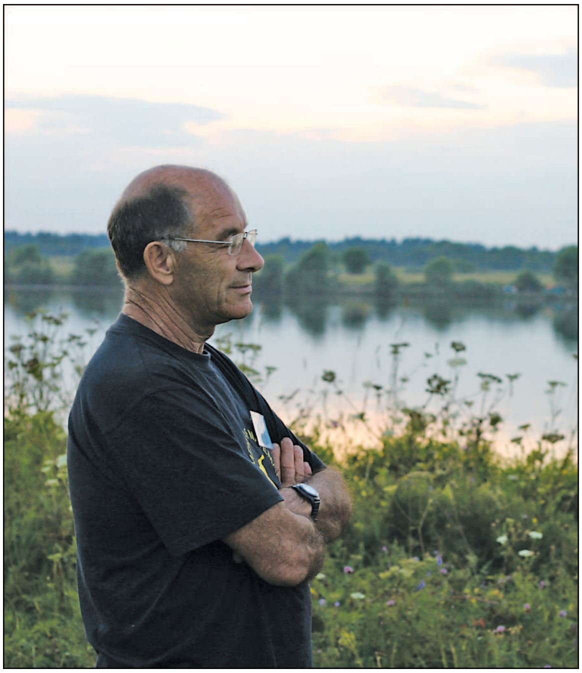
At Arnoldfest, Toronto, 1997.

If I had to name one characteristic feature of Arnold as I remember him, I would choose his agility. He walked fast at walkways of Moscow State (faster than most of the students, not to mention the faculty), his speech was fast and clear, his reaction to one's remark in a conversation was almost always instantaneous, and often utterly unexpected. His fantastic scientific productivity is well known, and so is his enthusiasm for sports.

V. I. always devoted a surprising amount of time and effort to his students. From time to time, he had rather weak students, but I do not recall a single case when he rejected even a struggling student. In the 1980s almost every meeting of his famous seminar at Moscow State he started with "harvesting": collecting notes of his students with sketches of their recent mathematical achievements or drafts of their papers (and Arnold returned the previously collected ones with his corrections and suggestions). After a seminar or a lecture, he often continued talking with participants for another 2–3 hours. Arnold's generosity was abundant. Many times, he gave long written mathematical consultations, even to people unknown to him, or wrote paper reports substantially exceeding the submitted papers.



Dubna, 2006.

In recent years, he used all his energy to stop a rapid deterioration of mathematical (and not only mathematical) education in Russia.

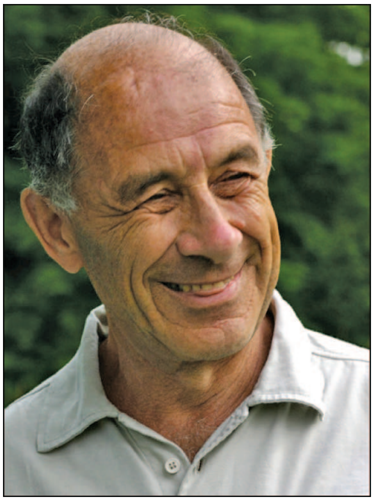
I tried to describe my experience of being V. I.'s student at Moscow State in [34]. I would like to emphasize here that Arnold did not follow any pattern in supervising his students. In some cases he would inform a student that there was a certain "uninhabited" corner in the vast mathematical land, and if the student decided to "settle" at that corner, then it was this student's task to find the main literature on the subject, to study it, to pose new problems, to find methods of their solution, and to achieve all this practically single-handedly. Of course, V. I. kept the progress under control. (I recall that, as a senior, I failed to submit my "harvest" for a long time, but finally made substantial progress. Arnold exclaimed, "Thank God, I have started fearing that I would have to *help* you!") But in other situations, Arnold would actively discuss a problem with his student and invite him to collaborate— this is how our joint paper [13] came about. When need be, V. I. could be rather harsh. Once I witnessed him telling a student, "You are working too slowly. I think it will be good if you start giving me weekly reports on your progress." Arnold never tried to spare one's self-esteem.

V. I. had a surprising feeling of the unity of mathematics, of natural sciences, and of all nature. He considered mathematics as being part of physics, and his "economics" definition of mathematics as a part of physics in which experiments are cheap is often quoted. (Let me add in parentheses that I would prefer to characterize mathematics as the natural science that studies the phenomenon of infinity by analogy with a little-known but remarkable definition of topology as the science that studies the phenomenon of continuity.) However, Arnold noted other specific features of mathematics: "It is a fair observation that physicists refer to the first author, whereas mathematicians to the latest one." (He considered adequate references to be of paramount importance and paid much attention to other priority questions; this was a natural extension of his generosity, and he encouraged his students to "over-acknowledge", rather than to "under-acknowledge".)

V. I. was an avid fighter against "Bourbakism", a suicidal tendency to present mathematics as a formal derivation of consequences from unmotivated axioms. According to Arnold, one needs mathematics to discover new laws of nature as opposed to "rigorously" justify obvious things. V. I. tried to teach his students this perception of mathematics and natural sciences as a unified tool for understanding the world. For a number of reasons, after having graduated from university, I had to work partially as a chemist, and after Arnold's school this caused me no psychological discomfort.

Fundamental mathematical achievements of Arnold, as well as those of his teacher, A. N. Kolmogorov, cover almost all mathematics. It well may be that V. I. was the last universal mathematician. My mathematical specialization is the KAM theory. V. I. himself described the contributions of the three founders; see, e.g., [15], [17]. For this reason, I shall only briefly recall Arnold's role in the development of the KAM theory.

KAM theory is the theory of quasiperiodic motions in nonintegrable dynamical systems. In 1954 Kolmogorov made one of the most astonishing discoveries in mathematics of the last century. Consider a completely integrable Hamiltonian system with n degrees of freedom, and let (I, φ) be the corresponding action-angle variables. The phase space of such a system is smoothly foliated into invariant n-tori $\{I = \text{const}\}$ carrying conditionally periodic motions $\dot{\varphi} = \omega(I)$. Kolmogorov showed that if $\det(\partial\omega/\partial I) \neq 0$, then (in spite of the general opinion of the physical community of that time) most of these tori (in the Lebesgue sense) are not destroyed by a small Hamiltonian perturbation but only slightly deformed in the phase space. To be more precise, a torus $\{I = I^0\}$ persists under a perturbation whenever the frequencies $\omega_1(I^0), \dots, \omega_n(I^0)$ are Diophantine (strongly incommensurable). The perturbed tori (later called Kolmogorov tori) carry quasiperiodic motions with the same frequencies. To prove this fundamental theorem, Kolmogorov proposed a new, powerful method of constructing an infinite sequence of canonical coordinate



Vladimir Arnold.

transformations with accelerated ("quadratic") convergence.

Arnold used Kolmogorov's techniques to prove analyticity of the Denjoy homeomorphism conjugating an analytic diffeomorphism of a circle with a rotation (under the condition that this diffeomorphism is close to a rotation and possesses a Diophantine rotation number). His paper [4] with this result contained also the first detailed exposition of Kolmogorov's method. Then, in a series of papers, Arnold generalized Kolmogorov's theorem to various systems with degeneracies. In fact, he considered two types of degeneracies often encountered in mechanics and physics: the *proper degeneracy*, where some frequencies of the perturbed tori tend to zero as the perturbation magnitude vanishes, and the *limit degeneracy*, where the unperturbed foliation into invariant tori is singular and includes tori of smaller dimensions. The latter degeneracy is modeled by a one-degree-of-freedom Hamiltonian system having an equilibrium point surrounded by invariant circles (the energy levels). These studies culminated in Arnold's famous (and technically extremely hard) result [5] on stability in planetary-like systems of celestial mechanics where both the degeneracies combine.

Kolmogorov and Arnold dealt only with analytic Hamiltonian systems. On the other hand, J. K. Moser examined the finitely smooth case. The acronym "KAM" was coined by physicists F. M. Izrailev and B. V. Chirikov in 1968.

Arnold always regarded his discovery of the universal mechanism of instability of the action variables in nearly integrable Hamiltonian systems with more than two degrees of freedom [6] as his main achievement in the Hamiltonian perturbation theory. He also constructed an explicit example where such instability occurs. Chaotic evolution of the actions along resonances between the Kolmogorov tori was called "Arnold's diffusion" by Chirikov in 1969. In the case of two degrees of freedom, the Kolmogorov 2-tori divide a three-dimensional energy level, which makes an evolution of the action variables impossible.

All these works by Arnold took place in 1958–1965. At the beginning of the eighties, he returned to the problem of quasiperiodic motions for a short time and examined some interesting properties of the analogs of Kolmogorov tori in reversible systems. That was just the time when I started my diploma work. So V. I. forced me to grow fond of reversible systems and KAM theory, for which I'll be grateful to him forever.

I would like to touch on yet one more side of Arnold's research. In spite of what is occasionally claimed, Arnold did not hate computers: he considered them as an absolutely necessary instrument of mathematical modeling when indeed large computations were involved. He initiated many computer experiments in dynamical systems and number theory and sometimes participated in them (see [17]). But of course he strongly disapproved of the aggressive penetration of computer technologies into all pores of society and the tendency of a man to become a helpless and mindless attachment to artificial intelligence devices. One should be able to divide 111 by 3 without a calculator (and, better still, without scrap paper).

V. I. had a fine sense of humor. It is impossible to forget his somewhat mischievous smile. In conclusion, here are a couple of stories which might help to illustrate the unique charm of this person.

I remember how a speaker at Arnold's seminar kept repeating the words "one can lift" (a structure from the base to the total space of a bundle). Arnold reacted: "Looks like your talk is about results in weight-lifting."

On another occasion, Arnold was lecturing, and the proof of a theorem involved tedious computations: "Everyone must make these computations once—but only once. I made them in the past, so I won't repeat them now; they are left to the audience!"

In the fall of 1987 the Gorbachev *perestroika* was gaining steam. A speaker at the seminar was drawing a series of pictures depicting the perestroika (surgery) of a certain geometrical object as depending on a parameter. Arnold: "Something is not quite right here. Why is your central stratum always the same? Perestroika always starts at the center and then propagates to the periphery."

Askold Khovanskii and Alexander Varchenko

Arnold's Seminar, First Years

In 1965–66, V. I. Arnold was a postdoc in Paris, lecturing on hydrodynamics and attending R. Thom's seminar on singularities. After returning to Moscow, Vladimir Igorevich started his seminar, meeting on Tuesdays from 4 to 6 p.m. It continued until his death on June 3 of 2010. We became Arnold's students in 1966 and 1968, respectively. The seminar was an essential part of our life. Among the first participants were R. Bogdanov, N. Brushlinskaya, I. Dolgachev, D. Fuchs, A. Gabrielov, S. Gusein-Zade, A. Kushnirenko, A. Leontovich, O. Lyashko, N. Nekhoroshev, V. Palamodov, A. Tyurin, G. Tyurina, V. Zakalyukin, and S. Zdravkovska.

V. I. Arnold had numerous interesting ideas, and to realize his plans he needed enthusiastic colleagues and collaborators. Every semester he started the seminar with a new list of problems and comments. Everyone wanted to be involved in this lively creative process. Many problems were solved, new theories were developed, and new mathematicians were emerging.

Here we will briefly describe some of the topics of the seminar in its first years, as well as the ski outings which were an integral part of the seminar.

Hilbert's 13th Problem and Arrangements of Hyperplanes

An algebraic function $x = x(a_1, \dots, a_k)$ is a multivalued function defined by an equation of the form

$$x^n + P_1(a_1, \dots, a_k)x^{n-1} + \cdots + P_n(a_1, \dots, a_k) = 0$$

where P_i's are rational functions.

Hilbert's 13th Problem: Show that the function $x(a, b, c)$, defined by the equation

$$x^7 + x^3 + ax^2 + bx + c = 0,$$

cannot be represented by superpositions of continuous functions in two variables.

A. N. Kolmogorov and V. I. Arnold proved that in fact such a representation does exist [3], thus solving the problem negatively. Despite this result it is still believed that the representation is impossible if one considers the superpositions of (branches of) algebraic functions only.

Can an algebraic function be represented as a composition of radicals and arithmetic operations? Such a representation does exist if and only

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V. Arnold, Yu. Chekanov, V. Zakalyukin, and A. Khovanskii at Arnoldfest, Toronto, 1997.

if the Galois group of the equation over the field of its coefficients is solvable. Hence, the general algebraic function of degree $k \geq 5$, defined by the equation $a_0x^k + a_1x^{k-1} + \cdots + a_k = 0$, cannot be represented by radicals.

In 1963, while teaching gifted high school students at Moscow boarding school No. 18, founded by Kolmogorov, V. I. Arnold discovered a topological proof of the insolvability by radicals of the general algebraic equation of degree ≥ 5, a proof which does not rely on Galois theory. Arnold's lectures at the school were written down and published by V. B. Alekseev in [1].

V. I. Arnold often stressed that when establishing the insolvability of a mathematical problem, topological methods are the most powerful and those best suited to the task. Using such topological methods, V. I. Arnold proved the insolvability of a number of classical problems; see [18], [14]. Inspired by that approach, a topological Galois theory was developed later; see [28]. The topological Galois theory studies topological obstructions to the solvability of equations in finite terms. For example, it describes obstructions to the solvability of differential equations by quadratures.

The classical formula for the solution by radicals of the degree four equation does not define the roots of the equation only. It defines a 72-valued algebraic function. V. I. Arnold introduced the notion of an exact representation of an algebraic function by superpositions of algebraic functions in which all branches of algebraic functions are taken into account. He proved that the algebraic function of degree $k = 2^n$, defined by the equation $x^k + a_1x^{k-1} + \cdots + a_k = 0$, does not have an exact representation by superpositions of algebraic functions in $< k - 1$ variables; see [8] and the references therein. The proof is again topological and based on the characteristic classes of algebraic functions, introduced for that purpose. The characteristic classes are elements of the cohomology ring of the complement to the discriminant of an algebraic function. To prove that theorem V. I. Arnold calculated the cohomology ring of the pure braid group.

Consider the complement in $\mathbb{C}^k$ to the union of the diagonal hyperplanes,

$$U = \{\, y \in \mathbb{C}^k \mid y_i \neq y_j \text{ for all } i \neq j\}.$$

The cohomology ring $H^*(U, \mathbb{Z})$ is the cohomology ring of the pure braid group on k strings. The cohomology ring $H^*(U, \mathbb{Z})$ was described in [7]. Consider the ring $\mathcal{A}$ of differential forms on U generated by the 1-forms $w_{ij} = \frac{1}{2\pi i} d\log(y_i - y_j)$, $1 \leq i, j \leq k$, $i \neq j$. Then the relations $w_{ij} = w_{ji}$ and

$$w_{ij} \wedge w_{jk} + w_{jk} \wedge w_{ki} + w_{ki} \wedge w_{ij} = 0$$

are the defining relations of $\mathcal{A}$. Moreover, the map $\mathcal{A} \to H^*(U, \mathbb{Z})$, $\alpha \mapsto [\alpha]$, is an isomorphism.

This statement says that each cohomology class in $H^*(U, \mathbb{Z})$ can be represented as an exterior polynomial in w_{ij} with integer coefficients and the class is zero if and only if the polynomial is zero. As an application, V. I. Arnold calculated the Poincaré polynomial $P_D(t) = \sum_{i=0}^{k} \operatorname{rank} H^i(D)\, t^i$,

$$P_\Delta(t) = (1+t)(1+2t)\cdots(1+(n-1)t).$$

Arnold's paper [7] was the beginning of the modern theory of arrangements of hyperplanes; see, for example, the book by P. Orlik and H. Terao.

Real Algebraic Geometry

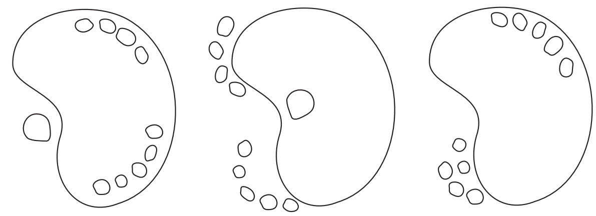
By Harnack's theorem, a real algebraic curve of degree n in the real projective plane can consist of at most $g+1$ ovals, where $g = (n-1)(n-2)/2$ is the genus of the curve. The M-curves are the curves for which this maximum is attained. For example, an M-curve of degree 6 has 11 ovals. Harnack proved that the M-curves exist.

If the curve is of even degree $n = 2k$, then each of its ovals has an interior (a disc) and an exterior (a Möbius strip). An oval is said to be positive if it lies inside an even number of other ovals and is said to be negative if it lies inside an odd number of other ovals. The ordinary circle, $x^2 + y^2 = 1$, is an example of a positive oval.

In his 16th problem, Hilbert asked how to describe the relative positions of the ovals in the plane. In particular, Hilbert conjectured that 11 ovals on an M-curve of degree 6 cannot lie external to one another. This fact was proved by Petrovsky in 1938 [33].

The first M-curve of degree 6 was constructed by Harnack, the second by Hilbert. It was believed for a long time that there were no other M-curves of degree 6. Only in the 1960s did Gudkov construct a third example and prove that there are only three types of M-curves of degree 6; see [26].

Experimental data led Gudkov to the following conjecture: If p and m are the numbers of positive and negative ovals of an M-curve of degree $2k$, then $p - m = k^2 \bmod 8$.



Harnack's, Hilbert's, and Gudkov's M-curves.

V. I. Arnold was a member of Gudkov's Doctor of Science thesis defense committee and became interested in these problems. V. I. Arnold related Gudkov's conjecture and theorems of divisibility by 16 in the topology of oriented closed four-dimensional manifolds developed by V. Rokhlin and others. Starting with an M-curve, V. I. Arnold constructed a four-dimensional manifold with an involution and using the divisibility theorems proved that $p - m = k^2 \bmod 4$; see [9]. Soon after that, V. A. Rokhlin, using Arnold's construction, proved Gudkov's conjecture in full generality.

This paper by V. I. Arnold began a revitalization of real algebraic geometry.

Petrovsky–Oleinik Inequalities

Petrovsky's paper [33] led to the discovery of remarkable estimates for the Euler characteristics of real algebraic sets, called Petrovsky-Oleinik inequalities. V. I. Arnold found in [10] unexpected generalizations of these inequalities and new proofs of the inequalities based on singularity theory.

Consider in $\mathbb{R}^{n+1}$ the differential one-form $\alpha = P_0 dx_0 + P_1 dx_1 + \cdots + P_n dx_n$, whose components are homogeneous polynomials of degree m. What are possible values of the index ind of the form α at the point $0 \in \mathbb{R}^{n+1}$?

Let us introduce Petrovsky's number $\Pi(n, m)$ as the number of integral points in the intersection of the cube $0 \leq x_0, \ldots, x_n \leq m-1$ and the hyperplane $x_0 + \cdots + x_n = (n+1)(m-1)/2$. V. I. Arnold proved in [10] that

$$|\mathrm{ind}| \leq \Pi(n, m) \qquad \text{and} \qquad \mathrm{ind} \equiv \Pi(n, m) \bmod 2.$$

His elegant proof of these relations is based on the Levin-Eisenbud-Khimshiashvili formula for the index of a singular point of a vector field.

Let P be a homogeneous polynomial of degree $m+1$ in homogeneous coordinates on $\mathbb{R}P^n$. Petrovsky-Oleinik inequalities give upper bounds for the following quantities:

a) $|\chi(P = 0) - 1|$ for odd n, where $\chi(P = 0)$ is the Euler characteristic of the hypersurface $P = 0$ in $\mathbb{R}P^n$, and

b) $|2\chi(P \leq 0) - 1|$ for even n and $m+1$, where $\chi(P \leq 0)$ is the Euler characteristic of the subset $P \leq 0$ in $\mathbb{R}P^n$.

V. I. Arnold noticed in [10] that in both cases a) and b) the estimated quantity equals the absolute value of the index at $0 \in \mathbb{R}^{n+1}$ of the gradient of P. Thus, the Petrovsky-Oleinik inequalities are particular cases of Arnold's inequalities for $\alpha = dP$.

Furthermore, Arnold's inequalities are exact (unlike the Petrovsky-Oleinik ones): for any integral value of ind with the properties $|\mathrm{ind}| \leq \Pi(n,m)$ and $\mathrm{ind} \equiv \Pi(n,m) \bmod 2$ there exists a homogeneous 1-form α (not necessarily exact) with this index (proved by Khovasnkii).

Critical Points of Functions

Critical points of functions was one of the main topics of the seminar in its first years. V. I. Arnold classified simple singularities of critical points in 1972, unimodal ones in 1973, and bimodal ones in 1975. Simple critical points form series A_n, D_n, E_6, E_7, E_8 in Arnold's classification. Already in his first papers V. I. Arnold indicated (sometimes without proofs) the connections of simple critical points with simple Lie algebras of the corresponding series. For example, the Dynkin diagram of the intersection form on vanishing cohomology at a simple singularity of an odd number of variables equals the Dynkin diagram of the corresponding Lie algebra, the monodromy group of the simple singularity equals the Weyl group of the Lie algebra, and the singularity index of the simple singularity equals $1/N$, where N is the Coxeter number of the Lie algebra.

One of the main problems of that time was to study characteristics of critical points. The methods were developed to calculate the intersection form on vanishing cohomology at a critical point (Gabrielov, Gusein-Zade), monodromy groups (Gabrielov, Gusein-Zade, Varchenko, Chmutov), and asymptotics of oscillatory integrals (Varchenko). The mixed Hodge structure on vanishing cohomology was introduced (Steenbrink, Varchenko), and the Hodge numbers of the mixed Hodge structure were calculated in terms of Newton polygons (Danilov, Khovanskii); see [12], [22] and the references therein.

The emergence of extensive new experimental data led to new discoveries. For example, according to Arnold's classification, the unimodal singularities form one infinite series $T_{p,q,r}$ and 14 exceptional families. Dolgachev discovered that the 14 exceptional unimodal singularities can be obtained from automorphic forms associated with the discrete groups of isometries of the Lobachevsky plane generated by reflections at the sides of some 14 triangles [23]. For the angles $\pi/p, \pi/q, \pi/r$ of such a triangle, the numbers p, q, r are integers, called Dolgachev's triple. According to Gabrielov [25], the intersection form on vanishing cohomology at an exceptional unimodal singularity is described by another triple of integers, called Gabrielov's triple. V. I. Arnold noticed that Gabrielov's triple of an exceptional unimodal singularity equals Dolgachev's triple of (in general) another exceptional unimodal singularity, while Gabrielov's triple of that other singularity equals Dolgachev's triple of the initial singularity. Thus, there is an involution on the set of 14 exceptional unimodal singularities, called Arnold's strange duality. Much later, after discovery of the mirror symmetry phenomenon, it was realized that Arnold's strange duality is one of its first examples.

Newton Polygons

While classifying critical point of functions, Arnold noticed that, for all critical points of his classification, the Milnor number of the critical point can be expressed in terms of the Newton polygon of the Taylor series of that critical point. Moreover, an essential part of Arnold's classification was based on the choice of the coordinate system simplifying the Newton polygon of the corresponding Taylor series. (According to Arnold, he used "Newton's method of a moving ruler (line, plane)".) V. I. Arnold formulated a general principle: in the family of all critical points with the same Newton polygon, discrete characteristics of a typical critical point (the Milnor number, singularity index, Hodge numbers of vanishing cohomology, and so on) can be described in terms of the Newton polygon.

That statement was the beginning of the theory of Newton polygons. Newton polygons were one of the permanent topics of the seminar. The first result, the formula for the Milnor number in terms of the Newton polygon, was obtained by Kouchnirenko in [30]. After Kouchnirenko's report at Arnold's seminar, Lyashko formulated a conjecture that a similar statement must hold in the global situation: the number of solutions of a generic system of polynomial equations in n variables with a given Newton polygon must be equal to the volume of the Newton polygon multiplied by $n!$. Kouchnirenko himself proved this conjecture. David Bernstein [21] generalized the statement of Kouchnirenko's theorem to the case of polynomial equations with different Newton polygons and found a simple proof of his generalization. Khovanskii discovered the connection of Newton polygons with the theory of toric varieties and using this connection calculated numerous characteristics of local and global complete intersections in terms of Newton polygons; see [11] and the references therein. Varchenko calculated the zeta-function of the monodromy and asymptotics of oscillatory integrals in terms of Newton polygons; see [12].

Nowadays Newton polygons are a working tool in many fields. Newton polygons appear in real and complex analysis, representation theory, and real algebraic geometry; and the Newton polygons provide examples of mirror symmetry and so on.

Skiing and Swimming

Every year at the end of the winter Arnold's seminar went to ski on the outskirts of Moscow. This tradition started in 1973. While the number of seminar participants was between twenty and thirty people, no more than ten of the bravest participants came out to ski. People prepared for this event the whole winter. The meeting was at 8 a.m. at the railway station in Kuntsevo, the western part of Moscow, and skiing went on until after sunset, around 6 p.m. The daily distance was about 50 km.

Usually Arnold ran in front of the chain of skiers, dressed only in swimming trunks. He ran at a speed a bit above the maximal possible speed of the slowest of the participants. As a result, the slowest participant became exhausted after an hour of such an outing and was sent back to Moscow on a bus at one of the crossroads. Then the entire process was repeated again and another participant was sent back to Moscow after another hour. Those who were able to finish the skiing were very proud of themselves.

Only one time was the skiing pattern different. In that year we were joined by Dmitri Borisovich Fuchs, a tall, unflappable man, who was at one time a serious mountain hiker. Early in the morning when Arnold started running away from the station with us, Dmitri Borisovich unhurriedly began to walk in the same direction. Soon he completely disappeared from our view, and Arnold stopped and began waiting impatiently for Fuchs to arrive. Arnold again rushed to run and Fuchs, again unperturbed, unhurriedly followed the group. So proceeded the entire day. That day none of the participants of the run were sent home in the middle of the day.

Several times we were joined by Olya Kravchenko and Nadya Shirokova, and every time they kept up the run as well as the best.

All participants of the ski-walk brought sandwiches, which they ate at a stop in the middle of the day. Before sandwiches there was bathing. In Moscow suburbs you will come across small rivers which are not frozen even in winter. We would meet at such a stream and bathe, lying on the bottom of the streambed as the water was usually only knee deep. We certainly did not use bathing suits, and there were no towels. The tradition of bathing in any open water at any time of the year Arnold had adopted from his teacher, Kolmogorov. This tradition was taken up by many participants of the seminar.

Summer expedition, 1960s.

Arnold thought that vigorous occupation with mathematics should be accompanied by vigorous physical exercise. He skied regularly in the winter (about 100 km per week), and in summer rode a bicycle and took long walks.

There is a funny story connected to the tradition of bathing in any available open water. In 1983 the Moscow mathematicians were taken out to the Mathematical Congress in Warsaw. This congress had been boycotted by Western mathematicians. The large Soviet delegation was supposed to compensate for the small number of Western participants. A special Moscow-Warsaw-Moscow train had been arranged, which delivered us to Poland with Arnold. Once, walking across Warsaw in the evening with Arnold, we arrived at a bridge across the Vistula. While on the bridge we decided to bathe, as required by tradition. We reached the water in total darkness and swam for a few minutes. In the morning we found, to our amazement, that we were floating more in mud than water.

Michael Berry

Memories of Vladimir Arnold

My first interaction with Vladimir Arnold was receiving one of his notoriously caustic letters. In 1976 I had sent him my paper (about caustics, indeed) applying the classification of singularities of gradient maps to a variety of phenomena in optics and quantum mechanics. In my innocence, I had called the paper "Waves and Thom's theorem". His reply began bluntly:

> Thank you for your paper. References:…

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There followed a long list of his papers he thought I should have referred to. After declaring that in his view René Thom (whom he admired) never proved or even announced the theorems underlying his catastrophe theory, he continued:

> I can't approve your system of referring to English translations where Russian papers exist. This has led to wrong attributions of results, the difference of 1 year being important—a translation delay is sometimes of 7 years…

and

> …theorems and publications are very important in our science (…at present one considers as a publication rather 2-3 words at Bures or Fine Hall tea, than a paper with proofs in a Russian periodical)

and (in 1981)

> I hope you'll not attribute these result [sic] to epigons.

He liked to quote Isaac Newton, often in scribbled marginal afterthoughts in his letters:

> A man must either resolve to put out nothing new, or to become a slave to defend it

and (probably referring to Hooke)

> Mathematicians that find out, settle and do all the business must content themselves with being nothing but dry calculators and drudges and another that does nothing but pretend and grasp at all things must carry away all the invention as well of those that were to follow him as of those that went before.

(I would not accuse Vladimir Arnold of comparing himself with Newton, but was flattered to be associated with Hooke, even by implication.)

I was not his only target. To my colleague John Nye, who had politely written "I have much admired your work…," he responded:

> I understand well your letter, your admiration have not led neither to read the [reference to a paper] nor to send reprints….

This abrasive tone obviously reflected a tough and uncompromising character, but I was never offended by it. From the beginning, I recognized an underlying warm and generous personality, and this was confirmed when I finally met him in the late 1980s. His robust correspondence arose from what he regarded as systematic neglect by Western scientists of Russian papers in which their results had been anticipated. In this he was sometimes right and sometimes not. And he was unconvinced by my response that scientific papers can legitimately be cited to direct readers to the most accessible and readable source of a result rather than to recognize priority with the hard-to-find original publication.

He never lost his ironic edge. In Bristol, when asked his opinion of perestroika, he declared: "Maybe the fourth derivative is positive." And at a meeting in Paris in 1992, when I found, in my conference mailbox, a reprint on which he had written: "to Michael Berry, admiringly," I swelled with pride—until I noticed, a moment later, that every other participant's mailbox contained the same reprint, with its analogous dedication!

In 1999, when I wrote to him after his accident, he replied (I preserve his inimitable style):

> …from the POINCARÉ hospital…the French doctors insisted that I shall recover for the following arguments: 1) Russians are 2 times stronger and any French would already die. 2) This particular person has a special optimism and 3) his humour sense is specially a positive thing: even unable to recognize you, he is laughing…. I do not believe this story, because it would imply a slaughtering of her husband for Elia, while I am still alive.

(Elia is Arnold's widow.)

There are mathematicians whose work has greatly influenced physics but whose writings are hard to understand; for example, I find Hamilton's papers unreadable. Not so with Arnold's: through his pellucid expositions, several generations of physicists came to appreciate the significance of pure mathematical notions that we previously regarded as irrelevant. "Arnold's cat" made us aware of the importance of mappings as models for dynamical chaos. And the exceptional tori that do not persist under perturbation (as Kolmogorov, Arnold, and Moser showed that most do) made us aware of Diophantine approximation in number theory: "resonant torus" to a physicist = "rational number" to a mathematician.

Most importantly, Arnold's writings were one of the two routes by which, in the 1970s, the notion of genericity slipped quietly into physics (the other route was critical phenomena in statistical mechanics, where it was called universality). Genericity emphasizes phenomena that are typical rather than the special cases (often with high symmetry) corresponding to exact solutions of the governing equations in terms of special functions. (And I distinguish genericity from abstract generality, which can often degenerate into what Michael Atiyah has called "general nonsense".)

This resulted in a shift in our thinking whose significance cannot be overemphasized.

It suddenly occurs to me that in at least four respects Arnold was the mathematical counterpart of Richard Feynman. Like Feynman, Arnold made massive original contributions in his field, with enormous influence outside it; he was a master expositor, an inspiring teacher bringing new ideas to new and wide audiences; he was uncompromisingly direct and utterly honest; and he was a colorful character, bubbling with mischief, endlessly surprising.

Acknowledgments

The photographs are courtesy of the Arnold family archive, F. Aicardi, Ya. Eliashberg, E. Ferrand, B. and M. Khesin, J. Pöschel, M. Ratner, S. Tretyakova, and I. Zakharevich.

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About the Cover

A mathematician's dinner table conversation

It is often said that mathematicians are eager to draw on napkins, but in our experience it is actually rather rare. The doodles on the cover are by the late mathematician Vladimir Arnold, the subject of articles in this and the next issue of the *Notices*. Emmanuel Ferrand saved the napkin and took the photograph. He writes,

"Arnold wrote on this napkin at the occasion of a private meal at the Institut des Hautes Études Scientifiques near Paris. As far as I remember, it was early in 2006, and this is consistent with the problems discussed on this napkin. Most of what is written here is related to the question of the enumeration of the topological types of Morse functions on surfaces. The two drawings with the letters A, B, C correspond to two of his favorite examples, the height functions of Stromboli and Etna, two famous volcanos in Italy. These are specifically referred to in Arnold's papers on the classification of Morse functions, and some of the problems he posed have been solved—some, for example, by Liviu Nicolaescu.

"It is important to note that Arnold's problems are one of his major contributions to mathematics. He wrote a list of about twenty open problems every year. He was not interested in leaving problems for the next generation like the Poincaré conjecture or the Riemann Hypothesis. On the contrary, the 'half life' of his problems was about five years. He also insisted not to take his questions too literally, but to consider them as a loose direction of research instead. He was not that much interested in questions whose answer is only 'yes' or 'no'. His seminars in Moscow and in Paris were structured around this problem list. Most of his problems were collected in the book *Arnold's Problems* from Springer-Verlag. There are still many interesting open questions waiting for the curious mathematician here!"

As anyone familiar with Arnold's writings knows, his line drawings, although not exactly requiring high technology to produce, were a charming and important part of his exposition. As the article in the next issue of the *Notices* points out, he insisted that figures play an important role in all of his students' work, as well.

—Bill Casselman
Graphics editor
(`notices-covers@ams.org`)

Interview with John Milnor

Martin Raussen and Christian Skau

John W. Milnor is the recipient of the 2011 Abel Prize of the Norwegian Academy of Science and Letters. This interview, conducted by Martin Raussen and Christian Skau, took place in Oslo in conjunction with the Abel Prize celebration on May 25, 2011, and originally appeared in the September 2011 issue of the *Newsletter of the European Mathematical Society*.

Raussen & Skau: *Professor John Milnor, on behalf of the Norwegian and Danish Mathematical Societies, we would like to congratulate you for being selected as the Abel Prize Laureate in 2011.*

Milnor: Thank you very much.

Student at Princeton University

R & S: *What kindled your interest in mathematics and when did you discover that you had an extraordinary aptitude for mathematics?*

Milnor: I can place that quite clearly. The first time that I developed a particular interest in mathematics was as a freshman at Princeton University. I had been rather socially maladjusted and did not have too many friends, but when I came to Princeton, I found myself very much at home in the atmosphere of the mathematics common room. People were chatting about mathematics, playing games, and one could come by at any time and just relax. I found the lectures very interesting. I felt more at home there than I ever had before and I have stayed with mathematics ever since.

R & S: *You were named a Putnam Fellow as one of the top scorers of the Putnam competition in mathematics in 1949 and 1950. Did you like solving mathematics problems and puzzles?*

Milnor: I think I always approached mathematics as interesting problems to be solved, so I certainly found that congenial.

R & S: *Your first important paper was accepted already in 1949 and published in 1950 in the prestigious journal* Annals of Mathematics*. You were only 18 years of age at the time and this is rather exceptional. The title of the paper was "On the Total Curvature of Knots". Could you tell us how you got the idea for that paper?*

Milnor: I was taking a course in differential geometry under Albert Tucker. We learned that Werner Fenchel, and later Karol Borsuk, had proved the following statement: the total curvature of a closed curve in space is always at least 2π, with equality only if the curve bounds a convex subset of some plane. Borsuk, a famous Polish topologist, had asked what one could say about total curvature if the curve was knotted? I thought about this for a few days and came up with a proof that the total curvature is always greater than 4π. (I think I did a poor job explaining the proof in the published paper, but one has to learn how to explain mathematics.) The Hungarian mathematician István Fáry had produced a similar proof at more or less the same time, but this was still a wonderful introduction to mathematics.

R & S: *That was quite an achievement! When you started your studies at Princeton in 1948, you met John Nash, three years your senior, who was a Ph.D. student. John Nash is well known through the book and movie* A Beautiful Mind*. Did you have any interaction with him? And how was it to be a Princeton student?*

Milnor: As I said, I spent a great deal of time in the common room, and so did Nash. He was a very interesting character and full of ideas. He also used to wander in the corridors whistling things like Bach, which I had never really heard before—a strange way to be introduced to classical music!

I saw quite a bit of him over those years and I also became interested in game theory, in which he was an important contributor. He was a very interesting person.

R & S: *At Princeton, you played Kriegspiel, Go, and a game called Nash?*

Milnor: That is true. Kriegspiel is a game of chess in which the two players are back-to-back and do not see each other's boards. There is a referee who tells whether the moves are legal or not. It is very easy for the referee to make a mistake, and it often happened that we could not finish because he got confused. In that case we said that the referee won the game! It was a marvelous game.

The game of Go was also very popular there. My first professor, Ralph Fox, was an expert in Go. So I learned something of it from him and also from

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many other people who played. The game that we called Nash had actually been developed earlier in Denmark by Piet Hein, but Nash invented it independently. This game, also called Hex, is based on topology. It is very interesting from a mathematical point of view. It is not hard to prove that the first player will always win if he plays correctly, but there is no constructive proof. In fact, when you play, it often happens that the first player does not win.

***R & S:** You even published some papers on game theory with John Nash?*

Milnor: We often talked about game theory, but there was only one joint paper. Together with C. Kalish and E. D. Nering, we carried out an experiment with a group of people playing a many-person game. This experiment convinced me that many-person game theory is not just a subject of mathematics. It is also about social interactions and things far beyond mathematics, so I lost my enthusiasm for studying it mathematically.

One paper written on my own described a theoretical model for the game of Go. This was further developed by Olof Hanner and much later by Berlekamp and Wolfe. (John Conway's construction of "surreal numbers" is closely related.)

Knot Theory

***R & S:** You wrote your Ph.D. thesis under the supervision of Ralph Fox; the title of the thesis was "Isotopy of links". Did you get the idea to work on this topic yourself? And what was the impact of this work?*

Milnor: Fox was an expert in knot theory, so I learned a great deal about knots and links from him. There were many people in the department then that were active in this area, although there were also other people at the department that considered it a low-class subject and not very interesting. I think it's strange that, although it wasn't considered a very central subject then, today it's a subject which is very much alive and active.

As one example, I often saw a quiet Greek gentleman Christos Papakyriakopoulus around the common room, but I never got to know him very well. I had no idea he was doing important work, but Fox had managed to find money to support him for many years, while he did research more or less by himself. He finally succeeded in solving a very important problem in knot theory which, perhaps, was the beginning of a rebirth of the study of three-dimensional manifolds as a serious part of mathematics. A paper in 1910 by Max Dehn had claimed to prove a simple property about knots. Essentially it said that if the fundamental group of the complement of a knot is cyclic, then the knot can be unknotted. This proof by Max Dehn had been accepted for almost twenty years until Hellmuth Kneser in 1929 pointed out there was a big gap in the argument. This remained a famous unsolved problem until 1957, when Papakyriakopoulus developed completely new methods and managed to give a proof of "Dehn's Lemma" and related theorems.

Photo by Knut Falch.

John Milnor

That was a big step in mathematics and an example of a case in which someone working in isolation made tremendous progress. There are relatively few examples of that. Andrew Wiles's proof of Fermat's last theorem is also an example of someone who had been working by himself and surprised everyone when he came up with the proof. Another example is Grigori Perelman in Russia who was working very much by himself and produced a proof of the Poincaré hypothesis. These are isolated examples. Usually mathematicians work in a much more social context, communicating ideas to each other. In fact, ideas often travel from country to country very rapidly. We are very fortunate that mathematics is usually totally divorced from political situations. Even at the height of the Cold War, we received information from the Soviet Union and people in the Soviet Union were eagerly reading papers from outside. Mathematics was much more open than most scientific subjects.

***R & S:** As a footnote to what you said: Max Dehn was a student of David Hilbert and he solved Hilbert's third problem about three-dimensional polyhedra of equal volume, showing that you cannot always split them up into congruent polyhedra. No wonder people trusted his proof because of his name.*

Milnor: It's a cautionary tale because we tend to believe in mathematics that when something is proved, it stays proved. Cases like Dehn's Lemma, where a false proof was accepted for many years, are very rare.

Manifolds

***R & S:** For several years after your Ph.D. your research concentrated on the theory of manifolds. Could you explain what a manifold is and why manifolds are important?*

Milnor: In low dimensions manifolds are things that are easily visualized. A curve in space is an example of a one-dimensional manifold; the surfaces of a sphere and of a doughnut are examples of two-dimensional manifolds. But for mathematicians the dimensions one and two are just the beginning; things get more interesting in higher dimensions. Also, for physicists manifolds are very important, and it is essential for them to look at higher-dimensional examples.

For example, suppose you study the motion of an airplane. To describe just the position takes three coordinates, but then you want to describe what direction it is going in, the angle of its wings, and so on. It takes three coordinates to describe the point in space where the plane is centered and three more coordinates to describe its orientation, so already you are in a six-dimensional space. As the plane is moving, you have a path in six-dimensional space, and this is only the beginning of the theory. If you study the motion of the particles in a gas, there are enormously many particles bouncing around, and each one has three coordinates describing its position and three coordinates describing its velocity, so a system of a thousand particles will have six thousand coordinates. Of course, much larger numbers occur, so mathematicians and physicists are used to working in large-dimensional spaces.

R & S: *The one result that made you immediately famous at age twenty-five was the discovery of different exotic structures on the seven-dimensional sphere. You exhibited smooth manifolds that are topologically equivalent to a seven-dimensional sphere but not smoothly equivalent, in a differentiable sense. Would you explain this result and also describe to us how you came up with the idea?*

Milnor: It was a complete accident, and certainly startled me. I had been working on a project of understanding different kinds of manifolds from a topological point of view. In particular, I was looking at some examples of seven-dimensional manifolds which were constructed by a simple and well-understood construction. They were explicit smooth objects which I would have thought were well understood, but looking at them from two different points of view, I seemed to find a complete contradiction. One argument showed that these manifolds were topological spheres and another very different argument showed that they couldn't be spheres.

Mathematicians get very unhappy when they have apparently good proofs of two contradictory statements. It's something that should never happen. The only way I could get out of this dilemma was by assuming there was an essential difference between the concept of a topological sphere (homeomorphic to the standard sphere) and the concept of a differentiable sphere (diffeomorphic to the standard sphere). This was something which hadn't been expected, and I am not aware that anybody had explicitly asked the question; we just assumed the answer was obvious. For some purposes one assumed only the topology and for other purposes one assumed the differentiable structure; but no one had really considered the possibility that there was a real difference. This result awakened a great deal of interest and a need for further research to understand exactly what was going on.

R & S: *You were certainly the driving force in this research area and you applied techniques both from differential geometry and topology, and also from algebraic topology, to shed new light on manifolds. It is probably fair to say that the work of European mathematicians, and especially French mathematicians like René Thom and Jean-Pierre Serre, who, by the way, received the first Abel Prize in 2003, made very fundamental contributions and made your approach possible. How did the collaboration over the Atlantic work at the time?*

Milnor: It was very easy to travel back and forth and I found French mathematicians very welcoming. I spent a great deal of time at the IHES [Institut des Hautes Études Scientifiques] near Paris. I hardly knew Serre (until much later), but I admired him tremendously, and still do. His work has had an enormous influence.

René Thom I got to know much better. He was really marvelous. He had an amazing ability to combine geometric arguments with hard algebraic topology to come up with very surprising conclusions. I was a great admirer of Thom and found he was also extremely friendly.

R & S: *Building on the work of, among others, Frank Adams from Britain and Stephen Smale from the United States, you, together with the French mathematician Michel Kervaire, were able to complete, to a certain extent, the classification of exotic structures on spheres. There are still some open questions concerning the stable homotopy of spheres, but at least up to those, we know what differentiable structures can be found on spheres.*

Milnor: That's true, except for very major difficulties in dimension four and a few problems in high dimensions (notably, the still unsolved "Kervaire Problem" in dimension 126). There are very classical arguments that work in dimensions one and two. Dimension three is already much more difficult, but the work of Bill Thurston and Grisha Perelman has more or less solved that problem. It was a tremendous surprise when we found, in the 1960s, that high dimensions were easier to work with than low dimensions. Once you get to a high enough dimension, you have enough room to move around so that arguments become much simpler. In many cases, one can make such arguments work even in dimension five, but dimension four is something else again and very difficult: neither high-dimensional methods nor low-dimensional methods work.

R & S: *One seems to need much harder pure analysis to work in dimensions three and four.*

Milnor: Well, yes and no. Michael Freedman first proved the topological Poincaré hypothesis in dimension four, and that was the very opposite of analysis. It was completely by methods of using very wild topological structures with no differentiability. But the real breakthrough in understanding differential 4-manifolds was completely based on methods from mathematical physics: methods of gauge theory and later Seiberg-Witten theory. Although motivated by mathematical physics, these tools turned out to be enormously useful in pure mathematics.

R & S: *Terminology in manifold theory is graphic and down to earth. Some techniques are known as "plumbing". Also "surgery" has become a real industry in mathematics, and you have written a* *paper on "killing", but of course just homotopy groups. May we ask to what extent you are responsible for this terminology?*

Milnor: To tell the truth, I'm not sure. I probably introduced the term "surgery", meaning cutting up manifolds and gluing them together in a different way (the term "spherical modification" is sometimes used for the same thing). Much later, the idea of quasiconformal surgery played an important role in holomorphic dynamics.

Simple graphic terminology can be very useful, but there are some words that get used so much that one loses track of what they mean (and they may also change their meaning over the years). Words like "regular" or "smooth" are very dangerous. There are very many important concepts in mathematics, and it is important to have a terminology which makes it clear exactly what you are talking about. The use of proper names can be very useful because there are so many possible proper names. An appropriate proper name attached to a concept often pins it down more clearly than the use of everyday words. Terminology is very important; it can have a very good influence if it's successfully used and can be very confusing if badly used.

R & S: *Another surprising result from your hand was a counterexample to the so-called Hauptvermutung , the "main conjecture" in combinatorial topology, dating back to Steinitz and Tietze in 1908. It is concerned with triangulated manifolds or, more generally, triangulated spaces. Could you explain what you proved at the time?*

Milnor: One of the important developments in topology in the early part of the twentieth century was the concept of homology, and later cohomology. In some form, they were already introduced in the nineteenth century, but there was a real problem making precise definitions. To make sense of them, people started by cutting a topological space up into linear pieces called simplexes. It was relatively easy to prove that homology was well defined on that level, and well behaved if you cut the simplexes into smaller ones, so the natural conjecture was that you really were doing topology when you defined things this way. If two simplicial complexes are homeomorphic to each other, then you should be able to cut them up into pieces that corresponded to each other. This was the first attempt to prove that homology was topologically invariant, but nobody could quite make it work. Soon they developed better methods and got around the problem. But the old problem of the Hauptvermutung, showing that you could always find isomorphic subdivisions, remained open.

I ran into an example where you could prove that it could not work. This was a rather pathological example, not about manifolds; but about ten years later, counterexamples were found even for nicely triangulated manifolds. A number of people worked on this, but the ones who finally built a really satisfactory theory were Rob Kirby and my student Larry Siebenmann.

R & S: *Over a long period of years after your thesis work, you published a paper almost every year, sometimes even several papers, that are known as landmark papers. They determined the direction of topology for many years ahead. This includes, apart from the themes we have already talked about, topics in knot theory, three-dimensional manifolds, singularities of complex hypersurfaces, Milnor fibrations, Milnor numbers, complex cobordism, and so on. There are also papers of a more algebraic flavor. Are there any particular papers or particular results you are most fond or proud of?*

Milnor: It's very hard for me to answer; I tend to concentrate on one subject at a time so that it takes some effort to remember precisely what I have done earlier.

Geometry, Topology, and Algebra

R & S: *Mathematics is traditionally divided into algebra, analysis, and geometry/topology. It is probably fair to say that your most spectacular results belong to geometry and topology. Can you tell us about your working style and your intuition? Do you think geometrically, so to speak? Is visualization important for you?*

Milnor: Very important! I definitely have a visual mind, so it's very hard for me to carry on a mathematical conversation without seeing anything written down.

R & S: *On the other hand, it seems to be a general feature, at least when you move into higher-dimensional topology, that real understanding arises when you find a suitable algebraic framework which allows you to formulate what you are thinking about.*

Milnor: We often think by analogies. We have pictures in small dimensions and must try to decide how much of the picture remains accurate in higher dimensions and how much has to change. This visualization is very different from just manipulating a string of symbols.

R & S: *Certainly, you have worked very hard on algebraic aspects of topology and also algebraic questions on their own. While you developed manifold theory, you wrote, at the same time, papers on Steenrod algebras, Hopf algebras, and so on. It seems to us that you have an algebraic mind as well?*

Milnor: One thing leads to another. If the answer to a purely topological problem clearly requires algebra, then you are forced to learn some algebra. An example: in the study of manifolds one of the essential invariants—perhaps first studied by Henry Whitehead—was the quadratic form of a four-dimensional manifold, or more generally a $4k$-dimensional manifold. Trying to understand this, I had to look up the research on quadratic forms. I found this very difficult until I found a beautiful exposition by Jean-Pierre Serre which provided exactly what was needed. I then discovered that the theory of quadratic forms is an exciting field on its own. So just by following my nose, doing what came next, I started studying properties of quadratic forms. In these years, topological K-theory was also developed, for example by Michael Atiyah, and was very exciting. There were beginnings of algebraic analogs. Grothen-dieck was one of the first. Hyman Bass developed a theory of algebraic K-theory, and I pursued that a bit further and discovered that there were relations between the theory of quadratic forms and algebraic K-theory. John Tate was very useful at that point, helping me work out how these things corresponded.

R & S: *John Tate was last year's Abel Prize winner, by the way.*

Milnor: I made a very lucky guess at that point, conjecturing a general relationship between algebraic K-theory, quadratic forms, and Galois cohomology. I had very limited evidence for this, but it turned out to be true and much later was proved by Vladimir Voevodsky. It's very easy to make guesses, but it feels very good when they turn out to be correct.

R & S: *That's only one of the quite famous Milnor conjectures.*

Milnor: Well, I also had conjectures that turned out to be false.

R & S: *Algebraic K-theory is a topic you already mentioned, and we guess your interest in that came through Whitehead groups and Whitehead torsion related to K_1.*

Milnor: That is certainly true.

R & S: *It is quite obvious that this is instrumental in the theory of nonsimply connected manifolds through the s-cobordism theorem. That must have aroused your interest in general algebraic K-theory where you invented what is today called Milnor K-theory. Dan Quillen then came up with a competing or different version with a topological underpinning….*

Milnor: Topological K-theory worked in all dimensions, using Bott periodicity properties, so it seemed there should be a corresponding algebraic theory. Hyman Bass had worked out a complete theory for K_0 and K_1, and I found an algebraic version of K_2. Quillen, who died recently after a long illness, provided a satisfactory theory of K_n for all values of n. Quillen's K_2 was naturally isomorphic to my K_2, although our motivations and expositions were different. I did construct a rather ad hoc definition for the higher K_n. This was in no sense a substitute for the Quillen K-theory. However, it did turn out to be very useful for certain problems, so it has kept a separate identity.

R & S: *Giving rise to motivic cohomology, right?*

Milnor: Yes, but only in the sense that Voevodsky developed motivic cohomology in the process of proving conjectures which I had posed.

R & S: *You introduced the concept of the growth function for a finitely presented group in a paper from 1968. Then you proved that the fundamental group of a negatively curved Riemannian manifold has exponential growth. This paved the way for a spectacular development in modern geometric group theory and eventually led to Gromov's hyperbolic group theory. Gromov, by the way, received the Abel Prize two years ago. Could you tell us why you found this concept so important?*

Milnor: I have been very much interested in the relation between the topology and the geometry of a manifold. Some classical theorems were well known. For example, Preismann had proved that if the curvature of a complete manifold is strictly negative, then any Abelian subgroup of the fundamental group must be cyclic. The growth function seemed to be a simple property of groups which would reflect the geometry in the fundamental group. I wasn't the first to notice this. Albert Schwarz in Russia had done some similar work before me, but I was perhaps better known and got much more publicity for the concept.

I can bring in another former Abel Prize winner Jacques Tits, who proved what is now called the "Tits alternative" for finitely generated subgroups of algebraic groups. He proved that either there was a free subgroup or the group was virtually solvable. All the finitely generated groups I was able to construct had this property: either they contained a noncyclic free subgroup or else they contained a solvable subgroup of finite index. Such groups always have either polynomial growth or exponential growth. The problem of groups of intermediate growth remained unsolved for many years until Grigorchuk in Russia found examples of groups that had less than exponential growth but more than polynomial growth. It is always nice to ask interesting questions and find that people have interesting answers.

Dynamics

R & S: *We jump in time to the last thirty years in which you have worked extensively on real and complex dynamics. Roughly speaking, this is the study of iterates of a continuous or holomorphic function and the associated orbits and stability behavior. We are very curious to hear why you got interested in this area of mathematics.*

Milnor: I first got interested under the influence of Bill Thurston, who himself got interested from the work of Robert May in mathematical ecology. Consider an isolated population of insects where the numbers may vary from year to year. If there get to be too many of these insects, then they use up their resources and start to die off, but if there are very few, they will grow exponentially. So the curve which describes next year's population as a function of this year's will have positive slope if the population is small and negative slope if the population gets too big. This led to the study of dynamical properties of such "unimodal" functions. When you look at one year after another, you get a very chaotic looking set of population data. Bill Thurston had gotten very interested in this problem and explained some of his ideas to me. As frequently happened in my interactions with Bill, I first was very dubious and found it difficult to believe what he was telling me. He had a hard time convincing me, but finally we wrote a paper together explaining it.

R & S: *This was a seminal paper. The first version of this paper dates from around 1977. The manuscript circulated for many years before it was published in the* Springer Lecture Notes *in 1988. You introduced a new basic invariant that you called the "kneading matrix" and the associated "kneading determinant". You proved a marvelous theorem connecting the kneading determinant with the zeta function associated to the map, which counts the periodic orbits. Browsing through the paper, it seems to us that it must have been a delight to write it. Your enthusiasm shines through!*

Milnor: You said that the zeta function describes periodic orbits, which is true, but it omits a great deal of history. Zeta functions were first made famous by Riemann's zeta function (actually first studied by Euler). Zeta functions are important in number theory, but then people studying dynamics found that the same mathematical formalism was very useful for counting periodic orbits. The catalyst was André Weil, who studied an analog of the Riemann zeta function for curves over a finite field, constructed by counting periodic orbits of the Frobenius involution.

So there is a continuous history here from pure number theory, starting with Euler and Riemann and then André Weil, to problems in dynamics in which one studies iterated mappings and counts how many periodic orbits there are. This is typical of something that makes mathematicians very happy: techniques that are invented in one subject turn out to be useful in a completely different subject.

R & S: *You must have been surprised that the study of a continuous map from an interval into itself would lead to such deep results?*

Milnor: Well, it was certainly a very enjoyable subject.

R & S: *Your work with Bill Thurston has been compared to Poincaré's work on circle diffeomorphisms 100 years earlier which led to the qualitative theory of dynamical systems and had a tremendous impact on the subject.*

Use of Computers in Mathematics

R & S: *This leads to another question. There is a journal called* Experimental Mathematics. *The first volume appeared in 1992 and the first article was written by you. It dealt with iterates of a cubic polynomial. The article included quite a lot of computer graphics. You later published several papers in this journal. What is your view on computers in mathematics?*

Photo by Eirik F. Baardsen.

Abel interview, from left to right: Martin Raussen, Christian Skau, and John Milnor.

Milnor: I was fascinated by computers from the very beginning. At first one had to work with horrible punch cards. It was a great pain, but it has gotten easier and easier. Actually, the biggest impact of computers in mathematics has been just to make it easier to prepare manuscripts. I always have had a habit of rewriting over and over, so in the early days I drove the poor secretaries crazy. I would hand in messy longhand manuscripts. They would present a beautiful typescript. I would cross out this, change that, and so on. It was very hard on them. It has been so much easier since one can edit manuscripts on the computer.

Of course, computers also make it much easier to carry out numerical experiments. Such experiments are nothing new. Gauss carried out many numerical experiments, but it was very difficult at his time. Now it's so much easier. In particular, in studying a difficult dynamical system it can be very helpful to run the system (or perhaps a simplified model of it) on a computer. Hopefully this will yield an accurate result. But it is dangerous. It is very hard to be sure that round-off errors by the computer, or other computing errors, haven't produced a result which is not at all accurate. It becomes a kind of art to understand what the computer can do and what the limitations are, but it is enormously helpful. You can get an idea quickly of what you can expect from a dynamical system and then try to prove something about it using the computer result as an indication of what to expect. At least, that's in the best case. There's also the other case where all you can do is to obtain the computer results and hope that they are accurate.

R & S: *In a sense, this mathematical discipline resembles what the physicists do when they plan their experiments and when they draw conclusions from the results of their experiments….*

Milnor: There is also the intermediate stage of a computer-assisted proof where (at least if you believe there are no mistakes in the computer program or no faults in the hardware) you have a complete proof.

But the assumption that there are no mistakes is a very important one. Enrico Bombieri had an experience with this. He was using a fancy new high-speed computer to make experiments in number theory. He found that in some cases the result just seemed wrong. He traced it back and traced it back and finally found that there was a wiring mistake in the hardware!

R & S: *Do you have examples from your own experience where all experiments you have performed indicate that a certain conjecture must be true but you don't have a way to prove it in the end?*

Milnor: In my experience, computer experiments seldom indicate that something is definitely true. They often show only that any possible exception is very hard to find. If you verify a number-theoretical property for numbers less than 10^{10}, who knows what would happen for 10^{11}. In dynamics, there may be examples where the behavior changes very much as we go to higher dimensions. There is a fundamental dogma in dynamics, saying that we are not interested in events which happen with probability zero. But perhaps something happens with probability 10^{-10}. In that case, you will never see it on a computer.

Textbooks and Expository Articles

R & S: *You have written several textbooks which are legendary in the sense that they are lucid and lead the reader quickly to the point, seemingly in the shortest possible way. The topics of your books deal with differential topology, algebraic K-theory, characteristic classes, quadratic forms, and holomorphic dynamics. Your books are certainly enjoyable reading. Do you have a particular philosophy when you write mathematical textbooks?*

Milnor: I think most textbooks I have written have arisen because I have tried to understand a subject. I mentioned before that I have a very visual memory and the only way I can be convinced that I understand something is to write it down clearly enough so that I can really understand it. I think the clarity of writing, to the extent it exists, is because I am a slow learner and have to write down many details to be sure that I'm right and then keep revising until the argument is clear.

R & S: *Apart from your textbooks and your research contributions, you have written many superb expository and survey articles which are a delight to read for every mathematician, expert or nonexpert.*

Two questions come to mind. Do you enjoy writing articles of a historical survey type? You certainly have a knack for it. Do you think it is important that articles and books on mathematics of a popular and general nature are written by prominent mathematicians like yourself?

Milnor: The answer to your first question is certainly yes. Mathematics has a rich and interesting history. The answer to the second question is surely no. I don't care who writes an article or a book. The issue is: is it clearly written, correct, and useful.

R & S: *Are you interested in the history of mathematics also—following how ideas develop?*

Milnor: I certainly enjoy trying to track down just when and how the ideas that I work with originated. This is, of course, a very special kind of history, which may concentrate on obscure ideas which turned out to be important, while ignoring ideas which seemed much more important at the time. History to most scientists is the history of the ideas that worked. One tends to be rather bored by ideas that didn't work. A more complete history would describe how ideas develop and would be interested in the false leads also. In this sense, the history I would write is very biased, trying to find out where the important ideas we have today came from—who first discovered them. I find *that* an interesting subject. It can be very difficult to understand old papers because terminology changes. For example, if an article written 100 years ago describes a function as being "regular", it is hard to find out precisely what this means. It is always important to have definitions which are clearly written down so that, even if the terminology does change, people can still understand what you were saying.

R & S: *Is it also important to communicate that to a wider mathematics audience?*

Milnor: It is important to communicate to a wide audience what mathematics is and does. However, my own expositions have always been directed to readers who already have a strong interest in mathematics. In practice, I tend to write about what interests me, in the hope that others will also be interested.

Academic Work Places

R & S: *You started your career at Princeton University and you were on the staff for many years. After some intermediate stages in Los Angeles and at MIT, you went back to Princeton but now to the Institute for Advanced Study. Can you compare the Institute and the university and the connections between them?*

Milnor: They are alike in some ways. They have close connections; people go back and forth all the time. The big difference is that at the university you have continual contact with students, both in teaching and with the graduate students, and there is a fair amount of continuity since the students stay around, at least for a few years. The Institute is much more peaceful, with more opportunity for work and more idyllic circumstances, but there is a continually rotating population, so almost before you get to know people, the year is over and they move on. So it's unsatisfactory in that way. But they are both wonderful institutions and I was very happy at both.

R & S: *In the late 1980s you left for Stony Brook, to the State University of New York, where you got* *in contact with students again, as an academic teacher.*

Milnor: Yes, that was certainly one strong motivation. I felt that the Institute was a wonderful place to spend some years, but for me it was, perhaps, not a good place to spend my life. I was too isolated, in a way. I think the contact with young people and students and having more continuity was important to me, so I was happy to find a good position in Stony Brook.

There were also domestic reasons: my wife was at Stony Brook and commuting back and forth, which worked very well until our son got old enough to talk. Then he started complaining loudly about it.

R & S: *A colleague of mine and I had an interview with Atle Selberg in Princeton in 2005. He told us, incidentally, that he thought Milnor would never move from the Institute because his office was so messy that just to clean it up would take a tremendous effort. But you moved in the end….*

Milnor: I don't know if the office ever got cleaned up. I think it was moved into boxes and stored in our garage.

Development of Mathematics

R & S: *Are there any mathematicians that you have met personally during your lifetime who have made a special, deep impression on you?*

Milnor: There are many, of course. There were certainly the professors at Princeton. Ralph Fox, Norman Steenrod, and Emil Artin all made a strong impression on me. Henry Whitehead, I remember, invited a group of very young topologists to Oxford. This was a wonderful experience for me when I was young. I mentioned René Thom. More recently Adrien Douady was a very important influence. He was an amazing person, always full of life and willing to talk about any mathematical subject. If you had a question and emailed him, you would always get an answer back within a day or so. These are the names that occur most prominently to me.

R & S: *When we observe mathematics as a whole, it has changed during your lifetime. Mathematics has periods in which internal development is predominant and other periods where a lot of momentum comes more from other disciplines, like physics. What period are we in currently? What influences from the outside are important now and how would you judge future developments?*

Milnor: I think the big mystery is how the relation between mathematics and biology will develop.

R & S: *You mentioned ecology as an example.*

Milnor: Yes, but that was a discussion of a very simplified mathematical model. It's clear that most biological problems are so complex that you can never make a total mathematical model. This is part of the general problem in applied mathematics; most things that occur in the real world are very complicated. The art is to realize what the essential variables are, in order to construct a simplified model that can still say something about the actual more complex situation. There has recently been tremendous success in the understanding of large data sets (also in statistical analysis). This is not a kind of mathematics I have ever done but, nevertheless, it's very important. The question of what kind of mathematics will be useful in biology is still up in the air, I think.

Work Style

R & S: *You have proved many results that are described as breakthroughs by mathematicians all around. May we ask you to recall some of the instances when an idea struck you that all of a sudden solved a problem you had been working on? Did that rather occur when you had been working on it very intensely or did it often happen in a relaxed atmosphere?*

Milnor: Here is one scenario. After a lot of studying and worrying about a question, one night you go to sleep wondering what the answer is. When you wake up in the morning, you know the answer. That really can happen. The other more common possibility is that you sit at the desk working and finally something works out. Mathematical conversations are definitely very important. Talking to people, reading other people's work, and getting suggestions are usually very essential.

R & S: *Talking, very often, makes ideas more clear.*

Milnor: Yes, in both directions. If you are explaining something to someone else, it helps you understand it better. And certainly, if someone is explaining something to you, it can be very important.

R & S: *Is the way you do mathematics today any different from how you did mathematics when you were thirty or forty?*

Milnor: Probably, yes.

R & S: *How many hours per day do you work on mathematics?*

Milnor: I don't know. I work a few hours in the morning, take a nap, and then work a few hours in the afternoon. But it varies. When I was younger I probably worked longer hours.

R & S: *Do you subscribe to Hardy when he said that mathematics is a young man's game? You seem to be a counterexample!*

Milnor: What can you say? Whatever age, do the best you can!

R & S: *In an article around fifteen years ago, you described several areas in mathematics that you first had judged as of minor interest but which later on turned out to be fundamental in solving problems that you had been working on yourself. I think Michael Freedman's work was one of the examples you mentioned. Do you have more examples and is there a general moral?*

Milnor: I think that one of the joys about mathematics is that it doesn't take an enormous grant and an enormous machine to carry it out. One person working alone can still make a big contribution. There are many possible approaches to most questions, so I think it's a big mistake to have everything concentrated in a few areas. The idea of having many people working independently is actually very useful because it may be that the good idea comes from a totally unexpected direction. This has happened often. I am very much of the opinion that mathematics should not be directed from above. People must be able to follow their own ideas.

R & S: *This leads to a natural question: what is mathematics to you? What is the best part of being a mathematician?*

Milnor: It is trying to understand things, trying to explain them to yourself and to others, to interchange ideas and watch how other people develop new ideas. There is so much going on that no one person can understand all of it, but you can admire other people's work even if you don't follow it in detail. I find it an exciting world to be in.

R & S: *What's the worst part of being a mathematician, if there is any? Is competition part of it?*

Milnor: Competition can be very unpleasant if there are several people fighting for the same goal, especially if they don't like each other. If the pressure is too great and if the reward for being the successful one is too large, it distorts the situation. I think, in general, most mathematicians have a fair attitude. If two different groups produce more or less the same results at more or less the same time, one gives credit to everyone. I think it's unfortunate to put too much emphasis on priority. On the other hand, if one person gets an idea and other people claim credit for it, that becomes very unpleasant. I think the situation in mathematics is much milder than in other fields, like biology, where competition seems to be much more ferocious.

R & S: *Do you have the same interest in mathematics now as you had when you were young?*

Milnor: I think so, yes.

Prizes

R & S: *You received the Fields Medal back in 1962, particularly for your work on manifolds. This happened in Stockholm at the International Congress and you were only thirty-one years old. The Fields Medal is the most important prize given to mathematicians, at least to those under the age of forty. The Abel Prize is relatively new and allows us to honor mathematicians regardless of age. Receiving the Fields Medal almost fifty years ago, do you remember what you felt at the time? How did receiving the Fields Medal influence your academic career?*

Milnor: Well, as you say, it was very important. It was a recognition and I was certainly honored by it. It was a marvelous experience going to Stockholm and receiving it. The primary motive is to understand mathematics and to work out ideas. It's gratifying to receive such honors, but I am not sure it had a direct effect.

R & S: *Did you feel any extra pressure when you wrote papers after you received the Fields Medal?*

Milnor: No, I think I continued more or less as before.

R & S: *You have won a lot of prizes throughout your career: the Fields Medal, the Wolf Prize, and the three Steele Prizes given by the American Mathematical Society. And now you will receive the Abel Prize. What do you feel about getting this prize on top of all the other distinctions you have gotten already?*

Milnor: It is surely the most important one. It is always nice to be recognized for what you have done, but this is an especially gratifying occasion.

R & S: *What do you generally feel about prizes to scientists as a means of raising public awareness?*

Milnor: It is certainly very successful at that. I'm not sure I like getting so much attention, but it doesn't do me much harm. If this is a way of bringing attention to mathematics, I'm all in favor. The danger of large prizes is that they will lead to the situations I described in biology. The competition can become so intense that it becomes poisonous, but I hope that will never happen in mathematics.

Personal Interests

R & S: *Having talked about mathematics all the time, may we finish this interview by asking about other things you are interested in: your hobbies, etc?*

Milnor: I suppose I like to relax by reading science fiction or other silly novels. I certainly used to love mountain climbing, although I was never an expert. I have also enjoyed skiing. Again I was not an expert, but it was something I enjoyed doing…. I didn't manage it this winter but I hope I will be able to take up skiing again.

R & S: *What about literature or music?*

Milnor: I enjoy music but I don't have a refined musical ear or a talent for it. I certainly enjoy reading, although, as I said, I tend to read nonserious things for relaxation more than trying to read serious things. I find that working on mathematics is hard enough without trying to be an expert in everything else.

R & S: *We would like to thank you very much for this most interesting interview. This is, of course, on the behalf of the two of us but also on behalf of the Danish, Norwegian, and European Mathematical Societies. Thank you very much!*

2011-2012 Faculty Salaries Report

Richard Cleary, James W. Maxwell, and Colleen Rose

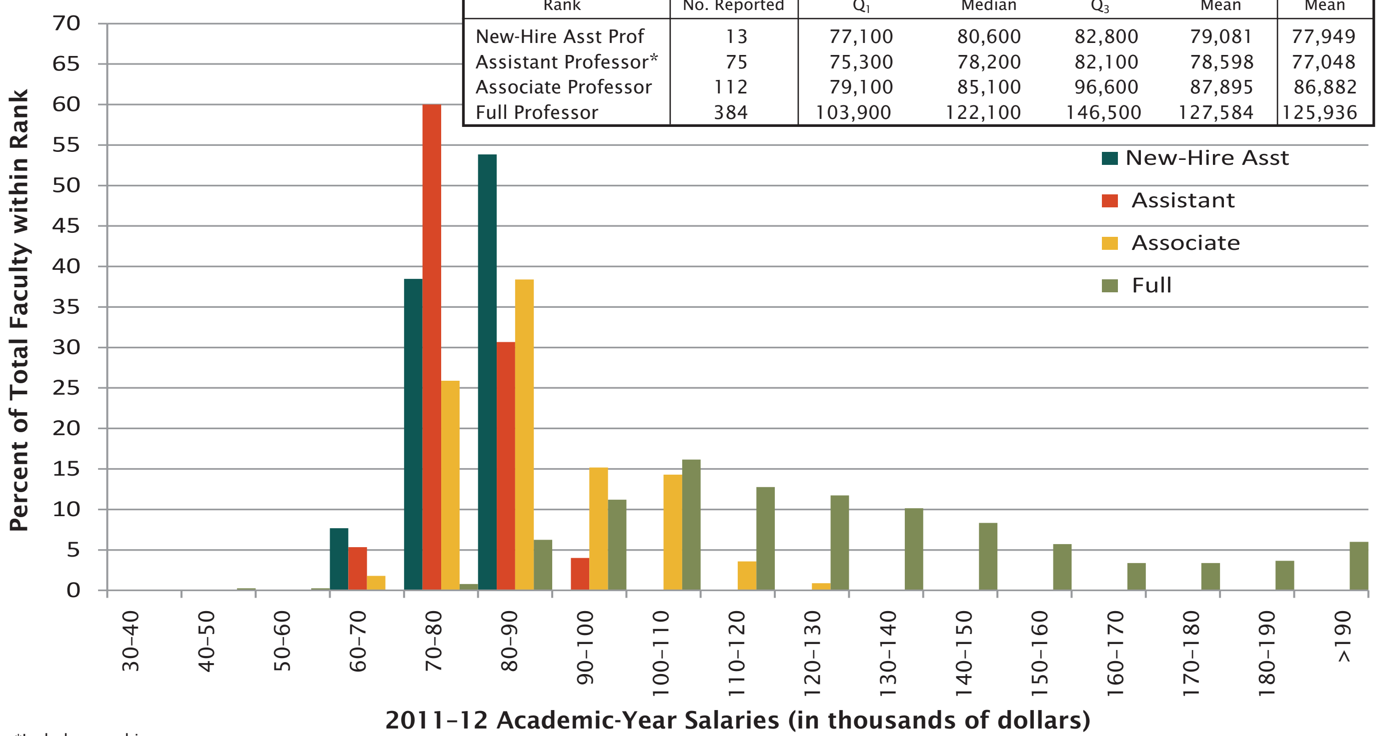
This report provides information on the distribution of 2011–2012 academic-year salaries for tenured and tenure-track faculty at four-year mathematical sciences departments in the U.S. by the departmental groupings used in the Annual Survey. (See page 415 for the definitions of the various departmental groupings.) Salaries are described separately by rank. Salaries are reported in current dollars (at time of data collection). Results reported here are based on the departments which responded to the survey with no adjustment for non-response.

Departments were asked to report for each rank the number of tenured and tenure-track faculty whose 2011–2012 academic-year salaries fell within given salary intervals. Reporting salary data in this fashion eliminates some of the concerns about confidentiality but does not permit determination of actual quartiles. The quartiles reported have been estimated assuming that the density over each interval is uniform.

When comparing current and prior year figures, one should keep in mind that differences in the set of responding departments may be one of the most important factors in the change in the reported mean salaries.

Group I (Public) Faculty Salaries
Doctoral degree-granting departments of mathematics
13 responses out of 25 departments (52%)

	2011–12					2010–11
Rank	No. Reported	Q_1	Median	Q_3	Mean	Mean
New-Hire Asst Prof	13	77,100	80,600	82,800	79,081	77,949
Assistant Professor*	75	75,300	78,200	82,100	78,598	77,048
Associate Professor	112	79,100	85,100	96,600	87,895	86,882
Full Professor	384	103,900	122,100	146,500	127,584	125,936



*Includes new hires.

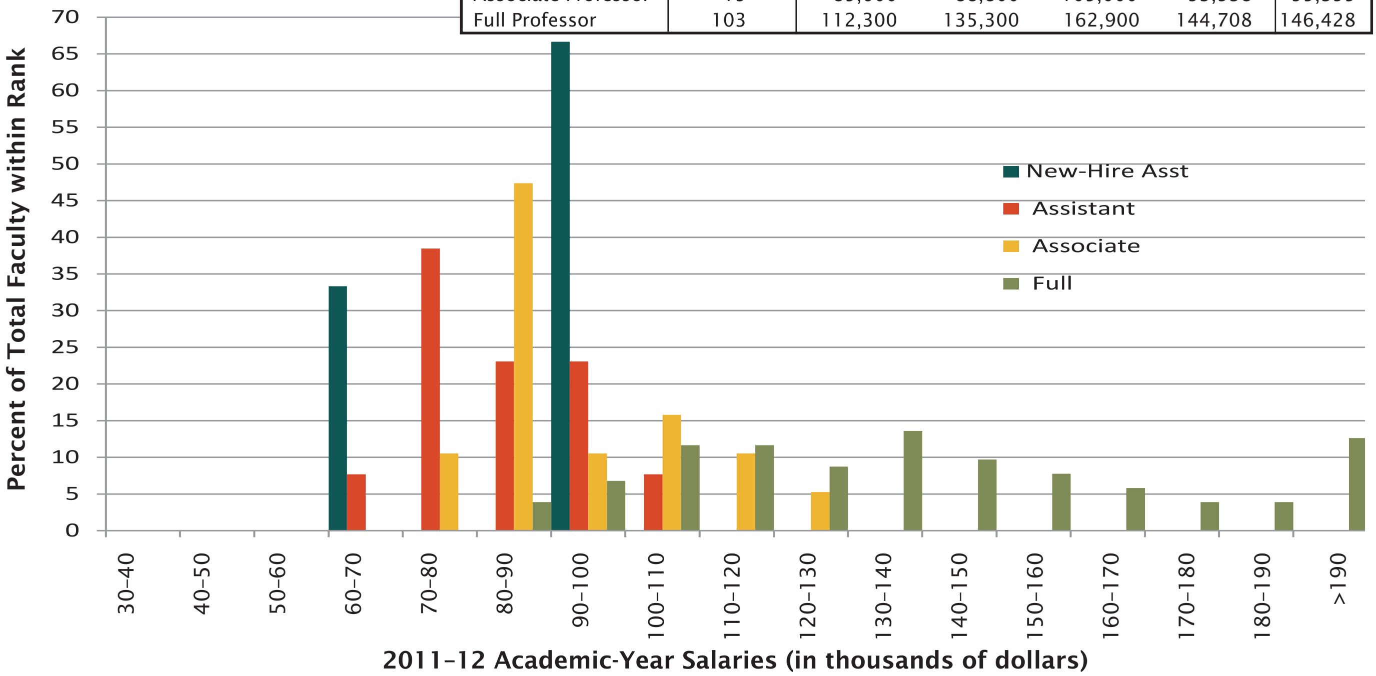
Richard Cleary is a professor in the Department of Mathematical Sciences at Bentley University. James W. Maxwell is AMS associate executive director for special projects. Colleen A. Rose is AMS survey analyst.

Group I (Private) Faculty Salaries

Doctoral degree-granting departments of mathematics

5 responses out of 23 departments (22%)

Rank	2011–12 No. Reported	2011–12 Q_1	2011–12 Median	2011–12 Q_3	2011–12 Mean	2010–11 Mean
New-Hire Asst Prof	3	Too few to report				
Assistant Professor*	13	76,500	81,700	91,900	83,528	76,571
Associate Professor	19	85,600	88,800	105,000	93,938	95,395
Full Professor	103	112,300	135,300	162,900	144,708	146,428

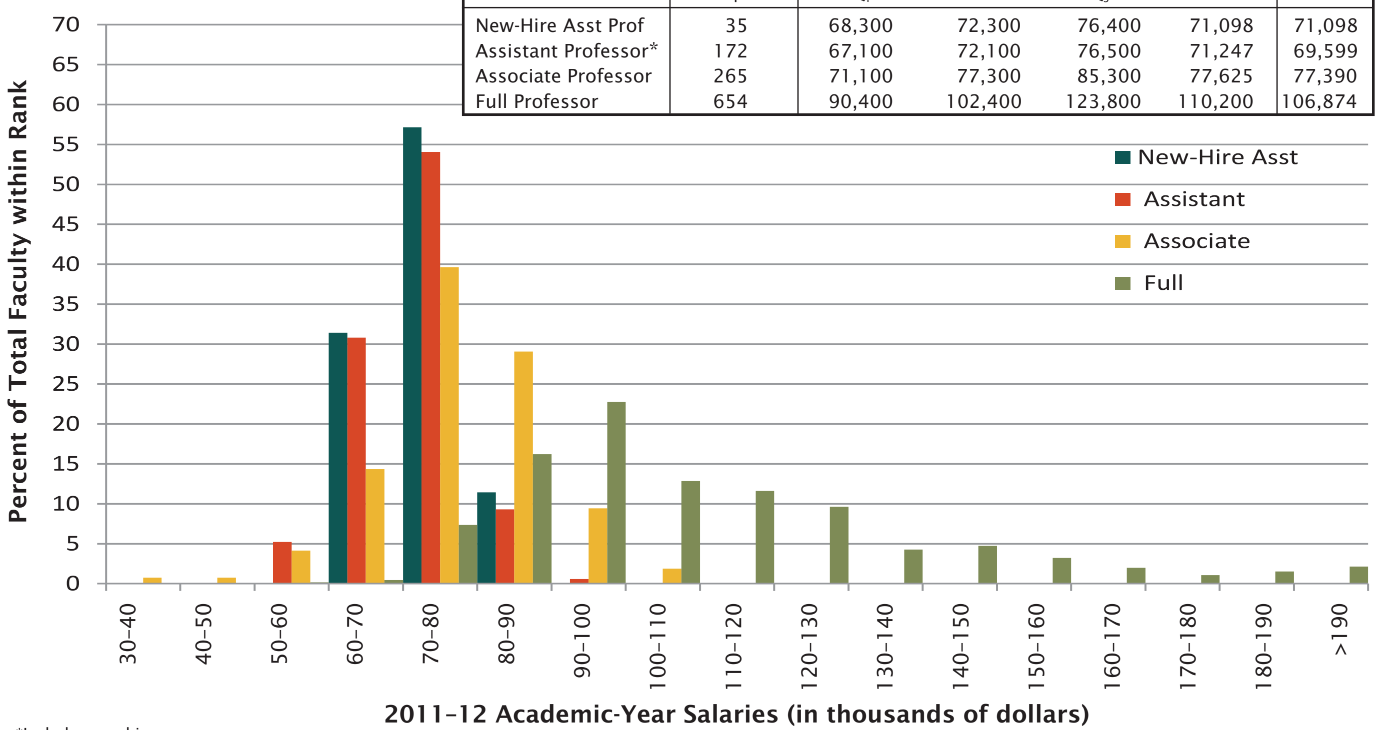


Group II Faculty Salaries

Doctoral degree-granting departments of mathematics

33 responses out of 56 departments (59%)

Rank	2011–12 No. Reported	2011–12 Q_1	2011–12 Median	2011–12 Q_3	2011–12 Mean	2010–11 Mean
New-Hire Asst Prof	35	68,300	72,300	76,400	71,098	71,098
Assistant Professor*	172	67,100	72,100	76,500	71,247	69,599
Associate Professor	265	71,100	77,300	85,300	77,625	77,390
Full Professor	654	90,400	102,400	123,800	110,200	106,874



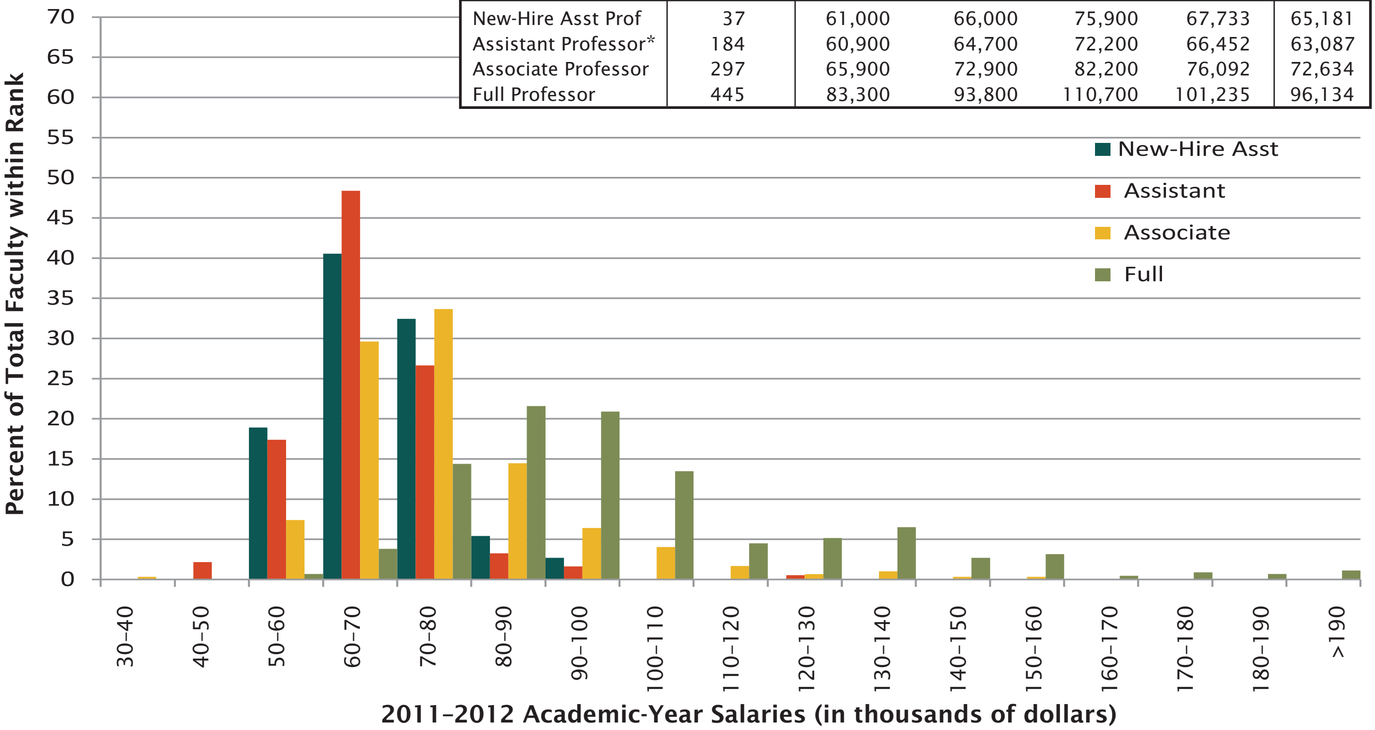
*Includes new hires.

Group III Faculty Salaries

Doctoral degree-granting departments of mathematics

43 responses out of 80 departments (54%)

Rank	2011–12					2010–11
	No. Reported	Q_1	Median	Q_3	Mean	Mean
New-Hire Asst Prof	37	61,000	66,000	75,900	67,733	65,181
Assistant Professor*	184	60,900	64,700	72,200	66,452	63,087
Associate Professor	297	65,900	72,900	82,200	76,092	72,634
Full Professor	445	83,300	93,800	110,700	101,235	96,134

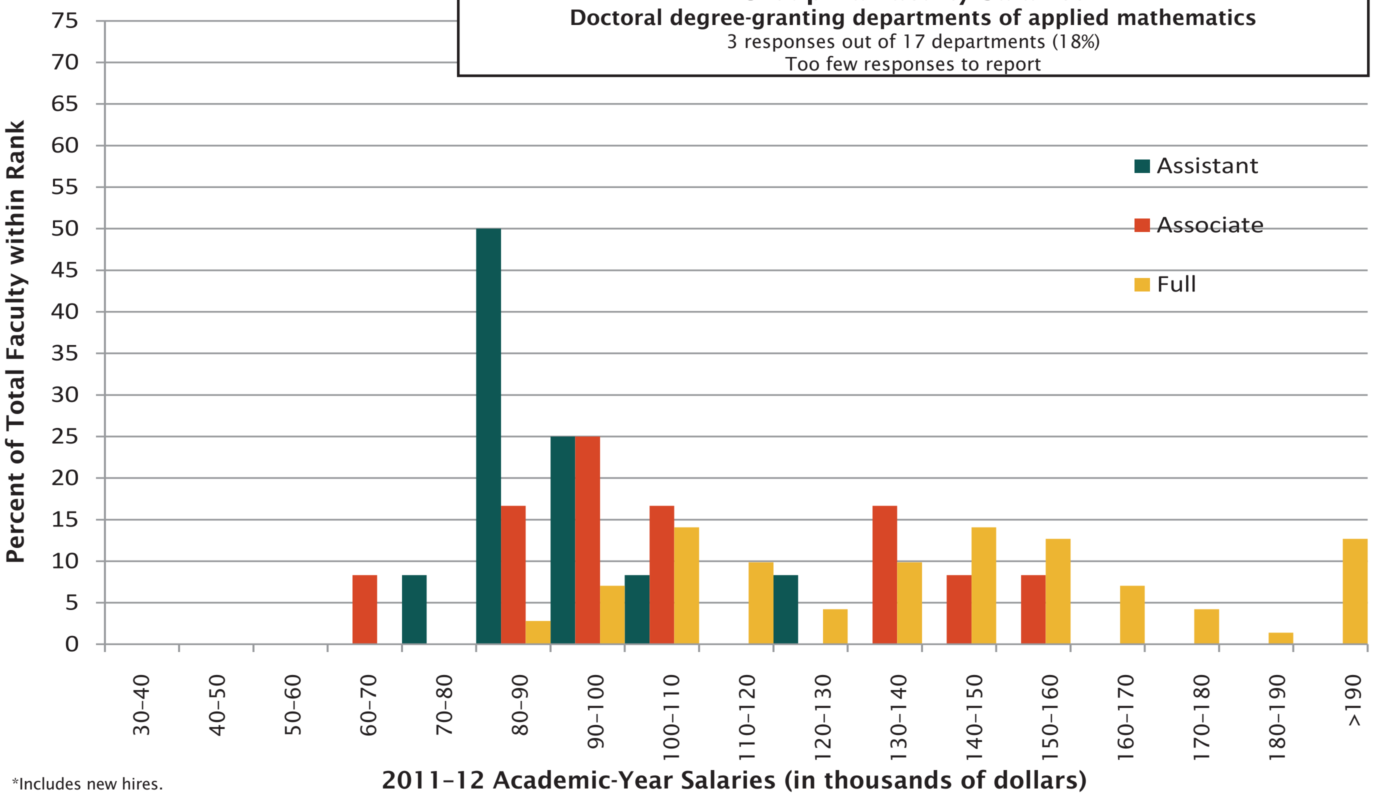


Group Va Faculty Salaries

Doctoral degree-granting departments of applied mathematics

3 responses out of 17 departments (18%)

Too few responses to report



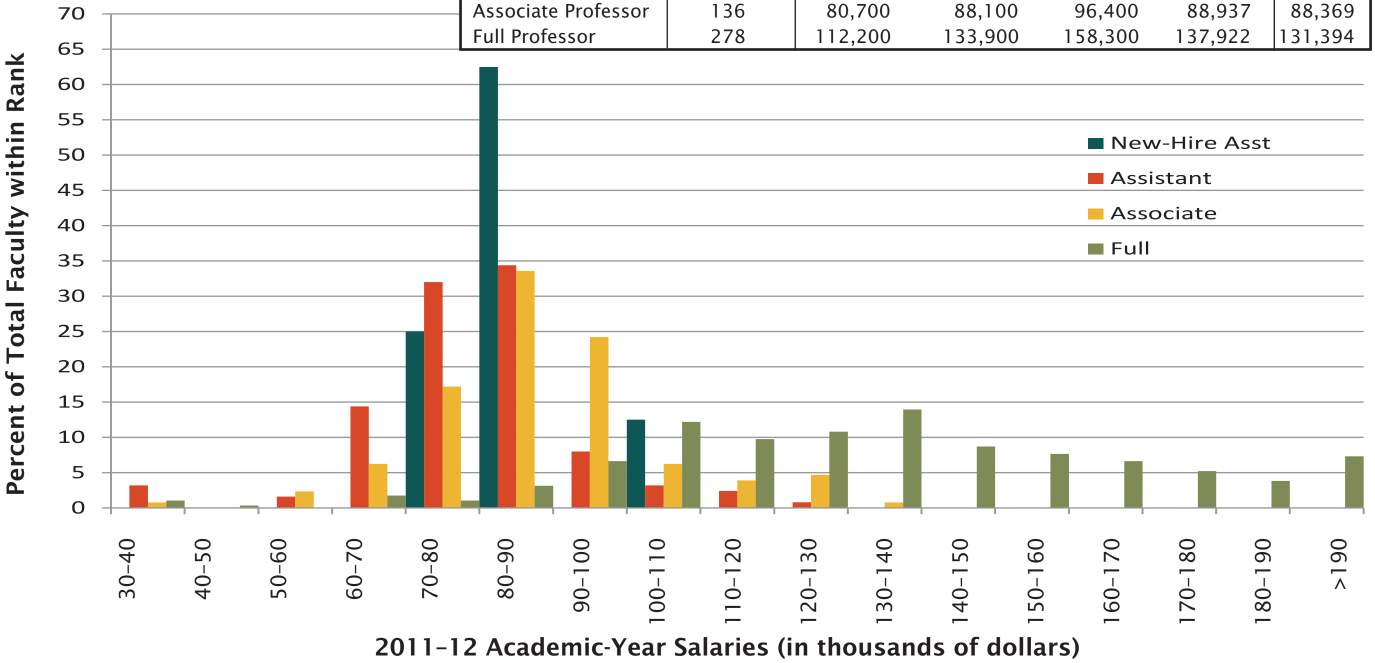
*Includes new hires.

Group IV Statistics Faculty Salaries*

Doctoral degree-granting departments of statistics

35 responses out of 57 departments (61%)

Rank	2011–12					2010–11
	No. Reported	Q_1	Median	Q_3	Mean	Mean
New-Hire Asst Prof	8	Too few to report				
Assistant Professor**	115	74,500	80,200	85,400	79,749	77,847
Associate Professor	136	80,700	88,100	96,400	88,937	88,369
Full Professor	278	112,200	133,900	158,300	137,922	131,394

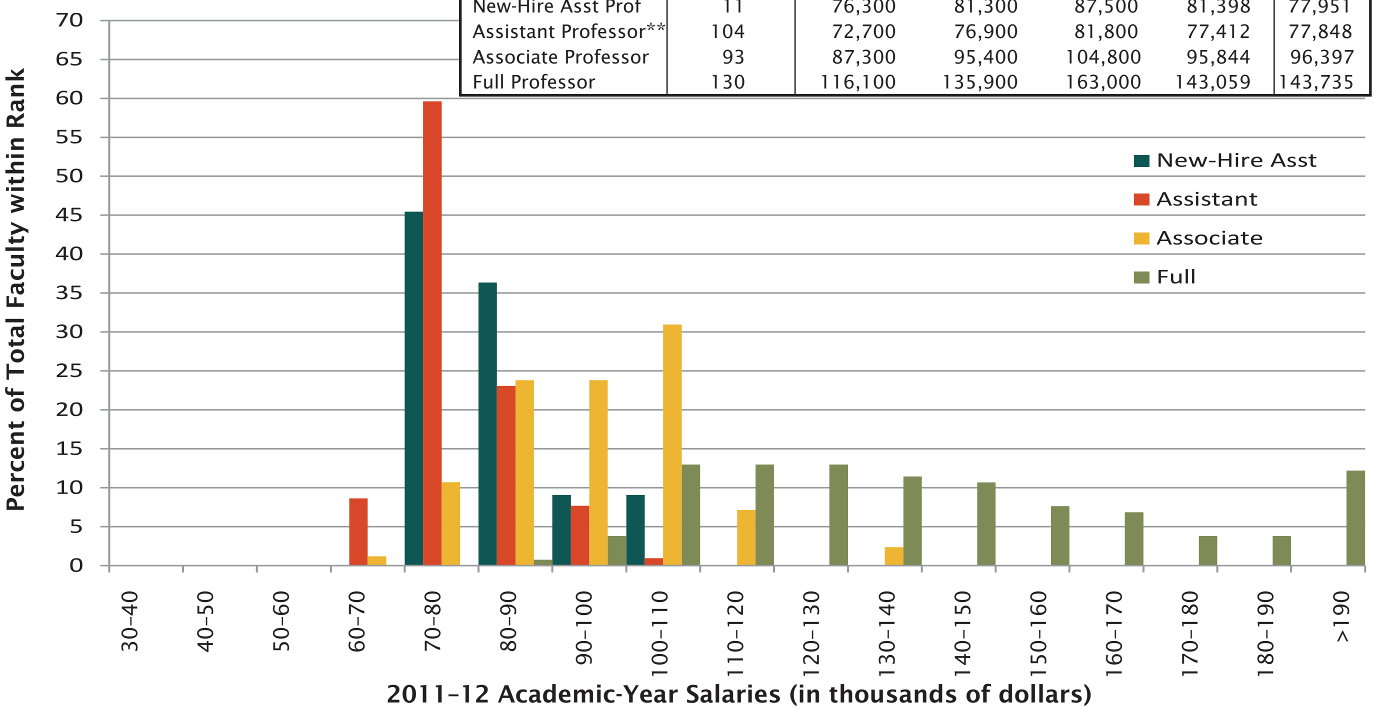


Group IV Biostatistics Faculty Salaries*

Doctoral degree-granting departments of biostatistics

19 responses out of 35 departments (54%)

Rank	2011–12					2010–11
	No. Reported	Q_1	Median	Q_3	Mean	Mean
New-Hire Asst Prof	11	76,300	81,300	87,500	81,398	77,951
Assistant Professor**	104	72,700	76,900	81,800	77,412	77,848
Associate Professor	93	87,300	95,400	104,800	95,844	96,397
Full Professor	130	116,100	135,900	163,000	143,059	143,735



*Faculty salary data provided by the American Statistical Asssoication.

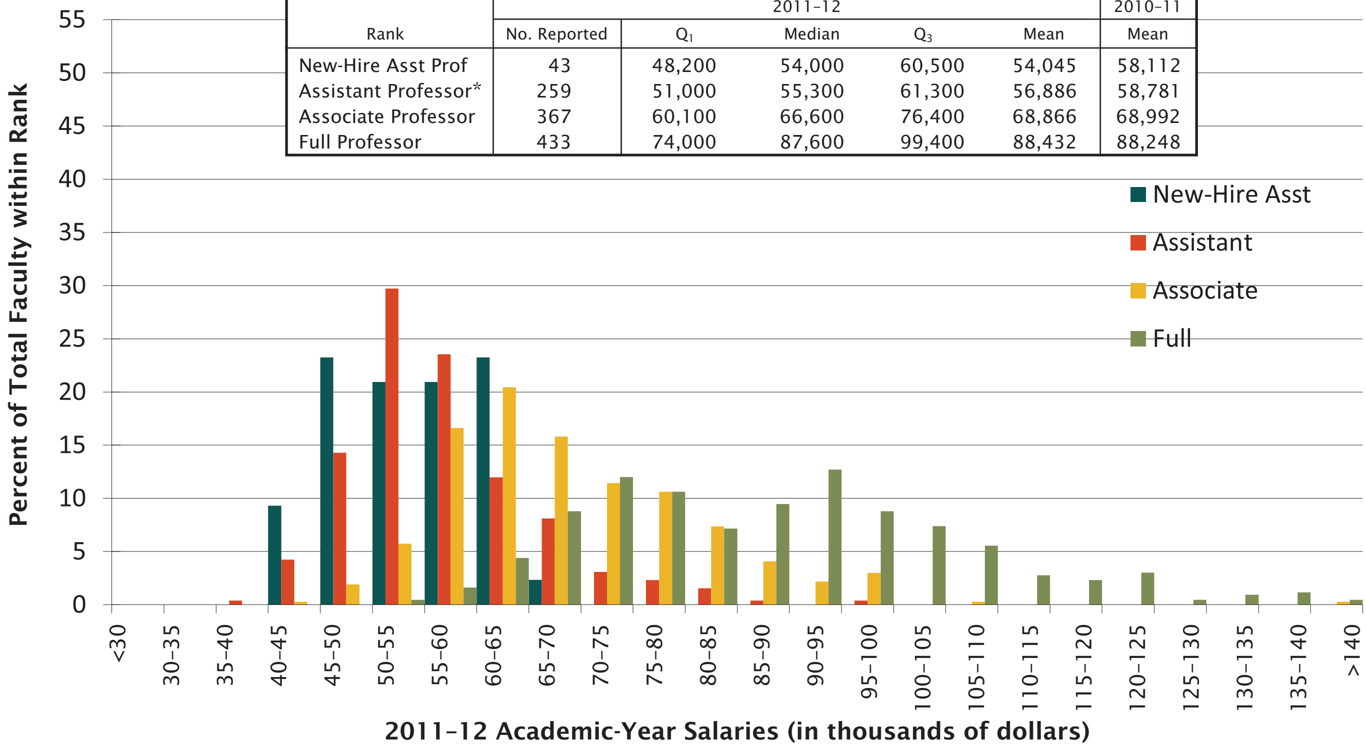
**Includes new hires.

Group M Faculty Salaries

Master's degree-granting departments of mathematics

58 responses out of 177 departments (33%)

Rank	2011–12					2010–11
	No. Reported	Q_1	Median	Q_3	Mean	Mean
New-Hire Asst Prof	43	48,200	54,000	60,500	54,045	58,112
Assistant Professor*	259	51,000	55,300	61,300	56,886	58,781
Associate Professor	367	60,100	66,600	76,400	68,866	68,992
Full Professor	433	74,000	87,600	99,400	88,432	88,248

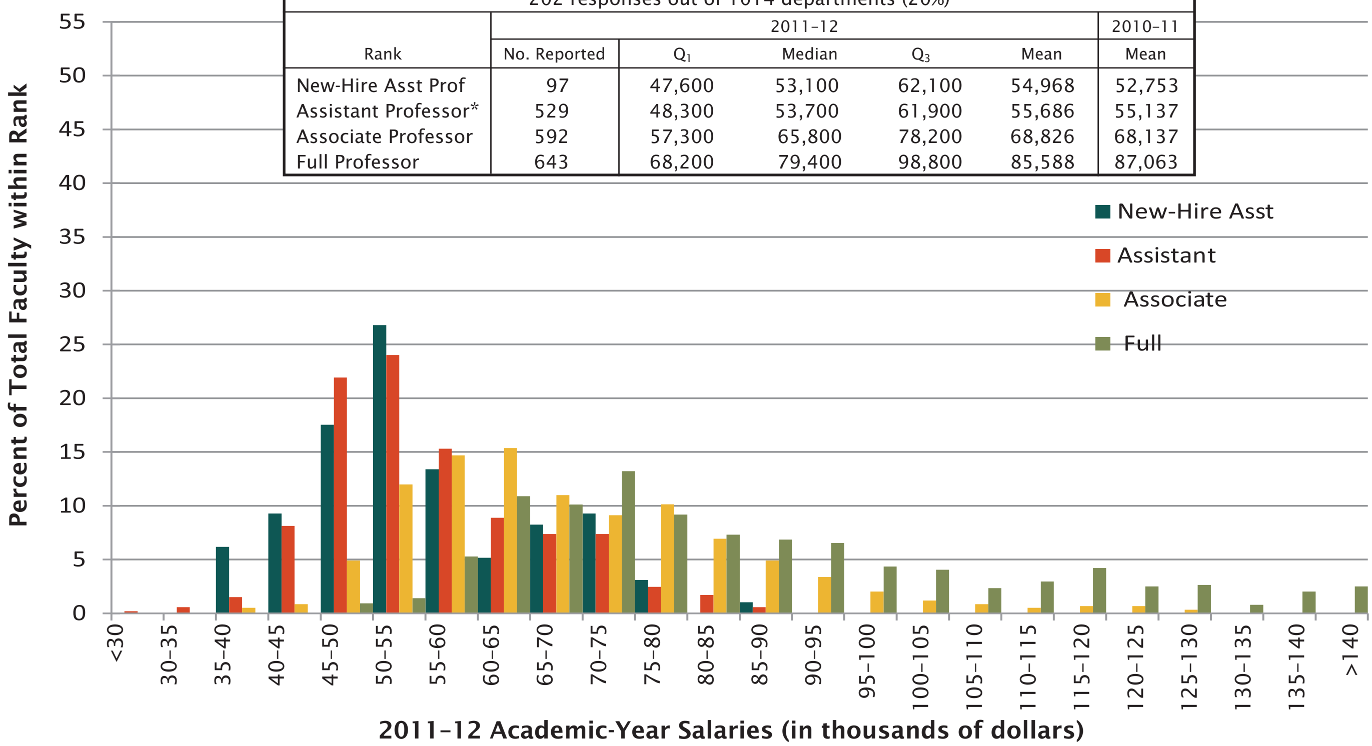


Group B Faculty Salaries

Bachelor's degree-granting departments of mathematics

202 responses out of 1014 departments (20%)

Rank	2011–12					2010–11
	No. Reported	Q_1	Median	Q_3	Mean	Mean
New-Hire Asst Prof	97	47,600	53,100	62,100	54,968	52,753
Assistant Professor*	529	48,300	53,700	61,900	55,686	55,137
Associate Professor	592	57,300	65,800	78,200	68,826	68,137
Full Professor	643	68,200	79,400	98,800	85,588	87,063



*Includes new hires.

Other Information

Obtain a Special Faculty Salaries Analysis

See how the salaries of your department's tenured/tenure-track faculty compare to those in similar departments. The only requirement is that your department must have responded to our latest Faculty Salary survey.

Send a list of your peer institutions (a minimum of 12 institutions is required) to `ams-survey@ams.org` along with the date the analysis is needed. (If not enough of your peer group have responded to the salary survey you'll be asked to provide additional institutions.) A minimum of two weeks is needed to complete a special analysis.

The analysis produced includes a listing of your peer group institutions with along their salary survey response status, a summary table including the rank (assistant, associate, and full professor), the number reported in each rank, the 1st quartile, median, 3rd quartile, and mean salaries for each along with bar graphs.

Acknowledgements

The Annual Survey attempts to provide an accurate appraisal and analysis of various aspects of the academic mathematical sciences scene for the use and benefit of the community and for filling the information needs of the professional organizations. Every year, college and university departments in the United States are invited to respond. The Annual Survey relies heavily on the conscientious efforts of the dedicated staff members of these departments for the quality of its information. On behalf of the Data Committee and the Annual Survey Staff, we thank the many secretarial and administrative staff members in the mathematical sciences departments for their cooperation and assistance in responding to the survey questionnaires.

Previous Annual Survey Reports

The 2010 Report on New Doctoral Recipients, Faculty Salaries, Academic Recruitment and Hiring, and the Departmental Profile Survey Reports were published in the *Notices of the AMS* in the February, August, May 2011, and February 2012 issues respectively. These reports and earlier reports, as well as a wealth of other information from these surveys, are available on the AMS website at `www.ams.org/annual-survey/survey-reports`.

Group Descriptions

Group I is composed of 48 departments with scores in the 3.00–5.00 range. Group I Public and Group I Private are Group I departments at public institutions and private institutions, respectively.

Group II is composed of 56 departments with scores in the 2.00–2.99 range.

Group III contains the remaining U.S. departments reporting a doctoral program, including a number of departments not included in the 1995 ranking of program faculty.

Group IV contains U.S. departments (or programs) of statistics, biostatistics, and biometrics reporting a doctoral program.

Group V contains U.S. departments (or programs) in applied mathematics/applied science, operations research, and management science which report a doctoral program.

Group Va is applied mathematics/applied science.

Group M contains U.S. departments granting a master's degree as the highest graduate degree.

Group B contains U.S. departments granting a baccalaureate degree only.

Listings of the actual departments which compose these groups are available on the AMS website at `www.ams.org/annual-survey/groups_des`.

Other Sources of Data

Visit the AMS website at `www.ams.org/annual-survey/other-sources` for a listing of additional sources of data on the Mathematical Sciences.

About the Annual Survey

The Annual Survey series, begun in 1957 by the American Mathematical Society, is currently under the direction of the Data Committee, a joint committee of the American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society of Industrial and Applied Mathematics. The current members of this committee are Pam Arroway, Richard Cleary (chair), Steven R. Dunbar, Susan Geller, Boris Hasselblatt, Abbe H. Herzig, Ellen Kirkman, Peter March, David R. Morrison, James W. Maxwell (ex officio), and Bart S. Ng. The committee is assisted by AMS survey analyst Colleen A. Rose. In addition, the Annual Survey is sponsored by the Institute of Mathematical Statistics. Comments or suggestions regarding this Survey Report may be directed to the committee.

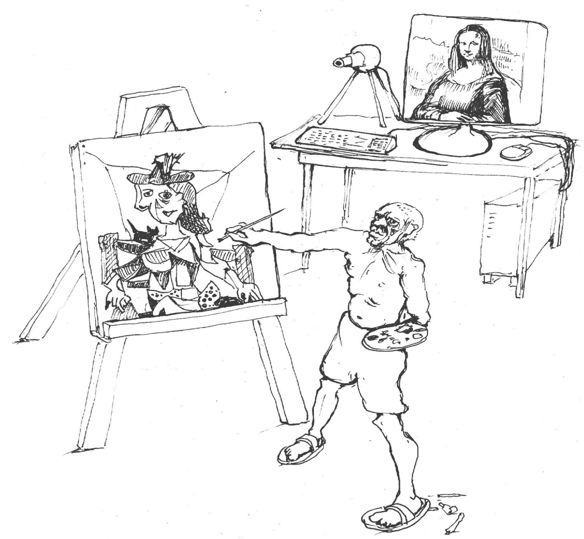
Organic Mathematics: Then and Now

Jonathan M. Borwein and Veselin Jungić

In the early 1990s a group of researchers, J. Borwein, P. Borwein, R. Corless, L. Jörgenson, and N. Sinclair, all then affiliated with the Center for Experimental and Constructive Mathematics at Simon Fraser University—as part of a National Telelearning Network—started the Organic Mathematics Project (OMP).[1] One of the main goals of the project was to achieve a more meaningful integration of the technologies then available, moving towards the ideal environment described as follows:

> A mathematician working in ideal conditions would be able to look at a fresh problem and easily access any related material, find all the work on simpler but similar problems, and quickly carry out any subcomputations needed for the solution of the fresh problem. Such a person would also be able to consult freely not only with colleagues but with experts with whom he or she/was not previously familiar.

The project culminated with the Organic Mathematics Workshop, held in Vancouver, B.C., on December 12–14, 1995. The editors of the online workshop proceedings[2] stated that they



"Computers are useless. They can only give you answers."

> want the information in the Proceedings of this workshop to form examples of "living documents", connected to their references, connected to each other, connected to algorithms for live mathematical work on the part of the reader. We want them to be, in a word, "organic".

The body of the OMP was the "math-activations"[3] of fifteen previously published and highly regarded papers by leading researchers such as George Andrews, Jeff Lagarias, Andrew Odlyzko, Ron Graham, and Andrew Granville.[4] This allowed

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[1] *See* `http://www.cecm.sfu.ca/organics/contents.html`.

[2] *Also published in the traditional hard-copy form:* Organic Mathematics *(Burnaby, BC, 1995), CMS Conf. Proc. 20, Amer. Math. Soc., Providence, RI, 1997.*

DOI: http://dx.doi.org/10.1090/noti805

[3] *Algorithms in Maple, graphics, audio, videos, simulations, annotation tools, etc. While we already used Java in our Centre, we did not exploit it as it was not then well established.*

[4] *See* `http://www.cecm.sfu.ca/organics/papers/` *for the papers, some of which had already won major prizes.*

the project to focus on issues of enhancement rather than traditional editing. There were an equal number of submitted papers which were similarly enhanced—in each case by teams of two students who worked with the author.[5] In the section "The future of these proceedings" some of the challenges and dilemmas were listed.

> Where will this volume be in ten years? How about ten months? Experience to date tells us that the potential for long-term functionality is quite limited. … Perhaps a more pertinent question is, supposing that someone will be able to read these proceedings at some time in the future, will they want to?[6]

Given that we were largely building a bridge as we walked upon it—since very few of today's Web design or management tools existed—it is fair to say that we have been pleased and surprised by the robustness and longevity of the OMP. Software licensing issues and antique servers have proven the main exceptions.

As one of many possible illustrations that some of the core ideas motivating the Organic Mathematics Project still intrigue mathematicians, we mention the Manifold Atlas Project[7] sponsored by the Hausdorff Center at the University of Bonn. The structure of the Project is described in the following "organic" way:

> The pages of the *Atlas* provide a public work space for topologists and other interested scientists to collaborate via the World Wide Web. *Atlas* pages are continually open for editing and development. However, they are not strongly scientifically citable. Once an *Atlas* page has reached maturity, it will be refereed. … *Atlas* pages which have been published in the *Bulletin* are still open for improvement, modification, and correction. When such a page again reaches maturity, it will be refereed again.

Contemporary researchers and publishers who aim for "a more meaningful use of the technologies available" face challenges depressingly similar to those of nearly twenty years ago. It is an interesting exercise to compare two documents, one[8] published in 1994 and the other[9] published in 2011, on the self-same topic of the future of the mathematics journals.

For a modern working mathematician it is increasingly difficult to follow developments in his or her field. The quantity of published papers or preprints, the number of relevant meetings, the steady stream of information coming from sources such as the arXiv or blogs,[10] and other Web-based sources,[11] the ever changing working environment, increasing demand in teaching and administrative duties, sharp competition to publish new results,[12] the exhausting process of applying for and reporting on research funds, and the search for high quality graduate students, can be nearly overwhelming for many members of our community.

Hence, for years to come, just to stay well informed "our hero" will be on the run like Alice's Red Queen[13] to access needed information, to grasp it at the necessary level as quickly as possible, and to trust the sources of that information. We believe that mathematical journals, in their capacity as verifiers, evaluators, disseminators, and keepers of mathematical knowledge, should play a major role in helping the community to face some of the challenges listed above. This will include resolving some of the issues that are consequences of the new role that technology plays in mathematical research and of the ways that mathematics is and will be communicated.

Current technology has forever changed the relationship between so-called pure and applied mathematics. Using mathematical technology to predict or check a mathematical fact, to visualize abstract objects and/or their properties, or to calculate with unprecedented precision, has become a standard in mathematical research. The computational sides of graph theory, topology or group theory, applications of number theory, experimental mathematics, and mathematical modeling of complex systems are just a few of

[5] *And got course credit for so doing.*

[6] *We suggest the curious reader visit* `http://www.cecm.sfu.ca/organics/` *for answers to these questions.*

[7] `http://www.map.him.uni-bonn.de/Main_Page.`

[8] *A. M. Odlyzko, Tragic loss or good riddance? The impending demise of traditional scholarly journals,* Journal of Universal Computer Science ***0****, 3–53.*

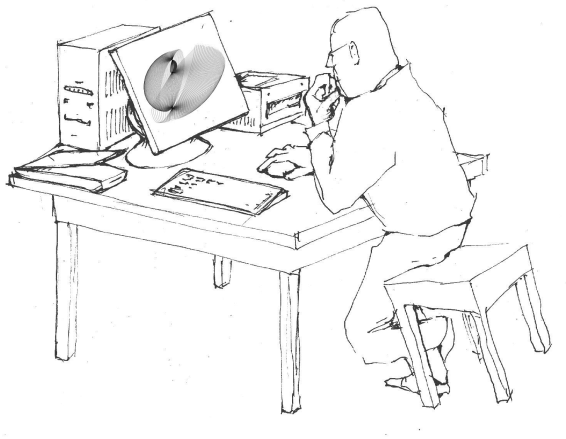
[9] Mathematics journals: What is valued and what may change. *Report of the workshop held at MSRI, Berkeley, California, on February 14–16, 2011,* `http://www.msri.org/attachments/workshops/587/MSRIfinalreport.pdf`.

[10] *From sublime, such as Tim Gower's Polymath Project* `http://michaelnielsen.org/polymath1/index.php?title=Main_Page`, *to ridiculous.*

[11] *See, for example,* `http://www.eg-models.de/`.

[12] *And even to prematurely announce them in the press.*

[13] *"Well, in our country," said Alice, still panting a little, "you'd generally get to somewhere else if you run very fast for a long time, as we've been doing." "A slow sort of country!" said the Queen. "Now, here, you see, it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!" From Lewis Carroll's* Alice Through the Looking Glass.



"The computer knows more than I do?"

the branches of mathematics that both depend heavily on the technology and inspire and demand its new developments.

Now contemporary natural and social scientists wish—rightly or wrongly—to quantify almost everything and need to recognize and record changes in their data. This gives a new responsibility to the mathematics community. This responsibility includes providing tools to analyze and understand collected data and to be able to correct theoretical errors, fill holes, and explain apparent paradoxes in the data. It follows that both mathematics driven by preexisting data and mathematics that produces new data are now part of everyday life in MathLand, and as such they deserve fair treatment in mathematics journals.

For example, we need to establish protocols as to how journals manage verifying, presenting, and storing various computational—or computationally checkable—components of newly created mathematics. As part of such protocols, standard suites of problems should be maintained for the validation of performance claims for new algorithms.[14] It should no longer be good enough for an author to assert that his or her method is "better" or "competitive with" current standard methods. It should also be possible for a referee intrigued by or suspicious of a formula to try it on the spot.

Also, maintaining the data that support mathematical discovery is an issue that is both technical and part of the emerging new relationship between publishers, editors, and authors. How can we preserve and keep data available to interested readers, an issue complicated by changing protocols and related technology? Whose responsibility should this be, publishers' or authors'? In these matters we can learn much from the biomedical sciences.

[14] *In many applied areas there are good suites of test problems. What is missing then is a protocol and infrastructure for their use.*

Given all this, we should like to further revisit the notion of *the ideal environment as it was envisioned by the Organic Mathematics Project in 1995.*[15] One of the reasons that we do this is because even now a paper published in a mathematical journal often looks worse than its arXiv version—cramped, unenhanced, and unloved. Moreover, the arXiv paper is often less fun and less informative than a good Beamer or PowerPoint presentation on the subject which has a human element—side comments, reflections, jokes—even when it lacks animations, simulations, movie clips, or the like, all of which have been possible for at least twenty years.

Now, it takes effort to add such enhancements, and while many of us already do so in part, it is unreasonable to expect this as a basic requirement for a mathematics paper. While the jobs of copyediting and typesetting have been added to the rest

"By golly—she does!"

of the academic remit, the journals (even those of nice guys like SIAM and the AMS) have responded by offering us even less for more.

So here is what we suggest: SIAM, the AMS, and other society publishers should build us a modern publishing template—a "Beamer without tears" if you will—and promise to expedite publication of well-enhanced articles.

We believe that this kind of change will greatly benefit mathematicians and the scientific community in general. By creating mathematical publications in the form of layered structures, the reader will get an opportunity to match his or her needs and expertise with the appropriate layer of the publication. This would make new mathematics more accessible to many researchers from other parts of mathematics and other scientific fields and will help remove the barrier established by the

[15] *What follows is partly based on a July 2011 letter in* SIAM Review *by the first author.*

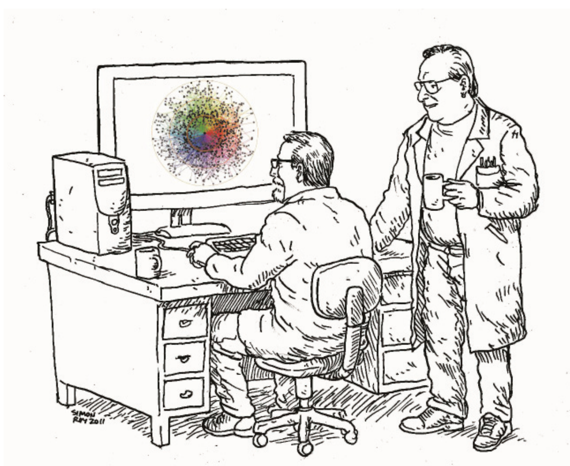
fact that most mathematical papers are written by experts for experts.

In its essence this idea is not new. In 1994 Leslie Lamport wrote:

> When first shown a detailed, structured proof, most mathematicians react: I don't want to read all those details; I want to read only the general outline and perhaps some of the more interesting parts. My response is that this is precisely why they want to read a hierarchically structured proof. The high-level structure provides the general outline; readers can look at as much or as little of the lower-level detail as they want.[16]

The main difference between *then* and *now* is in the technology. For Lamport "[t]he ideal tool for reading a structured proof would be a computer-based hypertext system." Today, ways to separate different levels of a proof are near limitless.

We are aware that there is another, equally "organic", publishers' side of this issue. Let us list some of the questions that are central to the enterprise of scientific publishing. How would the above proposed changes impact the existing business model for mathematics journals? Would further breaking with traditional modes of publishing be manageable for commercial publishers? Would the model "fit more than a few"? Would it be robust enough to keep the current level of mathematical rigor intact? Would it truly reach out of the experts' zone and, at the same time, justify the necessary investment in people and technology? Should the status quo be disturbed? Our answer to all of these questions is "yes".



"Sometimes it is easier to see than to say."

Who is to make these decisions: professional mathematicians, research librarians, academic publishers, governments through their granting agencies, or maybe its Majesty, the Market itself? Furthermore, would a mathematician, whose whole world had just crashed because he found a seemingly devastating bug in his proof, care about any of this?

The necessary components such as MathJax are all here if only recently. Yes, there are many annoying technical and organizational details, but all that is needed to overcome them is for academic publishers to step up to the plate for their own benefit and for the benefit of the whole mathematical and scientific community.

Acknowledgments. The final cartoon by Simon Roy shows recent computer-discovered work on random walks by the first author and his colleagues. All others are by Karl H. Hofmann and appeared originally in J. Borwein and K. Devlin, *The Computer as Crucible*, A K Peters, 2008.

[16] *L. Lamport, How to write a proof,* American Mathematical Monthly ***102*** *(7) (1994), 600–608.*

WHAT IS... an Affine Sphere?

Daniel J. F. Fox

Linear maps and translations generate the group Aff of *affine motions* comprising diffeomorphisms of $\mathbb{R}^{n+1}$ that preserve straight lines. The simplest interesting class of hypersurfaces preserved by Aff is the *affine spheres*. These are defined by the affinely invariant condition that their *affine normal* lines meet in a point, possibly at infinity. While affine spheres include nondegenerate quadrics, there are many others. A simple example is the surface $\{xyz = 1 : x > 0,\ y > 0\}$. It resembles a flattened hyperboloid asymptotic to the positive orthant bounded by the coordinate planes. As will be explained, an infinitude of inequivalent affine spheres results from solving certain PDEs of *Monge-Ampère* type, meaning that they involve the determinant of the Hessian of an unknown function. The construction of affine spheres has played an important role in the development of techniques for solving Monge-Ampère equations (see [4]), while affine spheres arise in studying such apparently different topics as flat projective structures (see [1]) and convex optimization (see [3]).

Most intuitive geometrical notions relate to describing position (e.g., distance, angle) and relative position (e.g., midpoints, parallels), size (e.g., volume, length), and shape (e.g., round or flat). What *geometry* means in a particular context is determined operationally by those features of space to be distinguished or identified. Rotations and translations generate the *Euclidean motions* Euc comprising affine motions preserving the standard inner product $\langle\ ,\ \rangle$. The *Euclidean* and *affine geometry* of $\mathbb{R}^{n+1}$ refer to those constructs preserved, respectively, by Euc and Aff. For example, Euclidean geometry distinguishes between a sphere and an ellipsoid, because their surfaces bend in different ways. Though they cannot be superimposed by a Euclidean motion, they can be by composing one with appropriate shears and dilations. Since Aff contains Euc, affine equivalence and invariance are coarser notions than their Euclidean counterparts. For example, any two ellipsoids or any two triangles are affinely equivalent. As Aff preserves the standard directional derivative operator D but not $\langle\ ,\ \rangle$, distance and angle have no sense in affine geometry, but straight lines, parallels, and midpoints do. Ordinary calculus (based on D) still makes sense, though there is no natural way to identify the differential of a function with a vector field.

A *smooth hypersurface* in $\mathbb{R}^{n+1}$ means a submanifold Σ locally representable as the graph $\Sigma_f = \{x^0 = f(x^1, \dots, x^n)\}$ of a smooth function f. Assume Σ is *two sided*, meaning that there is a nonvanishing vector field N everywhere *transverse* to Σ. This excludes examples such as the Möbius band, but always holds locally, and simplifies the discussion. For vector fields X and Y tangent to Σ, the part of the directional derivative D_XY in the direction of a transversal N has the form $h(X,Y)N$ for a field h of symmetric bilinear forms. If N is changed, h is multiplied by a nonvanishing function. The *second fundamental form* of Σ is the equivalence class $[h]$ of h under the identification of a field of bilinear forms with its multiple by a nonvanishing function. Since $[h]$ depends only on D, it is preserved by Aff. Henceforth the hypersurface Σ is assumed to be *nondegenerate*,

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meaning h is nondegenerate. If h is definite, then Σ is *locally uniformly convex*, in which case it is moreover locally strictly convex. For a graph Σ_f, the Hessian $f_{ij} = \frac{\partial^2 f}{\partial x^i \partial x^j}$ is the representative of $[h]$ corresponding to the transversal $\frac{\partial}{\partial x^0}$. Hence Σ_f is nondegenerate (resp. locally uniformly convex) if and only if f_{ij} is nondegenerate (resp. definite). For example, the saddle $z = xy$ is nondegenerate but not convex, while $z = x^4 + y^4$ is strictly convex but not uniformly so, for it is degenerate where $xy = 0$. That Aff preserves $[h]$ means that such local convexity properties of Σ_f are affinely invariant. The scaling of N, though not its direction, can be fixed by requiring that the corresponding h equal the *equiaffine* (or *Blaschke*) *metric* given locally by $|\det f_{ij}|^{-1/(n+2)} f_{ij}$. The prefix *equi* indicates a structure preserved by the group SAff of volume-preserving affine motions. Local uniform convexity implies that the intersection of Σ with a small ball B centered on $P \in \Sigma$ is contained in one of the half-spaces delimited by the tangent plane $T_P\Sigma$ at P and divides B into an *exterior* region intersecting $T_P\Sigma$ and an *interior* region not intersecting $T_P\Sigma$. In this case, h is positive definite if N is chosen to point to the interior.

The *shape operator* is the endomorphism S of $T_P\Sigma$ associating with a tangential vector field X the tangential part of the derivative $-D_X N$. Its eigenvalues are the *principal curvatures* of Σ. These encode how Σ bends and twists in $\mathbb{R}^{n+1}$ relative to N. A Euclidean invariant transversal is determined up to sign by the requirements that it be perpendicular to Σ and have unit norm. Via the Euclidean metric the corresponding shape operator S and bilinear form h are identified. In affine geometry there is no notion of orthogonality with which to single out a transversal. Nonetheless, it is possible to define a distinguished *affine normal* direction preserved by Aff, although the corresponding S and h are *not* naturally identified.

The following description of the affine normal, due to W. Blaschke, is based on the affine equivariance of the center of mass. If $[a, b] \subset \mathbb{R}$ and $x \in (a, b)$, then $|x-a|$ and $|x-b|$ are unchanged by a translation and scale identically under a dilation so the ratio $A(x) = |x-a|/|x-b|$ is unchanged by an affine transformation applied to a, b, and x. The unique $x \in (a, b)$ such that $A(x) = 1$ is the *midpoint* of $[a, b]$. This is the one-dimensional specialization of the usual notion of the *centroid* (also *center of mass* or *barycenter*) of a region. It follows from the change-of-variables formula for integrals that the centroid is affinely equivariant in the sense that the centroid of the image of a region under $g \in \mathsf{Aff}$ is the image under g of the centroid of the region. Consider a point P on a locally uniformly convex hypersurface $\Sigma \subset \mathbb{R}^{n+1}$ and the plane K tangent to Σ at P. Because of Σ's convexity, its intersection with the parallel translate $K(t)$ of K at distance t from P encloses an $(n-1)$-dimensional convex subset $\Omega(t) \subset K(t)$. The tangent at P to the curve $n(t)$ formed by the centroids of the $\Omega(t)$ spans the *affine normal subspace*. Since affine transformations preserve centroids and parallelism, this construction is manifestly affinely equivariant, though, since the distance t is not preserved by Aff, the vector tangent to $n(t)$ at $t = 0$ is not. However, for $s(t)$ equal to an appropriate function of the volume enclosed by $K(t)$ and Σ, the derivative at $s = 0$ of $n(t(s))$ is the manifestly SAff equivariant *equiaffine normal*. Equivalently, the scaling of the equiaffine normal is determined by the requirement that the corresponding h be the equiaffine metric. The corresponding S is the *equiaffine shape operator*.

For example, if Σ is a sphere, then its intersection with a parallel translate of its tangent plane at P is a round ball in this plane, and this ball's centroid is the plane's intersection with the radius of Σ through P. Thus a sphere's affine normals meet in its center. By affine invariance the same is true for any ellipsoid. The abscissa of the midpoint of the segment of any secant to the parabola $y = x^2$ demarcated by the parabola equals the abscissa of the point at which the tangent to the parabola is parallel to the secant. Extending this reasoning shows that the affine normals of an upward-opening elliptic paraboloid are vertical lines.

Basic tasks are to identify and characterize Euclidean or affinely invariant classes of hypersurfaces. Many such classes are described by algebraic conditions on the shape operator, e.g., constant trace or determinant. The most studied Euclidean invariant hypersurfaces are the *minimal* surfaces—the critical points of the surface area functional with respect to small variations in the normal direction. These arise physically as films spanning a wire frame dipped in soapy water. They are called *minimal* because when they are extremal, they are in fact minimizing. For a graph Σ_f, the Euclidean volume element is $(1 + |df|^2)^{1/2} dx$, and minimality of Σ_f is the nonlinear second-order PDE for f given by the vanishing of the mean curvature, by definition the average of the Euclidean principal curvatures. The *equiaffine volume* of Σ is the integral of $|\det h|^{1/2}$ over Σ for the equiaffine metric h. Its critical points with respect to small normally directed variations are given by the vanishing of the *affine mean curvature*, the average of the equiaffine principal curvatures. Affine mean curvature zero hypersurfaces had been for many years called *affine minimal*, until E. Calabi calculated that, when extremal, such hypersurfaces are locally volume *maximizing*; now they are called *affine maximal*. For a graph Σ_f, $|\det h|^{1/2} = |\det D^2 f|^{1/(n+2)} |dx|$, and that Σ_f be affine maximal is a nonlinear *fourth*-order PDE for f.

Among the simplest hypersurfaces are the *umbilical* ones, those for which the principal curvatures are all equal at every point. Equivalently, the shape operator is a multiple of the identity. This means, imprecisely, that at every one of its points such a hypersurface bends in the same way in every direction. In the Euclidean case these are simply either hyperplanes or spheres. It turns out that the affine mean curvature H of an affinely umbilical hypersurface Σ must be constant. This is equivalent to the geometric condition that the affine normals of Σ either all meet in a point, its *center*, or all are parallel, in which case Σ is said to have *center at infinity*. This fact motivates calling an affinely umbilic hypersurface an *affine sphere*. An affine sphere is *improper* or *proper* as its center is or is not at infinity. For example, for any $\phi \in C^{\infty}(\mathbb{R})$ the graph of $z = xy + \phi(x)$ is an improper affine sphere with equiaffine normal $\frac{\partial}{\partial z}$.

In the convex case strong results follow from a representation of an affine sphere as a solution of an elliptic Monge-Ampère equation. The graph Σ_f of a locally strictly convex function f is a mean curvature H affine sphere centered at the origin or infinity if and only if the Legendre transform u of f solves

$$\text{(1)} \qquad \det \frac{\partial^2 u}{\partial y_i \partial y_j} = \begin{cases} (Hu)^{-n-2}, & \text{if } H \neq 0, \\ 1, & \text{if } H = 0. \end{cases}$$

Recall that $u(y)$ is the strictly convex function on the image $\Omega \subset \mathbb{R}^n$ of the differential df defined implicitly by $u = \sum_{i=1}^{n} x^i \frac{\partial f}{\partial x^i} - f$ with respect to the coordinates $y_i = \frac{\partial f}{\partial x^i}$ on Ω. In the proper case the *radial graph* $\{u^{-1}(-1, y) \in \mathbb{R}^{n+1} : y \in \Omega \subset \mathbb{R}^n\}$ is again an affine sphere, with mean curvature H^{-1}. In the improper case, interchanging the roles of u and f in (1) shows that the ordinary graph of u is also an improper affine sphere. That affine spheres come in pairs in this way is a deep manifestation of the Legendre transform duality.

If a convex affine sphere is improper, it is also called *parabolic*, while if proper, it is called *elliptic* or *hyperbolic* as H is positive or negative (this means its center is contained in its interior or its exterior, respectively). Using basic results about the growth of solutions to Monge-Ampère equations like (1) due to Jörgens, Calabi, and Pogorelov, it can be proved that, under some technical hypotheses, an elliptic affine sphere is an ellipsoid and a parabolic affine sphere is an elliptic paraboloid. In the hyperbolic case there are more possibilities. For $\rho(x) = -z_0^2 + z_1^2 + \cdots + z_n^2$, the hyperboloid

$$\mathbb{H}_r = \{z \in \mathbb{R}^{n+1} : \rho(z) = -r^{2(n+2)/(n+1)}, z_0 > 0\}$$

is the radial graph over the unit ball of the solution $u = -r^{-(n+2)/(n+1)}(1 - |x|^2)^{1/2}$ of (1) with $H = -r^2$. The $\mathbb{H}_r$ are asymptotic to the null cone $\{\rho(z) = 0\}$ and fill out its interior $\{\rho(z) < 0\}$. From a theorem of S. Y. Cheng and S. T. Yau showing that on a bounded convex domain Ω there is for $H < 0$ a unique negative convex solution of (1) extending continuously to be 0 on the boundary of Ω, it follows that a similar geometric picture holds far more generally (see [2] and [4]). Namely, suppose the convex cone K is *sharp*, meaning it contains no full straight line (this rules out, e.g., a half-space). Then the interior of K is, in a unique way, a disjoint union $\bigcup_{r>0} L_r$ of mean curvature $-r^2$ hyperbolic affine spheres L_r asymptotic to K and with centers at the vertex of K. Alternatively, by other theorems of Cheng and Yau and of Mok and Yau there is on K a unique solution of the Monge-Ampère equation $\det \frac{\partial^2 F}{\partial z_I \partial z_J} = e^{2F}$ tending to $+\infty$ on the boundary of K and such that $\frac{\partial^2 F}{\partial z_I \partial z_J}$ is a complete Riemannian metric, and the affine spheres foliating K are the level sets of F. In fact, for the solution u of (1), F equals $-(n+1)\log|z_0 u(z_i/z_0)|$ plus a constant. For example, the radial graph of

$$\text{(2)} \qquad u = -\sqrt{n+1}\,(y_1 y_2 \cdots y_{n-1} y_n)^{1/(n+1)}$$

over the orthant $\Omega = \{y \in \mathbb{R}^n : y_i > 0\}$ is a mean curvature -1 affine sphere contained in a level set of $-\log|\prod_{I=1}^{n+1} z_I|$ asymptotic to $\{\prod_{I=1}^{n+1} z_I = 0\}$. Though this theorem yields an affine sphere for every sharp convex cone, such a sphere is difficult to describe explicitly except when K is homogeneous, meaning its automorphism group G acts on it transitively. In this case orbits of $G \cap \mathsf{SAff}$ are affine spheres asymptotic to K; e.g., (2) comes from the group of diagonal linear maps. A component of a nonzero level set of the discriminant

$$x_2^2 x_3^2 + 18x_1x_2x_3x_4 - 4x_1x_3^3 - 4x_2^3x_4 - 27x_1^2x_4^2$$

of the binary cubic form $f(u, v) = x_1u^3 + x_2u^2v + x_3uv^2 + x_4v^3$ is a striking example arising in this way. Although such constructions yield many examples, affine spheres with indefinite signature equiaffine metric are less well understood, in part because the theory for nonelliptic Monge-Ampère equations is inadequately developed. Also, explicit representations of the hyperbolic affine spheres asymptotic to inhomogeneous cones are not known, even for seemingly simple cases like polyhedral cones.

Affine spheres occur as models in some applied contexts. The flow evolving a hypersurface along its equiaffine normal is studied in mathematical imaging. Its self-similar (soliton) solutions are affine spheres. In another direction, the function $-\log|\prod_I z_I|$ is the prototype for the *self-concordant barrier functions* that play a key role in polynomial time interior point methods for solving convex programming problems; see [3].

The Cheng-Yau theorem associates an affine sphere with the cone over the universal cover of a manifold M carrying a *flat projective structure* (see

[1]), which is *convex* in the sense that it is the quotient of a convex domain by a group of projective transformations. The equiaffine metric descends to give a canonical metric on M analogous to the Kähler-Einstein metric on a compact Kähler manifold with negative first Chern class. This metric should be fundamental in better understanding convex projective structures. For example, J. Loftin showed how to use it to identify the deformation space of convex projective structures on a genus $g > 1$ compact orientable surface M with a vector bundle of total dimension $16g - 16$ over the Teichmüller space of M.

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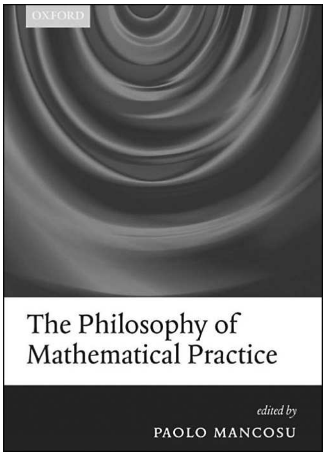
Book Review



The Philosophy of Mathematical Practice

Reviewed by Timothy Bays

The Philosophy of Mathematical Practice
Edited by Paolo Mancosu
Oxford University Press, 2008
US$125.00, 460 pages
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For mathematicians, modern philosophy of mathematics may seem somewhat puzzling. In the late nineteenth and early twentieth centuries, the borders between philosophy and mathematics were porous. Influential mathematicians like Poincaré, Brouwer, Ramsey, and Hilbert wrote extensively on philosophical topics, and philosophers like Russell, Wittgenstein, and Quine made serious philosophical use of contemporaneous mathematics. More importantly, the issues that concerned philosophers of mathematics were often continuous with developments in mathematics proper—e.g., the rigorization of analysis in the nineteenth century, the discovery of the set-theoretic paradoxes and Gödel's incompleteness theorems, or Bourbaki's project of systematizing mathematics as a whole.

In recent years, mathematics and the philosophy of mathematics have become somewhat more distant. The topics that most exercise philosophers of mathematics—realism and anti-realism, the metaphysics of structuralism, or arguments concerning the indispensability of mathematics to natural science—don't connect very well with the day-to-day concerns of practicing mathematicians. This is not to say that philosophy of mathematics has become insular and unmotivated. It's just that its connections tend to be to other areas of philosophy—to metaphysics, epistemology, and the philosophy of language in particular—and its motivating problems tend to be purely philosophical. Further, the mathematics one needs to address these problems is often quite rudimentary—in many cases, reflection on elementary arithmetic and geometry is enough to make philosophical progress. For philosophers, therefore, there's often little motivation to engage with the parts of mathematics which are of most interest to practicing mathematicians.[1]

Of course, some philosophers *have* worked on mainstream mathematical topics, and in some cases—Imre Lakatos and Philip Kitcher, in particular—their work has been well received by the larger philosophical community. But, in all too many cases, philosophers pursuing this kind of work have seemed more focused on discussing cutting-edge mathematical *topics* than on generating genuine philosophical results from those topics. This work has been less well received. Like mathematicians, philosophers can be pragmatists: it's not enough to *talk about* exciting mathematics; you have to show that there's a properly philosophical payoff to the discussion.

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[1] *This point needs to be emphasized. I've occasionally heard mathematicians suggest that certain questions in the philosophy of mathematics are ill-motivated (or even pointless!) simply because they lack immediate relevance to the day-to-day concerns of working mathematicians. In my view, this is a mistake. It's the equivalent of dismissing work in the philosophy of language on the grounds that such work often lacks immediate relevance to working novelists. More pointedly, I think it's the equivalent of dismissing work in more abstract branches of mathematics on the grounds that this work lacks immediate applications to, say, aerospace engineering.*

This brings us to Paolo Mancosu's new collection, *The Philosophy of Mathematical Practice.* The authors of the papers in this collection share a vision of the philosophy of mathematics that is at once attentive to the history and practice of working mathematicians *and* to the necessity of making this history and practice philosophically relevant. Their papers have several distinctive features. First, they address a collection of philosophical issues that have been underdiscussed in the recent philosophical literature—e.g., the role of visualization in mathematical reasoning, the idea that some proofs are more "explanatory" than others, or the relevance of category theory to philosophical structuralism. Second, they treat a broader range of mathematical history and practice than has been customary in philosophical contexts: not just logic and arithmetic, but category theory, complex analysis, algebraic geometry, and number theory as well.[2] Finally, they insist on the relevance of this mathematics, not only to the new topics this volume aims to introduce, but also to classic topics in traditional philosophy of mathematics.

The collection begins with a brief introduction in which Mancosu situates the project against some other recent attempts at developing a more mathematically sophisticated philosophy of mathematics.[3] For mathematicians, this introduction will provide a useful guide to some of the most interesting work in recent philosophy of mathematics along with a picture of the larger philosophical milieu in which that work (and this volume) are embedded. Following the introduction, the book divides into eight sections, each of which contains a general introduction to a topic followed by a more focused research paper that develops one aspect of that topic in more depth. For reasons of space, I will group these sections under three general headings, and I will say more about the research papers than about the introductions.

What We See

The first two sections concern the use of diagrams and visualization in mathematics. In practice, these are things we use all the time: we draw pictures on the blackboard for our students, we include diagrams in our papers, and we make sketches for ourselves when trying to think through a proof. Since the nineteenth century, however, both mathematicians and philosophers have questioned whether such practices should play any *formal* role in mathematics. After all, geometric diagrams can be both imperfect and atypical, and the use of such diagrams seems to have led traditional geometers to overlook important axioms—e.g., axioms of completeness. As even Roger Nelson, the author of *Proofs without Words,* has insisted: "of course, 'proofs without words' are not really proofs."

The first two papers, by Marcus Giaquinto (University College London), concern visual reasoning in mathematics. Giaquinto's introductory paper highlights some of the (many) ways we use visualization: to help us prove theorems, to discover and motivate new results, and to deepen our *understanding* of particular pieces of mathematics. In the end, he argues that visual reasoning can play a legitimate—and an important—role in grounding both proof and understanding. His research paper explores the more specific role that visualization plays in grasping *particular* mathematical structures. Using some recent work in cognitive science, Giaquinto claims that we can, in fact, use visualization to gain knowledge of small finite structures, and he argues that this process can be extended to get a grasp on the structure of the whole set of natural numbers. He then explores how far this kind of visual thinking can take us—probably to the ordinal ω^2, less probably to the ordinal ω^ω, but almost certainly not to the whole set of real numbers.

Ken Manders (Pittsburgh) focuses on classical Euclidean geometry. In his introductory paper, he notes that whatever worries we *now* have about diagrammatic reasoning, diagram-based geometry was an extraordinarily successful mathematical practice for over 2,000 years. The success of this practice—the fact that it didn't run aground on the problems we have now become aware of—is something that philosophers need to explain. Manders's research paper explores the kinds of "diagram discipline" that enabled traditional geometers to avoid fallacies and to *legitimately* draw inferences from their diagrams. So, for instance, Manders distinguishes between *exact* features of diagrams—features, like the straightness of a line

[2] *These last two points should not be misconstrued. Neither the philosophical nor the mathematical topics discussed in this book are entirely original. Most of them have been discussed before (and often by these very authors!). Instead, the book is an attempt to highlight this kind of work by collecting some of its best practitioners together in one volume.*

[3] *Mancosu focuses, in particular, on recent work by Lakatos, Kitcher, Corfield, and Maddy. Most mathematicians will, I suspect, be familiar with Lakatos. Kitcher emphasizes the importance of historical studies of the growth of mathematical knowledge and of the changes in mathematical methodology and practice over time. Corfield argues that philosophers should pay more attention to "real mathematics"—to the mathematics that leading mathematicians are most interested in and to the ways that the mathematical community structures and conceptualizes its most central research programs. Maddy highlights the significance of informal aspects of mathematical justification—e.g., the sophisticated but nonrigorous arguments that set theorists give for accepting and/or rejecting new set-theoretic axioms.*

or the equality of two angles, that are unstable under even slight perturbations of the diagram—and *coexact features*—features, like the inclusion of one region in another or the intersection of two lines, that remain stable under appropriate (small) perturbations. He notes that, in practice, traditional geometers inferred only coexact attributes from a diagram and that the restriction to co-exact inferences explains the reliability of classical geometric reasoning. He also provides an illuminating discussion of indirect reasoning in geometry—reasoning in which the diagrams are *by design* inaccurate since they are supposed to represent geometric situations that cannot be instantiated. This paper is a modern classic that has circulated informally since the mid-1990s and that is published here for the first time; it is rich and deep and will repay careful, repeated reading.

What We Care About

Many philosophers have a simple picture of mathematics that focuses almost entirely on the issues of truth and proof: we start with simple, self-evident axioms, and we then ask whether various theorems can be proved from those axioms. For this picture, the only interesting questions involve the security of our axioms, the reliability of our proofs, and the ultimate truth of our theorems. Of course, this story misses much of the texture of day-to-day mathematical practice: theorems can be deep or shallow, definitions can be natural or unnatural, results can be constructive or conceptual, proofs can be more or less explanatory, etc. The next three sections of Mancosu's book look at some of these less formal—but no less important—ways that mathematicians evaluate their work.

The first two papers, by Mancosu himself and by Mancosu and Johannes Hafner (Berkeley and NC State, respectively), look at the notion of mathematical explanation, both in the sense of using mathematics to explain results in other disciplines—e.g., physics or economics—and in the sense of explanation within mathematics itself. Mancosu's introductory paper provides a short, but very clear, survey of some of the best recent philosophical work on these topics. As with his introduction to the volume as a whole, this paper will be particularly useful as a guide to those mathematicians who would like to read further in contemporary philosophy of mathematics. Mancosu and Hafner's research paper focuses on a model of "explanation as unification" that was developed by Philip Kitcher (and was explicitly intended to be applicable to mathematics). Using a test case from real algebraic geometry, they argue that Kitcher's model fails to capture many of the judgments about explanation that mathematicians actually make.[4] They conclude that, even if Kitcher's model is on roughly the right track, it will need substantial revision, guided by far more detailed analyses of a far wider range of mathematical test cases, before it can provide an adequate account of mathematical explanation.

The next two papers, by Mic Detlefsen (Notre Dame) and Michael Hallett (McGill), involve an issue that has come to be known as "purity of method"—roughly, the desire that mathematicians sometimes express to prove results in a *particular* branch of mathematics using only techniques *proper to* that branch of mathematics. By way of example, consider mid-twentieth-century number theorists' interest in obtaining a purely number-theoretic proof of the prime number theorem—i.e., a proof that does not use complex analysis. Or, consider the desire of group theorists to obtain a group-theoretic proof of Burnside's $p^a q^b$ theorem—a proof that does not appeal to representation theory. Finally, and most famously, consider nineteenth-century concern over the *casus irreducibilis*—the fact that when we use radicals to extract real roots of a cubic polynomial with rational coefficients, we often have to detour through the complex plane (even in cases where all of the roots of our polynomial turn out to be real).

Mic Detlefsen's paper provides a nice history of purity concerns in mathematics. He starts with Greek attempts to eliminate mechanical reasoning from geometric proofs, works forward through nineteenth-century attempts to remove geometric reasoning from analysis, and ends with some contemporary cases where mathematicians have expressed an interest in purity (e.g., the prime number theorem example from the last paragraph or the search for an "elementary" proof of the Erdös-Mordell theorem). Along the way, he discusses the *different* reasons mathematicians have given for pursuing purity and the various formal and epistemological virtues that pure proofs might be thought to have. Michael Hallett's paper follows this discussion with a rich and detailed study of the role purity played in Hilbert's axiomatization of geometry. Clearly, Hilbert had some purity concerns here—e.g., he wanted to eliminate

[4] *The case involves using the Tarski-Seidenberg decision procedure and/or the Tarski-Seidenberg transfer principle to prove general theorems about real closed fields. The transfer principle, for instance, allows us to use special properties of the reals—say, the Bolzano-Weierstrass property or the least upper bound property—to prove theorems about the real field and to then "transfer" these theorems to other real closed fields. This works even when the other fields lack the special properties used in the initial proofs. Many geometers feel that these kinds of "transcendental" arguments are less explanatory than purely algebraic arguments that work uniformly in all real closed fields.*

certain kinds of intuitive and diagrammatic reasoning from geometry. Hallett highlights the subtle and complicated interplay between these kinds of purity concerns and Hilbert's larger project of incorporating metamathematical reasoning into geometry.

Finally, Jamie Tappenden (Michigan) discusses the significance of good definitions in mathematics. His first paper uses the introduction of the Legendre symbol as a case study for exploring the role that fruitful consequences play in explicating the notion of a mathematically "natural" definition.[5] It also discusses the ways definitions can *change* in response to deeper understanding of a field—e.g., the realization by algebraic number theorists that the notion of primality is more fundamentally captured by the property

$$n \text{ is prime iff } n|ab \Rightarrow n|a \text{ or } n|b$$

than by the traditional,

$$n \text{ is prime iff } a|n \Rightarrow a = 1 \text{ or } a = n.$$

Tappenden's research paper traces these kinds of issues through the work of Riemann and Dedekind, and it explores the relationship between natural definitions in mathematics and some recent philosophical discussions of "natural properties". (Along the way, he makes some illuminating criticisms of "structuralist" accounts of the natural numbers as given by, e.g., Benacerraf, Shapiro, and Sider.) I should note that, although these two papers' focus is clearly philosophical, some sections are likely to be more accessible to mathematicians than to philosophers. Tappenden clearly feels space constraints when summarizing the mathematical evidence for his conclusions, and those who already know the mathematical context in detail will find these sections easier going than those who don't.

Who We Talk To

The final three sections of the book concern questions arising from the interaction of modern mathematics with other academic disciplines: computer science, philosophy, and physics. Of course some such questions have been widely discussed among philosophers—e.g., the applicability of mathematics to physics or the relevance of recursion theory to computer science—but the particular questions examined here have been far less commonly addressed.

Jeremy Avigad (Carnegie Mellon) focuses on the growing interaction between mathematics and computer science. The first half of his introduction explores the role computers increasingly play in generating (informal) evidence for mathematical assertions—e.g., in testing conjectures numerically or using computer verification to check a long and complicated proof. The issue here is not the traditional: "do computer proofs count as full-fledged proofs?" Rather, it's the more complicated question of whether we can give *any* philosophical account of the notions of mathematical plausibility that are in play in these more informal cases.[6] The remainder of Avigad's papers involve the relevance of computer science to the study of mathematical *understanding.* The research paper, in particular, provides a richly detailed discussion of some of the difficulties that arise when training computers to follow certain families of mathematical proofs, proofs that most human mathematicians can follow quite readily. By focusing on such cases, Avigad hopes to provide new insights into the ways human mathematicians actually understand proofs and to thereby give a deeper account of the nature of mathematical knowledge.

Colin McLarty (Case Western) explores the relationship between category theory and mathematical structuralism. Very roughly, structuralism is the claim that mathematics is concerned, not with particular mathematical objects, but only with general patterns or structures. Particular objects are defined in terms of their positions within such structures—i.e., their relations to other elements of the structures—but they have no identity conditions outside of those structures.[7] McLarty argues that the *right* way to develop structuralism is through category theory. His paper provides a whirlwind tour through some central episodes in the history of mathematics, leading up to the development of the notion of a *scheme.* He then

[5] *At first blush, the definition of the Legendre symbol is a paradigm of an unnatural and cobbled-together definition. It's only after we do some mathematics with the symbol—to understand its algebraic properties, its useful consequences, and its various generalizations—that we come to understand how natural the symbol really is.*

[6] *To see the problem here, consider a simple Bayesian approach to the issue. Since Bayesian theories assign probability* 1 *to all logical tautologies, they can't be used to model plausible reasoning about whether a given statement is, in fact, a tautology. Similarly, suppose that Goldbach's conjecture is false. Then there is a proof of this fact which uses only elementary arithmetic. So, Bayesian theories would insist that we should reject Goldbach's conjecture with (at least) the same confidence as we accept elementary arithmetic. But, of course, we don't yet know that elementary arithmetic disproves Goldbach's conjecture (even though the relevant proof is out there somewhere, we haven't yet discovered it). So, this kind of Bayesian analysis won't capture the reasoning we actually use when we assess the plausibility of the conjecture.*

[7] *So, for instance, the number* 2 *is completely defined by its position in the ordering of the natural numbers, and there's nothing to say about its nature or essence outside of that pattern. While both* $\{\{\varnothing\}\}$ *and* $\{\varnothing, \{\varnothing\}\}$ *may play the role of* 2 *in some particular set-theoretic reduction of arithmetic, it makes no sense to ask which of them really is the number* 2.

draws some illuminating lessons for philosophical structuralists concerning, e.g., the distinction between morphisms and functions, the significance of the difference between isomorphism and equivalence in category theory, and the role that category theory can play in solving (or dissolving) puzzles concerning mathematical ontology. As in the case of Tappenden's paper, I expect that the mathematical prerequisites of McLarty's paper will make it (far) more accessible to mathematicians than to philosophers.

The final two papers in this collection, both by Alasdair Urquhart (Toronto), concern the relevance of physics to mathematics. Urquhart's primary interest is in the ways physicists shamelessly abuse mathematics and yet, with some regularity, manage to parlay their weird mathematics into strikingly good physics. The core of Urquhart's research paper consists of a series of case studies—involving infinitesimals, the umbral calculus, and the Dirac delta function—in which physicists developed new mathematical techniques that mathematicians were only later—and, in some cases, *much* later—able to put on any kind of rigorous footing. Urquhart urges mathematicians and philosophers to take the lesson of these studies to heart and to view physics as an important source of new mathematical ideas (even, or perhaps especially, when the physicists themselves cannot explain the mathematical foundations of what they are doing). He ends with a final case study involving the Sherrington-Kirkpatrick model of spin glasses, a case in which physicists are currently making surprising progress by employing some extremely dodgy—and at this point inexplicable—mathematics.

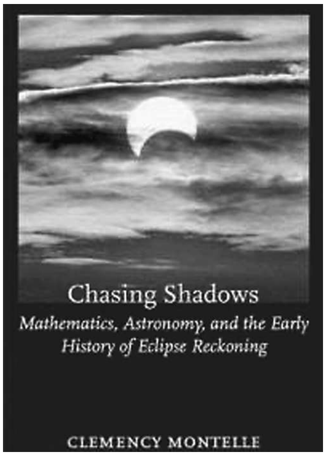
Concluding Remarks

This, then, gives us a picture of the details of Mancosu's book. Let me close with two somewhat more general comments. First, I want to emphasize just how good this book really is. The papers are clear and well written; the introductory surveys provide nice introductions to the relevant philosophical literature; the research papers address fresh topics which have been unduly neglected in recent philosophical discussion; and the mathematics in the book is both more varied and more central to mainstream mathematical practice than is typical in philosophical contexts. Taken as a whole, the book provides an excellent introduction to some of the most exciting and mathematically well-informed work in recent philosophy of mathematics, and it lays out an attractive agenda for future philosophical research.

Second, I want to issue a minor caution. In some cases, the papers feel just a bit too programmatic. Although they lay out interesting agendas for future work and take some preliminary steps towards fulfilling those agendas—asking important questions, clarifying key notions, and working through specific test cases—I often found myself wanting just a little more development and a little more argument in order to be fully convinced. Readers who are looking for airtight theses, completely detailed analyses, fully worked out arguments, and rigorously proved claims (in the philosopher's, not the mathematician's, sense of "rigorously proved"!) may find some sections of this book a bit frustrating.

That being said, this *caution* isn't intended as a genuine *criticism*. The purpose of this book is to lay out a vision of what the philosophy of mathematics could someday be—both by sketching the kinds of topics philosophers might turn their minds to and by making enough (local) progress on these topics to convince us that our efforts are likely to be rewarded. On these terms, the book succeeds splendidly. My desire for a more developed version of the project—say, a small bookshelf filled with monograph-length expansions of these papers—is merely a testament to this book's overall success.

Book Review



Chasing Shadows: Mathematics, Astronomy, and the Early History of Eclipse Reckoning

Reviewed by Christopher Linton

Chasing Shadows: Mathematics, Astronomy, and the Early History of Eclipse Reckoning
Clemency Montelle
Johns Hopkins University Press, 2011
US$75.00, 424 pages
ISBN-13: 978-0801896910

It is not hard to appreciate the enormous impact that a solar eclipse must have had on those witnessing such an event in times when the cause of eclipses was not understood. Lunar eclipses are much less dramatic but much more common (as far as an individual observer is concerned), and even here it is easy to imagine the sense of fear and wonder that they would have generated. The facts that a great deal of mythology grew up around eclipses, that people sought to be able to predict when they would occur, and that there was a desire to understand the causes behind the phenomena thus come as no surprise. But the details of how mathematics displaced superstition and how our physical understanding of the cosmos supplanted tradition and folklore are less well known.

There is an extensive literature on the history of ancient mathematical astronomy, much of it rather technical, and the contribution and influence of Otto Neugebauer (1899–1990) cannot be overstated. (Incidentally, Neugebauer founded *Mathematical Reviews* in 1940.) Neugebauer's monumental *History of Ancient Mathematical Astronomy*, published in three large volumes in 1975, set the standard for future scholarship and research. Neugebauer founded the History of Mathematics department at Brown University in 1947, and his and future generations of students have continued the program that he began. The author, Clemency Montelle, is a direct descendant in this line, having been supervised in her doctoral studies by Neugebauer's student David Pingree.

It was the Greeks who first understood eclipses as being caused by the spatial alignment of three bodies, and it is worth describing this geometry so as to introduce some terminology. The Moon's form changes over a period of about $29\frac{1}{2}$ days from thin crescent to full moon back to thin crescent again and then disappears for two or three nights. This time period is known as a lunation or synodic month, and it formed the basis of many ancient calendars. When the Moon is full, it is diametrically opposed to the Sun and we say that the Sun and the Moon are in opposition; whereas when the Sun and the Moon are in the same direction, at new moon, they are said to be in conjunction. The Moon's orbit is inclined to the plane of the Earth's orbit around the Sun (the ecliptic plane), and it crosses this plane at two points called the nodes, labelled as ascending or descending depending on whether the Moon is moving from south to north or vice versa. A lunar eclipse occurs when the Moon is in opposition to the Sun and simultaneously at (or near) one of its nodes. Similarly, a solar eclipse occurs when the Moon's passage through one of its nodes corresponds to a conjunction with the Sun.

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The period between successive ascending (or descending) nodes is known as the draconitic month, the word deriving from the Greek for dragon. In medieval times that part of the Moon's orbit south of the ecliptic was known as the dragon (which devoured the Moon during eclipses), and from this we get the terminology "dragon's head" for the ascending node and "dragon's tail" for the descending node. The speed with which the Moon moves across the sky relative to the stars varies between about 12° and 15° per day, and the period of time it takes for the Moon to return to the same speed is called the anomalistic month. This period is related to the elliptical nature of the lunar orbit and hence to variations in the apparent diameter of the Moon (which has a significant effect on eclipse phenomena).

Montelle's book focuses on the contributions of four different cultures to the understanding of eclipse phenomena: those of the ancient Near East, ancient Greece, India, and the Islamic Near East, all of which played a key role in shaping the broader tradition of "Western" astronomy. Attention is also given to the transmission of ideas and practice between the distinct scientific environments that existed. It is significant that the space devoted to Indian eclipse reckoning is more than double that afforded to any of the other cultures, which suggests that readers will find much more here than simply a rehash of material that is readily available elsewhere, and this is indeed the case. Most of the research which underpins the book was done during the past fifty years, and it is no exaggeration to say that our understanding of the achievements of Mesopotamian, Indian, and Islamic scholars has been transformed during that period. For example, Anton Pannekoek's classic *History of Astronomy*, originally published in 1951, brushes over Indian contributions in a couple of paragraphs within a short chapter covering the whole of Arabic astronomy.

Beginning in the ancient Near East, the omen compendium *Enūma Anu Enlil*, compiled roughly 4,000 years ago, contains about 7,000 omens, both auspicious and inauspicious, in the form *if* [something is observed], *then* [something will happen], which cover both astronomical and meteorological phenomena. Eclipses are a prominent theme, and many aspects of these phenomena are considered: time of impact, duration, even color. What is of particular interest though, from the perspective of the development of understanding, is the recognition that some aspects of eclipses are periodic (or at least roughly so) and therefore that they may plausibly be predicted. By the seventh century BCE, the facts that lunar eclipses can occur only on the fourteenth or fifteenth days of each lunation (counting from the new moon) while solar eclipses occur on the twenty-ninth or thirtieth day appear to have been established. Similarly, it was observed that lunar eclipses are spaced at six-monthly intervals, with occasional deviations from this rule.

The real flowering of the mathematical approach to heavenly phenomena in the ancient Near East came in the Seleucid era (311–125 BCE). The astronomers of that period had access to records of eclipses dating back over many centuries, and they appreciated that this was a rich source of data from which to extract patterns that could then be manipulated into predictive tools. The nature of the deviations from the six-month separation between lunar eclipses suggested that a more sophisticated cycle could be constructed for times at which eclipses might occur; in this cycle, a succession of six-month intervals was interspersed with an occasional five-month gap. This is particularly impressive, as it is now known that at no time during the centuries that these observations were made was there a pair of lunar eclipses separated by five months where both eclipses were potentially visible to the observers. The most notable such pattern is the so-called Saros cycle, containing 223 months and 38 eclipse possibilities, the accuracy of which is related to the rough equivalence between 223 synodic months, 242 draconitic months, and 239 anomalistic months. With modern average values, we have, in days,

$$\begin{aligned} 223 \times 29.531 &\approx 242 \times 27.212 \\ &\approx 239 \times 27.555 \approx 6585\tfrac{1}{3}. \end{aligned}$$

Montelle provides a detailed description of typical astronomical texts that were produced by Babylonian astronomers during the last three centuries BCE, showing how combinations of simple periodic schemes were used to model many recurring astronomical phenomena, including eclipses. Two distinct approaches were used: so-called system A, in which the Sun's nonuniform motion was modelled as a periodic step function; and the more sophisticated system B, in which alternate increasing and decreasing arithmetic progressions (often illustrated as a zig-zag function) were employed. The level of sophistication and predictive power of these techniques is hugely impressive, especially when one considers that there appears to have been no attempt to understand the causes of eclipses or to use a geometrical picture of the heavens to guide enquiry. It was ancient Greek thinkers who took on this challenge, but the legacy of observational records, astronomical conventions, and arithmetical techniques that the Greeks inherited from Mesopotamia were invaluable and instrumental to their success.

From a mathematical perspective, one of the most significant developments that arose from attempts to create a geometrical picture of the heavens was the emergence of what we now call trigonometry, beginning with the work of Hipparchus in the second century BCE. By the time Ptolemy wrote the *Almagest* three hundred years or so later, techniques of plane and spherical trigonometry were well understood. The *Almagest* includes complete geometrical models for the motion of the Sun, the Moon, and the planets, including tables with instructions for their use as well as details of their construction, and it dominated astronomical thought for more than a millennium. In terms of eclipses one of the key barriers to accurate predictions is the concept of parallax (the effect of the fact that the observer is displaced from the centre of the Earth), and the first detailed and complete account of parallax computations that we have is Ptolemy's.

The *Almagest* is an awe-inspiring work, but it is hardly user friendly, and Ptolemy appears to have appreciated this, since he later produced a much more practical reference for astronomers in the form of the *Handy Tables*. It appears that this handbook was constructed so as to be particularly useful in computing eclipses. The *Handy Tables* was commented on, reproduced, and modified many times over the centuries so as to adapt the contents to users' needs, but challenges to the fundamental theoretical basis on which it was built took a long time to materialize.

Outside of scholarly journals, relatively little has been written about Indian astronomy and virtually nothing which addresses in any depth the technical details contained within the considerable corpus of Sanskrit writings on the subject. *Chasing Shadows* is thus a significant contribution. Indian astronomy was built on a wealth of sources from other civilizations, initially Mesopotamian and Greek, but this material was expanded, modified and molded into a distinctly different cultural tradition. The format in which Indian astronomical works were preserved is certainly noteworthy, as they were written in verse. This would appear to have been to make them easier to memorize, as they were to be used in a largely oral environment, but as Montelle points out, there was a price to pay for this "scientific poetry". The author was forced to exclude key material or to use rather obscure expressions so as to conform to the strict metrical rules of the verse. For example, two systems were devised to represent numbers: the *bhūtasaṅkhyā* system, in which common objects were associated with numbers (eye = 2, gods = 33, for example), and the *kaṭapayādi* system, in which each number was assigned a set of letters so that meaningful words could be created to represent strings of numbers.

As well as a general discussion covering the various different Indian schools of astronomical thought that existed during the millennium from about 400 CE, the significance and impact of their work, and the general features of eclipse reckoning in India, Montelle gives a detailed description of a number of texts. For example, the *Pañcasiddhāntikā* of Varāhamihira (sixth century), a compilation of five earlier works, shows clear evidence of Greek and Babylonian sources, but the level of sophistication of the rules for accounting for parallax (which do not use the spherical trigonometry found in Ptolemy) compared with that in later Indian works shows that the transmission of ideas from the Near East to India was not limited to a single period in history. Other texts include Āryabhaṭa's *Āryabhaṭīya* (ca. 500 CE), the *Brāhmasphuṭasiddhānta* and the *Khaṇḍakhādyaka*, written about forty years apart during the seventh century by Brahmagupta, and Vaṫeśvara's *Vaṫeśvarasiddhānta* (ca. 900 CE). Montelle's elucidation of these texts is accompanied by numerous transliterations and translations of the original Sanskrit, many by the author herself.

Early trigonometry was built around the Greek chord function. Given a circle of radius R, the chord is related to the modern sine function via the relation $\operatorname{crd} 2\alpha = 2R \sin\alpha$. In particular, it is a length rather than a ratio. It was Indian astronomers who recognized that a lot of time could be saved in calculations by tabulating the half-chords of double angles ($\frac{1}{2}\operatorname{crd} 2\alpha$), a recognition that ultimately led to the modern sine function. They also introduced functions related to the modern cosine function and the now largely defunct versine ($\operatorname{versin}\alpha = 1 - \cos\alpha$). However, while these and a number of other technical improvements were implemented, there was little attempt to understand and improve the underlying models and parameters until perhaps the work of Parameśvara and the astronomical school that grew up in the southwestern region of Kerala in the fourteenth and fifteenth centuries.

Early Indian mathematical astronomy did exert a significant influence on the Islamic empire that grew up around the Mediterranean and flourished between the eighth and fourteenth centuries. The unifying feature of the scholarly tradition that emerged was the Arabic language, and a massive translation program was initiated very soon after the establishment of a new capital at Baghdad. Over a period of more than two hundred years, Indian, Persian, and, most significantly, Greek works were translated and assimilated. The *Almagest* was translated several times—the definitive version for the time being that of Isḥāq ibn Ḥunayn, as revised by Thābit ibn Qurra in the late ninth century—and served as the bedrock of Islamic astronomy.

Many astronomers produced works, each known as a *zīj*, a word of Persian origin which was used originally to mean a set of astronomical tables (of which the *Handy Tables* was the prototypical example) but which was later used to refer to any astronomical treatise. Thousands of these works survive to this day, most unedited and/or untranslated. Indian mathematical advances, such as their trigonometric functions, the decimal place value system, and the concept of zero, were quickly adopted, as were some other aspects of Indian astronomy, notably techniques for the calculation of the effects of parallax. Taken alongside Ptolemy's work, Islamic scholars were then equipped with the necessary tools to develop a sophisticated and accurate theory of eclipses. Equally significantly, they appreciated that judicious simplifications to the complicated machinery that had been developed could reduce computational effort (and hence increase utility) with minimal loss of accuracy.

Montelle discusses the contributions to Islamic eclipse reckoning of figures such as al-Khwārizmī (ninth century), from whose name we get the word algorithm and whose work was very much influenced by Brahmagupta and Vaṭeśvara, and Naṣīr al-Dīn al-Ṭūsī (thirteenth century), who was one of the most prominent figures in Islamic intellectual history and whose Ṭūsī couple (a geometrical device which produced motion in a straight line from a combination of circular motions) undermined the Aristotelian distinction between circular heavenly motion and rectilinear terrestrial motion.

The stated primary purpose of this book is to shed light on the ways in which knowledge about eclipses was originated, developed, preserved, and transmitted. Measured against this criterion, it is certainly a success. Parts of the work are heavy going—there is a limit to how many techniques for calculating the effect of parallax one can absorb at a single sitting—but one cannot fail to be impressed by Montelle's mastery of the subject matter and her commitment to increasing our understanding of such a rich history.

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Ethics for Undergraduate Researchers

Mike Axtell and Chad Westphal

The National Science Foundation (NSF) has been the primary funder of mathematics research experiences for undergraduates (REUs) in the nation since the 1960s. These REUs have allowed thousands of mathematics undergraduates to experience mathematical research years before they might encounter it during graduate school. In recent years, the NSF has also begun to push for an increased focus on the ethics of scientific inquiry. Since at least 2005 the NSF has funded ethics components in some mathematics REUs. Beginning in 2010 an ethics component became mandatory by law. Section 7009 of the America Creating Opportunities to Meaningfully Promote Excellence in Technology, Education, and Science (COMPETES) Act (42 U.S.C. 1862o-1) requires that

> *each institution that applies for financial assistance from the Foundation for science and engineering research or education describe in its grant proposal a plan to provide appropriate training and oversight in the responsible and ethical conduct of research to undergraduate students…participating in the proposed research project* [1].

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Wabash College has hosted an eight-week mathematics REU since 2005. From its inception, the Wabash REU has featured an ethics component, though the precise nature of this component has changed considerably. In the following we describe the format that we have found to be the most successful.

The secret is ice cream. In an attempt to more seamlessly integrate the ethics into the day-to-day running of the REU, we've developed a weekly Wednesday afternoon ethics session, known as *Ethics and Ice Cream*. Following the well-known student-behavior mantra "If you feed them, they will come," various ice cream novelties accompany each discussion, setting the scene for a comfortable but highly relevant minicourse on ethics. Each of the six ninety-minute ethics sessions is conducted as a group discussion involving the REU participants with two of the research mentors acting as guides.

The first two sessions feature a classic, and sometimes difficult, dose of Greek philosophy on the nature of *the good*. We begin with *The Meno* by Plato [2]. This text works well as an opener, and it has a lovely mathematical subtext to it. However, the main theme of the reading leads to a discussion on the nature of virtue, whether virtue can be taught, and what virtue (if any) is inherently common to us all. The goal here is to begin thinking of a foundation on which we may build a rationale for ethics.

The second session is, perhaps, the most challenging as we discuss the first book of Aristotle's *Nicomachean Ethics* [3]. Aristotle attempts to describe happiness and to determine the aim of ethics. This generates a good deal of spirited discussion among the participants (of all ages). While we may not all agree on a definition of *the good*, we attempt to identify some common ground.

These first two texts seek to provide a space to frame what ethics are and why we discuss them. With this background, we move forward in time to the present day and narrow our focus to the mathematical community. In the third session, the participants read and discuss the codes of conduct/ethics for a variety of professional organizations such as the AMS, MAA, and ACM, as well as statements on *Ethics in Publishing* from Elsevier; see [4], [5], [6], [7]. This discussion begins with questions on the purpose of these statements as well as attempts to distinguish similarities and differences in the documents. Surprisingly, these professional codes tend to be short and general, and simply asking how detailed a professional code of ethics should be exposes the difficulty of rigorously prescribing ethical behavior.

The final three sessions feature a discussion of various case studies consisting of ethical situations mathematicians might find themselves in at various stages of their career. For the first two sessions, we consider case studies that we, the research mentors, have developed. Students are given two or three case studies, and the discussion focuses on some subset of these as time allows. In particular, students wrestle with the tension between what is advantageous and what is ethical; for example:

- Is it ever ok to renege on accepting a graduate school or other job offer?
- Can one objectively referee a publication if there are competing motivations?
- What if you discover errors in your own work even if no one else does?
- How should one deal with authorship on publications when contributions are vastly unequal?

Very rarely do we agree on a simple solution. Our case studies are available at [8] and may be freely used and/or modified.

A side benefit of these case studies is the relevance of the context which provides opportunities to collectively and comfortably talk about topics such as the process of applying to graduate school; surviving and thriving in graduate school; the peer review process, both from the author and reviewer point of view; collaboration and coauthorship; tenure and promotion in an academic career; and balancing competing responsibilities under pressure. Many of the participants in an REU will pursue a career in the mathematical sciences, but few see clearly any details past finishing their undergraduate degree.

The final week of the ethics component features case studies again, but this time the students themselves create the situations. Students are asked to develop their own cases, which are combined into a single name-free document and distributed to all participants. The final ethics session then centers on the case studies they find most intriguing. These student-generated case studies prove to be a great way to end the ethics component, full of lively discussion, and by looking over all the case studies, we get a good look at the issues that are truly of concern to the participants. This knowledge can then be used to make the next summer's ethics component more engaging.

The participant response to this format has been quite positive. In the six years of conducting an ethics component within the REU, this method appears to be most meaningful to the participants. Below are two representative participant comments concerning the ethics component described above.

> *I really enjoyed the ethics component. I really liked that you brought up situations that could happen in my professional career that I had never thought about. It was nice to be able to talk about these potential situations as a group. I enjoyed hearing other peoples' opinions. Typically, I came into the room having my own thoughts on the situation, but then after listening to everyone else, I frequently left with an altered opinion about the situation.*

> *I thought at first that the ethics component was going to be a big waste of time. I generally dislike talking about ethics and philosophy, but the group and the professors really had some thought-provoking questions and interesting discussion. I doubted there could be any ethics involved in mathematics, but clearly there were many aspects of an academic career that I hadn't considered.*

An ethics component in a mathematics REU can be a difficult proposition. Mathematics, perhaps, has fewer inherent ethical issues than other sciences; however, there are certainly situations that can arise in mathematics that will put one's ethical code to the test. In addition, the very act of discussing what ethics are, why we have them, and how we make decisions in light of them is a healthy and invaluable practice for anyone in the mathematical sciences. The ethics component of our REU is designed to explore these issues in an open format. Its purpose is not to provide answers but to provide space to examine questions of ethics that may someday arise. Besides, defending your ideas to your peers in a friendly environment over ice cream is a pretty good way to spend a hot summer afternoon.

References

[1] http://edocket.access.gpo.gov/2009/E9-19930.htm, 2009.

[2] http://classics.mit.edu/Plato/meno.html.

[3] http://classics.mit.edu/Aristotle/nicomachaen.html.

[4] http://www.ams.org/about-us/governance/policy-statements/sec-ethics.

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[6] http://www.acm.org/about/code-of-ethics.

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SCRIPTA MANENT

...written words endure

Do Mathematicians Get the Author Rights They Want?

Kristine K. Fowler

"What do you want from your publisher?" is the way the IMU's CEIC Copyright Recommendations frames the issues an article author could consider [1], [2]. "How important is it to you to retain the author rights listed?" was a survey question I recently put to a random sample of mathematicians [3]. Whatever the wording, the underlying idea is that authors can manage the rights in their papers at a more granular level than may be apparent when offered a standard journal publication agreement. This column identifies the rights mathematicians say they want when publishing an article, which rights they often do not get, and how and why an author might keep the important ones.

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"Copyright" may be grammatically singular, but it is helpful to think of it as the plural "author rights". The author usually starts by owning all of them; when agreeing on publication terms, the separable rights in the copyright bundle may be divided in various ways between publisher and author. The author might, for example, give the publisher the exclusive right to publish the paper and distribute it in print. Author and publisher could somehow share the right to distribute it electronically, as might be specified in a clause allowing the author to post the paper on his/her website. The author might want to retain the right to use the content in his/her future articles or books (in legal terms, "preparing derivative works"). The CEIC recommends that the publisher authorize reprinting of the paper in collections [1], but this right might again be shared. There are myriad possible ways to apportion the various rights.

our REU is designed to explore these issues in an open format. Its purpose is not to provide answers but to provide space to examine questions of ethics that may someday arise. Besides, defending your ideas to your peers in a friendly environment over ice cream is a pretty good way to spend a hot summer afternoon.

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[6] http://www.acm.org/about/code-of-ethics.

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"Copyright" may be grammatically singular, but it is helpful to think of it as the plural "author rights". The author usually starts by owning all of them; when agreeing on publication terms, the separable rights in the copyright bundle may be divided in various ways between publisher and author. The author might, for example, give the publisher the exclusive right to publish the paper and distribute it in print. Author and publisher could somehow share the right to distribute it electronically, as might be specified in a clause allowing the author to post the paper on his/her website. The author might want to retain the right to use the content in his/her future articles or books (in legal terms, "preparing derivative works"). The CEIC recommends that the publisher authorize reprinting of the paper in collections [1], but this right might again be shared. There are myriad possible ways to apportion the various rights.

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Table 1. Rights Explicitly Retained by Authors in Standard Journal Publishing Agreements from Selected Mathematics Publishers							
	AMS	Cambridge U. Press	Elsevier	LMS	SIAM	Springer	Wiley-Blackwell
Email copies to others	x		x				x
Re-use part or all in future papers/books	x	x†	x		x	x	x†
Post author-created version on own website	x	x	x	x	x	x	x
Distribute photocopies to students	x	x	x	x			x
Others at university use copies in teaching	x		x	x			x
Deposit author-created version in the arXiv or other online repository	x	x†	x	x	x	x	x
Post publisher's version on own website	x	x			x		
Copyright	x*						

*Optional †Restricted

In practice, many journal publication agreements start with a blanket "copyright transfer" to the publisher, which then may grant back specified rights to the author(s). More than half the mathematicians I surveyed (over 600 respondents from a range of countries, research areas, and career stages [3]) deemed the following rights "very important" for an article author to have:

1. Email copies to others.
2. Post an author-created version on his/her own website.
3. Reuse part or all of the article in future papers/books.
4. Distribute photocopies to students.
5. Deposit an author-created version in the arXiv or other online repository.

An author wanting these rights should check for them in a journal publication agreement, since industry studies show that publishers usually permit some, but not all, of these rights. Posting an author-created version of a paper on the author's website is allowed by 60 percent of publishers [4], and the final published version by only 10 percent [5], yet these rights are considered at least somewhat important by over 80 percent of mathematicians [3]. Deposit in a subject repository like the arXiv presents an additional mismatch: 45 percent of publishers permit it [5], whereas 82 percent of mathematicians think it is at least somewhat important [3] even if they do not always exercise that right. A smaller majority (59 percent) consider it important to retain copyright as a whole [3], but still much larger than the 19 percent of publishers that allow it [4].

In mathematics journal publishing specifically, Table 1 shows the rights authors retain under the standard publishing agreements of seven selected publishers (American Mathematical Society (AMS), Cambridge University Press, Elsevier, London Mathematical Society (LMS), Society for Industrial and Applied Mathematics (SIAM), Springer, Wiley-Blackwell). All allow two of the rights mathematicians value highly—those of posting some author-created version of an article on the author's own website and in an institutional and/or subject repository—although some publishers require delays or otherwise restrict the latter, and almost all impose conditions such as crediting, but not posting, the published version. In some cases an author's future use of the article is also limited either as to the permitted quantity or type of use. Only one of these selected publication agreements allows the author to retain copyright, so there is the same mismatch with author desires as with the industry average. The top right mathematicians want, emailing copies to others, is mentioned by few; if, as has been suggested [5], most publishers actually allow this, it would be clearer for it to be explicitly stated, since less than half of mathematicians think it is allowed [3].

Some mathematicians have been remarkably effective in keeping the rights they want: among survey respondents who have negotiated with publishers to retain more rights, 92 percent report being usually or always successful [3]. Their most common approaches are attaching an addendum to or amending the terms of the contract. A source for the former is the Scholar's Copyright Addendum Engine, which provides several choices of modifications to attach to a standard publication agreement [6]. The latter may be as easy as crossing out terms in the agreement or writing in new ones. Other easy ways to retain rights include choosing journal publishers with author-friendly agreements (SHERPA RoMEO is a convenient website to check [8]) or taking advantage of options provided in a standard agreement, such as the steps for retaining copyright through the AMS form.

Despite the very high success rate, fewer than one in five mathematicians have acted to improve their rights position [3]. That does not mean that authors are satisfied with their publication agreements; some sign them reluctantly, and some sign without reading the terms carefully—or at all!

(More than a quarter of mathematicians have done this at least once [3].) Authors may be skeptical that it is worth paying attention to these contracts, since it is after all unlikely a publisher would sue a researcher for posting a paper on his/her website without permission. The major risks of ignoring copyright transfer terms are less about potential lawsuits from publishers and more about the publisher's ability to take actions that the author might morally expect to control but would not be legally entitled to. An author who wants to give—or refuse—permission for his/her paper to be included in a collected volume, for example, must take care not to sign away that right completely. The best-case scenario is to have a publication agreement both parties can truly agree to and abide by, that gives both publishers and authors the rights they feel are important. If a standard agreement does this, no effort has to be spent on adjustments. If not, an author need only take a small extra step, such as attaching an addendum; this could even forestall future rights negotiations, as "pressure from authors may lead a publisher to change his standard contract" [1]. Such moves would contribute to the Science Commons's goal: "Authors need to have the clear and unambiguous freedom to engage in their normal everyday scholarly activities without contending with complex technology, continuous amendments to contracts or the need for a lawyer" [7].

Journal publishing agreements have changed significantly, as have authors' and readers' ways of communicating, with the increase in electronic dissemination of research. Continued attention is needed by individual mathematicians, universities, scholarly societies, and publishers to fine-tune a normative balance of rights that best serves the mathematics community.

Acknowledgment

The author gratefully acknowledges the helpful advice of Carol Hutchins and Nancy Sims.

References

[1] International Mathematical Union Committee on Electronic Information and Communication, Copyright Recommendations, http://www.mathunion.org/ceic/Publications/Recommendations/7_copyright.shtml.

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[3] K. K. Fowler, Mathematicians' views on current publishing issues: A survey of researchers, *Issues in Science & Technology Librarianship* **67**, http://www.istl.org/11-fall/refereed4.html. Complete survey results available at http://purl.umn.edu/109309 .

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[5] S. Morris, Journal authors' rights: Perception and reality, PRC Summary Paper 5, 2009, http://www.publishingresearch.net/documents/JournalAuthorsRights.pdf.

[6] Science Commons, Scholar's Copyright Addendum Engine, http://sciencecommons.org/projects/publishing/scae/.

[7] Science Commons, Background Briefing: What would a solution look like? http://sciencecommons.org/projects/publishing/background-briefing .

[8] SHERPA RoMEO, Publisher copyright policies & self-archiving, http://www.sherpa.ac.uk/romeo/.

Mathematics People

Bhargava and Rodnianski Awarded Fermat Prize

Manjul Bhargava of Princeton University and Igor Rodnianski of the Massachusetts Institute of Technology have been awarded the 2011 Fermat Prize for Mathematics Research. Bhargava was honored "for his work on various generalizations of the Davenport-Heilbronn estimates and for his startling recent results (obtained with A. Shankar) on the average rank of elliptic curves," and Rodnianski was selected "for his fundamental contributions to the study of the equations of general relativity and of the propagation of light on curved space-times (obtained with M. Dafermos, S. Klainerman, and H. Lindblad)." Bhargava received his B.A. from Harvard University in 1996 and his Ph.D. from Princeton University in 2001. Rodnianski received his B.A. in physics from St. Petersburg University and his Ph.D. in mathematics from Kansas State University.

The Fermat Prize is awarded every two years by the Institut de Mathématiques de Toulouse to one or more mathematicians under the age of forty-five in one of the following fields: calculus of variations or, more generally, partial differential equations; probability; and number theory. The prize carries a cash award of 20,000 euros (approximately US$26,000).

—Elaine Kehoe

Wohlmuth Awarded Leibniz Prize

Barbara Wohlmuth of the University of Technology, Munich, has been awarded the 2012 Leibniz Prize by the Joint Committee of the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) for her research achievements in numerical analysis, which enable direct applications in scientific and engineering computing. The prize citation reads in part: "A focus of her research is the numerics of partial differential equations, to which she has made key contributions, especially with her theoretical study of mortar domain decomposition methods. With this work, and with its translation into practical techniques, she has achieved an internationally leading role in her field. Wohlmuth's research demonstrates an extraordinarily deep theoretical understanding that also produces better computational methods, for example in solid and fluid mechanics. In short, this new Leibniz Prize winner has advanced basic research with elegance, efficiency, and an eye on practical applications."

The Leibniz Prize is considered the most important research award given in Germany. It carries a cash award of 2.5 million euros (approximately US$3,300,000), which can be used for up to seven years at the prizewinner's own discretion for his or her scientific work.

—From a DFG announcement

Malek Awarded Marshall Sherfield Fellowship

Samar Malek of the Massachusetts Institute of Technology has been awarded a Marshall Sherfield Fellowship at the University of Bath, United Kingdom. A Ph.D. candidate in the Department of Civil and Environmental Engineering at MIT, she will conduct research on geometry and grid-shell structures at Bath. The fellowships, two of which are awarded each year, are funded by the Marshall Sherfield Fellowship Foundation and administered by the Marshall Commission; they enable American scientists or engineers to undertake postdoctoral research for a period of one to two academic years at a British university or research institute.

—Elaine Kehoe

Sontag Receives 2011 IEEE Control Systems Award

Eduardo D. Sontag of Rutgers University has been named the recipient of the 2011 Control Systems Award of the Institute of Electrical and Electronics Engineers (IEEE). He was honored for "fundamental contributions to nonlinear systems theory and nonlinear feedback control." His contributions include the control Lyapunov function (CLF), input-to-state stability (ISS), and related concepts that help in the design of stable nonlinear feedback systems. CLF provides control practitioners with the ability to make appropriate feedback control choices; the ISS concept helped tackle the difficulties presented by uncertainty in nonlinear systems. With ISS, he showed how to capture the effect of persistent disturbances in nonlinear systems, which has enabled engineers to solve many robust stabilization problems. The award is presented to an individual for outstanding contributions to control systems engineering, science or technology, and it consists of a bronze medal, certificate, and honorarium.

—From an IEEE announcement

Lasiecka Receives SIAM Reid Prize

Irena Lasiecka of the University of Virginia has received the 2011 W. T. and Idalia Reid Prize in Mathematics from the Society for Industrial and Applied Mathematics (SIAM). Established in 1993, this annual prize recognizes outstanding work in or other contributions to the broadly defined areas of differential equations and control theory. The prize fund was endowed by the late Mrs. Idalia Reid to honor her husband. The recipient of the prize receives a cash award of US$10,000 and an engraved medal.

Lasiecka was honored for "her fundamental contributions in control and optimization theory, particularly for dynamical systems governed by partial differential equations and their applications." She delivered a lecture at the SIAM Conference on Control and its Applications in July 2011.

—From a SIAM announcement

AAAS Fellows 2011

Six researchers have been elected fellows of the Section on Mathematics of the American Association for the Advancement of Science (AAAS). They are: Mark S. Alber, University of Notre Dame; Ingrid Daubechies, Duke University; Mark L. Green, University of California, Los Angeles; Claudia Neuhauser, University of Minnesota, Rochester; Richard A. Tapia, Rice University; and Roger Temam, Indiana University.

—From an AAAS announcement

2011 Siemens Competition

Several high school students whose work involves the mathematical sciences have won prizes in the Siemens Competition in Math, Science, and Technology.

Brian Kim, a senior at Stuyvesant High School, New York, New York, was awarded a US$50,000 scholarship for his project "Packing and Covering with Centrally Symmetric Disks". He was mentored by Dan Ismailescu of Hofstra University. For millennia, people have been interested in how we can efficiently pack more objects into an area. Brian Kim examined packing and covering geometric shapes, a topic that he says "could be understood and appreciated with a basic geometry background, but required power tools, particularly vectors, with which to make new ground." He was attracted to the idea of arranging shapes in space because this problem has been studied extensively by mathematicians. "The topic is simple yet at the same time extremely complex." Kim first recognized his passion for math after joining his school's math team. "There are no 'textbook problems' or solutions in math team, as ingenuity and cleverness are constant necessities." He enjoys running, golf, handball, and playing the guitar, piano, and trombone. He would like to major in applied mathematics or computer science and dreams of becoming a professor of mathematics at the Massachusetts Institute of Technology.

Sitan Chen, a senior at Northview High School, Duluth, Georgia, received a scholarship worth US$40,000 for his project "On the Rank Number of Graphs". He was mentored by Jesse Geneson of MIT. His mathematics project has potential applications in optimizing circuit design, finding errors in large data structures more efficiently, and manufacturing complex products in industrial systems more quickly. His research could potentially result in a new method of studying graphs, an important area of mathematics. He is inspired by mathematics because of "the power of a single new idea to change the way we look at the world around us." He was a National Finalist in the 2010 Siemens Competition. He is also an accomplished musician who has performed on piano and violin at Carnegie Hall. He is on the fencing team and organizes benefit concerts to raise funds for disaster relief. He hopes to study music or mathematics in college, with dreams of one day becoming a university professor.

The team of Andrew Xu, Lowell High School, San Francisco, California; Kevin Chang, Texas Academy of Mathematics and Science, Denton, Texas; and Kevin Tian, Westwood High School, Austin, Texas, were awarded a US$20,000 scholarship for their joint project "Determining the Existence of Graceful Valuations of Various Families of Graphs". They were mentored by Edward Early of St. Edward's University. The team developed three new algorithms to construct graceful labelings for several families of graphs. Using graph theory as a model, their results could provide an important contribution toward the Graceful Tree Conjecture, one of the most famous unsolved problems in graph labeling. Xu, a senior, is a winner of the Dong Lieu Science Prize and founder and president of ScienceDays, a program that brings hands-on science experiments to elementary schools. He works on creating worksheets for YouTube math videos (created by Vi Hart), and enjoys basketball, swimming, and playing the piano. He is exploring various majors and hopes to become a research mathematician. Chang, a junior, is a three-time Texas American Regions Math League Gold Team Member and has qualified multiple times for the U.S. Math and Junior Math Olympiads. He organized a MathStar club for elementary and middle school kids in his community and is president of the math club. He plans to major in math and business, with hopes of pursuing a career in one of those fields. Tian, a senior, was a Regional Finalist in last year's Siemens Competition. An accomplished musician, he is a viola player in his school orchestra and also plays the violin, piano, guitar, ukulele, and harmonica. He is fluent in Mandarin and proficient in French, active in community service, and enjoys playing basketball. He plans to major in economics or math and become a professor in mathematics or a related field.

—From a Siemens Competition announcement

Mathematics Opportunities

Call for Nominations for Prizes of the Academy of Sciences for the Developing World

The Academy of Sciences for the Developing World (TWAS) prizes are awarded to individual scientists in developing countries in recognition of outstanding contributions to knowledge in eight fields of science. Eight awards are given each year in the fields of mathematics, medical sciences, biology, chemistry, physics, agricultural sciences, earth sciences, and engineering sciences. Each award consists of a prize of US$15,000 and a plaque. Candidates for the awards must be scientists who have been working and living in a developing country for at least ten years.

The deadline for nominations for the 2012 prizes is **March 31, 2012**. Nomination forms should be sent to: TWAS Prizes, International Centre for Theoretical Physics (ICTP) Campus, Strada Costiera 11, 1-34151 Trieste, Italy; fax: 39 040 2240 7387; email: `prizes@twas.org`. Further information is available on the World Wide Web at `http://www.twas.org/`.

—From a TWAS announcement

AMS-Simons Travel Grants for Early-Career Mathematicians

The AMS is accepting applications for the second year of the AMS-Simons Travel Grants program. Each grant provides an early-career mathematician with US$2,000 per year for two years to reimburse travel expenses related to research. Sixty new awards will be made in 2012. Individuals who are not more than four years past the completion of the Ph.D. are eligible. The department of the awardee will also receive a small amount of funding to help enhance its research atmosphere.

The deadline for 2012 applications is **March 30, 2012**. Applicants must be located in the United States or be U.S. citizens. For complete details of eligibility and application instructions, visit `www.ams.org/programs/travel-grants/AMS-SimonsTG`.

—AMS announcement

For Your Information

Dynkin Interviews Digitized at Cornell

The Cornell University Library has acquired a collection of interviews of mathematicians conducted by Eugene Dynkin, Cornell's Emeritus A. R. Bullis Professor of Mathematics. Dynkin worked with the library's Division of Rare and Manuscript Collections and Digital Scholarship Services to organize and digitize his revolutionary conversations, many of which are interviews with Russian mathematicians. They are available online at `dynkincollection.library.cornell.edu`.

The interviews, which Dynkin recorded over more than half a century, are a rich source of information not only about mathematics but also about history, providing insight into academic life under a repressive Soviet regime. The collection contains nearly one hundred fifty audio and video recordings, plus biographical information about each mathematician and a select group of photographs.

Dynkin was born in Leningrad in 1924. He received a Ph.D. in 1948 from Moscow State University, where he was a faculty member for many years. Informal contact with Western colleagues was impossible during the Stalin era. "Western mathematical journals in the library were stamped 'Restricted Access. Only for Official Use'," he said. "Even after Stalin's death, like most Soviet mathematicians, I was not permitted to travel to Western countries. However, I was able to record a few conversations with foreign visitors to Moscow."

At the time that Dynkin and his wife immigrated to the United States in 1976, taking abroad any manuscript or audio recording needed the approval of an expert committee. Dynkin transferred his interviews from cassettes to small reels and left them with his friends, who later gave the reels to traveling American or Canadian colleagues to bring back to Dynkin at Cornell. He continued his interviews with mathematicians all over the world, although the Russian part of his collection was restricted to conversations with émigrés. After the end of the cold war Dynkin became able to interview former colleagues.

In the interviews, mathematicians discuss their family histories, other famous mathematicians, and current research. Although mathematics is the central focus of most of the interviews, a few contain hidden gems of mathematicians singing folk songs, performing operatic arias, and playing musical instruments.

Through the American Mathematical Society some funds were made available for the translation of the Russian-language material so that it would be accessible to the international community, but many more still need to be addressed before the site can assemble a complete English-language archive. To help in continuing to make the collection accessible to researchers, the library is asking those who listen to the interviews to contribute lists of the topics they contain, as well as transcripts or translations, to `rareref@cornell.edu`. More information can be found at `http://communications.library.cornell.edu/news/111107/dynkin`.

—From a Cornell University press release

Correction

The February issue of the *Notices* carried a review of the film *Top Secret Rosies*; the reviewer was Judy Green. Due to an editing error, the final footnote of the review was garbled. As a result, the first sentence of the last paragraph was imprecise. The point of that sentence is the following: While things are better for women, mathematicians and others, than they were in the 1940s, it took about fifty years for the original programmers of the ENIAC to be recognized.

The *Notices* regrets this error.

—Sandy Frost

Inside the AMS

AMS Hosts Congressional Briefing

Mathematics and stents was the subject of a congressional briefing hosted by the AMS on December 6, 2011. The Capitol Hill presentation, titled "Mathematics: Leading the Way for New Options in the Treatment of Coronary Artery Disease", was given by Suncica Canic of the University of Houston.

Coronary artery disease is a precursor for heart attack, the number one killer in the United States. Treatment of this disease entails inserting a stent to keep the coronary arteries open. Patient-specific decisions on the choice of a particular stent tailored to a given patient's anatomy are not common practice. This presentation showed how mathematics provides a quick and inexpensive way to make patient-specific decisions by testing the stent's behavior prior to the insertion into a patient's coronary artery. Prescribing mathematical and computer simulations, in addition to prescribing a blood test and angiogram, is the future of personalized medicine.

An AMS "Mathematical Moments", titled "Improving Stents" and encapsulating this research, is available at `http://www.ams.org/samplings/mathmoments/mm72-stent.pdf`.

The AMS holds annual congressional briefings as a means to communicate information to policymakers. Speakers discuss the importance of mathematics research and present their work in layman's terms to congressional staff as a way to inform members of Congress of how mathematics impacts today's important issues.

—Anita Benjamin, AMS Washington office

From the AMS Public Awareness Office

Mathematics History: AMS Books and Resources. See links to AMS Books (History of Mathematics Series, Collected Works Series, AMS Chelsea Publishing Series and Non-Series Books); Free Online Books (on American Mathematical Society and mathematical history); Free Online Resources (AMS Presidents: A Timeline, This Mathematical Month, Feature Column); and articles on mathematics history and mathematicians from *Notices of the AMS* at `http://www.ams.org/samplings/math-history/math-history`.

PhD + epsilon. Early-career mathematician Adriana Salerno blogs about her experiences and challenges. Recent topics include pretenure reviews, mathematicians in unlikely places, grading, and using Skype. She welcomes comments from mathematicians at all levels. See `http://blogs.ams.org/phdplus/`.

AMS on social networks. As part of the AMS commitment to the open flow of communications and community engagement, the Society uses several social networking tools to supplement the channels currently in place for press, members, and general communication. AMS members are invited to follow the AMS and connect with colleagues on AMS Facebook, AMS Twitter, and AMS LinkedIn. Link to all at `http://www.ams.org/about-us/social`.

—Annette Emerson and Mike Breen
AMS Public Awareness Officers
`paoffice@ams.org`

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.wustl.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

February 15, 2012: Applications for AMS Congressional Fellowship. See `http://www.ams.org/programs/ams-fellowships/ams-aaas/ams-aaas-congressional-fellowship` or contact the AMS Washington Office at 202-588-1100; `amsdc@ams.org`.

March 1, 2012: Applications for Summer Program for Women in Mathematics (SPWM). Contact the director, Murli M. Gupta, email: `mmg@gwu.edu`; telephone: 202-994- 4857; or visit the program's website at `http://www.gwu.edu/~spwm/`.

March 30, 2012: Applications for AMS-Simons Travel Grants for Early-Career Mathematicians. See "Mathematics Opportunities" in this issue.

March 31, 2012: Nominations for prizes of the Academy of Sciences for the Developing World (TWAS). See "Mathematics Opportunities" in this issue.

April 9, 2012: Applications for Math for America San Diego site. See the website at `http://www.mathforamerica.org/`.

May 1, 2012: Applications for National Academies Research Associateship Programs. See `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email `rap@nas.edu`.

May 1, 2012: Applications for National Academies Christine Mirzayan Graduate Fellowship Program for fall 2012. See the website `http://sites.nationalacademies.org/PGA/policyfellows/index.htm` or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202- 334-1667; email: `policyfellows@nas.edu`.

May 1, 2012: Applications for AWM Travel Grants. See `http://www.awm-math.org/travelgrants.html#standard`; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030; 703-934-0163; `awm@awm-math.org`.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

July 10, 2012: Full proposals for NSF Research Networks in the Mathematical Sciences. See `http://www.nsf.gov/pubs/2010/nsf10584/nsf10584.htm?WT.mc_id=USNSF_25&WT.mc_ev=click`.

August 1, 2012: Applications for National Academies Research Associateship Programs. See `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email `rap@nas.edu`.

October 1, 2012: Applications for AWM Travel Grants. See `http://www.awm- math.org/travelgrants.html#standard`; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030; 703-934-0163; `awm@awm-math.org`.

November 1, 2012: Applications for National Academies Research Associateship Programs. See `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email `rap@nas.edu`.

Board on Mathematical Sciences and Their Applications, National Research Council

The Board on Mathematical Sciences and Their Applications (BMSA) was established in November 1984 to lead activities in the mathematical sciences at the National Research Council (NRC). The mission of BMSA is to support and promote the quality and health of the mathematical sciences and their benefits to the nation. Following are the current BMSA members.

Tanya Styblo Beder, SB Consulting Corporation

Patricia Brennan, University of Wisconsin

Gerald G. Brown, Naval Postgraduate School

Ricardo Caballero, Massachusetts Institute of Technology

L. Anthony Cox, Cox Associates, Inc.

Brenda Dietrich, IBM Thomas J. Watson Research Center

Constantine Gatsonis, Brown University

Darryll Hendricks, UBS Investment Bank

Kenneth L Judd, Stanford University

C. David Levermore (Chair), University of Maryland

David Maier, Portland State University

James C. McWilliams, University of California, Los Angeles

Juan C. Meza, University of California, Merced

John W. Morgan, Stony Brook University

Vijayan N. Nair, University of Michigan

Claudia Neuhauser, University of Minnesota

J. Tinsley Oden, University of Texas at Austin

Donald Saari, University of California at Irvine

J. B. Silvers, Case Western Reserve University

George Sugihara, University of California, San Diego

Eva Tardos, Cornell University

Karen L. Vogtmann, Cornell University

Bin Yu, University of California, Berkeley

The postal address for BMSA is: Board on Mathematical Sciences and Their Applications, National Academy of Sciences, Room K974, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2421; fax: 202-334-2422; email: `bms@nas.edu`; website: `http://sites.nationalacademies.org/DEPS/BMSA/DEPS_047709`.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* `notices-booklist@ams.org`.

*Added to "Book List" since the list's last appearance.

The Adventure of Reason: Interplay between Philosophy of Mathematics and Mathematical Logic, 1900–1940, by Paolo Mancosu. Oxford University Press, January 2011. ISBN-13: 978-01995-465-34.

At Home with André and Simone Weil, by Sylvie Weil. (Translation of *Chez les Weils*, translated by Benjamin Ivry.) Northwestern University Press, October 2010. ISBN-13: 978-08101-270-43. (Reviewed May 2011.)

The Autonomy of Mathematical Knowledge: Hilbert's Program Revisited, by Curtis Franks. Cambridge University Press, December 2010. ISBN-13: 978-05211-838-95.

The Beginning of Infinity: Explanations That Transform the World, by David Deutsch. Viking Adult, July 2011. ISBN-13: 978-06700-227-55.

The Best Writing on Mathematics: 2010, edited by Mircea Pitici. Princeton University Press, December 2010. ISBN-13: 978-06911-484-10. (Reviewed November 2011.)

The Big Questions: Mathematics, by Tony Crilly. Quercus, April 2011. ISBN-13: 978-18491-624-01.

The Blind Spot: Science and the Crisis of Uncertainty, by William Byers. Princeton University Press, April 2011. ISBN-13:978-06911-468-43.

The Calculus Diaries: How Math Can Help You Lose Weight, Win in Vegas, and Survive a Zombie Apocalypse, by Jennifer Ouellette. Penguin, reprint edition, August 2010. ISBN-13: 978-01431-173-77.

The Calculus of Selfishness, by Karl Sigmund. Princeton University Press, January 2010. ISBN-13: 978-06911-427-53. (Reviewed January 2012.)

Chasing Shadows: Mathematics, Astronomy, and the Early History of Eclipse Reckoning, by Clemency Montelle. Johns Hopkins University Press, April 2011. ISBN-13: 978-08018-969-10. (Reviewed in this issue.)

The Clockwork Universe: Isaac Newton, the Royal Society, and the Birth of the Modern World, by Edward Dolnick. Harper, February 2011. ISBN-13: 978-00617-195-16.

Crafting by Concepts: Fiber Arts and Mathematics, by Sarah-Marie Belcastro and Carolyn Yackel. A K Peters/CRC Press, March 2011. ISBN-13: 978-15688-143-53.

Cycles of Time: An Extraordinary New View of the Universe, by Roger Penrose. Knopf, May 2011. ISBN-13: 978-03072-659-06.

Divine Machines: Leibniz and the Sciences of Life, by Justin E. H. Smith. Princeton University Press, May 2011. ISBN-13: 978-06911-417-87.

An Early History of Recursive Functions and Computability from Gödel to Turing, by Rod Adams. Docent Press, May 2011. ISBN-13: 978-09837-004-01.

**Emmy Noether's Wonderful Theorem,* by Dwight E. Neuenschwander. Johns Hopkins University Press, November 2010. ISBN-13: 978-08018-969-41.

The Evolution of Logic, by W. D. Hart. Cambridge University Press, August 2010. ISBN-13: 978-0-521-74772-1

Fascinating Mathematical People: Interviews and Memoirs, edited by Donald J. Albers and Gerald L. Alexanderson. Princeton University Press, October 2011. ISBN: 978-06911-482-98.

Gottfried Wilhelm Leibniz: The Polymath Who Brought Us Calculus, by M. B. W. Tent. AK Peters/CRC Press, October 2011. ISBN: 978-14398-922-20.

Hidden Harmonies (The Lives and Times of the Pythagorean Theorem), by Robert and Ellen Kaplan. Bloomsbury Press, January 2011. ISBN-13: 978-15969-152-20.

The History and Development of Nomography, by H. A. Evesham. Docent Press, December 2010. ISBN-13: 978-14564-796-26.

Hot X: Algebra Exposed, by Danica McKellar. Hudson Street Press, August 2010. ISBN-13: 978-15946-307-05.

**In Pursuit of the Unknown: 17 Equations that Changed the World,* by Ian Stewart. Basic Books, March 2012. ISBN-13: 978-04650-297-30.

The Information: A History, a Theory, a Flood, by James Gleick. Pantheon, March 2011. ISBN-13: 978-03754-237-27.

Knots Unravelled: From String to Mathematics, by Meike Akveld and Andrew Jobbings. Arbelos, October 2011. ISBN: 978-09555-477-20.

Le Operazioni del Calcolo Logico, by Ernst Schröder. Original German version of Operationskreis des Logikkalkuls and Italian translation with commentary and annotations by Davide Bondoni. LED Online, 2010. ISBN-13: 978-88-7916-474-0.

Loving + Hating Mathematics: Challenging the Myths of Mathematical Life, by Reuben Hersh and Vera John-Steiner. Princeton University Press, January 2011. ISBN-13: 978-06911-424-70.

Magical Mathematics: The Mathematical Ideas that Animate Great Magic Tricks, by Persi Diaconis and Ron Graham. Princeton University Press, November 2011. ISBN: 978-06911-516-49.

Mathematics and Reality, by Mary Leng. Oxford University Press, June 2010. ISBN-13: 978-01992-807-97.

Mathematics Education for a New Era: Video Games as a Medium for Learning, by Keith Devlin. A K Peters/CRC Press, February 2011. ISBN-13: 978-1-56881-431-5.

The Mathematics of Life, by Ian Stewart. Basic Books, June 2011. ISBN-13: 978-04650-223-80. (Reviewed December 2011.)

Mathematics, Religion and Ethics: An Epistemological Study, by Salilesh Mukhopadhyay. Feasible Solution LLC, September 2010. ISBN: 978-1-4507-3558-2.

Mysteries of the Equilateral Triangle, by Brian J. McCartin. Hikari, August 2010. ISBN-13: 978-954-91999-5-6. Electronic copies available for free at `http://www.m-hikari.com/mccartin-2.pdf`.

NIST Handbook of Mathematical Functions, Cambridge University Press, Edited by Frank W. J. Olver, Daniel W. Lozier, Ronald F. Boisvert, and Charles W. Clark. Cambridge University Press, May 2010. ISBN-13: 978-05211-922-55 (hardback plus CD-ROM); ISBN-13: 978-05211-406-38 (paperback plus CD-ROM). (Reviewed September 2011.)

**The Noether Theorems: Invariance and Conservation Laws in the Twentieth Century,* by Yvette Kosmann-Schwarzbach. Springer, December 2010. ISBN-13: 978-03878-786-76.

Numbers: A Very Short Introduction, by Peter M. Higgins. Oxford University Press, February 2011. ISBN 978-0-19-958405-5. (Reviewed January 2012.)

One, Two, Three: Absolutely Elementary Mathematics [Hardcover] David Berlinski. Pantheon, May 2011. ISBN-13: 978-03754-233-38.

Origami Inspirations, by Meenakshi Mukerji. A K Peters, September 2010. ISBN-13: 978-1568815848.

The Perfect Swarm: The Science of Complexity in Everyday Life, by Len Fisher. Basic Books, March 2011 (paperback). ISBN-13: 978-04650-202-49.

**The Philosophy of Mathematical Practice,* Paolo Mancosu, Editor. Oxford University Press, December 2011. ISBN: 978-01996-401-02. *(Reviewed in this issue.)*

Problem-Solving and Selected Topics in Number Theory in the Spirit of the Mathematical Olympiads, by Michael Th. Rassias. Springer, 2011. ISBN-13: 978-1-4419-0494-2.

Proof and Other Dilemmas: Mathematics and Philosophy, edited by Bonnie Gold and Roger A. Simons. Mathematical Association of America, July 2008. ISBN-13:978-08838-556-76. (Reviewed December 2011.)

The Proof is in the Pudding: A Look at the Changing Nature of Mathematical Proof, by Steven G. Krantz. Springer, May 2011. ISBN: 978-03874-890-87.

Proofiness: The Dark Arts of Mathematical Deception, by Charles Seife. Viking, September 2010. ISBN-13: 978-06700-221-68.

The Quants: How a New Breed of Math Whizzes Conquered Wall Street and Nearly Destroyed It, by Scott Patterson. Crown Business, January 2011. ISBN-13: 978-03074-533-89. (Reviewed May 2011.)

Roads to Infinity: The Mathematics of Truth and Proof, by John C. Stillwell. A K Peters/CRC Press, July 2010. ISBN-13: 978-15688-146-67.

Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving, by Sanjoy Mahajan. MIT Press, March 2010. ISBN-13: 978-0-262-51429-3. (Reviewed August 2011.)

Survival Guide for Outsiders: How to Protect Yourself from Politicians, Experts, and Other Insiders, by Sherman Stein. BookSurge Publishing, February 2010. ISBN-13: 978-14392-532-74.

The Theory That Would Not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy, by Sharon Bertsch McGrayne. Yale University Press, April 2011. ISBN-13: 978-03001-696-90.

Top Secret Rosies: The Female Computers of World War II. Video documentary, produced and directed by LeAnn Erickson. September 2010. Website: http://www.topsecretrosies.com. (Reviewed February 2012.)

Towards a Philosophy of Real Mathematics, by David Corfield. Oxford University Press, April 2003. ISBN-13: 0-521-81722-6. (Reviewed November 2011.)

Train Your Brain: A Year's Worth of Puzzles, by George Grätzer. A K Peters/CRC Press, April 2011. ISBN-13: 978-15688-171-01.

Viewpoints: Mathematical Perspective and Fractal Geometry in Art, by Marc Frantz and Annalisa Crannell. Princeton University Press, August 2011. ISBN-13: 978-06911-259-23.

Visual Thinking in Mathematics, by Marcus Giaquinto. Oxford University Press, July 2011. ISBN-13: 978-01995-755-34.

What's Luck Got to Do with It? The History, Mathematics and Psychology of the Gambler's Illusion, by Joseph Mazur. Princeton University Press, July 2010. ISBN: 978-0-691-13890-9. (Reviewed February 2012.)

Why Beliefs Matter: Reflections on the Nature of Science, by E. Brian Davies. Oxford University Press, June 2010. ISBN13: 978-01995-862-02.

Classified Advertisements

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SabbaticalHomes.com. Do you have a home to rent? Are you looking for housing while on sabbatical? We are the academic community's resource for home rentals and home swaps worldwide.

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CHILE

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
Department of Mathematics

The Department of Mathematics invites applications for three tenure-track positions at the Assistant Professor level beginning either March or August 2013. Applicants should have a Ph.D. in mathematics, proven research potential either in pure or applied mathematics, and a strong commitment to teaching and research. The regular teaching load for assistant professors consists of three one-semester courses per year, reduced to two during the first two years. The annual salary will be US$47,000 (calculated at the current exchange rate of 500 chilean pesos per dollar).

Please send a letter indicating your main research interests, potential collaborators in our department (www.mat.puc.cl), detailed curriculum vitae, and three letters of recommendation to:

Monica Musso
Director
Pontificia Universidad Católica
de Chile
Av. Vicuña Mackenna 4860
Santiago, Chile;
fax: (56-2) 552-5916;
email: mmusso@mat.puc.cl

For full consideration, complete application materials must arrive by June 30, 2012.

000023

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2011 rate is \$3.25 per word. No discounts for multiple ads or the same ad in consecutive issues. For an additional \$10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: April 2012 issue–January 30, 2012; May 2012 issue–February 28, 2012; June/July 2012 issue–April 30, 2012; August 2012 issue–May 29, 2012; September 2012 issue–June 28, 2012; October 2012 issue–July 26, 2012.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 667 (vol. 56).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to classads@ams.org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

The Theory That Would Not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy, by Sharon Bertsch McGrayne. Yale University Press, April 2011. ISBN-13: 978-03001-696-90.

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Please send a letter indicating your main research interests, potential collaborators in our department (`www.mat.puc.cl`), detailed curriculum vitae, and three letters of recommendation to:

Monica Musso
Director
Pontificia Universidad Católica de Chile
Av. Vicuña Mackenna 4860
Santiago, Chile;
fax: (56-2) 552-5916;
email: `mmusso@mat.puc.cl`

For full consideration, complete application materials must arrive by June 30, 2012.

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Leroy P. Steele Prizes

Call for Nominations

The selection committee for these prizes requests nominations for consideration for the 2013 awards. Further information about the prizes can be found in the January 2012 *Notices*, pp. 79–100 (also available at http://www.ams.org/prizes-awards).

Three Leroy P. Steele Prizes are awarded each year in the following categories: (1) the Steele Prize for Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) the Steele Prize for Mathematical Exposition: for a book or substantial survey or expository-research paper; and (3) the Steele Prize for Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. In 2013 the prize for Seminal Contribution to Research will be awarded for a paper in logic.

Nomination with supporting information should be submitted to http://www.ams.org/profession/prizes-awards/nominations. Include a short description of the work that is the basis of the nomination, including complete bibliographic citations. A curriculum vitae should be included. Those who prefer to submit by regular mail may send nominations to the secretary, Robert J. Daverman, American Mathematical Society, 238 Ayres Hall, Department of Mathematics, University of Tennessee, Knoxville, TN 37996-1320. Those nominations will be forwarded by the secretary to the relevant prize selection committee.

Deadline for nominations is April 30, 2012.

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at http://www.ams.org/cgi-bin/mathcal-submit.pl.
The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

March 2012

1–3 **Trends in Mathematical Analysis**, Dipartimento di Matematica "F.Brioschi" Politecnico di Milano, Milan, Italy. (Feb. 2012, p. 337)

* 2–3 **Interuniversity Seminar on Research in the Mathematical Sciences (SIDIM) XXVII**, University of Puerto Rico, Mayagüez, Puerto Rico.
Description: The SIDIM is an annual gathering where researchers, professors, teachers, graduate and undergraduate students share and discuss their recent work in mathematics, computer science and related fields. Its diverse and intense scientific program provides an interesting cross section of the current status of mathematical research in Puerto Rico. Math faculty and students from all the higher learning institutions in Puerto Rico, as well as distinguished visiting mathematicians will be among the 250 expected participants of the meeting.
Information: http://www.sidim.pr.

3–4 **AMS Western Section Meeting**, University of Hawaii, Honolulu, Hawaii. (May 2011, p. 744)

* 3–4 **Redbud Topology Conference – Rank vs. Genus**, Oklahoma State University, Tulsa, Oklahoma.
Description: This conference is focused around Tao Li's recent examples of hyperbolic 3-manifolds whose Heegaard genera are strictly greater than the rank of their fundamental groups.
Information: http://www.math.okstate.edu/~jjohnson/redbud/.

5–9 **5th International Conference on High Performance Scientific Computing**, Institute of Mathematics, Vietnam Academy of Science and Technology, 18 Hoang Quoc Viet Road, Hanoi, Vietnam. (Dec. 2010, p. 1498)

* 6–10 **Anomalous Statistics, Generalized Entropies, and Information Geometry**, Nara Women's University, Nara City, Japan.
Description: The aim of this workshop is to focus on recent developments on the anomalous statistics in the wide scientific framework. More specifically, nonextensive statistical mechanics in physics and information geometry in mathematics have been recognized to have close theoretical relations. Their collaborations for not only foundations of statistical mechanics but also applications in many fields are strongly expected. Their applications include statistical and information sciences in addition to complex systems in physics, chemistry, economics, geophysics, astrophysics, biology, networks and quantum systems.
Information: http://sites.google.com/site/next2012nara/home.

9–10 **International Conference on Advances in Computing and Emerging E-Learning Technologies (ICAC2ET 2012)**, Hotel Marina, Singapore, Singapore. (Jan. 2012, p. 102)

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

10–11 **AMS Southeastern Section Meeting**, University of South Florida, Tampa, Florida. (May 2011, p. 744)

11–14 **Fourth International Conference on Mathematical Sciences, ICM2012**, UAE University, Al-Ain, United Arab Emirates. (Dec 2011, p. 1615)

11–31 **Branching Laws**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Jan. 2012, p. 102)

12–June 15 **Computational Methods in High Energy Density Plasmas**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Mar. 2011, p. 497)

12–16 **AIM Workshop: Classifying fusion categories**, American Institute of Mathematics, Palo Alto, California. (May 2011, p. 744)

12–16 **ICERM Workshop: Global Arithmetic Dynamics**, ICERM, Providence, Rhode Island. (Jun/Jul. 2011, p. 862)

13–14 **International Conference on Internet & Cloud Computing Technology (ICICCT 2012)**, Hotel Marina, Singapore, Singapore. (Jan. 2012, p. 102)

13–16 **Computational Methods in High Energy Density Plasmas: Tutorials**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Feb. 2012, p. 337)

14–16 **IAENG International Conference on Operations Research 2012**, Royal Garden Hotel, Kowloon, Hong Kong. (Sept. 2011, p. 1184)

* 16–17 **LONI Pipeline Training Workshop**, Cincinnati Children's Hospital, 3333 Burnet Avenue, Cincinnati, Ohio.
Description: The LONI Pipeline environment is a free distributed workflow application that enables users to quickly create computational processing protocols, execute these as graphical workflows, monitor the state of their analyses, and broadly disseminate the detailed provenance of data and processing protocols. This workshop will provide hands-on training on remote Pipeline server installation, client-server interfaces, utilization of Grid resources using the distributed Pipeline computational infrastructure, design of new and modification of existent heterogeneous data analysis protocols, sharing of data and pipeline workflows, integration of imaging, demographic and meta-data. The organizers will provide a detailed training handbook and supplementary electronic media with all of the necessary software and test data. Who may be interested in this event? Neuroimaging researchers, computational scientists, bioinformaticians.
Information: `http://pipeline.loni.ucla.edu/training/cchmc2012/`.

* 16–18 **ICMMSC2012**, Gandhigram Rural Institute, Deemed University, Dindigul, Tamilnadu/India.
Description: Computational Science is a main pillar of most of the present research, industrial and commercial activities. Logic and Information play a unique role which are the base ideas of day to day human life. In exploiting Information and Communication Technologies as innovative technologies, science relies on the use of these, its controllability according to the human needs. The ICMMSC 2012 is aimed to offer a real opportunity to discuss new issues, tackle complex problems and find advanced enabling solutions able to shape new trends in Computational and Information Sciences
Information: `http://www.mathgru.com`.

* 17 **Bernd Math Day**, University of California, Berkeley, California.
Description: The 24th Bay Area Discrete Math Day in honor of Bernd Sturmfels' 50th birthday (Bernd Math Day) will take place at the University of California, Berkeley, in Evans Hall, Room 10 on Saturday, March 17, 2012, between 9AM and 5PM. There is no registration fee. All are welcomed.
Information: `http://www.msri.org/people/members/chillar/badmathday/`.

17–18 **AMS Eastern Section Meeting**, George Washington University, Washington, District of Columbia. (May 2011, p. 744)

19–21 **Topology: Quantitative and Applied**, Tulane University, New Orleans, Louisiana. (Jan. 2012, p. 102)

22–24 **46th Annual Spring Topology and Dynamics Conference**, Universidad Nacional Autonoma de Mexico, Mexico City, Mexico. (Feb. 2012, p. 337)

24–25 **36th Annual SIAM Southwastern Atlantic Section Conference**, University of Alabama in Huntsville, Huntsville, Alabama. (Sept. 2011, p. 1184)

25–26 **International Conference on Information Technology, System & Management (ICITSM 2012)**, Abu Dubai, United Arab Emirates. (Jan. 2012, p. 102)

25–28 **Conference on Partial Differential Equations and Applications**, Vietnam National University, Hanoi, Vietnam. (Jun/Jul. 2011, p. 862)

* 26–29 **Ischia Group Theory 2012**, Grand Hotel delle Terme Re Ferdinando, Ischia, Naples, Italy.
Description: The meeting will consist of talks given by invited speakers and a permanent poster session. The scientific programme will be dedicated to the memory of Narain D. Gupta on Wednesday and to the memory of James Wiegold on Thursday. Conference proceedings will be published by the University of Isfahan as an Issue or Volume of *International Group Theory Conference*. The social programme will include a welcome cocktail on monday late afternoon, a recital of classical Neapolitan songs on tuesday evening, a social trip on wednesday morning, and the conference dinner on thursday evening.
Information: `http://www.dipmat.unisa.it/ischiagroup-theory`.

26–30 **AIM Workshop: Cohomological methods in abelian varieties**, American Institute of Mathematics, Palo Alto, California. (Aug. 2011, p. 1012)

26–30 **Computational Challenges in Hot Dense Plasmas**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Feb. 2012, p. 337)

26–30 (NEW DATE) **IMA Workshop: Machine Learning: Theory and Computation**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Jan. 2011, p. 85)

26–30 **Statistical Mechanics and Conformal Invariance**, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2011, p. 1616)

30–31 **Information and Econometrics of Networks**, American University, Washington, District of Columbia. (Mar. 2011, p. 497)

30–31 **International Conference on Human Computer Interaction & Learning Technologies (HCILT 2012)**, Abu Dubai, United Arab Emirates. (Jan. 2012, p. 102)

30–April 1 **AMS Central Section Meeting**, University of Kansas, Lawrence, Kansas. (May 2011, p. 744)

30–April 1 **Around scattering by obstacles and billiards**, University of Aveiro, Aveiro, Portugal. (Dec. 2011, p. 1616)

30–April 1 **Call for Papers: 2012 International Conference on e-Commerce, e-Administration, e-Society, e-Education, and e-Technology**, Hong Kong, Japan. (Dec. 2011, p. 1616)

April 2012

1–4 **The 8th International Conference on Scientific Computing and Applications (SCA2012)**, University of Nevada Las Vegas (UNLV), Las Vegas, Nevada. (Aug. 2011, p. 1012)

2–5 **SIAM Conference on Uncertainty Quantification (UQ12)**, Raleigh Marriott City Center Hotel, Raleigh, North Carolina. (Aug. 2011, p. 1012)

2–6 **AIM Workshop: Vector equilibrium problems and their applications to random matrix models**, American Institute of Mathematics, Palo Alto, California. (Aug. 2011, p. 1013)

4–6 **Advances in Scientific Computing, Imaging Science and Optimization: Stan Osher's 70th Birthday Conference**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Feb. 2012, p. 338)

8–9 **International Conference on Information Systems, Engineering & Management Science (ICISEMS 2012)**, Hong Kong, Japan. (Jan. 2012, p. 102)

9–13 **AIM Workshop: Nonlinear solvers for high-intensity focused ultrasound with application to cancer treatment**, American Institute of Mathematics, Palo Alto, California. (Nov. 2011, p. 1494)

* 9–14 **Workshop on "Stochastic Analysis and Applications"**, Ksar Kaissar, El Kelaa Mgouna (City of Roses).
Description: The workshop is organized within the framework of the Marie Curie Initial Training Network (ITN). Deterministic and Stochastic Controlled Systems and Applications. (PITN-GA-2008-213841) It focuses on stochastic analysis and related topics including applications in control, finance and insurance. The workshop will be held in Kasr Kaisar El Kala Mgouna. We plan invited talks of 50–60 minutes and contributed talks of 30–45 minutes.
Information: `http://www.ucam.ac.ma/itn-marrakech/event.html`.

13–15 **Underrepresented Students in Topology and Algebra Research Symposium (USTARS)**, University of Iowa, Iowa City, Iowa. (Jan. 2012, p. 102)

* 14–15 **Great Lakes Geometry Conference 2012**, Ohio State University, Columbus, Ohio.
Description: The Great Lakes Geometry Conference is a well established conference series held annually in the Great Lakes region, rotating among different universities.
Aim: The aim of this conference series is to bring together distinguished speakers in a variety of areas in geometry, topology, and mathematical physics.
Topics: The 2012 edition of the Conference will feature topics related to complex geometry, geometric analysis, and mirror symmetry.
Speakers: Bennett Chow (UCSD), Matthew J. Gursky (Notre Dame), Naichung Conan Leung (CUHK), Zhiqin Lu (UC Irvine), Duong Hong Phong (Columbia), Valentino Tosatti (Columbia), Eric Zaslow (Northwestern), Steve Zelditch (Northwestern).
Information: If you plan to attend the conference, please register by sending an e-mail to `glgc2011@yahoo.com` with your name, title, and affiliation. Partial support is available for graduate students, post docs, and unsupported faculties. Contact the organizers Bo Guan, Hsian-Hua Tseng, Damin Wuat at `glgc2011@yahoo.com`; `http://www.math.osu.edu/~tseng.109/glgc12_main.html`.

16–17 **2nd International Conference on E-Learning & Knowledge Management Technology (ICEKMT 2012)**, Hotel Corus, Kuala Lumpur, Malaysia. (Feb. 2012, p. 338)

16–20 **Computational Challenges in Magnetized Plasma**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Feb. 2012, p. 338)

16–20 **ICERM Workshop: Moduli Spaces Associated to Dynamical Systems**, ICERM, Providence, Rhode Island. (Jun/Jul. 2011, p. 862)

18–21 **Fourth Discrete Geometry and Algebraic Combinatorics Conference**, South Padre Island, Texas. (Feb. 2012, p. 338)

19–21 **XIV International Scientific Conference Devoted to the memory of academician M. Kravchuk (1892–1942)**, National Technical University of Ukraine, KPI., Institute of Mathematics of NASU, National Shevchenko University, National Drahomanov Pedagogical University Kyiv, Ukraine. (Feb. 2012, p. 338)

20–22 **The Fifteenth Riviere-Fabes Symposium on Analysis and PDE**, School of Mathematics, University of Minnesota, Minneapolis, Minnesota. (Sept. 2011, p. 1185)

22–28 **Variational Analysis and Its Applications**, Paseky nad Jizerou, Czech Republic. (Jan. 2012, p. 102)

23–25 **5TH IMA International Conference on Analytical Approaches to Conflict**, Royal Military Academy, Sandhurst, United Kingdom. (Oct. 2011, p. 1324)

23–27 **Spring School in Probability**, Inter-University Center, Dubrovnik, Croatia. (Nov. 2011, p. 1494)

27–28 **International Conference on Web Information System and Computing Education**, Bangkok, Thailand. (Feb. 2012, p. 338)

29–May 2 **4th International Interdisciplinary Chaos Symposium on "Chaos and Complex Systems"**, Antalya, Turkey. (Nov. 2011, p. 1494)

30–May 5 **Random Walks and Random Media**, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2011, p. 1013)

May 2012

* 4–10 **Annual Spring Institute on Noncommutative Geometry and Operator Algebras**, Vanderbilt University, Nashville, Tennessee.
Description: The Tenth Annual Spring Institute on Noncommutative Geometry and Operator Algebras will take place at Vanderbilt University under the direction of Vaughan Jones. The topic of this year's conference/school is "Conformal Field Theory and von Neumann Algebras".
Information: `http://www.math.vanderbilt.edu/~ncgoa12/`.

7–11 **AIM Workshop: Motivic Donaldson-Thomas theory and singularity theory**, Renyi Institute, Budapest, Hungary. (Oct. 2011, p. 1325)

7–11 **IMA Workshop: User-Centered Modeling**, Institute for Mathematics and its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Oct. 2011, p. 1185)

7–11 **Mathematical and Computer Science Approaches to High Energy Density Physics**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Feb. 2012, p. 338)

10–11 **MCAG 2012: Michigan Computational Algebraic Geometry 2012**, Oakland University, Rochester, Michigan. (Dec. 2011, p. 1616)

10–12 **25th Cumberland Conference on Combinatorics, Graph Theory, and Computing**, East Tennessee State University, Johnson City, Tennessee. (Jan. 2012, p. 103)

10–12 **International Conference on Functional Equations and Geometric Functions and Applications (ICFGA 2012)**, Department of Mathematics, Payame Noor University, Tabriz, Iran. (Feb. 2012, p. 338)

14–18 **AIM Workshop: ACC for minimal log discrepancies and termination of flips**, American Institute of Mathematics, Palo Alto, California. (Aug. 2011, p. 1013)

* 14–18 **Microeconometrics with Focus on Panel Data and Discrete Choice: Theory and Practice**, American University, Washington, DC.
Description: Microeconometrics with Focus on Panel Data and Discrete Choice: Theory and Practice, May 14–18 (Monday-Friday), 2012. Instructor: William Greene (NYU). Details to be announced shortly on Info-Metrics Summer Classes web page.
Information: `http://www.american.edu/cas/economics/info-metrics/econometrics.cfm`.

14–25 **School and Workshop on Random Polymers and Related Topics**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Jan. 2012, p. 103)

17–19 **International Conference on "Applied Mathematics and Approximation Theory 2012"**, TOBB University of Economics and Technology, Ankara, Turkey. (Aug. 2011, p. 1013)

17–19 **Symmetries of Differential Equations: Frames, Invariants and Applications. A conference in honor of the 60th birthday of Peter Olver**, University of Minnesota, Minneapolis, Minnesota. (Dec. 2011, p. 1616)

20–22 **SIAM Conference on Imaging Science (IS12)**, Doubletree Hotel Philadelphia, Philadelphia, Pennsylvania. (Sept. 2011, p. 1185)

20–25 **7th European Conference on Elliptic and Parabolic Problems**, Gaeta, Italy. (May 2011, p. 744)

21–25 **AIM Workshop: Contact topology in higher dimensions**, American Institute of Mathematics, Palo Alto, California. (Aug. 2011, p. 1013)

21–25 **CANT 2012—School and Conference on Combinatorics, Automata and Number Theory**, CIRM–Centre International de Rencontres Mathématiques: International center of Mathematical Meetings, Marseille, France. (Jan. 2012, p. 103)

21–25 **Computational Challenges in Warm Dense Matter**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Feb. 2012, p. 338)

* 21–25 **International Topological Conference "Alexandroff Readings"**, Moscow State University, Moscow, Russia.
Description: The conference is traditionally organized at the Moscow State University in honour of P. S. Alexandroff (1896–1982).
Topics: K-Theory, algebraic and geometric topology, toric topology and combinatorics, knot theory, symplectic geometry and topology, dynamics of hamiltonian systems, geometric methods in non-linear equations, geometric variational problems, topological groups and transformation groups, geometric topology, discrete computational geometry, computer geometry, modern differential geometry, set-theoretic topology, dimension theory.
Information: `http://mech.math.msu.su/~manuilov/Alexandroff.html`.

28–June 1 **Workshop on Nonlinear Partial Differential Equations on the occasion of the sixtieth birthday of Patrizia Pucci**, Perugia, Italy. (Jan. 2012, p. 103)

28–June 2 **BALWOIS 2012 Conference (Fifth International Scientific Conference on Water, Climate and Environment**, Ohrid, Macedonia. (Jan. 2012, p. 103)

28–June 3 **International Conference "Theory of Approximation of Functions and its Applications"**, Kamianets-Podilsky Ivan Ohienko National University, Kamianets-Podilsky, Ukraine. (Jun/Jul. 2011, p. 862)

29–June 2 **The 2012 NSF-CBMS Conference on Mathematical Methods of Computed Tomography**, Department of Mathematics, University of Texas at Arlington, Arlington, Texas. (Feb. 2012, p. 339)

* 30–June 1 **ICERM Topical Workshop: Heterostructured Nanocyrstalline Materials**, ICERM, Providence, Rhode Island.
Description: The theme of this workshop is the computation, modeling, and mathematical analysis of heterostructured nanocyrstalline materials. This includes quantum dots, nanowires, graphene, and grain boundaries. These various phenomena will be discussed in the context of modeling and computation on different scales ranging from density functional theory to continuum mechanics. The workshop will also address various techniques that allow one to combine models on different scales to yield efficient computational methods.
Information: `http://icerm.brown.edu/tw12-3-hnm`.

30–June 2 **12th Viennese Workshop on Optimal Control, Dynamic Games and Nonlinear Dynamics**, Vienna University of Technology, Vienna, Austria. (Sept. 2011, p. 1186)

* 30–June 6 **Spring School in Nonlinear Partial Differential Equations**, Université Libre de Bruxelles, Brussels, Belgium.
Description: The spring school consists in mini courses given by - Professor Peter A. Markowich, University of Cambridge; Professor Sir John Ball, University of Oxford; Professor Nassif Ghoussoub, University of British Columbia; Professeur Didier Smets, Université Pierre et Marie Curie (Paris 6); Professor Kazunaga Tanaka, Waseda University; Professor Paolo Piccione, Universidade de Sao Paulo; and plenary talks by invited speakers. A Workshop will be held June 2. The participants will have the opportunity to present a short talk. Submission should be done before April 15, 2012.
Information: `http://pde2012.ulb.ac.be/`.

June 2012

1–22 **Financial Time Series Analysis: High-dimensionality, Non-stationarity and the Financial Crisis**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Jan. 2012, p. 103)

3–30 **Clay Mathematics Institute 2012 Summer School "The Resolution of Singular Algebraic Varieties"**, Obergurgl, Tyrolean Alps, Austria. (Jan. 2012, p. 103)

4–8 **BIOCOMP2012 - Mathematical Modeling and Computational Topics in Biosciences**, Hotel Lloyd's Baia, Vietri sul Mare, Italy. (Jan. 2012, p. 103)

4–8 **Probability, Control and Finance**, Columbia University, New York, New York. (Jan. 2012, p. 104)

6–9 **EUROMECH Colloquium 535 Similarity and symmetry methods in Solid Mechanics**, Varna, Bulgaria. (Dec. 2011, p. 1617)

7–9 **Quantum invariants of 3-manifolds**, Institut de Recherche Mathématiquée Avancé, University of Strasbourg, France. (Nov. 2011, p. 1494)

8–10 **The International Conference on the Frontier of Computational and Applied Mathematics: Tony Chan's 60th Birthday Conference**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Feb. 2012, p. 339)

8–13 **38th International Conference "Applications of Mathematics in Engineering and Economics" AMEE'12**, Leisure House of the Technical University of Sofia, Sozopol, Bulgaria. (Jan. 2012, p. 104)

8–13 **XIVth International Conference on Geometry, Integrability and Quantization**, Sts. Constantine and Elena resort (near Varna, Bulgaria). (Dec. 2011, p. 1617)

* 10–16 **Probability and Analysis**, Mathematical Research and Conference Center of the Polish Academy of Sciences, Bedlewo, Poland.
Description: The conference will bring together mathematicians from the areas of Analysis and Probability, in particular those whose works relate to the life-long scientific interests of Professor Stanisław Kwapień. The meeting is in honor of his 70th anniversary.
Information: `http://www.mimuw.edu.pl/~probanal/`.

11–15 **Arrangements in Pyrenees: School and Workshop on "Hyperplane arrangements and related topics"**, Universite de Pau et des Pays de l'Adour, Pau, France. (Feb. 2012, p. 339)

* 11–15 **NSF/CBMS Conference: Finite Element Exterior Calculus (FEEC)**, ICERM, Providence, Rhode Island.
Description: FEEC is a recent advance in the mathematics of finite element methods that employs differential complexes to construct stable numerical schemes for several important types of application problems. It has aroused great interest because it both presents interesting mathematical problems and shows great potential for application in computational science and engineering. The concentrated sequence of lectures in this program will provide participants with an understanding of the mathematical tools required to fully grasp the concepts in FEEC. ICERM is pleased to host this NSF-CBMS Regional Research Conference.

Principal speaker: Douglas Arnold, University of Minnesota.
Information: `http://icerm.brown.edu/tw12-2-cbms`.

11–22 **Mathematical Modeling on Ecology and Epidemiology**, University of Wyoming, Laramie, Wyoming. (Jan. 2012, p. 104)

12–15 **5th Chaotic Modeling and Simulation International Conference (CHAOS2012)**, Athens, Greece. (Nov. 2011, p. 1494)

12–15 **"The Incomputable" — A workshop of the 6-month Isaac Newton Institute programme — "Semantics and Syntax: A Legacy of Alan Turing" (SAS)**, Kavli Royal Society International Centre, Chicheley Hall, Newport Pagnell MK16 9JJ, United Kingdom. (Aug. 2011, p. 1013)

* 13–15 **USENIX ATC '12: 2012 USENIX Annual Technical Conference**, Sheraton Boston Hotel in Boston, Massachusetts.
Description: USENIX ATC has always been the place to present groundbreaking research and cutting-edge practices in a wide variety of technologies and environments. USENIX ATC '12 will be no exception.
Information: `http://www.usenix.org/events/atc12/`.

13–16 **SIAM Conference on Nonlinear Waves and Coherent Structures (NW12)**, The University of Washington, Seattle, Washington. (Sept. 2011, p. 1186)

17–22 **BIOMATH 2012: International Conference on Mathematical Methods and Models in Biosciences**, Bulgarian Academy of Sciences, Sofia, Bulgaria. (Dec. 2011, p. 1617)

18–22 **AIM Workshop: Dynamics of the Weil-Petersson geodesic flow**, American Institute of Mathematics, Palo Alto, California. (Sept. 2011, p. 1186)

18–22 **NSF/CBMS Regional Conference in the Mathematical Sciences: Hodge Theory, Complex Geometry, and Representation Theory**, Texas Christian University, Fort Worth, Texas. (Dec. 2011, p. 1617)

18–23 **Turing Centenary Conference (CiE 2012): How the World Computes**, University of Cambridge, Cambridge, United Kingdom. (Aug. 2011, p. 1013)

18–29 **Noncommutative Algebraic Geometry**, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2011, p. 1617)

* 18–29 **St. Petersburg School in Probability and Statistical Physics**, Chebyshev Laboratory, St. Petersburg State University, Russia.
Description: The School is devoted to recent advances in probability and statistical physics as well as to classical topics with current increase of activity. It targets graduate students and postdocs, but undergraduate students and advanced researchers are also welcome to attend. We plan four longer courses: Gerard Ben Arous, Alexei Borodin, Yuval Peres, Ofer Zeitouni; as well as mini-courses and talks by Yuri Bakhtin, Sourav Chatterjee, Alice Guionnet, Greg Lawler, Andrei Okounkov (tbc), Mikhail Sodin and others.
Support: We will be able to cover accommodation and possibly travel costs for a number of participants, with priority given to graduate students and recent Ph.D.s. If you are interested in participating, please submit your application with a short CV to: `spspsp@chebyshev.spb.ru` and provide for a letter of recommendation to be sent to the same address. The applications must be submitted by March 1, 2012 (and by February 12, if you request financial support).
Information: `http://spspsp.chebyshev.spb.ru/`.

18–August 10 **SummerICERM: Geometry and Dynamics**, Institute for Computational and Experimental Research in Mathematics (ICERM), 121 South Main Street, Providence, Rhode Island. (Feb. 2012, p. 339)

18–August 15 **Random Matrix Theory and its Applications II**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Jan. 2012, p. 104)

* 19–22 **Financial Engineering Summer School 2012**, Madrid Stock Exchange, Madrid, Spain.
Description: Analistas Financieros Internacionales and the Centre de Recerca Matemàtica (CRM) present the fifth Financial Engineering Summer School.
Aim: To bring together practitioners and academics working in the area of quantitative finance to learn about issues of current interest from some of the world's foremost experts.
Topics: The programme will consist of four short courses, each 4.5 hours.
Speakers: Topics in counterparty credit risk, Jon Gregory, Solum Financial; Robust Risk Management, Paul Glasserman, Columbia Business School; An introduction to the longevity risk market, Enrico Biffis, Imperial College Business School; Systemic risk: theory and policy, Jon Danielsson, London School of Economics.
Information: `http://fess.afi.es`.

19–24 **FRG Workshop and Conference on Characters, Liftings, and Types**, American University, Washington, District of Columbia. (Feb. 2012, p. 339)

* 20–24 **International Conference on Applied Analysis and Algebra (ICAAA2012)**, Davutpasa Campus, Yildiz Technical University, Istanbul, Turkey.
Description: Jointly organized by Yildiz Technical University, University Putra Malaysia and Malaysian Mathematical Sciences Society. Send us your abstracts through the online registration form for oral or poster presentations. The talks given by plenary speakers are to be 50 minutes long including a ten minute question session while the talks from the other contributors are to be 20 minutes long including the question session.
Aim: To bring together mathematicians working in the new trends of applications of math in a wonderful city of the world, Istanbul. This conference is dedicated to Professor Ravi P. Agarwal on his 65th birth anniversary. Please note that the conference language is English.
Information: `http://www.ica12.yildiz.edu.tr`.

25–28 **5th Podlasie Conference on Mathematics**, Bialystok University of Technology, Bialystok, Poland. (Feb. 2012, p. 339)

25–29 **AIM Workshop: Hypergeometric motives**, International Centre for Theoretical Physics, Trieste, Italy. (Nov. 2011, p. 1494)

25–29 **3rd European Seminar on Computing (ESCO 2012)**, Pilsen, Czech Republic. (Nov. 2011, p. 1494)

* 25–29 **The Fourteenth International Conference on "Hyperbolic Problems: Theory, Numerics, Applications" (HYP2012)**, Università di Padova, Italy.
Description: The Fourteenth International Conference on "Hyperbolic Problems: Theory, Numerics and Applications" (HYP2012) will be the XIV meeting in this bi-annual international series, which has the objective of bringing together researchers and students with interests in theoretical and computational aspects of hyperbolic PDEs and of related time-dependent models in the applied sciences. Particular attention will be devoted to the following topics: Singular limits and dispersive equations in mathematical physics, nonlinear wave patterns in multi-dimensions, particle/molecular dynamics, theory and numerics of multiphases and interfaces, transport in complex environments, control problems for hyperbolic PDEs and differential games, general relativity and geometric PDEs.
Information: `http://www.hyp2012.eu`.

25–30 **IVth Workshop on Coverings, Selections, and Games in Topology**, Department of Mathematics, Seconda Università di Napoli, Caserta, Italy. (Jan. 2012, p. 104)

* 27–29 **International Conference on Special Functions and their Applications and Symposium on Life and Works of Ramanujan (XIth Annual Conference of Society of Special function and their Applications)**, Department of Applied Mathematics and Humanities, S.V. National Institute of Technology, Surat 395 007, Gujarat, India.

Description: The Society for Special Functions and their Applications (SSFA) was founded in 1998 with its headquarter at Jaunpur, India. Later, to facilitate its functioning, the Society was reregistered with headquarters at Lucknow in 1999. The Society has interaction with the SIAM OPSF. Under the agreement the permission has been granted to SSFA for mutual exchange of information between Newsletter of SSFA and SIAM OPSF. The three day Conference is jointly organized by SVNIT, Surat and SSFA. This conference will provide a common platform for interaction, exchange of ideas and latest development in the field of Special Functions and various related fields of Mathematical Physics. The day to day activities during the conference are designed to be interactive, involving sessions like Plenary Lectures, Invited Talks and Paper Presentation Sessions, covering a wide range of topics including Special Functions, Lie Theory, Orthogonal Polynomials, Fractional Calculus, Number Theory, Combinatorics etc.
Information: `http://www.ssfaindia.webs.com/conf.htm`.

* 27–29 **Low dimensional conformal structures and their groups**, Institute of Mathematics, Gdansk University, Gdansk, Poland.
Description: This is a Satellite Workshop of 6th European Congress of Mathematics (ECM 2012 in Krakow) organized in Gdansk (Poland) by Emilio Bujalance (Madrid), Grzegorz Gromadzki (Gdansk) and Ruben Hidalgo (Valparaiso) which will be devoted to compact Riemann surfaces, their automorphisms and generalizations, which include also Fuchsian groups, non-Euclidean crystallographic groups (NEC), Kleinian and Schottky groups, Hecke groups, mapping class groups, Teichmüller and moduli spaces of compact Riemann surfaces, maps and graphs on surfaces and generalizations.
Information: `http://mat.ug.edu.pl/conformal/`.

July 2012

1–5 **9th AIMS conference on Dynamical Systems, Differential Equations and Applications**, Orlando, Florida. (Sept. 2011, p. 1186)

1–7 **Workshop on geometric structures on manifolds and their applications**, Castle Rauischholzhausen near Marburg, Germany. (Nov. 2011, p. 1494)

1–16 **Special Case for Interpolation by Using Bézier Curve Numerically**, Al-Muthana University, College of Science, Department of Mathematics & Computer Applications, Muthana, Iraq. (Feb. 2012, p. 339)

* 2–4 **Superluminal Physics & Instantaneous Physics – as new trends in research (electronic conference)**, University of New Mexico, 200 College Road, Gallup, New Mexico.
Description: In a similar way as passing from Euclidean Geometry to Non-Euclidean Geometry, we can pass from Subluminal Physics to Supraluminal Physics, and further to Instantaneous Physics (instantaneous traveling). In the lights of two consecutive successful CERN experiments with superluminal particles in the Fall of 2011, we believe these two new fields of research should begin developing. A physical law has a form in Newtonian physics, another form in the Relativity Theory, and different forms at Superluminal theory and at Instantaneous (infinite) speeds – according to the S-Denying Theory spectrum. First one extends the physical laws, formulas, and theories to superluminal traveling and to instantaneous traveling. Afterwards one founds a general theory that unites all theories at: law speeds, relativistic speeds, superluminal speeds, and instantaneous speeds – as in the S-Multispace Theory.
Deadline: Papers should be sent by July 1, 2012, to Professor Florentin Smarandache at `smarand@unm.edu`.
Information: `http://fs.gallup.unm.edu/Superluminal-Physics.htm`.

2–6 **12th International Conference on p-Adic Functional Analysis**, University of Manitoba, Winnipeg, Canada. (Dec. 2011, p. 1618)

2–6 **Model Theory in Algebra, Analysis and Arithmetic**, Cetraro, Italy. (Jan. 2012, p. 104)

2–7 **24th Conference in Operator Theory**, West University, Timisoara, Romania. (Nov. 2011, p. 1495)

* 2–20 **RTG Summer School on Inverse Problems & Partial Differential Equations**, University of Washington, Seattle, Washington.
Description: The Research Training Group in the Department of Mathematics at the University of Washington will host a summer school for advanced undergraduates and beginning graduate students on Inverse Problems & Partial Differential Equations. Students will attend lectures in the morning and problem sessions in small groups with mentors in the afternoon. On-campus accommodation and meals will be provided, plus a travel allowance of up to $600. Two mini-courses will be given: "The Radon transform and the X-Ray Transform" (Robin Graham, Peter Kuchment, and Leonid Kunyansky) and "A mathematical approach to cancer growth" (John Lowengrub, Ami Radunskaya, and Tatiana Toro).
Registration: Apply online by April 1. (The Summer School is supported by an NSF Research Training Grant. Support is restricted to U.S. citizens/permanent residents. Applications from international students may be considered, but see link for details regarding their support.)
Information: `http://www.math.washington.edu/ipde/summer/`.

3–5 **2nd International Conference on Mathematical Applications in Engineering 2012, ICMAE'12**, Kuala Lumpur, Malaysia. (Dec. 2011, p. 1618)

4–6 **The 2012 International Conference of Applied and Engineering Mathematics**, Imperial College London, London, United Kingdom. (Jan. 2012, p. 104)

* 4–6 **Numerical Software: Design, Analysis and Verification**, Universidad de Cantabria, Santander, Spain.
Description: The workshop aims to review and discuss recent advances and research trends in the design, analysis and verification of numerical software. The workshop is opened to computational scientists and mathematicians with interest in different areas of numerical software. Selected contributions will be invited for a special issue of the journal "Science of Computer Programming".
Deadlines: Contributed session proposals: February 1, 2012. Abstracts for contributed talks and posters: April 1, 2012.
Information: `http://personales.unican.es/segurajj/numsoft12`.

4–6 **Workshop: Statistical Inference in Complex/High-Dimensional Problems**, University of Vienna, Vienna, Austria. (Jan. 2012, p. 104)

8–11 **Trends in Set Theory**, Stefan Banach International Mathematical Center, Warsaw, Poland. (Dec. 2011, p. 1618)

8–15 **International Conference on Wavelets and Applications**, Euler International Mathematical Institute, St. Petersburg, Russia. (Jan. 2012, p. 105)

9–11 **Third Workshop on Mathematical Cryptology**, International Centre for Mathematical Meetings (CIEM), Castro Urdiales (Cantabria), Spain. (Jan. 2012, p. 105)

9–13 **Additive Combinatorics in Paris 2012 — Combinatoire Additive à Paris 2012**, Institut Henri Poincaré, Paris, France. (Jan. 2012, p. 105)

9–13 **EVEQ 2012: International Summer School on Evolution Equations**, Prague, Czech Republic. (Jan. 2012, p. 105)

9–15 **The 10th International Conference on Fixed Point Theory and its Applications (ICFPTA-2012)**, Faculty of Mathematics and Computer Science, Babes, Bolyai University, Cluj-Napoca, Romania. (Jun/Jul. 2011, p. 862)

9–20 **Mathematical General Relativity**, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2011, p. 1618)

9–27 **Graduate Summer School: Deep Learning, Feature Learning**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Feb. 2012, p. 339)

10–14 **3Quantum: Algebra, Geometry, Information**, Tallinn University of Technology, Tallinn, Estonia. (Feb. 2012, p. 340)

11–13 **Third International Conference on Symbolic Computation and Cryptography**, International Centre for Mathematical Meetings (CIEM), Castro Urdiales (Cantabria), Spain. (Jan. 2012, p. 105)

16–December 21 **Topological Dynamics in the Physical and Biological Sciences**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (April 2011, p. 629)

16–18 **SING8: 8th Spain-Italy-Netherlands Meeting on Game Theory**, Institute of Economics, Hungarian Academy of Sciences and Corvinus University, Budapest, Hungary. (Nov. 2011, p. 1495)

* 16–20 **CRM Conference on Applications of Graph Spectra in Computer Science**, CRM, Barcelona, Spain.
Description: The spectra of matrices associated with a network are widely used to characterize its properties and to extract structural information. Applications of graph spectral methods may be easily found in combinatorial optimization (e.g., Goemans-Williamson MAXCUT algorithm), computer science (e.g., spectral clustering, expander graphs, pattern recognition) or complex networks (e.g., web page ranking, synchronization, epidemic thresholds). The aim of this conference is to bring together the diverse collection of researchers from graph theory, computer science and complex networks, who are interested in the theory and applications of graph spectra to discuss current trends and future directions in this area. The conference is expected to provide cross-fertilization of ideas, create more awareness among disparate groups of researchers and foster an increase in collaborations between theory- and application-oriented studies.
Information: `http://www.crm.es/Activitats/Activitats/2011-2012/GraphSpectra/web-graphspectra/`.

17–27 **The 5th Mathematical Society of Japan Seasonal Institute, 2012 International Summer School and Conference on Schubert Calculus**, Osaka City University, Osaka, Japan. (Jan. 2012, p. 105)

22–27 **Vibration and structural acoustics measurement and analysis**, Universidade do Porto, Porto, Portugal. (Oct. 2011, p. 1325)

23–27 **Algebraic Topology: applications and new directions Stanford Symposium 2012**, Stanford University, Palo Alto, California. (Oct. 2011, p. 1325)

23–August 3 **Model Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2011, p. 1618)

* 23–August 3 **Poisson 2012: Poisson Geometry in Mathematics and Physics (Summer School and Conference)**, Utrecht University, Utrecht, The Netherlands.
Description: This will be a 2-week event: the first week is a “Summer School”, and the second one is a “Conference”. This is the 8th event of a series of biennial meetings that started in 1998 (see `http://poissongeometry.org/`). For more information (list of speakers, poster, etc), please consult: `http://www.uu.nl/poisson2012`. For junior participants, we plan to offer financial assistance towards living expenses.
Important Deadlines: General registration deadline: June 15, 2012, financial support deadline: May 14, 2012 (but we very strongly encourage early applications).
Organizing committee: Marius Crainic (chair), Eckhard Meinrenken (chair of the scientific committee).
Information: `http://www.uu.nl/poisson2012`.

23–August 17 **Spectral Theory of Relativistic Operators**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Sept. 2011, p. 1187)

* 27–August 5 **Summer School and Workshop on Cohomology and Support in Representation Theory and Related Topics**, University of Washington, Seattle, Washington.
Description: The theme of this two-week event will be a survey of the state of the art in the use of cohomology and support in the study of representation theory, commutative algebra, triangulated categories, and various related topics. The first week (July 27-30) will be a preparatory summer school intended for graduate students and recent doctoral students. The summer school will consist of three mini-courses on (1) Descent techniques in modular representation theory by P. Balmer (UCLA); (2) Modules of constant Jordan type and vector bundles on projective spaces by D. Benson (Univ. of Aberdeen); and (3) Commutative algebra for modular representations of finite groups by S. Iyengar (Univ. of Nebraska, Lincoln). The second week (August 1–5) will be a workshop consisting of talks on the latest developments in the field.
Funding/deadline: Funding is available to support the participation of graduate students and recent doctoral students. The deadline to apply for funding is March 19, 2012.
Information: `http://www.math.washington.edu/~pischool/`.

30–August 3 **The 24th International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC.12)**, Nagoya University, Nagoya, Japan. (Nov. 2011, p. 1495)

30–August 3 **Iwasawa 2012**, Heidelberg University, Heidelberg, Germany. (Feb. 2011, p. 336)

* 30–August 3 **Variational Methods for Evolving Objects**, Room 3-309, Faculty of Science Building #3, Hokkaido University, Sapporo, Japan.
Description: The purpose of the workshop is to bring together leading experts and researchers working on various aspects of nonsmooth solutions of important nonlinear partial differential equations describing evolving shape and pattern. The fields which will be covered are the theory of viscosity solutions, variational analysis, asymptotic analysis, real analysis and stochastic analysis. The participants are expected to have an overview of up-to-date status of research on the respective subject. There will be a poster session during the workshop, which will provide an ideal platform for young researchers to interact with senior scientists.
Organizers: L. Ambrosio (Pisa), Y. Giga (Tokyo), P. Rybka (Warsaw), Y. Tonegawa (Sapporo).
Course Lecturers: L. Ambrosio (Pisa), Y. Brenier (Nice), R. Jerrard (Toronto).
Invited Speakers: A. Chambolle (Palaiseau), K. Ecker (Berlin) - to be confirmed, W. Gangbo (Atlanta), P. Guidotti (Irvine), P. Mucha (Warsaw), M. Novaga (Padova), D. Pallara (Lecce), N. Wickramasekera (Cambridge).
Information: Interested graduate students and post-doctoral research fellows should contact `cri@math.sci.hokudai.ac.jp` for further inquiries; `http://www.math.sci.hokudai.ac.jp/sympo/120730/index_en.html`.

30–August 11 **Workshop and Conference on Holomorphic Curves and Low-Dimensional Topology**, Stanford University, Palo Alto, California. (Jan. 2012, p. 105)

August 2012

6–8 **The Sixth Global Conference on Power Control and Optimization PCO 2012**, Monte Carlo Resort and Casino, Las Vegas, Nevada. (Oct. 2011, p. 1187)

6–11 **XVII International Congress on Mathematical Physics (ICMP12)**, Aalborg Kongress og Kultur Center, Europa Plads 4, 9000 Aalborg, Denmark. (Aug. 2011, p. 1013)

* 8–10 **21st USENIX Security Symposium (USENIX Security ‘12)**, Hyatt Regency Bellevue in Bellevue, Washington.

Description: The USENIX Security Symposium brings together researchers, practitioners, system administrators, system programmers, and others interested in the latest advances in the security of computer systems and networks. The Symposium will span three days, with a technical program including refereed papers, invited talks, posters, panel discussions, and Birds-of-a-Feather sessions. Workshops will precede the symposium on August 6 and 7. Co-located workshops include: 5th Workshop on Cyber Security Experimentation and Test (CSET'12), 3rd USENIX Workshop on Health Security and Privacy (HealthSec '12)
Information: `http://www.usenix.org/events/sec12/`.

13–26 **Meeting the Challenges of High Dimension: Statistical Methodology, Theory, and Applications**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Jan. 2012, p. 105)

20–24 **AIM Workshop: Invariants in convex geometry and Banach space theory**, American Institute of Mathematics, Palo Alto, California. (Aug. 2011, p. 1013)

20–24 **Spectral Theory and Differential Equations**, B. Verkin Institute for Low Temperature Physics & Engineering, V. Karazin Kharkiv National University, Kharkiv, Ukraine. (Dec. 2011, p. 1618)

20–December 21 **Cluster Algebras Program**, Mathematical Sciences Research Institute, Berkeley, California. (Oct. 2011, p. 1325)

22–24 **Connections for Women: Discrete Lattice Models in Mathematics, Physics, and Computing**, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2011, p. 1619)

* 27–30 **PADGE 2012: Conference on Pure and Applied Differential Geometry**, Department of Mathematics, K. U. Leuven, Belgium.
Description: Conference on differential geometry, Special topics: Minimal submanifolds, Lagrangian submanifolds and related topics, affine differential geometry, Lorentzian geometry
Information: `http://wis.kuleuven.be/Events/padge2012`.

27–31 **8th International Symposium on Geometric Function Theory and Applications**, Ohrid, Republic of Macedonia. (Feb. 2012, p. 340)

27–September 7 **Joint Introductory Workshop: Cluster Algebras and Commutative Algebra**, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2011, p. 1013)

* 29–September 2 **Semigroups and Applications**, Department of Mathematics, Uppsala University, Uppsala, Sweden.
Invited Speakers: James East (University of Western Sydney), Victoria Gould (University of York), Mark Kambites (University of Manchester), Ganna Kudryavtseva (University of Ljubljana), Mark Lawson (Heriot-Watt University), Stuart Margolis (Bar Ilan University), László Márki (Hungarian Academy of Sciences), Nik Ruskuc (University of St. Andrews), Benjamin Steinberg (City College of New York), Rick Thomas (University of Leicester), Mikhail Volkov (Ural Federal University).
Contact: email: `semi2012@math.uu.se`.
Organizer: Volodymyr Mazorchuk.
Deadline: June 15, 2012.
Information: `http://www.math.uu.se/Semi2012/`.

September 2012

3–7 **International Conference on Differential-Difference Equations and Special Functions**, University of Patras, Patras, Greece. (Aug. 2011, p. 1014)

* 3–7 **1st International Conference on Mathematical Sciences and Applications (IECMSA)**, Prishtine University, Prishtine, Kosovo.
Description: The goal of the conference is to contribute to the development of mathematical sciences and its applications and to bring the mathematics community, interdisciplinary researchers, educators, mathematicians, statisticians and engineers from all over the world. The conference will present new results and future challenges, in series invited and short talks, poster presentations. The presentations can be done in the language of Albanian, Turkish and English. Also, original, unpublished papers are invited for presentation in the conference IECMSA. All presented paper's abstracts will be published in the conference proceeding. Moreover, selected and peer review articles will be published in the following journals: *Applied and Computational Mathematics* (SCI), *TWMS Journal of Pure and Applied Mathematics*, *TWMS Journal of Applied and Engineering Mathematics*, *International Electronic Journal of Geometry*, *Mathematical Sciences and Applications*, *e-Notes*. Looking forward to see you in IECMSA.
Information: `http://www.iecmsa.org`.

5–7 **ICERM Semester Program: Computational Challenges in Probability**, ICERM, Providence, Rhode Island. (Jan. 2012, p. 106)

* 5–9 **Lie Algebras and Applications**, Uppsala University, Uppsala, Sweden.
Invited speakers: Peter Fiebig (Universität Erlangen-Nürnberg), Maria Gorelik (Weizmann Institute), Jonas Hartwig (Stanford University), Daniel Nakano (University of Georgia), Maxim Nazarov (University of York), Ivan Penkov (Jacobs University Bremen), Alistair Savage (University of Ottawa), Vera Serganova (University of California at Berkeley), Eric Vasserot (Université Paris VII), Weiqiang Wang (University of Virginia), Geordie Williamson (MPIM Bonn), Kaiming Zhao (Wilfrid Laurier University).
Organizer: Volodymyr Mazorchuk.
Registration: Deadline: June 15, 2012.
Information: email: `lie2012@math.uu.se`. `http://www.math.uu.se/Lie2012/`.

10–12 **3rd IMA Conference on Numerical Linear Algebra and Optimisation**, University of Birmingham, United Kingdom. (Feb. 2012, p. 340)

10–December 14 **Materials Defects: Mathematics, Computation and Engineering**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Oct. 2011, p. 1325)

* 12–14 **Algebra Geometry and Mathematical Physics – 8th workshop – Brno 2012**, Faculty of mechanical engineering, Brno University of Technology, Czech Republic.
Description: AGMP is the traditional scientific meeting on contemporary topics in Algebra, Geometry and Mathematical Physics, with an emphasis on the interface between them. The aim of the conference is to bring together researchers in these disciplines. The meeting is organized in Brno, the city in the geographical center of Europe. All participants are encouraged to present a contributed talk. The proceedings of the conference is planned to be published as a special number of an established journal.
Information: `http://agmp.eu/brno12/general.php`.

17–21 **ICERM Workshop: Bayesian Nonparametrics**, ICERM, Providence, Rhode Island. (Jan. 2012, p. 106)

* 19–25 **ICNAAM2012: 3rd Symposium on Semigroups of Linear Operators and Applications**, Kypriotis Hotels and Conference Center, Kos, Greece.
Description: The 3rd Symposium on Semigroups of Linear Operators and Applications brings together researchers from all over the world to present new results in the theory of semigroups of linear operators and its applications. Besides scheduling talks from established mathematicians, we will give opportunity to junior researchers to present their works.
Information: `http://www.fih.upt.ro/personal/dan.lemle/conferinte/Lemle_Simpozion_2012.pdf`.

20–22 **Lie and Klein; the Erlangen program and its impact on mathematics and physics**, Institut de Recherche Mathématiqué Avancé, University of Strasbourg, France. (Nov. 2011, p. 1495)

22–23 **AMS Eastern Section Meeting**, Rochester Institute of Technology, Rochester, New York. (Sept. 2011, p. 1187)

October 2012

1–5 **New trends in Dynamical Systems**, Salou, Catalonia, Spain. (Feb. 2012, p. 340)

1–April 30 **Semester on Curves, Codes, Cryptography**, Sabanci University, Istanbul, Turkey. (Feb. 2012, p. 340)

2–4 **SIAM Conference on Mathematics for Industry: Challenges and Frontiers (MI12)**, The Curtis, A Doubletree by Hilton, Denver, Colorado. (Aug. 2011, p. 1014)

3–6 **International Conference on Applied and Computational Mathematics (ICACM)**, Middle East Technical University (METU), Ankara, Turkey. (Oct. 2011, p. 1325)

8–12 **ICERM Workshop: Uncertainty Quantification**, ICERM, Providence, Rhode Island. (Jan. 2012, p. 106)

13–14 **AMS Southeastern Section Meeting**, Tulane University, New Orleans, Louisiana. (Aug. 2011, p. 1014)

20–21 **AMS Central Section Meeting**, University of Akron, Akron, Ohio. (Aug. 2011, p. 1014)

* 24–26 **International Conference in Modeling Health Advances 2012**, San Francisco, California.
Description: The conference ICMHA'12 is held under the World Congress on Engineering and Computer Science WCECS 2012. The WCECS 2012 is organized by the International Association of Engineers (IAENG), a non-profit international association for the engineers and the computer scientists. The congress has the focus on the frontier topics in the theoretical and applied engineering and computer science subjects. The last IAENG conference attracted more than five hundred participants from over 30 countries. All submitted papers will be under peer review and accepted papers will be published in the conference proceeding (ISBN: 978-988-19251-6-9). The abstracts will be indexed and available at major academic databases.
Deadline: Draft Paper Submission Deadline: July 2, 2012.
Information: `http://www.iaeng.org/WCECS2012/ICMHA2012.html`.

27–28 **AMS Western Section Meeting**, University of Arizona, Tucson, Arizona. (Aug. 2011, p. 1014)

29–November 2 **ICERM Workshop: Monte Carlo Methods in the Physical and Biological Sciences**, ICERM, Providence, Rhode Island. (Jan. 2012, p. 106)

November 2012

9–10 **Blackwell Tapia Conference 2012**, Institute for Computational and Experimental Research in Mathematics (ICERM), 121 South Main Street, Providence, Rhode Island. (Feb. 2012, p. 340)

* 17 **Info-Metrics and Nonparametric Inference**, University of California Riverside, Riverside, California.
Description: The one-day conference is organized jointly by the Info-Metrics Institute, American University, and the Department of Economics of University of California, Riverside. Interest in nonparametric estimation and inference goes back half a century but has rapidly increased recently (especially with recent advances in computing power) with many new directions of research that cover a vast range of applications in different disciplines. Ongoing research on information-theoretic estimation and inference methods is similarly inter-disciplinary, involving information theory, engineering, mathematical statistics, econometrics and the natural sciences. This one-day conference will explore recent advances in the area of nonparametric estimation and inference and in info-metrics, which may help current and future research combining nonparametric procedures with information-theoretic methods. For more information, please visit our Info-Metrics Institute web page.
Information: `http://www.american.edu/cas/economics/info-metrics/workshop/workshop-2012-november.cfm`.

December 2012

10–14 **36ACCMCC—The 36th Australasian Conference on Combinatorial Mathematics and Combinatorial Computing**, University of New South Wales, Sydney, Australia. (Feb. 2012, p. 340)

16–22 **Commutative rings, Integer-valued polynomials and Polynomial functions**, Graz University of Technology, Graz, Austria. (Nov. 2011, p. 1495)

17–20 **9th IMA International Conference on Mathematics in Signal Processing**, Austin Court, Birmingham, United Kingdom. (Nov. 2011, p. 1495)

17–21 **International Conference on the Theory, Methods and Applications of Nonlinear Equations**, Department of Mathematics, Texas A&M University-Kingsville, Kingsville, Texas. (Feb. 2012, p. 340)

27–30 **Eighth International Triennial Calcutta Symposium on Probability and Statistics**, Department of Statistics, Calcutta University, Kolkata, West Bengal, India. (Nov. 2011, p. 1495)

January 2013

7–12 **Iwasawa Theory, Representations, and the p-adic Langlands program**, University of Münster, Münster, Germany. (Jan. 2012, p. 106)

14–May 24 **Noncommutative Algebraic Geometry and Representation Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Oct. 2011, p. 1325)

24–25 **Connections for Women: Noncommutative Algebraic Geometry and Representation Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2011, p. 1014)

28–February 1 **Introductory Workshop: Noncommutative Algebraic Geometry and Representation Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2011, p. 1014)

28–May 3 **ICERM Semester Program: Automorphic Forms, Combinatorial Representation Theory and Multiple Dirichlet Series**, ICERM, Providence, Rhode Island. (Jan. 2012, p. 106)

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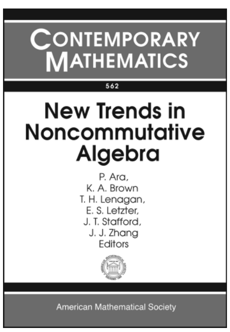
Algebra and Algebraic Geometry



New Trends in Noncommutative Algebra

P. Ara, *University Autonoma de Barcelona, Spain*, **K. A. Brown**, *University of Glasgow, United Kingdom*, **T. H. Lenagan**, *University of Edinburgh, United Kingdom*, **E. S. Letzter**, *Temple University, Philadelphia, PA*, **J. T. Stafford**, *University of Manchester, United Kingdom*, and **J. J. Zhang**, *University of Washington, Seattle, WA*, Editors

This volume contains the proceedings of the conference "New Trends in Noncommutative Algebra", held at the University of Washington, Seattle, in August 2010, in honor of Ken Goodearl's 65th birthday.

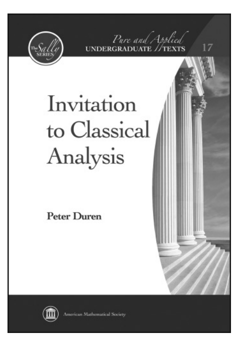
The articles reflect the wide interests of Goodearl and will provide researchers and graduate students with an indispensable overview of topics of current interest. Specific fields covered include: noncommutative algebraic geometry, representation theory, Calabi–Yau algebras, quantum algebras and deformation quantization, Poisson algebras, growth of algebras, group algebras, and noncommutative Iwasawa algebras.

Contents: G. Abrams and **K. M. Rangaswamy**, Row-finite equivalents exist only for row-countable graphs; **K. Ardakov**, The controller subgroup of one-sided ideals in completed group rings; **J.P. Bell**, **K. Casteels**, and **S. Launois**, Enumeration of torus-invariant strata with respect to dimension in the big cell of the quantum minuscule Grassmannian of type B_n; **J. P. Bell**, **L. W. Small**, and **A. Smoktunowicz**, Primitive algebraic algebras of polynomially bounded growth; **D. Chan** and **C. Ingalls**, Conic bundles and Clifford algebras; **M. Chlouveraki**, **I. Gordon**, and **S. Griffeth**, Cell modules and canonical basic sets for Hecke algebras from Cherednik algebras; **E. Coskun**, **R. S. Kulkarni**, and **Y. Mustopa**, On representations of Clifford algebras of ternary cubic forms; **D. Goldstein** and **R. M. Guralnick**, Certain subgroups of Weyl groups are split; **K. R. Goodearl** and **T. H. Lenagan**, Primitive ideals in quantum SL_3 and GL_3; **B. Huisgen-Zimmermann** and **K. R. Goodearl**, Irreducible components of module varieties: Projective equations and rationality; **D. A. Jordan** and **S.-Q. Oh**, Poisson brackets and Poisson spectra in polynomial algebras; **L.-Y. Liu**, **Q.-S. Wu**, and **C. Zhu**, Hopf action on Calabi-Yau algebras; **M. Musson**, Finitely generated, non-Artinian monolithic modules; **D. Rogalski** and **J. J. Zhang**, Regular algebras of dimension 4 with 3 generators; **D. Izychev** and **O. Venjakob**, Galois invariants of K_1-groups of Iwasawa algebras; **M. Yakimov**, Strata of prime ideals of the De Concini-Kac-Procesi algebras and Poisson geometry; **A. Yekutieli**, Twisted deformation quantization of algebraic varieties.

Contemporary Mathematics, Volume 562

February 2012, 297 pages, Softcover, ISBN: 978-0-8218-5297-2, LC 2011041733, 2010 *Mathematics Subject Classification:* 16-XX; 17B37, 20C15, 20G42, **AMS members US$79.20**, List US$99, Order code CONM/562

Analysis



Invitation to Classical Analysis

Peter Duren, *University of Michigan, Ann Arbor, MI*

This book gives a rigorous treatment of selected topics in classical analysis, with many applications and examples. The exposition is at the undergraduate level, building on basic principles of advanced calculus without appeal to more sophisticated techniques of complex analysis and Lebesgue integration.

Among the topics covered are Fourier series and integrals, approximation theory, Stirling's formula, the gamma function, Bernoulli numbers and polynomials, the Riemann zeta function, Tauberian theorems, elliptic integrals, ramifications of the Cantor set, and a theoretical discussion of differential equations including power series solutions at regular singular points, Bessel functions, hypergeometric functions, and Sturm comparison theory. Preliminary chapters offer rapid reviews of basic principles and further background material such as infinite products and commonly applied inequalities.

This book is designed for individual study but can also serve as a text for second-semester courses in advanced calculus. Each chapter concludes with an abundance of exercises. Historical notes discuss the evolution of mathematical ideas and their relevance to physical applications. Special features are capsule scientific biographies of the major players and a gallery of portraits.

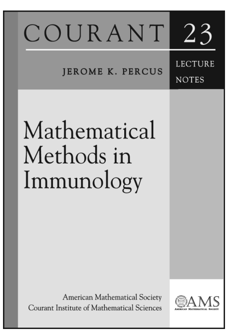
Although this book is designed for undergraduate students, others may find it an accessible source of information on classical topics that underlie modern developments in pure and applied mathematics.

Contents: Basic principles; Special sequences; Power series and related topics; Inequalities; Infinite products; Approximation by polynomials; Tauberian theorems; Fourier series; The gamma function; Two topics in number theory; Bernoulli numbers; The Cantor set; Differential equations; Elliptic integrals; Index.

Pure and Applied Undergraduate Texts, Volume 17

March 2012, approximately 388 pages, Hardcover, ISBN: 978-0-8218-6932-1, LC 2011045853, 2010 *Mathematics Subject Classification:* 26-01, 33-01, 34-01, 40-01, 41-01, 42-01, 11-01, 11B68, 40E05, **AMS members US$59.20**, List US$74, Order code AMSTEXT/17

Applications



Mathematical Methods in Immunology

Jerome K. Percus, *Courant Institute of Mathematics, New York, NY, and Department of Physics, New York University, NY*

Any organism, to survive, must use a variety of defense mechanisms. A relatively recent evolutionary development is that of the adaptive immune system, carried to a quite sophisticated level by mammals. The complexity of this system calls for its encapsulation by mathematical models, and this book aims at the associated description and analysis. In the process, it introduces tools that should be in the armory of any current or aspiring applied mathematician, in the context of, arguably, the most effective system nature has devised to protect an organism from its manifold invisible enemies.

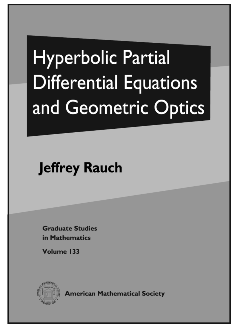
Titles in this series are co-published with the Courant Institute of Mathematical Sciences at New York University.

Contents: The HIV pandemic; Basic facts of immunology; Quantifying the immune response (assays); Modeling humoral immune responses; Modeling cell-mediated response; Control of immune response; Viewpoint of the virus; General references; Index.

Courant Lecture Notes, Volume 23

March 2012, 111 pages, Softcover, ISBN: 978-0-8218-7556-8, LC 2011045038, 2010 *Mathematics Subject Classification:* 92-XX, **AMS members US$25.60**, List US$32, Order code CLN/23

Differential Equations



Hyperbolic Partial Differential Equations and Geometric Optics

Jeffrey Rauch, *University of Michigan, Ann Arbor, MI*

This book introduces graduate students and researchers in mathematics and the sciences to the multifaceted subject of the equations of hyperbolic type, which are used, in particular, to describe propagation of waves at finite speed.

Among the topics carefully presented in the book are nonlinear geometric optics, the asymptotic analysis of short wavelength solutions, and nonlinear interaction of such waves. Studied in detail are the damping of waves, resonance, dispersive decay, and solutions to the compressible Euler equations with dense oscillations created by resonant interactions. Many fundamental results are presented for the first time in a textbook format. In addition to dense oscillations, these include the treatment of precise speed of propagation and the existence and stability questions for the three wave interaction equations.

One of the strengths of this book is its careful motivation of ideas and proofs, showing how they evolve from related, simpler cases. This makes the book quite useful to both researchers and graduate students interested in hyperbolic partial differential equations. Numerous exercises encourage active participation of the reader.

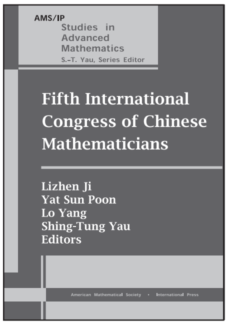
The author is a professor of mathematics at the University of Michigan. A recognized expert in partial differential equations, he has made important contributions to the transformation of three areas of hyperbolic partial differential equations: nonlinear microlocal analysis, the control of waves, and nonlinear geometric optics.

Contents: Simple examples of propagation; The linear Cauchy problem; Dispersive behavior; Linear elliptic geometric optics; Linear hyperbolic geometric optics; The nonlinear Cauchy problem; One phase nonlinear geometric optics; Stability for one phase nonlinear geometric optics; Resonant interaction and quasilinear systems; Examples of resonance in one dimensional space; Dense oscillations for the compressible Euler equations; Bibliography; Index.

Graduate Studies in Mathematics, Volume 133

April 2012, approximately 373 pages, Hardcover, ISBN: 978-0-8218-7291-8, 2010 *Mathematics Subject Classification:* 35A18, 35A21, 35A27, 35A30, 35Q31, 35Q60, 78A05, 78A60, 78M35, 93B07, **AMS members US$51.20**, List US$64, Order code GSM/133

General Interest



Fifth International Congress of Chinese Mathematicians

Lizhen Ji, *University of Michigan, Ann Arbor, MI*, **Yat Sun Poon**, *Tsinghua University, Beijing, China*, **Lo Yang**, *Chinese Academy of Sciences, Beijing, China*, and **Shing-Tung Yau**, *Harvard University, Cambridge, MA*, Editors

This two-part volume represents the proceedings of the Fifth International Congress of Chinese Mathematicians, held at Tsinghua University, Beijing, in December 2010. The Congress brought together eminent Chinese and overseas mathematicians to discuss the latest developments in pure and applied mathematics. Included are 60 papers based on lectures given at the conference.

Titles in this series are co-published with International Press, Cambridge, MA.

Contents: *Part 1:* **B. Andrews**, Gradient and oscillation estimates and their applications in geometric PDE; **J. A. Chen** and **M. Chen**, On canonical and explicit classification of algebraic threefolds; **C.-Y. Chi**, Canonical pseudonorms on pluricanonical spaces; **J. Coates**, The enigmatic Tate-Shafarevich group; **X. Dai**, Eta invariants for even dimensional manifolds; **F. Fang** and **Z. Zhang**, Ricci flow on 4-manifolds and Seiberg-witten equations; **L. Fargues** and **J.-M. Fontaine**, Vector bundles and p-adic Galois representations; **B. Fu**, Geometry of nilpotent orbits: Results and conjectures; **L. Fu**, Integrable connections and Galois representations; **A. Futaki**, Asymptotic Chow polystability in Kähler geometry; **W. T. Gan**, Representations of metaplectic groups; **C. Fang** and **X. He**, Notes on partial conjugation; **K.-W. Lan**, Geometric modular forms and the cohomology of torsion automorphic sheaves; **N. C. Leung**, SYZ transformations for toric varieties; **B. Guo** and **H. Li**, Some variational problems in conformal geometry; **T. Draghici**, **T.-J. Li**, and **W. Zhang**, Geometry of tamed almost complex structures on 4-dimensional manifolds; **W.-C. W. Li**, The arithmetic of noncongruence modular forms; **Y.-P. Lee**, **H.-W. Lin**, and **C.-L. Wang**, Analytic continuations of quantum cohomology; **K. Liu** and **P. Peng**, Mathematical aspects of string duality; **X. Guo** and **H. Qin**, The tame kernels of number fields; **B. Sun**, Notes on MVW-extensions; **F. Chen** and **S. Tan**, Vertex operator representations for a class of BC_ν-graded Lie algebras; **H.-H. Tseng**, Notes on orbifold Gromov-Witten theory; **L.-S. Tseng**, Cohomologies and elliptic operators on symplectic manifolds; **M.-T. Wang**, Quasilocal mass from a mathematical perspective; **S. Wang**, On dimension data, local VS global conjugacy; **X.-J. Wang** and **B. Zhou**, Variational problems of Monge-Ampère type; **S. Wu**, Wellposedness of the two and three dimensional full water wave problem; **H.-W. Xu**, Recent developments in differentiable sphere theorems; **R. Du** and **S. Yau**, New invariants for complex manifolds, singularities, and CR manifolds with applications; **W. Zhang**, Gross-Zagier formula and arithmetic fundamental lemma; **J. Zhou**, Integrality properties of mirror maps; **X-Y. Zhou** and **L. Zhu**, Ohsawa-Takegoshi L^2 extension theorem: Revisited; *Part 2:* **B.-L. Chen**, Regularity of Einstein spacetimes; **K.-C. Chen**, A survey on retrograde and prograde orbits of the three-body problem by variational methods; **D. X. Gu**, **W. Zeng**, **L. M. Lui**, **F. Luo**, and **S.-T. Yau**, Recent development of computational conformal geometry; **B.-Y. Guo**, **C. Zhang**, and **T. Sun**, Some developments in spectral methods; **L.-H. Huang**, On the center of mass in general relativity; **Z. Huang**, Tailored finite point method for numerical simulation of partial differential equations; **T. Lam**, Loop symmetric functions and factorizing matrix polynomials; **A. Laptev**, Spectral inequalities for partial differential equations and their applications; **E. K-W. Chu**, **T.-S. Huang**, and **W.-W. Lin**, Structured doubling algorithms for solving g-palindromic quadratic eigenvalue problems; **Y. Lin**, Ricci curvature and functional inequalities on graphs; **P. Lu**, Complexity dichotomies of counting problems; **L. M. Lui**, **T. W. Wong**, **W. Zeng**, **X. Gu**, **P. M. Thompson**, **T. F. Chan**, and **S.-T. Yau**, A survey on recent development in computational quasi-conformal geometry and its applications; **T. Luo**, Dynamics of shock fronts for some hyperbolic systems; **L. Han** and **J.-S. Pang**, Time-stepping methods for linear complementarity systems; **C.-W. Shu**, A brief survey on high order accurate maximum principle satisfying and positivity preserving discontinuous Galerkin and finite volume schemes for conservation laws; **J. Smoller** and **B. Temple**, A one parameter family of expanding wave solutions of the Einstein equations that induces an anomalous acceleration into the standard model of cosmology; **G. Strang**, Banded matrices with banded inverses and $A - LPU$; **G. Wahba**, Dissimilarity data in statistical model building and machine learning; **R.-H. Wang**, Some progress on computational geometry; **J. Wei**, Geometrization program of semilinear elliptic equations; **Z. Zhu**, **A. M.-C. So**, and **Y. Ye**, Fast and near-optimal matrix completion via randomized basis pursuit; **J. Yin**, Mathematical questions of quantum dilute gases; **X. Yuan**, Algebraic dynamics, canonical heights and Arakelov geometry; **B.-Y. Zhang**, Well-posedness and control of the Korteweg-de Vries equation on a bounded domain; **Y. Jiang**, **H. Zhang**, and **W. Zhu**, Statistical analysis in genetic association studies of mental illnesses; **H. Zhao**, Compressible Navier-Stokes equations with large density oscillation; **W. Zou**, Some results on variational and topological methods.

AMS/IP Studies in Advanced Mathematics, Volume 51

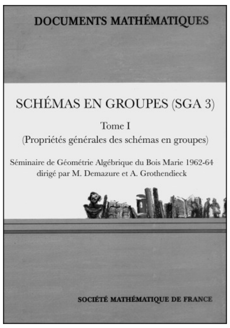
Part 1: March 2012, approximately 497 pages, Softcover, ISBN: 978-0-8218-7586-5, 2010 *Mathematics Subject Classification*: 05-XX, 08-XX, 11-XX, 14-XX, 22-XX, 35-XX, 37-XX, 53-XX, 58-XX, 62-XX, 65-XX, 20-XX, 30-XX, 80-XX, 83-XX, 90-XX, **AMS members US$104**, List US$130, Order code AMSIP 51.1

Part 2: March 2012, approximately 498 pages, Softcover, ISBN: 978-0-8218-7587-2, 2010 *Mathematics Subject Classification*: 05-XX, 08-XX, 11-XX, 14-XX, 22-XX, 35-XX, 37-XX, 53-XX, 58-XX, 62-XX, 65-XX, 20-XX, 30-XX, 80-XX, 83-XX, 90-XX, **AMS members US$104**, List US$130, Order code AMSIP 51.2

Set: March 2012, approximately 995 pages, Softcover, ISBN: 978-0-8218-7555-1, 2010 *Mathematics Subject Classification:* 05-XX, 08-XX, 11-XX, 14-XX, 22-XX, 35-XX, 37-XX, 53-XX, 58-XX, 62-XX, 65-XX, 20-XX, 30-XX, 80-XX, 83-XX, 90-XX, **AMS members US$176**, List US$220, Order code AMSIP/51

New AMS-Distributed Publications

Algebra and Algebraic Geometry



Schémas en Groupes (SGA 3)

Volume I (Propriétés Générales des Schémas en Groupes)

Michel Demazure and **Alexandre Grothendieck**

This volume is an updated edition of "Schémas en Groupes (SGA 3), volume I (Propriétés Générales des Schémas en Groupes)", Lecture Notes in Mathematics, 151, Springer-Verlag, Berlin-Heidelberg-New York, 1970, by Michel Demazure, Alexandre Grothendieck et al.

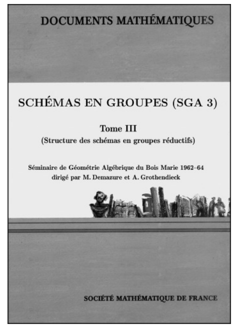
This volume introduces the language of representable functors and sheaves and proves general results about group schemes (Exp. I to VII_A) and formal groups (Exp. VII_B).

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: **M. Demazure**, Structures algébriques. Cohomologie des groupes; **M. Demazure**, Fibrés tangents—Algèbres de Lie; **M. Demazure**, Extensions infinitésimales; **M. Demazure**, Topologies et faisceaux; **P. Gabriel**, Construction de schémas quotients; **P. Gabriel**, Généralités sur les groupes algébriques; **J.-E. Bertin**, Généralités sur les schémas en groupes; **P. Gabriel**, Étude infinitésimale des schémas en groupes; Index.

Documents Mathématiques, Number 7

October 2011, 638 pages, Hardcover, ISBN: 978-2-85629-323-2, 2010 *Mathematics Subject Classification:* 14A15, 14B12, 14D15, 14F20, 14F35, 14K99, 14L15, 14L30, 17B22, 20G10, 20G35, **Individual member US$121.50**, List US$135, Order code SMFDM/7



Schémas en Groupes (SGA 3)

Volume III (Structure des Schémas en Groupes Réductifs)

Michel Demazure and **Alexandre Grothendieck**

This volume is an updated edition of "Schémas en Groupes (SGA 3), volume III (Structure des Schémas en Groupes Réductifs), Lecture Notes in Mathematics, 153, Springer-Verlag, Berlin-Heidelberg-New York, 1970, by Michel Demazure, Alexandre Grothendieck et al.

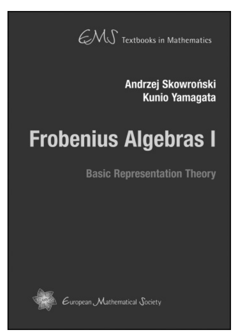
This volume gives the structure of reductive S-group schemes (Exp. XIX to XXV), and their parabolic subgroups (Exp. XXVI), over an arbitrary base scheme S.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Groupes réductifs—Généralités; Groupes réductifs de rang semi-simple 1; Données radicielles; Groupes réductifs: déploiements, sous-groupes, groupes quotients; Groupes réductifs: unicité des groupes épinglés; Automorphismes des groupes réductifs; Le théorème d'existence; Sous-groupes paraboliques des groupes réductifs; Index.

Documents Mathématiques, Number 8

October 2011, 337 pages, Hardcover, ISBN: 978-2-85629-324-9, 2010 *Mathematics Subject Classification:* 14A15, 14B12, 14D15, 14F20, 14F35, 14K99, 14L15, 14L30, 17B22, 20G10, 20G35, **Individual member US$81**, List US$90, Order code SMFDM/8



Frobenius Algebras I

Basic Representation Theory

Andrzej Skowroński, *Nicolaus Copernicus University, Torun, Poland*, and **Kunio Yamagata**, *Tokyo University of Agriculture and Technology, Fuchu, Japan*

This is the first of two volumes which will provide a comprehensive introduction to the modern representation theory of Frobenius algebras. The first part of the book serves as a general introduction to basic results and techniques of the modern representation theory of finite dimensional associative algebras over fields, including the Morita theory of equivalences and dualities and the Auslander–Reiten theory of irreducible morphisms and almost split sequences.

The second part is devoted to fundamental classical and recent results concerning the Frobenius algebras and their module categories. Moreover, the prominent classes of Frobenius algebras, the Hecke algebras of Coxeter groups, and the finite dimensional Hopf algebras over fields are exhibited.

This volume is self contained and the only prerequisite is a basic knowledge of linear algebra. It includes complete proofs of all results presented and provides a rich supply of examples and exercises.

The text is primarily addressed to graduate students starting research in the representation theory of algebras as well mathematicians working in other fields.

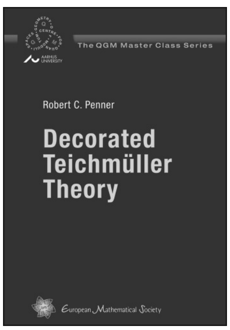
A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Algebras and modules; Morita theory; Auslander–Reiten theory; Selfinjective algebras; Hecke algebras; Hopf algebras; Bibliography; Index.

EMS Textbooks in Mathematics, Volume 12

December 2011, 661 pages, Hardcover, ISBN: 978-3-03719-102-6, 2010 *Mathematics Subject Classification:* 16-01, 13E10, 15A63, 15A69, 16Dxx, 16E30, 16G10, 16G20, 16G70, 16K20, 16W30, 51F15, **AMS members US$62.40**, List US$78, Order code EMSTEXT/12

Analysis



Decorated Teichmüller Theory

Robert C. Penner, *Aarhus University, Denmark, and California Institute of Technology, Pasadena, CA*

There is an essentially "tinker-toy" model of a trivial bundle over the classical Teichmüller space of a punctured surface, called the decorated Teichmüller space, where the fiber over a point is the space of all tuples of horocycles, one about each puncture. This model leads to an extension of the classical mapping class groups called the Ptolemy groupoids and to certain matrix models solving related enumerative problems, each of which has proved useful both in mathematics and in theoretical physics. These spaces enjoy several related parametrizations leading to a rich and intricate algebro-geometric structure tied to the already elaborate combinatorial structure of the tinker-toy model. Indeed, the natural coordinates give the prototypical examples not only of cluster algebras but also of tropicalization.

This interplay of combinatorics and coordinates admits further manifestations, for example, in a Lie theory for homeomorphisms of the circle, in the geometry underlying the Gauss product, in profinite and pronilpotent geometry, in the combinatorics underlying conformal and topological quantum field theories, and in the geometry and combinatorics of macromolecules.

This volume gives the story a wider context of these decorated Teichmüller spaces as developed by the author over the last two decades in a series of papers, some of them in collaboration. Sometimes correcting errors or typos, sometimes simplifying proofs, and sometimes articulating more general formulations than the original research papers, this volume is self contained and requires little formal background. Based on a master's course at Aarhus University, it gives the first treatment of these works in monographic form.

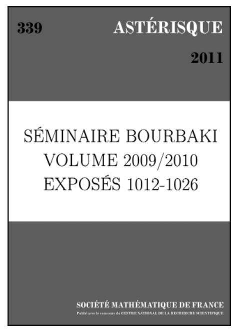
A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: The basics; Lambda lengths in finite dimensions; Lambda lengths in infinite dimensions; Decomposition of the decorated spaces; Mapping class groupoids and moduli spaces; Further applications; Epilogue; Appendix A. Geometry of Gauss product; Appendix B. Dual to the Kähler two form; Appendix C. Stable curves and screens; Bibliography; List of notation; Index.

The QGM Master Class Series, Volume 1

January 2012, 377 pages, Hardcover, ISBN: 978-3-03719-075-3, 2010 *Mathematics Subject Classification:* 30-02, 30F60, 32G15, 30F10, 30Fxx, **AMS members US$62.40**, List US$78, Order code EMSQGM/1

General Interest



Séminaire Bourbaki

Volume 2009/2010 Exposés 1012–1026

As in the preceding volumes of this seminar, at which more than one thousand talks have been presented, this volume features fifteen survey lectures on topics of current interest: five lectures about group theory, three about mathematical physics, two related to Langlands' program, two on algebraic geometry, one about differential geometry, one on clusters algebras, and one about random matrices.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: *Novembre 2009:* **J.-L. Colliot-Thélène**, Groupe de Chow des zéro-cycles sur les variétés p-adiques; **M. Emerton**, p-adic families of modular forms; **B. Keller**, Algèbres amassées et applications; **S. Klainerman**, Linear stability of black holes; **A. Kupiainen**, Ergodicity of two dimensional turbulence; *Mars 2010:* **L. Berger**, La correspondance de Langlands locale p-adique pour $\mathrm{GL}_2(\mathbf{Q}_p)$; **O. Biquard**, Métriques kählériennes extrémales sur les surfaces toriques; **A. Guionnet**, Grandes matrices aléatoires et théorèmes d'universalité; **B. Oliver**, La classification des groupes p-compacts; **B. Rémy**, Groupes algébriques pseudo-réductifs et applications; *Juin 2010:* **M. Burger**, Fundamental groups of Kähler manifolds and geometric group theory; **F. Paulin**, Sur les automorphismes de groupes libres et de groupes de surface; **S. Serfaty**, Lois de conservation et régularité par compensation pour les systèmes antisymétriques et les surfaces de Willmore; **B. Totaro**, The ACC conjecture for log canonical thresholds; **J. S. Wilson**, Finite index subgroups and verbal subgroups in profinite groups.

Astérisque, Number 339

December 2011, 408 pages, Softcover, ISBN: 978-2-85629-326-3, 2010 *Mathematics Subject Classification:* 11G25, 14C25, 14G20, 14C35, 11F33, 11F80, 16S99, 05E15, 22E46, 16G20, 18E30, 35J10, 37A25, 37A60, 37N10, 37L55, 76F55, 76F20, 60H15, 60H07, 35R60, 60H30, 76B03, 35J60, 11Fxx, 11Sxx, 22Exx, 53C55, 32Q26, 15B52, 55R35, 55P35, 20F55, 20Gxx, 20G15, 14L15, 20G30, 20G35, 14F35, 20F65, 32J27, 20E08, 20E36, 20E05, 20F69, 20G20, 53C42, 35J40, 35D10, 58E20, 14B05, 14E30, 14E15, 20E18, 20F12, 20F10, 20D99, **Individual member US$94.50**, List US$105, Order code AST/339

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Honolulu, Hawaii

University of Hawaii at Manoa

March 3–4, 2012

Saturday – Sunday

Meeting #1078

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: December 2011
Program first available on AMS website: January 26, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Zhiqin Lu, University of California Irvine, *Geometry of Calabi-Yau moduli*.

Peter Schröder, California Institute of Technology, *Conformal editing of surface meshes*.

Pham Tiep, University of Arizona, Tucson, *Representations of finite groups: Conjectures, reductions, and applications*.

Lauren Williams, University of California, Berkeley, *Combinatorics of the real Grassmannian and shallow water waves*.

Special Sessions

Algebraic Combinatorics, **Federico Ardila**, San Francisco State University, **Sara Billey**, University of Washington, and **Kelli Talaska** and **Lauren Williams**, University of California, Berkeley.

Algebraic Geometry: Singularities and Moduli, **Jim Bryan**, University of British Columbia, and **Jonathan Wise**, Stanford University.

Algebraic Number Theory, Diophantine Equations and Related Topics, **Claude Levesque**, Université de Laval, Quebec, Canada.

Applications of Nonstandard Analysis, **Tom Lindstrom**, University of Oslo, Norway, **Peter Loeb**, University of Illinois at Urbana-Champaign, and **David Ross**, University of Hawaii, Honolulu.

Arithmetic Geometry, **Xander Faber**, **Michelle Manes**, and **Gretel Sia**, University of Hawaii.

Asymptotic Group Theory, **Tara Davis**, Hawaii Pacific University, **Erik Guentner**, University of Hawaii, and **Michael Hull** and **Mark Sapir**, Vanderbilt University.

Automorphic and Modular Forms, **Pavel Guerzhoy**, University of Hawaii, and **Zachary A. Kent**, Emory University.

C-algebras and Index Theory*, **Erik Guentner**, University of Hawaii at Manoa, **Efren Ruiz**, University of Hawaii at Hilo, and **Erik Van Erp** and **Rufus Willett**, University of Hawaii at Manoa.

Computability and Complexity, **Cameron E. Freer**, Massachusetts Institute of Technology, and **Bjorn Kjos-Hanssen**, University of Hawaii at Manoa.

Geometry and Analysis on Fractal Spaces, **Michel Lapidus**, University of California, Riverside, **Hung Lu**, Hawaii Pacific University, **John A. Rock**, California State Polytechnic University, Pomona, and **Machiel van Frankenhuijsen**, Utah Valley University.

Holomorphic Spaces, **Hyungwoon Koo**, Korea University, and **Wayne Smith**, University of Hawaii.

Kaehler Geometry and Its Applications, **Zhiqin Lu**, University of California Irvine, **Jeff Streets**, Princeton University, **Li-Sheng Tseng**, Harvard University, and **Ben Weinkove**, University of California San Diego.

Kernel Methods for Applications on the Sphere and Other Manifolds, **Thomas Hangelbroek**, University of Hawaii at Manoa.

Knotting in Linear and Ring Polymer Models, **Tetsuo Deguchi**, Ochanomizu University, **Kenneth Millett**, University of California, Santa Barbara, **Eric Rawdon**, University of St. Thomas, and **Mariel Vazquez**, San Francisco State University.

Linear and Permutation Representations, **Robert Guralnick**, University of Southern California, and **Pham Huu Tiep**, University of Arizona.

Mathematical Coding Theory and its Industrial Applications, **J. B. Nation**, University of Hawaii, and **Manabu Hagiwara**, National Institute of Advanced Industrial Science and Technology, Japan.

Mathematical Teacher Preparation, **Diane Barrett** and **Roberto Pelayo**, University of Hawaii at Hilo.

Model Theory, **Isaac Goldbring**, University of California Los Angeles, and **Alice Medvedev**, University of California Berkeley.

New Techniques and Results in Integrable and Near-Integrable Nonlinear Waves, **Jeffrey DiFranco**, Seattle University, and **Peter Miller**, University of Michigan.

Noncommutative Algebra and Geometry, **Jason Bell**, Simon Fraser University, and **James Zhang**, University of Washington.

Nonlinear Partial Differential Equations at the Common Interface of Waves and Fluids, **Ioan Bejenaru** and **Vlad Vicol**, University of Chicago.

Nonlinear Partial Differential Equations of Fluid and Gas Dynamics, **Elaine Cozzi**, Oregon State University, and **Juhi Jang** and **Jim Kelliher**, University of California Riverside.

Singularities, Stratifications and Their Applications, **Terence Gaffney**, Northeastern University, **David Trotman**, Université de Provence, and **Leslie Charles Wilson**, University of Hawaii at Manoa.

Transformation Groups in Topology, **Karl Heinz Dovermann**, University of Hawaii at Manoa, and **Daniel Ramras**, New Mexico State University.

Universal Algebra and Lattice Theory, **Ralph Freese**, **William Lampe**, and **J. B. Nation**, University of Hawaii.

Tampa, Florida

University of South Florida

March 10–11, 2012

Saturday – Sunday

Meeting #1079

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: January 2012
Program first available on AMS website: February 2, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Anne Condon, University of British Columbia, *Some why's and how's of programming DNA molecules.*

Mark Ellingham, Vanderbilt University, *Beyond the Map Color Theorem.*

Mauro Maggioni, Duke University, *Digital data sets: Geometry, random walks, multiscale analysis, and applications.*

Weiqiang Wang, University of Virginia, *What is super in representation theory of Lie superalgebras?*

Special Sessions

Algebraic and Combinatorial Structures in Knot Theory, **J. Scott Carter**, University of South Alabama, and **Mohamed Elhamdadi** and **Masahico Saito**, University of South Florida.

Analysis in Metric Spaces, **Thomas Bieske**, University of South Florida, and **Jason Gong**, University of Pittsburgh.

Applications of Complex Analysis in Mathematical Physics, **Razvan Teodorescu**, University of South Florida, **Mihai Putinar**, University of California, Santa Barbara, and **Pavel Bleher**, Indiana University-Purdue University Indianapolis.

Asymptotic Properties of Groups, **Alexander Dranishnikov**, University of Florida, and **Mark Sapir**, Vanderbilt University.

Combinatorics: Algebraic and Geometric, **Drew Armstrong**, University of Miami, and **Benjamin Braun**, University of Kentucky.

Complex Analysis and Operator Theory, **Sherwin Kouchekian**, University of South Florida, and **William Ross**, University of Richmond.

Computational Algebraic Geometry and Applications, **Tony Shaska**, Oakland University, and **Artur Elezi**, American University.

Dirac Analysis, **Craig Nolder**, Florida State University, and **John Ryan**, University of Arkansas.

Discrete Mathematics and Geometry, **Eunjeong Yi** and **Cong X. Kang**, Texas A&M University Galveston.

Discrete Models in Molecular Biology, **Alessandra Carbone**, Université Pierre et Marie Curie and Laboratory of Microorganisms Genomics, **Natasha Jonoska**, University of South Florida, and **Reidun Twarock**, University of York.

Extremal Combinatorics, **Linyuan Yu**, University of South Carolina, and **Yi Zhao**, Georgia State University.

Finite Fields and Their Applications, **Xiang-dong Hou**, University of South Florida, and **Gary Mullen**, Pennsylvania State University.

Graph Theory, **Mark Ellingham**, Vanderbilt University, and **Xiaoya Zha**, Middle Tennessee State University.

Hopf Algebras and Galois Module Theory, **James Carter**, College of Charleston, and **Robert Underwood**, Auburn University Montgomery.

Interaction between Algebraic Combinatorics and Representation Theory, **Mahir Can**, Tulane University, and **Weiqiang Wang**, University of Virginia.

Inverse Problems in Partial Differential Equations, **Xiaosheng Li**, Florida International University, and **Alexandru Tamasan**, University of Central Florida.

Low-Dimensional Topology, **Peter Horn**, Columbia University, and **Constance Leidy**, Wesleyan University.

Modeling Crystalline and Quasi-Crystalline Materials, **Mile Krajcevski** and **Gregory McColm**, University of South Florida.

Nonlinear Partial Differential Equations and Applications, **Netra Khanal**, University of Tampa.

Recent Developments of Finite Element Methods for Partial Differential Equations, **Bo Dong**, Drexel University, and **Wei Wang**, Florida International University.

Representations of Algebraic Groups and Related Structures, **Joerg Feldvoss** and **Cornelius Pillen**, University of South Alabama.

Solvability and Integrability of Nonlinear Evolution Equations, **Wen-Xiu Ma**, University of South Florida, and **Ahmet Yildirim**, Ege University and University of South Florida.

Spectral Theory, **Anna Skripka** and **Maxim Zinchenko**, University of Central Florida.

Stochastic Analysis and Applications, **Sivapragasam Sathananthan**, Tennessee State University, and **Gangaram Ladde**, University of South Florida.

Stochastic Partial Differential Equations and Random Global Dynamics, **Yuncheng You**, University of South Florida, and **Shanjian Tang**, Fudan University.

Washington, District of Columbia

George Washington University

March 17–18, 2012

Saturday – Sunday

Meeting #1080

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: January 2012
Program first available on AMS website: February 9, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Jim Geelen, University of Waterloo, *Matroid minors*.

Boris Solomyak, University of Washington, *Some recent advances in tiling dynamical systems*.

Gunther Uhlmann, University of California, Irvine and University of Washington, *Cloaking: Science meets science-fiction* (Einstein Public Lecture in Mathematics).

Anna Wienhard, Princeton University, *Deformation spaces of geometric structures*.

Special Sessions

Analysis of Wavelets, Frames, and Fractals, **Keri Kornelson**, University of Oklahoma, and **Judy Packer**, University of Colorado Boulder.

Computable Mathematics (in honor of Alan Turing), **Douglas Cenzer**, University of Florida, **Valentina Harizanov**, George Washington University, and **Russell Miller**, Queens College and Graduate Center - CUNY.

Convex and Discrete Geometry, **Jim Lawrence** and **Valeriu Soltan**, George Mason University.

Difference Equations and Applications, **Michael Radin**, Rochester Institute of Technology.

Dynamics of Complex Networks, **Yongwu Rong**, **Guanyu Wang**, and **Chen Zeng**, George Washington University.

Homology Theories Motivated by Knot Theory, **Jozef H. Przytycki**, George Washington University, **Radmila Sazdanovic**, University of Pennsylvania, and **Alexander N. Shumakovitch** and **Hao Wu**, George Washington University.

Mathematical Methods in Disease Modeling, **Shweta Bansal**, Georgetown University and National Institutes of Health, and **Sivan Leviyang**, Georgetown University.

Mathematics Applied in the Sciences: From Statistics to Topology, **James Carroll** and **Hanna Makaruk**, Los Alamos National Laboratory, and **Robert Owczarek**, Enfitek, Inc. and University of New Mexico.

Matroid Theory, **Joseph E. Bonin**, George Washington University, and **Sandra Kingan**, Brooklyn College.

Nonlinear Dispersive Equations, **Manoussos Grillakis**, University of Maryland, **Justin Holmer**, Brown University, and **Svetlana Roudenko**, George Washington University.

Optimization: Theory and Applications, **Roman Sznajder**, Bowie State University.

Relations between the History and Pedagogy of Mathematics, **David L. Roberts**, Prince George's Community College, and **Kathleen M. Clark**, Florida State University.

Self-organization Phenomena in Reaction Diffusion Equations, **Xiaofeng Ren**, George Washington University, and **Junping Shi**, College of William and Mary.

Structural and Extremal Problems in Graph Theory, **Daniel Cranston**, Virginia Commonwealth University, and **Gexin Yu**, College of William & Mary.

Symmetric Functions, Quasisymmetric Functions, and the Associated Combinatorics, **Nicholas Loehr**, Virginia Tech, and **Elizabeth Niese**, Marshall University.

The Legacy of Goedel's Second Incompleteness Theorem for the Foundations of Mathematics, **Karim J. Mourad**, Georgetown University.

Tilings, Substitutions, and Bratteli-Vershik Transformations, **E. Arthur Robinson**, George Washington University, and **Boris Solomyak**, University of Washington.

Topics in Geometric Analysis and Complex Analysis, **Zheng Huang** and **Marcello Lucia**, City University of New York, Staten Island.

Lawrence, Kansas

University of Kansas

March 30 – April 1, 2012

Friday – Sunday

Meeting #1081

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: February 2012
Program first available on AMS website: March 8, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Frank Calegari, Northwestern University, *Studying algebraic varieties through their cohomology—Recent progress in the Langlands program*.

Christopher Leininger, University of Illinois at Urbana-Champaign, *On complexity of surface homeomorphisms*.

Alina Marian, Northeastern University, *Title to be announced*.

Catherine Yan, Texas A&M University, *Enumerative combinatorics with fillings of polyominoes*.

Special Sessions

Algebraic Geometry and its Applications, **Yasuyuki Kachi**, **B. P. Purnaprajna**, and **Sarang Sane**, University of Kansas.

Combinatorial Commutative Algebra, **Christopher Francisco** and **Jeffrey Mermin**, Oklahoma State University, and **Jay Schweig**, University of Kansas.

Complex Analysis, Geometry and Probability, **Pietro Poggi-Corrandini** and **Hrant Hakobyan**, Kansas State University.

Dynamics and Stability of Nonlinear Waves, **Mat Johnson** and **Myunghyun Oh**, University of Kansas.

Enumerative and Geometric Combinatorics, **Margaret Bayer**, University of Kansas, **Joseph P. King**, University of North Texas, **Svetlana Poznanovik**, Georgia Institute of Technology, and **Catherine Yan**, Texas A&M University.

Geometric Representation Theory, **Zongzhu Lin**, Kansas State University, and **Zhiwei Yun**, Massachusetts Institute of Technology.

Geometric Topology and Group Theory, **Richard P. Kent IV**, University of Wisconsin-Madison, **Christopher J. Leininger**, University of Illinois Urbana-Champaign, and **Kasra Rafi**, University of Oklahoma.

Geometry of Moduli Spaces of Sheaves, **Alina Marian**, Northeastern University, and **Dragos Oprea**, University of California San Diego.

Harmonic Analysis and Applications, **Arpad Benyi**, Western Washington University, **David Cruz-Uribe**, Trinity College, and **Rodolfo Torres**, University of Kansas.

Interplay between Geometry and Partial Differential Equations in Several Complex Variables, **Jennifer Halfpap**, University of Montana, and **Phil Harrington**, University of Arkansas.

Invariants of Knots, **Heather A. Dye**, McKendree University, and **Aaron Kaestner** and **Louis H. Kauffman**, University of Illinois at Chicago.

Mathematical Statistics, **Zsolt Talata**, University of Kansas.

Mathematics of Ion Channels: Life's Transistors, **Bob Eisenberg**, Rush Medical Center at Chicago, **Chun Liu**, Penn State University, and **Weishi Liu**, University of Kansas.

Mirror Symmetry, **Ricardo Castano-Bernard**, Kansas State University, **Paul Horja**, Oklahoma State University, and **Zheng Hua** and **Yan Soibelman**, Kansas State University.

Nonlinear Dynamical Systems and Applications, **Weishi Liu** and **Erik Van Vleck**, University of Kansas.

Numerical Analysis and Scientific Computing, **Weizhang Huang**, **Xuemin Tu**, **Erik Van Vleck**, and **Honggou Xu**, University of Kansas.

Partial Differential Equations, **Milena Stanislavova** and **Atanas Stefanov**, University of Kansas.

Singularities in Commutative Algebra and Algebraic Geometry, **Hailong Dao**, University of Kansas, **Lance E. Miller**, University of Utah, and **Karl Schwede**, Pennsylvania State University.

Stochastic Analysis, **Jin Feng**, **Yaozhong Hu**, and **David Hualart**, University of Kansas.

Topics in Commutative Algebra, **Hailong Dao**, **Craig Huneke**, and **Daniel Katz**, University of Kansas.

Undergraduate Research, **Marianne Korten** and **David Yetter**, Kansas State University.

University Mathematics Education in an Online World, **Andrew G. Bennett** and **Carlos Castillo-Garsow**, Kansas State University.

Rochester, New York

Rochester Institute of Technology

September 22–23, 2012

Saturday – Sunday

Meeting #1082

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: May 2012

Program first available on AMS website: July 19, 2012
Program issue of electronic *Notices*: September 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: February 22, 2012
For consideration of contributed papers in Special Sessions: May 15, 2012
For abstracts: July 10, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Steve Gonek, University of Rochester, *Title to be announced*.

James Keener, University of Utah, *Title to be announced*.

Dusa McDuff, Barnard College, *Title to be announced*.

Peter Winkler, Dartmouth College, *Title to be announced*.

Special Sessions

Analytic Number Theory (Code: SS 5A), **Steve Gonek**, University of Rochester, and **Angel Kumchev**, Towson University.

Continuum Theory (Code: SS 3A), **Likin C. Simon Romero**, Rochester Institute of Technology.

Financial Mathematics (Code: SS 1A), **Tim Siu-Tang Leung**, Columbia University.

Inverse Problems and Nonsmooth Optimization: Celebrating Zuhair Nashed's 75th Birthday (Code: SS 7A), **Patricia Clark**, **Baasansuren Jadama**, and **Akhtar A. Khan**, Rochester Institute of Technology, and **Hulin Wu**, University of Rochester.

Microlocal Analysis and Nonlinear Evolution Equations (Code: SS 2A), **Raluca Felea**, Rochester Institute of Technology, and **Dan-Andrei Geba**, University of Rochester.

Modern Relativity (Code: SS 6A), **Manuela Campanelli** and **Yosef Zlochower**, Rochester Institute of Technology.

New Advances in Graph Theory (Code: SS 9A), **Jobby Jacob**, Rochester Institute of Technology, and **Paul Wenger**, University of Colorado Denver.

Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs (Code: SS 8A), **Bernard Brooks**, **Darren Narayan**, and **Tamas Wiandt**, Rochester Institute of Technology.

Special Session on Operator Theory and Function Spaces (Code: SS 4A), **Gabriel T. Prajitura** and **Ruhan Zhao**, State University of New York at Brockport.

New Orleans, Louisiana

Tulane University

October 13–14, 2012

Saturday – Sunday

Meeting #1083

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: June 2012
Program first available on AMS website: September 6, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: March 13, 2012
For consideration of contributed papers in Special Sessions: July 3, 2012
For abstracts: August 28, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Anita Layton, Duke University, *Title to be announced*.

Lenhard Ng, Duke University, *Title to be announced*.

Henry K. Schenck, University of Illinois at Urbana-Champaign, *From approximation theory to algebraic geometry: The ubiquity of splines*.

Milen Yakimov, Louisiana State University, *Title to be announced*.

Akron, Ohio

University of Akron

October 20–21, 2012

Saturday – Sunday

Meeting #1084

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: August 2012
Program first available on AMS website: September 27, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: March 22, 2012
For consideration of contributed papers in Special Sessions: July 10, 2012
For abstracts: September 4, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Tanya Christiansen, University of Missouri, *Title to be announced.*

Tim Cochran, Rice University, *Title to be announced.*

Ronald Solomon, Ohio State University, *Title to be announced.*

Ben Weinkove, University of California San Diego, *Title to be announced.*

Special Sessions

Extremal Graph Theory (Code: SS 2A), **Arthur Busch**, University of Dayton, and **Michael Ferrara**, University of Colorado Denver.

Groups, Representations, and Characters (Code: SS 1A), **Mark Lewis**, Kent State University, **Adriana Nenciu**, Otterbein University, and **Ronald Solomon**, Ohio State University.

Tucson, Arizona

University of Arizona, Tucson

October 27–28, 2012

Saturday – Sunday

Meeting #1085

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2012
Program first available on AMS website: October 4, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: March 27, 2012
For consideration of contributed papers in Special Sessions: July 17, 2012
For abstracts: September 11, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael Hutchings, University of California Berkeley, *Title to be announced.*

Kenneth McLaughlin, University of Arizona, Tucson, *Title to be announced.*

Ken Ono, Emory University, *Title to be announced* (Erdős Memorial Lecture).

Jacob Sterbenz, University of California San Diego, *Title to be announced.*

Goufang Wei, University of California, Santa Barbara, *Title to be announced.*

Special Sessions

Dispersion in Heterogeneous and/or Random Environments (Code: SS 2A), **Rabi Bhattacharya**, Oregon State University, Corvallis, and **Edward Waymire**, University of Arizona, Tucson.

Geometric Analysis and Riemannian Geometry (Code: SS 4A), **David Glickenstein**, University of Arizona, **Guofang Wei**, University of California Santa Barbara, and **Andrea Young**, Ripon College.

Geometrical Methods in Mechanical and Dynamical Systems (Code: SS 3A), **Akif Ibragimov**, Texas Tech University, **Vakhtang Putkaradze**, Colorado State University, and **Magdalena Toda**, Texas Tech University.

Harmonic Maass Forms and q-Series (Code: SS 1A), **Ken Ono**, Emory University, **Amanda Folsom**, Yale University, and **Zachary Kent**, Emory University.

Representations of Groups and Algebras (Code: SS 5A), **Klaus Lux** and **Pham Huu Tiep**, University of Arizona.

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday

Meeting #1086

Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Oxford, Mississippi

University of Mississippi

March 1–3, 2013

Friday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Chestnut Hill, Massachusetts

Boston College

April 6–7, 2013

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 6, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Ames, Iowa

Iowa State University

April 27–28, 2013

Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: April 2013
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 27, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions

Operator Algebras and Topological Dynamics (Code: SS 1A), **Leslie Hogben**, Iowa State University and American Institute of Mathematics, and **Bryan Shader**, University of Wyoming.

Zero Forcing, Maximum Nullity/Minimum Rank, and Colin de Verdiere Graph Parameters (Code: SS 2A), **Leslie Hogben**, Iowa State University and American Institute of Mathematics, and **Bryan Shader**, University of Wyoming.

Alba Iulia, Romania

June 27–30, 2013

Thursday – Sunday
First Joint International Meeting of the AMS and the Romanian Mathematical Society, in partnership with the "Simion Stoilow" Institute of Mathematics of the Romanian Academy.
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Louisville, Kentucky

University of Louisville

October 5–6, 2013

Saturday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 5, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Philadelphia, Pennsylvania

Temple university

October 12–13, 2013

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 12, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

St. Louis, Missouri

Washington University

October 18–20, 2013

Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 20, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Riverside, California

University of California Riverside

November 2–3, 2013

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15–18, 2014

Wednesday – Saturday
Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Tel Aviv, Israel

Bar-Ilan University, Ramat-Gan and Tel-Aviv University, Ramat-Aviv

June 16–19, 2014

Monday – Thursday
The 2nd Joint International Meeting between the AMS and the Israel Mathematical Union.
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday
Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015

Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Porto, Portugal

University of Porto

June 11–14, 2015

Thursday – Sunday
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday
Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday
Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday
Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2017
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: `lapidus@math.ucr.edu`; telephone: 951-827-5910.

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: `benkart@math.wisc.edu`; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: `steve.weintraub@lehigh.edu`; telephone: 610-758-3717.

Southeastern Section: Matthew Miller, Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: `miller@math.sc.edu`; telephone: 803-777-3690.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** `www.ams.org/meetings/`**.**

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 111 in the this issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit `http://www.ams.org/cgi-bin/abstracts/abstract.pl`. Questions about abstracts may be sent to `abs-info@ams.org`. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see `http://www.ams.org/meetings/` for the most up-to-date information on these conferences.)

March 11–14, 2012: Fourth International Conference on Mathematical Sciences, United Arab Emirates (held in cooperation with the AMS). Please see `http://icm.uaeu.ac.ae/` for more information.

June 10–June 30, 2012: MRC Research Communities, Snowbird, Utah. (Please see `http://www.ams.org/amsmtgs/mrc.html` for more information.)

CALL FOR CONFERENCE PROPOSALS

2014 Mathematics Research Communities

The American Mathematical Society invites individuals and groups of individuals to serve as organizers of the conferences of the Mathematics Research Communities (MRC) program to be held in Snowbird, Utah, during the summer of 2014.

The goal of the MRC program is to create research cohorts of early career mathematicians that will sustain themselves over time, fostering joint research and coherent research programs. The MRC program aims to achieve this goal through:

- One-week conferences in each topic area in summer 2014
- AMS Special Sessions at the Joint Mathematics Meetings in January 2015
- Discussion networks
- Longitudinal study

Additional information about the MRC program and guidelines for proposal preparation can be found at http://www.ams.org/programs/research-communities/mrc-proposals-14.

The 2014 MRC program is contingent on renewed funding from the National Science Foundation.

SEND INQUIRIES AND PROPOSALS TO:

Mathematics Research Communities
American Mathematical Society

by email: mrc2014@ams.org
by mail: 201 Charles Street, Providence, RI 02904
by fax: 401-455-4001

Deadlines for proposals for the 2014 MRCs:
September 15, 2012

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American Mathematical Society

Congratulations!

We are proud to recognize our authors who received AMS awards at this year's Joint Mathematics Meetings.

2012 AMS Steele Prize for Mathematical Exposition

The Classification of Finite Simple Groups

Groups of Characteristic 2 Type

Michael Aschbacher, *California Institute of Technology, Pasadena, CA*, **Richard Lyons**, *Rutgers University, Piscataway, NJ*, **Stephen D. Smith**, *University of Illinois at Chicago, IL*, and **Ronald Solomon**, *The Ohio State University, Columbus, OH*

Mathematical Surveys and Monographs, Volume 172; 2011; 347 pages; Hardcover; ISBN: 978-0-8218-5336-8; List US$94; SALE US$71; Order code SURV/172

Classifying Spaces of Sporadic Groups

David J. Benson, *University of Aberdeen, Scotland, United Kingdom*, and **Stephen D. Smith**, *University of Illinois at Chicago, IL*

Mathematical Surveys and Monographs, Volume 147; 2008; 289 pages; Hardcover; ISBN: 978-0-8218-4474-8; List US$88; SALE US$66; Order code SURV/147

Abstract Algebra

TEXTBOOK

Ronald Solomon, *Ohio State University, Columbus, OH*

Pure and Applied Undergraduate Texts, Volume 9; 2003; 227 pages; Hardcover; ISBN: 978-0-8218-4795-4; List US$62; SALE US$47; Order code AMSTEXT/9

2012 JPBM Communications Award

What's Happening in the Mathematical Sciences, Volume 8

Dana Mackenzie

Major mathematical developments made understandable to the general reader, bringing alive the presence of mathematics in the everyday world

What's Happening in the Mathematical Sciences, Volume 8; 2011; 129 pages; Softcover; ISBN: 978-0-8218-4999-6; List US$23; SALE US$17; Order code HAPPENING/8

2012 AMS Cole Prize in Algebra

Cohomological Invariants in Galois Cohomology

Skip Garibaldi, *Emory University, Atlanta, GA*, **Alexander Merkurjev**, *University of California, Los Angeles, CA*, and **Jean-Pierre Serre**, *Collège de France, Paris, France*

University Lecture Series, Volume 28; 2003; 168 pages; Softcover; ISBN: 978-0-8218-3287-5; List US$39; SALE US$29; Order code ULECT/28

The Algebraic and Geometric Theory of Quadratic Forms

Richard Elman, *University of California, Los Angeles, CA*, **Nikita Karpenko**, *Université Pierre et Marie Curie - Paris 6, France*, and **Alexander Merkurjev**, *University of California, Los Angeles, CA*

A comprehensive study of the algebraic theory of quadratic forms that combines classical and modern approaches to the subject

Colloquium Publications, Volume 56; 2008; 435 pages; Hardcover; ISBN: 978-0-8218-4329-1; List US$81; SALE US$61; Order code COLL/56

2012 AMS-SIAM Birkhoff Prize

Large-Scale Computations in Fluid Mechanics, Part 1*

Applied Mathematics

Bjorn E. Engquist, **Stanley Osher**, and **Richard C. J. Somerville**, Editors

Lectures in Applied Mathematics, Volume 22; 1985; 370 pages; Hardcover; ISBN: 978-0-8218-1129-0; List US$110; SALE US$85; Order code LAM/22.1

Large-Scale Computations in Fluid Mechanics, Part 2*

Applied Mathematics

Bjorn E. Engquist, **Stanley Osher**, and **Richard C. J. Somerville**, Editors

Lectures in Applied Mathematics, Volume 22; 1985; 409 pages; Hardcover; ISBN: 978-0-8218-1130-6; List US$110; SALE US$85; Order code LAM/22.2

*Quantities are limited on these volumes

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