# Letters to the Editor

## Word Problems

In a recent article ("Modeling the journey from elementary word problems to mathematical research"), Chris Sangwin aptly describes how word problems help developing mathematical aptitudes. His quote from Pólya, "the most important single task of mathematical instruction in the secondary school is to teach the setting up of equations to solve word problems", appears controversial but can be seen as an incentive for teaching how to solve problems via equations with *awareness* (about strengths, limitations, pitfalls, style, etc.). Equations delegate substantial parts of reasoning (especially the tricky ones) to symbolic calculations. However, this requires some mathematical literacy.

Therefore, and since diversity enhances understanding, it is helpful to consider methods suitable in primary school. These can serve later as a *sanity check* when learning to work with equations (before reversing the roles). An example is Sangwin's Example 4:

"A dog starts in pursuit of a hare at a distance of thirty of his own leaps from her. He takes five leaps while she takes six, but covers as much ground in two as she in three. In how many leaps of each will the hare be caught?"

Setting up an equation requires "careful work on the part of the student." Here follows a third-grade-level solution.

The dog advances 5 dog leaps whenever the hare advances 6 hare leaps, which is 4 dog leaps. So, the dog gains 1 dog leap with every 5 leaps. To annihilate the initial distance of 30 dog leaps, the dog must make $5 \times 30$ or 150 leaps. The hare has then made $6 \times 30$ or 180 leaps.

Sangwin also notes that "problems involving rates are particularly difficult." In an interview (*Notices of the AMS*, April 1997), Vladimir Arnol'd mentions such a problem he solved at age 12:

"Two old women started at sunrise and each walked at constant velocity. One went from A to B and the other from B to A. They met at noon and, continuing with no stop, arrived respectively at B at 4 p.m. and at A at 9 p.m. At what time was the sunrise on this day?"

In secondary school, one would set up an equation. In primary school, many learn the *rule of three*, a systematic method from the sixth century BCE for dealing with rates without using symbols, which (with just one nonstandard step) also yields the solution.

Another invaluable sanity check for equations is dimensional analysis.

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## Committee on Education Offers Qualified Endorsement of Common Core Standards

The Committee on Education recognizes and commends the efforts of those who produced the Common Core State Standards in Mathematics. We see substantial benefits to be gained from a successful implementation of these standards. One would be an approximate synchronization of mathematics learning in schools across the country, which has obvious advantages in a mobile society. More important, we endorse the goals of focus, logic, coherence, thoroughness, and grade-to-grade continuity that have guided the writing of the standards. We acknowledge the concern of some mathematicians that an attempt to adhere to these standards without a sufficiently large corps of mathematically knowledgeable teachers will not be beneficial, and in fact could do significant harm. Well-qualified teachers are absolutely essential in implementing ambitious standards like these. The adoption of the Common Core State Standards in Mathematics by more than forty states affords mathematicians the opportunity and the responsibility to involve ourselves in efforts to strengthen the mathematical knowledge of both pre-service and in-service teachers.

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## Response to Quinn

I found particularly useful Frank Quinn's article "A revolution in mathematics? What really happened a century ago and why it matters today", published in the January issue of the *Notice*s. I have also read Dr. Tevian Dray's justified criticism (in the same issue of *Notices*) of a previous work authored by Quinn ["A science-of-learning approach to mathematics education", *Notices*, October 2011]. However, there is something particularly important that should be said about Quinn's "A Revolution in mathematics?..." I think that any discussion about mathematical education today, in particular about teaching strategies of proofs, should take into account the modernist transformation of mathematics that took place between 1890 and 1930. F. Quinn's recent article does a very useful service to the mathematical community: it merges Jeremy Gray's very insightful contribution from his 2008 monograph *Plato's Ghost* (one of the most useful books ever written in the history of mathematics) with the needs of contemporary pre-college education. F. Quinn states that the main point of his article "is not that a revolution occurred, but that there are penalties for not being aware of it." The matter is sensitive today for the mathematical community, as the topic borders on the main dilemma of the calculus wars. Quinn spells it out clearly when he mentions that "the precollege-education community was, and remains, antagonistic to the new methodology." Perhaps integrating the study of history of mathematics in our contemporary teaching of undergraduate courses could be particularly useful to close, even partially, this gap. For example, I found it extremely useful to embed Jeremy Gray's description of the modernist transformation of mathematics into the Foundation of Geometry course that I teach for math major undergraduate students who are planning a career in education. I am using the very well written *The Foundations of Geometry*, by Gerard Venema, as the regular textbook, but I additionally present in detail the evolution of various axiomatic systems in plane geometry. One can present the chronological sequence and important developments of various axiomatic systems (D. Hilbert, O. Veblen, G. D. Birkhoff) as an evolution within the ample change of modernist revolution. Quinn's recent article points out how describing such a historical evolution could make a difference and matter a lot for contemporary pre-college education, as well as for the K–12 education community.

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## Response to Quinn

In response to the article by Dr. F. Quinn in the January 2012 *Notices*: You may be right in what you say. However, one thing I know is that mathematics is some unknown process taking place in the human brain. Mathematics is not floating around in spacetime. At least I haven't been struck and knocked down by a flying theorem lately. If I only understood what the brain is doing when it thinks 1 + 1 = 2, I would think I'm in heaven. Better than that would be to understand how the brain gets from one step in one of your "core math" proofs to the next step in a proof with a finite number of steps. Is it some sort of quantum jump between energy levels or does the brain "slide" in some manner between the steps? But best of all would be to understand what is going on in your brain when you think that your "core math" is superior to the old-fashioned math done by us old-time intuitionists. Maybe one of these "hostile" scientists will tell me how the brain uses experience to modify the strengths of the NMDA synapses. Then I could model the brain and perhaps come to an understanding of mathematics by running special cases on a computer using floating point arithmetic?

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## Reformers Operating in a Vacuum

In his February 2012 *Doceamus* piece, Alan H. Schoenfeld provided some fascinating anecdotes concerning the current state of mathematics pseudo-education, and then reminisced about teaching precalculus at the University of Rochester. In the last paragraph, he proposed "that we revise the entire curriculum." Unfortunately, as is the case with many self-styled math reformers, Schoenfeld continues to operate in a complete historical vacuum. For example, with respect to precalculus, he could have pointed out how our textbooks have degenerated.

When I was in the twelfth grade, we used the textbook by William E. Kline, Robert O. Oesterle, Leroy M. Willson, *Foundations of Advanced Mathematics*, The American Book Company, New York (1959), 519 pp., which is described at: http://mathforum.org/kb/thread.jspa?forumID=206&threadID=478525.

In the late 1960s this was replaced by Mary P. Dolciani's (et al.), *Modern Introductory Analysis*, Houghton Mifflin (1964), 651 pp., which was influenced by the new-math strand developed by the School Mathematics Study Group and is described briefly at: http://mathforum.org/kb/thread.jspa?forumID=206&threadID=1728344&messageID=6179351.

When my son was in the eleventh grade, he used Merilyn Ryan's (et al.), *Advanced Mathematics: A Precalculus Approach*, Prentice Hall (1993), 946 pp., which was based on the above and included all the empty slogans of the NCTM "Standards".

Four years ago I received unsolicited desk copies of the previous editions of the following 1,000-page doorstops by Michael Sullivan and Michael Sullivan III:

http://www.amazon.com/Precalculus-Concepts-Functions-Approach-Trigonometry/dp/0321644875/

http://www.amazon.com/Precalculus-Concepts-Functions-Triangle-Trigonometry/dp/0321645081/.

I was appalled to see two distinct doorstops for the two approaches to trigonometry, whose equivalence was demonstrated in a few pages of my high school textbook. As long as we continue to operate in a historical vacuum, and continue to produce bloated doorstops, the pseudo-education of American students will continue unabated.

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## Correction

Authors Barbara and Robert Reys ("Supporting the next generation of 'stewards' in mathematics education", *Notices*, February 2012) were incorrectly identified as being Curators' Professors of Mathematics at the University of Missouri–Columbia. They are, in fact, Curators' Professors in the Department of Learning, Teaching, and Curriculum at the University of Missouri–Columbia.

*—Sandy Frost*