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of the American Mathematical Society

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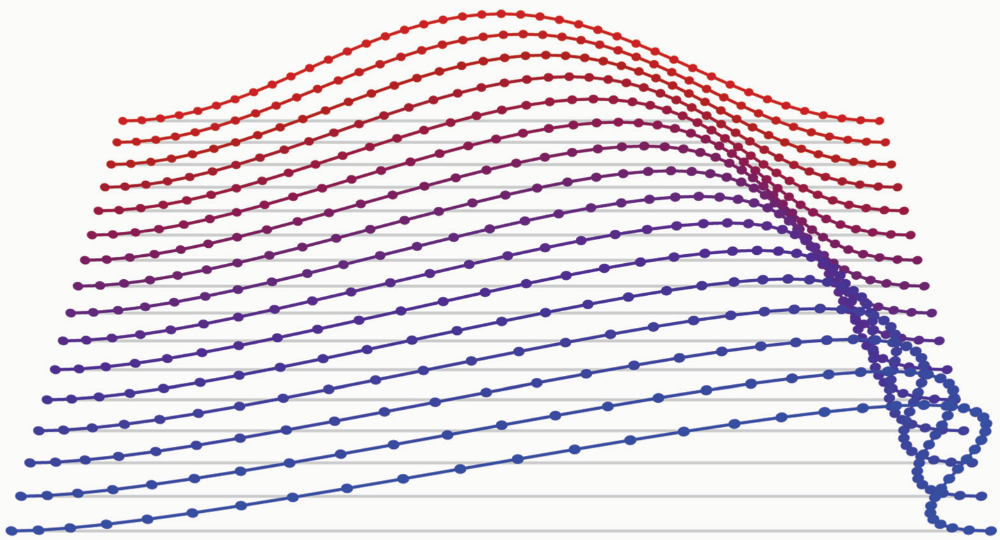

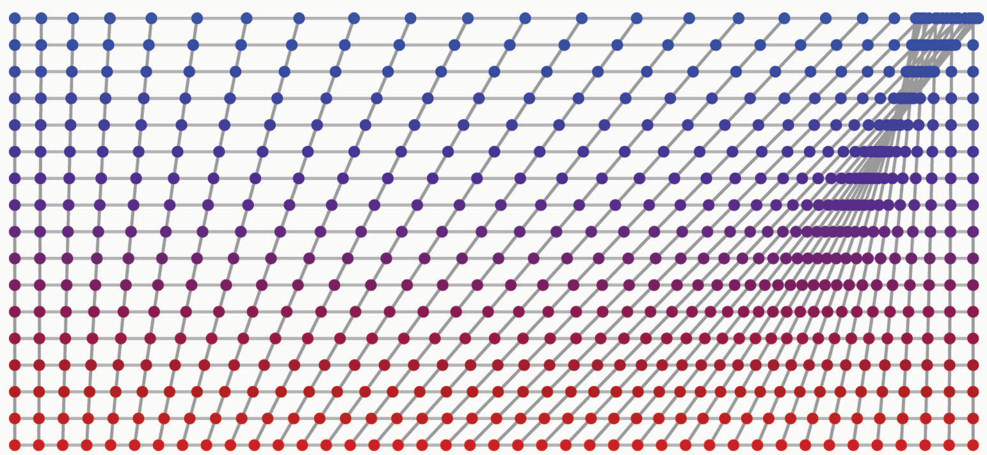

About the Cover:
Two views of Burgers' equation
See page 654

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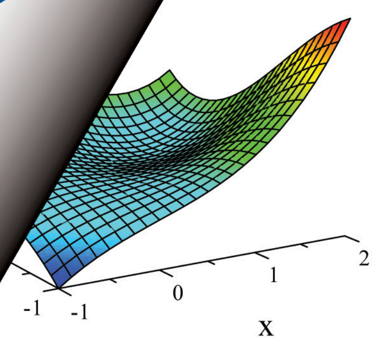
$x^3 - 3xy + y^3$



$y + y^3 = 0$ goes through the point $(0,0,0)$.
vel curve, make a 2D plot of $x^3 - 3xy + y^3 =$
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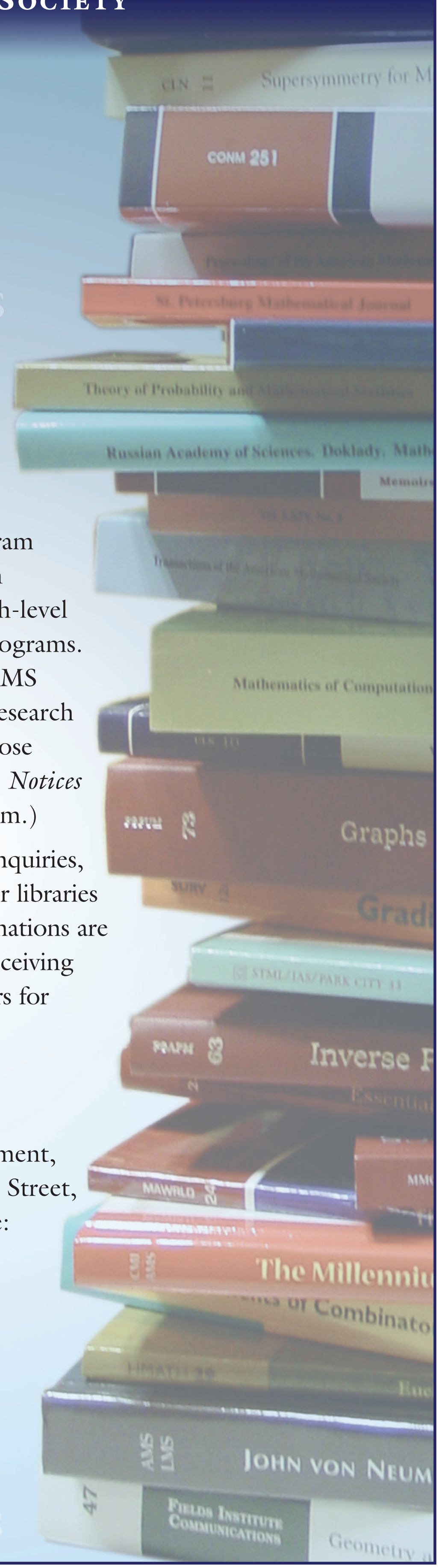

Solve the differential equation.

$$t \ln t \frac{dr}{dt} + r = 7te^t$$

$$\boxed{r = \frac{7e^t + C}{\ln t}}$$ ✓

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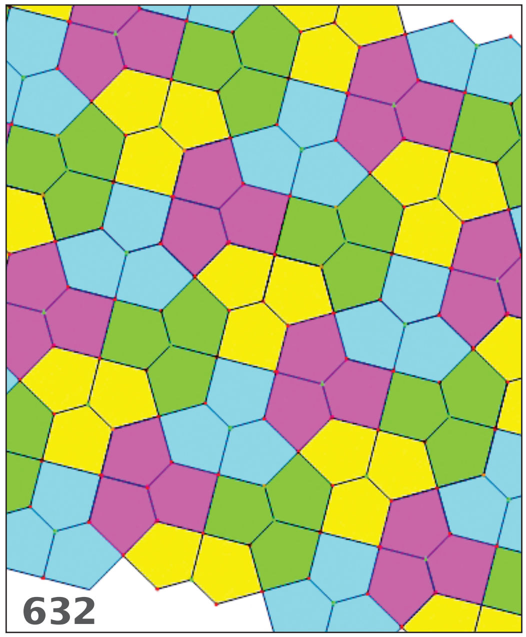
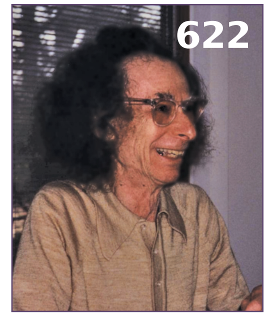
Notices
of the American Mathematical Society

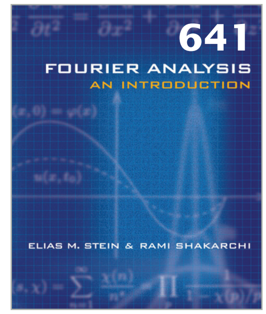
May 2012

Communications

Commentary







May brings an incisive review of the important quartet of analysis books by E. M. Stein and R. Shakarchi. There is also a fascinating article on tilings, and an exploration of partial differential equations with discontinuous solutions. Finally we offer a remembrance of the distinguished mathematician Herbert Federer. As usual, the *Doceamus* and *Scripta Manent* and other Communications round out the mathematical picture for May.

—*Steven G. Krantz, Editor*

Features

Notices
of the American Mathematical Society

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Departments

From the AMS Secretary

Opinion

Mathematical Congress of the Americas

The Mathematical Congress of the Americas (MCA) will be held, for the first time, in Guanajuato, Mexico, on August 5–9, 2013. It is anticipated that the MCA will be a quadrennial event held in different countries of the Americas. Its goals are ambitious: to highlight the excellence of mathematical achievements in the Americas, within the context of the international arena, and to foster the scientific integration of all mathematical communities in the two continents. This article contains a brief explanation of how the Congress came to be and an outline of the preparations to ensure that it will achieve the scientific excellence and the wide participation of mathematicians and students from all over the region that are necessary to attain such goals.

The decision to launch the MCA was made at a meeting held in New Orleans on January 6, 2011, at the invitation of the AMS. The meeting was attended by representatives of several mathematical institutes and societies including SIAM, the national mathematical societies of Brazil (SBM), Canada (CMS), and Mexico (SMM), and the Mathematical Union for Latin America and the Caribbean (UMALCA). Following the New Orleans meeting, the six founding societies (AMS, SBM, CMS, SMM, SIAM, and UMALCA) were invited to nominate representatives to form the MCA 2013 Steering Committee (SC). Support for the MCA initiative was then unanimously reiterated in a meeting hosted by the Brazilian Mathematical Society in Rio de Janeiro on May 9–10, 2011, which was attended by representatives of most national mathematical societies in North, Central, and South America and the Caribbean, as well as of major mathematical institutes in the region.

One of the SC's first tasks was to issue a call for proposals to organize the MCA 2013. By the April 7, 2011, deadline, the SC had received two excellent proposals: from the Universidad de los Andes in Bogotá, Colombia, and from the Centro de Investigación en Matemáticas (CIMAT) at Guanajuato, Mexico. Both locations were visited by SC representatives, to assess the facilities available for the Congress. After a long discussion, where all aspects were carefully weighed, the SC decided to award the realization of the inaugural MCA to CIMAT, Guanajuato. The SC also deliberated on the structure of the scientific program, which is to include five plenary lectures, twenty invited lectures, about twenty-five special sessions, and a number of additional activities, such as general public lectures.

The SC appointed the Program Committee (PC), whose task is to select the plenary speakers and invited speakers. The PC is chaired jointly by Dusa McDuff (Columbia University, USA) and Jaime San Martin (CMM, Santiago, Chile). The choice of speakers is based on excellence in research and very good expository skills. Five outstanding mathematicians have been selected as plenary speakers at the MCA 2013: James Arthur (University of Toronto, Canada), Artur Avila (IMPA/Brazil and CNRS/France), Manjul Bhargava (Princeton University, USA), Luis Caffarelli (University of Texas, USA), and Ingrid Daubechies (Duke University, USA). The list of the invited speakers can be found at `http://www.mca2013.org`.

Proposals for special sessions should be submitted by email to `mca2013.sessions@gmail.com` by July 31, 2012. The topics should be broad and fairly well represented throughout the Americas. The co-organizers must represent at least two different countries in the region. Preference will be given to proposals whose list of suggested speakers represents diversity in various aspects. More information can be obtained at `http://www.mca2013.org`.

The SC established the following mathematical awards to acknowledge accomplishments that are of special relevance to the goals of the Congress:

The MCA Prize: Five prizes of US$1,000 each will be awarded to mathematicians who are no more than twelve years past the Ph.D. in August 2013. Eligibility for consideration of nominees requires that they either received their graduate education or currently hold a position in the Americas.

The Americas Prize: One prize of US$5,000 will be awarded to an individual or a group in recognition of work to enhance collaboration and the development of research that links mathematicians in several countries in the Americas.

The Solomon Lefschetz Medal: Two medals with an award of US$5,000 will be given to mathematicians in recognition of their excellence in research and their contributions to the development of mathematics in the Americas.

Nominations for these prizes should be made by email to `mca2013.prizes@gmail.com` by January 31, 2013. Complete information on the nomination procedures can be found at `http://www.mca2013.org`.

The realization of the MCA 2013 at CIMAT, Guanajuato, is made possible by a grant from the CONACyT, the national research council of Mexico. Other sponsors include the founding societies and such institutes as AIM, CAMS at USC, IMPA, and MSRI. The AMS has been providing support and publicity for the MCA 2013 through such means as the *Notices* and the AMS website. The Brazilian Mathematical Society and the Brazilian Society for Applied and Computational Mathematics have committed BRL30,000 (about US$17,000) to support the participation of Brazilian mathematicians and students. Other mathematical societies and institutes in North and South America are expected to follow suit. A proposal is being submitted to the NSF for travel support for U.S. mathematicians to attend the MCA.

We are optimistic that the MCA 2013 will be most successful and that such a quadrennial Congress will take place as a regular event in the mathematical calendar.

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Letters to the Editor

Uncritical Use of Citation Database

This is to express my deep concern about publication in the *Notices* of the paper "Influential mathematicians: Birth, education, affiliation", vol. 59, no. 2, by John Panaretos and Chrisovalandis Malesios.

The authors discuss statistics based on the "list of Highly Cited Researchers" (HCR) published by Thomson Scientific.

I am not considering here the question whether the number of citations is a reasonable measure of "influence" of a mathematician (for example, someone who wrote a handbook in medical statistics can have a higher citation rate than any "pure" mathematician).

I would like to bring to your attention another, much simpler matter. Suppose that we are indeed interested in the number of citations of mathematicians. Every mathematician who ever cared to look at the Thomson database knows that this database is nonadequate and almost useless for this purpose.

To see this, it is enough to compare it with the MathSciNet citation database (which is by far the most complete citation database for mathematicians).

For example, the list of HCR in Mathematics contains Alan Gelfand but does not contain Israel Gelfand. According to MathSciNet, Alan has 398 citations while Israel has 5,269. The HCR list contains Douglas Arnold but does not contain Vladimir Arnold. According to MathSciNet, Douglas has 1,934 citations and Vladimir 5,880.

Among the first thirty-four people in HCR, twelve have less than 1,000 MathSciNet citations, however the HCR list *does not* contain the names Erdős (Paul Erdős, 8,301 references in MathSciNet), Hörmander (Lars Hörmander, 6,752), Milnor (John Milnor, 6,558), Lang (Serge Lang, 4,714), Hartshorne (Robin Hartshorne, 4,580), Tao (Terence Tao, 4,304), and so on. None of these names is in the HCR list. Can this be called a list of most cited mathematicians? Or of most influential mathematicians?

This shows that the HCR list is meaningless.

There are at least two evident reasons why the Thomson database cannot be used for making such a list.

1. It does not distinguish between people with similar names. But most importantly:

2. The Thomson database contains the data for *all* sciences. As a percentage of the whole of science literature, mathematics is negligible. Mathematicians and mathematical journals have much lower citation rates than the average for the whole of science. For this reason, the data on mathematics in the Thomson database are marginal and have no statistical significance.

Uncritical use of this HCR list for the discussion of demography of "influential mathematicians" is an example of poor scholarship. For example, what is the meaning of the fact that there is one HCR mathematician in Turkey, and none in Russia? The whole contents of the paper is discussion of such facts.

Unfortunately, one statement in the paper is correct: "*This interest is not purely academic, these rankings have caught the attention of policy makers.*"

That's why I am so much concerned that such a paper could be accepted in the *Notices*.

—Alexandre Eremenko
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eremenko@math.purdue.edu

(Received February 5, 2012)

Consumer Alert: Mead Flashcards with Errors

Mead division flashcards (© 2010) have cards with errors. Mistakes were made involving division by zero. For instance one card asked for the answer to $2 \div 0$ and then gives the answer as zero.

MeadWestvaco was contacted about the problem in the fall of 2011. New printings of the cards have corrected the errors and the company purged the faulty inventory in their warehouse; however, the faulty cards were not removed from stores and are still being sold. For instance, in early March I found that the faulty cards were still on the shelf at my local Walmart. Few consumers likely know of the problem. Other than product reviews on such sites as Amazon and a couple of notices submitted to some education journals, no other notification has been given of the problem, and so most of the cards sold are likely still being used. I am asking those in the mathematical community who have contact with schools to take the time to notify them of this problem. Elementary teachers could be asked to contact parents as well. (In Wisconsin, the State Department of Public Instruction was contacted and notification of the problem is being sent out in a publication for teachers.) If someone has a box of Mead flashcards ($24 \div 3$ card pictured on the box), they should contact MWV Consumer and Office Products Consumer Affairs Department at 1-800-648-6323 and provide the representative with the item number or UPC (bar code) from the product, as well as your name and mailing address, and a new set of flashcards will be sent. (Note that corrected boxes will have a $48 \div 6$ card pictured on the box.)

—Susan Kelly
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(Received March 2, 2012)

Efficient Algorithms for Solving Partial Differential Equations with Discontinuous Solutions

Chi-Wang Shu

Partial differential equations (PDEs) arise in numerous scientific and engineering applications. Mathematical modeling in various applications often ends up with a set of PDEs. Mathematical techniques can help to understand many crucial properties of the PDEs, such as existence and uniqueness of solutions under suitable initial and/or boundary conditions, and the well-posedness of the problem, which refers to the fact that the change of the solution for later time is controlled by the change of the initial condition or the change of the solution inside the domain is controlled by the change of the boundary data. However, for the vast majority of PDEs from applications, it is not possible to obtain explicit formulas for their solutions. Thus, in scientific and engineering applications, it is necessary to solve the PDEs numerically, that is, using certain algorithms to obtain approximate solutions to the PDEs on a computer.

Hyperbolic PDEs and Discontinuous Solutions

PDEs are categorized into different types, including elliptic, parabolic, and hyperbolic PDEs. In this article we concentrate our discussion on hyperbolic PDEs. These PDEs describe various wave propagation phenomena, such as water waves, electromagnetic waves, and waves in gas dynamics.

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The simplest hyperbolic PDE is the equation of linear convection

$$u_t + au_x = 0 \tag{1}$$

where $u = u(x,t)$ is the solution which depends on the spatial location x and time t, a is a constant which is the speed of wave propagation, and subscripts refer to partial derivatives, e.g., $u_t = \frac{\partial u}{\partial t}$. If we provide an initial condition

$$u(x,0) = g(x), \tag{2}$$

then it is easy to verify that the unique solution of the PDE (1) is

$$u(x,t) = g(x - at). \tag{3}$$

That is, the solution is simply a shift of the initial condition with speed a. We can imagine that this is the kind of waves we may observe on a lake with a mild wind. For this problem, we do have the analytical formula for the solution in (3); hence there is no need to resort to numerical algorithms. However, such simple examples with known exact solutions are often used to verify the performance of different algorithms since the numerical error can be computed exactly. The exact solution (3) implies that the solution u remains smooth for later time t if it is smooth initially. This is a common feature of linear hyperbolic PDEs: discontinuities in the solution u can only arise from the imposed initial and/or boundary conditions.

If we change the PDE "slightly" to the Burgers' equation

$$u_t + uu_x = 0, \tag{4}$$

or more appropriately written in conservation form

$$u_t + \left(\frac{u^2}{2}\right)_x = 0, \tag{5}$$

which is a simple model of fluid flow, the situation changes dramatically. With the same initial condition (2), we can no longer easily write down the solution for later time analytically. Moreover, even if we take a very smooth initial condition $g(x) = \sin(x)$, the solution evolves into a sharp front and finally, at $t = 1$, the front becomes vertical and the solution becomes discontinuous; see e.g., [17]. The solution for later time must be understood in the weak sense [17] since it no longer makes sense to require the PDE (5) to be literally satisfied at the point of discontinuity. For nonlinear hyperbolic equations, discontinuous solutions are generic: regardless of the smoothness of the given initial and/or boundary conditions, the solution to the PDE may become discontinuous.

The main theme of this article is the discussion of efficient numerical algorithms to solve hyperbolic PDEs when the solution becomes discontinuous.

Numerical Schemes for Smooth Solutions

Before discussing discontinuous solutions, we first describe the basic concepts of numerical schemes for smooth solutions. There are many types of numerical schemes in applications, such as finite difference schemes, finite element schemes, and spectral methods; see e.g., [8], [2], and [10]. For the simple PDE (1) or (5), assuming that we would like to solve it over the interval $x \in [0, 1]$ with a periodic boundary condition, the finite difference scheme would start with a choice of grid points $0 = x_1 < x_2 < \cdots < x_N = 1$, which are assumed uniform with $h = x_{j+1} - x_j = \frac{1}{N}$ for simplicity, and a time discretization $0 = t^0 < t^1 < t^2 < \cdots$. We can then write down the scheme satisfied by the numerical solution u_j^n, which approximates the solution u of the PDE at the location (x_j, t^n). The finite element and spectral method would start with an N-dimensional function space $V_N = \mathrm{span}\{\varphi_1(x), \varphi_2(x), \ldots, \varphi_N(x)\}$. We can then seek the numerical solution $u_N(x,t)$ approximating $u(x,t)$ from the space V_N. That is,

$$u_N(x,t) = \sum_{j=1}^{N} a_j(t)\varphi_j(x),$$

and the scheme solves for the coefficients $a_j(t)$ at the time levels t^n for $n = 0, 1, 2, \ldots$. The difference between the finite element method and the spectral method is that V_N consists of piecewise polynomial functions for the former and of global polynomials or trigonometric polynomials for the latter.

We can see that for all these methods, the computer stores and processes N pieces of information for each time step (u_j^n for $1 \le j \le N$ for the finite difference method and $a_j(t^n)$ for $1 \le j \le N$ for the finite element and spectral methods). The processing cost (the number of operations such as additions and multiplications) per time step is different for different methods, but for finite difference and finite element methods it is typically $O(N)$ and for the spectral method using fast Fourier transforms (FFT) it is $O(N \log N)$. Since for most practical computations N is not that large, $\log N$ would be a very modest number compared with N; hence all three methods have costs roughly of the same order with respect to N per time step.

The purpose of *numerical analysis* is to analyze the stability and errors of these schemes when N changes (increases). One measurement of computational efficiency for a numerical algorithm is the order of accuracy. For the three types of methods described above, if the error e_h, which for a finite difference scheme is $(e_h)_j^n = u(x_j, t^n) - u_j^n$ and for a finite element or spectral method is $e_h(x, t^n) = u(x, t^n) - u_N(x, t^n)$, measured by a suitable norm such as the L^p norm with $1 \le p \le \infty$, is of the order $O(N^{-k})$, then k is referred to as the order of the scheme. If k is large, the scheme is called a "high-order" scheme (usually k would need to be at least 3 in order for the method to be referred to as a high-order scheme). When the number N is doubled, the error of a first-order method would only be reduced by a factor of 2, but the error of a third-order method would be reduced by a factor of $2^3 = 8$. Since the cost per time step is always $O(N)$ (of course for higher-order methods the coefficient in the $O(N)$ term is larger, but this coefficient relative to N does not change), apparently a high-order method is more efficient when N is large.

It is relatively easy to design algorithms which are high-order accurate when the solution of the PDE is smooth. For example, for the finite difference scheme, a two-point finite difference

$$\frac{u_{j+1} - u_j}{h} = u_x(x_j) + O(h) \tag{6}$$

would be first-order accurate, while a three-point finite difference

$$\frac{u_{j+1} - u_{j-1}}{2h} = u_x(x_j) + O(h^2) \tag{7}$$

would be second-order accurate (we have dropped the superscript n when considering spatial accuracy). Here we are measuring the errors locally, namely by putting the exact smooth solution u of the PDE into the finite difference scheme and measuring its remainder, which is referred to as the *local truncation error*. We would like to emphasize that for a kth-order local truncation error analysis such as (6) or (7) to hold, the solution u should have $k+1$ continuous derivatives since we are using Taylor expansions to obtain the order of accuracy. The Lax equivalence theorem and its nonlinear generalization by Strang (see e.g., [8]) provide us with assurance that for a stable scheme, such easily measured local truncation error and the global error e_h defined above are of the same order with respect to N.

Comparing with finite difference and finite element methods which are usually of a fixed order of accuracy, spectral methods have errors which are smaller than $O(N^{-k})$ for *any* fixed k; thus they are "infinite-order" schemes. In fact, the error for spectral methods approximating analytic solutions of PDEs can be exponentially small; namely, the error decays as $O(e^{-\alpha N})$ for some constant $\alpha > 0$. If the constant α is such that $e^{-\alpha} = \frac{1}{2}$, then the error would be reduced by a factor of 2 when N is only increased by 1 to $N+1$. This compares sharply with a first-order scheme, for which N would need to be doubled to $2N$ in order for the error to be reduced by the same factor of 2.

Of course, different schemes have different costs per time step (that is, the coefficients in the $O(N)$ term could be different), and the coefficient in the error $O(N^{-k})$ could also be different for different schemes. A more practical measurement of efficiency is the CPU cost to achieve the desired level of error. Figure 1, taken from [18], plots the L^1 error (in logarithm scale) versus CPU time in seconds for three finite difference schemes, of first-, third- and fifth-order accuracy, respectively, solving the eikonal equation which is a nonlinear hyperbolic PDE. It can be seen clearly that, for this example, the higher-order schemes are much more efficient in CPU cost than the lower-order ones to achieve the same level of L^1 error.

A quantitative study on the efficiency of high-order schemes for linear hyperbolic PDEs can be found in, e.g., [8].

Linear PDEs with Discontinuous Solutions

As we mentioned before, the solutions to hyperbolic PDEs may be discontinuous. For linear PDEs such as (1), the discontinuities in the solution come from the prescribed initial and/or boundary conditions. For nonlinear PDEs such as (5), the discontinuities in the solution may be generated spontaneously even if the initial and boundary conditions are infinitely smooth.

For such discontinuous solutions, the performance of high-order accurate schemes, such as the spectral method and high-order finite difference schemes, will degrade dramatically. Convergence will be completely lost in the strong L^∞ norm, and it is at most first-order in average norms

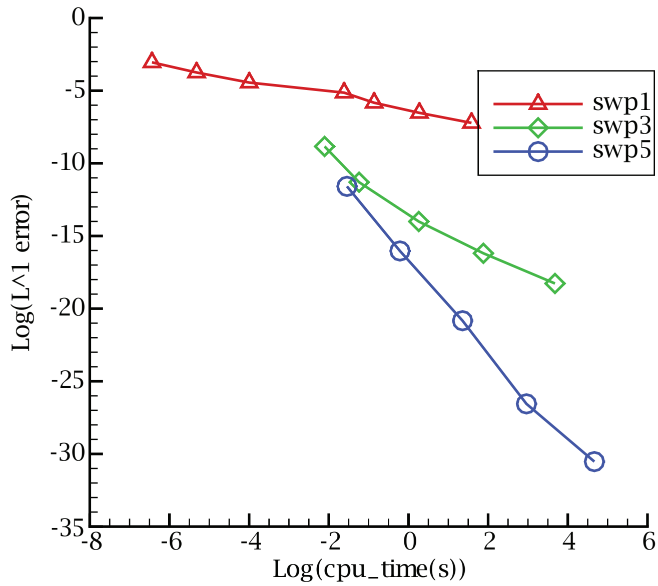

Figure 1. Reproduced from [18]. L^1 error versus CPU time. "swp1" is a first-order finite difference scheme, "swp3" a third-order one, and "swp5" a fifth-order one.

such as the L^1 norm. This problem exists already at the approximation level; namely, even the approximation to the initial condition cannot be high order. A simple example is the Fourier spectral solution for the linear equation (1) with $a = 1$ and an initial condition $u(x, 0) = x$ for $-1 \le x < 1$ with periodic boundary conditions. The approximation to the initial condition is shown in Figure 2, taken from [5]. The initial condition

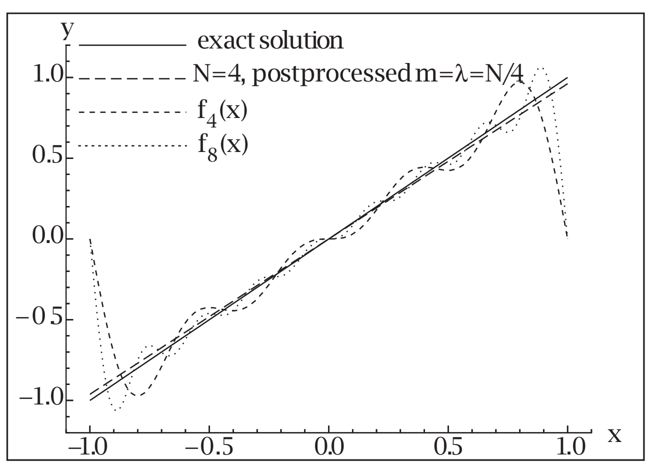

Figure 2. Reproduced from [5]. The function $f(x) = x$ (solid line), the Fourier partial sum $f_N(x)$ with $N = 4$ (short dashed line) and $N = 8$ (dotted line), and the approximation through the Gegenbauer post-processing for $N = 4$ (long dashed line).

has a discontinuity at $x = \pm 1$ when viewed as a periodic function. We can see clearly that there are significant oscillations for the spectral approximation (for $N = 4$ and $N = 8$, denoted by $f_4(x)$ and $f_8(x)$ in the figure) near the discontinuity. These are called the *Gibbs oscillations*, and these oscillations are polluted throughout the computational domain, although their magnitudes are smaller away from the discontinuity. Modern nonoscillatory schemes, e.g., the weighted essentially nonoscillatory (WENO) schemes [13], [16], can remove these spurious oscillations and produce sharp, monotone shock transitions. However, with transition point(s) across the discontinuity, which cannot be avoided by conservative shock capturing schemes, the error measured by the L^1 norm still cannot be higher than first order.

Therefore, when measured by the errors in the L^p norms, a high-order accurate scheme seems to have little advantage over a first-order accurate scheme whenever the solution contains discontinuities. This would seem to be a major difficulty in justifying the design and application of high-order schemes for discontinuous problems.

One possible way to address this difficulty is to measure the error *away from the discontinuities*. For example, in Table 1, again taken from [18], the error measured 0.15 away from the singularity $(0,0)$ achieves the designed order of accuracy for third- and fifth-order WENO schemes, which are finite difference schemes, for solving the eikonal equation. For many problems in applications, such high-order accuracy would be desirable and would justify the usage of high-order schemes.

However, such measurement of error is not global, leaving open the theoretical issue of whether a high-order scheme produces solutions which are globally high-order accurate. The proof of high-order accuracy away from the discontinuities is difficult, and for coupled hyperbolic systems, in regions between characteristic lines, the error may be only first-order even though we measure it away from the discontinuities [15].

A major contribution of mathematics to the design and understanding of algorithms in such a situation is the discovery that many high-order schemes are still high-order accurate for discontinuous solutions if we measure the error in a weaker norm, namely, the so-called *negative Sobolev norm*. We recall that the negative Sobolev norm is defined by

$$||u||_{-k} = \max_{v \in H^k, v \neq 0} \frac{(u,v)}{||v||_k}$$

where H^k is the space of all functions with finite kth order Sobolev norm defined by

$$||v||_k^2 = \sum_{\ell=0}^{k} \int_a^b \left(\frac{d^\ell v}{dx^\ell} \right)^2 dx$$

Table 1. Reproduced from [18]. L^1 and L^∞ errors and orders of accuracy for finite difference first-order (swp1), third-order WENO (swp3), and fifth-order WENO (swp5) fast sweeping schemes solving the eikonal equation. The errors are measured outside the circle $\sqrt{x^2 + y^2} \leq 0.15$ which contains the singularity $(0,0)$.

swp5	L^1 error	order	L^∞	order
80	6.442E-08	–	5.628E-06	–
160	1.367E-09	5.558	4.525E-07	3.637
320	2.122E-11	6.010	1.515E-09	8.222
640	6.681E-13	4.989	5.146E-11	4.880
swp3	L^1 error	order	L^∞	order
80	1.544E-06	–	1.283E-04	–
160	2.890E-07	2.418	4.052E-06	4.984
320	4.697E-08	2.621	1.220E-06	1.731
640	6.161E-09	2.930	1.609E-07	2.923
swp1	L^1 error	order	L^∞	order
80	5.413E-03	–	1.302E-02	–
160	2.045E-03	1.404	7.684E-03	0.761
320	1.131E-03	0.854	4.543E-03	0.758
640	6.106E-04	0.890	2.355E-03	0.948

where (a,b) is our computational interval. A classical example of a function which has a small negative norm but finite strong norms is

$$v^n(x) = \sin nx$$

for large n. It can be easily verified that

$$||v^n||_{-k} = O(n^{-k})$$

for any positive integer k, yet

$$||v^n||_{L^p} = O(1)$$

for any $1 \leq p \leq \infty$. This classical example demonstrates a typical feature of functions with small negative norms but large regular norms: they are typically rapidly oscillatory functions. An easy way to understand convergence in such negative norms is to say that the convergence is "in the moments", namely in integrals against smooth functions. Thus the error e_h may not be small at any specific point, that is, $||e_h||$ is no better than $O(h)$ when the norm is taken as the standard L^p norm, but the integral

$$\int_a^b e_h(x)v(x)dx$$

is small (of the size $O(h^k)$ for a large k) for *any* smooth function $v(x)$. It can be proved, for example in [12], [14], and [15], that a high-order scheme is still high-order accurate for a *linear* hyperbolic PDE, measured in a suitable negative norm, with suitable handling of smoothly cutting off high frequencies. For example, a fourth-order accurate scheme is still fourth-order accurate measured in the $||\cdot||_{-4}$ norm, and a spectral method is accurate of kth order for any k in the negative $||\cdot||_{-k}$ norm.

For a mathematician, it is reassuring to know that high-order accurate schemes are indeed high-order accurate even for discontinuous solutions; only we must measure the error in a particular norm (in this case, the negative norm). However, for an engineer performing scientific computing, it is difficult to argue for the practical value of the scheme by simply knowing that the error is small in some fancy norm. After all, for practical problems one cares about errors in strong norms, such as the L^2 norm. Fortunately, there are many post-processing techniques [1], [4], [6], [12], [15] which can recover high-order accuracy in strong norms, such as the usual L^2 or L^∞ norm, in smooth regions of the solution, for any sequence of numerical solutions which converges in the negative norm with high-order accuracy to a discontinuous but piecewise smooth solution. In Figure 1, the post-processed solution is a much better approximation to the exact solution than the one before post-processing.

Thus we can conclude that for a linear hyperbolic PDE with discontinuous but piecewise smooth solutions, a good computational strategy is still to use a high-order accurate numerical method. The numerical solutions may be oscillatory and may converge poorly in strong norms, but they do converge in high-order accuracy measured in suitable negative norms. A good post-processor can then be applied to recover high-order accuracy in strong norms in smooth regions of the solution.

Nonlinear PDEs with Discontinuous Solutions

When the hyperbolic PDE is nonlinear, for example the Burgers' equation (5), the situation becomes more complicated. On the one hand, for a nonlinear hyperbolic PDE, as in the Burgers' equation, the solution will become discontinuous in finite time even if the initial condition is smooth. Therefore, it is even more important to consider numerical methods for discontinuous solutions as they are generic. On the other hand, for nonlinear hyperbolic PDEs, it is still an open problem mathematically whether or in what sense the numerical solutions of a high-order scheme are high-order accurate. We can certainly measure the error in a region away from the discontinuous, as in Table 1, which is for the numerical solution of a nonlinear hyperbolic PDE. However, this is difficult to prove mathematically, may not always be correct (especially for hyperbolic systems of PDEs), and is not an indication of the global error.

Lax [11] argued that, for a nonlinear system, high-order information is retained by a high-order scheme and may be extracted by post-processing. In fact, Lax's argument indicates that more high-order information is retained in high-order solutions of nonlinear systems than of linear ones since in the nonlinear case the solution operator is contractive. However, it is not clear mathematically in what form such high-order information is retained and how it can be extracted from the numerical solution.

Numerical evidence does indicate that high-order accurate schemes, especially those modern high-order accurate, "high-resolution" schemes which have nonlinear mechanisms to control spurious oscillations near discontinuities, such as WENO schemes [13], [16], are suitable choices for solving nonlinear PDEs with discontinuous solutions, especially when the solutions contain

Table 2. Euler equations for the nozzle flow. The maximum norm errors to the right of the shock before and after post-processing. Fifth-order WENO computation with N grid points.

N	before error	λ	m	after error	order
600	1.37×10^{-3}	3	3	8.16×10^{-4}	
800	1.09×10^{-3}	3	4	3.33×10^{-4}	3.11
1000	1.27×10^{-3}	4	5	1.66×10^{-4}	3.13
1200	1.19×10^{-3}	5	6	8.19×10^{-5}	3.8
1400	1.27×10^{-3}	6	7	4.09×10^{-5}	4.5
1600	1.19×10^{-3}	6	8	1.72×10^{-5}	6.46
1800	1.19×10^{-3}	7	9	8.95×10^{-6}	5.54

Table 3. Reproduced from [3]. Fourth-order residual distribution WENO scheme for the nozzle flow problem. Errors outside three cells around the shock and numerical orders of accuracy for the density ρ on nonsmooth meshes with N cells.

sub-cell integration in the shocked cell				
	before shock		after shock	
N	L^∞ error	order	L^∞ error	order
21	7.09E-07	-	3.35E-05	-
41	6.13E-08	3.53	2.19E-06	3.93
81	4.85E-09	3.66	6.61E-08	5.05
161	3.11E-10	3.96	3.00E-09	4.46
321	2.00E-11	3.96	1.36E-10	4.46
641	1.20E-12	4.05	4.92E-12	4.80
regular integration in the shocked cell				
	before shock		after shock	
N	L^∞ error	order	L^∞ error	order
21	7.09E-07	-	6.42E-05	-
41	6.13E-08	3.53	1.42E-05	2.18
81	4.85E-09	3.66	3.37E-06	2.07
161	3.11E-10	3.96	7.29E-07	2.21
321	2.00E-11	3.96	1.77E-07	2.04
641	1.22E-12	4.04	2.52E-08	2.81

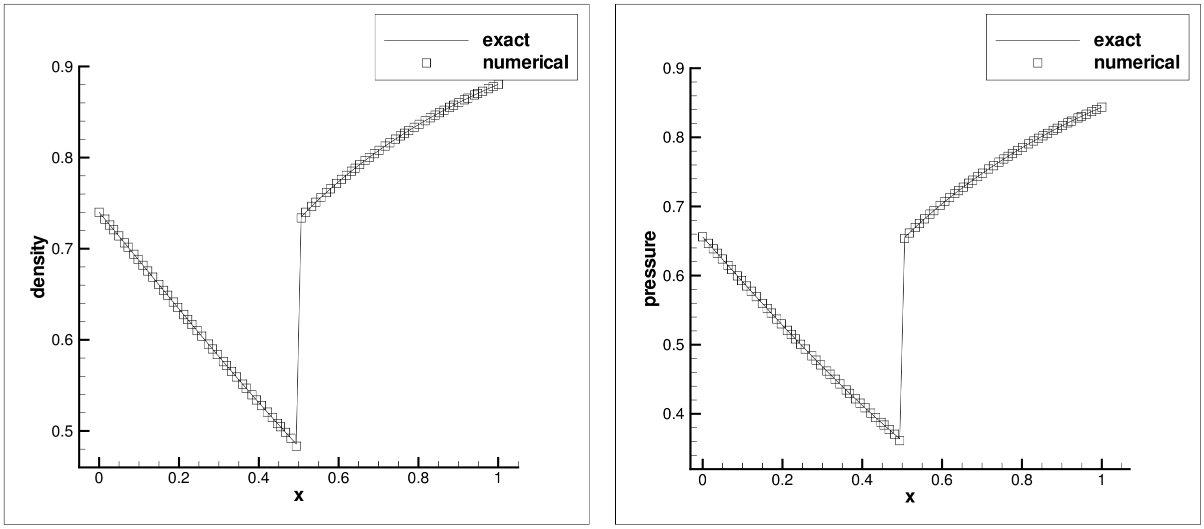

Figure 3. Reproduced from [3]. Fourth-order residual distribution WENO scheme for the nozzle flow problem. Nonsmooth mesh with 81 cells. Solid lines: exact solution; symbols: numerical solution. Left: density ρ. Right: pressure p.

both discontinuities and complicated smooth region structure between discontinuities.

In [7], Gottlieb, Gottlieb, and Shu presented numerical experiments in post-processing numerical solutions of the Euler equation of gas dynamics (a nonlinear system of hyperbolic PDEs) obtained by a fifth-order WENO scheme. The order of accuracy for the numerical solution is diminished in a region downstream of the shock. It is demonstrated that, for steady-state solutions near the shock, the application of the Gegenbauer post-processing method [4] recovers the designed order of accuracy in the L_2 and (emulated) maximum norms (by "emulated" we mean that the maximum error is taken over all points excluding only one near the discontinuity; the domain in which the error is measured therefore gets closer to the discontinuity as the total number of grid points N increases, thus emulating uniform convergence). Table 2 shows the maximum norm errors for the density in the entire region to the right of the shock before and after post-processing. The maximum norm errors of the numerical solution before post-processing do not decay when the grid is refined. After post-processing the region to the right of the shock, high-order accuracy is recovered. The parameters λ and m in Table 2 are the ones used in Gegenbauer post-processor [4].

For this nozzle problem, if a certain "sub-cell resolution" technique [9] is applied in the cell containing the discontinuity, then it is possible to obtain the designed high-order accuracy of the numerical scheme without any post-processing. We demonstrate this by the simulation using a fourth-order residual distribution WENO scheme in [3]. In Figure 3, we observe good resolution of the WENO numerical solution in comparison with the exact solution. In Table 3, we observe that the designed fourth-order accuracy is achieved for both upstream and downstream of the shock if a sub-cell integration technique is applied in the shocked cell, while the accuracy downstream of the shock has lower than the designed fourth-order accuracy if no special treatment is performed in the shocked cell.

It is a challenging mathematical problem to investigate high-order schemes for discontinuous solutions of nonlinear hyperbolic PDEs. Is the numerical solution high-order accurate measured in a suitable norm? Can the high-order information be extracted to yield high-order accuracy in a strong norm? An affirmative answer to these questions will help in the design and application of efficient numerical methods for nonlinear hyperbolic PDEs which are very important in applications.

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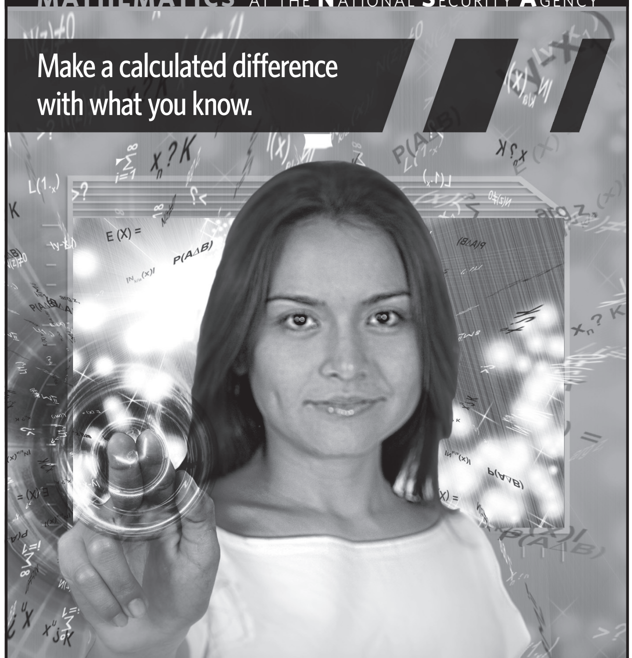

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Remembering Herbert Federer (1920–2010)

Harold Parks, Coordinating Editor


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Herbert Federer

Leslie Vaaler

Herbert Federer taught me about life, scholarship, and the world of mathematics; he was my father. When I was a little girl, my father and I would go on walks and he would talk to me. As I remember this communication, he always respected my ability to understand adult topics, so long as they were presented with careful explanation. He spoke deliberately, taking the time to choose words he felt conveyed just what he was trying to say. (Those who knew Herbert Federer will recognize this precision with language.) On our walks, my father was pleased to be asked questions and encouraged further queries by treating them as intelligent responses. In this manner, he gave me the roots of intellectual self-confidence.

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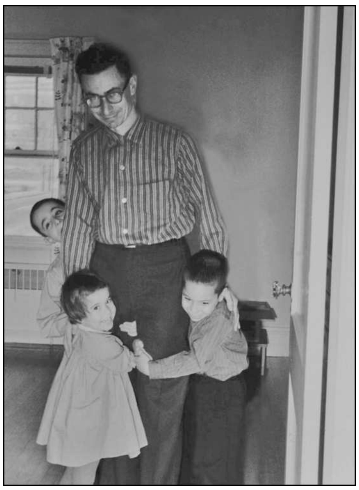
My father wanted me to understand his world, and so he talked to me about teaching, about the Brown University mathematics department, about mathematicians he admired, and about the joys and frustration of being a mathematician. I remember my father telling me about visiting Princeton as a young mathematician and walking with Steenrod on the golf course. He talked to me about working on his thesis the summer before he became a graduate student. He shared with me thoughts about mathematicians being familiar with areas of mathematics other than just their own.

In a 1976 description of his mathematical career to date, he wrote, "I have worked hard to transform this subject from a collection of isolated results into a cohesive body of knowledge. However, my main effort has been directed towards a deeper understanding of concepts significantly related to some classical properties in other parts of mathematics. These interests also led me to write two papers on group theory and homotopy theory."

Thanks to my father, even before I understood any significant mathematics, I understood that mathematics was an art as well as a science.

My father liked to work at home. I was brought up knowing that it was important not to disturb him, but I also knew that should I knock on his study door, he would always stop and talk with me. One day, as I sat in his study, he explained to me the importance of a mathematician having a "big wastebasket" so that many paths could be tried out, the majority of which would not turn out to be useful. He placed a paper model of a surface in a decorative animal clip I had given him, and when I asked him about it, he talked of minimal surfaces. Prior to the publication of *Geometric Measure Theory* (in 1969 when I was eleven years old), he talked to me about the importance of good mathematical notation, making a bibliography, and proof sheets. Years later, when I wrote a book, these lessons were useful.

My father shared with me his hopes for his book *Geometric Measure Theory*. As stated in the preface, he wished it to serve as a "comprehensive treatise" on the subject for "mature mathematicians" as well as a textbook for very "able students". It was certainly his hope that the book would bring more attention to the subject. My father lamented that certain other areas of mathematics were more fashionable than geometric measure theory and blamed himself for not being a sufficiently good politician.



Herb and his children, Andrew, Wayne, and Leslie.

At some point, after the book was published, most likely when I was an undergraduate student and took a particular interest in algebra, my father took pride in showing me that *Geometric Measure Theory* began with an explanation of exterior algebras. He once wrote that his scientific effort was "directed to the development of geometric measure theory, with its roots and applications in classical geometry and analysis, yet in the functorial spirit of modern topology and algebra."

Professor Federer enjoyed teaching graduate analysis using the second chapter of his book. He was very pleased when he found a hardworking student with talent to understand the material. He always had high standards for himself and for his family, and I am sure he was a demanding teacher.

My father was born on July 23, 1920, in Vienna, Austria. He immigrated to the United States in 1938 and became a naturalized citizen in 1944. He chose never to travel to Europe, and his domestic travel was also quite limited.

Herbert Federer began his undergraduate education at Santa Barbara and then transfered to Berkeley, receiving the degrees B.A. in mathematics and physics in 1942 and Ph.D. in mathematics in 1944. During 1944 and 1945, he served in the U.S. Army at the Ballistic Research Laboratory in Aberdeen. Beginning in 1945, he was a member of the mathematics department at Brown University. He became a full professor in 1951, a Florence Pirce Grant University Professor in 1966, and professor emeritus in 1985. He supervised the Ph.D. theses of ten students.

Herbert Federer and his daughter, Leslie, in 1984.

Herbert Federer joined the American Mathematical Society in 1943. He served on the invitations committee for the 1958 summer institute, as associate secretary during 1967 and 1968, and as Representative on the National Research Council from 1966 to 1969. He delivered an invited address (New York City, 1951) and was the colloquium lecturer at the August 1977 meeting in Seattle. My father and Wendell Fleming received the 1987 Steele Prize for their 1960 paper "Normal and integral currents".

Professor Federer was an Alfred P. Sloan Research Fellow (1957–1960), a National Science Foundation Senior Postdoctoral Fellow (1964–1965), and a John Guggenheim Memorial Fellow (1975–1976). He became a fellow of the American Academy of Arts and Sciences in 1962 and a member of the National Academy of Sciences in 1975.

My father was a private man. Mathematics and his family were Herbert's two loves. I believe he would not want me to share further personal details of his life, but he would be pleased if this memoir attracted mathematicians to learn more about geometric measure theory, the subject he loved so dearly.

Herbert Federer taught me about life, scholarship, and the world of mathematics. He taught me about love and responsibility. He was a wonderful mathematician and father.

John Wermer

Herb Federer was a remarkable man. He was passionately committed to mathematics and had a very personal approach to all issues, including notation.

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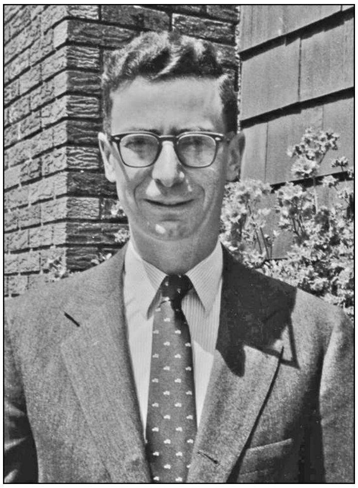

Herbert Federer, 1958.

When I came to Brown University as Herb's junior colleague in 1954, it greatly impressed me that Herb had professional knowledge of and gave graduate courses in and wrote up lecture notes for algebraic topology, differential geometry, algebraic geometry, besides his central fields of interest in real analysis and geometric analysis. When visitors came to speak at Brown, in different areas, they were often eager to consult with Herb on all kinds of mathematical questions.

I remember that when Iz Singer gave a colloquium at Brown on the Atiyah–Singer theorem in its early stages, I understood little of the talk, but Herb understood it very well and realized that something important had happened.

Together with his coworker Wendell Fleming, Herb developed a theory of currents which became a powerful tool in modern geometric analysis, and he wrote his monumental book *Geometric Measure Theory*, which has been very influential.

When I came to Brown in 1954, Herb was very friendly towards me, and I remember him fondly.

William Allard

When I first arrived at Brown University in the fall of 1963, I wasn't sure what to expect. Even though I didn't know much mathematics, I was pretty sure I wanted to study it. I began my graduate career by taking courses in real analysis, taught by Bob Accola; algebra, taught by Than Ward; and complex analysis taught by Herb Federer. These courses, as I thought at the time and as I now realize even more, were taught very well, for which I am now very grateful.

On the first day of class Herb said that only one in five of us would earn a degree; to this day I do not know why he said that—perhaps it was to remind us that graduate study in mathematics was not a cakewalk. He also noticed some ashtrays (yes, ashtrays; this was 1963) in the first or second row of desks and proceeded to deposit them in the wastebasket, making a remark or two as to the low opinion he had of smokers, a group of people which at the time included me.

Herb taught a splendid complex analysis course. He used notes he had developed over the years. Indeed, when Herb decided to learn a subject, he started from the beginning and worked everything out his own way, constantly and laboriously reorganizing as he understood more. He did this, as far as I know, with algebraic topology and algebraic geometry as well as, needless to say, geometric measure theory. It shortly became clear to me how concisely and elegantly he presented the material. More important than this perhaps was his superb organization of the material.

In the class were some talented undergraduates among whom were Blaine Lawson and Joel Pasternack. Herb held office hours every Friday afternoon, which Joel and I nearly always attended. They were wonderful. In spite of the fact that Herb was feared by many students, he was very welcoming to anyone who cared about mathematics and who took the trouble to get to know him. Among other things, I distinctly remember him elaborating on the construction of the universal covering space during one of these office hours; this was something that was fantastic to me at the time. Herb's course lasted for one year. I still have the notes and am contemplating writing them up; I wouldn't be surprised if some of the material, particularly on Riemann surfaces, is not easily accessible in the literature.

Now at this time Herb was fairly well along in the writing of *Geometric Measure Theory*. It turned out that he wanted someone to check it very carefully. He asked me to do this during my second year at Brown. What an opportunity! Of course I agreed and proceeded to read his beautifully handwritten notes for the next three and a half years.

Over the years a number of people, many of whom are excellent mathematicians, have complained about the book being too difficult or not having enough motivation or not being friendly to the reader. This continues to puzzle me. I guess I believe that anyone who wants to learn geometric measure theory will have to suffer in doing so because of the inherent technicality of the subject. But I believe Herb's book affords the diligent and patient reader a path to the goal of learning large parts of the subject which minimizes the pain. I must admit, however, that there were several times when I would suggest that he ought to say a bit more than he did in various proofs. He never accepted my suggestions, saying words like, "But don't you see; I have said all that needs to be said." Oh, well. Then there is the famous Theorem 4.5.9 which has thirty-one parts! I have come to believe that there are many different ways

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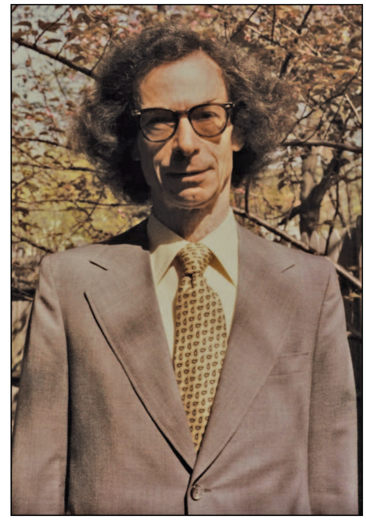
to approach learning and inventing mathematics and that, for some, Herb's way won't work. But I remain convinced that his book is a remarkable and extremely valuable part of the literature. It represents the culmination of many years of work by a talented craftsman absolutely dedicated to his work.

It turns out that, during the last five years or so, after having left the field around twenty years ago to do other things, I have again been working on geometric measure theory, so I have had many occasions to revisit *Geometric Measure Theory* and have been amazed by how clear and efficient the presentation is.

I would now like to elaborate on some of the material in *Geometric Measure Theory*. At this point in time, the most important parts of the book are Chapters Two, Three, and Four, entitled "General measure theory", "Rectifiability", and "Homological integration theory", respectively. In Chapters Two and Three we find a beautiful and efficient development of, among other things, Hausdorff measure and everything one might want to know about the images and level sets of Lipschitz functions on Euclidean space. Chapter Two ends with the statement and proof of the famous Besicovitch–Federer theorem on rectifiability and nonrectifiability. One also finds a treatment of Haar measure as well as the fine structure of real analytic and semianalytic sets. I must also mention the marvelous Morse–Sard–Federer theorem on the regularity of the level sets of highly differentiable functions.

In Chapter Four we find the theory of currents. (I must admit I have always found the title "Homological integration theory" to be a bit pretentious.) Of course currents were introduced by de Rham many years earlier. But de Rham did not treat the *rectifiable* currents; these form natural spaces in which one finds the solution of many variational problems like, most notably, the Plateau problem of minimizing area with a prescribed boundary. Rectifiable and integral currents first appeared in the landmark 1960 paper "Normal and integral currents" by Federer and Fleming. I have reread large parts of this chapter recently and have been delighted by the clarity and efficiency of the presentation. In my opinion, this chapter remains the best reference for this material today.

Chapter One is entitled "Grassmann algebra". Here we find a beautiful treatment of *metric* multilinear algebra including exterior algebra. Again, I don't believe there is a better treatment of this subject.



Herbert Federer, 1979.

Finally, we come to Chapter Five, "Applications to the calculus of variations". Here we find a treatment of Almgren's regularity theory for elliptic variational problems which had been published right before Federer wrote this chapter. We also find a treatment of Simons's work on minimizing cones which appeared in 1968 as well as the De Giorgi–Federer dimension reduction trick for applying regularity theory for the area integrand in dimension n to obtain regularity results in dimension $n+1$. As Federer himself predicted, the results and techniques in this chapter have been superseded by later work. Thus, perhaps a bit sadly, I have to say that Chapter Five is not where one goes to study regularity theory. This is not the place to give the many relevant references for the state of the art in this area.

The aforementioned Besicovitch–Federer rectifiability theory was used in "Normal and integral currents" as well as in Chapter Four to obtain the fundamental compactness results for integral currents which in turn give existence results in the calculus of variations. Owing to the work of many people, most notably Almgren, this rectifiability theory is no longer necessary to obtain compactness theorems. Indeed, I find this later work more appealing geometrically.

In closing, let me point out that in the last twenty years or so there has been a flowering of work in geometric measure theory not just in the United States but also in Europe. Herb Federer, perhaps as much as anyone, laid the foundations.

Robert Hardt

Since 1967 Herbert Federer was an inspiring scholar and excellent mentor to me. His outstanding works have had a crucial influence on the development of geometric calculus of variations and the study of rectifiable sets and geometric measures. It was my great fortune to have had him as a teacher and Ph.D. advisor at Brown University from 1967 to 1971. These were turbulent years globally and locally with the Cold War, the Vietnam War, the student protests, and the reforms at the universities. Nevertheless, it was also a period of

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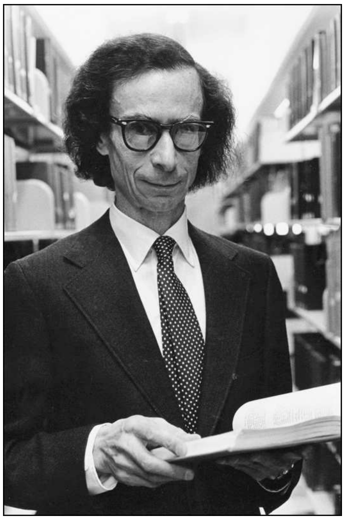


Brown University Archives, John Foraste, photographer.

Herbert Federer, mid-1970s.

great mathematics, and I felt the excitement of the coming-of-age of geometric calculus of variations in the beautiful works of Ennio De Giorgi, E. R. “Peter" Reifenberg, Herbert Federer, and Wendell Fleming. Their papers introduced various natural higher-dimensional generalizations of the classical two-dimensional Plateau problem of finding a surface (e.g., soap film) of least area spanning a given boundary curve. The objects discovered in these papers include various geometric weak limits of manifolds and polyhedra and have proven to have wide applications. The work by Federer and Fleming on normal and integral currents is still wonderful reading, whether in the Steele Prize-winning paper [FF60] or embedded in Federer’s fundamental book [Fed69].

As another sample of Federer’s insights, I want to call attention to the delightful 1965 paper [Fed65] which I believe has created linkages between Riemannian, complex, and algebraic geometry. His proof of the mass minimality of arbitrary complex subvarieties of Kähler manifolds greatly facilitated the birth of the now widely studied subject of *calibration theory*, in which many different special closed, possibly singular, forms provide variational information on associated geometric objects. In [Fed59], Federer introduced the important co-area formula, which involves fiber integration for changing variables with a Lipschitz map from one manifold to another one of a smaller dimension. In [Fed65], Federer generalized this to rectifiable and normal currents, where densities and orientations are involved. He recognized that the consequent theory of *slicing* could describe and be useful for numerous intersection theory phenomena in algebraic topology and in differential and algebraic geometry.

As is indicated in the paper [Fed65] and was a part of the spirit of all of his publications, one of the characteristics of Federer’s work was his love of and his dedication to many kinds of mathematics. Certainly he was expert in all types of analysis, old and new, but few analysts know about his 1946 and 1956 papers that treat free groups and spectral sequences or about his string of outstanding papers that solved numerous open problems in the then-popular theory of Lebesgue area. Whenever Federer became interested in a new subject (e.g., algebraic topology or algebraic geometry), he would go to the library and load up on large stacks of classic and modern books and journals. He’d then spend many weeks reading them, teach a graduate course, and ultimately produce a large collection of (unfortunately unpublished) notes. The various lecture notes that I have seen were extensive, likely to the chagrin of some students. Yet to most students he was an inspiration through his hard work and the reach of his research. He had one principle point of advice for students, and this he indicated by the only sign on his door—a long, vertically stacked series of small stickers that said “Read, Read, Read, ...” In contrast to the narrow reading habits of most mathematicians, Federer once said, “I never read *any* mathematics that I didn’t eventually use.” I believe that Federer was well influenced by Hassler Whitney in having this breadth of mathematical interests as well as in the direction of his research. See Federer’s enthusiastic review [Fed58] of Whitney’s book and Federer’s later paper [Fed75].

Herbert Federer had a strong sense of scholarship, as is evident in all of his writings. He was extremely careful and not too quick to publish. He once advised that after completing a paper, one should put it in the desk drawer for one month and then bring it out and reread it to find mistakes “as if you were the author’s worst enemy.” I remember when he pulled the paper [Fed70] out of the drawer. This now well-known paper involved estimating the dimension of the singular set of solutions of the codimension-one oriented Plateau problem. He had written it some time earlier but waited to submit it until he was sure that the singular set could in fact be nonempty. This was established by Bombieri, De Giorgi, and Giusti. In retrospect, this delay in publication was probably not a good idea because the important technique of this paper, now referred to as *Federer induction*, has proven to have wide applicability, not only to other area-minimizing problems but also to energy-minimizing harmonic maps and other systems of elliptic PDE’s.

Federer was a real stickler for precision, organization, and referencing. His notation was logical, even if it wasn’t always common. All these characteristics are evident in his seminal book *Geometric Measure Theory*. Its appearance in 1969 was timely, as it brought together earlier studies of geometric Hausdorff-type measures, work on rectifiability of sets and measures of general dimension, and the fast developing theory of geometric higher-dimensional calculus of variations. All of the arguments in his text exhibited exceptional completeness. That said, this book is not for the casual reader because his writing tends to be particularly concise. Forty years after the book’s publication, the richness of its ideas continue to make it both a profound and indispensable work. Federer once told me that, despite more than a decade of his work, the book was destined to become obsolete in the next twenty years. He was wrong. This book was just like his car, a Plymouth Fury wagon, purchased in the early 1970s that he somehow managed to keep going for almost the rest of his life. Today, the book *Geometric Measure Theory* is still running fine and continues to provide thrilling rides for the youngest generation of geometric measure theorists.

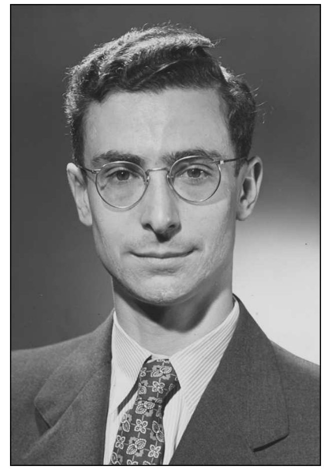
William P. Ziemer

I was both shocked and deeply saddened to learn of the death of Herbert Federer. I was shocked because to me Federer was a giant and giants are supposed to go on forever. I was deeply saddened because one of my primary sources of inspiration was to be no more.

In fact, Federer was considered a giant by many mathematicians because of his profound influence in geometric analysis. Federer, one of the creators of geometric measure theory (GMT), is perhaps best known for his fundamental development of the subject, which culminated in his publication of a treatise in 1969, with the same name, [Fed69]. The book, nearly 700 pages, is written in a manner which commands both admiration and respect because of its virtually flawless presentation of a wide range of mathematical subjects and is written in a style that is unique to Federer. The book, as well as all of his work, was carefully prepared in handwritten notes and includes an extensive bibliography of approximately 230 items. The manuscript is about ten inches thick and is characterized by the degree to which it attains perfection. This is an attribute that is shared with all of his writings. I know of only one small errata sheet.

I first met Federer in 1958 when I entered Brown University as a graduate student. (Coincidentally, this was the same time that Wendell Fleming, my Ph.D. mentor, joined the faculty at Brown.) I was impressed by how friendly and warm he was to my wife, Suzanne, and me. In fact, shortly after our first meeting, he insisted that I call him "Herb", something that I had difficulty in doing for a long time.

Despite the fact that Herb is best known for his work in geometric measure theory, this occupied only the second half of his career. The first half, from 1943–1960, was also a highly productive period with several of his papers laying the groundwork for his subsequent work in GMT. In fact, most of his papers in this period were devoted to area theory, a subject which has been lost to most researchers today. Because of its intrinsic beauty and because several of the fundamental advances in GMT can be traced to his ideas in area theory, I will focus on his achievements in this field. His first published paper, in this period and in his career, was the result of his asking A. P. Morse, who later turned out to be his Ph.D. mentor, for a problem to test whether he was capable of being a research mathematician. The answer became abundantly clear in the joint paper with Morse that Federer had the right stuff [FM43].


Brown University Library.

Herbert Federer, circa 1948–1949.

The problem of what should constitute the area of a surface confounded researchers for many years. In 1914 Carathéodory defined a k-dimensional measure in $\mathbb{R}^n$ in which he proved that the length of a rectifiable curve coincides with its one-dimensional measure. In 1919 Hausdorff, developing Carathéodory's ideas, constructed a continuous scale of measures. After this, it became obvious that area should be regarded as a two-dimensional measure and should establish the well-known integral formulas associated with area. Later Lebesgue's definition, somewhat modified by Frechet, of the area as being the lower limit of areas of approximating polyhedra became the dominant one. It became dominant partly because of its successful application in the solution of the classical Plateau problem. It had the notable feature of lower semicontinuity, which is crucial in the calculus of variations.

Federer's next two papers, [Fed44a], [Fed44b], mark the beginning of his research on Lebesgue area, a field that was dominated by two influential mathematicians, Lamberto Cesari and Tibor Radó.

In [Fed44b], Federer considers the problem that is perhaps the central question in area theory, the answer to which had been sought by many researchers.

(1) It asks for the type of multiplicity function that, when integrated over the range of f with respect to Hausdorff measure, will yield the Lebesgue area of f.

In this paper his results imply that if all the partial derivatives of f exist everywhere in a region T, then the Lebesgue area can be represented as the integral of the crude multiplicity function

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$N(f, T, y)$, which denotes the number of times in T that f takes the value y.

The paper [Fed47] really lays the foundation for the development of GMT. Up to the time of this paper, A. S. Besicovitch had studied the geometric properties of plane sets of finite Carathéodory linear measure and these studies were extended by A. P. Morse and J. F. Randolph. The corresponding problems for two-dimensional measures over three-dimensional space are connected with the theory of surface area. This paper contains a discussion of these properties for a large class of k-dimensional (outer) measures over n-dimensional space and also develops some of the fundamental tools of GMT. For example, he shows that any set $E \subset \mathbb{R}^n$ with finite k-dimensional Hausdorff measure can be decomposed into rectifiable and nonrectifiable parts. Then Federer applies the preceding theory to show that the Hausdorff measure of a two-dimensional nonparametric surface in $\mathbb{R}^3$ equals the Lebesgue area of the map defining the surface.

The problem of finding a suitable multiplicity function such that its integral over the range of f will yield the Lebesgue area of f remained intractable until Federer brought some notions of algebraic topology to bear. In [Fed46], an area is defined for all continuous k-dimensional surfaces in terms of the stable values of their projections into k-dimensional subspaces; the area thus defined is lower semicontinuous. Its relation to Lebesgue area is only partially settled in this paper.

Then, in [Fed48], results were announced which represent generalizations to n dimensions of previous material known only in the two-dimensional case. The topological index, which had been used as a principal tool in the two-dimensional case, is replaced by the topological degree, expressed in terms of Čech cohomology groups, and the use of the Hopf Extension Theorem, which allows the stable multiplicity function to be determined by merely counting the number of essential domains of $f^{-1}(U)$, where U is a domain in $\mathbb{R}^n$. The techniques of algebraic topology are fully applied.

The key to extending the theory of Lebesgue area from two-dimensional surfaces in $\mathbb{R}^3$ to surfaces in $\mathbb{R}^n$ was the generalization of Cesari's inequality from $\mathbb{R}^3$ to $\mathbb{R}^n$ [Ces42]. That inequality states that the Lebesgue area of a mapping $f : X \to \mathbb{R}^3$ is dominated by the sum of the areas of its projection onto the three coordinate planes. Here X denotes a finitely triangulable subset of the plane. In [Fed55], Federer proved the extension of this inequality to $\mathbb{R}^n$, which was a monumental achievement as it necessitated the complete development of the length of light mappings defined on an arbitrary metric space, thus foretelling the directions of modern day GMT. Here, the length of a light mapping $f : X \to Y$, where X is assumed to be a locally compact, separable metric space and Y an arbitrary metric space, is defined as the supremum of $\sum \operatorname{diam}[f(C)]$ where the supremum is taken over all countable disjoint families of nondegenerate continua in X. So, with this result, the theory of Lebesgue area for surfaces in $\mathbb{R}^3$ can be essentially generalized to surfaces in $\mathbb{R}^n$.

The paper [Fed55] is one of Federer's best efforts in area theory. In particular, it contains the basic idea that led to the fundamental result, the Deformation Theorem of GMT [FF60, §5]. It appears in the *Annals of Mathematics* because it was rejected for publication by the *Transactions* despite the fact that it was of the highest quality. This bothered Federer considerably and he contemplated leaving the field. Fortunately, he did not, and thus his best work was yet to come. For example, he and Demers went on to improve the results in [Fed55] by showing that in the case of a flat mapping, a mapping in which both the domain space and range space are of the same dimension, the k-dimensional Lebesgue measure equals the integral of a new multiplicity function which is defined in terms of norms of cohomology classes [DF59].

The paper [Fed59] establishes a very useful result in GMT, known as the co-area formula. In its most elementary form, it states that if f is real-valued, then the total variation of f can be expressed in terms of integration of f over the fibers of f with respect to $(n-1)$-dimensional Hausdorff measure. In its more general form, the formula is valid for any Lipschitz mapping from X to Y where X and Y are separable Riemannian manifolds of class 1 with respective dimensions n and k, $n \geq k$.

This result has generated great interest and has led to many applications and generalizations. For example, [FR60] established a co-area formula for $f \in BV(\mathbb{R}^n)$, while [MSZ03] proved it for a suitable class of Sobolev mappings.

The paper [Fed60] establishes that the Lebesgue area of a nonparametric surface in $\mathbb{R}^n$ is equal to the $(n-1)$-dimensional Hausdorff measure of its graph. This was proved previously in [Fed47] when $n = 3$, and thus this answers the question that Federer pursued in his first publication in area theory [Fed44a].

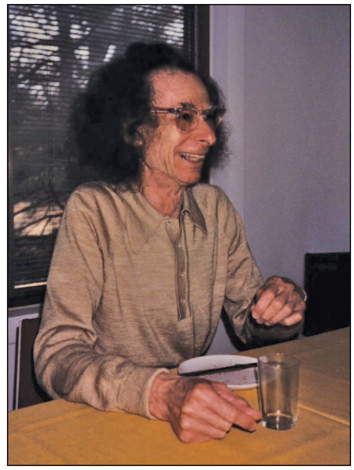
As for the question that was posed in (1), the answer was provided in his last publication on the subject [Fed61]. Let $f : X \to \mathbb{R}^n$ be a continuous mapping where X is a compact manifold of dimension $k \leq n$. Assuming that f has finite Lebesgue area and that either $k = 2$ or that the range of f has $(k+1)$-dimensional Hausdorff measure 0, Federer proves that there exists a unique current-valued measure μ defined over M_f, the middle space associated with f, such that the total variation of μ is equal to the Lebesgue area of f. Moreover, the density of μ, with respect to k-dimensional Hausdorff measure, yields a multiplicity function that provides the answer to the question posed in (1). While Herb was writing this paper, he said that he intended to write it very concisely because he knew that area theory was a dying field and that the paper would not generate much interest. By that time, he was already consumed with the development of GMT.

Even for the casual reader of Herbert Federer's work, it becomes clear that he brings an incredible arsenal of tools to bear on the problem at hand. It is also clear that his determination to learn essentially everything about a problem is highly unusual, for example, taking a period of seventeen years to answer the question raised in (1). He once told me that he uses everything he has learned in his work. This becomes apparent to virtually anyone who has studied his papers. Consider the following quote from G. Bailey Price in his review of Federer's paper [Fed46]: "The paper as a whole is characterized by the treatment of problems and the employment of methods of great generality. The author uses many results from two of his previous papers [Fed44a], [Fed44b]. In addition, he employs a wide variety of powerful tools selected freely from the theory of topological groups, measure theory, integration theory, the theory of functions of real variables, topology and other fields of modern mathematics." As an indication of how he has inspired others to carry on his work, one may note that the number of citations to his book in *Mathematical Reviews* is nearly 1,500, and one should look at the recent work of those who have extended Federer's work to metric spaces; cf. [AK00a], [AK00b], [Mal03], and the references therein.

Herb told me that while he was writing his book, he "was inscribing his epitaph on his tombstone." Indeed, he has and it is our good fortune that he has done it so indelibly because his legacy will be the source of inspiration into the far distant future.

Wendell Fleming

Herbert Federer is remembered for his many deep and original contributions to geometric measure theory (GMT) beginning with his 1945 paper on the Gauss–Green theorem [Fed45]. His work has had a profound influence. It is difficult to imagine that the rapid growth of GMT beginning in the 1960s, as well as its subsequent influence on other areas of mathematics and applications, could have happened without Federer's groundbreaking efforts. His book [Fed69] is a classic reference. He gave the Colloquium Lectures at the 1977 Summer AMS Meeting in Seattle. The manuscript for those lectures appears as [Fed78] and provides a summary of results in GMT through the late 1970s, including historical background. Nonspecialists may find [Fed78] a useful complement to the more detailed development in [Fed69].



Herbert Federer, 1990.

I first met Herb Federer at the 1957 Summer AMS Meeting at Penn State. Afterwards, he suggested to the mathematics department at Brown that I might be offered an assistant professorship. An offer was made, which I accepted. Upon our arrival in Providence in the fall of 1958, my wife and I were warmly welcomed by Herb and Leila Federer. The academic year 1958–1959 was the most satisfying time of my mathematical life. Our joint work on normal and integral currents was done then. This involved many blackboard sessions at Brown, as well as evening phone calls at home. Both Herb and I had heavy teaching loads (by present day standards) and families with young children. Herb undertook the task of organizing our results into a systematic, coherent form, which appeared as [FF60].

During the 1960s there was a lot of activity in GMT at Brown. We had strong Ph.D. students and several visitors. Among the visitors were Peter Reifenberg, who was at Brown in the summer of 1963, and Ennio De Giorgi who visited during the spring semester of 1964. Reifenberg had found another highly original approach to the higher-dimensional Plateau problem [Rei60]. Unfortunately, his promising career ended when he died in a mountaineering accident in 1964.

Herbert Federer set very high standards for his mathematical work and expected high quality work from his students. He was fair-minded and very careful to give proper credit to the work of other people. He was generous with his time when serious mathematical issues were at stake. Federer was the referee for John Nash's 1956 *Annals of*

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Herb and his wife, Leila, in 1979.

Mathematics paper "The imbedding problem for Riemannian manifolds". This involved a collaborative effort between the author and referee over a period of several months. In the final accepted version, Nash stated, "I am profoundly indebted to H. Federer, to whom may be traced most of the improvements over the first chaotic formulation of this work."

By the 1970s I had left GMT to work on stochastic control. When Herb and I met in later years, we didn't discuss mathematics very much, but we always exchanged updates about our children.

In the 1950s Lebesgue area theory had reached a mature state. It had succeeded in providing existence theorems for two-dimensional geometric problems of the calculus of variations, including the Plateau problem. In the area theory formulation, the minimum is achieved among surfaces of a prescribed topological type which have as boundary a given curve. A very different formulation would be needed to study calculus of variations problems with $k \geq 2$, in which only the $(k-1)$-boundary but not the topological type of the k-dimensional comparison surfaces is given. One such formulation is in terms of L. C. Young's generalized surfaces [You51]. Young was my Ph.D. advisor. I came to Brown expecting to continue working in a generalized surface setting. However, Federer soon convinced me of the advantages of developing instead a theory expressed in terms of de Rham's theory of currents. His wisdom and foresight in this regard have been amply justified by developments in GMT which followed our joint paper [FF60].

The k-dimensional Plateau problem in n-dimensional Euclidean $\mathbb{R}^n$ is to minimize k-dimensional area in a suitably defined class of objects with given $(k-1)$-dimensional boundary. The objects which Federer and I considered are called integral currents. Our paper provided a theorem about the existence of k-area minimizing integral currents. There remained the notoriously difficult "regularity question", which is to prove smoothness of the support of an integral current which minimizes k-area, except at points of a singular set of lower Hausdorff dimension. Examples show that in dimensions $1 < k < n-1$, the singular set can have Hausdorff dimension $k-2$. The earliest partial regularity results were due to De Giorgi [DG61] and Reifenberg [Rei64]. Federer's Ph.D. student Fred Almgren and coauthors later made remarkable further progress on the regularity problem for a larger class of elliptic variational integrands [Whi98], [Tay99]. This required persistent, courageous efforts. References [Fed69, Chapter 5] and [Fed78, Section 10] also give systematic accounts of results for the regularity problem up to 1977.

In codimension one ($k = n-1$) it seemed at first that area minimizing currents might have no singular points. This turned out to be correct for $n \leq 7$ by results of De Giorgi, Almgren, and Simons. However, Bombieri, De Giorgi, and Giusti [BDGG69] gave an example of a cone in $\mathbb{R}^8$ which provides a seven-dimensional area-minimizing integral current with a singularity at the vertex. In [Fed70], Federer showed that, for codimension one, this example is generic in the sense that the singular set can have Hausdorff dimension at most $n-8$.

In the integral current formulation, orientations are assigned to tangent k-spaces. These orientations vary continuously on the regular part of the support of any k-area minimizing integral current. Another formulation of the Plateau problem is in terms of Whitney-type flat chains with coefficients in the group Z_2 of integers mod 2. This formulation, in effect, ignores orientations. Federer showed in [Fed70] that for this "nonoriented" version of the Plateau problem, the singular set has Hausdorff dimension at most $k-2$ for arbitrary k. This is essentially the best possible result for the nonoriented version.

Federer also made notable contributions to the theory of weakly differentiable functions on $\mathbb{R}^n$ with applications to Fourier analysis [Fed68], [Fed69, Section 4.5], [Fed78, Section 5]. These include sharp results which extend to $n > 1$ the fact that a function of one variable of bounded variation has everywhere finite left and right limits which differ only on a countable set. Federer's results are included in the lengthy Theorem 4.5.9 in [Fed69]. The statement of that theorem provides a comprehensive list of properties of functions on $\mathbb{R}^n$ with first-order partial derivatives which are measures (in the Schwartz distribution sense). Any such function with compact support corresponds to a normal current of dimension $k = n$.

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Isoperimetric Pentagonal Tilings

Ping Ngai Chung, Miguel A. Fernandez, Yifei Li, Michael Mara, Frank Morgan, Isamar Rosa Plata, Niralee Shah, Luis Sordo Vieira, Elena Wikner

In 2001 Thomas Hales ([H]; see [M1, Chap. 15]) proved the Honeycomb Conjecture, which says that regular hexagons provide a least-perimeter tiling of the plane by unit-area regions. In this paper we seek perimeter-minimizing tilings of the plane by unit-area pentagons. The regular pentagon has the least perimeter, but it does not tile the plane. There are many planar tilings by a single irregular pentagon or by many different unit-area pentagons; for some simple examples see Figure 3. Which of them has the least average perimeter per tile? Our main theorem, Theorem 3.5, proves that the Cairo and Prismatic tilings of Figure 1 minimize perimeter, assuming that the pentagons are convex. We conjecture that this convexity assumption is unnecessary. The Cairo and Prismatic tiles have identical perimeter $2\sqrt{2+\sqrt{3}} \approx 3.86$, less than a unit square's (4) but more than the regular pentagon's (≈ 3.81). There are infinitely many equally efficient tilings by mixtures of Cairo and Prismatic tiles, as in Figure 2 (see also Remark 2.3 and Figures 5–16).

Tilings of the plane by pentagons have intrigued mathematicians and have been the subject of numerous studies (see the beautiful survey by Schattschneider [S3] and recent work of Sugimoto and Ogawa [SO]). Only in 1985 [HH] were the five types of *equilateral* convex pentagons that tile the plane proved complete. There are fourteen known types of convex pentagons that tile the plane, pictured in Figure 3, and more tilings by nonconvex pentagons and mixtures of pentagons. For some appearances of pentagons throughout the world, see `http://sites.williams.edu/Morgan/?p=802`.

Along the way we prove an isoperimetric result we were not able to find in the literature (see Proposition 3.1 and Figure 17):

For n given angles $0 < a_i \leq \pi$ summing to $(n-2)\pi$, the n-gon circumscribed about the unit circle maximizes area for given perimeter.

This complements the well-known result that, for given *edge lengths*, the n-gon *inscribed* in a circle maximizes area. (Idea of short proof: otherwise take the inscribed n-gon, with the surmounted circular arcs of the circle, and deform it to enclose more area. With the surmounted circular arcs, it now beats the circle, a contradiction.)

The proof that the Cairo and Prismatic tilings minimize perimeter begins with the Euler

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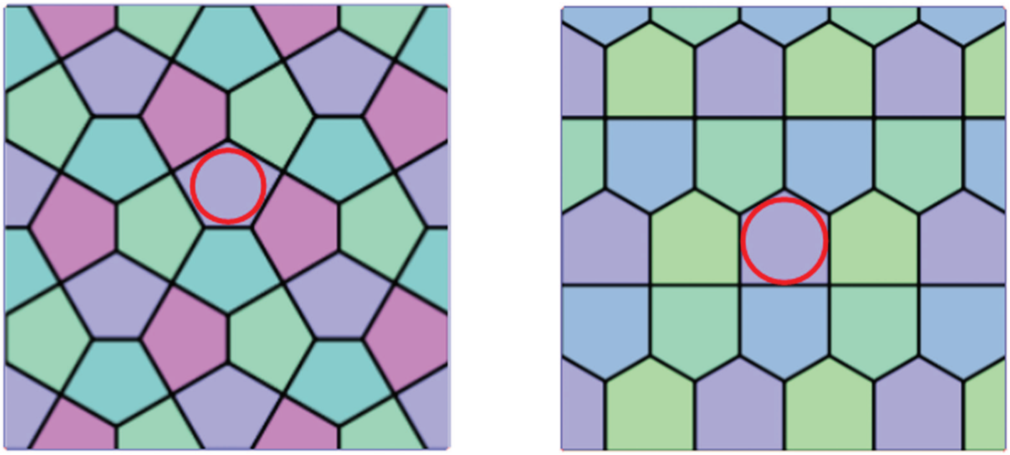

Cairo Pentagonal Tiling

Prismatic Pentagonal Tiling

Figure 1. Minimal convex pentagonal tilings. Each tile has two right angles and three angles of $2\pi/3$ and is circumscribed about a circle.

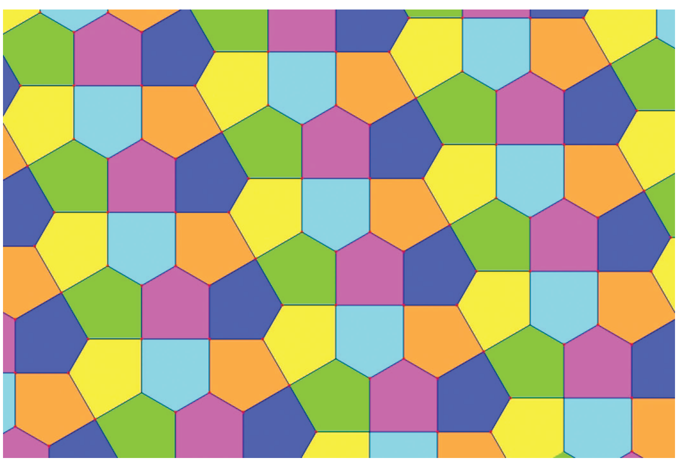

Figure 2. One of infinitely many equally efficient tilings by mixtures of Cairo and Prismatic pentagons.

characteristic formula, which implies that on average at least three angles per pentagon belong to vertices of the tiling of degree three. Thus at least $3/5$ of the angles are "large" angles with average measure $2\pi/3$. Certain convexity arguments show it best for three angles of each pentagon to equal $2\pi/3$ and for the others to equal $\pi/2$, yielding the Cairo and Prismatic tiles. In the problematic case that one or two of the angles is π, the pentagon becomes a quadrilateral or a triangle and we need to enlarge our category to convex n-gons with $n \leq 5$, not necessarily meeting edge to edge (along entire edges). The Euler consequence generalizes to the statement that, on average, the number of large angles is at least $3n - 12$. Lower bounds for the perimeter contributed separately by triangles, quadrilaterals, and pentagons are compared via linear approximations.

Tilings

2.1. Definitions. A polygonal *tiling* is a decomposition of the plane as a union of polygonal regions which meet only along their boundaries. The tiling is *edge to edge* if the tiles meet only along entire edges. A *monohedral* tiling is a tiling by congruent copies of a single *prototile*. We do not restrict ourselves to monohedral tilings.

The *perimeter ratio* of a planar tiling is defined as the limit superior as R approaches infinity of the perimeter inside an R-disc about the origin divided by the area πR^2, counting all edges and portions of edges inside the R-disc as in Figure 4.

Although the terms are sometimes used more broadly, we define the *Cairo* and *Prismatic* pentagons as unit-area pentagons circumscribed about a circle with two right angles and three angles of $2\pi/3$, as in Figure 1. Proposition 3.1

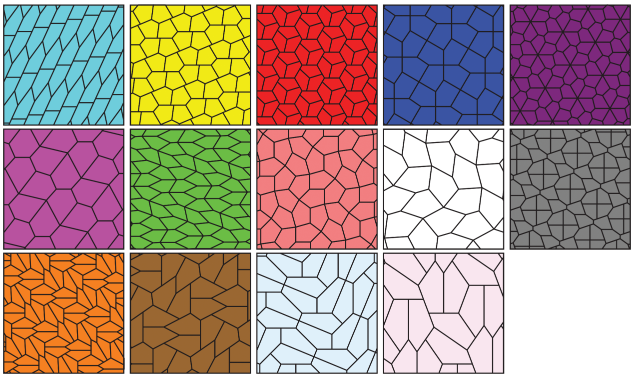

Figure 3. The fourteen known types of convex pentagons that tile the plane. `http://en.wikipedia.org/wiki/Pentagon_tiling`.

shows that circumscribed polygons minimize perimeter for given angles. The Cairo tile has one shorter edge of length $a = (2/3)\sqrt{6 - 3\sqrt{3}} \approx .5977$ and four equal longer edges of length $b = (3 + \sqrt{3})\sqrt{2 - \sqrt{3}}/3 \approx .8165$. The Prismatic tile has two adjacent shorter edges of length a (forming the roof of the house), two edges of length b (the walls), and a still longer base of length $c = 2\sqrt{2 - \sqrt{3}} \approx 1.0353$.

Proposition 2.2. *There is a unique planar tiling by Cairo tiles. There is a unique edge-to-edge planar tiling by Prismatic tiles.*

We call these the Cairo and Prismatic tilings (pictured in Figure 1).

Proof. For the Cairo tiles, the shorter edges have vertices of degree 3, so every tiling must be edge to edge. Working out from a vertex of degree 4, it is easy to see that an edge-to-edge tiling by the Cairo or the Prismatic tile is unique.

Remark 2.3 *(Nonuniqueness).* The first version of this paper asked whether a tiling of the plane by a mixture of Cairo and Prismatic tiles, which we now call a Cairo-Prismatic tiling, exists. During the application process for Frank Morgan's 2011 NSF "SMALL" undergraduate research Geometry Group, a freshman at MIT, shortly recruited as a coauthor (P. N. Chung), discovered uncountably many tilings by mixtures of Cairo and Prismatic tiles. In Figure 2, there are alternating diagonal rows of Cairo and Prismatic tiles. The Cairo tiles are grouped in fours into hexagons, while the Prismatic tiles are grouped in pairs. Uncountably many other such tilings may be obtained by placing arbitrarily many rows of one type before rows of the other type. Most of these tilings are nonperiodic. One other example is shown in Figure 5.

After writing this paper, we came across the older, nonperiodic example of Figure 6. It was discovered by the now famous housewife Marjorie Rice [R], featured in Doris Schattschneider's article "In Praise of Amateurs" [S1, Fig. 15]. Rice had no interest in minimizing perimeter; apparently she simply found it a beautiful tiling by two pentagonal prototiles. Our original goal had been to prove that the tilings shown in Figure 2 were the only tilings by mixtures of Cairo and Prismatic Pentagons. In attempting a proof and by trial and error with Geometer's Sketchpad, we found many more Cairo-Prismatic tilings, some with symmetries of four of the seventeen wallpaper groups (Figures 7–11) and others with fewer or no symmetries (Figures 12–16); see [C] for more. We wonder whether there are examples for the other wallpaper groups. For a chart of the seventeen wallpaper groups and their respective symmetries, see [S2] or [W].

Daniel Huson [Hu] has found tilings by many other pairs of prototiles.

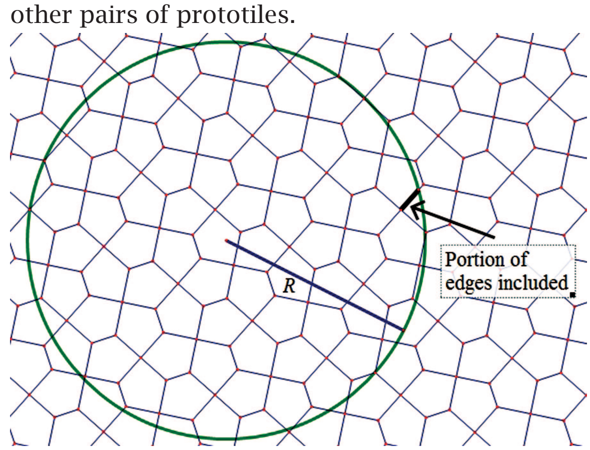

Figure 4. The perimeter ratio is the limit superior as R approaches infinity of the perimeter inside an R-disc, including portions of edges, divided by the area πR^2.

Isoperimetric Pentagonal Tilings

Our main theorem, Theorem 3.5, proves that the Cairo and Prismatic tilings minimize perimeter among all tilings by unit-area convex polygons with at most five sides. We begin with the isoperimetric Proposition 3.1, which we have not been able to find in the literature, perhaps because it depends on knowledge of the ideal "Wulff shape" for crystals [M2, 10.6]. Note that, for given ordered angles or equivalently given orientation of the edges, there is a unique n-gon as in Figure 17 circumscribed about the unit circle by just taking the edges tangent to the circle; this is the n-gon which minimizes perimeter for given area.

Proposition 3.1. *For n given angles $0 < a_i \leq \pi$ summing to $(n-2)\pi$, the n-gon circumscribed*

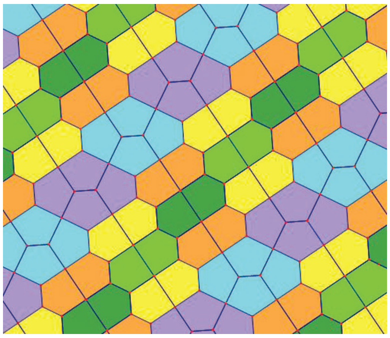

Figure 5. Another tiling by a mixture of Cairo and Prismatic tiles, in which the blue-purple diagonal rows of Cairo tiles are separated by three diagonal rows of Prismatic tiles.

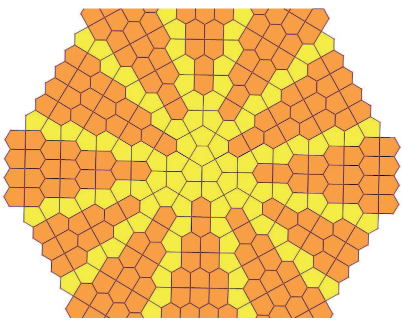

Figure 6. A nonperiodic tiling by a mixture of Cairo and Prismatic tiles due to Marjorie Rice ([R] or [S1, Fig. 15]).

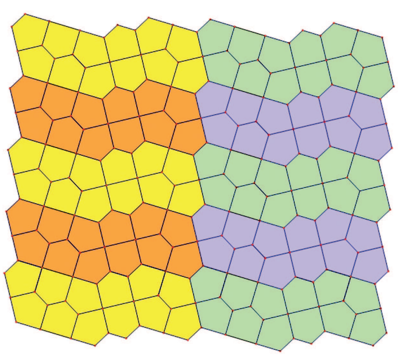

Figure 7. "Sardines" Cairo-Prismatic tiling with wallpaper group *p*1.

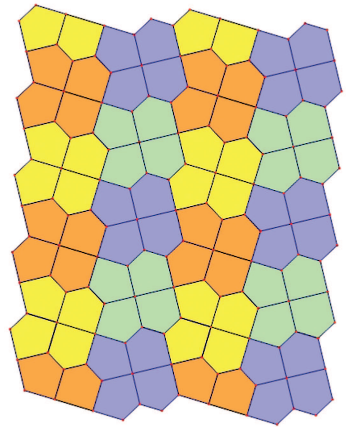

Figure 8. "Stripes" Cairo-Prismatic tiling with wallpaper group *p*2.

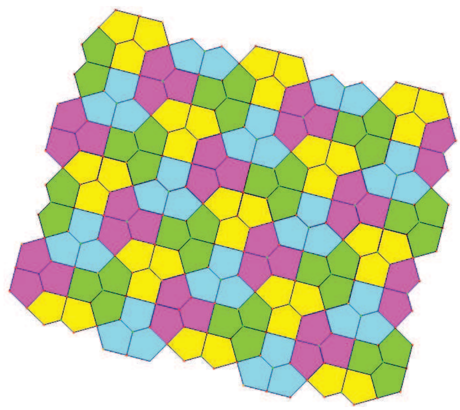

Figure 9. "Pills" Cairo-Prismatic tiling with wallpaper group *p*4*g*.

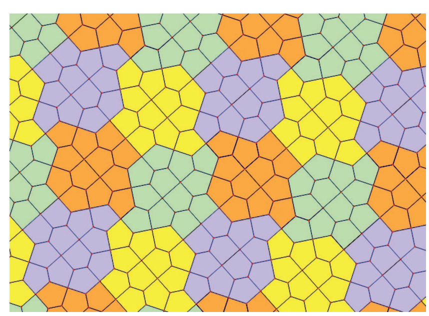

Figure 10. "Spaceship" Cairo-Prismatic tiling with wallpaper group *p*4*g*.

about the unit circle as in Figure 17 is uniquely perimeter minimizing for its area. Scaled to unit area, its perimeter is

$$2\sqrt{\cot(a_i/2)}. \tag{1}$$

Since cotangent is strictly convex up to $\pi/2$, the more nearly equal the angles, the smaller the perimeter.

Proof. (See [M2, 10.6].) The celebrated isoperimetric theorem says that, for the Euclidean norm on $\mathbf{R}^2$ (or $\mathbf{R}^n$), balls provide the least-perimeter way to enclose given area (or volume). There is a less familiar generalization to other (not necessarily even) norms Φ, under which the length of a curve is weighted by Φ applied to the unit normal. Then the least-perimeter way to enclose given area is

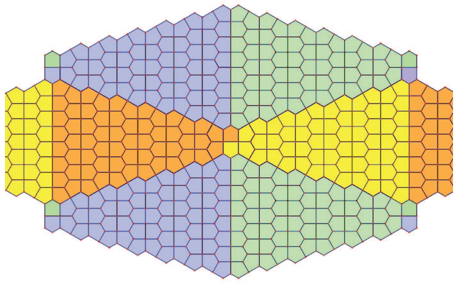

Figure 11. "Christmas Tree" Cairo-Prismatic tiling with wallpaper group *cmm*.

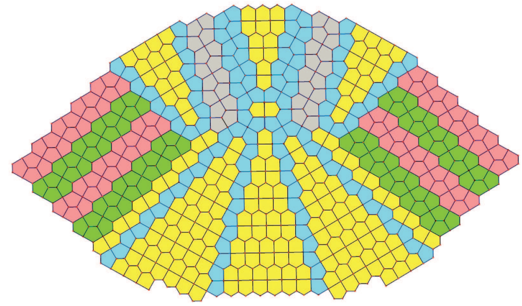

Figure 12. "Bunny" Cairo-Prismatic tiling with symmetry group D_1.

given by scalings of the "Wulff shape" or unit ball in the dual norm Φ^*, defined by

$$\Phi^*(x) = \sup\{x \cdot y \colon \Phi(y) \le 1\}.$$

In particular, a polygon P circumscribed about the unit circle is the unit Φ^* ball, where the unit Φ ball is the inscribed polygon with vertices at the points of tangency of P. Note that Φ is 1 on the sides of P and some irrelevant larger value in other directions.

Therefore P minimizes Euclidean length for the given orientations of its sides, as desired. By simple geometry, its perimeter P_0 and area A_0 satisfy

$$P_0 = 2A_0 = 2\sum \cot(a_i/2).$$

The final formula for unit area follows by scaling.

Remarks. If you allow angles between π and 2π, the circumscribed polygon still minimizes perimeter for sides in the associated directions, but they may necessarily occur in a different order and produce different angles and a smaller perimeter. If you require the prescribed order to keep the prescribed angles, some sides may become infinitesimally small, thus omitted, replacing pairs of external angles with their sum and reducing our equation (1). For $n > 3$, the least-perimeter unit-area n-gon with one small prescribed angle is not convex; for $n = 4$ it is close to being an equilateral triangle with a small nonconvex protrusion near one of the three vertices, as in Figure 18.

The proof of Theorem 3.5, which shows that certain features of tiles which hold on average ideally hold exactly, depends on the following convexity arguments, summarized in Corollary 3.4. By Proposition 3.1, the $f(u)$ of Lemma 3.2 is proportional to the perimeter of a unit-area n-gon with k angles of 2θ and k' angles of $2\theta'$. Convexity means that the closer together these angle sizes, the better. Lemma 3.3 allows k and k' to vary and shows furthermore that ideally all n-gons have the same number k of large angles.

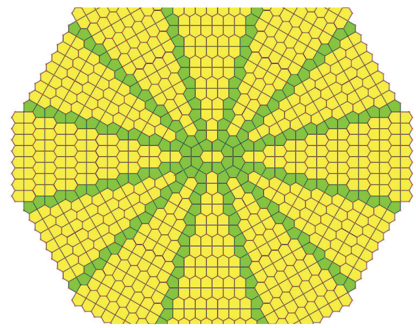

Figure 13. "Plaza" Cairo-Prismatic tiling with symmetry group D_2.

Lemma 3.2. *For positive integers $k + k' = n$, consider the positive function*

$$f^2(u) = g(u) = kt + k't'$$

where $t = \cot\theta$, $t' = \cot\theta'$, $0 < \theta, \theta' < \pi/2$, $\theta = k'u$, $\theta' = (n-2)\pi/2k' - ku$. Then f is strictly convex.

Proof. It suffices to show that $2gg'' - g'^2$ is positive (where primes on g denote differentiation with respect to u). Writing $s = \csc\theta$, $s' = \csc\theta'$, we compute that

$$g' = -kk's^2 + kk's'^2,$$
$$g'' = 2kk'^2s^2t + 2k^2k's'^2t',$$

and after discarding some positive terms

$$2gg'' - g'^2 > 2k^2k'^2s^2t^2 + 2k^2k'^2s'^2t'^2 - k^2k'^2s^4 - k^2k'^2s'^4 + 2k^2k'^2s^2s'^2,$$

which is proportional to

$$\begin{aligned} &2s^2t^2 + 2s'^2t'^2 - s^4 - s'^4 + 2s^2s'^2 \\ &\quad > 2s^2t^2 + 2s'^2t'^2 - s^4 - s'^4 + s^2 + s'^2 \\ &\quad = 2s^2t^2 + 2s'^2t'^2 - s^2t^2 - s'^2t'^2 > 0. \end{aligned}$$

Lemma 3.3. *For positive constants n, c, c' and nonnegative constants d, d', consider the positive function*

$$f^2(k) = g(k) = kt + k't',$$

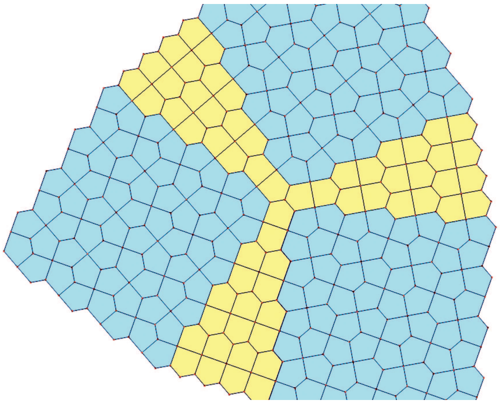

Figure 14. "Waterwheel" Cairo-Prismatic tiling with symmetry group D_3.

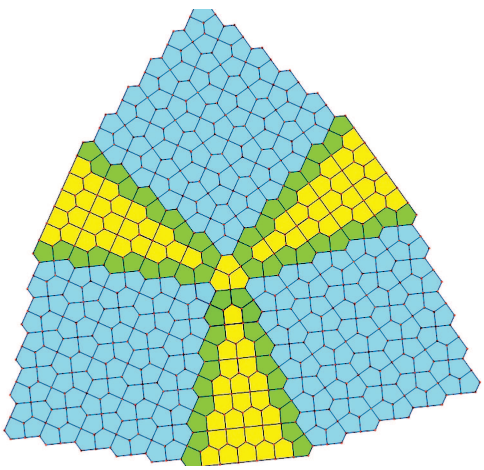

Figure 15. "Windmill" Cairo-Prismatic tiling with symmetry group D_3.

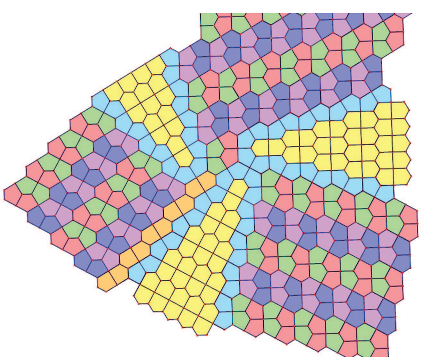

Figure 16. "Chaos" Cairo-Prismatic tiling with no symmetry.

where $0 < k < n$, $k' = n - k$, $t = \cot\theta$, $t' = \cot\theta'$, $0 < \theta, \theta' < \pi/2$, $\theta = d - c/k$, $\theta' = d' - c'/k'$, *and* $\min\{\theta, \theta'\} \le \pi/3$. *Then f is strictly convex.*

Proof. It suffices to show that $2gg'' - g'^2$ is positive. Write $s = \csc\theta$, $s' = \csc\theta'$. By symmetry we

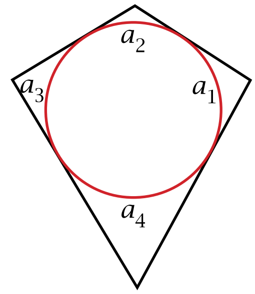

Figure 17. For given angles the least-perimeter unit-area polygon is circumscribed about a circle.

may assume that $\theta \le \theta'$, so $c/k \ge c'/k'$, $t \ge t'$, and $s \ge s'$. We compute that

$$\begin{aligned} g' &= t - ks^2(c/k^2) - t' + k's'^2(c'/k'^2) \\ &= t - t' - (s^2c/k - s'^2c'/k'), \end{aligned}$$

so

$$-s^2c/k \le g' \le t - t'.$$

Since

$$t - t' \le s^2(\theta' - \theta) \le s^2c/k,$$

we see that

$$|g'| \le s^2c/k.$$

We further compute that

$$g'' = 2s^2tc^2/k^3 + 2s'^2tc'^2/k'^3.$$

Hence

$$2gg'' - g'^2 > 2(kt)(2s^2tc^2/k^3) - s^4c^2/k^2,$$

which is proportional to $4t^2 - s^2$, which is nonnegative because $\theta \le \pi/3$.

Corollary 3.4. *For integers $n \ge 3$, the positive function f_n defined by*

$$f_n^2(k, k\theta) = k\cot\theta + k'\cot\theta',$$

where k' and θ' are defined by $k + k' = n$, $k\theta + k'\theta' = (n-2)\pi/2$, $0 < k, k' < n$, $0 < \theta, \theta' < \pi/2$, and $\min\{\theta, \theta'\} \le \pi/3$ *is strictly convex.*

Proof. On a vertical line in the domain (k constant), f_n is strictly convex by Lemma 3.2. On a nonvertical line, f_n is strictly convex by Lemma 3.3.

Theorem 3.5. *Perimeter-minimizing planar tilings by unit-area convex polygons with at most five sides are given by Cairo and Prismatic tiles (as in Figures 1, 2, 5–16).*

Remarks. There is never uniqueness, since compact variations as in Figure 19 do not change the limiting perimeter ratio.

For doubly periodic or monohedral tilings, however, there are no others, although the Prismatic tilings need not be edge to edge.

If nonconvex tiles are allowed, the result remains conjectural (except for monohedral tilings,

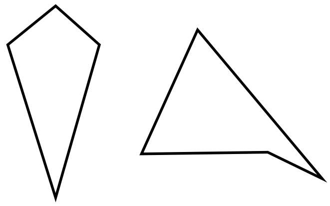

Figure 18. The least-perimeter unit-area quadrilateral with one small prescribed angle is not the convex quadrilateral with the other angles equal but a triangle with a small nonconvex protrusion.

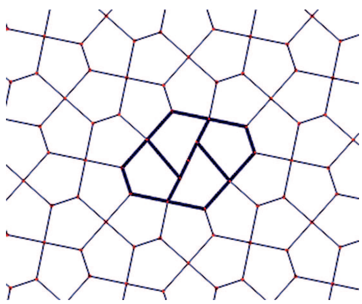

Figure 19. An example of a compact variation of the Cairo tiling, which remains perimeter minimizing.

when such a large angle gives the prototile more perimeter than a unit square).

We are not assuming that the tiling is monohedral or edge to edge.

The proof of Theorem 3.5 reduces the question about tilings to a question about abstract finite collections of polygons satisfying certain conditions (Step III). In Steps IV and V, Corollary 3.4 is applied to get a sharp lower bound on the perimeter, with strict inequality for triangles and 4-gons and equality for Cairo and Prismatic pentagons. The proof begins with careful truncation to reduce the question to collections of N polygons with nice limit properties as N approaches infinity.

Proof of Theorem 3.5. Consider a planar tiling with bounded perimeter ratio ρ by unit-area convex polygons with at most five sides. We may assume that every angle is strictly less than π by eliminating vertices at angles of π from polygons. (Recall that we are not assuming that the tilings are edge to edge.)

Step I: *Truncation.* To control truncation we claim that, for some sequence of R going to infinity, the circle about the origin of radius R meets the tiling in $o(R^2)$ points (i.e., the ratio to R^2 goes to 0). Otherwise, for almost all large R, for some $\varepsilon > 0$, since P grows at a rate at least equal to the number of such points (see [M1, 15.3] for details),

$$dP/dr \geq \varepsilon R^2 \geq (\varepsilon/\rho)P.$$

This implies that perimeter and hence area grow exponentially. Since area equals πR^2, this is a contradiction.

For such a sequence of R, consider the tiles inside the disc D_R of radius R. The ratio of their area to πR^2 approaches 1, while the number of adjacent discarded tiles is $o(R^2)$, which is negligible. Therefore the perimeter ratio inside D_R equals the average cost of the tiles inside D_R up to $o(1)$.

Step II: *The fraction f_n of polygons with n edges, the number e of edges per polygon, and the number of vertices v_d per polygon of degree d.* We will examine more carefully our sequence as R approaches infinity. By taking a subsequence if necessary, we may assume that the fraction of polygons of n edges approaches a limit f_n, hence that the number of edges per polygon approaches $e = \sum nf_n$, and that the number of vertices per polygon of degree d in the tiling approaches a limit v_d. Thus

$$\sum v_d = e. \tag{1}$$

By Euler, for the tiles inside D_R,

$$V = E - F + 1 \geq eF/2 - F + 1 = (e/2 - 1)F + 1, \tag{2}$$

with equality up to $o(1)$ if the tiling is edge to edge. Hence, if the tiling is edge to edge, the following equality holds up to $o(1)$ for the tiles inside D_R:

$$\sum v_d/d = V/F = e/2 - 1. \tag{3}$$

If the tiling is not edge to edge, we count a vertex where $k - 1$ other edges meet in the interior of an edge as contributing to v_{2k} instead of v_k, because these polygon angles are $2\pi/2k$ on the average. Such a vertex counts just 1/2 on the left side of (3). Let V^* be the number of such vertices. Then

(4)

$$\sum v_d/d = (V - V^*/2)/F = (E - V^*/2 - F)/F$$
$$= ((eF + V^*)/2 - V^*/2 - F)/F = e/2 - 1,$$

because each such vertex breaks a polygonal edge into two edges, replacing eF with $eF + V^*$. In the limit, equation (4) holds exactly. By (1) and (4),

$$v_3/3 + (e - v_3)/4 \geq e/2 - 1,$$

$$v_3 \geq 3e - 12. \tag{5}$$

Step III: *The reduction from tilings to abstract collections of polygons with k_n large angles averaging $2\theta_n$.* Now it suffices to prove that, for every collection of N convex unit-area polygons with at most five edges with $(3e - 12)N - N_1$ angles of average measure at least $2\pi/3$ with $N_1 = o(N)$, the average perimeter is at least nearly that of the

Cairo and Prismatic pentagons. Among all such collections of N pentagons, choose one to minimize the average perimeter. By Proposition 3.1, they are all circumscribed about circles. Since, by Proposition 3.1, reducing large angles reduces perimeter, the $(3e-12)N-N_1$ "large" angles have average measure exactly $2\pi/3$ and are all at least as large as the remaining "small angles", all at most $2\pi/3$. Up to $o(N)$, the number of small angles is $(12-2e)N$, their sum is $(6-e)\pi N$, so their average is $\pi/2+o(1)$, and all large angles are at least that large.

For each $3 \le n \le 5$, we may assume that the average large angle in an n-gon approaches a limit $2\theta_n$ and that the number of large angles per n-gon approaches a limit k_n. Then

(6)
$$\sum f_n k_n = 3e-12, \qquad \sum f_n k_n \theta_n = (\pi/3) \sum f_n k_n.$$

Since $e = \sum n f_n$, we see that

(7) $$f_3(k_3+3)+f_4k_4+f_5(k_5-3)=0,$$
$$f_3(k_3\theta_3+\pi)+f_4k_4\theta_4+f_5(k_5\theta_5-\pi)=0.$$

Note that each quadrilateral has at least one large angle or it could be replaced by a Cairo pentagon, increasing both $(3e-12)N-N_1$ and the number of large angles by three and decreasing perimeter.

By Proposition 3.1, in each n-gon the large angles are all equal and the small angles are all equal.

Step IV: *The consequences of convexity*. By convexity (Corollary 3.4), for each $3 \le n \le 5$, in the limit the average perimeter is bounded below by the values of the perimeter function $P_n(k,q)$ at k_n and $k_n\theta_n$, where

$$P_n(k,q) = 2(k\cot q/k + k'\cot q'/k')^{1/2},$$

$k+k'=n$, $q+q'=(n-2)\pi/2$. This function corresponds to the perimeter of an n-gon with k large angles, each of measure $2q/k$. In particular, $P_5(3,\pi)$ corresponds to the perimeter of a Cairo or Prismatic pentagon. We compute to four decimal places that

(8)
$$\begin{aligned}
(\partial P_3/\partial k)(1/2,\pi/8) &\approx -.7217,\\
(\partial P_3/\partial q)(1/2,\pi/8) &\approx 1.2265,\\
(\partial P_4/\partial k)(1,\pi/3) &\approx -.4455,\\
(\partial P_4/\partial q)(1,\pi/3) &\approx .5334,\\
(\partial P_5/\partial k)(3,\pi) &\approx -.3091,\\
(\partial P_5/\partial q)(3,\pi) &\approx .3451.
\end{aligned}$$

Note that $f_5 > 0$; otherwise, by (7) $f_4 = 1$, $k_4 = 0$, and the square tiling provides a lower bound, while we know that the Cairo tiling is better. Also by (7), $k_5 \le 3$. Since the average small angle is $\pi/2$, every large angle $2\theta_n$ is at least $\pi/2$, $k_3 < 2$, and $k_4 < 3$. Since the sum of the angles of an n-gon is $(n-2)\pi$, $k_n\theta_n + k'_n\theta'_n = (n-2)\pi/2$, $\theta_5 \ge 3\pi/10$ and $\theta'_n \le (n-2)\pi/2n$.

Step V: *The lower bound on perimeter*. Since, by Corollary 3.4, each f_n is convex, we can bound the perimeter P per tile from below by the linear approximations:

$$\begin{aligned}
P &= f_3P_3(k_3,k_3\theta_3)+f_4P_4(k_4,k_4\theta_4)+f_5P_5(k_5,k_5\theta_5)\\
&\ge f_3[P_3(1/2,\pi/8)+(\partial P_3/\partial k)(1/2,\pi/8)(k_3-1/2)\\
&\qquad +(\partial P_3/\partial q)(1/2,\pi/8)(k_3\theta_3-\pi/8)]\\
&\quad + f_4[P_4(1,\pi/3)+(\partial P_4/\partial k)(1,\pi/3)(k_4-1)\\
&\qquad +(\partial P_4/\partial q)(1,\pi/3)(k_4\theta_4-\pi/3)]\\
&\quad + f_5[P_5(3,\pi)+(\partial P_5/\partial k)(3,\pi)(k_5-3)\\
&\qquad +(\partial P_5/\partial q)(3,\pi)(k_5\theta_5-\pi)].
\end{aligned}$$

Using (7) to substitute for $f_5(k_5-3)$ and $f_5(k_5\theta_5-\pi)$, we obtain

$$P \ge f_5P_5(3,\pi)+f_3Q_3+f_4Q_4,$$

where (rounding appropriately)

$$\begin{aligned}
Q_3 &> P_3(1/2,\pi/8)+(1/2)(.7216)\\
&\quad -(\pi/8)(1.2266)+3(.3090)-\pi(.3452)\\
&\quad +k_3(-.7218+.3090)+k_3\theta_3(1.2264-.3452)\\
&> 4.6503+(1/2)(.7216)-(\pi/8)(1.2266)\\
&\quad +3(.3090)-\pi(.3452)\\
&\quad +k_3(-.7218+.3090)+k_3\theta_3(1.2264-.3452)\\
&> 4.3718+k_3(-.4128+(\pi/4)(.8812))\\
&> P_5(3,\pi)
\end{aligned}$$

and

$$\begin{aligned}
Q_4 &> P_4(1,\pi/3)+.4454-(\pi/3)(.5335)\\
&\quad +k_4(-.4456+.3090)+k_4\theta_4(.5333-.3452)\\
&> 3.9622+k_4(-.1366+(\pi/4)(.1881))\\
&> P_5(3,\pi).
\end{aligned}$$

Therefore the perimeter per tile is at least the perimeter $P_5(3,\pi)$ of the Cairo and the Prismatic tiles, as desired.

For doubly periodic tilings, the argument can be done on the torus without limits or truncation error, and every tile must be Cairo or Prismatic.

Remark. Theorem 3.5 holds for a monohedral edge-to-edge planar tiling by a curvilinear polygon P (embedded rectifiable curves meeting only at vertices) with at most five edges. Because the tiling is monohedral and edge-to-edge, one can replace P with an immersed rectilinear n-gon with the same vertices, as in Figure 20. If it is not embedded, it has more perimeter than a square.

There remains the opening conjecture that the Cairo and Prismatic tilings remain perimeter minimizing if one allows mixing in some nonconvex pentagonal tiles (along with lots of nearly regular pentagonal tiles).

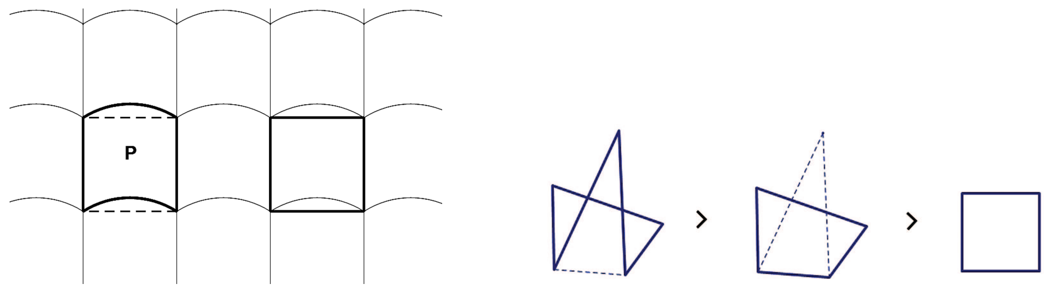

Figure 20. Each tile in a monohedral edge-to-edge planar tiling by a curvilinear polygon P can be replaced by an immersed rectilinear *n*-gon with the same vertices (left). If the *n*-gon is not embedded, it has more perimeter than a square (right).

Acknowledgments

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Princeton Lectures in Analysis by Elias M. Stein and Rami Shakarchi—*A Book Review*

Reviewed by Charles Fefferman and Robert Fefferman with contributions from Paul Hagelstein, Nataša Pavlović, and Lillian Pierce

Comments by Charles Fefferman and Robert Fefferman

For the last ten years, Eli Stein and Rami Shakarchi have undertaken a labor of love, producing a sequence of intensive undergraduate analysis courses and an accompanying set of four books, called the *Princeton Lectures in Analysis*. The individual titles are:

- *Fourier Analysis: An Introduction*
- *Complex Analysis*
- *Real Analysis: Measure Theory, Integration, and Hilbert Spaces*

and

- *Functional Analysis: Introduction to Further Topics in Analysis*.

All four books are now available; all four books bear the unmistakable imprint of Eli Stein.

Every mathematician knows Stein as an analyst of unsurpassed originality and impact. A few dozen of us have had the privilege of writing a Ph.D. thesis under his supervision. We know firsthand how Eli conveys the essential unity of many seemingly disparate ideas. To him, analysis is always an organic whole. Even more remarkably, his own enthusiasm for the subject instills great optimism in all who learn from him. First-rate math is right in front of us, ready to be discovered. Eli turns us into research mathematicians by encouraging us to become active participants in the process.

Many who have not enjoyed the privilege of studying under Stein or collaborating with him have nevertheless benefited greatly by studying from his classic books *Singular Integrals*, *Introduction to Fourier Analysis in Euclidean Spaces* (with Guido Weiss), and *Harmonic Analysis*. People far removed from Princeton have been able to read those books and then go on to do significant work. Eli succeeded in putting on the printed page the kernel of what he conveyed as a teacher and research collaborator.

Before proceeding further, we should say a few words about Rami Shakarchi. Shakarchi is a remarkable man in his own right. He is, among other things, a passionate and accomplished pilot. He is now an active worker in the financial sector. As a graduate student, Rami volunteered to help Eli to plan the sequence of courses and to write the four books.

The collaboration was a great success. Eli and Rami got along famously and communicated perfectly. Rami earned his Ph.D. under one of us (CF) and took a demanding finance job in London. Even when his firm, Lehman Brothers, declared bankruptcy, he stayed with the Stein project and saw it through to the end.

The Stein-Shakarchi books constitute an extraordinary achievement. They are accessible (with a lot of work) to any math student who has had a rigorous one-variable calculus course and a little linear algebra, yet they cover an astonishing range of material, including (in alphabetical order) Brownian motion, the Brunn-Minkowski inequality,

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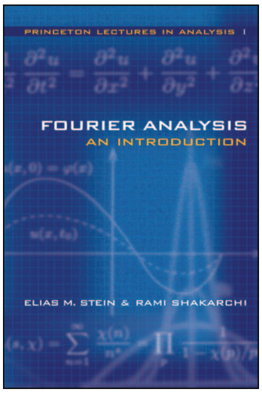
Dirichlet's principle, Dirichlet's theorem on primes in an arithmetic progression, elliptic functions, the ergodic theorem (maximal, mean, and pointwise), the gamma function, the Hardy-Littlewood maximal theorem, Hausdorff dimension, the isoperimetric inequality, the Kakeya problem, the partition function, the prime number theorem, representations of positive integers as sums of two squares and as sums of four squares, the Riemann zeta function, the Runge approximation theorem, Stirling's formula, theta functions, and a lot more.



Moreover, these topics appear, not as a zoo of isolated wonders, but quite naturally as part of a unified picture. For instance, results from analytic number theory and probability give the books a relevance that reaches beyond analysis to other branches of mathematics and science.

The material is explained with the perfect clarity, focus on essentials, and stress upon the interconnection of ideas that one expects from Eli Stein. Numerous challenging problems encourage active audience participation. By working the problems, the student earns an understanding of key ideas.

Another remarkable feature of the Stein-Shakarchi books is that they take very seriously the historical development of analysis. The authors boldly present Fourier analysis before passing from the Riemann to the Lebesgue integral. This makes it possible for the student to start doing interesting mathematics right away, without getting bogged down in unmotivated technicalities. When the time comes to start discussing measure and integration, the subject is introduced with the following words:

> Starting in about 1870, a revolutionary change in the conceptual framework of analysis began to take shape, one that ultimately led to a vast transformation and generalization of the understanding of such basic objects as functions, and such notions as continuity, differentiability and integrability.
>
> The earlier view that relevant functions in analysis were given by formulas or other "analytic" expressions, that these functions were by their nature continuous (or nearly so), that by necessity such functions had derivatives at most points, and moreover these were integrable by the accepted methods of integration—all of these ideas began to give way under the weight of various examples and problems that arose in the subject, which could not be ignored and required new concepts to be understood. Parallel with these developments came new insights that were at once more geometric and more abstract: a clearer understanding of the nature of curves, their rectifiability and their extent; also the beginnings of the theory of sets, starting with subsets of the line, the plane, etc., and the "measure" that could be assigned to each.

One sees here the vast and penetrating scope of Stein's view of analysis and how he is able to weave it into his teaching.

The question remains, of course, how such an ambitious set of books can work in practice. To answer this query, we asked Paul Hagelstein, Nataša Pavlović, and Lillian Pierce to comment on their own experiences. Paul taught the Stein-Shakarchi courses as a VIGRE postdoc. Nataša taught Complex Variables à la Stein-Shakarchi at Princeton. Lillian took the Stein-Shakarchi courses as an undergraduate, then went on to serve as a TA for Nataša's class. We are grateful to Paul, Nataša, and Lillian for their thoughtful comments, which appear below.

It remains for us only to add that, in our view, the Stein-Shakarchi books will be immensely valuable for any undergraduate or graduate math student, for a wide audience of working mathematicians, and for many science or engineering students and researchers with a mathematical bent. Even those few mathematicians who thoroughly know the contents of all four books will find pleasure in the beauty of their unified presentation of a vast subject.

Comments by Lillian B. Pierce

The Courses

In the fall of 1999 a murmur spread through the community of math students at Princeton: a new course would be offered in the spring, the first of four intensive courses in analysis, taught by Professor Elias M. Stein. Professor Stein had already acquired status in our eyes, as it happened that most of my cohort had taken his Introduction to Single Variable Real Analysis course in our first semester at Princeton. So as the rumor of the new course rumbled through our ranks, it was clear that we'd all sign up for it. It was also clear from the first lectures that we were in for something unlike anything we'd ever experienced before.

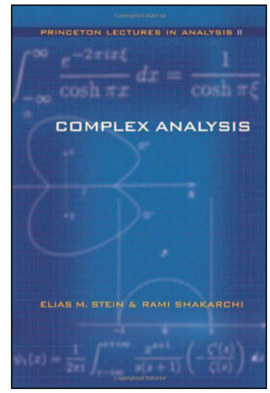
At the beginning of each of the four courses, Professor Stein started by saying that he would work very hard on the course and that the course would in turn require us to work very hard. Then he expressed his gratitude, ahead of time, for all our ensuing hard work. This was a striking attitude of enfranchisement, which I certainly never encountered in any other course.

Indeed, we did have to work hard. Of course, substantial courses in mathematics are often described as "hard", and this superficial descriptor can conceal a multitude of sins or blessings. Are the lectures simply unclear, the assigned problems ill chosen? Or are the lectures deeply insightful, the assigned problems a methodology for building a versatile toolbox? In this analysis sequence, the lectures were fluid and painstakingly prepared; they possessed Stein's trademark of logical progressions of big ideas blended with clarifying close-up examinations. In particular, he was careful to lift each concept up from first principles, so that we could start from our existing knowledge (as second-year undergraduates) and follow him deeply into the material. It was as though he was successfully accelerating a car full of passengers, having first checked that all our seatbelts were fastened!

The problem sets were memorably substantial (one group of students estimated that they sometimes spent about thirty hours per week on the courses). But, unlike some "hard" problem sets in ambitious courses, these did not feel punitive. The problems were carefully posed and often with a hint to get us on the right track. The goal of each problem was always some interesting property or beautiful expression that was appealing enough to motivate us to push through to a solution. Along the way, we learned how to use classical analytic techniques.[1]

The courses set a rapid pace, but the lectures did not feel hurried, and they did not sweep details under the rug. Glimpses of the historical development of the field were contained in the presentation of the theory. One day, at the start of class, Professor Stein read aloud some writing of Fourier in the French original and then gave an off-the-cuff translation. I remember feeling at that moment a thrilling connection to the origins of the deep ideas of Fourier analysis. Students are used to their own comprehension of a concept starting out fuzzy, eventually sharpening with thought and practice. But students can also be startled when they first realize that the concepts themselves started out fuzzy and were sharpened by a progression of refinements by a series of mathematicians. The courses conveyed this historical sense, highlighting key contributions of individual mathematicians (while avoiding a digression into blow-by-blow timelines).

Looking back, I realize that we students did not feel that we were merely inputting stale mathematics into our heads; the beautiful presentations in the lectures and the intense work on the problem sets made us feel like we were part of a creative force, (re)discovering mathematics. I am sure we had this sense of creation partly because the courses were being developed into books, but I think that it also came from the respect given to the work we were doing, conveyed by the high standards and high expectations of the courses. We felt like fledgling mathematicians, and this gave us a corresponding pleasure and responsibility, which we tried to live up to. I suspect that few people have ever learned material so naturally and so deeply as we did in these courses.



The Books

As a student, the first time the four courses were taught I did not of course benefit from the books in their final, published form. But the intensity of the courses imbued the material with an indelible presence, and reading the books feels like a homecoming.

Readers of Stein's more advanced books in analysis (say *Harmonic Analysis* from 1993) will be familiar with the precision and clarity of his presentations. Mathematical writing can be too detailed, submerging big ideas in muddy, over-notated details. Or it can be cruelly sparse, failing to give the reader necessary hand- and footholds up the cliff of comprehension. Stein's writing generally avoids both of these flaws; the *Lectures in Analysis* series, aimed at undergraduates, is no exception. The mathematics is deep, and of course reading the books (and solving the problems) requires sharp attention and thought, but the pace from sentence to sentence, equation to equation is so carefully and evenly planned that the learning process is consistently rewarding. As one student said, "It's like reading a novel, but by the end you know some mathematics."

The clarity is heightened because concepts are developed from first principles, initially avoiding advanced language (although not sacrificing rigor). For example, the Fourier transform is derived without explicitly calling upon L^p spaces. This allows undergraduates to be drawn directly into the power and beauty of Fourier analysis before proceeding through the technical aspects of L^p spaces. The definitions and derivations in the text seem so "right" that it is possible to forget that they could ever be described otherwise. Students who go on to graduate studies in math will likely need to review the

[1] *The mechanics of the courses contributed to a comfortable, noncompetitive atmosphere. Each week the problem set consisted of a dozen or more required problems, with a few optional problems printed on a separate sheet of colored paper to emphasize their optionality. Hard problems were starred, so we knew not to mind being stymied. Each week we could select one required problem that we were having trouble with and defer handing in the solution until later. We were encouraged to collaborate on problems. These collaborations, which got us "talking math" and learning how to engage in the experimental back-and-forth of creative mathematics, led to some of my best times at college. Our solutions were not graded against a strict rubric of potential points to be lost, but were instead labeled α, β, or γ—which encouraged us to be creative in our approaches. The exams were not stressful, but instead made sure that we understood the ideas in the main theorems (such as the role each hypothesis played) and gave us a chance to apply these ideas in a few choice, beautiful problems.*

concepts from a more technical perspective, if only to be able to speak of the concepts with the shortcuts that fancier technical language allows. But, by initially learning the concepts free from obscure technical language, they will view the principles of the derivations almost as second nature. Some math texts are handy learning guides or excellent references, but often one has to translate the symbols on the page into one's own internal language in order to store the concepts as retrievable thoughts. The *Princeton Lectures in Analysis* is much more readily digested.

As in the courses, the "Exercises and Problems" in the books are integral to the experience of learning. The range in difficulty allows for the possibility of both undergraduate and graduate use. The problems are clearly very carefully designed: some codify definitions, some develop dexterity with key theorems, some prepare for deeper material in analysis, some elaborate on classical examples or counterexamples, some push toward applications in other fields. All of them are posed clearly in a self-contained manner that makes them easy to reference (especially by means of the careful index). A reader who solves all the problems will develop a fluid proficiency in analytic methods as well as an arsenal of useful facts and tools. The more casual reader will not be irritated by the relegation of important developments to unsolved "Exercises for the Reader".

One of the goals of the series is to transcend the barriers that arise when the broad field of analysis is artificially broken down into separate subjects in separate textbooks by separate authors. This is certainly achieved: each volume of the series presents a highly organized linear progression of ideas, while the four volumes as a collection are closely interconnected (without repetition) and well cross-referenced. Somewhat unusually, the authors emphasize the interaction between analysis and number theory. Volume I (*Fourier Analysis*) culminates in a proof of Dirichlet's theorem on primes in arithmetic progressions; Volume II (*Complex Analysis*) ends in a proof of the prime number theorem and results on theta functions and representations of integers as a sum of two or four squares; Volume IV (*Functional Analysis*) includes applications to counting lattice points, into which form many problems in number theory may be converted. Some of these presentations are novel, while others are expositions of known proofs, but all share a sharply analytic rigor.

This explicitly analytic presentation of results in number theory absolutely captured my attention as a student; it ultimately is what led me to do graduate work at the intersection of number theory and analysis. Even if one is not interested in number theory per se, an important lesson of the books is that if one is at home with the tools of analysis, one can work "as an analyst" (say, with a concern for integrability, deltas, epsilons, measurability, applying the tricks of contour integration, oscillatory integrals, etc.), yet prove results originating from a genuinely distinct field (number theory, physics, radiology, and so on). Analysis becomes a mode of working rather than just a set of facts.

Rami Shakarchi (left) and Elias M. Stein.

While I have used the texts as a student and teaching assistant, I have not yet had the pleasure to teach a full course from them. I look forward to doing so. When proving a theorem, Stein sometimes summons up what he calls the main ingredients of the proof. In these texts, the ingredients of analysis are developed in pure and vivid form, and recipes for combining them into expert dishes are beautifully presented. When recipes are this good, I for one will enjoy cooking from them again and again, probably for the rest of my life.

Comments by Paul Hagelstein

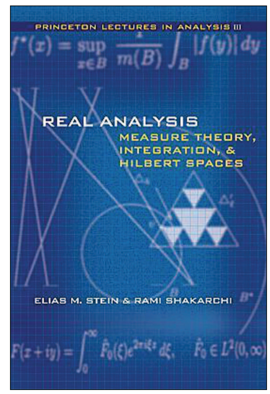
A fresh Ph.D. from the University of Chicago, I arrived at Princeton University in September 2000 with a three-year VIGRE postdoctoral fellowship. In my letter of appointment, I had been informed that my teaching responsibilities at Princeton would consist of assisting E. M. Stein with the teaching and development of a sequence of undergraduate courses in analysis. As a graduate student I had studied Stein's *Fourier Analysis on Euclidean Spaces* (coauthored with Guido Weiss), *Singular Integrals and Differentiability Properties of Functions*, and *Harmonic Analysis* in great detail and knew them to be masterpieces, the worthy sequels of Zygmund's *Trigonometric Series.* I knew Stein to be a towering figure in analysis, a man who had done seminal work in numerous important areas: harmonic analysis, partial differential equations, representation theory, and several complex variables, for example. That being said, my advisor, Robert Fefferman, had told me that Stein was working on a new series of books that would be very different from the ones he had written in the past; they would be a series of texts addressing fundamentals of analysis from a highly mature perspective, somewhat akin to what Feynman had done in his *Lectures on Physics.* I was certainly very excited at having the opportunity to participate in this undertaking, which promised to be a unique experience. Immediately after my arrival at Princeton, I introduced myself to Stein, and he enthusiastically welcomed me to the project. Friendly, receptive, and perfectly willing to answer my questions regarding the courses he was teaching, as well as my personal research, he demonstrated immediately that his reputation for graciousness and generosity is well deserved.

On my second day at Princeton, Professor Stein introduced me to Rami Shakarchi, and they filled me in on what was happening with the project. They were collaboratively working on what was to be a series of four books on analysis. The first book was almost completed and was on Fourier analysis. The second book was to be devoted to complex analysis, the third to measure theory and Hilbert spaces, and the last to functional analysis and further topics. The first undergraduate course involving this sequence had been taught the prior spring semester. In the fall of 2000, while Stein was teaching complex analysis, he and Rami would work on the complex analysis text. The sequence of courses would be continued and finished during the 2001–2002 academic year. In the spring of 2001, I was to teach the sequence myself starting with the course in Fourier analysis. In doing so, I would use upper-level drafts of the texts and provide yet another thorough proofreading before publication.

That fall I sat in on all of Professor Stein's lectures in complex analysis and held office hours to answer student questions regarding the course. The pace of play was fast. Stein prepared transparencies outlining the main definitions and theorems of each lecture, initially using the chalkboard to write down the associated proofs. When we realized that the students were having difficulty writing everything down, we decided that copies of the transparencies should be made available to them before each lecture.

The homework sets, divided into so-called Exercises and Problems, were fascinating, a definite strongpoint of this course and indeed of the entire sequence. The Exercises typically involved basic and important calculations (such as the computation of contour integrals) and also introduced students to fundamental mathematical objects and results such as Abel's theorem, Blaschke products, and the Mellin transform. There were some unexpected jewels as well—mathematical and pedagogical surprises that are pleasurably illuminating. The Problems were typically much more involved, sometimes outlining alternative proofs of results in the texts (such as Koebe's proof of the Riemann mapping theorem) and frequently requiring students to get their hands dirty in order to find a solution. I found then, and I continue to find, that even the best students struggle with a number of these, but in the process they are not only learning how to partition problems into more tractable pieces but also getting a better feel for handling chains of inequalities. Of course, Stein and Shakarchi designed these problems well aware that our students are going to need these skills if they wish to be successful researchers in analysis. The Princeton students who were involved in this project were outstanding and exceedingly hardworking. Several of them now have Ph.D.s in mathematics and are already making notable contributions in research. Certainly the excitement and enthusiasm they felt for these courses was much appreciated by Professor Stein, Rami, and me.

According to the original plan, I started independently teaching the sequence of courses in the spring of 2001. Drafts of the texts were few suggestions regarding the texts and are available at this point, and the students and I marked our copies with corrections that were later forwarded to Eli and Rami. I had a particularly exceptional group of six students for the first two courses in the sequence. They made quite a responsible for many of the hints that are now included with the "Exercises and Problems".



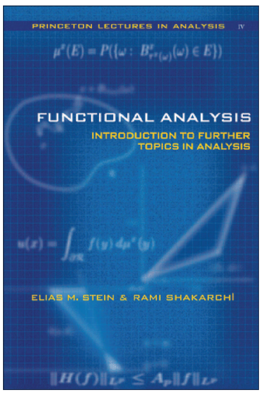
A few words about *Lectures in Analysis* as a whole. Zygmund, after emphasizing in his *Trigonometric Series* the impact of harmonic analysis on other areas of mathematics, once joked about his "mathematical imperialism". This theme of imperialism[2] is clearly evident in the *Lectures in Analysis*: in the volume *Fourier Analysis*, Stein and Shakarchi establish the fundamentals and central questions of Fourier analysis as a base camp for many of their further operations. Young analysts should take heed and thoroughly assimilate how basic problems involving heat flow and vibrating strings motivate the notion of a Fourier series and how making precise this notion necessitates the introduction of families of good kernels and the concepts of pointwise and norm convergence. Having done so, they will be in a much better position to appreciate subsequent, more abstract, results regarding the convergence of sequences and series and the Lebesgue theory of integration. They will also be able to validate these results against personal experience gained in the study of elementary Fourier series.

Having established this base camp in *Fourier Analysis*, the authors take advantage of it repeatedly in later volumes. In *Complex Analysis*, they show how the Fourier transform provides a condition that guarantees a function f defined on the real line can be extended to a holomorphic function in the complex plane. In *Real Analysis* they show that the heat and Fejér kernels provide approximations

[2]*For Zygmund's views on the matter, see the epigraph in the chapter on distributions in* Functional Analysis.

to the identity; they relate orthogonality considerations in Fourier series to Hilbert spaces and use the Radon transform to show why Besicovitch sets in $\mathbb{R}^2$ have dimension two. In *Functional Analysis*, issues involving the pointwise divergence of Fourier series illustrate the uniform boundedness principle, and the desire to extend the Fourier L^2 theory to L^p spaces motivates the Riesz interpolation theorem, which in turn is used to develop the L^p theory of the Hilbert transform. In the same volume, Stein and Shakarchi also consider issues involving the restrictions of Fourier transforms and how they relate to dispersive partial differential equations.

Finally, the incredibly broad mathematics for which Stein is so well known is beautifully reflected in this series. A surprising amount of number theory



is treated, including Dirichlet's theorem on primes in arithmetic progressions, the two and four square theorems, the prime number theorem, and results related to counting lattice points (along the lines of the Dirichlet divisor problem). Not only are the basics of complex analysis and measure theory covered, but the series also introduces students to fundamentals of ergodic theory, spectral theory, probability theory, Brownian motion, and several complex variables. One particularly nice aspect of the topics treated is that students who reach the end of the last volume have made significant headway toward the research frontier in a number of areas, including those involving Bochner-Riesz summability, the dimension of Besicovitch sets in $\mathbb{R}^n$, regularity properties for X-ray like transforms, and lattice point counting. These research areas have become quite central, and even seasoned analysts who wish to learn about these topics will find much of interest here.

Who is the target audience for these books? The four volumes of Stein and Shakarchi require surprisingly little in prerequisites—essentially a solid background in calculus with a rudimentary exposure to linear algebra. I recommend that aspiring mathematicians get copies into their hands as soon as possible. The books are clearly written and are eminently suitable for self-study. Moreover, many of the important topics covered in these volumes are only intermittently taught in even the best graduate programs. Stein and Shakarchi have served young mathematicians well in providing an introduction to these topics in such a cohesive manner.

Functional Analysis will likely become standard reading for graduate students in harmonic analysis. In many respects it provides a useful companion volume to Stein's *Singular Integrals* and *Harmonic Analysis*. Like these earlier volumes but at a more leisurely pace, *Functional Analysis* treats the Hilbert transform, the Riesz interpolation theorem, Hardy spaces, BMO, and oscillatory integrals and their applications to dispersive equations. Moreover, since there are profound connections between harmonic analysis and the all-too-often unfamiliar realm of several complex variables, students of the former will greatly appreciate the chapter on the latter; it not only introduces the subject but also clearly indicates where to go to learn it further.

Analysis is a beautiful and extremely wide-ranging subject, extending from the oscillations of vibrating strings to number theory and from the splashing of waves to the pricing of stock options. A gestalt overview is difficult to come by; the fabled seven years is an understatement. After decades of study and research, E. M. Stein is one of the very few who have attained such a wide-ranging perspective. We are extremely fortunate that he and Shakarchi share it with us in these *Lectures in Analysis*.

Comments by Nataša Pavlović

I truly enjoy teaching complex analysis. The power and elegance of its theorems have never ceased to fascinate me (even after I learned that some of them fit into the broader context of the theory of elliptic PDEs). I like to share this excitement with my students. Stein and Shakarchi's book *Complex Analysis* helps me to do so. From the fact that a holomorphic function is infinitely differentiable to the phenomenon of analytic continuation, complex analysis is (especially for the student accustomed to real variables) rife with magic. And that is how Stein and Shakarchi see the results of complex analysis. In the introduction the authors say: "When we begin the study of complex analysis we enter a marvelous world, full of wonderful insights. We are tempted to use the adjective 'magical', or even 'miraculous', when describing the first theorems we learn; and, in pursuing the subject, we continue to be astonished by the elegance and sweep of the results."

Stein and Shakarchi lead the reader through this magical world of fundamental concepts in complex analysis and some of its important applications in Fourier analysis, number theory, and combinatorics in a gentle and thought-inspiring way. For example, they start by introducing functions in the complex plane and the concept of integration along curves, and in the subsequent chapter they link these two notions via Cauchy's theorem. Their approach is gradual and as such it is perfect for the classroom—like a chorus in a Greek play, at the beginning of a chapter they give the reader a quick preview of what is coming as well as a glimpse of applications of the results to be discussed. Then the story develops from simpler building blocks to more involved results. For example, in the above-mentioned chapter on Cauchy's theorem, the authors first familiarize the reader with the special case of Cauchy's theorem for a triangle and then use this as a starting point to derive many other results. The chapter proceeds with a detailed evaluation of some integrals (among them is a classical example that shows that $e^{-\pi x^2}$ is its own Fourier transform) and concludes with applications, beginning with the fact that a holomorphic function in an open set Ω has a power series expansion for all points in a disc whose closure is contained in Ω and proceeding to Liouville's theorem and an elegant proof of the fundamental theorem of algebra based upon it. A gradual approach applies to all subsequent chapters as well as to the entire book. Proofs throughout are elegant and clearly written—it is a joy to follow them.

The book differs significantly from many other undergraduate or beginning graduate textbooks, because Stein and Shakarchi continually remind the reader of links with previously discussed topics and offer a perspective as to why these results are relevant for further applications. Hence a student can learn about motivations and connections in greater detail than he/she generally does in lectures.

Another difference with many other complex analysis textbooks, which makes this one almost a page-turner, is the choice of more advanced topics. For example, one chapter is devoted to establishing a connection between the 1D theory of the Fourier transform and complex analysis, while another chapter focuses on the zeta function and a proof of the prime number theorem. When I used the book at Princeton University to teach an advanced undergraduate class, students really appreciated these connections with Fourier analysis and number theory.

At Princeton, the Complex Analysis course consisted of two components: lectures and weekly problem sessions. Lillian Pierce, who did a wonderful job leading the problem sessions affiliated with my lectures, discussed problems which mainly came from the book itself. She was able to do so because, in addition to thoroughly solved examples in the main text, a nice selection of exercises follows each chapter. Among them are important types of problems which include both easy and more elaborate ones (for which a hint is often given). Also most chapters have a list of more challenging exercises, entitled "Problems".

Many different audiences can benefit from this book. At Princeton the textbook is used for an advanced undergraduate course, while at some other universities it is used as a beginning graduate textbook. With its clear presentation and broad range of exercises, this text is a "must read". I myself recommend it as supplemental reading for students in a beginning undergraduate course in complex analysis.[3] Since it is less dense and provides a wealth of applications, the book is more accessible than Ahlfors's *Complex Analysis* or Rudin's *Real and Complex Analysis*. On the other hand, with its clear presentation of fundamental concepts of complex analysis, as well as their applications to algebra, Fourier analysis, number theory, and combinatorics, and a wealth of important as well as interesting exercises, I like to recommend the book as essential and valuable to mathematics and physics graduate students seeking a classical book on complex analysis. It is such a beautifully written book, with a wonderful balance between insight and technical rigor, that even research mathematicians will be proud to have it on their bookshelves. Those contemplating writing a mathematical textbook might like to have an additional copy on their nightstand too.

Closing Remarks

The books of Stein and Shakarchi are a remarkable achievement, the crowning glory of a very distinguished and important career in analysis. They pass on to future generations a wealth of collected wisdom and a fusillade of techniques that will help to launch many a productive research career. Our lives are all enriched by these books.

[3] *For example, when trying to teach my students that one cannot compare complex numbers, I typically refer to the exercise in the book of Stein and Shakarchi that guides students in showing that it is impossible to define a total ordering in $\mathbb{C}$ compatible with its structure as a field.*

Mathematical Aesthetics in a Beautiful Town: Bridges Coimbra 2011

Kristóf Fenyvesi

Interdisciplinary discussions on the relations between mathematics and the arts, science, aesthetics, and artistic practice have a long history. At the present time, various cultural phenomena under the influence of mathematics and the arts continue to inspire people working in different fields of science. Similarly, several artists and scholars share a common interest in combining creative thinking, intellectual curiosity, and aesthetic sensibility in their work and research. The Bridges Conferences (`http://www.BridgesMathArt.org`), founded and lead by Reza Sarhangi (Towson University) and running annually since 1998, aim to initiate a dialogue between the mathematical and the artistic points of view in various fields where artistic and mathematical thinking and practice merge. Unique components of the Bridges Conferences, in addition to formal presentations, are gallery displays of visual art; hands-on workshops designed for professional educators as well as for families and children; working sessions with artists and mathematicians who are crossing the mathematics-arts boundaries; mathematical poetry readings; and musical, th;atrical, and movie events in the evenings.

Coimbra: The City of Sciences, Arts, and Bridges

The 2011 Bridges Conference, codirected by Penousal Machado, was held at the University of Coimbra, Portugal, during July 27–31, 2011. The University of Coimbra was established in 1290 as the first university in Portugal and the first Portuguese language university in the world. After the much larger metropolitan areas of Lisbon and Porto, Coimbra is the most important urban focal point of the central part of the country. It is situated on the Rio Mondego, the longest river located exclusively in Portugal. The river, which flows through the city, provides a scenic setting for the town, which invites all to walk along its river banks and across its four bridges. It was a beautiful setting for the Bridges Conference!

Math Goes Fashion

Since last year when Bridges visited Hungary,[1] the conference has grown again and set new records. The conference received almost 300 paper, artwork, and proposal submissions, more than ever before, presenting intriguing ideas in mathematics, in the arts, and in several other cultural domains. This year the keynote paper was coauthored by Fields medalist William P. Thurston (Cornell University) and was presented by Kelly Delp (Buffalo State College). The spectacular presentation described a process that was inspired by modern fashion design and that can be adapted to construct many different surfaces out of paper and craft foam.

Another plenary paper was presented by Paulus Gerdes, the president of ISGEm, the International Study Group on Ethnomathematics and vice president of Southern Africa in the African Academy of Sciences. Gerdes analyzed instances of mathematical ideas interwoven in the artistic decoration of handbags, hats, mats, and other products of basketry from several regions of Mozambique. He shared instructive stories about "field encounters" with local handcrafters and touched upon such exciting phenomena as the tradition of geometrical

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[1] *Paul Gailiunas, Bridges Pécs 2010,* Notices of the American Mathematical Society **58***, no. 2 (February 2011), 289–290.*

imagination and the cultural role of the so-called "mental geometry" and their links to the future of mathematics education in Africa.

Among the many memorable presentations, it is also important to mention the joint paper by artist Jim R. Paulsen and Reza Sarhangi on Paulsen's impressive sculptures *Sentinels*. As an addition to the keynote talks, George W. Hart, a member of the Bridges board of directors and the chief of content at the Museum of Mathematics (MoMath) in New York City, offered exclusive insight for the conference participants into MoMath's most interesting innovations of interactive presentation techniques of mathematical contents in the museum.

The local organizers also realized a Portuguese math/art day, which featured invited talks by Manuel Arala Chaves (University of Porto, a member of the directing board of the Atractor Project-Interactive Mathematics), Paulus Gerdes, and João Paulo Xavier (University of Porto), complemented by workshops on the symmetry of plane patterns, on Roman architecture in Portugal, and on Lunda art, inspired by sona geometry from Angola.

Mathematics Is Art

An exhibition of mathematical art has been an annual feature of Bridges since 2001, and it has grown steadily over the years under the dedicated leadership of Robert Fathauer. This year, work was submitted by more than seventy artists from twenty countries. Diverse artistic media were represented, including wood, metal, and stone sculptures; beadwork; fabric; and a variety of two-dimensional media. Mathematical ideas at play in the art exhibition encompassed tilings, fractals, polyhedra, hyperbolic geometry, anamorphosis, knots, topology, and magic squares.

On the threshold of the digital age, it was a great pleasure to see the acrylic, watercolor, pigment, and oil paintings of Aurora, Anita Chowdry, and János Saxon-Szász at the exhibition. The process of creation and the pure sensation of the color, just like the materiality of the medium, play a central role in their art: "I do not use a calculator in order to create these paintings; I use my mind to do the math. The value of this process is that, as the painting is completed, I embody these patterns and concepts and carry them within myself." So writes Aurora in her artist's statement. Chowdry's techniques are based on her research into the methods and materials of painting and illuminating in Indian and Persian manuscripts.

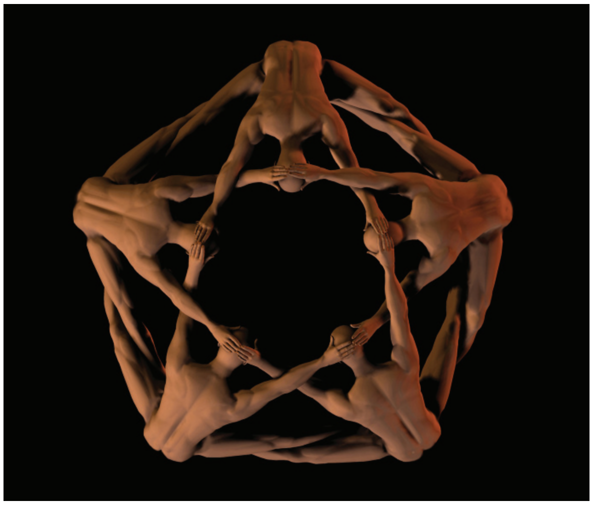
Browsing among the great variety of 3D artworks, the visitors could play with Xavier De Clippeleir's tricky transforming bodies and admire Nicholas Durnan's carved alabaster sculptures (a Möbius and two variations on the theme of Borromean rings) and Bente Simonsen's beautifully mirroring steel objects. They could also study the magical skillfulness of woodcarving by Bjarne Jespersen.[2] From the exhibited photographs of Nick Sayers's artworks, we could also get a taste of using mathematics in contemporary conceptual "bricolage-art".

Nick Sayers: Show Home, 2011.

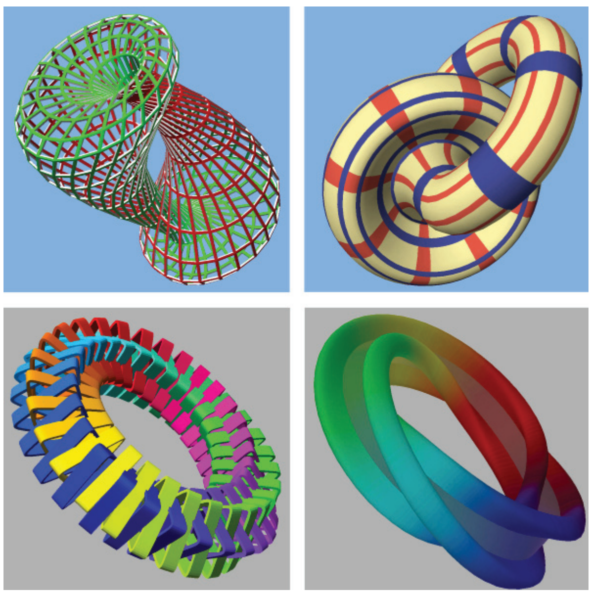
Nicholas Durnan: Borromean Rings 2, 50 x 220 x 220 mm, English alabaster, 2011.

Unlike much mathematical art that is purely abstract, Sayers uses recognizable household objects to create work that is accessible, real, and fun. The largest work he presented, as a photograph, at the exhibit was *Show Home*: a four-meter-in-diameter, three-meter-high geodesic shelter built from 135 estate agent (realtor) "For Sale" and "To Let" boards. The piece made a statement about homelessness, the housing market, and sustainable architecture.

[2] *Bjarne Jespersen,* Woodcarving Magic. How to Transfer a Single Block of Wood Into Impossible Shapes, *East Petersburg, PA: Fox Chapel Publishing, 2012.*



Mike Naylor: Pentamen, 16” x 12”, digital print, 2002.



Carlo H. Séquin, University of California, Berkeley.

Carlo Séquin: The World of Wild and Wonderful Tori, 24” x 24”, composite of computer images, 2011.

After seeing the pieces of Mike Naylor’s geometric body art, the audience could be convinced about the naked truth of the artist’s simple statement: “Mathematics is part of us, and we are part of mathematics.” Moving away from the mathematically obsessed nude people towards the dressed ones, we could enjoy remarkable examples of mathematically inspired fashion design by Jasmin Schaitl and Eunsuk Hur. By the fusion of mathematics and fashion Schaitl’s model shapes and defines the body in a new context and opens up a variety of exciting design processes. Hur’s modular textile pieces also encourage the end user to participate in the design process by allowing the user freedom to personalize the design through playful experimentation.

Computer-generated graphics and animation art were also represented on a high-quality level. Mingjang Chen used the Structural Cloning Method (SCM) and the Leaping Iterated Function System (LIFS) to explore abstract and landscape paintings. Gary Greenfield’s *Transport Network Overlay* series are based on simulations that are inspired by mathematical models of physical and biological processes. Mehrdad Garousi used Mandelbulb 3D for adventurous explorations into his extended, three-dimensional fractal worlds. Daniela Rinaudo, in a philosophical piece called *Geometric Man*, used computer animation to offer a virtual journey for the spectator into “imaginary time” in search of another dimension into space. Nathan Selikoff experimented with chaotic attractors. Carlo Séquin (University of California, Berkeley), a plenary speaker, who together with Reza Sarhangi, took a major role in the organization of the scientific program of the conference, exhibited his 3D artworks. He also exhibited his computer graphics to visually support his plenary talk, the “Tori-Story”. His presentation elaborated on the classification of all topological tori into four regular homotopy classes, where the members in one class cannot be smoothly transformed into members of another class. His art submissions depicted some intriguing structures that topologically are torus surfaces but with enough surprising contortions so that most people would not immediately see them as your everyday donut.

We saw innovative approaches in uniting fractals and tessellation art by Robert Fathauer, through the combination of traditional art forms like photography with digital techniques to create tree-like and knotted spiral designs that have an intriguing blend of complexity and beauty.

Design works are always in high favor at the Bridges exhibits, especially when broadly celebrated designers like Fabien Vienne from France bring the honor of their presence to the exhibition. Vienne’s beautiful creations convey the notion that geometry is no mere tool of composition; it’s an epistemology. Each of his works seeks to employ the principle of economy to find the essential in a problem and to then “substantiate” it in a least action solution, which brings that essence to light and to life.

Mathematics Is Culture

In 2011, for the second time in the Bridges’ history, the music night was organized by Dmitri Tymoczko, a composer and music theorist at Princeton University. Coimbra’s professional orchestra, the Orquestra Clássica do Centro, had graciously agreed to play mathematically inspired and inspiring works, both new and old. The concert featured a newly composed piano concerto by Dmitri Tymoczko based on the idea of cellular automata, a recent piece by Giovanni Albini based on the idea

Jasmin Schaitl: Body-Index-Cloth III, 83 x 50 x 30 cm, fashion, 100 percent cotton with satin ribbon, 2011.

Above: © Artpiece: Jasmin Schaitl; Photo: Jasmin Schaitl and Debora Däubl.

of symmetry, and an older piece by Tom Johnson in which a mathematics theorem is proved before one's very ears! Maestro Artur Pinho's orchestra performed magnificently.

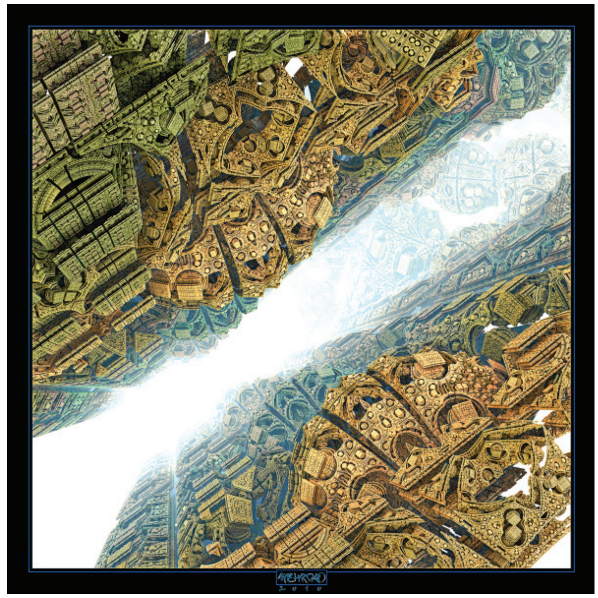
The theater night was orchestrated by Steve Abbott (Middlebury College), a researcher of the intersections of mathematics and theater and the coeditor of *Math Horizons*. The performance focused on a single dramatic piece: *The Physicists*, by Friedrich Dürrenmatt, performed in the form of a staged reading by the conference participants themselves.

A rapidly growing number of artists and educators are using movies, videos, and animations for applications spanning education, industry, and the arts. The Bridges math/art short movie festival, which was directed by Amy Christie and Nathan Selikoff, screened many short movies, including a virtual reenactment of Escher's *Drawing Hands*, a demonstration of the bubble-sort algorithm via Hungarian folk dance, and an animation, the *Adventures of the Klein Bottle*, by Konstantin Weixelbaum and Ilkay Sakalli, supervised by Konrad Polthier (Free University, Berlin).

Poetry day was a new addition this year to the Bridges Conference. This event was coordinated by Sarah Glaz, a mathematics professor from the University of Connecticut and a poet. There was a reading of poems with strong links to mathematics. Reading their own poems were Emily Grosholz, JoAnne Growney, Amy Uyematsu, and Sarah Glaz. Saeed Ghahramani (a mathematician and the dean of arts and sciences at Western New England University) read translated modern Iranian poems. Coimbra University mathematician and translator Francisco Craveiro was joined by other poets for a bilingual reading of mathematical poems that he translated into Portuguese. Craveiro read the Portuguese translations, while the other poets read the English originals. The event ended with an "open microphone" period in which Bridges participants could share their own mathematical poems with the audience.

Mathematics Is Fun

Mathematics can be scientific, and it can be artistic as well. But can mathematics be playful? Can



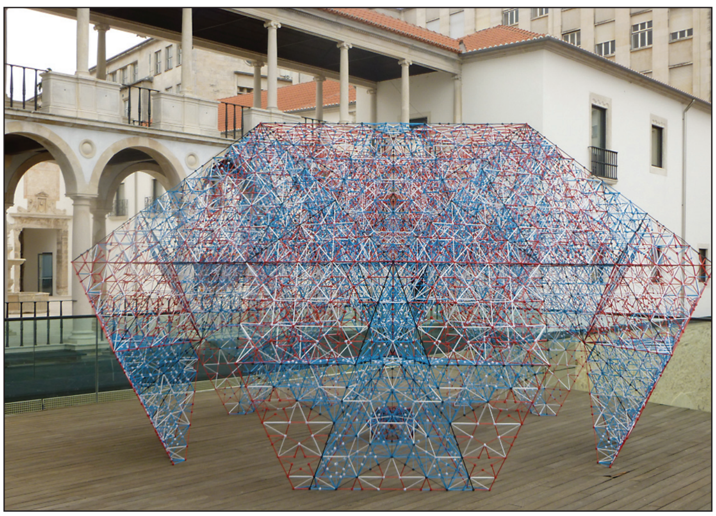
Mehrdad Garousi: Space Shelter, 20" x 20", digital print, 2010.

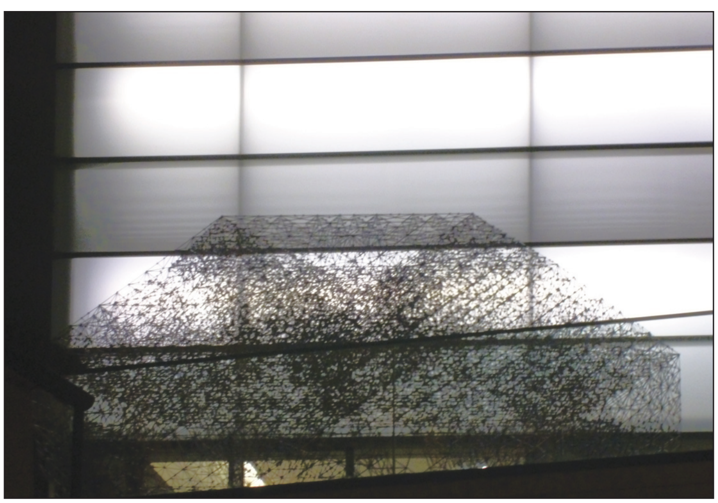
it be fun? We gave it a try at Bridges family day (`http://www.familyday.hu`). The participating children, parents, and all the visitors had the possibility, during the workshops, to test themselves as math-fashion designers with the leadership of the enthusiastic artist team of Ruth Mateus-Berr from the University of Applied Arts in Vienna; or as mathematical sculptors with Nicholas Durnan; or as weavers or quilters with Elaine Krayenke Ellison. We made geometry with our own bodies at the Human Geometry Workshop led by Mike Naylor and Vi Hart; enjoyed the miracle of kaleidoscopes with Curtis Palmer; practiced mathematical origami with Wojtek Burczyk; learned some tricks from a real mathe-magician, Fernando Blasco; played amazing puzzles with the internationally renowned designer Jean-Marc Castera; took a quick tour in the "Vasarely Playhouse" with Slavik Jablan; played with the giant triangles of Simon Morgan and Jacqueline Sack; worked with clay under the direction of Jouko Koskinen; immersed ourselves in the secrets of Islamic mosaic design with Sarah Abdellahi and Tom Goris; and tried the Zometool modeling kit with Paul Hildebrandt, Mike Stranahan, and Samuel Verbiese. The Zometool modeling kit was in use during the conference in the Castro Machado Museum of Coimbra to build up the ambitious Pentigloo: an incredible construction of 44,771 parts with a diameter of almost nineteen feet by Fabien Vienne, the noted 86-year-old designer who was present and worked together with the construction team (led by Paul Hildebrandt and Jim Hausman) during the construction process.

Such a rich and mosaic-like event, which brought participants from more than thirty countries in the world, could not be put together by just a few official chair persons. In addition to the 2011 Bridges Program Committee of about forty individuals and the organizers of the many events described above, I should mention the important roles of the organizing team in Coimbra, including Ana Almeida from the Department of Physics and Mathematics, ISEC-Instituto Superior de Engenharia de Coimbra; Amílcar Cardoso, the president of the Centre for Informatics and Systems, University of Coimbra, Portugal; and Penousal Machado from the Department of Computer Science, University of Coimbra, Portugal.

The next Bridges event will take place on July 25–29, 2012, at Towson University, located in the Baltimore metropolitan area, Maryland, USA. Please find more information at `http://www.BridgesMathArt.org/Bridges-2012`.

The Zometool Pentigloo construction team at work. Derek Vorthman (left), László Vörös (left, background), Scott Vorthman (middle), George W. Hart (right), Kelly Delp (right, foreground). (Photo by Samuel Verbiese.)





Above: Bridges around the clock: Fabien Vienne's Pentigloo at day and at night in the Museu Nacional de Machado de Castro, Coimbra. (Photos by Samuel Verbiese.)

Tribute to Lawrence E. Payne

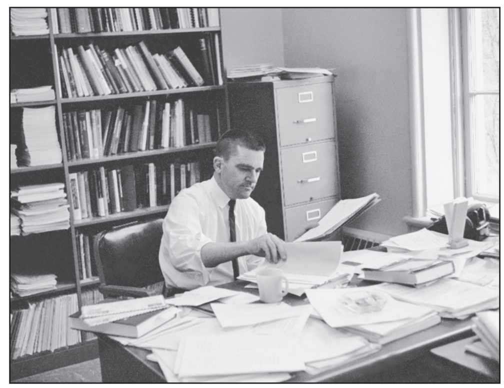
Howard A. Levine

Lawrence E. Payne, who received his B.S. in mechanical engineering in 1946 and an M.S. and Ph.D. in applied mathematics from Iowa State University in 1950 under D. L. Holl, died August 11, 2011, of acute lymphoma. He was born in McCleansboro, Illinois, a small farming community near Carbondale in 1923. His first position was at the University of Arizona, followed by fourteen years at the University of Maryland in the Institute for Fluid Dynamics and Applied Mathematics. He joined the faculty of Cornell University as a full professor in January 1965.

Although he had a very distinguished academic career, he was fond of telling his friends and students that his first employer was the Merit Shoe Company in Detroit where he was first employed in July 1941 as a shoe salesman. In his usual self-effacing style, he neglected to mention that by the time he left the shoe business in February 1943 he was the assistant manager. He then joined the U.S. Navy, serving a little over three years before returning to Ames for graduate school. He never forgot his Iowa State University connections and always spoke appreciatively and highly of the education he received there.

Larry, as he preferred to be called, was a recognized international leader in partial differential equations, especially in isoperimetric inequalities and improperly posed problems of mathematical physics. (Improperly posed problems are those for which the solution either fails to exist, fails to be unique when it does exist, or fails to depend continuously on the data.) He authored or coauthored nearly three hundred articles and two books. He held visiting research positions for extended periods of time at the University of Genoa, the University of Florence, the University of Berlin, the University of Dublin, the University of Glasgow, Heriot Watt University, the University of Newcastle upon Tyne, the University of Virginia, and the Swiss Federal Technical University (ETH) in Zurich. He was the major professor for fifteen Ph.D. students, some of whom came from such faraway places as Iran, South Korea, and South Africa.

He was primarily responsible for rebuilding the applied mathematics program within the Cornell mathematics department and for the establishment of the Center for Applied Mathematics there. His quiet, persuasive manner enabled him to succeed where individuals with more dynamic but less political personalities had failed. This center as well as the group in numerical analysis and partial differential equations came to be considered by many as among the best in the world.



Lawrence E. Payne, 1966.

Photo taken at Cornell University by H. A. Levine.

Among his awards was the prestigious Steele Prize from the American Mathematical Society. This prize is given for an expository paper (his paper is "Isoperimetric inequalities and their applications", *Soc. Ind. Appl. Math. Rev.* **9** (1967), 453–488) that has made a significant and lasting impact on the field in which it is written. In 1990 he received an honorary Doctor of Science degree from the National University of Ireland. In 1992 he received the Citation of Merit from the College of Liberal Arts and Sciences at Iowa State University. A three-day conference in his honor was held at Cornell in October 1990. It was attended by friends and colleagues from all over the world.

In addition to his numerous scientific achievements, his colleagues the world over held him in the highest esteem as a friend, advisor, and colleague. Jim Bramble, Larry's first student, remarked, "I learned so much from him, particularly in terms of his approach to problem solving and discovery. We were great friends." He gave selflessly of his time and energy and was one of the kindest and most patient people I ever met. These are, without a doubt, sentiments with which all of us who knew him will agree.

Howard A. Levine is professor of liberal arts and sciences at Iowa State University. His email address is `halevine@iastate.edu`.

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About the Cover

Two views of Burgers' equation

This month's cover portrays two ways of visualizing a solution to Burgers' equation, which is one of the topics Chi-Wang Shu discusses briefly in his article "Efficient algorithms for solving partial differential equations with discontinuous solutions" in this issue. Burgers' equation is the simplest model of many phenomena appearing in the solution of nonlinear hyperbolic partial differential equations, and exhibits in a particularly simple manner the way in which solutions become singular.

What this means is that solutions, even with very reasonable initial conditions such as the example

$$u(x,0) = \cos x \quad (|x| \le \pi)$$

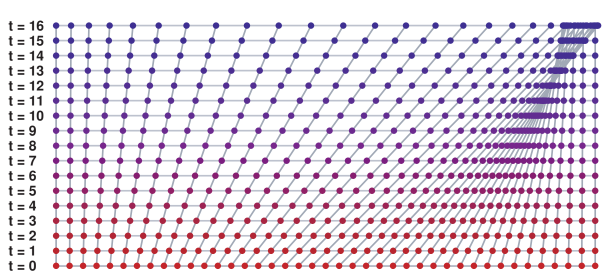
shown on the cover, cease to be honest functions at some point. This phenomenon is commonly called discontinuity and, as Shu mentions, it is ubiquitous in nonlinear PDEs.

There is much discussion of Burgers' equation on the Internet, but we haven't seen any that mentioned the very simple and intuitive interpretation in terms of traffic flow. We learned about it from Chapitre 2 of the book by the late Vladimir Arnold whose title in French is *Chapitres supplémentaires de la théorie des équations différentielles ordinaires*. (Author query: Has it been translated into English?) In this model, at initial time t=0 a collection of particles is assembled on the x-axis with velocity given by a function $u(x, 0)$ depending only on position. As time proceeds, the function $u(x,t)$ specifies the velocity of the particles at position x (but they are no longer the particles originally there), and it satisfies Burgers' equation

$$u_t + u\,u_x = 0\,.$$

If the initial function has a negative slope at any point, then of course the faster particles eventually catch up to the slower ones, bringing about a singularity of sorts, and at this time $u(x,t)$ ceases to be a function of x. The top image on the cover shows plots for various times as a graph in 3D, while the bottom one and the figure below show in a different way how the particles with different velocities eventually give rise to a caustic in the plot, where the particles accumulate to infinite density.

—*Bill Casselman*
Graphics editor
(notices-covers@ams.org)



Philip Schaefer

It was my privilege to be one of Larry Payne's Ph.D. students when he was on the faculty at the University of Maryland. Twenty-five years later we began collaborating on overdetermined and nonstandard boundary value problems, growth and decay results, energy and pointwise bounds, and lower (upper) bounds for blow-up time in partial differential equations and systems. Much of this work was accomplished when Larry spent the winter months of his retirement in the milder climate of Tennessee. On my last visit to Cornell, we did not learn of the 9/11 tragedy until 10:45 a.m. because we were "hard at work". In 1987 we hosted the "Maximum Principles and Eigenvalue Problems in Partial Differential Equations" conference (*Pitman Research Notes* **175**) at the University of Tennessee, which reunited Larry with Murray Protter and Hans Weinberger and with other of his collaborators such as Cornelius Horgan, Howard Levine, and Brian Straughan. A final conference with international participants which honored Larry's contributions to mathematics was held in Cagliari, Sardinia, in 2003. Without a doubt, Larry Payne was a "man for all seasons"—a gentleman and a scholar—a prolific researcher and dedicated teacher and mentor. He will be sadly missed by many people that he touched.

Simon Levin

When I was a graduate student at the University of Maryland, Larry Payne—then one of my professors and later to be a valued colleague—gave me a copy of a manuscript he was ready to submit for publication. He said that he knew that I was working on similar ideas for my thesis and that if I found anything in his paper that would compete with my results, I should let him know so that he could remove them from his manuscript. I was young and naive, but even so I recognized that this was a remarkable gesture from a remarkable person. Although there was nothing that needed removing, this incident had an indelible effect upon me as a lesson for how to deal with others, especially one's students, and I have tried to remember the lesson in my own mentoring. I learned too that this was not an isolated behavior from Larry, who was not only a great and generous mathematician but a model of how one should live one's life.

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WHAT IS... an Approximate Group?

Ben Green

Let A be a nonempty finite subset of a group G. Before saying what it means for A to be an approximate subgroup of G, let us consider the easier question of what it means for A to be an actual subgroup of G. Throughout this article we adopt the following standard notation. If $A, B \subseteq G$ we write $A^{-1} := \{a^{-1} : a \in A\}$, $AB := \{ab : a \in A, b \in B\}$ and $A^n := \{a_1 \dots a_n : a_1, \dots, a_n \in A\}$. We say that A is *symmetric* if $A^{-1} = A$.

Here, then, are three easily proven characterisations of what it means to be a subgroup:

(i) If $x, y \in A$, then $xy^{-1} \in A$;
(ii) A is symmetric, contains the identity, and $|A^2| = |A|$;
(iii) A is symmetric, contains the identity, and A^2 coincides with some right-translate Ax of A.

Approximate group theory is concerned with what happens when we try to relax these statements. Let $K \geqslant 1$ be a parameter; the bigger K is, the more relaxed we are going to be. Consider the following properties that a set A may have:

(i) If x, y are selected randomly from A, then $xy^{-1} \in A$ with probability at least $1/K$;
(ii) A is symmetric and $|A^2| \leqslant K|A|$;
(iii) A is symmetric and A^2 can be covered by K right-translates of A.

Each of these is a reasonable notion of approximate group, but (iii) has become standard.

Definition (Tao)**.** Let A be a symmetric subset of a group G. Then we say that A is a K-approximate group if A^2 is covered by K right- (or left-) translates of A.

Rather surprisingly, it hardly matters which of (i), (ii), or (iii) one chooses as "the" definition so long as one is only interested in the "rough" nature of A. For example, if A is symmetric and satisfies (i) then there is a set $\tilde{A} \subseteq A^4$ satisfying (iii) with parameter $\tilde{K}$, and with $\frac{1}{\tilde{K}} \leqslant \frac{|\tilde{A}|}{|A|} \leqslant \tilde{K}$, where $\tilde{K}$ is bounded polynomially in K. This result, which is not at all obvious, is essentially the Balog-Szemerédi-Gowers (BSG) theorem. Other equivalences of a similar type between (i), (ii), and (iii) were described by Tao, building on fundamental work of Ruzsa.

Let us give some examples of approximate groups.

Example 1. Any genuine subgroup A is a 1-approximate group.

Example 2. Any geometric progression $A = \{g^n : -N \leqslant n \leqslant N\}$, $g \in G$, is a 2-approximate group.

Example 3. Let $x_1, \dots, x_d \in \mathbb{Z}$. Then the d-dimensional *generalised arithmetic progression* $A = \{n_1 x_1 + \cdots + n_d x_d : |n_i| \leqslant N_i\}$ is a 2^d-approximate subgroup of $\mathbb{Z}$ (written with additive notation).

Example 4. If

$$S = \{ \left(\begin{smallmatrix} 1 & n_1 & n_3 \\ 0 & 1 & n_2 \\ 0 & 0 & 1 \end{smallmatrix}\right) : |n_1|, |n_2| \leqslant N, |n_3| \leqslant N^2 \},$$

then $A := S \cup S^{-1}$ is a 100-approximate group. This is an example of a *nilprogression*.

The definition of approximate group is rather combinatorial, but the above examples have an algebraic flavour. The *rough classification problem* for approximate groups is to understand the extent to which an arbitrary approximate group A looks roughly like an algebraic example such as one of those described above.

A solution to the rough classification problem for approximate subgroups of $\mathbb{Z}$ was given by

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Freiman and (later with a simpler proof) by Ruzsa. They showed that every K-approximate group A is contained in P, a d-dimensional generalised arithmetic progression, where $d \leqslant K$ and $|P|/|A| \leqslant f_1(K)$ for some function f_1. Very recently a solution to the rough classification problem in general was given in [1], building on a major breakthrough (using model theory) by Hrushovski and influenced by Gromov's theorem that groups of polynomial growth are virtually nilpotent. [1] shows that any approximate group is contained in a "coset nilprogression" P with $|P|/|A| \leqslant f_2(K)$: roughly speaking, an object built from examples such as the four described above.

These results are rather qualitative in nature. Whilst $f_1(K)$ can be taken to be merely exponential in K, no effective bound is known for $f_2(K)$ because [1] relies on an ultrafilter argument and an appeal to "infinitary" analysis results connected with Hilbert's fifth problem. In certain specific situations good quantitative results are known. In a seminal paper, Helfgott showed that if A is a K-approximate subgroup of $G = \mathrm{SL}_2(\mathbb{F}_p)$, then either $|A|/|G| \geqslant K^{-C}$ or else at least $K^{-C}|A|$ elements of A are contained in a soluble group (for example, the upper-triangular matrices). He later obtained an appropriate generalisation of this to $\mathrm{SL}_3(\mathbb{F}_p)$, and subsequent work of Pyber-Szabó and Breuillard-Green-Tao further generalised this to $\mathrm{SL}_n(\mathbb{F}_p)$ and other linear groups.

Where do approximate groups arise? We give two examples. The first is in connection with the topic of growth in groups. Let G be a group generated by a finite symmetric set S. If G is a free group (say), then $|S^n|$ will grow exponentially in n. At the other extreme we have the notion of *polynomial growth*, where $|S^n| \leqslant n^d$ for all large n. In this case there are infinitely many n for which S^n is a 10^d-approximate group.

By combining this observation with the rough classification, one obtains certain extensions of Gromov's theorem. Perhaps future developments will lead to the conclusion that G is virtually nilpotent under much weaker assumptions such as $|S^n| \leqslant \exp(n^c)$ for infinitely many n.

The second example comes from *expanders* [2], [3]. If G is a finite group then a symmetric set S of generators has the *expansion property with constant* ε if whenever $A \subseteq G$ is a set with $|A| < |G|/2$, we have $|AS| \geqslant (1+\varepsilon)|A|$. Bourgain and Gamburd used Helfgott's work to find new families of generators for $\mathrm{SL}_2(\mathbb{F}_p)$ and other groups with the expansion property. For example, answering a question of Lubotzky, they showed that the set $S = \{A, A^{-1}, B, B^{-1}\}$ has this property with $\epsilon > 0$ independent of p, where $A = \left(\begin{smallmatrix} 1 & 3 \\ 0 & 1 \end{smallmatrix}\right)$ and $B = \left(\begin{smallmatrix} 1 & 0 \\ 3 & 1 \end{smallmatrix}\right)$. We give a rough sketch of their argument.

It is known that the expansion property is equivalent to the rapid equidistribution, in time $\sim \log|G|$, of the random walk with generating set S. Suppose that X_n is the G-valued random variable describing the nth step of this walk. Thus, in our example, X_1 takes each of the values A, A^{-1}, B, B^{-1} with probability $\frac{1}{4}$, and X_n is distributed as the product of n independent copies of X_1.

By an application of representation theory due to Sarnak and Xue it suffices to prove the weaker statement that X_n is "somewhat" uniform at time $n \sim \log|G|$.

Now it is not hard to show that X_n becomes "smoother" as n increases. For each n there is a dichotomy: either X_{2n} is "much" smoother than X_n or $X_{2n} \approx X_n$ in some sense. If the former option occurs frequently, then X_n will rapidly become somewhat uniform on G, thereby concluding the proof. Suppose, by contrast, that $X_{2n} \approx X_n$; then the product of two independent copies of X_n has almost the same distribution as X_n. This basically implies that the support $\mathrm{Supp}(X_n)$ of X_n satisfies property (i) above with some smallish value of K. By the BSG theorem a large chunk of $\mathrm{Supp}(X_{4n})$ satisfies property (iii) and so is a $\tilde{K}$-approximate group. This is how approximate groups arise in the study of expanders.

Applying Helfgott's result we conclude that either $\mathrm{Supp}(X_{4n})$ is almost all of G, which implies that X_{4n} is somewhat uniform on G, or else a large part of $\mathrm{Supp}(X_{4n})$ generates a soluble group. This second possibility, however, may be ruled out. In fact, for $n \leqslant \frac{1}{100}\log|G|$ the random walk X_{4n} behaves like a random walk on a free group, whilst if a large chunk of $\mathrm{Supp}(X_{4n})$ were soluble one would have many commutation relations $[[M_1, M_2], [M_3, M_4]] = I$.

Let me conclude by stating one of my favourite open problems, now known as the *Polynomial Freiman-Ruzsa conjecture*. Suppose that $f : \mathbb{F}_2^n \to \mathbb{F}_2^n$ is a function which is weakly linear in the sense that $f(x+y) - f(x) - f(y)$ takes only K different values as x, y range over $\mathbb{F}_2^n$. Is $f(x) = g(x) + h(x)$, where g is linear and $|\operatorname{im} h| \leqslant K^C$? Ruzsa showed that this is equivalent to a good quantitative classification of the approximate subgroups of $\mathbb{F}_2^n$.

This is easy to achieve with $|\operatorname{im} h| \leqslant 2^K$. In deep recent work Sanders, building on work of Schoen and Croot-Sisask, showed that we can have $|\operatorname{im} h| \leqslant e^{C(\log K)^4}$, the current state of the art.

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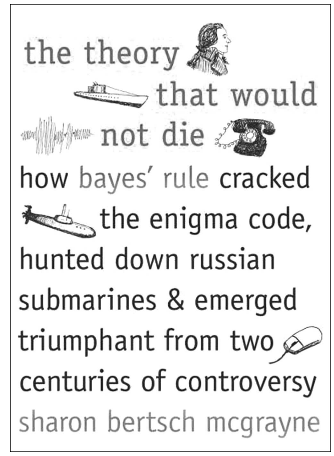
The Theory That Would Not Die

Reviewed by Andrew I. Dale



The Theory That Would Not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy
Sharon Bertsch McGrayne
Yale University Press, April 2011
US$27.50, 336 pages
ISBN-13: 978-03001-696-90

In the early 1730s Thomas Bayes (1701?–1761) was appointed minister at the Presbyterian Meeting House on Mount Sion, Tunbridge Wells, a town that had developed around the restorative chalybeate spring discovered there by Dudley, Lord North, in 1606. Apparently not one who was a particularly popular preacher, Bayes would be recalled today, if at all, merely as one of the minor clergy of eighteenth-century England, who also dabbled in mathematics. How is it, then, that Roger Farthing, author of an excellent history of Mount Sion, could describe Bayes as "to my mind, the greatest man to have lived in Tunbridge Wells" ([5, p. 167])? The answer is fairly simple: *Bayesian statistics*.

In 1763 Richard Price forwarded to the Royal Society of London an essay by Bayes [2] in which the following problem was addressed:

> *Given* the number of times in which an unknown event has happened and failed: *Required* the chance that the probability of its happening in a single trial lies between any two degrees of probability that can be named.

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The solution, given *more geometrico* as Proposition 10 in [2], can be written today in a somewhat compressed form as

$$\text{posterior probability} \propto \text{likelihood} \times \text{prior probability}.$$

Price himself added an appendix in which he used the proposition in a prospective sense to find the probability of the sun's rising tomorrow given that it has arisen daily a million times. Later use was made by Laplace, to whom one perhaps really owes modern Bayesian methods.

The degree to which one uses, or even supports, Bayes's Theorem (in some form or other) depends to a large extent on one's views on the nature of probability. Setting this point aside, one finds that the Theorem is generally used to update (to justify the updating of?) information in the light of new evidence as the latter is received, resulting in a strengthening of one's belief. Bayes's Theorem provides a codification of "learning from experience", and *The Theory That Would Not Die* is concerned with the investigation and exposition of situations in which such learning has been both required and achieved.

A globally acceptable definition of *Bayesianism*, or *the Bayesian method*, seems hardly possible, for there are perhaps almost as many definitions as there are practitioners of the art. A useful and generally acceptable description is given as follows by Anthony O'Hagan: "The Bayesian method briefly comprises the following principal steps. *Likelihood …Prior …Posterior …Inference*" ([8, p. 10]). It is also considered essential that the prior probability distributions be explicitly given (if these distributions are estimated from the data, the method, developed in the 1950s, is known as *Empirical Bayesianism*).

Laplace initially gave Bayes's Theorem in a simplified form as

$$\Pr[A_i|E] = \Pr[E|A_i] \,/\, \sum_1^n \Pr[E|A_j],$$

where, for instance, $\Pr[E|A_i]$ is the probability of the event E given the cause A_i. This formula was developed in his later work to the continuous form one knows today, and he immediately applied it in his scientific investigations. A large study of births (more accurately baptisms) in France suggested to Laplace with strong probability that the chance of the birth of a boy exceeded that of a girl in Paris (and similarly in London).

Most statisticians these days are acquainted with the theoretical aspects of Bayesianism. Less well known, I suspect, are certain areas of practical application, and McGrayne's book is a "find" in this respect. So many, indeed, are the different applications considered that we cannot mention more than a few here.

Despite Laplace's work, his successors saw little virtue in Bayesian methods and almost succeeded in killing them off. One who kept the flickering flame alight was Joseph Louis François Bertrand with his work on projectiles for artillery field officers, and later, in the famous Dreyfus affair, the defence witness Henri Poincaré strongly supported the updating of the probability, in the light of later evidence, of the truth of a hypothesis as to whether a certain letter was a forgery.

Come the Great War, statisticians in the United States found themselves using Bayesian methods in connection with the making of decisions about injured workers and telephone communications. Yet these methods did not meet with general approval: "Early in the twentieth century" writes McGrayne, "theoreticians would change their attitudes toward Bayes' rule from tepid toleration to outright hostility" (p. 45)[1].

Sporadic work continued after the war, and in 1925 Egon Sharpe Pearson published a long paper [9] in which he sought to put the theoretical rule to the test—"the most extensive exploration of Bayesian methods conducted between Laplace in the 1780s and the 1960s" (p. 49). Before the candle could be entirely guttered, the flame spluttered up in the work of Èmile Borel, Frank Plumpton Ramsey, and Bruno de Finetti and in the face of strong opposition from Ronald Aylmer Fisher and Jerzy Neyman. Only the sterling work of Harold Jeffreys—largely and undeservedly ignored by statisticians—at Cambridge kept the resuscitation on the go.

Pearson's idea ran as follows: let a sample of size n (say of taxicabs in London streets) be observed in which the number of taxis having registration LX is p and that without is q. Then observe a further number m with the two numbers being r and s, respectively. Now repeat this experiment (with the same numbers n and m) in different situations, and see whether "the distribution of observed values of r could be compared with the theoretical distributions which on Bayes' Theorem the knowledge of n and p should enable us to predict" ([9, p. 396]).

The damage done by German U-boats to Allied shipping during the Second World War made it imperative for the Allies to fathom the workings of the Enigma machines. These machines, used by the Germans to send encoded messages, became more and more sophisticated and complicated as the war progressed. Intensive research by a dedicated team under the leadership of Alan Turing at Bletchley Park in England eventually resulted in the cracking of the codes (the difficulty was considerably eased when a code book of encrypting tables was obtained from a sinking German submarine off Egypt). McGrayne writes "Turing was developing a homegrown Bayesian system. Finding the Enigma settings that had encoded a particular message was a classic problem in the inverse probability of causes" (p. 68).

During the war Turing paid a visit to the United States to discuss the work done at Bletchley Park with the U.S. Navy cryptographers. However "he tried in vain to explain the general principle that confirming inferences suggested by a hypothesis would make the hypothesis itself more probable" (pp. 75–6). Here Turing met Claude Shannon, who was working intensively on information theory. "Roughly speaking, if the posterior in a Bayesian equation is quite different from the prior, something has been learned; but when a posterior is basically the same as the prior guess, the information content is low" (p. 77). Turing's work was not altogether dismissed, however, for Eisenhower later said that "Bletchley Park's decoders had shortened the war in Europe by at least two years" (p. 81).

However, the success achieved by Bayesian methods during the war was not generally appreciated, it having been decided by those in authority that such results were to be kept classified. Bayes's Theorem was still seen as suspect: "'Bayes' still meant equal priors and did not yet mean making inferences, conclusions, or predictions based on updating observational data" (p. 87).

Part III of McGrayne's book records "the glorious revival". The first chapter here is devoted to the work of Arthur Bailey, who made extensive and important use of Bayes's Theorem in his work on credibility in casualty insurance as chief actuary at the New York State Insurance Department. His reason for so doing was made quite clear: "It

[1] *References to page numbers without a citation are to the book being reviewed.*

will be realized that all of the problems in which credibilities are used are problems in statistical estimation" ([1, p. 8]).

Chapter 8 details the entry of Bayesian methods into medical research, chiefly through the efforts, McGrayne claims, of Jerome Cornfield. The matter treated here is the connection of smoking with lung cancer and, later, heart disease. Small studies had been carried out before the Second World War, but the retrospective large study [3] published by Richard Doll and Austin Bradford Hill in 1950 gave firm (conclusive?) evidence of the connection between cigarette smoking and carcinoma of the lung. Doll and Hill, while identifying a seeming association, were careful to write that, "This is not necessarily to say that smoking causes carcinoma of the lung" ([3, p. 746]). In 1962, extending his important work, Cornfield studied the most critical risk factors for cardiovascular disease.

Bayesianism was later to be of further use in the medical context. For example, in the 1990s it was shown that mass screening of a large population for a rare disease (AIDS and HIV) would be unprofitable. As another example, while the test for the presence in men of high blood levels of prostate-specific antigen may be very efficient in identifying men who actually have prostate cancer, the disease is relatively so rare that in many, if not most, cases in which positive test results are returned the patient is found not to have cancer.

One of the most important situations in which Bayesianism scores heavily over frequentism is when there has been no past experience to draw on. (Here one might mention Good's paper [6] on the estimation of the population frequencies of species. Based on work done at Bletchley Park during the war, the paper was written in this vein to avoid Good's transgressing the Official Secrets Act). In Chapter 9 McGrayne looks at one important issue: the probability of a thermonuclear device exploding by mistake. While information was sparse, data were available on "the more dramatic incidents" (p. 122) involving nuclear weapons between 1950 and 1958. Results of such studies showed that there was certainly a positive probability of an accident, and some of the safety features suggested by the researchers Fred Charles Iklé and Albert Madansky were eventually implemented by the military.

While Bayesian and frequency methods sometimes result in the same conclusions, this is not always the case, and in [4] Edwards, Lindman, and Savage showed convincingly that in even moderate sample sizes Bayesian methods (with reasonable priors) and frequency p-values could lead to widely differing conclusions.

As the century progressed, Bayesian methods became more widely used. In the business field the prime developers were Howard Raiffa and Robert Osher Schlaifer. The emphasis was now switching from (merely) analyzing data to making decisions, and McGrayne notes that "some business Bayesians even dropped the prior odds called for by Bayes's rule" (p. 149). Whether such a move would be accepted as Bayesian might in fact be denied by many: Jeffreys, for instance, "was interested in making inferences from scientific evidence, not in using statistics to guide future action. To him, decision making …was irrelevant" (p. 57).

Chapter 12 deals mainly with the investigation into the authorship of some papers published in *The Federalist* in 1787 and 1788. Twelve of these, published anonymously, were known to be by Alexander Hamilton or James Madison. McGrayne details the intensive undertaking by Mosteller and Wallace in the 1950s to attribute these papers to their correct authors, a task that took a decade's worth of work on word frequencies, the identification of "markers" (e.g., the presence of "while" or of "whilst") and Bayesian analysis. The prior odds were found, surprisingly, to be of little effect, and Mosteller and Wallace showed in [7] that, with "satisfyingly fat" odds, all twelve papers could be attributed to Madison.

John Tukey's use of Bayesian methods in psephology in the United States is dealt with in Chapter 13. While irritated by the "lack of methodology for quantifying Bayes' initial prior" (p. 168), Tukey (perhaps unstatedly) used a form of Empirical Bayesianism, with past election results being used to construct the prior. The analysis, carried out during several elections, presented forecasts of an election's final outcome as the results were coming in to NBC and turned out to be surprisingly accurate.

In the 1970s Norman Rasmussen undertook a study into the safety aspects of the nuclear power industry. A report issued in 1974 contained what was clearly a Bayesian investigation using probability distributions about equipment failure rates and human error. The report seemed to be of little effect: in 1979 the core of the Three Mile Island nuclear-generating Unit 2 was severely damaged. The Rasmussen report, McGrayne notes, now seemed prescient (p. 180).

Not all studies initially conceived as being ideally treated by Bayesian methods were in fact so eventually carried out. For example, on January 17, 1966, an American B-52 jet carrying four nonactivated bombs disintegrated in the air while refueling over the southeastern coast of Spain. While three of the bombs were found within twenty-four hours, the fourth proved more elusive.

John Craven, called in to handle the recovery operation, set up seven hypotheses (to which priors were assigned) as to what could have happened to the missing bomb—e.g., it had fallen free and would not be found in the debris of the plane; it had been carried far out to sea as a result of the deploying of both of the bomb's parachutes; a Spanish fisherman had seen the bomb fall into the sea.

Craven hired a team of mathematicians who soon discovered from their meetings with the military authorities that they had not been employed to use Bayes's Theorem and update their prior distribution. What was wanted was not an expectation that the bomb could be found: rather, "If the H-bomb was not found, the navy wanted to be able to prove statistically that it was not there" (p. 188). The prior probabilities of the hypotheses were never updated, despite data resulting from a search of the ocean floor from a ship. Fortunately optimal search methods eventually resulted in the desired finding of the bomb.

The final section of the book begins with a chapter in which a number of cases in which Bayesian methods have been used are fairly briefly discussed, cases in which the large amount of experimental data involved urged the development of computers and techniques (including efficient numerical integration procedures) that could handle such quantities of results. Long and difficult integrations, it was later realized, could be replaced by suitable sampling, an observation that led to the development of Markov chain Monte Carlo methods. Here a class of algorithms is taken for sampling from a probability distribution and based on the construction of a Markov chain having a desired equilibrium distribution. As a sample of the desired distribution, the state of the chain after a large number of steps is used.

In 1983 the U.S. Air Force sponsored a review of the estimates made by NASA of the probability of a shuttle failure. Using Bayesian methods, the researchers concluded that the odds on a failure were 1 in 35, NASA's estimate being 1 in 100,000. On January 28, 1986, the shuttle *Challenger* exploded on launch, a bitter vindication of Bayesianism.

The reader of *The Theory That Would Not Die* will realize that little is said here of prior distributions *per se*: most of the applications are concerned with the gathering of data and subsequent prognostication. Nevertheless, the effect of the prior generally diminishes with an increase in the amount of data.

McGrayne's giving the references in separate sections for each chapter is perhaps unfortunate. Sometimes things "fall through the cracks": for instance, Note 10 to Chapter 16 refers one to "Smith (1984)", an item not listed in the bibliography to that chapter. Also, the system adopted makes it difficult to find a reference only vaguely remembered (was it in the list for Chapter N or Chapter M?).

For the student who is being exposed to Bayesian statistics for the first time, McGrayne's book provides a wealth of illustrations to whet his or her appetite for more. It will broaden and deepen the field of reference of the more experienced statistician, and the general reader will find an understandable, well-written, and fascinating account of a scientific field of great importance today.

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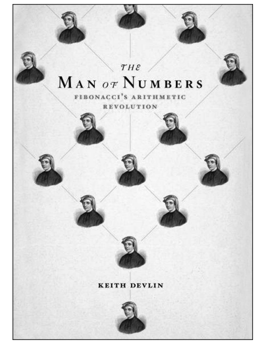
The Man of Numbers: Fibonacci's Arithmetic Revolution

Reviewed by John Hannah



The Man of Numbers: Fibonacci's Arithmetic Revolution
Keith Devlin
Walker and Company, 2011
US$25.00, 192 pages
ISBN-10: 0802778127
ISBN-13: 978-0802778123

The Fibonacci numbers are among the most popular topics in mathematics—appealing to students, mathematics enthusiasts, and research mathematicians as well. As Devlin points out in the book under review (p. 143), their simple recurrence relation

$$a_{n+1} = a_n + a_{n-1}$$

makes them an ideal context in which school children can practice arithmetic in the decimal place-value system. They also lend themselves to attractive pictorial representation, either through tilings of the plane (Figure 1) or through their connection to the golden ratio.

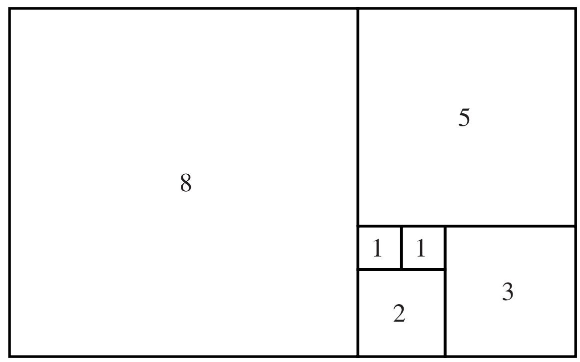


Figure 1. A tiling using successive Fibonacci numbers.

By the time students come to university, textbook authors can assume familiarity with the Fibonacci numbers, using them as a basis for friendly examples in a diverse range of topics, including linear algebra, numerical methods, and engineering mathematics.

But who was Fibonacci? What did he actually do? Where does his work fit into the story of the development of mathematics?

What we know about Fibonacci as a person can be stated quite briefly. The name Fibonacci is actually more of a nickname, conferred on him six hundred years later by the historian Guillaume Libri. His real name, Leonardo Pisano, tells us that he came from Pisa in northern Italy. In the dedication at the start of the 1228 version of his most famous book *Liber Abbaci*, Fibonacci offers a few precious details of his life:

> When my father, appointed by his homeland, held the post of public notary in the custom-house of Bejaia [on the coast of modern Algeria] for the Pisan merchants frequenting it, he arranged for me to come to him when I was a boy and, because he thought it would be useful and appropriate for me, he wanted me to spend a few days there in the *abbaco* school, and to be taught there. Here I was introduced to that art [the *abbaco*] by the wonderful kind of teaching that used the nine figures of the Indians. Getting to know the *abbaco* pleased me far beyond all else and I set my mind to it, to such an extent that I learnt, through much study and the cut and thrust of disputation, whatever study was devoted to it in Egypt, Syria, Greece, Sicily and Provence, together with their different methods, in the course of my subsequent journeys to these

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places for the sake of trade. (Translation by Burnett in [2, p. 87].)

Fibonacci's reputation led to an invitation to a mathematical challenge at the court of the Holy Roman emperor Frederick II (he wrote up some of the problems and their solutions in another book *Flos*) but the only other official acknowledgment of his life that we know about is a decree of the Republic of Pisa in 1240 awarding an annual salary of twenty pounds plus expenses for services rendered.

Although we know only a little about Fibonacci's life, we do at least seem to have a significant fraction of his mathematical output [13, p. 605]. His most substantial writings are

- *Liber Abbaci*, an extensive account of computation in the decimal system, along with commercial arithmetic, problem solving and second-degree algebra,
- *Practica Geometriae*, a book about practical and theoretical geometry, and
- *Liber Quadratorum*, the book of squares, described by Vogel as "a first-rate scientific achievement" showing Fibonacci "as a major number theorist" [13, p. 610].

Vogel's *Dictionary of Scientific Biography* article [13] gives an excellent, but necessarily brief, overview of all these works. However, until quite recently, it has been quite difficult for English-speaking readers to get firsthand experience of Fibonacci's works. The books have come down to us in a handful of Latin manuscripts that lay ignored in European libraries until being published by Boncompagni in the nineteenth century [1]. English translations of the above three books have appeared only in the last twenty-five years [7], [10], and [11].

Unfortunately, even with these translations now available, there are still significant hurdles facing the curious mathematician (or lay reader) wanting to know what Fibonacci actually did. Foremost among these is the lack of any symbolism. Problems and solutions are presented entirely in words. Moreover, this isn't always in the most elegant English, as some translators eschew style in favor of an almost literal word-for-word correspondence between the original Latin and the final English. Thus, even when Fibonacci is doing something we might find familiar, like using an efficient elimination method to solve a linear problem involving four unknowns [11, pp. 415–417], it can be difficult to understand what the problem is asking or which method Fibonacci is using, and the calculation takes two pages of solid text. But there are lots of interesting things to be found, not just isolated examples like the famous rabbit problem that gives rise to the Fibonacci numbers [11, pp. 404–405], but also extended treatments of topics that we might now see as early developments in solving systems of linear equations [12] or early acknowledgment of negative solutions [9]. Even when he uses methods that we now see as obsolete, such as simple false position, we recognize a fellow mathematician as he seeks proofs for techniques that seem to have been discovered experimentally [3].

With so much raw material, we can be fairly confident that we know what mathematics Fibonacci actually did, even if further study is needed for us to fully appreciate it. Fibonacci's role in the development of mathematics is harder to describe with any confidence, whether we are interested in his dependence on potential sources or his influence on those who followed him. Our understanding of these influences is handicapped by the medieval custom of not acknowledging sources, so that writers borrowed freely from whichever available manuscripts seemed relevant to their topic. Thus to identify Fibonacci's sources we have to fall back on comparisons of his texts with earlier texts that share common families of problems. On this basis, it is generally accepted that Fibonacci knew about the algebra of al-Khwārizmī and Abū Kāmil [13, p. 611] from the ninth and early tenth centuries, but that it is unlikely that he knew Al-Karajī's work in the late tenth century. Indeed, his knowledge of Arabic mathematics seems to be over two hundred years out of date [8]. Similar comparisons of Fibonacci's texts with later texts have led to the view that, in his introduction of decimal arithmetic and its applications to commercial life, he became the teacher of the masters of computation and surveyors over the next two or three hundred years [13, p. 612]. However, sharing families of problems need not indicate dependence on one another, but merely dependence on some other common source, and so Høyrup has even been able to argue that Fibonacci may have been recording an already existing culture, rather than starting a new one [6]. On the other hand, Fibonacci's more advanced work on number theory was essentially lost, with comparable material not appearing in European mathematics until the rediscovery of Diophantus in the late fifteenth century and Fibonacci's own writings not becoming well known until their publication by Boncompagni in the nineteenth century [1].

Clearly there is material enough here for several books about Fibonacci and, of course, there are several different audiences such books could be aimed at. In the introduction to his book (Chapter 0: Your days are numbered), Devlin pitches his tent in the popular market, seeking readers who are well-educated lay people rather than, say, expert mathematicians or historians. He reminds us about the ubiquity of numbers in modern life and about the decimal representation of numbers that we now take for granted. It is good to stop occasionally to think about things we take for granted, and this book is going to do that for the decimal system. Where did this system come from? Or more precisely, how did the decimal system reach the West? Devlin identifies Fibonacci, and more particularly his *Liber Abbaci*, as the key influence in the development of this aspect of modern culture, ranking him alongside Copernicus and Galileo (p. 5) in terms of revolutionizing our world. After a brief account of Fibonacci and *Liber Abbaci*, to be expanded later in the book, Devlin admits that the lack of appropriate sources makes a straight chronicle of Fibonacci's life impossible, and so this book will have to focus mainly on what he did and his modern legacy.

Chapter 1 begins with a digression on titles, examining the meaning of *abbaci* as used by Fibonacci and of *Fibonacci* as a nickname for Leonardo of Pisa. The main theme of the chapter, though, is a very brief history of number systems, from prehistoric tally marks through Roman numerals and on to the Hindus' positional decimal system. Hindu-Arabic numerals reached Europe long before Fibonacci (p. 21), with a Spanish monk, Vigila, reporting the following in 976:

> We must know that the Indians have a most subtle talent and all other races yield to them in arithmetic and geometry and the other liberal arts. And this is clear in the 9 figures with which they are able to designate each and every degree of each order (of numbers).[1]

These early appearances of Hindu-Arabic numerals seem to have been merely as labels for counters on an abacus board. For Devlin, it was Fibonacci who first realized the practical significance of these nine figures in the world of commerce.

As already mentioned, we have very little biographical information about Fibonacci. But the culture he lived in has left much other evidence behind, and Chapters 2 and 3 give a nice description of life in Pisa and of the role of Pisa in Mediterranean commerce during Fibonacci's lifetime, along with a tentative description of his education both in Pisa and in Bejaia. The central spread of eight pages of illustrations includes some color photos of modern Pisa that helped bring these descriptions to life, for me at least, but placing Pisa in a geographic and political context might have been a bit easier if Devlin had included one or two maps.[2] An assertion here that Fibonacci wrote *Liber Abbaci* primarily for merchants (p. 35) may mislead some readers, although Devlin corrects himself later in the book, admitting that some sections were aimed more at scholars (p. 87) and that the dense pages of *Liber Abbaci* were too challenging even for some abbacus authors (p. 120).

In Chapter 4 Devlin discusses Fibonacci's possible sources for *Liber Abbaci*, particularly the initial chapters introducing decimal arithmetic and, at greater length, the final section of the last chapter, dealing with algebra. Much of the discussion focuses on al-Khwārizmī and his work, and there is a good description of the techniques of *al-jabr* (or restoration) and *al-muqabala* (or balancing) that constituted algebra in the ninth century and that eventually gave algebra the name it now has.

Chapter 5 begins with a brief summary of *Liber Abbaci* and then discusses in more detail some typical problems. It would always be difficult to summarize such a large book (over 600 pages) in a mere twenty-six pages, but surprisingly little is said about the aspects that are supposed to have revolutionized our world: the decimal system and its adoption in commercial arithmetic. If *Liber Abbaci* did indeed change the way merchants worked, then it must surely be because of the chapters about arithmetic and about its applications to pricing and barter, to sharing company profits, and to the value of coins alloyed from copper and silver (Chapters 1 to 11). These chapters are rich in social detail (particularly regarding commodities being traded and their ports of origin) but relatively lightweight in terms of mathematical content, with the main tool being the proportional thinking that gives rise to the rule of three. At first sight these chapters consist solely of recipes to be learned by doing repeated examples, but Fibonacci seems to be teaching for understanding as he follows his recipes by explanations in the style of Euclid. He also makes links between apparently different methods, drawing parallels, for example, between companies and alloys [11, p. 242] that might confuse more literal minded students.

The later material in *Liber Abbaci* (Chapters 12 to 15) has always attracted more attention from mathematicians, probably because it is here that we can imagine seeing the origins of modern ideas like negative numbers or symbolic algebra or methods for solving systems of linear equations. Devlin focuses on a few methods which recur throughout

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[1] *Quoted without source, p. 21. Occasionally Devlin forgets to credit sources for his quotes, even though these sources would sometimes be quite good "Further Reading" for interested readers. In this case a quick search in Google found* [5].

[2] *This deficiency is of course easily remedied by using Google.*

these chapters: the rule of three, the methods of false position and double false position, and the direct method (a kind of rhetorical algebra for linear problems). These are explained in modern terms but he also lets Fibonacci speak for himself, with excerpts usually taken from Sigler's rather stilted translation [11].[3]

Chapter 6 deals with the episode at the court of Frederick II. In the absence of direct eye witness accounts, the description has to rely on averted vision: who was Frederick and what did he do? How did Fibonacci report the mathematical problems and their solutions later on in *Flos*? For the reader interested in Fibonacci's mathematics, a highlight of this chapter is an extended excerpt (pp. 95–97) showing how he used the direct method to solve an indeterminate linear problem in four unknowns. Without symbols this is heavy going, but Devlin offers a symbolic translation that clarifies the method. It is awe inspiring to see how far medieval mathematicians could progress using such primitive tools.

The last three chapters of Devlin's book look at how Fibonacci might have influenced later generations. Chapter 7 looks at the spread of abbacus texts and abbacus schools in the three centuries following Fibonacci's death, looking for traces of his influence in terms of shared problems or similar syllabi. Chapter 8 reports recent work by Franci [4] that tries to make a stronger case for direct borrowing from Fibonacci. Unfortunately Franci's argument depends on us knowing the content of a long lost shorter version of *Liber Abbaci*, written by Fibonacci specifically for merchants. Franci deduces this content by applying "medieval literary fornesics" (p. 138), a careful comparison of crucial manuscripts from the years immediately after Fibonacci's death. But some scholars remain sceptical that such a direct link has been proven [6].

All this might be described as Fibonacci's hidden heritage, his contribution to our modern culture that was borrowed without acknowledgment and absorbed into a fabric that we now take for granted. This includes not just the use of decimal arithmetic and its application to commerce, but also the beginnings of algebra. The final chapter looks at Fibonacci's more visible heritage. It begins with a short account of the Fibonacci numbers and his relatively minor role in their story. But the chapter also gives the stories of two rather more tangible reminders of Fibonacci: first, a commemorative statue erected in Pisa in the nineteenth century and second, the precious surviving manuscript copies of his texts. Perhaps the sensual highlight of this book is the inclusion of three color photos of pages from these manuscripts. This is as close as we will ever get to hearing Fibonacci speak in his own words.

By the end of the book, Devlin has told a good story. There isn't a lot of mathematical detail, just enough to give a flavor of Fibonacci's thinking. A math-phobe could skip these bits, but realistically the target audience is probably someone with a good grasp of high school algebra. Mathematicians will enjoy the book, but probably the best people to give it to would be keen mathematics students, whether at senior high school level or at undergraduate level. It will open their eyes to an important part of mathematics that we all take for granted.

[3] *An exception occurs (pp. 77–78) where he seems to have preferred my own translation of the archetypal false position problem* [3, p. 309]. *Naturally I am quite pleased, if this is the case, but it is unacknowledged and it would have been nice to see this paper included in the bibliography for readers who wanted more detail about Fibonacci's treatment of the method.*

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2012 Mathematics Programs That Make a Difference

Each year the AMS Committee on the Profession (CoProf) selects outstanding programs to be designated as Mathematics Programs That Make a Difference. For 2012 CoProf selected the Mathematical Sciences Research Institute in Berkeley for its programs promoting diversity in the mathematics profession.

CoProf created the Mathematics Programs That Make a Difference designation in 2005 as a way to bring recognition to outstanding programs that successfully address the issue of underrepresented groups in mathematics. Each year CoProf identifies one or two exemplary programs that:

1. aim to bring more individuals from underrepresented minority backgrounds into some portion of the pipeline, beginning at the undergraduate level and leading to an advanced degree in mathematics, or to retain them in the pipeline;
2. have achieved documentable success in doing so; and
3. are replicable models.

Previously designated Mathematics Programs That Make a Difference are: the graduate program at the University of Iowa and the Summer Institute in Mathematics for Undergraduates/Research Experience for Undergraduates at Universidad de Puerto Rico, Humacao (2006); Enhancing Diversity in Graduate Education (EDGE) and the Mathematical Theoretical Biology Institute (2007); the Mathematics Summer Program in Research and Learning (Math SPIRAL) at the University of Maryland and the Summer Undergraduate Mathematical Science Research Institute at Miami University (Ohio) (2008); the Department of Statistics at North Carolina State University and the Department of Mathematics at the University of Mississippi (2009); the Department of Computational and Applied Mathematics at Rice University and the Summer Program in Quantitative Sciences, Harvard School of Public Health (2010); and the Center for Women in Mathematics at Smith College and the Department of Mathematics at North Carolina State University (2011).

Citation

Be it resolved that the American Mathematical Society and its Committee on the Profession recognize the Mathematical Sciences Research Institute (MSRI) for its significant efforts to encourage students from underrepresented groups to continue in the study of mathematics.

The program at MSRI has been a leader for many years in efforts to promote diversity in mathematics. Their research workshops showcase the accomplishments of women and encourage their participation in mathematical research. MSRI coordinates career development and outreach programs for all levels, from K–12 teachers to full professors. The conference for African American Researchers in the Mathematical Sciences was founded at MSRI in 1995. This conference has been credited by many African American graduate students as providing the spark and mentoring needed to carry them through to the Ph.D.

The AMS commends the administration at MSRI for their high level of commitment and successful efforts to improve diversity in the profession of mathematics in the United States.

About the Program

MSRI is known and appreciated by mathematicians the world over as one of the largest and most active mathematics research institutes. What might be less well known is that MSRI has also been a leader in promoting diversity in mathematics. By devoting serious attention and resources to diversity issues, MSRI has not only made a difference with the programs and events it has organized but has also sent a powerful signal within the mathematical community that these issues matter.

When he served as MSRI director, William Thurston established in 1993 the Human Resources Advisory Committee (HRAC), making MSRI the first mathematics institute in the U.S. to have such a committee. Today, nearly all U.S. institutes have one. This committee helps to implement MSRI's policy to actively seek diversity in the gender and ethnicity of program and workshop participants. While proposals for MSRI programs are ultimately selected on the basis of scientific merit, every proposal must include a section that addresses diversity issues. Once a program has been accepted, an HRAC liaison is appointed to help the program committee keep diversity issues on its agenda. The liaison need not be an HRAC member but must work in an area close to the program topic. In addition, the HRAC reviews the proposed composition of program organizing committees and might suggest additional members who can reach out to underrepresented groups. Program committees are urged to pay careful attention to ensuring diversity in the program participants right from the start of the planning for the program.

MSRI has several established activities aimed at groups underrepresented in mathematics. One of these is the Conference for African American Researchers in the Mathematical Sciences (CAARMS), which MSRI first organized in 1995. Now an annual event, CAARMS has been held several times at MSRI, as well as at universities and other mathematics institutes. The conference showcases current mathematical sciences research done primarily, though not exclusively, by African Americans. Another highly successful and visible event is the biennial Blackwell-Tapia Conference, which is named after the African American statistician David Blackwell and the Hispanic mathematician Richard Tapia. First held in 2000 and sponsored jointly by MSRI and Cornell University, the conference provides a mix of activities designed to inform the next generation of students about career opportunities in mathematics and to provide a chance for them to network with other students and with mathematical scientists who play a leadership role in their communities. The 2002 Blackwell-Tapia Conference marked the first awarding of the Blackwell-Tapia Prize, which honors a mathematical scientist who has contributed significantly to his or her field of expertise and who has made efforts to address the underrepresentation of minorities in mathematics.

For some time the HRAC had been discussing how to increase participation by minority mathematicians in the institute's programs. Growing out of those discussions are the Modern Mathematics Workshops, which began in 2003. The idea is to have organizers of upcoming MSRI research programs present expository introductions to the areas of the programs. The workshops are open to all but are especially aimed at mathematicians and students from underrepresented minorities, with the goal of sparking their interest in MSRI programs and encouraging them to participate. The workshops were initially sponsored by MSRI and held at minority-serving institutions. Since 2006 they have been held just prior to the annual conference of SACNAS, the Society for the Advancement of Chicanos and Native Americans in Science. In addition, the workshops are now sponsored by all the U.S.-based mathematics institutes, with each institute bringing in speakers to describe its upcoming programs. The workshop reaches a diverse audience of graduate students, postdocs, and midcareer faculty. Most recently, MSRI played a key role developing a collaboration among eight mathematics institutes to run a coordinated series of workshops related to mathematics and diversity. The list of these workshops may be found at `http://www.mathinstitutes.org/diversity.php`.

In addition to the conferences and workshops that occur on a regular basis, MSRI has organized various one-off events that put the spotlight on diversity issues. One of the most important of these was Promoting Diversity at the Graduate Level in Mathematics: A National Forum, held in October 2008. The purpose of the forum was to stimulate, identify, and disseminate successful models that improve retention of underrepresented groups in graduate programs in mathematics. With 130 participants, 20 percent of them graduate students, the forum reached a wide range of individuals across the spectrum of the mathematical sciences, allowing them to share experiences and information and discuss concrete ways to implement diversity programs at their own institutions. The proceedings of the forum appear in a 28-page booklet available on the MSRI website. An article about the forum, "Revisiting the Question of Diversity: Faculties and Ph.D. Programs", by H. G. Grundman, appeared in the October 2009 issue of the *Notices*.

Along with its efforts aimed specifically at minorities underrepresented in mathematics, MSRI also has paid significant attention to encouraging women in mathematics. In 2005 the institute started a two-day workshop called Connections for Women, and this workshop has now become an established event associated with each MSRI research program. The purpose of the workshop is to bring together the women who will participate in the program, as well as others from outside MSRI with the same scientific interests, so that they can get to know each other and start forming a working cohort. Another important effort has been MSRI's graduate summer schools. MSRI's sponsoring institutions can nominate two students per summer to attend the school or three if at least one is a woman or a member of an underrepresented group. As a result, MSRI has maintained female representation of 30 percent in its summer schools for the last seven to eight years. This figure is in line with the proportion of women receiving Ph.D.'s in mathematics in the U.S., which has been around 30 percent for the last ten years.

The MSRI Undergraduate Program, called MSRI-UP for short, aims to identify talented students, especially those from underrepresented groups, who are interested in mathematics. Although MSRI-UP shares some similarities with Research Experiences for Undergraduates programs, it is more comprehensive, providing not only research opportunities but also skills and information needed to navigate the path from the math major to graduate school to a career in research. Since its inception in 2006, MSRI-UP has reached over eighty minority students and will likely have a significant impact on the face of the mathematics profession in the future.

By putting its prestige and expertise at the service of groups underrepresented in mathematics, MSRI has helped to make the mathematical community more stimulating and diverse, one that welcomes a wide range of individuals and helps them participate and thrive.

2012 Award for an Exemplary Program or Achievement in a Mathematics Department

The Award for an Exemplary Program or Achievement in a Mathematics Department was established by the AMS Council in 2004 and was given for the first time in 2006. The purpose is to recognize a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community internally or in relation to the rest of society. Departments of mathematical sciences in North America that offer at least a bachelor's degree in mathematical sciences are eligible. Through the generous support of an anonymous donor, the award carries a cash prize of US$5,000.

The award is presented by the AMS Council acting on the recommendation of a selection committee. For the 2012 award, the members of the selection committee were: Carlos Castillo-Chavez, Annalisa Crannell, Phil Kutzko (chair), Suzanne M. Lenhart, and Francis E. Su.

The previous recipients of the award are Harvey Mudd College (2006), the University of California, Los Angeles (2007), the University of Iowa (2008), the University of Nebraska, Lincoln (2009), North Carolina State University (2010), and the Math Center at the University of Arizona (2011).

The recipient of the 2012 Award for an Exemplary Program or Achievement in a Mathematics Department is the Department of Mathematics at Bryn Mawr College. What follows is the selection committee's citation.

Citation

The American Mathematical Society is pleased to recognize the Department of Mathematics at Bryn Mawr College with the 2011 Award for an Exemplary Program or Achievement by a Mathematics Department. The math department at Bryn Mawr has a long history of encouraging women to pursue careers in mathematics and, more generally, of providing women with an environment in which they may thrive in this pursuit. Indeed, it was Bryn Mawr that Emmy Noether chose as her new home when she was forced to leave her position in Germany in 1933, a choice that is emblematic of the role that the department played in those difficult days. In the 1970s the department made the decision to encourage all students to take as much mathematics as they can, regardless of their ability or career path, while at the same time providing the students with the support they need to be successful. During the 1990s roughly 5 percent of Bryn Mawr graduates majored in math; during the period 2001–2010 the rate rose to almost 9 percent. To put this in perspective, this percentage is 11 times the national average for the percentage of women graduating with an undergraduate degree in mathematics and places Bryn Mawr second among all colleges and universities in the U.S. Although the department has substantially increased Bryn Mawr's percentage of math majors, it has maintained its tradition for sending its students on to doctoral programs. The department ranks 10th nationally in the percentage of female graduates going on to earn the Ph.D. in mathematics and, in the past three years, has produced four of the few female winners of the NSF Graduate Fellowship in mathematics. Bryn Mawr is known nationally as a college with a tradition of inclusiveness and the math department exemplifies this. Over the last five years almost 10 percent of the department's graduates are from African American or Hispanic backgrounds.

As is typical with Exemplary Program awardees, these achievements came about as a result of careful planning and great dedication. The department has worked to create a vertically integrated mathematical community. For example, students in the department's small Ph.D. program play a supporting role in advanced undergraduate courses. As a result, fully 4 percent of the Bryn Mawr student body took real analysis last semester. Community is also encouraged by a variety of activities which range from colloquia and math clubs to more unusual activities such as the Math Shakespeare Reading Group and the weekly Distressing Math Collective at which, "…upper class students and graduate students give presentations and lead discussions on distressingly weird math problems." In addition to building community, the department has carefully considered the role of its curriculum, continually assessing its performance and making adjustments as warranted. For example, the differential equations course has been revised to include a dynamical systems approach focusing on mathematical models of environmental issues.

Given the passion with which the Bryn Mawr math faculty has built this exemplary program, it will come as no surprise that they are leaders nationally. Rhonda Hughes, working with Sylvia Bozeman, Professor of Mathematics at Spelman College, founded the highly successful and award-winning EDGE program, a program that supports women and minorities as they make the transition from undergraduate to graduate school. Other faculty have taught in the EDGE program or at the IAS Women and Mathematics Program. In sum, this small faculty, consisting of eight permanent members, has played a national role well out of proportion to its size.

The purpose of the Award for an Exemplary Program or Achievement by a Mathematics Department is to recognize a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. The department at Bryn Mawr, by playing such a prominent and effective role, both at Bryn Mawr and nationally, in the effort to increase the number of women and students from underrepresented backgrounds who major in math certainly meets these criteria. Further, the department at Bryn Mawr provides a model that may be implemented at other departments with similar demographics and missions. It is for these reasons that the department is so deserving of this award.

DOCEAMUS

—***doceamus*** . . . let us teach

Inquiry-Based Learning: Yesterday and Today

Ronald G. Douglas

A little more than fifty years ago, I entered a classroom at the Illinois Institute of Technology (IIT), where I was a freshman, and had an experience that changed my life. The professor, Pasquale Porcelli, was teaching calculus using an inquiry-based approach I learned later was called the Moore Method. In the class we were being asked to develop rigorously the basic concepts of calculus, together with the theorems, along a carefully crafted path developed by him. Moreover, we had to present and defend our ideas to the rest of the class.

Over the remainder of my studies at IIT, about half of my mathematics classes were taught in this manner and the other half by the traditional lecture method. Upon graduation I followed Porcelli to Louisiana State University (LSU), where I obtained my doctorate under his direction. Again, about half of my studies at LSU followed the Moore Method approach and half by way of lectures. What were the benefits of the inquiry-based approach to learning and why do I say it changed my life?

While I had been interested in mathematics before, these Moore Method courses prompted me to decide not only to major in mathematics as an undergraduate but also to pursue a doctorate. Further, I found that I understood the basics in most areas and how to approach questions so well that I had a head start in learning new mathematics as well as in doing research. But that was not all. A mastery of clear, critical thinking carried over to

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Members of the Editorial Board for Doceamus *are: David Bressoud, Roger Howe, Karen King, William McCallum, and Mark Saul.*

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other parts of my life, including my administrative career. Knowing how to approach problems and defend the value and correctness of my insights and analyses were invaluable in my roles as department chair, dean, provost, and executive vice president, and in a leadership position in the wider mathematics community.

But why am I relating to you this "ancient history" concerning my life and career? After all, it is not surprising that a mathematically talented student such as I was might thrive in the atmosphere of a Moore Method classroom. Let me tell you why.

Several years ago, because of this background, I joined a project to develop and promote approaches to mathematics instruction closely related to the Moore Method—methods which are called inquiry-based learning or IBL. While related in spirit to the Moore Method, IBL is broader and can be characterized as a student-centered approach that focuses on solving, communicating, and discussing meaningful mathematics problems in a purposeful sequence and, further, to construct and understand mathematical concepts. (And constructing a purposeful sequence is critical.)

After several years of effort, the IBL project decided that an impartial, objective evaluation and assessment was needed to determine the effects of IBL at the four universities where the project was focused. The data collected have been tabulated and analyzed, and the results are now available.[1] They did not surprise or disappoint. First and foremost, there were positive benefits for almost all students. However, two findings were particularly welcome and we believe important.

Women students reported a more positive experience and much greater learning gains in the IBL courses as compared to lecture courses. In particular, whereas the gains in IBL courses were largely independent of gender, that was not the case in lecture courses. Moreover, the same was true for students whose previous performance in mathematics was below average. These students reported a more positive experience and developed a better understanding of mathematics. Finally, we did not see the increasingly negative attitude toward mathematics that was formed by students in the lecture courses!

What lessons can be drawn from these data? I am not suggesting that the data support that all mathematics be taught using an IBL approach. However, having some IBL courses, particularly early in their studies, seems to provide a more positive experience for women students and improves their attitude toward mathematics. It also seems to help below-average-achieving students catch up or, at least, perform at a level which would enable them to go on in mathematics-related disciplines.

Now I am not certain my experience at IIT and LSU can be used to evaluate an IBL approach for current students. That is why the evaluation/assessment study is so important. We all seek to use our own experience as a guide to understand what we see in today's classrooms. Sometimes this works and sometimes it doesn't.

Given the great penchant of today's students to communicate their thoughts with each other in "real time", it is not clear to me how that fits with the lecture method. And I doubt that their intense desire to interact with all information sources, as their blogs and constant use of ubiquitous social media make clear, supports the lecture approach either. In particular, it seems to me that an IBL approach would resonate more strongly with today's students.

In evaluating and assessing the various approaches to mathematics instruction, we must be careful that we agree on the goals we have for them. Learning mathematics is certainly most important. But developing a positive attitude toward mathematics and acquiring the ability to learn and use mathematics are key in a student's future studies and career.

In conclusion, I believe the assessment study strongly supports the notion that to achieve these goals for today's students, experiencing an IBL approach in several mathematics courses is superior to a program based only on lectures.

[1] http://www.colorado.edu/eer/research/steminquiry.html.

of its curriculum, continually assessing its performance and making adjustments as warranted. For example, the differential equations course has been revised to include a dynamical systems approach focusing on mathematical models of environmental issues.

Given the passion with which the Bryn Mawr math faculty has built this exemplary program, it will come as no surprise that they are leaders nationally. Rhonda Hughes, working with Sylvia Bozeman, Professor of Mathematics at Spelman College, founded the highly successful and award-winning EDGE program, a program that supports women and minorities as they make the transition from undergraduate to graduate school. Other faculty have taught in the EDGE program or at the IAS Women and Mathematics Program. In sum, this small faculty, consisting of eight permanent members, has played a national role well out of proportion to its size.

The purpose of the Award for an Exemplary Program or Achievement by a Mathematics Department is to recognize a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. The department at Bryn Mawr, by playing such a prominent and effective role, both at Bryn Mawr and nationally, in the effort to increase the number of women and students from underrepresented backgrounds who major in math certainly meets these criteria. Further, the department at Bryn Mawr provides a model that may be implemented at other departments with similar demographics and missions. It is for these reasons that the department is so deserving of this award.

DOCEAMUS

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Inquiry-Based Learning: Yesterday and Today

Ronald G. Douglas

A little more than fifty years ago, I entered a classroom at the Illinois Institute of Technology (IIT), where I was a freshman, and had an experience that changed my life. The professor, Pasquale Porcelli, was teaching calculus using an inquiry-based approach I learned later was called the Moore Method. In the class we were being asked to develop rigorously the basic concepts of calculus, together with the theorems, along a carefully crafted path developed by him. Moreover, we had to present and defend our ideas to the rest of the class.

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Over the remainder of my studies at IIT, about half of my mathematics classes were taught in this manner and the other half by the traditional lecture method. Upon graduation I followed Porcelli to Louisiana State University (LSU), where I obtained my doctorate under his direction. Again, about half of my studies at LSU followed the Moore Method approach and half by way of lectures. What were the benefits of the inquiry-based approach to learning and why do I say it changed my life?

While I had been interested in mathematics before, these Moore Method courses prompted me to decide not only to major in mathematics as an undergraduate but also to pursue a doctorate. Further, I found that I understood the basics in most areas and how to approach questions so well that I had a head start in learning new mathematics as well as in doing research. But that was not all. A mastery of clear, critical thinking carried over to other parts of my life, including my administrative career. Knowing how to approach problems and defend the value and correctness of my insights and analyses were invaluable in my roles as department chair, dean, provost, and executive vice president, and in a leadership position in the wider mathematics community.

But why am I relating to you this "ancient history" concerning my life and career? After all, it is not surprising that a mathematically talented student such as I was might thrive in the atmosphere of a Moore Method classroom. Let me tell you why.

Several years ago, because of this background, I joined a project to develop and promote approaches to mathematics instruction closely related to the Moore Method—methods which are called inquiry-based learning or IBL. While related in spirit to the Moore Method, IBL is broader and can be characterized as a student-centered approach that focuses on solving, communicating, and discussing meaningful mathematics problems in a purposeful sequence and, further, to construct and understand mathematical concepts. (And constructing a purposeful sequence is critical.)

After several years of effort, the IBL project decided that an impartial, objective evaluation and assessment was needed to determine the effects of IBL at the four universities where the project was focused. The data collected have been tabulated and analyzed, and the results are now available.[1] They did not surprise or disappoint. First and foremost, there were positive benefits for almost all students. However, two findings were particularly welcome and we believe important.

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The New Publishing Scene and the Tenure Case: An Administrator's View

Daniele C. Struppa

When I received my Ph.D. back in 1981, the rules were simple. You published in peer reviewed journals (impact factors, though introduced by Thomson Reuters in the sixties, were not yet in everybody's vision), and that's about all you needed to know.

As I aged, I did not lose my passion for mathematics but became more involved with administration, becoming first chair, then dean, then provost, and now chancellor (I hope, like Palpatine, to be promoted to emperor soon). In this new capacity, I find myself in charge of the last and often key evaluation in the tenure process, and the many changes that have occurred recently have made this process more challenging than in the past. The case of mathematics, in particular, presents three new challenges that I will briefly discuss here.

Electronic Publishing

Though many journals now live in cyberspace (I rarely go to the library to read a paper; rather I do so at home on my computer), we are still able to offer a first rough evaluation of the value of a paper by looking at its placement. However, a new trend has emerged that makes this more difficult. In the last several years, the number of papers that are simply posted on arXiv, for example, has grown significantly, and it is not infrequent for a CV to contain papers that only appear on arXiv. Should one disregard them because they have not been vetted through the revered peer review process?

This seems hardly fair. After all, we are supposed to read the papers, not to blindly accept the referee's opinion. Those of us who have been involved in refereeing have plenty of horror stories to share. I once recommended a paper for publication, and the journal sent the authors a heavily redacted subset of my review, as an excuse not to publish the work. I have also seen many less than professional reviews, where a paper is accepted by somebody who clearly has not really read the paper or where a paper is rejected by somebody who is prejudiced against a certain field.

Thus we cannot count only on placement, and we need to actually read the works before us. Well, this is easy to say for us in mathematics, but (I hope you will forgive my arrogance) how can we expect a dean or provost from the humanities to actually read a paper in mathematics or in physics? So, the advice I would give to any faculty is to certainly use electronic databases such as arXiv (particularly useful to protect priority when working in a rapidly developing field), but I would also

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encourage authors to move their work steadily from arXiv to refereed publications in good quality journals. This brings us to the second issue.

Impact Factors and Other Metrics

When I was young, I sought publication in journals that I knew were reputable because I had read interesting papers in them. Thus, for example, I published my dissertation in the *Memoirs of the AMS* because that's where my scientific grandfather (Leon Ehrenpreis) had published his. Now, whenever I choose a publishing venue, I look at its metrics in MathSciNet. When I review tenure files, I see reports that include, for every journal, a variety of metrics, including, for the candidate, a citation list that tells me how many times his work has been cited. How relevant are those numbers? Taken individually, every one of those numbers is irrelevant. For example, it is very well known that impact factors can be manipulated and that they are not really suitable for individual evaluations. In this case, it has been said, the h-index (another measure based on citations and introduced also by Thomson Reuters) is a more useful instrument. In my opinion, one has to exercise caution and prudence. A publication in a journal with very low impact factor should not condemn a tenure case, just as a single publication in a high impact factor journal should not be considered the stamp of approval on the case. More important is the balance, and especially (when I look at faculty from different disciplines) the realization that different bibliographical and stylistic conventions render cross-disciplinary comparisons untenable (for example, *Nature* and *Science* have impact factors in the 30s, while the most prominent mathematics journals have impact factors between 2 and 3). My recommendation, which I am sure is already being taken into account by most administrators, is to look at the balance of the resumé. If none of the papers appear in a good journal (the definition of "good" being subjective, but I believe we would find substantial agreement among experts), then we do have a problem. Either the papers are not very good or the author is lazy (and prefers to send papers to journals where acceptance is more readily obtained) or the author has low self-esteem. The decision as to whether the lack of prestigious journals is an indication of low quality or timid behavior is a burden that the chair, the dean, and the provost need to bear. We get now to the last issue I want to discuss.

Multiple Coauthors

In biology, chemistry, physics, and most other natural sciences, there is a long tradition of publications with multiple authors. Because of the natural need for scientists to work in large teams, we have now a well-established convention. The first author is the one who did the majority of the work in the paper. The last author is the scientist who is responsible for the laboratory where the work is being done. Thus, these are the two prestigious positions in an article. Everybody in the middle has probably contributed in a more limited way (if at all) to the paper. It is quite possible that the middle authors are just members of the lab and their contributions are minimal. In mathematics we behave differently, in what appears to be a more "democratic" fashion. Authors are almost invariably listed in alphabetical order, and an unaware dean or provost may incorrectly interpret the value of the contributions of his or her faculty. The alphabetical usage is now being extended to other disciplines, as we see, for example, in the recent articles which contain the first results from the European Large Hadron Collider: a cursory look at arXiv will show articles with literally hundreds of authors in alphabetical order. One wonders how such articles may be evaluated in the course of a tenure process. This did not used to be a significant problem, but the average number of authors on a mathematical paper seems to be growing each year. If one takes a cursory look at MathSciNet, it will be evident that the large majority of papers in 1955, for example, were single authored. Compare this with 2005, where the large majority of papers have at least two authors. In fact, a recent article in the *Notices* shows that while in the 1940s more than 90% of the articles in mathematics were single authored, now the percentage has declined to about 50%, and more than 10% have three or more authors. How are we to judge our candidates for tenure? Once again, I make a plea for balance and coherence. An author who has no single-authored publications may raise some suspicions, but if (s)he has many different collaborators, this can simply be a sign of his or her collaborative style and preference. Generally speaking, I would advise a junior faculty member to establish his or her reputation with a few well-placed, single-author contributions, and then (if that's their pleasure) they should feel free to engage with other scholars and publish as they prefer.

It is clear that the nature and form of scholarly publication and scholarly discourse in general are changing and evolving, and our methods for evaluating scholarly work should grow and change with it.

Bryn Mawr Department Receives Award

Allyn Jackson

We have a very positive outlook on doing math. It's inclusive. We don't turn away people because they don't meet an artificial criterion of what one is supposed to be to do math.

— Paul Melvin,
Chair, Bryn Mawr mathematics department

When you open wide the doors to mathematics, a broad spectrum of individuals enters and they find something to engage their minds. Maybe they finally shake off the "math anxiety" that followed them in school. Maybe after a couple of introductory courses they gain appreciation for the usefulness of the field; maybe, their interest piqued, they opt for a minor in math. Maybe for the first time in their lives they realize that mathematics is a field they can love, and they pursue a math major.

The Bryn Mawr mathematics department has such wide-open doors. By cultivating a special atmosphere—fun-loving yet serious, supportive yet challenging—the department has been enormously successful in attracting a diverse group of students. Some are already committed to mathematics and seek out Bryn Mawr as a great place to prepare for graduate school; others never imagined they might actually like mathematics. As a women's college, and as one of the few liberal arts colleges to have a graduate program in mathematics, Bryn Mawr has played a significant role in increasing the participation of women in mathematics. It also offers an example that many other departments can emulate. For its unique profile and outstanding performance, the Bryn Mawr mathematics department has been selected as the 2012 recipient of the AMS Award for an Exemplary Program or Achievement in a Mathematics Department.

Evidence of Success

The success of the Bryn Mawr mathematics department grows from deep roots. Its graduate program was started almost simultaneously with the founding of the college in 1885. When Emmy Noether fled Europe in 1933, she joined the Bryn Mawr faculty. Although she lived only another two years, her intellectual legacy left a profound impact on the college.

Up to the 1970s, the main purpose of the Bryn Mawr mathematics major was preparation for doctoral work. Around that time, the department began making a conscious effort to encourage all students to study as much mathematics as they could. During the 1990s, the number of majors rose to an average of thirteen per year, or about 5 percent of the graduating class. The numbers continued to increase over the succeeding decade, with an average of twenty-seven majors per year between 2001 and 2010—almost 9 percent of the graduating class. Many Bryn Mawr math majors excel in the subject, with about a dozen of them annually taking part in Research Experiences for Undergraduates programs across the country and some continuing on to graduate school. Currently, four Bryn Mawr math major alumnae hold Graduate Research Fellowships from the National Science Foundation.

The Bryn Mawr mathematics department stands out among liberal arts colleges for its research profile. It has a total of six faculty either with tenure or on the tenure track, and all of them are active in research. With a steady-state of just six doctoral students, the graduate program is too small to have been included in the last ranking of graduate programs by the National Research Council. Nevertheless, one can calculate where the department would have landed had it been ranked, and the results are impressive. Its publication rate for the years 2000–2006 was one paper per year per faculty member, which would have placed the department in the top half of ranked departments. The faculty's citation rate of approximately 1.5 citations per paper each year would have been in the top quarter, and in the last few years that rate rose to 2.2 citations per paper per year.

Helping Students Succeed

The most basic ingredient is that the faculty sincerely want to help students of all levels succeed.

— Victor Donnay,
Bryn Mawr mathematics department

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The Bryn Mawr mathematics faculty is united by a love of mathematics and a desire to open the subject to all students. We "really do believe that a very broad range of students can learn to enjoy and succeed in mathematics," said faculty member Helen G. Grundman. This is in contrast to many mathematics departments, where attention is trained on those students who are "math stars", while others pass through largely unnoticed. "We see potential in a much broader sense and strongly encourage all students to pursue more mathematics," commented faculty member Lisa Traynor. "We have a number of students who are self-proclaimed math nerds, but we also have many in our community who never dreamed they would be math majors."

According to Bryn Mawr alumna Rebecca Rebhuhn-Glanz, now a math graduate student at the University of Michigan, the best thing about the Bryn Mawr mathematics department is the way individual development is prized and nurtured. "[T]he professors, while helping you find your abilities, don't try to push you into a single career path," she commented. "The department trains people who go into various fields of work, and they do very well wherever they are." Bryn Mawr math majors have gone on to successful careers in areas such as teaching, consulting, finance, government, and medicine, as well as on to graduate school in various subjects.

Another key aspect of the department's success is its sense of community. "We have developed a welcoming and supportive community where all members—faculty, graduate students, and undergraduates—have an important role," said Traynor. The students interact in a noncompetitive environment where working together and mutual support are encouraged, and the faculty members put a great deal of personal effort into making all students feel welcome and valued. Indeed, one feels the sense of community when walking the halls of the department; one sees it in the open and lively interactions between faculty and students; one hears it in students' comments about the department. One of them summed it up simply: "The Bryn Mawr math department is a home to me."

Activities, Serious and Silly

A variety of events bring together the departmental community at Bryn Mawr. One example is the Distressing Mathematics Collective (DMC), which started about fifteen years ago. Grundman and some students dreamed up the idea and the quirky name, which is intended to call to mind those small tidbits of mathematics that distress in their unexpectedness—counterexamples that run against intuition, mind-bending paradoxes, or just results that have a strange twist or are off the beaten track of most mathematics courses. Each week a student talks for thirty to sixty minutes in a

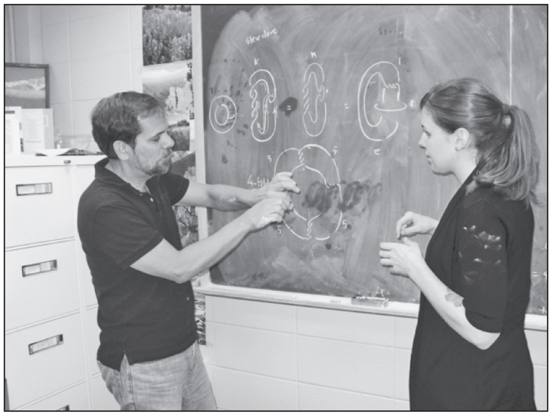
Professor Lisa Traynor (right) in discussion with Shuning Yan '13.

All photos courtesy of the Department of Mathematics, Bryn Mawr College.

casual, relaxed atmosphere where jokes, interruptions, and, as Grundman put it, "general silliness," are encouraged. The session usually ends with a game of cards. Despite the lighthearted tone, the DMC has some serious payoffs. Students get exposed to a wide range of mathematics in a friendly atmosphere, and they learn to speak up and ask questions, so that mathematics becomes more than a spectator sport. Those who speak in the DMC have to deal with interruptions, so they learn ways to get and hold the audience's attention. Said Grundman, "More than once, I've heard students who are preparing to give a talk in a class or at a student conference say something like 'After giving talks in DMC, I can give a talk anywhere!'"

For some activities that are common in mathematics departments, Bryn Mawr has added innovative twists that increase the activities' effectiveness. For example, students are enticed to attend colloquium talks through the offer of bonus course points, which they can obtain by writing a paragraph about the talk. Once a semester, the traditional departmental tea is expanded into an afternoon celebration dedicated to recruiting new majors: Math majors have their pictures taken (the pictures are later posted on a department bulletin board) and mingle with potential majors. In addition to providing career advice to students, each spring the department holds a panel discussion featuring alumnae who speak about how the math major has been useful in their careers.

One successful departmental activity actually has little to do with mathematics: the Mathematics Shakespeare Reading Group. The group meets a few times a year to read a Shakespeare play aloud. To keep seriousness low and fun high, parts are assigned at random so no one can prepare ahead of time. Mathematics enters through the back door: Whenever a word with a mathematical meaning is read—group, ring, field, prime, integral—the readers echo it. The group was started about twenty years ago by a math major who liked to ask,



Bryn Mawr chair Paul Melvin (left) at the blackboard with graduate student Laura Mansfield.

"How often do English majors get together to read some algebra?" In fact, the reading group is open to anyone, not just math majors, and attracts a broad range of students.

A few years ago, the Bryn Mawr mathematics department established a student chapter of the Mathematical Association of America. Each year, over twenty math majors become members of the chapter's board of directors and take on such tasks as helping with the Distressing Math Collective, preparing the department newsletter, planning excursions to math conferences, and serving as captain of the department's Putnam Exam Club. The student chapter builds community among the students and gives them a stake in ensuring the success of departmental activities.

Advantages of a Graduate Program

Not many smaller schools have a serious Ph.D. program in mathematics. Bryn Mawr does, and it has an incredibly positive effect on our undergraduate majors.

—Helen G. Grundman, Bryn Mawr mathematics department

All the Bryn Mawr mathematics professors are active in research, and they cover a broad swath of the field. Bryn Mawr graduate students therefore can receive training comparable to that at a much larger university, but in a special atmosphere—one that is friendlier, more supportive, and less competitive.

In many ways, Bryn Mawr's graduate program is central to the department's identity. For one thing, it makes the department appealing to high-caliber mathematicians. "All of our hires have said that the graduate program is a factor in their coming here," Melvin noted. Bryn Mawr attracts a special breed of mathematician who is dedicated to both teaching and research, though finding time for both is not easy. Melvin noted that the Bryn Mawr mathematicians have coped in part by becoming adept at seizing any opportunity they can to involve students in research. ("And," he conceded, "we don't get any sleep.")

Another advantage of the graduate program is that highly talented math majors can take graduate courses, so that they get a head start on graduate school. They also have the option of a combined A.B./M.A. program, in which they can write a master's thesis and complete both degrees in four years. The department's doctoral students serve as teaching assistants and work with undergraduates one-on-one and in small groups, allowing for plenty of personal interaction. "This makes it possible for us to have the sort of welcoming, inclusive major that is our goal," Grundman commented. Operating in an environment that values excellent teaching, the graduate students are inspired to take their own teaching seriously. They also serve as role models for undergraduates who are considering graduate work in mathematics.

The success of Bryn Mawr's math graduate program has fed into a national program that has had a substantial impact on the participation of women and minorities in mathematics: the EDGE program. The name stands for Enhancing Diversity in Graduate Education and refers to the idea of helping to give women an "edge" in Ph.D. programs. Founded in 1998 by Rhonda Hughes, now retired from Bryn Mawr, and Sylvia Bozeman, Spelman College, EDGE is geared towards female students—particularly those from underrepresented minority groups—who have excelled in early mathematics courses but may have had limited exposure to advanced mathematics. The aim is to reduce attrition in the first year of graduate school by providing exposure to graduate-level mathematics and to the culture of graduate school, along with a support network and positive feedback. EDGE has since expanded to include "mentoring clusters" that support students through graduate school and beyond. At the time of this writing, a total of forty EDGE alumnae had received Ph.D.s in the mathematical sciences. In 2007 EDGE received the Mathematics Programs That Make a Difference Award, presented by the AMS Committee on the Profession. In 2004 Hughes received the Lifetime Mentor Award from the American Association for the Advancement of Science; Bozeman received it in 2008.

After initially alternating between Bryn Mawr and Spelman, EDGE became a national program by forming partnerships with several colleges and universities, and those institutions have hosted the program since 2003. In addition to Hughes, several Bryn Mawr faculty have been active in EDGE. In the fall of 2011, after thirteen years of running the program, Bozeman and Hughes turned over the directorship to Ami Radunskaya of Pomona College and Ulrica Wilson of Morehouse College, both of whom have had a long association with EDGE.

Educational Innovations

As the Bryn Mawr mathematics department opened its doors wider to include a greater diversity of students, it began to make innovations in its course offerings to better meet students' needs. For example, when the department found that many students were having trouble moving from courses focused on calculation to those focused on proofs, Grundman developed a sophomore-level course called "Transitions to Higher Mathematics", which orients students towards active reading of mathematics and communicating through proofs. To show students that mathematics is not just a collection of techniques but has a special beauty, Melvin and Traynor developed a sophomore-level course on knot theory. Because many students are interested in applying math to the real world, Leslie Cheng developed courses in financial mathematics, and Victor Donnay revised the differential equations course to include a dynamical systems approach focusing on mathematical modeling for environmental issues. In addition, Donnay taught a service-learning seminar on math and sustainability.

The Bryn Mawr mathematics department also refined its "senior capstone" course. "A lot of colleges struggle with what's right to do with this course," Melvin commented. "Ours seems to work very well." The course, called the "Senior Conference" is organized like a seminar, so that students give lectures and work together in groups—in other words, they receive in-depth practice in communicating about mathematics. "It's a lot of fun, and the students have a say in the direction of the seminar," Melvin commented. For students with a sufficiently strong academic record, there is also the option of writing a senior thesis.

Some Bryn Mawr students are interested in teaching, but the college's small education program has no specialist in mathematics or science. To fill this gap, Donnay developed and teaches regularly a course called "Changing Pedagogies in Math and Science Education". He has also been active in regional and national efforts to improve mathematics education. He was the coprincipal investigator of the Mathematics and Science Partnership of Greater Philadelphia, which aimed to improve math and science education in secondary schools, colleges, and universities. This US$12 million project, funded by the National Science Foundation, brought together higher education faculty from thirteen institutions and math and science teachers from forty-six school districts. Donnay was coprincipal investigator for a pilot project to support teachers and school districts in teaching environmental sustainability, and this year he is teaching a two-week summer institute for local teachers on math and sustainability.

Winners: Both the Stars and the Struggling

Bryn Mawr turned me into a mathematician. I owe absolutely everything that I've done mathematically to the wonderful professors who mentored me there.

— Jaclyn Lang,
Bryn Mawr alumna and
UCLA math graduate student

Discussing mathematics are (from left) Meagan Neal '13, Sarah Hockley '11, and Katie Link '12.

Some of the magic of the Bryn Mawr mathematics department is due to its being in a women's college. "It's a gentler atmosphere," Melvin acknowledged. But he also pointed out that many aspects of what Bryn Mawr does are replicable in other places. One of the main aspects is having wide-open doors that welcome all students. "Don't put students down if they don't achieve what you think they are supposed to," he said. "Find out what they are good at, and nurture that. Try to empower the students." Indeed, one of the biggest achievements of the Bryn Mawr math department has been its success in transforming attitudes towards mathematics in students who come in with the mindset that math is their worst subject.

The open-doors attitude is also appreciated by Bryn Mawr's math stars—like Jaclyn Lang. After graduating from Bryn Mawr, Lang received a Churchill fellowship to study mathematics at the University of Cambridge. Today she is pursuing a Ph.D. at the University of California, Los Angeles, supported by an NSF Graduate Fellowship. "The [Bryn Mawr] professors all care about the confused calculus and statistics students who come to their office hours just as they do the majors and graduate students with whom they discuss deeper mathematics," she said. "They listen to a student, figure out what she knows and what she struggles with, and then they think of a way to keep her challenged and interested in mathematics. The fact that they do this with students at all levels is what I find amazing, and I think this personal attention makes a huge impact on the students."

Bourgain and Tao Awarded 2012 Crafoord Prize

JEAN BOURGAIN of the Institute for Advanced Study, Princeton and TERENCE TAO of the University of California, Los Angeles have been awarded the 2012 Crafoord Prize in Mathematics of the Royal Swedish Academy of Sciences "for their brilliant and groundbreaking work in harmonic analysis, partial differential equations, ergodic theory, number theory, combinatorics, functional analysis and theoretical computer science."

Jean Bourgain

Photo by Reed Hutchinson/UCLA.

Terence Tao

The prize citation reads in part: "This year's Crafoord Prize Laureates have solved an impressive number of important problems in mathematics. Their deep mathematical erudition and exceptional problem-solving ability have enabled them to discover many new and fruitful connections and to make fundamental contributions to current research in several branches of mathematics. On their own and jointly with others, Jean Bourgain and Terence Tao have made important contributions to many fields of mathematics—from number theory to the theory of nonlinear waves. The majority of their most fundamental results are in the field of mathematical analysis. They have developed and used the toolbox of analysis in groundbreaking and surprising ways. Their ability to change perspective and view problems from new angles has led to many remarkable insights, attracting a great deal of attention among researchers worldwide."

Bourgain has proved several groundbreaking results on well-posedness of nonlinear differential equations, such as the Schrödinger equation of quantum mechanics and the Korteweg–de Vries equation of wave propagation. These theorems guarantee that the solutions to these equations evolve in a unique way, no matter how irregular the initial state of the system might be. He has made contributions in many areas, such as his 2010 result in Apollonian gaskets—infinite collections of circles that just touch without overlapping. In these arrangements, the radius of each bubble is 1 divided by an integer. Together with Elena Fuchs, Bourgain showed that, if you pick any integer n, there is a nonzero probability that some bubble in the Apollonian foam has radius equal to $1/n$. Bourgain was awarded the Fields Medal in 1994 and has also been awarded the Salem Prize (1983), the Damry-Deleeuw-Bourlart Prize (Belgian Science Prize, 1985), the Langevin (1985) and Cartan (1990) Prizes of the French Academy, the Ostrowski Prize (1991), and the Shaw Prize (2010).

Tao is a world-renowned mathematician working in a number of branches of mathematics, including harmonic analysis, partial differential equations, combinatorics, number theory, and signal processing. He is known for his highly original solutions of very difficult and important problems and for his technical brilliance in the use of the necessary mathematical machinery. He and Ben Green proved that there are arbitrarily long arithmetic progressions of prime numbers—a result now known as the Green–Tao theorem—which had been a major unsolved problem in number theory for centuries. He is also known for such practical work as his invention, with Emmanuel Candes, of compressed sensing. His areas of research include harmonic analysis, partial differential equations, combinatorics, and number theory. He received the Fields Medal in 2006 for his contributions in these fields. He has held Sloan Foundation (1999–2001), Packard (1999–2006), and MacArthur (2007) fellowships. He has also received the Salem Prize (2000), the Bôcher Prize (2002), the SASTRA Ramanujan Prize (2006), the Ostrowski Prize (2007), the Waterman Award (2008), and the King Faisal Prize (2010).

"Bourgain and Tao have similar and very unique strengths," says Peter Sarnak of the Institute for

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Advanced Study. “Their ability to see through technical difficulties that prevent almost anyone else to move forward has allowed them to resolve long-standing problems in an amazingly diverse set of areas.”

They will share the monetary award of four million Swedish kronor, approximately US$580,000.

The Anna-Greta and Holger Crafoord Fund was established in 1980, and the first prize was awarded in 1982. The prize is intended to promote international basic research in the disciplines of astronomy, mathematics, the geosciences, the biosciences (with particular emphasis on ecology), and polyarthritis (rheumatoid arthritis).

Previous recipients of the Crafoord Prize in Mathematics are Maxim Kontsevich and Edward Witten (2008), Alain Connes (2001), Simon Donaldson and Shing-Tung Yau (1994), Pierre Deligne and Alexander Grothendieck (1988), and V. I. Arnold and Louis Nirenberg (1982).

—Elaine Kehoe

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Hopkins Receives NAS Award in Mathematics

MICHAEL J. HOPKINS of Harvard University has received the 2012 NAS Award in Mathematics from the National Academy of Sciences. He was honored "for his leading role in the development of homotopy theory, which has both reinvigorated algebraic topology as a central field in mathematics and led to the resolution of the Kervaire invariant problem for framed manifolds."

The NAS Award in Mathematics was established by the AMS in commemoration of its centennial, which was celebrated in 1988. The award is presented every four years in recognition of excellence in research in the mathematical sciences published within the preceding ten years. The award carries a cash prize of US$5,000. Previous recipients are Robert P. Langlands (1988), Robert MacPherson (1992), Andrew J. Wiles (1996), Ingrid Daubechies (2000), Dan Virgil Voiculescu (2004), and Clifford H. Taubes (2008).

The Work of Michael Hopkins

Hopkins's first major contribution to mathematics was his proof (in joint work with Ethan Devinatz and Jeff Smith) of the nilpotence theorem: if $f: K \to K$ is a pointed map from a finite CW complex to itself that is trivial on a complex bordism, then some iterate of f is stably nullhomotopic. This is a foundational result, which marked the beginning of the modern era in stable homotopy theory. Using it, Hopkins and his collaborators were able to carry out a massive program (envisioned by Jack Morava, Doug Ravenel, and others) establishing deep connections between algebraic topology and the arithmetic of formal group laws. The resulting theory (sometimes called chromatic homotopy theory) has led to powerful new insights on old problems and is presently the best available tool for understanding the large-scale structural phenomena in the stable homotopy groups of spheres.

The most recent triumph of Hopkins's career is his resolution (in joint work with Mike Hill and Doug Ravenel) of the Kervaire invariant problem. This problem has many formulations, but perhaps the simplest is this: given a framed n-manifold M, does there always exist a (framed) bordism from M to a sphere? Hill, Hopkins, and Ravenel have shown that the answer is affirmative provided that $n \neq 2$, 6, 14, 30, 62, 126. This problem arose originally in the work of Kervaire and Milnor in the study of exotic smooth structures and was later shown by Browder to be intimately connected with the calculation of the stable homotopy groups of spheres. Despite its central role in both geometric and algebraic topology, the Kervaire invariant question remained unanswered for almost fifty years, until it was recently addressed by Hopkins and his collaborators using an array of new insights from equivariant algebraic topology.

Biographical Sketch

Michael Hopkins was born in Alexandria, Virginia, in 1958. He received both his bachelor's degree (1979) and his Ph.D. (1984) in mathematics from Northwestern University. He also received his D.Phil. in mathematics in 1984 from the University of Oxford. He held an NSF postdoctoral fellowship at Princeton University from 1984 to 1987, then advanced from instructor to assistant professor at Princeton before taking a professorship at the University of Chicago in 1988. He moved to the Massachusetts Institute of Technology in 1989 and has been professor at Harvard since 2005. He has been the recipient of a Rhodes Scholarship (1979–1982), a Presidential Young Investigator Award (1987–1995), and an Alfred P. Sloan Fellowship (1987–1992). He was awarded the Veblen Prize in Geometry in 2001. He is an associate editor of the *Journal of the American Mathematical Society* and of several other mathematics journals. He is a member of the American Academy of Arts and Sciences and a foreign member of the Royal Danish Academy of Sciences and Letters.

— Elaine Kehoe

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JPBM Communications Award

The 2012 Communications Award of the Joint Policy Board for Mathematics (JPBM) was presented at the Joint Mathematics Meetings in Boston, Massachusetts, in January 2012.

The JPBM Communications Award is presented annually to reward and encourage journalists and other communicators who, on a sustained basis, bring mathematical ideas and information to nonmathematical audiences. JPBM represents the American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. The award carries a cash prize of US$1,000.

Previous recipients of the JPBM Communications Award are: James Gleick (1988), Hugh Whitemore (1990), Ivars Peterson (1991), Joel Schneider (1993), Martin Gardner (1994), Gina Kolata (1996), Philip J. Davis (1997), Constance Reid (1998), Ian Stewart (1999), John Lynch and Simon Singh (special award, 1999), Sylvia Nasar (2000), Keith J. Devlin (2001), Claire and Helaman Ferguson (2002), Robert Osserman (2003), Barry Cipra (2005), Roger Penrose (2006), Steven H. Strogatz (2007), Carl Bialik (2008), George Csicsery (2009), Marcus du Sautoy (2010), and Nicolas Falacci and Cheryl Heuton (2011).

Citation

The 2012 JPBM Communications Award is presented to Dana Mackenzie.

Over the last fifteen years Mackenzie has produced a remarkably broad and deep body of writing for experts and nonexperts alike. The work focuses largely on mathematics itself but also touches geology, climate change, astronomy, academic mathematics as a profession, and even the game of chess—at which Mackenzie competes at the USCF National Master level.

Mackenzie's authorship of Volumes 6–8 of *What's Happening in the Mathematical Sciences*, published by the American Mathematical Society, illustrates his knowledge, versatility, and expository skill. These lucid, informative, and witty volumes showcase the importance and applicability of up-to-the-moment developments in mathematics in fields ranging from the geometry of surfaces to signal processing to the history of mathematics in antiquity. When *Science* recognized Grigory Perelman's proof of the Poincaré conjecture as "Breakthrough of the Year" in 2006, Mackenzie was chosen to write the cover article. In these and other works, Mackenzie reveals, celebrates, and illustrates the excitement and vitality of learning, using, and discovering excellent mathematics.

Dana Mackenzie

Biographical Sketch

Dana Mackenzie was born in 1958 and wrote his first book at the age of five. While *The Littlest Inchwarm* [sic] never got past its original limited edition of one copy, it did foreshadow a lifetime love of writing. Mackenzie's love of mathematics also surfaced at an early age. He eagerly read every book by Martin Gardner, discovered the formula for triangular numbers in sixth grade, and in the ninth grade discovered that i has a square root. Fortunately, his teachers didn't tell him that these things were already known.

After majoring in mathematics at Swarthmore College, Mackenzie earned his Ph.D. from Princeton University in 1983. He taught for six years at Duke University and seven years at Kenyon College. During that time, he received the 1993 George Pólya Award from the Mathematical Association of America.

In 1996, while surfing the newfangled World Wide Web, Mackenzie found out about the Science Communication Program at the University of California at Santa Cruz. It was another "eureka" moment, as he saw for the first time how to combine his passions for mathematics and writing. The program taught him the ropes of journalism and launched him on a new career as a freelance mathematics and science writer.

Since 1997 Mackenzie has written for such magazines as *Science, New Scientist, American Scientist*, and *Smithsonian.* His first trade book, *The Big Splat, or How Our Moon Came to Be*, appeared on *Booklist* magazine's best-of list for 2003 and

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was one of Audible.com's audiobooks of the year for 2010. While he enjoyed writing about other sciences, he still felt his portfolio was incomplete without a book about the subject he knew best. Princeton University Press will publish his first full-length popular book about mathematics, titled *The Universe in Zero Words*, in 2012. Mackenzie still lives in Santa Cruz with his wife, Kay, and an ever-changing array of foster animals.

Response from Dana Mackenzie

I like to tell people that my job gives me the opportunity to get free lessons from the smartest people in the country every week. For someone who likes learning new things, journalism— especially science journalism—is like a never-ending trip to the candy store. I never know what I'm going to turn up next.

Winning the JPBM Communications Award evokes two predictable emotions, which are both sincere in spite of their predictability. First, I am humbled by the list of previous winners. I see names of people who inspired me, such as Martin Gardner, Constance Reid, Ian Stewart, and Sylvia Nasar. I also see people like last year's winners, Nicolas Falacci and Cheryl Heuton, who have done much more than I ever could to bring mathematics to the masses.

The second sincere emotion is gratitude. I would like to thank several people, some well known and others not, without whom this journey would never have happened. Barry Cipra helped me get started in the writing business. Editors like Robert Coontz at *Science* and Brian Hayes and Rosalind Reid at *American Scientist* taught me the craft. John Wilkes had the vision to start the Science Communication Program at UCSC and gave me the last slot in the class of 1997, when I barely applied before the deadline. Ed Dunne and Jim Maxwell at the American Mathematical Society entrusted me with the *What's Happening in the Mathematical Sciences* series, so well begun by Barry Cipra. And Martin Gardner and George Pólya, neither of whom I ever met, were my greatest inspirations.

— Elaine Kehoe

MAA Prizes Presented in Boston

At the Joint Mathematics Meetings in Boston, Massachusetts, in January 2012, the Mathematical Association of America (MAA) presented several prizes.

Gung and Hu Award for Distinguished Service

The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award made by the MAA. It honors distinguished contributions to mathematics and mathematical education, in one particular aspect or many, whether in a short period or over a career.

The 2012 Gung and Hu Award was presented to John Ewing, president of Math for America.

John Ewing received his Ph.D. in mathematics from Brown University in 1971 and, after a two-year postdoctoral appointment at Dartmouth, he joined the mathematics department faculty at Indiana University. He quickly became known on campus not only as a very good teacher and researcher but also as a team player and leader who worked to improve the department, the campus, and the field of mathematics. At Indiana, he began his administrative career as director of undergraduate studies (1978–1980) and later served as chair of the mathematics department (1986–1989 and 1992–1995). During this period, while serving as the editor of the *Monthly* and the editor of Graduate Texts at Springer-Verlag, he played a key role in the expansion of the department, the renovation of two department buildings, and the strengthening of the department in both teaching and research. Also, while at Indiana, he published approximately thirty-five research papers in algebraic topology and related areas, supervised three Ph.D. students, and held visiting positions at the University of Virginia, Newcastle University, and the University of Göttingen.

Outside of Indiana University, Ewing first turned his considerable administrative skills to publication. In addition to work on editorial boards too numerous to list, he served as editor of the *American Mathematical Monthly* (1992–1996) and of the *Mathematical Intelligencer* (1980–1986). While he was editor, these journals were distinguished by articles containing engaging, accessible, and important mathematics. In 1994 Ewing edited the MAA publication *A Century of Mathematics: Through the Eyes of the Monthly*. Underwood

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Dudley's review in the *Intelligencer* captures Ewing's editorial style: "This is a rich and fascinating book. It has everything, and everything that it has is delightful, curious, enlightening, engrossing, interesting, informative, funny, stirring, poignant, or some combination of the preceding."

Between 1995 and 2009 Ewing served as executive director of the American Mathematical Society. His decisions within the AMS were always guided by a commitment to serve not only the specific interests of the AMS but also those of the broader mathematical sciences community. He is very much a "big tent" mathematician, always maintaining strong and cordial working relationships with his professional society colleagues, especially at the MAA and SIAM. New joint projects of the time that benefited from the mutual cooperation of the mathematics organizations included public awareness activities and government outreach.

As an example of his big tent vision, in *Science* in 1997, he called attention to the importance of Project NExT: "By bringing young mathematicians to meetings several years in a row, you show them the value of contacts. Most young mathematicians learn that slowly, over many years, or never learn it at all. Project NExT fellows have it handed to them for free." In 2001 the AMS began funding six Project NExT fellows per year.

In 1999 an AMS task force produced the influential and no-holds-barred *Towards Excellence: Leading a Mathematics Department in the 21st Century*, which was edited by Ewing and is available at no cost on the Internet. This volume quotes deans who report that mathematics departments tend to be too insular, and it emphasizes the importance of mathematics departments building good relationships with other departments across campus. The MAA's *Guidelines for Programs and Departments in Undergraduate Mathematical Sciences* calls *Towards Excellence* "an excellent reference for a planning and evaluation process" and David Bressoud, past president of the MAA, calls it "one of the most useful resources for making the case for greater support for curricular and instructional improvement." In 2002 Ewing presented the ideas in *Towards Excellence* at an MAA PREP workshop, Leading the Academic Department: A Workshop for Chairs of Mathematical Science Departments.

Ewing is a recognized expert in scholarly publishing and has been a strong, articulate, balanced, and visible voice about the role of professional society publishers in working for the long-term survivability of peer-reviewed journals as an essential tool for research and scholarship. He spoke eloquently, effectively, and sensibly at national and international venues on the issues that surround scholarly publishing, especially nonprofit publishers, and wrote an important series of commentaries on the state of publishing in the mathematical sciences. While he has not been an advocate of totally free access, which he finds unsustainable, he has worked tirelessly to improve access to the mathematical literature. He has been committed to low-cost, high-quality electronic publishing, with author-friendly copyright policies, and to databases that make an exhaustive literature search possible. The Digital Mathematics Registry, which went online in 2006, is a complete list of digitized publications in the mathematical sciences. The AMS maintains this registry as a public service, which is, itself, in the public domain. *Mathematical Reviews*, online through MathSciNet since 1996, is one of the indispensable tools of the working mathematician. Ewing's contributions to the growth and enhancement of *Mathematical Reviews* were substantial. With the advice of the senior MR staff and oversight committees, he worked first and foremost setting the agenda for improvements to the database. He is a detail-oriented person, and so his role was rather direct. He is very knowledgeable on the role of technology in building and serving information. He instituted a pricing scheme with the result that the number of institutions that could subscribe to MR doubled in a decade. He arranged for all reviews back to the first issue in 1940 to be digitized. There is no doubt that he was key to moving the project forward to produce the MR tools and database that we have today. Ewing chaired the Joint IMU/ICIAM/IMS Committee on Quantitative Assessment of Research Citation Statistics. Their report, a critical analysis of the use and misuse of citation statistics in science, was published as the lead article in *Statistical Science* in 2009. The report was featured in the *Wall Street Journal* and on MathDL.

In 2009 Ewing became president of Math for America. The mission of the nonprofit Math for America is "to improve mathematics education in U.S. public secondary schools by recruiting, training, and retaining outstanding mathematics teachers." When he became president, he said, "After three years, roughly forty percent of the teachers of mathematics [in U.S. schools] are gone.... You can't sustain a profession if you have that kind of attrition.... What Math for America does is concentrate on that one part of the problem. At the moment, it's bringing through something like forty to fifty new teachers a year. Our hope is to double that number in the next couple of years" (*Science*, January 2009). Since Ewing became president, Math for America has enlarged to about 420 participants, expanded to new cities, and developed new programs, including one for science teachers. Recently, he has again served on several MAA committees, including the Pólya Lecturer Committee and as chair of the 2010 search committee for the editor of the *Monthly*.

He has received the following awards: the Lester R. Ford Award (1975); SERC, Great Britain, Research Fellowship (1981); SFB Fellowship, Federal Republic of Germany (Göttingen) (1984); the MAA George Pólya Lectureship (1991–1992; 1992–1993); Honorary Doctorate of Science, St. Lawrence University (1995); the George Pólya Award (1996); Fellow, American Association for the Advancement of Science (2005).

Throughout his career, John Ewing has been an effective and firm, but gentle, leader. As Jonathan Borwein, a former MAA governor, wrote about serving with Ewing on the Committee on Electronic Information and Communication (CEIC) of the International Mathematical Union, "John is an enormously hard-working man—this is not a secret—who wears his remarkable erudition and breadth of knowledge very lightly. He is patient, hard to ruffle, and even harder to alienate. The CEIC was formed with many passionate members, all knowledgeable about some bits of the puzzle. It had only one expert: John Ewing. John's patience and generosity in educating the rest of us about the many pitfalls and subtleties was extraordinary. His care in trying to distinguish his role as committee member from that as AMS executive director (which could have made him the eight-hundred-pound gorilla on the committee) was remarkable."

Chauvenet Prize

The Chauvenet Prize recognizes a member or members of the MAA for the writing of an outstanding expository article. First awarded in 1925, the prize is named for William Chauvenet, who was a professor of mathematics at the United States Naval Academy and Washington University in St. Louis (where he founded the mathematics department).

The 2012 Chauvenet Prize has been awarded to Dennis DeTurck and Herman Gluck, both of the University of Pennsylvania; Daniel Pomerleano of the University of California, Berkeley; and David Shea Vela-Vick of Columbia University for their joint paper "The four vertex theorem and its converse", *Notices of the American Mathematical Society* **54** (2007), no. 2, 192–207. The four vertex theorem is a beautiful result in global differential geometry. It says that any smooth simple closed curve in the plane must have at least four "vertices"—local extrema for the curvature function. The vertices of a (noncircular) ellipse, for example, are located where it meets its major and minor axes. The four vertex theorem was proved for convex curves in 1909 by Syamadas Mukhopadhyaya and for general curves in 1912 by Adolf Kneser. Remarkably, the converse of the four vertex theorem is also true. Any continuous real-valued function on the circle with at least two local maxima and two local minima is the curvature function for some simple closed curve in the plane. The converse was proved for positive functions in 1971 by Herman Gluck and for arbitrary functions in 1997 by Björn Dahlberg.

The authors of this excellent expository article sketch Robert Osserman's 1985 proof of the four vertex theorem and Dahlberg's proof of the converse. The article ends with some generalizations of the four vertex theorem and biographical sketches of Mukhopadhyaya, Kneser, and Dahlberg. This carefully crafted survey has enough mathematical details to give the reader a sense of the proofs, but not so many as to obscure the big picture. Experts and nonexperts alike are sure to enjoy and understand this well-written and well-illustrated article. It is also a wonderful tribute to Björn Dahlberg, who passed away in early 1998, with his unpublished proof of the full converse still on his desk.

Euler Book Prize

The Euler Book Prize is given to the author(s) of an outstanding book about mathematics. Mathematical monographs at the undergraduate level, histories, biographies, works of mathematical fiction, and anthologies are among those types of books eligible for the prize. The prize was given for the first time in 2007, the 300th anniversary of the birth of Leonhard Euler.

Daina Taimiņa of Cornell University has been awarded the 2012 Euler Book Prize for *Crocheting Adventures with Hyperbolic Planes* (A K Peters, Ltd., 2009).

This book is unlike any previously considered for the Euler Prize. Indeed, it is unlike any book on hyperbolic geometry previously written, and it is in a different universe from any book on crochet previously written. But, when you look at it, the idea makes such perfect sense that it seems inevitable. Eugenio Beltrami, who in 1868 first modeled the non-Euclidean geometry of Bolyai and Lobachevsky by surfaces of negative curvature, actually toyed with the idea of building such surfaces. He made a small fragment of such a surface out of paper, and the idea was taken up again by William Thurston in the 1970s. But the idea did not take off, let alone reach a wide audience, until Daina Taimiņa wrote this book. By bringing crochet technology to the subject, she makes it easy and fun to construct hyperbolic surfaces that vividly illustrate essential features of non-Euclidean geometry. The book is elegant, from both a visual and mathematical point of view.

Thus *Crocheting Adventures with Hyperbolic Planes* is a novel approach to geometry that has brought a whole new audience to mathematics. In this respect it has greater outreach potential than any book we have previously considered. But it is much more than that; it is perfectly capable of standing on its mathematical feet as a clear, rigorous, and beautifully illustrated introduction to hyperbolic geometry. It is truly a book where art, craft, science, and mathematics come together in perfect harmony.

Beckenbach Book Prize

The Beckenbach Book Prize, established in 1986, is the successor to the MAA Book Prize, which was established in 1982. It is named for the late Edwin Beckenbach, a long-time leader in the publications program of the association and a well-known professor of mathematics at the University of California at Los Angeles. This prize is awarded to an author of a distinguished, innovative book published by the MAA. The award is not given on a regularly scheduled basis but is given only when a book appears that is judged to be truly outstanding.

The Beckenbach Book Prizes for 2012 were awarded to Dan Kalman (American University) and Nathan Carter (Bentley University)

Kalman was honored for *Uncommon Mathematical Excursions: Polynomia and Related Realms* (The Dolciani Mathematical Expositions, Vol. 35, Mathematical Association of America, 2009). Just how interesting can a book be whose only topic is polynomials? It doesn't take long to find out. By page 3 in Kalman's book, we encounter Lill's ingenious paper-folding technique for visualizing real roots of a polynomial, with an elegant proof based on Horner's method to evaluate a polynomial. Now most mathematicians are vaguely aware of Horner's method, but try to find one who knows Lill's technique, or one that knows that the first mention of Lill's idea in the United States appeared in an 1879 pamphlet by a lieutenant in the Army Corps of Engineers. By Chapter 2 we find that if a cubic $p(x)$ has three noncollinear roots in the complex plane, the unique ellipse inscribed in the triangle formed by those roots has the roots of $p'(x)$ as its foci (Marden's theorem) and that a polynomial $p(x)$ with positive integer coefficients is completely determined by the value of $p(1) = b$ and $p(b)$ (think base b). There is much, much more: palindromials, polynomial interpolation, symmetric functions, Newton's identities, and a brief history of Cardano's formula for the roots of a cubic (again, who knew Mark Kac sharpened his teeth at age fifteen on his own method of getting Cardano's formula?); and that is only the first third of the book! The next part covers max/min problems, beginning with a careful analysis of the Lagrange fallacy, then moving on to the Milkmaid problem, rotating ellipses, the ladder-around-the-corner (done with envelopes!), and the old shortest-time path through-or-around a field but with a discontinuous objective function. The last section focuses more on calculus questions, the wonder of elementary functions, and the expansion of the toolbox with special functions to solve, for example, $e^x = cx$. The exposition is perfect: expansive, relaxed, and detailed (almost nothing is stated without being proved). Although knowledge of calculus is needed, any good undergraduate mathematics major can read the entire book, as could many talented high school students. The book is a gold mine of topics for undergraduate talks. Kalman received his Ph.D. from the University of Wisconsin, Madison. He has been a frequent contributor to all of the MAA journals and has served on the editorial boards of both MAA book series and journals. He has served the national and regional MAA in several capacities, including a term as associate executive director for programs and as the current governor for the Maryland-DC-Virginia section.

Carter was honored for *Visual Group Theory* (Classroom Resource Materials Series, Mathematical Association of America, Washington, D.C., 2009). Burnside's classic text *Theory of Groups of Finite Order* was the first book written on group theory; it begins by defining a group as a collection of operators closed under multiplication and inverses (no associative law). Groups don't just sit there, they do something, they act on something, as permutations or symmetries or isometries. This is where groups come from, and it is the standard viewpoint both of users in physics, geometry, analysis, topology, and combinatorics, and also of specialists in permutation groups, representations, and geometric group theory. Unfortunately, this viewpoint is almost absent from present undergraduate textbooks. Nathan Carter's eye-opening textbook has a mission to fix that. Using a Rubik's Cube as a motivating example, he returns to Burnside's definition, only with "actions" instead of "operators". This is followed by Cayley diagrams to show how a group looks; Dehn's term *grüppenbilder*, or group pictures, was always the better name. Next come motivating examples: symmetries of polygons, crystals, frieze patterns, contra dancing, roots of polynomials. Chapter 4, entitled "Algebra at last", finishes with the "classic" definition of a group, although the action definition is really the classic one. Lastly, much of the traditional content of an undergraduate text on group theory can be found in Carter's text: the alternating and symmetric groups, abelian groups, subgroups, products and quotients, homomorphisms, Sylow theory, and a little Galois theory. The presentation is never traditional. Turn to any page and a figure jumps out: Cayley diagrams, colored multiplication tables, permutations as arrows from letter to letter—even Sylow theory is accompanied by conceptual sketches outlining the proofs. The exposition is breezy and leisurely: theorems are "taken out for a test drive," a proposition "gives us a headstart," a statement is "dense with notation" or "hairy". Steps are explained, proof strategies are analyzed—anything to loosen the seductive (after the fact) but intimidating (before the fact) concision and formality of algebra. The result is a textbook that reads like a conversation. Nathan Carter's *Visual Group Theory* is an original breakthrough that has a chance of transforming a staple of a mathematics major's diet. Carter received his

Ph.D. in mathematical logic from Indiana University in 2004. He uses computer science to advance mathematics.

Haimo Awards for Teaching

The Deborah and Franklin Tepper Haimo Awards for Distinguished College or University Teaching were established in 1991. These awards honor college or university teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has been shown to have had influence beyond their own institutions.

The 2012 Haimo Awards were presented to Matthew DeLong (Taylor University), Susan Loepp (Williams College), and Cynthia Wyels (California State University, Channel Islands).

Matthew DeLong is a passionate and reflective teacher who challenges his students, from the math-anxious to the most able, to higher levels of accomplishment. His personal qualities of integrity, creativity, caring, and patience contribute to his ability to connect with students on an individual basis and encourage them to do their best work. He has also become a leader in professional development for collegiate mathematics teaching and in student-centered instruction. While a Ph.D. student at the University of Michigan, he was the first graduate student put solely in charge of a course of ten sections of precalculus, received the Rackham Pedagogy Award for materials designed to facilitate the pedagogical development of graduate student instructors, and was responsible for helping plan and run the professional development week for graduate students and postdocs. He is the 2005 recipient of the MAA's Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Faculty Member. At Taylor University, DeLong started and leads the monthly School of Natural and Applied Sciences Educational Issues Seminar and is Taylor University's first Center for Teaching and Learning Excellence Fellow. He is dynamic in the classroom and implements progressive and innovative teaching strategies. A typical class session involves students presenting homework solutions, mini-lectures, and group problem-solving activities from handouts. He keeps the class moving quickly and on task, while encouraging the students to explore the "why" questions and to understand the mathematics. Students meet in groups outside of class to solve "team homework" problems, and they take turns writing the solutions to these problem sets. DeLong expects students to read the textbook before class by employing a blog for students to post questions on their reading. DeLong developed a liberal arts general education course at Taylor University that engages students in meaningful activities involving the application of mathematics skills and content as part of real-world problem solving. DeLong has published several articles on student-centered instruction and has given many regional and national professional presentations on teaching mathematics. He also coauthored the MAA book *Learning to Teach and Teaching to Learn Mathematics: Resources for Professional Development* (MAA Note Series, no. 57, 2001). This book presents a model for training college mathematics instructors in a collegiate program. Reviews say it "offers a treasure chest of ideas" and that this "high-quality comprehensive resource belongs in every mathematics instructor's hands." DeLong has led Taylor University's Mathematics Contest Team to a top-three finish seven times in the last ten years of the Indiana Collegiate Mathematics Competition, and he initiated an undergraduate research program at Taylor in which he has supervised students, two of whom have recently had their research published in *Mathematics Magazine*. In 1993 DeLong received his B.A. in mathematics and economics from Northwestern University and in 1998 he received his Ph.D. in mathematics from the University of Michigan. His mathematical interests are number theory and knot theory; other interests include faculty development, coaching little league sports, singing and conducting sacred choral music, performing and directing in musical theater, and singing in his faculty quartet, Quadrivium.

Susan Loepp has a profound influence on her students, challenging them to reach their full potential through high standards and talented encouragement. Her mentoring of a diverse group of students has inspired passion and encouraged many to major in mathematics. She also advises numerous undergraduates to successfully publish research and has created new courses and a concomitant book. She received an unprecedented three teaching awards as a graduate student at the University of Texas, Austin, and also the 2007 Alumni Award from her alma mater, Bethel College in Kansas. Her teaching evaluations at Austin, the University of Nebraska, and Williams College shine above most others. At Williams College, Loepp challenges and encourages a diverse group of students, men and notably women, athletes and scholars, the anxious and the overconfident. Loepp has high standards, assigning both daily and weekly problem sets, about which students have been known to brag how long it took and that they were able to finish. She says, "I am passionate about setting high standards for students and then helping them struggle to reach their potential." Loepp's colleagues write that she exudes energy, interest, confidence, and knowledge; "has a lively, friendly, inviting teaching style, but it's clear that she means business"; that she has the "courage to explore tough/pointless/all student questions, [even] if it [means] departing from her lesson plan"; and that she has an impressive ability to conduct class discussion: "She drew perceptive and eager responses from her students…due, in part, to her engaging them in vital mathematical conversation and exploration. Susan knows the importance of giving the students time to think." Loepp empowers students, connecting with them individually and inspiring mathematical zeal, encouraging many to become mathematics majors. Students are clearly infected by her enthusiasm. She also guides numerous students in undergraduate research in commutative algebra: she has seven joint papers with undergraduate students, and six additional research papers have been published by her students. She has advised thirty-two summer research students from across the country in the NSF "SMALL" undergraduate research project, and many of these have given student talks at conferences. A colleague writes, "It is remarkable that Loepp is able to make this very technical material accessible to undergraduate students. To bring students to the point of producing publishable results is truly incredible." Additionally, Loepp has advised thirty-five student colloquia at Williams College. Finally, she has created three courses that emphasize applications: a senior seminar on algebraic error-correcting codes, a course on quantum cryptography (which utilizes a book she cowrote with William K. Wootters), and an applied version of a core requirement in abstract algebra, including encryption on elliptic curves. Loepp received a B.A. in mathematics and a B.S. in physics from Bethel College (North Newton, Kansas) in 1989. She earned her Ph.D. in mathematics from the University of Texas at Austin in 1994. Her research area is commutative algebra. She is coauthor with William K. Wootters, an expert in quantum information theory, of the book *Protecting Information: From Classical Error Correction to Quantum Cryptography* (Cambridge University Press, Cambridge, 2006).

Cynthia Wyels is committed to student success. She works tirelessly to facilitate student learning, not only for her own students but also for students of her colleagues, students across her university, and students at other universities, including in other countries. Her devotion to supporting learning extends to students of all backgrounds and abilities. Wyels invests considerable time, creativity, and enthusiasm in developing, employing, and assessing innovative teaching practices. One example is her development of computer laboratory activities that lead students to understand and explore key concepts in a variety of undergraduate mathematics courses. Another example is her creation of in-class worksheets that help students to concentrate on the big picture of mathematical ideas being presented. More recently, she has begun to use "proof portfolios" to track students' abilities to construct complete, correct, and elegant proofs as they progress through their undergraduate studies. Wyels has shared all of her teaching innovations with her departmental colleagues and also more broadly through various MAA conferences and publications. Providing students with enriching research experiences is a particular passion of hers. She has led Research Experience for Undergraduates programs that have emphasized participation from students in underrepresented groups, particularly native Spanish speakers and first-generation college students. She has regularly donated her own faculty stipend in order to support participation of students from a Mexican university. She has mentored sixty-three undergraduates in research, including thirty-eight from underrepresented groups, with several of these projects leading to coauthored research papers. Wyels has also led efforts to institutionalize student research on her campus and to create an annual student research symposium. Wyels's dedication to effective teaching extends across her campus, as does her commitment to providing productive learning environments for all students. She founded the Critical Friends Group at CSU, Channel Islands, which colleagues report as having fostered a cultural shift in attitudes toward teaching, especially with regard to supporting students who initially find university culture to be confusing and alienating. Students and colleagues alike attest to the tremendous influence that she has had as a mentor, not only on their study of mathematics and their careers in mathematics and education but also in developing their self-confidence, persistence to succeed, and professionalism for handling all situations. Testimonials bear witness to her high standards and meticulousness and also to her generosity, enthusiasm, kindness, and selflessness. Wyels received her Ph.D. at the University of California, Santa Barbara. She has directed the graduate program at CSU, Channel Islands, since its inception. She is an advocate of undergraduate research and believes it is particularly meaningful for students from nontraditional backgrounds. Her research interests are in combinatorics, most recently in graph pebbling and graph labeling.

Certificates for Meritorious Service

Each year the MAA presents Certificates of Meritorious Service for service at the national level or for service to a section of the MAA. Those honored in 2012 are: David Kerr (Eckerd College), Florida Section; Joe Yanik (Emporia State University), Kansas Section; Ruth Favro (Lawrence Technological University), Michigan Section; Frank Ford (Providence College), Northeastern Section; Hortensia Soto-Johnson (University of Northern Colorado), Rocky Mountain Section; and Minerva Cordero-Epperson (University of Texas, Arlington), Texas Section.

— Elaine Kehoe

was one of Audible.com's audiobooks of the year for 2010. While he enjoyed writing about other sciences, he still felt his portfolio was incomplete without a book about the subject he knew best. Princeton University Press will publish his first full-length popular book about mathematics, titled *The Universe in Zero Words*, in 2012. Mackenzie still lives in Santa Cruz with his wife, Kay, and an ever-changing array of foster animals.

Response from Dana Mackenzie

I like to tell people that my job gives me the opportunity to get free lessons from the smartest people in the country every week. For someone who likes learning new things, journalism— especially science journalism—is like a never-ending trip to the candy store. I never know what I'm going to turn up next.

Winning the JPBM Communications Award evokes two predictable emotions, which are both sincere in spite of their predictability. First, I am humbled by the list of previous winners. I see names of people who inspired me, such as Martin Gardner, Constance Reid, Ian Stewart, and Sylvia Nasar. I also see people like last year's winners, Nicolas Falacci and Cheryl Heuton, who have done much more than I ever could to bring mathematics to the masses.

The second sincere emotion is gratitude. I would like to thank several people, some well known and others not, without whom this journey would never have happened. Barry Cipra helped me get started in the writing business. Editors like Robert Coontz at *Science* and Brian Hayes and Rosalind Reid at *American Scientist* taught me the craft. John Wilkes had the vision to start the Science Communication Program at UCSC and gave me the last slot in the class of 1997, when I barely applied before the deadline. Ed Dunne and Jim Maxwell at the American Mathematical Society entrusted me with the *What's Happening in the Mathematical Sciences* series, so well begun by Barry Cipra. And Martin Gardner and George Pólya, neither of whom I ever met, were my greatest inspirations.

— Elaine Kehoe

MAA Prizes Presented in Boston

At the Joint Mathematics Meetings in Boston, Massachusetts, in January 2012, the Mathematical Association of America (MAA) presented several prizes.

Gung and Hu Award for Distinguished Service

The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award made by the MAA. It honors distinguished contributions to mathematics and mathematical education, in one particular aspect or many, whether in a short period or over a career.

The 2012 Gung and Hu Award was presented to John Ewing, president of Math for America.

John Ewing received his Ph.D. in mathematics from Brown University in 1971 and, after a two-year postdoctoral appointment at Dartmouth, he joined the mathematics department faculty at Indiana University. He quickly became known on campus not only as a very good teacher and researcher but also as a team player and leader who worked to improve the department, the campus, and the field of mathematics. At Indiana, he began his administrative career as director of undergraduate studies (1978–1980) and later served as chair of the mathematics department (1986–1989 and 1992–1995). During this period, while serving as the editor of the *Monthly* and the editor of Graduate Texts at Springer-Verlag, he played a key role in the expansion of the department, the renovation of two department buildings, and the strengthening of the department in both teaching and research. Also, while at Indiana, he published approximately thirty-five research papers in algebraic topology and related areas, supervised three Ph.D. students, and held visiting positions at the University of Virginia, Newcastle University, and the University of Göttingen.

Outside of Indiana University, Ewing first turned his considerable administrative skills to publication. In addition to work on editorial boards too numerous to list, he served as editor of the *American Mathematical Monthly* (1992–1996) and of the *Mathematical Intelligencer* (1980–1986). While he was editor, these journals were distinguished by articles containing engaging, accessible, and important mathematics. In 1994 Ewing edited the MAA publication *A Century of Mathematics: Through the Eyes of the Monthly*. Underwood

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Dudley's review in the *Intelligencer* captures Ewing's editorial style: "This is a rich and fascinating book. It has everything, and everything that it has is delightful, curious, enlightening, engrossing, interesting, informative, funny, stirring, poignant, or some combination of the preceding."

Between 1995 and 2009 Ewing served as executive director of the American Mathematical Society. His decisions within the AMS were always guided by a commitment to serve not only the specific interests of the AMS but also those of the broader mathematical sciences community. He is very much a "big tent" mathematician, always maintaining strong and cordial working relationships with his professional society colleagues, especially at the MAA and SIAM. New joint projects of the time that benefited from the mutual cooperation of the mathematics organizations included public awareness activities and government outreach.

As an example of his big tent vision, in *Science* in 1997, he called attention to the importance of Project NExT: "By bringing young mathematicians to meetings several years in a row, you show them the value of contacts. Most young mathematicians learn that slowly, over many years, or never learn it at all. Project NExT fellows have it handed to them for free." In 2001 the AMS began funding six Project NExT fellows per year.

In 1999 an AMS task force produced the influential and no-holds-barred *Towards Excellence: Leading a Mathematics Department in the 21st Century*, which was edited by Ewing and is available at no cost on the Internet. This volume quotes deans who report that mathematics departments tend to be too insular, and it emphasizes the importance of mathematics departments building good relationships with other departments across campus. The MAA's *Guidelines for Programs and Departments in Undergraduate Mathematical Sciences* calls *Towards Excellence* "an excellent reference for a planning and evaluation process" and David Bressoud, past president of the MAA, calls it "one of the most useful resources for making the case for greater support for curricular and instructional improvement." In 2002 Ewing presented the ideas in *Towards Excellence* at an MAA PREP workshop, Leading the Academic Department: A Workshop for Chairs of Mathematical Science Departments.

Ewing is a recognized expert in scholarly publishing and has been a strong, articulate, balanced, and visible voice about the role of professional society publishers in working for the long-term survivability of peer-reviewed journals as an essential tool for research and scholarship. He spoke eloquently, effectively, and sensibly at national and international venues on the issues that surround scholarly publishing, especially nonprofit publishers, and wrote an important series of commentaries on the state of publishing in the mathematical sciences. While he has not been an advocate of totally free access, which he finds unsustainable, he has worked tirelessly to improve access to the mathematical literature. He has been committed to low-cost, high-quality electronic publishing, with author-friendly copyright policies, and to databases that make an exhaustive literature search possible. The Digital Mathematics Registry, which went online in 2006, is a complete list of digitized publications in the mathematical sciences. The AMS maintains this registry as a public service, which is, itself, in the public domain. *Mathematical Reviews*, online through MathSciNet since 1996, is one of the indispensable tools of the working mathematician. Ewing's contributions to the growth and enhancement of *Mathematical Reviews* were substantial. With the advice of the senior MR staff and oversight committees, he worked first and foremost setting the agenda for improvements to the database. He is a detail-oriented person, and so his role was rather direct. He is very knowledgeable on the role of technology in building and serving information. He instituted a pricing scheme with the result that the number of institutions that could subscribe to MR doubled in a decade. He arranged for all reviews back to the first issue in 1940 to be digitized. There is no doubt that he was key to moving the project forward to produce the MR tools and database that we have today. Ewing chaired the Joint IMU/ICIAM/IMS Committee on Quantitative Assessment of Research Citation Statistics. Their report, a critical analysis of the use and misuse of citation statistics in science, was published as the lead article in *Statistical Science* in 2009. The report was featured in the *Wall Street Journal* and on MathDL.

In 2009 Ewing became president of Math for America. The mission of the nonprofit Math for America is "to improve mathematics education in U.S. public secondary schools by recruiting, training, and retaining outstanding mathematics teachers." When he became president, he said, "After three years, roughly forty percent of the teachers of mathematics [in U.S. schools] are gone.... You can't sustain a profession if you have that kind of attrition.... What Math for America does is concentrate on that one part of the problem. At the moment, it's bringing through something like forty to fifty new teachers a year. Our hope is to double that number in the next couple of years" (*Science*, January 2009). Since Ewing became president, Math for America has enlarged to about 420 participants, expanded to new cities, and developed new programs, including one for science teachers. Recently, he has again served on several MAA committees, including the Pólya Lecturer Committee and as chair of the 2010 search committee for the editor of the *Monthly*.

He has received the following awards: the Lester R. Ford Award (1975); SERC, Great Britain, Research Fellowship (1981); SFB Fellowship, Federal Republic of Germany (Göttingen) (1984); the MAA George Pólya Lectureship (1991–1992; 1992–1993); Honorary Doctorate of Science, St. Lawrence University (1995); the George Pólya Award (1996); Fellow, American Association for the Advancement of Science (2005).

Throughout his career, John Ewing has been an effective and firm, but gentle, leader. As Jonathan Borwein, a former MAA governor, wrote about serving with Ewing on the Committee on Electronic Information and Communication (CEIC) of the International Mathematical Union, "John is an enormously hard-working man—this is not a secret—who wears his remarkable erudition and breadth of knowledge very lightly. He is patient, hard to ruffle, and even harder to alienate. The CEIC was formed with many passionate members, all knowledgeable about some bits of the puzzle. It had only one expert: John Ewing. John's patience and generosity in educating the rest of us about the many pitfalls and subtleties was extraordinary. His care in trying to distinguish his role as committee member from that as AMS executive director (which could have made him the eight-hundred-pound gorilla on the committee) was remarkable."

Chauvenet Prize

The Chauvenet Prize recognizes a member or members of the MAA for the writing of an outstanding expository article. First awarded in 1925, the prize is named for William Chauvenet, who was a professor of mathematics at the United States Naval Academy and Washington University in St. Louis (where he founded the mathematics department).

The 2012 Chauvenet Prize has been awarded to Dennis DeTurck and Herman Gluck, both of the University of Pennsylvania; Daniel Pomerleano of the University of California, Berkeley; and David Shea Vela-Vick of Columbia University for their joint paper "The four vertex theorem and its converse", *Notices of the American Mathematical Society* **54** (2007), no. 2, 192–207. The four vertex theorem is a beautiful result in global differential geometry. It says that any smooth simple closed curve in the plane must have at least four "vertices"—local extrema for the curvature function. The vertices of a (noncircular) ellipse, for example, are located where it meets its major and minor axes. The four vertex theorem was proved for convex curves in 1909 by Syamadas Mukhopadhyaya and for general curves in 1912 by Adolf Kneser. Remarkably, the converse of the four vertex theorem is also true. Any continuous real-valued function on the circle with at least two local maxima and two local minima is the curvature function for some simple closed curve in the plane. The converse was proved for positive functions in 1971 by Herman Gluck and for arbitrary functions in 1997 by Björn Dahlberg.

The authors of this excellent expository article sketch Robert Osserman's 1985 proof of the four vertex theorem and Dahlberg's proof of the converse. The article ends with some generalizations of the four vertex theorem and biographical sketches of Mukhopadhyaya, Kneser, and Dahlberg. This carefully crafted survey has enough mathematical details to give the reader a sense of the proofs, but not so many as to obscure the big picture. Experts and nonexperts alike are sure to enjoy and understand this well-written and well-illustrated article. It is also a wonderful tribute to Björn Dahlberg, who passed away in early 1998, with his unpublished proof of the full converse still on his desk.

Euler Book Prize

The Euler Book Prize is given to the author(s) of an outstanding book about mathematics. Mathematical monographs at the undergraduate level, histories, biographies, works of mathematical fiction, and anthologies are among those types of books eligible for the prize. The prize was given for the first time in 2007, the 300th anniversary of the birth of Leonhard Euler.

Daina Taimiņa of Cornell University has been awarded the 2012 Euler Book Prize for *Crocheting Adventures with Hyperbolic Planes* (A K Peters, Ltd., 2009).

This book is unlike any previously considered for the Euler Prize. Indeed, it is unlike any book on hyperbolic geometry previously written, and it is in a different universe from any book on crochet previously written. But, when you look at it, the idea makes such perfect sense that it seems inevitable. Eugenio Beltrami, who in 1868 first modeled the non-Euclidean geometry of Bolyai and Lobachevsky by surfaces of negative curvature, actually toyed with the idea of building such surfaces. He made a small fragment of such a surface out of paper, and the idea was taken up again by William Thurston in the 1970s. But the idea did not take off, let alone reach a wide audience, until Daina Taimiņa wrote this book. By bringing crochet technology to the subject, she makes it easy and fun to construct hyperbolic surfaces that vividly illustrate essential features of non-Euclidean geometry. The book is elegant, from both a visual and mathematical point of view.

Thus *Crocheting Adventures with Hyperbolic Planes* is a novel approach to geometry that has brought a whole new audience to mathematics. In this respect it has greater outreach potential than any book we have previously considered. But it is much more than that; it is perfectly capable of standing on its mathematical feet as a clear, rigorous, and beautifully illustrated introduction to hyperbolic geometry. It is truly a book where art, craft, science, and mathematics come together in perfect harmony.

Beckenbach Book Prize

The Beckenbach Book Prize, established in 1986, is the successor to the MAA Book Prize, which was established in 1982. It is named for the late Edwin Beckenbach, a long-time leader in the publications program of the association and a well-known professor of mathematics at the University of California at Los Angeles. This prize is awarded to an author of a distinguished, innovative book published by the MAA. The award is not given on a regularly scheduled basis but is given only when a book appears that is judged to be truly outstanding.

The Beckenbach Book Prizes for 2012 were awarded to Dan Kalman (American University) and Nathan Carter (Bentley University)

Kalman was honored for *Uncommon Mathematical Excursions: Polynomia and Related Realms* (The Dolciani Mathematical Expositions, Vol. 35, Mathematical Association of America, 2009). Just how interesting can a book be whose only topic is polynomials? It doesn't take long to find out. By page 3 in Kalman's book, we encounter Lill's ingenious paper-folding technique for visualizing real roots of a polynomial, with an elegant proof based on Horner's method to evaluate a polynomial. Now most mathematicians are vaguely aware of Horner's method, but try to find one who knows Lill's technique, or one that knows that the first mention of Lill's idea in the United States appeared in an 1879 pamphlet by a lieutenant in the Army Corps of Engineers. By Chapter 2 we find that if a cubic $p(x)$ has three noncollinear roots in the complex plane, the unique ellipse inscribed in the triangle formed by those roots has the roots of $p'(x)$ as its foci (Marden's theorem) and that a polynomial $p(x)$ with positive integer coefficients is completely determined by the value of $p(1) = b$ and $p(b)$ (think base b). There is much, much more: palindromials, polynomial interpolation, symmetric functions, Newton's identities, and a brief history of Cardano's formula for the roots of a cubic (again, who knew Mark Kac sharpened his teeth at age fifteen on his own method of getting Cardano's formula?); and that is only the first third of the book! The next part covers max/min problems, beginning with a careful analysis of the Lagrange fallacy, then moving on to the Milkmaid problem, rotating ellipses, the ladder-around-the-corner (done with envelopes!), and the old shortest-time path through-or-around a field but with a discontinuous objective function. The last section focuses more on calculus questions, the wonder of elementary functions, and the expansion of the toolbox with special functions to solve, for example, $e^x = cx$. The exposition is perfect: expansive, relaxed, and detailed (almost nothing is stated without being proved). Although knowledge of calculus is needed, any good undergraduate mathematics major can read the entire book, as could many talented high school students. The book is a gold mine of topics for undergraduate talks. Kalman received his Ph.D. from the University of Wisconsin, Madison. He has been a frequent contributor to all of the MAA journals and has served on the editorial boards of both MAA book series and journals. He has served the national and regional MAA in several capacities, including a term as associate executive director for programs and as the current governor for the Maryland-DC-Virginia section.

Carter was honored for *Visual Group Theory* (Classroom Resource Materials Series, Mathematical Association of America, Washington, D.C., 2009). Burnside's classic text *Theory of Groups of Finite Order* was the first book written on group theory; it begins by defining a group as a collection of operators closed under multiplication and inverses (no associative law). Groups don't just sit there, they do something, they act on something, as permutations or symmetries or isometries. This is where groups come from, and it is the standard viewpoint both of users in physics, geometry, analysis, topology, and combinatorics, and also of specialists in permutation groups, representations, and geometric group theory. Unfortunately, this viewpoint is almost absent from present undergraduate textbooks. Nathan Carter's eye-opening textbook has a mission to fix that. Using a Rubik's Cube as a motivating example, he returns to Burnside's definition, only with "actions" instead of "operators". This is followed by Cayley diagrams to show how a group looks; Dehn's term *grüppenbilder*, or group pictures, was always the better name. Next come motivating examples: symmetries of polygons, crystals, frieze patterns, contra dancing, roots of polynomials. Chapter 4, entitled "Algebra at last", finishes with the "classic" definition of a group, although the action definition is really the classic one. Lastly, much of the traditional content of an undergraduate text on group theory can be found in Carter's text: the alternating and symmetric groups, abelian groups, subgroups, products and quotients, homomorphisms, Sylow theory, and a little Galois theory. The presentation is never traditional. Turn to any page and a figure jumps out: Cayley diagrams, colored multiplication tables, permutations as arrows from letter to letter—even Sylow theory is accompanied by conceptual sketches outlining the proofs. The exposition is breezy and leisurely: theorems are "taken out for a test drive," a proposition "gives us a headstart," a statement is "dense with notation" or "hairy". Steps are explained, proof strategies are analyzed—anything to loosen the seductive (after the fact) but intimidating (before the fact) concision and formality of algebra. The result is a textbook that reads like a conversation. Nathan Carter's *Visual Group Theory* is an original breakthrough that has a chance of transforming a staple of a mathematics major's diet. Carter received his

Ph.D. in mathematical logic from Indiana University in 2004. He uses computer science to advance mathematics.

Haimo Awards for Teaching

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Matthew DeLong is a passionate and reflective teacher who challenges his students, from the math-anxious to the most able, to higher levels of accomplishment. His personal qualities of integrity, creativity, caring, and patience contribute to his ability to connect with students on an individual basis and encourage them to do their best work. He has also become a leader in professional development for collegiate mathematics teaching and in student-centered instruction. While a Ph.D. student at the University of Michigan, he was the first graduate student put solely in charge of a course of ten sections of precalculus, received the Rackham Pedagogy Award for materials designed to facilitate the pedagogical development of graduate student instructors, and was responsible for helping plan and run the professional development week for graduate students and postdocs. He is the 2005 recipient of the MAA's Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Faculty Member. At Taylor University, DeLong started and leads the monthly School of Natural and Applied Sciences Educational Issues Seminar and is Taylor University's first Center for Teaching and Learning Excellence Fellow. He is dynamic in the classroom and implements progressive and innovative teaching strategies. A typical class session involves students presenting homework solutions, mini-lectures, and group problem-solving activities from handouts. He keeps the class moving quickly and on task, while encouraging the students to explore the "why" questions and to understand the mathematics. Students meet in groups outside of class to solve "team homework" problems, and they take turns writing the solutions to these problem sets. DeLong expects students to read the textbook before class by employing a blog for students to post questions on their reading. DeLong developed a liberal arts general education course at Taylor University that engages students in meaningful activities involving the application of mathematics skills and content as part of real-world problem solving. DeLong has published several articles on student-centered instruction and has given many regional and national professional presentations on teaching mathematics. He also coauthored the MAA book *Learning to Teach and Teaching to Learn Mathematics: Resources for Professional Development* (MAA Note Series, no. 57, 2001). This book presents a model for training college mathematics instructors in a collegiate program. Reviews say it "offers a treasure chest of ideas" and that this "high-quality comprehensive resource belongs in every mathematics instructor's hands." DeLong has led Taylor University's Mathematics Contest Team to a top-three finish seven times in the last ten years of the Indiana Collegiate Mathematics Competition, and he initiated an undergraduate research program at Taylor in which he has supervised students, two of whom have recently had their research published in *Mathematics Magazine*. In 1993 DeLong received his B.A. in mathematics and economics from Northwestern University and in 1998 he received his Ph.D. in mathematics from the University of Michigan. His mathematical interests are number theory and knot theory; other interests include faculty development, coaching little league sports, singing and conducting sacred choral music, performing and directing in musical theater, and singing in his faculty quartet, Quadrivium.

Susan Loepp has a profound influence on her students, challenging them to reach their full potential through high standards and talented encouragement. Her mentoring of a diverse group of students has inspired passion and encouraged many to major in mathematics. She also advises numerous undergraduates to successfully publish research and has created new courses and a concomitant book. She received an unprecedented three teaching awards as a graduate student at the University of Texas, Austin, and also the 2007 Alumni Award from her alma mater, Bethel College in Kansas. Her teaching evaluations at Austin, the University of Nebraska, and Williams College shine above most others. At Williams College, Loepp challenges and encourages a diverse group of students, men and notably women, athletes and scholars, the anxious and the overconfident. Loepp has high standards, assigning both daily and weekly problem sets, about which students have been known to brag how long it took and that they were able to finish. She says, "I am passionate about setting high standards for students and then helping them struggle to reach their potential." Loepp's colleagues write that she exudes energy, interest, confidence, and knowledge; "has a lively, friendly, inviting teaching style, but it's clear that she means business"; that she has the "courage to explore tough/pointless/all student questions, [even] if it [means] departing from her lesson plan"; and that she has an impressive ability to conduct class discussion: "She drew perceptive and eager responses from her students…due, in part, to her engaging them in vital mathematical conversation and exploration. Susan knows the importance of giving the students time to think." Loepp empowers students, connecting with them individually and inspiring mathematical zeal, encouraging many to become mathematics majors. Students are clearly infected by her enthusiasm. She also guides numerous students in undergraduate research in commutative algebra: she has seven joint papers with undergraduate students, and six additional research papers have been published by her students. She has advised thirty-two summer research students from across the country in the NSF "SMALL" undergraduate research project, and many of these have given student talks at conferences. A colleague writes, "It is remarkable that Loepp is able to make this very technical material accessible to undergraduate students. To bring students to the point of producing publishable results is truly incredible." Additionally, Loepp has advised thirty-five student colloquia at Williams College. Finally, she has created three courses that emphasize applications: a senior seminar on algebraic error-correcting codes, a course on quantum cryptography (which utilizes a book she cowrote with William K. Wootters), and an applied version of a core requirement in abstract algebra, including encryption on elliptic curves. Loepp received a B.A. in mathematics and a B.S. in physics from Bethel College (North Newton, Kansas) in 1989. She earned her Ph.D. in mathematics from the University of Texas at Austin in 1994. Her research area is commutative algebra. She is coauthor with William K. Wootters, an expert in quantum information theory, of the book *Protecting Information: From Classical Error Correction to Quantum Cryptography* (Cambridge University Press, Cambridge, 2006).

Cynthia Wyels is committed to student success. She works tirelessly to facilitate student learning, not only for her own students but also for students of her colleagues, students across her university, and students at other universities, including in other countries. Her devotion to supporting learning extends to students of all backgrounds and abilities. Wyels invests considerable time, creativity, and enthusiasm in developing, employing, and assessing innovative teaching practices. One example is her development of computer laboratory activities that lead students to understand and explore key concepts in a variety of undergraduate mathematics courses. Another example is her creation of in-class worksheets that help students to concentrate on the big picture of mathematical ideas being presented. More recently, she has begun to use "proof portfolios" to track students' abilities to construct complete, correct, and elegant proofs as they progress through their undergraduate studies. Wyels has shared all of her teaching innovations with her departmental colleagues and also more broadly through various MAA conferences and publications. Providing students with enriching research experiences is a particular passion of hers. She has led Research Experience for Undergraduates programs that have emphasized participation from students in underrepresented groups, particularly native Spanish speakers and first-generation college students. She has regularly donated her own faculty stipend in order to support participation of students from a Mexican university. She has mentored sixty-three undergraduates in research, including thirty-eight from underrepresented groups, with several of these projects leading to coauthored research papers. Wyels has also led efforts to institutionalize student research on her campus and to create an annual student research symposium. Wyels's dedication to effective teaching extends across her campus, as does her commitment to providing productive learning environments for all students. She founded the Critical Friends Group at CSU, Channel Islands, which colleagues report as having fostered a cultural shift in attitudes toward teaching, especially with regard to supporting students who initially find university culture to be confusing and alienating. Students and colleagues alike attest to the tremendous influence that she has had as a mentor, not only on their study of mathematics and their careers in mathematics and education but also in developing their self-confidence, persistence to succeed, and professionalism for handling all situations. Testimonials bear witness to her high standards and meticulousness and also to her generosity, enthusiasm, kindness, and selflessness. Wyels received her Ph.D. at the University of California, Santa Barbara. She has directed the graduate program at CSU, Channel Islands, since its inception. She is an advocate of undergraduate research and believes it is particularly meaningful for students from nontraditional backgrounds. Her research interests are in combinatorics, most recently in graph pebbling and graph labeling.

Certificates for Meritorious Service

Each year the MAA presents Certificates of Meritorious Service for service at the national level or for service to a section of the MAA. Those honored in 2012 are: David Kerr (Eckerd College), Florida Section; Joe Yanik (Emporia State University), Kansas Section; Ruth Favro (Lawrence Technological University), Michigan Section; Frank Ford (Providence College), Northeastern Section; Hortensia Soto-Johnson (University of Northern Colorado), Rocky Mountain Section; and Minerva Cordero-Epperson (University of Texas, Arlington), Texas Section.

— Elaine Kehoe

AWM Awards Given in Boston

The Association for Women in Mathematics (AWM) presented three awards at the Joint Mathematics Meetings in Boston, Massachusetts, in January 2012.

Shafer Prize

The Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman was established in 1990. The prize is named in honor of Alice T. Schafer, one of the founders of AWM and one of its past presidents. Schafer passed away in September of 2009.

The 2012 Schafer Prize was awarded to Fan Wei of the Massachusetts Institute of Technology. She is a senior at MIT who has distinguished herself both by her outstanding coursework and by the excellence and unusually broad range of her research. She has authored or coauthored five upcoming papers in fields as diverse as number theory, combinatorics, statistics, and tropical geometry. She has participated in multiple undergraduate research projects at MIT and in two summer REU programs. Of the latter, the first was at Williams College (summer of 2010), where she cowrote a paper investigating the properties of Rikuna polynomials. The second one was at the University of Minnesota–Twin Cities (summer of 2011), where she produced two papers: one on a connection between the evacuation of Young tableaux and chip-firing and the second on tropical properties for general chain graphs. The latter paper is single authored.

She has already presented her results at two conferences: Young Mathematician's Conference, Ohio State University (2010), and Permutation Patterns, Dartmouth College (2010). Her work is being described as "elegant," "intricate," "very creative," "quite surprising," and "having stirred up a lot of interest [in the area]." According to her mentors, she is expected to have a very successful career as a research mathematician because "she learns very quickly" and has "an excellent instinct for seeing what needs to be done and then doing it." In addition to her varied research projects, her coursework at MIT is absolutely outstanding: she has earned the top grade in twenty-one advanced mathematics courses, five of which were at the graduate level. Her MIT instructors describe her as "incredibly bright," "truly outstanding," "one of the best students I have ever had in the course," and "destined to excel." Aside from her research and coursework, Wei was part of a Meritorious Winner Team for the Mathematical Contest in Modeling (2010); she is a mentor for the Girl's Angle Math Club in Cambridge; and she has served on the board of MIT's Society of Women Engineers.

Louise Hay Award

Established in 1991, the Louise Hay Award for Contributions to Mathematics Education recognizes outstanding achievements in any area of mathematics education. Louise Hay was widely recognized for her contributions to mathematical logic and her devotion to students.

The 2012 award was presented to Bonnie Gold of Monmouth University for her long career of dedicated service to mathematics and mathematics education. Trained in mathematical logic (Ph.D., Cornell University, 1976), she found her true calling not only in teaching university-level mathematics but also in writing about and working for mathematics and mathematics education in the areas of assessment and philosophy of mathematics, in developing and directing New Jersey's Project NExT (New Experiences in Teaching), and in serving as the founding chair of the Special Interest Group of the Mathematical Association of America (MAA) on the Philosophy of Mathematics (POMSIGMAA). She has won local and MAA section teaching awards, has served as chair of two very different mathematics departments, and has developed a huge variety of courses, ranging from calculus for the biological sciences to Platonic dialogues as drama. Her publication contributions are similarly wide ranging: from coediting the books *Assessment Practices in*

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Undergraduate Mathematics and *Proof and Other Dilemmas: Mathematics and Philosophy* to contributing articles to a variety of MAA publications to writing insightful reviews of numerous books on mathematical philosophy.

Gold has given generously and extensively of her time to professional service. In addition to Project NExT and POMSIGMAA, she has served on and chaired MAA committees ranging from the Committee on Assessment to the Coordinating Council for Education and the Committee on the Teaching of Undergraduate Mathematics. Roger Simon writes eloquently of the "very high standards of quality and thoroughness" that she brings to all that she does. He notes that she has been an outstanding teacher of mathematics, a department chair of two very different departments, a "sustained contributor of service" to the profession, and a "leader in developing departmental assessment techniques," noting that "Louise Hay's career had the same kind of highlights." He goes on to note that her professional work with POMSIGMAA has resulted in "sustained, effective efforts to rekindle mathematicians' interests in the philosophy of mathematics." She has done all this with two major motivations: one is "to get many more mathematicians to think about philosophical issues"; the other "is that she believes that our understanding of what mathematics is affects the way we teach or should teach."

M. Gweneth Humphreys Award for Mentorship of Undergraduate Women in Mathematics

This award is named for M. Gweneth Humphreys (1911–2006). Humphreys graduated with honors in mathematics from the University of British Columbia in 1932, earning the prestigious Governor General's Gold Medal at graduation. After receiving her master's degree from Smith College in 1933, Humphreys earned her Ph.D. at age twenty-three from the University of Chicago in 1935. She taught mathematics to women for her entire career. This award, funded by contributions from her former students and colleagues at Randolph-Macon Woman's College, recognizes her commitment to and her profound influence on undergraduate students of mathematics.

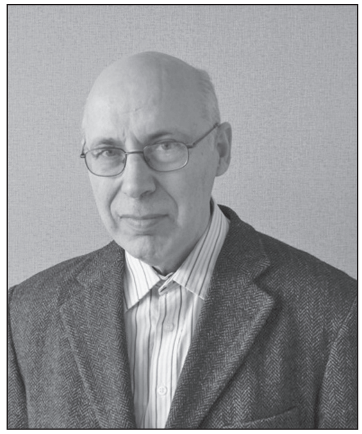
The 2012 award was presented to Deanna Haunsperger of Carleton College. Her nomination letters describe the amazing community of women in mathematics that she has created and nurtured for many years. She is a dedicated mentor, going out of her way to help young women make connections in the mathematical world. Together with Stephen Kennedy, she conceived of the Summer Mathematics Program (SMP) to mentor talented women early in their undergraduate studies. They have directed it nearly every summer since 1995, with Haunsperger playing the primary role in mentoring the participants. This program is different from other mathematics programs for women because it is intended for mathematically talented students in their first or second year of college who are uncertain about their future mathematical trajectories. Many are from small colleges from which few students go on to earn a Ph.D. in mathematics. The program gives these students a community of women who are serious about mathematics, and in the end many pursue graduate studies in mathematics.

Haunsperger has brought her energy and leadership to other projects as well. Colleagues at Carleton credit her with helping to build and sustain the strong community of mathematics majors there (the number of majors has doubled in the seventeen years since her arrival). She served as coeditor of *Math Horizons* and as second vice president of the MAA and chaired a key strategic planning group on MAA activities for students. More than fifty SMP graduates already have Ph.D.'s, and fifty more are currently in mathematics graduate programs. Her enthusiasm and dedication make the program and community the great success that they are.

— Elaine Kehoe

Mathematics People

Henkin Awarded 2011 Bergman Prize



Gennadi Henkin

Gennadi Henkin of the University of Paris has been awarded the 2011 Stefan Bergman Prize. Established in 1988, the prize recognizes mathematical accomplishments in the areas of research in which Stefan Bergman worked. The prize consists of one year's income from the prize fund, which was US$24,000 for 2011. The previous Bergman Prize winners are: David W. Catlin (1989), Steven R. Bell and Ewa Ligocka (1991), Charles Fefferman (1992), Yum Tong Siu (1993), John Erik Fornæss (1994), Harold P. Boas and Emil J. Straube (1995), David E. Barrett and Michael Christ (1997), John P. D'Angelo (1999), Masatake Kuranishi (2000), László Lempert and Sidney Webster (2001), M. Salah Baouendi and Linda Preiss Rothschild (2003), Joseph J. Kohn (2004), Elias M. Stein (2005), Kengo Hirachi (2006), Alexander Nagel and Stephen Wainger (2007-2008), and Ngaiming Mok and Duong H. Phong (2009). On the selection committee for the 2011 prize were Harold P. Boas, Carlos E. Kenig, and Linda Preiss Rothschild (chair).

Citation

G. M. Henkin has made fundamental contributions to the theory of functions on complex manifolds, integral representations in several complex variables, and the multidimensional Cauchy-Riemann equations. Henkin is a pioneer of the construction of explicit integral representations adapted to the geometry of a domain in a complex manifold. Sophisticated analysis of these concrete integrals leads to refined solutions of classical problems of several complex variables with precise control on uniform, Hölder, and L^p norms. Applications include uniform estimates for solutions of the inhomogeneous multidimensional Cauchy-Riemann equations, uniform estimates for extensions of holomorphic functions from submanifolds, and uniform approximation of holomorphic functions that are continuous on the closure of a domain. The Bergman kernel function is a canonical but nonexplicit holomorphic reproducing kernel. At the end of the 1960s Henkin and E. Ramírez de Arellano independently constructed on strongly pseudoconvex domains in $\mathbb{C}^n$ a noncanonical but explicit holomorphic reproducing kernel, the so-called Henkin-Ramírez kernel. This seminal work has seen a vast development at the hands of numerous authors, including Henkin and his many collaborators. The book *Theory of Functions on Complex Manifolds* by Henkin and J. Leiterer contains an account of the construction of global integral representations for differential forms in the general setting of strongly pseudoconvex polyhedra in Stein manifolds. A subsequent book by the same authors, *Andreotti-Grauert Theory by Integral Formulas*, contains an exposition of constructive integral representations on a class of complex manifolds intermediate between Stein manifolds and compact manifolds. The resulting uniform interpolation and approximation results for $\bar\partial$ ∂-cohomology classes have applications in the theory of holomorphic vector bundles and in the theory of the tangential Cauchy-Riemann equations. The Bergman theory concerns square-integrable holomorphic functions. In an influential paper with M. Gromov and M. Shubin, Henkin proved the infinite-dimensionality of the space of square-integrable holomorphic functions on pseudoconvex manifolds equipped with a suitable group action. Some of Henkin's work at the interface of complex analysis, partial differential equations, and geometry concerns the paradoxical question of solvability of unsolvable problems. Hans Lewy's famous example of an unsolvable partial differential equation is based on the local unsolvability in top degree of the tangential Cauchy-Riemann equations on a strongly pseudoconvex boundary. Henkin found a necessary and sufficient condition for global solvability, together with L^p estimates for the solution. A general compact, strongly pseudoconvex, three-dimensional CR-manifold cannot be embedded into complex Euclidean space, nor can a generic deformation of an embeddable one. In a remarkable paper with C. L. Epstein, Henkin revealed the subtle structure of embeddable deformations of embeddable three-dimensional CR-structures. Henkin has made many other noteworthy contributions, including the application of multidimensional integral representations to objects of mathematical physics, such as the complex Radon transform and inverse scattering problems; results on compactness and subellipticity of the $\bar{\partial}$-Neumann problem on domains with nonsmooth boundary; variations of the edge-of-the-wedge theorem; a study of zero sets of functions in the multidimensional Nevanlinna class; a proof of the biholomorphic inequivalence of analytic polyhedra and strongly pseudoconvex domains; and theorems on nonisomorphism of Banach spaces of holomorphic functions. G. M. Henkin's research is profound, insightful, and groundbreaking.

Biographical Sketch

Gennadi Markovich Henkin was born in Moscow in 1942. He received his Ph.D. in 1967 from Moscow State University, under the direction of Anatolii G. Vitushkin. In 1973 Henkin received the degree of Doctor of Sciences in Physics and Mathematics, also from Moscow State University, with a thesis entitled *Integral representations in some problems of functions in several complex variables.* Henkin has been a professor of mathematics at the University Paris VI since 1991 and a leading scientific researcher at the Central Economical Institute of the Russian Academy of Sciences since 1973. At the International Congress of Mathematicians in Warsaw in 1983, he gave a 45-minute invited address, "Tangent Cauchy-Riemann equations and the Yang-Mills, Higgs and Dirac fields". He received the Prize of the Moscow Mathematical Society in 1970, for works on the problem of isomorphic classification of Banach spaces of smooth and of holomorphic functions. In 1992 he was awarded the Kondratiev Prize of the Russian Academy of Sciences in Mathematical Economics, for works on Shumpeterian dynamics and nonlinear wave theory. Henkin has more than 130 publications in various areas, including complex and functional analysis, mathematical economics and evolution equations, integral geometry, and inverse problems.

About the Prize

The Bergman Prize honors the memory of Stefan Bergman, best known for his research in several complex variables, as well as the Bergman projection and the Bergman kernel function that bear his name. A native of Poland, he taught at Stanford University for many years and died in 1977 at the age of 82. He was an AMS member for 35 years. When his wife died, the terms of her will stipulated that funds should go toward a special prize in her husband's honor. The AMS was asked by Wells Fargo Bank of California, the managers of the Bergman Trust, to assemble a committee to select recipients of the prize. In addition the Society assisted Wells Fargo in interpreting the terms of the will to assure sufficient breadth in the mathematical areas in which the prize may be given. Awards are made every one or two years in the following areas: (1) the theory of the kernel function and its applications in real and complex analysis; and (2) function-theoretic methods in the theory of partial differential equations of elliptic type with attention to Bergman's operator method.

—*Allyn Jackson*

Schoenfeld and Radford Receive 2011 ICMI Medals

The International Congress on Mathematical Education (ICME) of the International Mathematical Union (IMU) has awarded two major prizes for 2011. ALAN SCHOENFELD of the University of California Berkeley has received the Felix Klein Medal for Lifetime Achievement, and LUIS RADFORD of Université Laurentienne, Sudbury, Canada, was awarded the Hans Freudenthal Medal for a major cumulative program of research.

From the Citation for Alan Schoenfeld

The Felix Klein Medal for 2011 is given to Alan H. Schoenfeld, University of California Berkeley, in recognition of his more than thirty years of sustained, outstanding lifetime achievements in mathematics education research and development. Alan Schoenfeld developed a keen interest in mathematics education early in his career and emerged as a leader in research on mathematical problem solving. He shows a lifelong pursuit of deeper understanding of the nature and development of mathematical learning and teaching. His work has helped to shape research and theory development in these areas, making a seminal impact on subsequent research. Alan Schoenfeld has also done fundamental theoretical and applied work that connects research and practice in assessment, mathematical curriculum, diversity in mathematics education, research methodology, and teacher education. He has more than 200 highly cited publications in mathematics education, mathematics, educational research, and educational psychology. His scholarship is of the highest quality, reflected in esteemed recognition over the years.

Alan Schoenfeld has nurtured a generation of new scholars who generate increasing impact on mathematics education research. He has undertaken a remarkable amount of outstanding work for national, regional, and international communities in education, mathematics, and mathematics education, providing leadership in professional associations and joint research endeavors, and has been an invited keynote speaker at numerous conferences around the globe.

Alan Schoenfeld began his career as a research mathematician. After obtaining a B.A. in mathematics from Queen's College, New York, in 1968 and an M.S in mathematics from Stanford University in 1969, he earned a Ph.D. in mathematics at Stanford in 1973. He became a lecturer at the University of California at Davis in 1973, and in 1975 a lecturer and research mathematician in the Graduate Group in Science and Mathematics Education (SESAME) at the University of California Berkeley. After academic appointments at Hamilton College (1978–1981) and the University of Rochester (1981–1984), Schoenfeld was invited back to U.C. Berkeley in 1985 to develop the mathematics education group. He has been a full professor since 1987 and now has a named chair in education and is an affiliated professor in the mathematics department. He has also been a Special Professor of the University of Nottingham since 1994.

He has been an elected member of the U.S. National Academy of Education since 1994, a member of its executive board in 1995, and vice president in 2001. He also served as the president of the American Educational Research Association (AERA) in 1998–1999. In 2000 he led the writing team for *Principles and Standards for School Mathematics* for the National Council of Teachers of Mathematics.

Among Alan Schoenfeld's many publications we mention his highly cited, groundbreaking book, *Mathematical Problem Solving* (1985); his chapter on cognition and metacognition, "Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics" (in the 1992 *Handbook for Research on Mathematics Teaching and Learning*); his rigorous study of the development and learning of a complex mathematical idea, *Learning* (1993, coauthored with J. P. Smith and A. A. Arcavi); his finely detailed work on teacher decision making, "Toward a theory of teaching-in-context" (published in *Issues in Education* in 1998); and his most recent book, *How We Think* (2010). Alan Schoenfeld's seminal theoretical contributions are all based on, and buttressed by, long sequences of carefully designed experiments and their exhaustive analysis.

From the Citation for Luis Radford

The Hans Freudenthal Medal for 2011 is given to Luis Radford, Université Laurentienne, Canada, in recognition of the theoretically well-conceived and highly coherent research program over the past two decades which has had a significant impact on the community. His development of a semiotic-cultural theory of learning has been anchored in detailed observations of students' algebraic activity. His research has been documented extensively in renowned scientific journals, books, and handbooks, as well as in numerous invited keynote presentations. The impact of Luis Radford's program of research has led to significant new insights in algebra teaching and learning and more broadly with his development of a widely applicable theory of learning.

Luis Radford has given many mentoring workshops for graduate students in Italy, Spain, Denmark, Colombia, Mexico, and Brazil. He has influenced teachers, teacher educators, and curriculum developers. He has served as associate editor of *For the Learning of Mathematics* and is currently an associate editor of *Educational Studies in Mathematics*.

Luis Radford graduated from the Universidad de San Carlos in Guatemala in 1977 with a degree in civil engineering. He then taught at that university's engineering school, followed by studies at Université Louis Pasteur I, Strasbourg, France, where he obtained a Licence in Mathematics and Fundamental Applications in 1981, a Diplôme of Advanced Studies in Mathematical Didactics in 1983, and a Doctorat de troisième cycle in Mathematical Didactics in 1985. He then returned to Guatemala, where he taught as an associate professor at the Universidad de San Carlos in the Humanities Faculty. In 1992 he moved to Canada, where he obtained a position in the School of Education at Université Laurentienne, Sudbury, Ontario, as full professor.

Luis Radford's research program can be traced back to the early 1990s, when he initiated a study that examined the role of historical-epistemological analyses of learning within a sociocultural perspective. His work continued to evolve, drawing upon the works of Vygotsky, Bakhtin, and Voloshinov to develop a semiotic-cultural framework to investigate the ways in which students use signs and endow them with meaning in their initial encounters with algebra. In further development he elaborated the notion that thinking is a sensuous and sign-mediated reflective activity embodied in the corporeality of actions, gestures, and artifacts, leading to a formulation of knowing and being as mutually constitutive. Luis Radford has more than 170 publications, many of them highly cited. Luis Radford's research program was ranked first in three consecutive competitions of the Social Sciences and Humanities Research Council of Canada (Education 1): 2004–2007, 2007–2010, and 2010–2013.

—From an ICME announcement

CMI Awards Announced

The Clay Mathematics Institute (CMI) has announced several awards for 2012.

Jeremy Kahn of Brown University and Vladimir Markovic of the California Institute of Technology have been named recipients of the 2012 Clay Research Award "for their work in hyperbolic geometry: (1) their proof that a closed hyperbolic three manifold has an essential immersed hyperbolic Riemann surface, i.e., the map on fundamental groups is injective; (2) their solution of the Ehrenpreis conjecture: that given any two compact hyperbolic Riemann surfaces, there are finite covers of the two surfaces which are arbitrarily close in the Teichmüller metric."

Ivan Corwin and Jack Thorne have been named Research Fellows for four and five years, respectively. Corwin received his Ph.D. in 2011 from New York University. His research involves computing exact formulas for the statistics of the solution to the Kardar-Parisi-Zhang nonlinear stochastic PDE. He presently holds the Schramm Memorial Postdoctoral Fellowship at Microsoft Research New England and the Massachusetts Institute of Technology. Thorne will receive his Ph.D. this year from Harvard University. His primary research interests are algebraic number theory and representation theory and the diverse connections between them. Most recently he has been interested in using automorphy lifting techniques to establish new cases of the Fontaine-Mazur conjecture.

Roman Travkin has been appointed a Clay Research Scholar for a period of three years. He will receive his Ph.D. this year from the Massachusetts Institute of Technology. His thesis proved the "generic part" of the quantum geometric Langlands duality for the group $GL(n)$ over a base field of positive characteristic, showing that in this context it is a twisted version of the Fourier-Mukai transform.

The awards will be presented at the 2012 Clay Research Conference, to be held June 18–19 at Oxford University in the Martin Wood Lecture Theatre of the Physics Department. Both Kahn and Markovic will speak on their work at that occasion. Other confirmed speakers are: Artur Avila (IMPA/CNRS), Francis Brown (CNRS), Stavros Garoufalidis (Georgia Tech), Marc Lackenby (Oxford), and Peter Scholze (Univ. of Bonn). Registration is free but strongly recommended. Please see `http://www.claymath.org/`.

—From a CMI announcement

McKibbin Awarded ANZIAM Medal

Robert McKibbin of Massey University has been awarded the 2012 ANZIAM (Australian and New Zealand Industrial and Applied Mathematics group) Medal for his work in applied and industrial mathematics. According to the prize citation, "he has been one of the preeminent applied mathematicians in New Zealand, with a particular focus on geophysical, geothermal and industrial applications. His mathematical work ranges from geothermal fluid dynamics and hydrothermal eruptions to the modeling of ground subsidence and aluminium smelting cells." The medal is awarded every two years for wide-ranging contributions to the discipline on the basis of a combination of research achievements, activities enhancing applied or industrial mathematics or both, and contributions to ANZIAM.

—From an ANZIAM announcement

Long Awarded Michler Prize

Ling Long of Iowa State University has been awarded the 2012–2013 Ruth I. Michler Memorial Prize by the Association for Women in Mathematics (AWM). Her research involves modular forms for finite index subgroups of the modular group. These groups play an important role in Grothendieck's program of *dessins d'enfants* (children's drawings). In 1997 she earned a B.Sc. from Tsinghua University, Beijing, China, majoring in mathematics with a minor in computer science and engineering. Long received her Ph.D. in mathematics from the Pennsylvania State University in 2002, where she studied modularity of elliptic surfaces under the direction of Wen-Ching Winnie Li and Noriko Yui. At Cornell she plans to work with Ravi Ramakrishna on Galois representations attached to noncongruence modular forms based on the pioneering work of Anthony Scholl and her joint work with Oliver Atkin, Li, and Tong Liu.

The Michler Prize grants a midcareer woman in academia a residential fellowship in the Cornell University mathematics department without teaching obligations.

—From an AWM announcement

Jona-Lasinio Receives Heineman Prize

Giovanni Jona-Lasinio of the University of Rome has been awarded the 2012 Dannie Heineman Prize for Mathematical Physics. He was honored "for contributions to the interaction between statistical mechanics, field theory and the theory of elementary particles, including spontaneous symmetry breaking, critical phenomena and a general theory of dissipative systems."

The prize carries a cash award of US$10,000 and is presented in recognition of outstanding publications in the field of mathematical physics. The prize was established in 1959 by the Heineman Foundation for Research, Educational, Charitable, and Scientific Purposes, Inc., and is administered jointly by the American Institute of Physics (AIP) and the American Physical Society (APS). The prize is presented annually.

—From an APS announcement

National Academy of Engineering Elections

The National Academy of Engineering (NAE) has elected sixty-six new members and ten foreign associates. Michael S. Waterman, professor of biological sciences, computer science, and mathematics at the University of Southern California, was elected "for development of computational methods for DNA and protein sequence analyses." Frank P. Kelly, professor of the mathematics of systems and master of Christ's College, University of Cambridge, was elected "for contributions to the theory and optimization of communication networks."

—From an NAE announcement

Mathematics Opportunities

Math for America Early Career Fellowship

Math for America (MfA) is a nonprofit organization with a mission to improve mathematics education in U.S. public secondary schools by recruiting, training, and retaining outstanding mathematics teachers and leaders. MfA offers fellowships for new and experienced teachers and school leaders. The Math for America Early Career Fellowship is awarded to public secondary school mathematics teachers early in their careers. MfA Early Career Fellows exhibit outstanding potential, a dedication to professional development, and an interest in collaboration with the Math for America community. The program provides professional support and growth opportunities for new teachers. The MfA Early Career Fellowship requires a commitment of four years. Applications are being accepted for the Early Career Fellowship in New York City. The deadline is **May 4, 2012**. For more information and to apply, see `http://www.mathforamerica.org/web/guest/apply`.

—From an MfA announcement

NSF Postdoctoral Research Fellowships

The National Science Foundation (NSF) awards Mathematical Sciences Postdoctoral Research Fellowships (MSPRF) for appropriate research in areas of the mathematical sciences, including applications to other disciplines. Awardees are permitted to choose research environments that will have maximal impact on their future scientific development. Awards are made in the form of either Research Fellowships or Research Instructorships. The Research Fellowship option provides full-time support for any eighteen academic-year months in a three-year period, in intervals not shorter than three consecutive months. The Research Instructorship option provides either two academic years of full-time support or one academic year of full-time and two academic years of half-time support. Under both options, the award includes six summer months; however, no more than two summer months of support may be received in any calendar year. Under both options, the stipend support for twenty-four months (eighteen academic-year months plus six summer months) will be provided within a forty-eight-month period.

The deadline for proposals is **October 17, 2012**. See `http://www.nsf.gov/pubs/2008/nsf08582/nsf08582.htm`.

—From an NSF announcement

Call for Nominations for SASTRA Ramanujan Prize

The Shanmugha Arts, Science, Technology Research Academy (SASTRA) invites nominations for the 2012 SASTRA Ramanujan Prize. The prize carries a cash award of US$10,000. The deadline for nominations is **June 30, 2012**.

The recipient will be invited to speak at the International Conference on the Legacy of Srinivasa Ramanujan at SASTRA University in Kumbakonam to be held December 14–16, 2012. The prize will be given during the celebration of the 125th anniversary of Ramanujan's birth. For more information, email `sastraprize@math.ufl.edu`, or see the website `http://www.math.ufl.edu/sastra-prize/nominations-2012.html`.

—Krishnaswami Alladi
University of Florida

Call for Nominations for 2012 Parzen Prize for Statistical Innovations

To promote the dissemination of statistical innovation, the Emanuel and Carol Parzen Prize for Statistical Innovation is awarded in even-numbered years to North American statisticians who have made outstanding and influential contributions by the development of innovative statistical methods and who received their Ph.D.'s more than twenty-five years ago. The Parzen Prize is awarded by the Department of Statistics at Texas A&M University to a nominee selected by the members of the Parzen Prize Committee. The prize consists of an honorarium of US$1,000 and travel to College Station, Texas, to present a lecture at the prize ceremony. The deadline for nominations for the

2012 prize is **June 15, 2012**. Nominations should be sent to Thomas Wehrly, Department of Statistics, 3143 TAMU, Texas A&M University, College Station, TX 77843-3143.

—From a Texas A&M announcement

International Mathematics Competition for University Students

The Nineteenth International Mathematics Competition (IMC) for University Students will be held July 26 through August 1, 2012, at American University in Blagoevgrad, Bulgaria. Participating universities are invited to send several students and one teacher; individual students are welcome. Students completing their first, second, third, or fourth years of university education are eligible. The competition will consist of two sessions of five hours each. Problems will come from the fields of algebra, analysis (real and complex), geometry, and combinatorics. The working language will be English. See the website `http://www.imc-math.org.uk/` or contact John Jayne, University College London, Gower Street, London WC1E 6BT, United Kingdom; telephone: +44 (0)77-40304010; email: `j.jayne@ucl.ac.uk`.

—John Jayne
University College London

News from IPAM

The Institute for Pure and Applied Mathematics (IPAM), an NSF math institute located at the University of California Los Angeles, offers programs that encourage cross-disciplinary collaboration. IPAM holds long- and short-term research programs and workshops throughout the academic year for junior and senior mathematicians and scientists who work in academia, the national laboratories, and industry.

Currently, IPAM is in the midst of its long program, "Computational Methods in High Energy Density Plasmas". Researchers from mathematics, physics, and engineering are in residence at IPAM, and a series of workshops are in progress. The final workshop, to be held the week of May 21, is entitled "Computational Challenges in Warm Dense Matter". IPAM sponsors two summer programs. Students in our undergraduate program "Research in Industrial Projects for Students" (RIPS) will work in teams on industry-sponsored research projects. Our summer school, "Deep Learning, Feature Learning", will be held July 9–27, 2012. The application deadlines for these programs have passed, but you may find more information on IPAM's website.

The Science Advisory Board will meet in November to consider workshop proposals for winter 2014, summer school proposals for summer 2014, and long program proposals for academic year 2014–2015. Program proposals from the community are welcome; instructions are available on our website.

IPAM's upcoming programs are listed below. Please go to `www.ipam.ucla.edu` for detailed information and to find application and registration forms.

September 10–December 14, 2012. Materials Defects: Mathematics, Computation, and Engineering. You may apply online for support to be a core participant for the entire program or to attend any of the following individual workshops.

September 11–14, 2012: Tutorials.

October 1–5, 2012: Workshop I: Quantum and Atomistic Modeling of Materials Defects.

October 22–26, 2012: Workshop II: Atomistic and Mesoscale Modeling of Materials Defects.

November 13–16, 2012: Workshop III: Mesoscale and Continuum Scale Modeling of Materials Defects.

December 3–7, 2012: Workshop IV: Computational Methods for Multiscale Modeling of Materials

January 14–March 8, 2013. Winter Workshops. You may apply for support or register for each workshop online.

January 14–18, 2013: Structure and Randomness in System Identification and Learning.

January 28–February 1, 2013: Adaptive Data Analysis and Sparsity.

February 11–15, 2013: Convex Relaxation Methods for Geometric Problems in Scientific Computing.

March 4–8, 2013: Multimodal Neuroimaging.

March 11–June 14, 2013. Interactions between Analysis and Geometry. You may apply online for support to be a core participant for the entire program or to attend any of the following individual workshops.

March 12–15, 2013: Tutorials.

March 18–22, 2013: Workshop I: Analysis on Metric Spaces.

April 8–12, 2013: Workshop II: Dynamics of Groups and Rational Maps.

April 29–May 3, 2013: Workshop III: Non-Smooth Geometry.

May 20–24, 2013: Workshop IV: Quasiconformal Geometry and Elliptic PDEs.

—From an IPAM announcement

Inside the AMS

Epsilon Awards for 2012

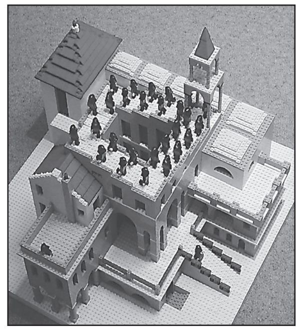
The AMS Epsilon Fund for Young Scholars was established in 1999 to provide financial assistance to summer programs in the United States and Canada for mathematically talented high school students. These programs have provided mathematically talented youngsters with their first serious mathematical experiences. The name for the fund was chosen in remembrance of the late Paul Erdős, who was fond of calling children "epsilons".

The AMS has chosen thirteen summer mathematics programs to receive Epsilon grants for activities in the summer of 2012. The grants will support program expenses and student scholarships and, in some cases, scholarships only. The programs were chosen on the basis of mathematical excellence and enthusiasm. Award amounts were governed by the varying financial needs of each program. The 2012 grants are awarded to: Canada/USA Mathcamp, University of Puget Sound, Tacoma, Washington; Governor's Institutes of Vermont: Mathematical Sciences, University of Vermont; Hampshire College Summer Studies in Mathematics (HCSSiM), Hampshire College, Amherst, Massachusetts; Lamar Achievement in Mathematics Program (LAMP), Lamar University, Beaumont, Texas; MathPath, Mount Holyoke College, South Hadley, Massachusetts; Mathworks Honors Summer Math Camp, Texas State University, San Marcos, Texas; PROMYS, Boston University; PROTaSM (Puerto Rico Opportunities for Talented Students in Mathematics), University of Puerto Rico, Mayagüez Campus; Research Science Institute, Massachusetts Institute of Technology; Ross Mathematics Program, The Ohio State University; Stanford University Mathematics Camp (SUMaC), Stanford University; Summer Program in Mathematical Problem Solving, Bard College, New York; Young Scholars Program, University of Chicago. The grants for summer 2012 are paid for by the AMS Epsilon Fund for Young Scholars. The AMS Epsilon Fund for Young Scholars has been funded by contributions of AMS members and friends; the goal of the endowment is to provide at least US$100,000 in support each summer.

For further information about the Epsilon Fund for Young Scholars, visit the website `http://www.ams.org/giving-to-ams/` or contact `development@ams.org`. Information about how to apply for Epsilon grants is available at `http://www.ams.org/programs/edu-support/epsilon/emp-epsilon/`. A fairly comprehensive listing of summer programs for mathematically talented high school students (including those with and without Epsilon grants) is available at `http://www.ams.org/employment/mathcamps.html`.

—AMS Development Office

From the AMS Public Awareness Office



Lipson's Lego® Sculptures. This new album on Mathematical Imagery includes constructions of M. C. Escher drawings and other mathematical forms. You can also send the images as e-postcards: `http://www.ams.org/mathimagery/thumbnails.php?album=29`.

Mathematics Events at the 2012 AAAS Meeting. Read about some of the events related to mathematics that took place at the 2012 annual meeting of the American Association for the Advancement of Science in Vancouver, BC, Canada, February 16–20, including Who Wants to Be a Mathematician: `http://www.ams.org/meetings/aaas2012`.

—Annette Emerson and Mike Breen
AMS Public Awareness Officers
`paoffice@ams.org`

AMS Holds Workshop for Department Chairs

The AMS held its annual workshop for department chairs prior to the Joint Mathematics Meeting in Boston, Massachusetts, in January 2012. This one-day session focused on a range of issues facing departments, including balancing faculty workload, the role of a department chair as a steward of the discipline, alternate forms of course delivery, and understanding the mathematical background of students in entry-level service courses and how to address their needs while upholding expectations and standards. The meeting is designed in a workshop format to stimulate discussion and facilitate the sharing of ideas and experiences among attending department chairs, which allows attendees to address departmental challenges from new perspectives.

The 2012 workshop was led by Timothy Hodges, University of Cincinnati; John Meakin, University of Nebraska-Lincoln; Helen Roberts, Montclair State University; and Alex Smith, University of Wisconsin-Eau Claire.

—Anita Benjamin
AMS Washington Office

Deaths of AMS Members

Donald L. Arenson, of Skokie, Illinois, died on January 21, 2010. Born on June 15, 1926, he was a member of the Society for 59 years.

M. Salah Baouendi, professor, University of California San Diego, died on December 24, 2011. Born on October 12, 1937, he was a member of the Society for 39 years.

Mario Benedicty, of Palo Alto, California, died on April 8, 2011. Born on July 16, 1922, he was a member of the Society for 47 years.

Ellen F. Buck, of Madison, Wisconsin, died on January 9, 2011. Born on August 25, 1919, she was a member of the Society for 67 years.

Frederic Cunningham Jr., professor, Bryn Mawr College, died on September 28, 2011. Born on September 6, 1921, he was a member of the Society for 63 years.

Torsten Ekedahl, of Stockholm, Sweden, died on November 23, 2011. Born on August 11, 1955, he was a member of the Society for 24 years.

John M. Howie, professor, University of Saint Andrews, died on December 26, 2011. Born on May 23, 1936, he was a member of the Society for 46 years.

Marvin I. Knopp, professor, Temple University, died on December 24, 2011. Born on January 4, 1933, he was a member of the Society for 54 years.

Solomon Leader, professor, Rutgers University, died on August 13, 2011. Born on November 14, 1925, he was a member of the Society for 59 years.

Eugene H. Lehamn, of Quebec, Canada, died on December 15, 2011. Born on January 26, 1913, he was a member of the Society for 49 years.

John M. Marr, professor, Kansas State University, died on May 3, 2011. Born on June 15, 1920, he was a member of the Society for 61 years.

Charles N. Maxwell, of Carbondale, Illinois, died on June 28, 2010. Born on October 27, 1927, he was a member of the Society for 56 years.

Meinhard E. Mayer, professor, University of California Irvine, died on December 11, 2011. Born on March 18, 1929, he was a member of the Society for 43 years.

Paul Meier, of New York, New York, died on August 7, 2011. Born on July 24, 1924, he was a member of the Society for 65 years.

Norman Oler, professor, University of Pennsylvania, died on November 1, 2011. Born on July 12, 1929, he was a member of the Society for 50 years.

Baburao Govindrao Pachpatte, of Aurangabad, India, died on August 3, 2011. Born on November 21, 1943, he was a member of the Society for 17 years.

Israel H. Rose, of Hastings-on-Hudson, New York, died on October 30, 2011. Born on May 17, 1917, he was a member of the Society for 68 years.

Robert J. Silverman, of Durham, New Hampshire, died on February 25, 2010. Born on November 24, 1922, he was a member of the Society for 58 years.

Edward John Specht, of Bloomington, Indiana, died on November 9, 2011. Born on July 29, 1915, he was a member of the Society for 68 years.

Paul F. Wacker, of Redlands, California, died on September 7, 2010. Born on May 25, 1914, he was a member of the Society for 51 years.

Herbert S. Wilf, professor, University of Pennsylvania, died on January 7, 2012. Born on June 13, 1931, he was a member of the Society for 59 years.

For Your Information

Kavli Foundation Endows Tokyo Institute

The University of Tokyo has announced the establishment of an endowment by the Kavli Foundation to support the Institute for the Physics and Mathematics of the Universe (IPMU).

The Institute, which will now be known as the Kavli Institute for the Physics and Mathematics of the Universe (Kavli IPMU), probes the biggest mysteries in modern cosmology: How did the universe begin, and how will it end? What is it made of, and what laws govern its behavior? How did we come to exist? The Institute is seeking answers through collaborative research conducted by a wide range of scientists, including mathematicians, theoretical physicists, experimental physicists, and astronomers.

The Kavli IPMU comprises about two hundred researchers from fifteen fields, with almost half coming from outside Japan. Reflecting its dedication to multidisciplinary collaboration, the Institute is embodied in a five-story research building at the Kashiwa campus, outside of Tokyo in Chiba prefecture, where researchers from different fields typically alternate offices, and the hallways gradually ramp from floor to floor to encourage informal connections.

The Kavli IPMU director is Professor Hitoshi Murayama, a particle physicist from the University of California Berkeley. Among the principal investigators are four in mathematics: Alexey Bondal, Toshiyuki Kobayashi, Toshitake Kohno, and Kyoji Saito. Hirosi Ooguri is a principal investigator in mathematical physics.

IPMU was originally established in the fall of 2007 as part of the World Premier International Research Center Initiative, a program of the Japanese government to promote interdisciplinary science in Japan, its international visibility, and globalization of the Japanese universities.

—From a Kavli Foundation news release

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.wustl.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

April 30, 2012: Nominations for AWM Gweneth Humphreys Award. See `www.awm-math.org`, telephone: 703-934-0163, or email: `awm@awm-math.org`.

May 1, 2012: Letters of intent for NSF Integrative Graduate Education and Research Training (IGERT) program. See the website `http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=12759`.

May 1, 2012: Applications for National Academies Research Associateship Programs. See `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2760; fax: 202-334-2759; email: `rap@nas.edu`

May 1, 2012: Applications for National Academies Christine Mirzayan Graduate Fellowship Program for fall 2012. See the website `http://sites.nationalacademies.org/PGA/policyfellows/index.htm` or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667; email: `policyfellows@nas.edu`.

May 1, 2012: Applications for AWM Travel Grants. See `http://www.awm-math.org/travelgrants.html#standard` or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030; 703-934-0163; `awm@awm-math.org`.

May 4, 2012: Applications for Math for America Early Career Fellowship in New York City. See "Mathematics Opportunities" in this issue.

May 15–June 15, 2012: Proposals for NSF DMS Workforce Program in the Mathematical Sciences. See `http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=503233`.

June 15, 2012: Deadline for nominations for 2012 Parzen Prize for Statistical Innovations. See "Mathematics Opportunities" in this issue.

June 30, 2012: Deadline for nominations for SASTRA Ramanujan Prize. See "Mathematics Opportunities" in this issue.

July 2, 2012: Full proposals for NSF Integrative Graduate Education and Research Training (IGERT) program.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

See http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=12759.

July 10, 2012: Full proposals for NSF Research Networks in the Mathematical Sciences. See http://www.nsf.gov/pubs/2010/nsf10584/nsf10584.htm?WT.mc_id=USNSF_25&WT.mc_ev=click.

August 1, 2012: Applications for National Academies Research Associateship Programs. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2760; fax: 202-334-2759; email: rap@nas.edu.

August 14, 2012: Full proposals for NSF Scholarships in Science, Technology, Engineering, and Mathematics (S-STEM) program. See http://www.nsf.gov/pubs/2012/nsf12529/nsf12529.htm.

September 15, 2012: Applications for spring 2013 semester of Math in Moscow. See http://www.mccme.ru/mathinmoscow or write to: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: mim@mccme.ru. Information and application forms for the AMS scholarships are available on the AMS website at http://www.ams.org/programs/travel-grants/mimoscow or by writing to: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2294; email: student-serv@ams.org.

October 1, 2012: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html#standard or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030; 703-934-0163; awm@awm-math.org.

October 17, 2012: Proposals for NSF Postdoctoral Research Fellowships. See “Mathematics Opportunities” in this issue.

November 1, 2012: Applications for National Academies Research Associateship Programs. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2760; fax: 202-334-2759; email: rap@nas.edu.

MPS Advisory Committee

Following are the names and affiliations of the members of the Advisory Committee for Mathematical and Physical Sciences (MPS) of the National Science Foundation. The date of the expiration of each member's term is given after his or her name. The website for the MPS directorate may be found at www.nsf.gov/home/mps/. The postal address is Directorate for the Mathematical and Physical Sciences, National Science Foundation, 4201 Wilson Boulevard, Arlington, VA 22230.

Taft Armandroff (09/12)
W. M. Keck Observatory
Kamuela, Hawaii

James Berger (chair) (09/14)
Department of Statistical Science
Duke University

Daniela Bortoletto (09/14)
Department of Physics
Purdue University

Emery N. Brown (09/14)
Massachusetts Institute of Technology

R. Paul Butler (09/13)
Department of Terrestrial Magnetism
Carnegie Institution of Washington

Kevin Corlette (09/12)
Department of Mathematics
University of Chicago

Eric A. Cornell (09/13)
JILA
University of Colorado

George W. Crabtree (09/13)
Materials Science Division
Argonne National Laboratory

Juan J. de Pablo (09/12)
Department of Chemical and Biological Engineering
University of Wisconsin-Madison

Joseph M. DeSimone (09/12)
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University of North Carolina at Chapel Hill

Francis J. DiSalvo Jr. (09/14}
Department of Chemistry
Cornell University

Bruce Elmegreen (09/14)
IBM Watson Research Center

Barbara J. Finlayson-Pitts (09/14)
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Irene Fonseca (09/14)
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Carnegie Mellon University

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Department of Chemical Engineering
University of Michigan

Naomi J. Halas (09/13)
ECE Department
Rice University

Elizabeth Lada (09/14)
Department of Astronomy
University of Florida

Jerzy Leszczynski (09/12)
Department of Chemistry and Biochemistry
Jackson State University

Dennis L. Matthews (09/13)
College of Engineering and School of Medicine
University of California, Davis

Juan C. Meza (09/13)
University of California, Merced

Michael L. Norman (09/13)
Department of Physics
University of California, San Diego

Luis Orozco (09/12)
Department of Physics
University of Maryland, College Park

Eugenia Paulus (09/13)
Department of Chemistry
North Hennepin Community College

Elsa Reichmanis (09/14)
School of Chemical and Biomolecular Engineering
Georgia Institute of Technology

Fred S. Roberts (09/12)
DIMACS
Rutgers University

Esther S. Takeuchi (09/13)
State University of New York, Buffalo

Geoffrey West (09/14)
Santa Fe Institute
Santa Fe, NM

Book List

The Book List highlights recent books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. Suggestions for books to include on the list may be sent to notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

**Adventures in Group Theory: Rubik's Cube, Merlin's Machine, and Other Mathematical Toys*, by David Joyner. Johns Hopkins University Press (second edition), December 2008. ISBN: 978-08018-9013-0.

The Adventure of Reason: Interplay between Philosophy of Mathematics and Mathematical Logic, 1900–1940, by Paolo Mancosu. Oxford University Press, January 2011. ISBN-13: 978-01995-465-34.

**The Annotated Turing: A Guided Tour Through Alan Turing's Historic Paper on Computability and the Turing Machine*, by Charles Petzold. Wiley, June 2008. ISBN-13: 978-04702-290-57. (Reviewed September 2011.)

The Autonomy of Mathematical Knowledge: Hilbert's Program Revisited, by Curtis Franks. Cambridge University Press, December 2010. ISBN-13: 978-05211-838-95.

The Beginning of Infinity: Explanations That Transform the World, by David Deutsch. Viking Adult, July 2011. ISBN-13: 978-06700-227-55. (Reviewed April 2012.)

The Best Writing on Mathematics: 2010, edited by Mircea Pitici. Princeton University Press, December 2010. ISBN-13: 978-06911-484-10. (Reviewed November 2011.)

The Big Questions: Mathematics, by Tony Crilly. Quercus, April 2011. ISBN-13: 978-18491-624-01.

The Blind Spot: Science and the Crisis of Uncertainty, by William Byers. Princeton University Press, April 2011. ISBN-13:978-06911-468-43.

The Calculus of Selfishness, by Karl Sigmund. Princeton University Press, January 2010. ISBN-13: 978-06911-427-53. (Reviewed January 2012.)

Chasing Shadows: Mathematics, Astronomy, and the Early History of Eclipse Reckoning, by Clemency Montelle. Johns Hopkins University Press, April 2011. ISBN-13: 978-08018-969-10. (Reviewed March 2012.)

Crafting by Concepts: Fiber Arts and Mathematics, by Sarah-Marie Belcastro and Carolyn Yackel. A K Peters/CRC Press, March 2011. ISBN-13: 978-15688-143-53.

The Crest of the Peacock: Non-European Roots of Mathematics, by George Gheverghese Joseph. Third edition. Princeton University Press, October 2010. ISBN: 978-0-691-13526-7.

Cycles of Time: An Extraordinary New View of the Universe, by Roger Penrose. Knopf, May 2011. ISBN-13: 978-03072-659-06.

Divine Machines: Leibniz and the Sciences of Life, by Justin E. H. Smith. Princeton University Press, May 2011. ISBN-13: 978-06911-417-87.

An Early History of Recursive Functions and Computability from Gödel to Turing, by Rod Adams. Docent Press, May 2011. ISBN-13: 978-09837-004-01.

**Elegance with Substance*, by Thomas Colignatus. Dutch University Press, 2009. ISBN 978-90361-013-87.

Emmy Noether's Wonderful Theorem, by Dwight E. Neuenschwander. Johns Hopkins University Press, November 2010. ISBN-13: 978-08018-969-41.

The Evolution of Logic, by W. D. Hart. Cambridge University Press, August 2010. ISBN-13: 978-0-521-74772-1

Experimental and Computational Mathematics: Selected Writings, by Jonathan Borwein and Peter Borwein. PSIpress, 2011. ISBN: 978-19356-380-56.

Excursions in the History of Mathematics, by Israel Kleiner. Birkhäuser, 2012. ISBN-13: 978-08176-826-75.

Fascinating Mathematical People: Interviews and Memoirs, edited by Donald J. Albers and Gerald L. Alexanderson. Princeton University Press, October 2011. ISBN: 978-06911-482-98.

**Flatland*, by Edwin A. Abbott, with notes and commentary by William F. Lindgren and Thomas F. Banchoff. Cambridge University Press, November 2009. ISBN-13: 978-05217-599-46.

**Gösta Mittag-Leffler: A Man of Conviction*, by Arild Stubhaug (translated by Tiina Nunnally). Springer, November 2010. ISBN: 978-36421-167-11.

Gottfried Wilhelm Leibniz: The Polymath Who Brought Us Calculus, by M. B. W. Tent. A K Peters/CRC Press, October 2011. ISBN: 978-14398-922-20.

The History and Development of Nomography, by H. A. Evesham. Docent Press, December 2010. ISBN-13: 978-14564-796-26.

I Want to Be a Mathematician: A Conversation with Paul Halmos. A film by George Csicsery. Mathematical Association of America, March 2009. ISBN-13: 978-08838-590-94. (Reviewed June/July 2011.)

In Pursuit of the Unknown: 17 Equations that Changed the World, by Ian Stewart. Basic Books, March 2012. ISBN-13: 978-04650-297-30.

In Service to Mathematics: The Life and Work of Mina Rees, by Amy Shell-Gellasch. Docent Press, December 2010. ISBN: 978-0-9837004-1-8.

The Information: A History, a Theory, a Flood, by James Gleick. Pantheon, March 2011. ISBN-13: 978-03754-237-27.

Knots Unravelled: From String to Mathematics, by Meike Akveld and Andrew Jobbings. Arbelos, October 2011. ISBN: 978-09555-477-20.

**Late Style: Yuri I. Manin Looking Back on a Life in Mathematics.* A DVD documentary by Agnes Handwerk and Harrie Willems. Springer, March 2012. ISBN NTSC: 978-3-642-24482-7; ISBN PAL: 978-3-642-24522-0.

Lost in a Cave: Applying Graph Theory to Cave Exploration, by Richard L. Breisch. National Speleological Society, January, 2012. ISBN: 978-1-879961-43-2.

The Lost Millennium: History's Timetables Under Siege, by Florin Diacu. Johns Hopkins University Press (second edition), November 2011. ISBN-13: 978-14214-028-88.

Magical Mathematics: The Mathematical Ideas that Animate Great Magic Tricks, by Persi Diaconis and Ron Graham. Princeton University Press, November 2011. ISBN: 978-06911-516-49.

The Man of Numbers: Fibonacci's Arithmetic Revolution, by Keith Devlin. Walker and Company, July 2011. ISBN 978-08027-781-23. (Reviewed in this issue.)

**A Mathematical Nature Walk*, by John A. Adam. Princeton University Press, October 2011 (paperback edition). ISBN: 978-06911-526-53.

Mathematics and Reality, by Mary Leng. Oxford University Press, June 2010. ISBN-13: 978-01992-807-97.

Mathematics Education for a New Era: Video Games as a Medium for Learning, by Keith Devlin. A K Peters/CRC Press, February 2011. ISBN-13: 978-1-56881-431-5.

**Mathematics Emerging: A Sourcebook 1540–1900*, by Jacqueline Stedall. Oxford University Press, November 2008. ISBN-13: 978-01992-269-00. (Reviewed June/July 2011.)

The Mathematics of Life, by Ian Stewart. Basic Books, June 2011. ISBN-13: 978-04650-223-80. (Reviewed December 2011.)

Mathematics, Religion and Ethics: An Epistemological Study, by Salilesh Mukhopadhyay. Feasible Solution LLC, September 2010. ISBN: 978-1-4507-3558-2.

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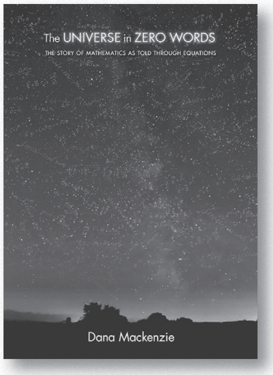

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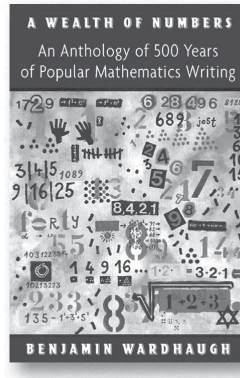

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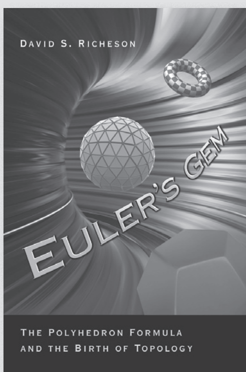

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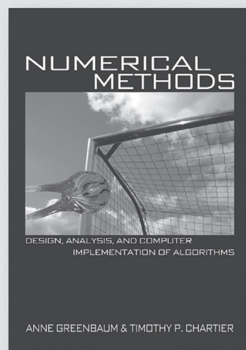

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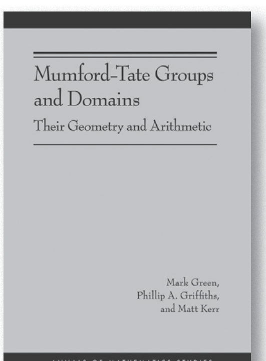

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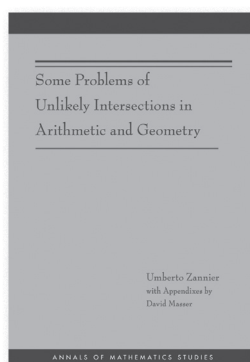

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ε* Nicholas J. Bezak
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ε Richard P. Brent
ε* Joseph Edward Brierly
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ε* Robert F. Brown
ε W. Dale Brownawell
Gloria Brown-Brook
ε* Andrew M. Bruckner
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ε Robert R. Bruner
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ε Nicholas P. Buchdahl
ε* Joseph T. Buckley
ε Daniel Buehler
ε Ioan Sebastian Buhai
Lutz Bungart
ε R. B. Burckel
* Krzysztof Burdzy
ε Richard H. Burkhart
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ε* Erio A. Castagnoli
ε German Tellez Castillo
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ε* Gulbank D. Chakerian
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ε Dawne M. Charters
ε Ronald J. Chase
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ε* Kwan-Wei Chen
ε* Herman Chernoff
ε William A. Cherry
Robert Chew
Peter H. S. Chiang
ε* Theodore S. Chihara
ε David R. J. Chillingworth
* Choong Yun Cho
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ε Todd E. Cochrane
ε John Coffey
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ε* Frederick R. Cohen
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* Richard M. Cohn
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ε* Daniel Comenetz
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ε Kumar Eswaran
ε* Edward R. Fadell
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ε Edson de Faria
ε Howard D. Fegan
ε* Mark E. Feighn
ε* Norman Feldman
ε Barry Fell
ε* José Humberto Ferreira Rosa
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ε Ludovico Fischer
ε William T. Fishback
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ε Lidia Gonzalez
ε* Kenneth R. Goodearl
ε* Roe W. Goodman
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ε Susan M. Gorter
ε* Yasuhiro Goto
ε Franklin R. Gould
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ε* Kevin A. Grasse
ε* Larry K. Graves
ε William Robert Green
ε* Curtis Greene
ε Peter H. Greene
ε* Frederick P. Greenleaf
* Stanley J. Greif
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ε* Richard A. Kanner
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ε Paul Levy
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ε* H. L. Lewis
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ε* Alexander Nagel
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Kanji Namba
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ε Manmath Nayak
ε Jan V. Neerven
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ε* Csaba Nemethi
ε Walter D. Neumann
ε Lee P. Neuwirth
Siu-Hung Ng
ε Liviu I. Nicolaescu
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ε Gerhard Niklasch
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ε* Richard C. Potter
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ε* Colin Smith
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ε* William M. Snyder Jr.
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Call for Nominations

The selection committees for these prizes request nominations for consideration for the 2013 awards, which will be presented at the Joint Mathematics Meetings in San Diego, CA, in January 2013. Information about past recepients of these prizes may be found in the January 2012 issue of the *Notices*, pp. 79–100 and at http://www.ams.org/prizes-awards.

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The Ruth Lyttle Satter Prize is presented every two years in recognition of an outstanding contribution to mathematics research by a woman in the previous six years.

E. H. MOORE RESEARCH ARTICLE PRIZE

Among other activities, E. H. Moore founded the Chicago branch of the AMS, served as the Society's sixth president (1901–2), delivered the Colloquium Lectures in 1906 and founded and nurtured the *Transactions of the AMS*. This prize was established in 2002 to honor his extensive contributions to the discipline and to the Society. It is awarded for an outstanding research article published in one of the AMS primary research journals (namely, the *Journal of the AMS*, *Proceedings of the AMS*, *Transactions of the AMS*, *Memoirs of the AMS*, *Mathematics of Computation*, *Electronic Journal of Conformal Geometry and Dynamics*, and the *Electronic Journal of Representation Theory*) during the calendar years 2004–2009. The US$5000 prize is awarded every three years.

LEVI L. CONANT PRIZE

The Levi L. Conant Prize, first awarded in January 2001, is presented annually for an outstanding expository paper published in either the *Notices* or the *Bulletin of the American Mathematical Society* during the preceding five years.

Each of these US$5,000 prizes below is awarded every three years.

DAVID P. ROBBINS PRIZE

This prize was established in 2005 in memory of David P. Robbins by members of his family. Robbins was a long-time member of the Institute for Defense Analysis Center for Communications Research and a prolific mathematician whose work (much of it classified) was in discrete mathematics. The prize is for a paper with the following characteristics: it shall report on novel research in algebra, combinatorics or discrete mathematics and shall have a significant experimental component; it shall be on a topic which is broadly accessible and shall provide both a clear statement of the problem and clear exposition of the work. The nomination should include a complete bibliographic citation for that work, supplemented with brief remarks explaining what aspects make it particularly suited for this prize.

OSWALD VEBLEN PRIZE IN GEOMETRY

The Oswald Veblen Prize in Geometry, which was established in 1961 in honor of Professor Veblen, is awarded in recognition of a notable research memoir in geometry or topology published in the preceding six years. To be considered, either the nominee should be a member of the Society or the memoir should have been published in a North American journal.

NORBERT WIENER PRIZE IN APPLIED MATHEMATICS

The Norbert Wiener Prize was established in 1967 in honor of Professor Wiener and was endowed by a fund from the Department of Mathematics of the Massachusetts Institute of Technology. The prize is awarded for an outstanding contribution to applied mathematics in the highest and broadest sense and is made jointly by the American Mathematical Society and the Society for Industrial and Applied Mathematics. The recipient must be a member of one of these societies.

Nomination with supporting information should be submitted to http://www.ams.org/profession/prizes-awards/nominations. Include a short description of the work that is the basis of the nomination, including complete bibliographic citations when appropriate. A brief curriculum vitae should be included for the nominee. Those who prefer to submit by regular mail may send nominations to the secretary, Robert J. Daverman, American Mathematical Society, 238 Ayres Hall, Department of Mathematics, University of Tennessee, Knoxville, TN 37996-1320. Those nominations will be forwarded by the secretary to the relevant prize selection committees.

Deadline for nominations is June 30, 2012.

AMS
AMERICAN MATHEMATICAL SOCIETY

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Except for the members at large of the Council, the month and year of the first term and the end of the present term are given. For members at large of the Council, the last year of the present term is listed.

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2013
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Estelle Basor
Patricia Hersh
Tara S. Holm
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2014
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Bryna Kra 2/10–1/14
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***Colloquium* Editorial Committee**
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***Proceedings* Editorial Committee**
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***Transactions* and *Memoirs* Editorial Committee**
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Call for Nominations

2013

Frank and Brennie Morgan AMS-MAA-SIAM Prize

for Outstanding Research in Mathematics by an Undergraduate Student

The prize is awarded each year to an undergraduate student (or students having submitted joint work) for outstanding research in mathematics. Any student who is an undergraduate in a college or university in the United States or its possessions, or Canada or Mexico, is eligible to be considered for this prize.

The prize recipient's research need not be confined to a single paper; it may be contained in several papers. However, the paper (or papers) to be considered for the prize must be submitted while the student is an undergraduate; they cannot be submitted after the student's graduation. The research paper (or papers) may be submitted for consideration by the student or a nominator. All submissions for the prize must include at least one letter of support from a person, usually a faculty member, familiar with the student's research. Publication of research is not required.

The recipients of the prize are to be selected by a standing joint committee of the AMS, MAA, and SIAM. The decisions of this committee are final. The 2013 prize will be awarded for papers submitted for consideration no later than June 30, 2012, by (or on behalf of) students who were undergraduates in December 2011.

Questions may be directed to:

Barbara T. Faires
Secretary
Mathematical Association of America
Westminster College
New Wilmington, PA 16172

Telephone: 724-946-6268
Fax: 724-946-6857
Email: faires@westminster.edu

Nominations and submissions should be sent to:

Morgan Prize Committee
c/o Robert J. Daverman, Secretary
American Mathematical Society
238 Ayres Hall
Department of Mathematics
University of Tennessee
Knoxville, TN 37996-1320

THE MATHEMATICAL ASSOCIATION OF AMERICA

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AMS EXEMPLARY PROGRAM PRIZE

At its meeting in January 2004, the AMS Council approved the establishment of a new award called the AMS Award for an Exemplary Program or Achievement in a Mathematics Department. It is to be presented annually to a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. Examples might include a department that runs a notable minority outreach program, a department that has instituted an unusually effective industrial mathematics internship program, a department that has promoted mathematics so successfully that a large fraction of its university's undergraduate population majors in mathematics, or a department that has made some form of innovation in its research support to faculty and/or graduate students, or which has created a special and innovative environment for some aspect of mathematics research.

The prize amount is $5,000. All departments in North America that offer at least a bachelor's degree in the mathematical sciences are eligible.

The Prize Selection Committee requests nominations for this award, which will be announced in Spring 2013. Letters of nomination may be submitted by one or more individuals. Nomination of the writer's own institution is permitted. The letter should describe the specific program(s) for which the department is being nominated as well as the achievements that make the program(s) an outstanding success, and may include any ancillary documents which support the success of the program(s). The letter should not exceed two pages, with supporting documentation not to exceed an additional three pages.

All nominations should be submitted to the AMS Secretary, Robert J. Daverman, American Mathematical Society, 238 Ayres Hall, University of Tennessee, Knoxville, TN 37996-1320. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations when appropriate. The nominations will be forwarded by the Secretary to the Prize Selection Committee, which will make the final decision on the award.

Deadline for nominations is September 14, 2012.

AMS Award for Mathematics Programs *That Make a Difference*

Deadline: September 14, 2012

This award was established in 2005 in response to a recommendation from the AMS's Committee on the Profession that the AMS compile and publish a series of profiles of programs that:

1. aim to bring more persons from underrepresented backgrounds into some portion of the pipeline beginning at the undergraduate level and leading to advanced degrees in mathematics and professional success, or retain them once in the pipeline;
2. have achieved documentable success in doing so; and
3. are replicable models.

Preference will be given to programs with signifigant participation by underrepresented minorities.

One or two programs are highlighted annually.

Nomination process: Letters of nomination may be submitted by one or more individuals. Nomination of the writer's own institution is permitted. The letter should describe the specific program(s) for which the department is being nominated as well as the achievements that make the program(s) an outstanding success, and may include any ancillary documents which support the success of the program(s). The letter of nomination should not exceed two pages, with supporting documentation not to exceed three more pages. Up to three supporting letters may be included in addition to these five pages.

Send nominations to:
Programs That Make a Difference
c/o Ellen Maycock
American Mathematical Society
201 Charles Street
Providence, RI 02904
or via email to ejm@ams.org

Recent Winners:

2012: Mathematical Sciences Research Institute.

2011: Center for Women in Mathematics, Smith College;
Department of Mathematics, North Carolina State University.

2010: Department of Compuational and Applied Mathematics (CAAM), Rice University;
Summer Program in Quantitative Sciences, Harvard School of Public Health.

2009: Department of Mathematics at the University of Mississippi; Department of Statistics at North Carolina State University.

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Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at http://www.ams.org/cgi-bin/mathcal-submit.pl.

The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

May 2012

* 4–6 **Chico Topology Conference**, California State University, Chico, California.
Invited Speakers include: Chris Herald (UN, Reno), Sergio Macias (UNAM, Mexico), Van Nall (University of Richmond), and Janusz Prajs (CSU, Sacramento). Researchers at all levels are invited to present 30-minute contributed talks in any area of topology. To apply, please send a title and abstract to Thomas Mattman (TMattman@CSUChico.edu) by April 1, 2012.
Information: http://www.csuchico.edu/~tmattman/CTC.html.

* 4–7 **50th Cornell Topology Festival**, Cornell University, Ithaca, New York.
Description: The program will feature talks by: Francis Bonahon, University of Southern California; David Gabai, Princeton University; Allen Hatcher, Cornell University; Peter May, University of Chicago; Dusa McDuff, Barnard College/Columbia University; John Milnor, Stony Brook University; Jacob Lurie, Harvard University; Tom Mrowka, Massachusetts Institute of Technology; Walter Neumann, Columbia University; Hee Oh, Brown University; John Pardon, Stanford Ronald Stern, University of California at Irvine; Peter Teichner, University of California at Berkeley; William Thurston, Cornell University.
Information: http://www.math.cornell.edu/~festival/.

* 10–12 **Young Women in PDEs**, Department of Applied Mathematics, University of Bonn, Bonn, Germany.
Description: This workshop is addressed to young female researchers in PDEs, the Calculus of Variations, and their applications. The aim of the conference is to provide a platform for scientific discussion and exchange of ideas, and to promote equal opportunity of women in the mathematical sciences.
Senior speakers: A. Garroni, University of Rome "La Sapienza", Italy; N. Uraltseva, St. Petersburg State University, Russia; M. Westdickenberg, RWTH Aachen, Germany. Ph.D. students and postdocs are invited to submit an abstract for a contributed talk by March 15th.
Organizing Commitee: L. Beck, C. Geldhauser, C. Zeppieri.
Information: http://www.iam.uni-bonn.de/ywipde.

* 12 **ECCAD 2012: East Coast Computer Algebra Day**, Oakland University, Rochester, Michigan.
Description: The East Coast Computer Algebra Day (ECCAD) is a one-day meeting for those interested in computer algebra and symbolic mathematical computation. It provides opportunities to learn and to share new results and current work in progress. The schedule includes prominent invited speakers along with contributed posters and software demonstrations. Plenty of time is allowed for unstructured interaction among the participants. Researchers, teachers, students, and users of computer algebra are all welcome!
Information: http://www.oakland.edu/math/eccad2012.

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

* 23–25 **Workshop on Semigroups 2012**, Caul, Lisbon, Portugal.
Description: The Workshop on Semigroups 2012 will be a special meeting to remember Professor John M. Howie.
Main Speakers: John Fountain, Univ. York, UK; Victoria Gould, Univ. York, UK; Peter M. Higgins, Univ. Essex, UK; Donald B. McAlister, Northern Illinois Univ., USA; Jean-Éric Pin, Univ. Paris 7, France; Jim Renshaw, Univ. Southampton, UK; Nik Ruskuc, Univ. St. Andrews, UK; Mária B. Szendrei, Univ. Szeged, Hungary.
Organizers: Gracinda M.S. Gomes, CAUL/FCTUNL, Portugal; Vítor H. Fernandes, CAUL/FCUL, Portugal.
Information: http://caul.cii.fc.ul.pt/WS2012/.

* 23–27 **History of Mathematics and Teaching of Mathematics Conference**, University of Miskolc, Sarospatak, Hungary.
Description: You and your colleagues are invited to participate in our conference.
Information: For more information please visit the web site: http://www.uni-miskolc.hu/hmtm.

* 30–June 2 **Workshop for Women in Analysis and PDE**, The Institute for Mathematics and its Applications (IMA), Minneapolis, Minnesota.
Description: The workshop will concentrate on central developments of modern Harmonic Analysis and Elliptic PDEs. It will introduce the participants to the methods and techniques which have emerged in recent years and outline important open problems, current progress, and main challenges. Some of the focal points include: the theory of weights, dyadic harmonic analysis, singular integral operators, Haar shifts and Bellman function, Carlson measures and their role in analysis and PDEs, elliptic PDEs on non-smooth domains, multilinear Fourier analysis, as well as connections to geometric measure theory, additive combinatorics, analytic number theory, several complex variables.
Information: http://www.ima.umn.edu/2011-2012/SW5.3-0-6.2.12.

June 2012

* 1–July 31 **Geometric and Analytic Techniques in Calculus of Variations and Partial Differential Equations**, Centro di Ricerca Matematica "Ennio De Giorgi", Piazza dei Cavalieri 3, 56100 Pisa, Italy.
Description: The intensive period will concentrate upon four topics. Each week of June will be devoted to one topic, with mini-courses and invited seminars at a doctoral/post-doctoral level. Seminars at a more specialized level will be organized in July. The weekly courses in June will be given by Dorin Bucur, Bernd Kawhol, Michel Pierre, Tadeusz Iwaniec, Pekka Koskela, Jan Maly, Xavier Cabre, Camillo De Lellis, Julio Rossi, Jean Dolbeaut, Giuseppe Mingione and Panagiota Daskalopoulos. Three workshops will be organized in July by Juan Luis Vazquez, Aldo Pratelli and Nicola Fusco, Antoine Henrot. A detailed scheme about planned activities is available at http://crm.sns.it/event/233/activities.html#title.
Financial support (board and lodging): For young participants will be provided upon selection.
Deadline: For applications for financial support is March 15th, 2012. For detailed information about the application procedure please link to http://crm.sns.it/event/233/financial.html.
Information: http://www.crm.sns.it/event/233/index.html#title.

* 4–8 **AIM Workshop: Cohomology bounds and growth rates**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to questions associated to the following 1984 conjecture of Guralnick's: There exists a "universal constant" C which bounds 1-cohomology.
Information: http://www.aimath.org/ARCC/workshops/cohombounds.html.

* 6–8 **Joint Conference of the Belgian, Royal Spanish, and Luxembourg Mathematical Societies**, University of Liege, Belgium.
Description: This Joint meeting of the Belgian, Royal Spanish, and Luxembourg mathematical societies will gather mathematicians around 7 plenary speakers (http://nalag.cs.kuleuven.be/research/workshops/BSL2012/plenary.shtml) and in 8 special sessions (Algebra, Algebraic and Symplectic Geometry, Discrete Mathematics, Functional and Harmonic Analysis, Mathematical Logic, Numerical Analysis, Orthogonal Polynomials and Special Functions, Statistics and Probability Theory). It will be the occasion to listen to lectures and to have discussions between senior, post-doc and Ph.D. students around several subjects of present research in maths. It will also be the occasion to discover the University and city of Liege in Belgium.
Information: http://nalag.cs.kuleuven.be/research/workshops/BSL2012/.

* 6–9 **Banach Spaces Workshop**, University of Birmingham, Birmingham, United Kingdom.
Speakers: Frederic Bayart, Universite Bordeaux; Pandelis Dodos, University of Athens; Vladimir Fonf, Ben-Gurion University; Petr Hajek, Academy of Science of the Czech Republic; Richard Haydon, University of Oxford; William Johnson, Texas A&M University; Gilles Lancien, Universite de Franche Comte; Maria Roginskaya, Goeteborgs Universitet; Gideon Schechtman, Weizmann Institute of Science; Thomas Schlumprecht, Texas A&M University; Jaroslav Tiser, Czech Technical University; Ludek Zajicek, Charles University.
Organizers: Olga Maleva, University of Birmingham; David Preiss, University of Warwick.
Information: http://tinyurl.com/banach-workshop.

* 11–13 **"Games and Strategy in Paris", held on the occasion of Sylvain Sorin's 60th birthday**, Institut Henri Poincaré, Paris, France.
Registration: Registration is free but mandatory on our website. Ph.D. students and young researchers willing to present a poster may apply for partial financial support. They are invited to submit an extended abstract (max 2 pages) to: conf.sorin@gmail.com before March 16, 2012. Notification of acceptance will be issued on March 30, 2012.
Information: https://sites.google.com/site/sorin60th/.

* 11–14 **Ninth Advanced Course in Operator Theory and Complex Analysis**, Faculty of Mathematics, University of Seville, Seville, Spain.
Description: Courses and talks will cover topics on complex analysis, operator theory and related areas of functional analysis.
Information: http://congreso.us.es/ceacyto/2012/index.html.

* 11–14 **Operator Theory, Analysis and Mathematical Physics**, Centre de Recerca Matemàtica, Bellaterra, Barcelona.
Description: This conference continues the series of OTAMP conferences organized every second year in Europe. It is devoted to recent achievements in analysis lying on the border between operator theory and mathematical physics. The following special sessions will be organized, Spectral theory: self-adjoint and non-self-adjoint problems; Orthogonal polynomials, Jacobi and CMV matrices; Random and quasi-periodic operators; Quantum graphs; Non-self-adjoint quantum mechanics.
Information: http://www.crm.cat/coptam.

* 11–15 **Mathematical Problems in Industry (MPI) Workshop 2012**, University of Delaware, Newark, Delaware.
Description: This one-week workshop is the twenty-eighth in the series, and will be held at UD for the sixth time. Modeled on the Oxford Study Group in the UK, the MPI workshop is a problem-solving workshop that attracts leading applied mathematicians and scientists from universities, industry, and national laboratories. During the workshop, engineers and scientists from industry interact closely with the academic participants on problems of interest to their companies.

Information: http://www.math.udel.edu/MPI/.

* 12–14 **Ninth Edition of the Advanced Course in Operator Theory and Complex Analysis**, Sevilla, Spain.
Description: As a part of attending the course, you have the opportunity to deliver a contributed talk.
Invited Lectures: By Håkan Hedenmalm, KTH (Sweden); Alexei Poltoratski, Texas A&M University.
Invited speakers: Wolfgang Arendt, Universität Ulm, Germany; Joseph A. Ball, Virginia Tech; Isabelle Chalendar, Université Lyon-1, France; Sjoerd Verduyn Lunel, Universiteit Leiden, Netherlands; Armen G. Sergeev, Steklov Mathematical Institute of Moscow, Russia.
Information: http://congreso.us.es/ceacyto/2012.

* 18–22 **Conference on Geometry and Quantization of Moduli Spaces (2012 VBAC Conference)**, Centre de Recerca Matemàtica, Bellaterra, Barcelona.
Scientific Committee: Peter Newstead (Chair), Usha Bhosle, Steven Bradlow, Leticia Brambila-Paz, Ugo Bruzzo, Carlos Florentino, Oscar Garcia-Prada, Peter Gothen, Daniel Hernandez Ruiperez, Alastair King, Herbert Lange, Ignasi Mundet i Riera, Christian Pauly, Alexander Schmitt, Andras Szenes.
Information: http://www.crm.cat/cmodulispaces.

* 25–29 **European Seminar on Computing (ESCO 2012)**, Pilsen, Czech Republic.
Description: ESCO 2012 is the 3rd event in a successful series of interdisciplineary meetings dedicated to modern methods and practices of scientific computing.
Main thematic areas: Multiphysics coupled problems, Higher-order computational methods, computing with python, GPU computing, and cloud computing. Theoretical results as well as applications are welcome. Application areas include, but are not limited to: Computational electromagnetics, civil engineering, nuclear engineering, mechanical engineering, nonlinear dynamics, fluid dynamics, climate and weather modeling, computational ecology, wave propagation, acoustics, geophysics, geomechanics and rock mechanics, hydrology, subsurface modeling, biomechanics, bioinformatics, computational chemistry, stochastic differential equations, uncertainty quantification, and others.
Information: http://esco2012.femhub.com/.

* 26–30 **International Conference "Probability Theory and its Applications" in Commemoration of the Centennial of Boris Vladimirovich Gnedenko**, Moscow State University, Moscow, Russia.
Description: The conference is held jointly with Steklov Mathematical Institute and Moscow Institute of Electronics and Mathematics. During the conference six one-hour general plenary talks by Russian and foreign scientists are planned.
Thematic section list: Limit theorems, Stochastic extreme value theory, Queueing theory, Mathematical reliability theory, Methods of actuarial mathematics, History of mathematics, Teaching of mathematics.
Information: http://gnedenko100conference.ru.

* 27–29 **Differential Geometry Days. In honour of Luis A. Cordero**, Departamento de Xeometría e Topoloxía, Facultade de Matematicas, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain.
Description: Luis A. Cordero has been a full professor of Geometry and Topology since 1976 at La Laguna Univ. and at Santiago de Compostela Univ. (USC, Spain) since 1979. The Institute of Math. of the USC in colaboration with Basque Country Univ. (UPV/EHU, Spain) and CSIC (Spain), is organizing the "Differential Geometry Days" on the occasion of his 65th birthday.
Topics: Current research topics in Differential Geometry will be treated during this conference.
Confirmed invited speakers: J. C. Díaz Ramos (USC, Spain), C.T.J. Dodson (Manchester Univ., UK), P. Gilkey (Oregon Univ., USA), S. Ivanov (Univ. of Sofia "St. Kliment Ohridski", Bulgaria), J. C. Marrero (Univ. de La Laguna, Spain), P.E. Parker (Wichita State Univ., USA), S. Salamon (King's College London, UK), L. Ugarte (Univ. de Zaragoza, Spain), I. Vaisman (Haifa Univ., Israel).
Deadline: Registration and Contribution (Poster), June 1st, 2012. The registration includes the possibility of submitting a contribution (poster).
Information: Contact: eduardo.garcia.rio@usc.es; http://xtsunxet.usc.es/cordero2012/.

July 2012

* 1–5 **Mathematical Modeling of Microbiological Systems (M3S)**, Marburg, Germany.
Description: The aim of the conference is to present the latest research in the field of computational biology with focus on mathematical methods for modeling, analysis, simulation and model optimization in system and cell biology. The program will comprise invited plenary lectures and contributed talks addressing new developments in the field of mathematical modeling of microbiological systems.
Information: email: rashkov@mathematik.uni-marburg.de; http://www.uni-marburg.de/synmikro/mathematik_tagung.

* 2–13 **Summer School on Algebraic and Enumerative Combinatorics**, Centro de estudos Camilianos, S. Miguel de Seide, Guimarães, Portugal.
Description: The summer school will be held in a building of Álvaro Siza, the 1992 Laureate of the Pritzker Architecture Prize. The Centro de estudos Camilianos is at S. Miguel de Seide, near to GuimarThees, Portugal, where the participants are expected to be lodged. The school will focus on four courses, given by Francesco Brenti, Christian Krattenthaler, Marc Noy and Vic Reiner.
Topics: The topics to be addressed by the speakers are, respectively, Combinatorics of Coxeter groups, map enumeration, asymptotic enumeration of topological graphs and reflection group counting and q-counting, and the courses are mainly directed to graduate and post-graduate students, as well as researchers. There will also be time for some contributed short talks by participants.
Information: http://www2.fc.up.pt/pessoas/agoliv/SC/default.htm.

* 5–10 **Mathematical Physics in Bahia: Algebraic Analysis, Quantization and Representations**, Instituto de Matematica, UFBA, Salvador, Brazil.
Description: The conference will cover various recent advances in the field Mathematical Physics with focus on: Algebraic analysis, mathematical aspects of quantization, noncommutative geometry in relation with quantization, topology and geometry of QFT, string theory and mirror symmetry, representation theory.
Registration: Is now open and should be done online.
Deadline: For registration: June 1st, 2012. The number of participants will be limited to 70.
Information: http://monge.u-bourgogne.fr/gdito/MPB2012/Home.html.

* 6–7 **National Conference on Mathematical and Computational Sciences**, Adikavi Nannaya University, Rajahmundry, Andhra Pradesh, India.
Description: In recent days, exploring the applications on the existing mathematical concepts has become the emerging area in mathematical research. adding up it is essential to focus on concrete mathematical structures and theories for the future applications. Hence, the conference is focused to share the views of new mathematical theories and the applications on the existing mathematical concepts.
Information: http://www.nannayauniversity.info/MACS.pdf.

* 9–27 **2012 Summer School on Geometry and Data**, Washington State University & University of Idaho, held in Moscow, Idaho.

Description: Geometric measure theory and geometric analysis offer powerful tools and insights which are just beginning to be exploited for their data analysis potential. This summer school will explore and explain aspects of geometric measure theory, applied harmonic analysis and computational methods focused on illuminating various data inspired problems.
Information: `http://geometricanalysis.org/Workshops/2012SummerSchool`.

* 12–19 **International Summer School on Fundamental Algorithms and Computable Modeling for High-Performance and Multi-scale Scientific/Engineering Computing**, Nankai University, School of Mathematical Sciences, No. 94 Weijin Road, Nankai District, Tianjin 300071, People's Republic of China.
Description: Since great achievements in the area of scientific and engineering computing have been made to provide plenty of new methods and outcomes from physical and engineering subjects and disciplines, such as Navier-Stokes equations from fluid dynamics, Maxwell equations from electromagnetism, solid and structural mechanics, etc. in the past few years, and since the needs of the scientific and engineering computing talented people are becoming more and more pressing, and since the way that mankind explores Nature has turned to scientific and engineering computing—the third way of scientific research, as well as the classical two scientific research ways theoretical research and physical experiment, by using supercomputers to run large-scale scientific and engineering computing in very extensive applications arising from Natural sciences and even social sciences, such as petroleum exploration, biomedicine,weather forecast, ocean and marine environment modelling, government affairs.
Information: `http://www.math.nankai.edu.cn/conference/summerschool/indexen.html`.

* 16–20 **Applications of Graph Spectra in Computer Science**, Centre de Recerca Matemàtica, Bellaterra, Barcelona.
Description: In graph theory, the spectra of matrices associated with a graph are widely used to characterize its properties and to extract structural information. There are several graph matrix representations such as the adjacency matrix, combinatorial Laplacian, normalized Laplacian and signless Laplacian. Spectral graph theory has also many applications in other scientific fields such as chemistry, theoretical physics, and quantum mechanics. The aim of this workshop is to foster the connections between spectral graph theory and computer science.
Information: `http://www.crm.cat/wkgraphspectra`.

* 16–20 **23rd International Workshop on Operator Theory and its Applications (IWOTA 2012)**, University of New South Wales, Sydney, Australia.
Description: For the past 30 years, IWOTA has brought together mathematicians working in a variety of fields that share operator theory as a common theme. In 2012, it is Australia's turn to host IWOTA. This meeting will emphasise three directions: Operator algebras, Harmonic analysis of differential operators. Control theory. Of course, the workshop will not be restricted to these three themes, and colleagues working on any aspect of operator theory, understood in a very broad sense, are encouraged to participate.
Plenary speakers: Current list: Pascal Auscher (Université Paris 11 Orsay), Joseph A. Ball (Virginia Tech), Qui Bui (University of Canterbury), Raul Curto (University of Iowa), Ron Douglas (Texas A&M University), Xuan Duong (Macquarie), Fritz Gesztesy (University of Missouri Columbia), Gilles Godefroy (Université Paris 6), Il Bong Jung (Kyungpook National University), Rien Kaashoek (VU Amsterdam), Jerry Kaminker (University of California Davis), Igor Klep (University of Auckland), Woo Young Lee (Seoul National University), Christian Le Merdy (Université de Franche-Comt), Shahar Mendelson (ANU/Technion), Jan van Neerven (TU Delft), Ben de Pagter (TU Delft), Carlos Perez (Universidad de Sevilla), Iain Raeburn (University of Otago).
Thematic Sessions: Continuous and discrete Clifford analysis (Swanhild Bernstein, Fabrizio Colombo, Uwe Kahler, Paul Leopardi), Toeplitz operators and their applications (Sergei Grudsky, Nikolai Vasilevski), Complex geometry and operator theory (Tirthankar Bhattacharyya, Nicholas Young), Operators on spaces of analytic functions (Carl Cowen), Dynamics and operator algebras (Sooran Kang, Aidan Sims), Systems and control theory (Hendra Nurdin), Harmonic analysis of differential operators (Pierre Portal), Topics in noncommutative analysis (Anna Skripka), Operator theory, function theory, and linear systems (Joe Ball, Rien Kaashoek), General session.
Organizing committee: T. ter Elst (University of Auckland), P. Portal (Australian National University/Universite Lille 1), D. Potapov (University of New South Wales).
Registration: Is now open, and can be completed online at: `http://conferences.science.unsw.edu.au/IWOTA2012/register.html`.
Information: `http://conferences.science.unsw.edu.au/IWOTA2012/`

* 16–28 **XV Summer Diffiety School**, Pomorski Park Naukowo-Technologiczny, Gdynia, Poland.
Description: The aim of this permanent school is to introduce undergraduate and Ph.D. students in Mathematics and Physics as well as post-doctoral researchers in a recently emerged area of Mathematics and Theoretical Physics: SECONDARY CALCULUS. A DIFFIETY is a new geometrical object that properly formalizes the concept of the solution space of a given system of (nonlinear) PDEs, much as an algebraic variety does with respect to solutions of a given system of algebraic equations. Secondary Calculus is a natural diffiety analogue of the standard calculus on smooth manifolds, and as such leads to a very rich general theory of nonlinear PDEs. Moreover, it appears to be a natural language for quantum physics, just as the standard calculus is the natural language for classical physics.
Information: `https://sites.google.com/site/levicivitainstitute/Activities/DiffietySchools/xv-summer-diffiety-school`.

* 18–August 1 **Summer School on Discrete Morse Theory and Commutative Algebra**, Institut Mittag-Leffler, Stockholm, Sweden.
Description: The summer school will focus on recent developments in combinatorial topology and discrete geometry, with an emphasis on the interaction with toric geometry and commutative algebra. A promising contemporary method for getting simple explicit descriptions of topological spaces is discrete Morse theory. Its applications range from real world problems, such as shape recognition, to theoretical studies of topological spaces, which encode important invariants from algebra, geometry and topology. Program participants will learn how to use these state-of-the-art tools to investigate a variety of topics such as: Complements of hyperplane arrangements, resolutions of monomial and toric ideals, knots in triangulated manifolds, metric structures on simplicial complexes, spaces realizing desired cohomology rings, and topological representations of matroids.
Accommodations/Travel: Free accommodation and meals are provided. Limited travel funds. Apply by April 15 on `http://www.mittag-leffler.se` or follow the link.
Information: `http://www.mittag-leffler.se/summer2012/summerschools/discrete_morse_theory/`.

August 2012

* 1–15 **Artificial General Intelligence Summer School 2012**, Reykjavik University, Reykjavik, Iceland.
Description: This summer school focuses on issues related to the original goal of artificial general intelligence, namely that of building machines capable of operating in a range of different environments and domains, and doing a range of unrelated tasks in a coordinated manner, with a special focus on architectural and integrative issues. The summer school is targeted to those with a background in artificial intelligence and computer science, and may also be of interest

to students in philosophy, psychology and cognitive science with a background in software development or mathematics. It is open to all graduate students in all countries.

Language: The teaching language is English.

Admission: To apply for admission, send an email with a short cover letter and a CV to the following address: summerschool-2012@cadia.ru.is. A limited amount of support may be provided to a subset of applicants; to apply, please indicate this in your cover letter, and clearly state reasons for requesting the support.

Information: http://wiki.humanobs.org/public:events:agi-summerschool-2012; http://www.ru.is.

* 22–24 **Berlin PUM Workshop 2012**, Humboldt University, Berlin, Germany.

Description: The aim of this workshop is to provide an opportunity for researchers and practitioners to discuss recent research results that may support a wide applicability in PUM related approaches. To build a foundation for these discussions, a number of experts have been invited to talk about their research. The covered topics will range from theoretical analysis of PUM-based methods to applications and aspects of implementation. Anybody interested to participate is cordially invited to join the discussions and to give a presentation on his/her own research.

Invited speakers: Uday Banerjee (Syracuse University, USA), Stéphane Bordas (Cardiff School of Engineering, UK), Armando Duarte (University of Illinois, USA), Thomas Fries (RWTH Aachen, Germany), Markus Melenk (TU Wien, Austria), Yves Renard (INSA de Lyon, France), Alexander Schweitzer (Universität Stuttgart, Germany).

Information: http://www.math.hu-berlin.de/~berlin-pum-workshop2012/.

* 22–25 **The 20th Conference on Applied and Industrial Mathematics CAIM 2012**, Tiraspol State University, Chisinau, Republic of Moldova.

Description: The conference is organized by the Romanian Society of Applied and Industrial Mathematics (ROMAI), the Mathematical Society of the Republic Moldova, the Tiraspol State University, the Institute of Mathematics and Informatics of the Academy of Sciences of Moldova, the Moldova State University and the Academy of Economical Studies of Moldova.

Sections: Mathematical analysis and differential equations; Algebra and logic; Geometry and topology; Analytical and numerical methods in partial differential equations; Computer sciences; Mathematical models in industry, Physics and biology; Education.

Information: http://www.romai.ro/index.php?option=com_content&view=article&id=207&Itemid=498.

* 23–26 **International Congress in Honour of Professor H. M. Srivastava**, The Auditorium at the Campus of Uludag University, Bursa, Turkey.

Description: The forthcoming International Congress in Honour of Professor H. M. Srivastava is motivated essentially by the remarkable popularity and success of the well-attended four-day International Congress Dedicated to Professor Srivastava on the Occasion of his 70th Birth Anniversary, which was held in August 2010 at Hotel Karinna in Mount Uludag (Bursa) under the auspices of Uludag University. This congress will be organized at the Gorukle Campus of Uludag University in the fourth largest city, Bursa in Turkey in honour of Professor Dr. Hari Mohan Srivastava. It will cover a wide range of topics of Mathematics, Statistics and related sciences. Visit http://srivastava2012.uludag.edu.tr/.

Information: http://srivastava2012.uludag.edu.tr/.

September 2012

* 3–7 **Dynamical Systems: 100 years after Poincaré**, University of Oviedo, Gijn, Spain.

Description: The conference is organized by the Dynamical Systems Group at the University of Oviedo. The aim is to bring together a broad group of scientists working in the field of dynamical systems on the occasion of the 100th anniversary of the death of Henri Poincaré. All topics related to dynamical systems are considered but focusing on local and global bifurcations in discrete and continuous dynamical systems, planar vector fields and celestial mechanics. Applications to real-world problems will be highlighted. The conference will promote the diffusion of recent developments and future perspectives. There will be ten keynote speakers, sixty-four 30-minute contributed talks in two parallel sessions and also two poster sessions. Plenary speakers are experts chosen from different areas of Dynamical Systems.

Information: http://www.unioviedo.es/ds100Poincare/.

* 5–7 **Complex patterns in wave functions: Drums, graphs, and disorder**, Kavli Royal Society International Centre, Chicheley Hall, Chicheley, Newport Pagnell, Buckinghamshire MK16 9JJ, UK.

Description: Wave functions display complex patterns which are intensively studied in many branches of Mathematics and Physics. Their value distributions, nodal sets, extreme values, and localization properties – to cite a few examples – are investigated using diverse methods developed within a network of fields whose connectivity leaves a lot to be desired. This conference gives a unique opportunity to discuss these common questions, and present different points of view and methods, yet in a single high-level forum. A set of world-leading researchers has been invited to lecture on their recent contributions to the field. The participants will have the opportunity to present contributions in a poster session and discuss future directions across discipline borders.

Information: http://royalsociety.org/events/Complex-patterns-in-wave-functions/

* 7–12 **Workshop on Stochastic and PDE Methods in Financial Mathematics**, Yerevan State University, Yerevan, Armenia.

Description: Institute of Mathematics of the National Academy of Sciences in association with Yerevan State University and American University of Armenia is organizing an international workshop in Stochastic and PDE Methods in Financial Mathematics, September 7–12, 2012.

Objective: The objective of this workshop is to bring together experts in the area of partial differential equations and stochastic analysis, working within financial mathematics to encourage and give an opportunity to young mathematicians in the region, to initiate contacts with experts in this area, and to stimulate contacts between theoretical and applied science and financial companies of the region having interests in development of mathematical modelling and research. The program of the workshop will consist of invited 50-minute plenary lectures and contributed 20-minute talks, poster sessions as well as short presentations.

Information: http://math.sci.am/conference/sept2012/index.html.

* 10–14 **6th International Conference on Stochastic Analysis and its Applications**, The Mathematical Research and Conference Center of the Institute of Mathematics of the Polish Academy of Sciences, Bedlewo, Poland.

Description: The conference will be the 6th in a series of international conferences on Stochastic Analysis and Its Applications. The previous ones were held at Seattle (2006), Seoul (2008), Beijing (2009), Osaka (2010) and Bonn (2011). The main topics of the conference are 1. Dirichlet forms and stochastic analysis; 2. Jump processes; 3. Stochastic partial differential equations; 4. Stochastic analysis and geometry; 5. Optimal transport and allocation problems; 6. Potential theory; 7. Random media, percolation clusters and fractals; 8. Stochastic models in physics and biology

Information: http://bcc.impan.pl/6ICSA/.

* 11–14 **Workshop on Operator Theory and Operator Algebras 2012–WOAT 2012**, Instituto Superior Técnico, Universidade Técnica de Lisboa, Portugal.
Description: The main scientific goal of WOAT 2012 is to present developments in operator theory, operator algebras and their applications, and to promote research exchanges in the operator theory and operator algebras areas. This workshop is dedicated to Professor António Ferreira dos Santos.
Information: http://www.math.ist.utl.pt/~woat2012.

* 20–October 20 **ERC research period on Diophantine Geometry**, Centro di Ricerca Matematica "Ennio De Giorgi", Scuola Normale Superiore, Piazza dei Cavalieri 3, 56100 Pisa, Italy.
Description: This event is part of the European Research Council programme. Beyond the natural scientific exchange among participants, the activities should consist of seminars. The event should focus on integral points, algebraic dynamics, unlikely intersections.
Scientific committee: Enrico Bombieri (Institute for Advanced Study), David Masser (University of Basel), Lucien Szpiro (Graduate Center, CUNY), Gisbert Wuestholz (ETH, Zürich) and Shou-Wu Zhang (Columbia University and Princeton University).
Local committee: Pietro Corvaja (University of Udine), Roberto Dvornicich (University of Pisa), Umberto Zannier (Scuola Normale Superiore, ERC coordinator).
Attendance/Registration/Financial Support: Is free, but registration is required. To register please link to http://www.crm.sns.it/event/242/registration.html. Financial support is available for invited participants only.
Information: http://www.crm.sns.it, crm@sns.it; http://www.crm.sns.it/event/242/.

* 24–27 **Eighth National Congress on Finite Element Method**, School of Mathematical Sciences, Nankai University 94 Weijin Road, Nankai District, Tianjin 300071, China.
Description: The Eighth National Congress on Finite Element Method will be held in Nankai University in Tianjin on September 24-27, 2012. This congress aims at providing a forum for computational mathematicians, scientists and engineers to meet and exchange ideas, research results and state-of-the-art themes and topics in various scientific and engineering disciplines related to finite element methods.
Information: http://www.math.nankai.edu.cn/conference/fem8/8thfem_Eg/Home.

* 24–28 **Mathematics and Physics of Moduli Spaces**, Heidelberg University, Germany.
Description: The main thematic focus is higher Teichmueller spaces and the various contexts in which they arise in theoretical physics. The goal of this activity is to make recent developments accessible to both mathematicians and physicists. The program will consist of lecture series given by Vladimir Fock, Edward Frenkel, Sergei Gukov, Francois Labourie, Greg Moore (tbc) and Joerg Teschner and a few additional talks. There will be plenty of time for informal discussions. Young researchers can apply for financial support to attend the workshop at the webpage.
Information: http://www.match.uni-heidelberg.de/MPMS/.

October 2012

* 22–26 **AIM Workshop: Lipschitz metric on Teichmüller space**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to recent developments and new directions in Teichmüller theory from the point of view of Thurston's Lipschitz metric.
Information: http://aimath.org/ARCC/workshops/lipschitzteich.html.

* 29–31 **The 17th edition of the Symposium on Solid and Physical Modeling**, University of Burgundy, Dijon, France.
Description: The Symposium on Solid and Physical Modeling 2012 (SPM'2012) is organized in cooperation with SIAM and in cooperation with ACM SIGGRAPH (pending).
Focus: The focus of the conference is on the mathematical and computational issues that arise in generating, analyzing, and processing geometric information in applications such as: mechanical design, process planning, manufacturing, bio-medical, games, animation, geology, and virtual reality. The proceedings of SPM2012, including full papers and short papers, will be published as a special issue of the *Journal of Computer Aided Design* (Elsevier). Technical papers should present previously unpublished, original results that are not simultaneously submitted elsewhere. All papers will be rigorously peer-reviewed by members of the international program committee.
Important Dates: Abstracts Due: March 25, 2012. Paper Submissions Due: April 2, 2012.
Information: http://spm12.u-bourgogne.fr; http://spm12.u-bourgogne.fr.

* 29–November 2 **Cluster Algebras in Combinatorics, Algebra, and Geometry**, Mathematical Sciences Research Institute, Berkeley, California.
Description: Cluster algebras provide a unifying algebraic/combinatorial framework for a wide variety of phenomena in settings as diverse as quiver representations, Teichmuller theory, Poisson geometry, Lie theory, discrete integrable systems, and polyhedral combinatorics. The workshop aims at presenting a broad view of the state-of-the-art understanding of the role of cluster algebras in all these areas, and their interactions with each other.
Information: http://www.msri.org/web/msri/scientific/workshops/all-workshops/show/-/event/Wm570

November 2012

* 1–3 **The 13th International Conference of Mathematics and its Applications ICMA2012**, Department of Mathematics, "Politehnica" University of Timisoara, City of Timisoara, Romania.
Description: The aim of the conference is to bring together mathematicians, engineers, economists, physicians from all over the world, with research interests in mathematics or in its applications and to attract original papers.
Topics: Mathematical Analysis and Applications, Algebra and Geometry, Computer Algebra Systems in Research, Applied Mathematics in Engineering, and Economics Probability and Statistics, Applications in Health and Clinical Research.
Information: http://www.mat.upt.ro/ICMA2012/index.htm.

* 16–18 **Special Functions, Partial Differential Equations and Harmonic Analysis, a conference in honor of Calixto P. Calderón**, Roosevelt University, 425 Wabash Ave, Chicago, Illinois.
Description: A group of friends of Calixto P. Calderón have decided to organize a conference to celebrate his research and academic archivements in his long academic career.
Confirmed main speakers: Carlos Kenig, University of Chicago; Mario Milman, Florida Atlantic University; Yoram Sagher, Florida Atlantic University; Ahmed Zayed, DePaul University; Marshall Ash, DePaul University; Alberto Torchinsky, Indiana University; Richard Wheeden, Rutgers University; Robert Fefferman, University of Chicago; Jeff Lewis, University of Illinois at Chicago; Richard Askey, University of Wisconsin; Rodolfo Torres, University of Kansas; James Moller, University of Illinois at Chicago; Alexandra Bellow, Northwestern University.
Information: http://www.roosevelt.edu/calderon.

* 29–December 1 **2012 Third International Conference on Emerging Applications of Information Technology (EAIT 2012)**, Indian Statistical Institute, Kolkata, India.
Description: The Computer Society of India (CSI) has been instrumental in guiding the Indian IT industry since its formative years. The mission of CSI is to facilitate research, knowledge sharing, learn-

ing and career enhancement for all categories of IT professionals, while simultaneously inspiring and nurturing new entrants into the industry and helping them to integrate into the IT community. Encouraged by the earlier events, EAIT 2006 (proceedings published by Elsevier) and EAIT 2011 (proceedings published by IEEE CS and Xplore), CSI Kolkata Chapter is organizing the Third International Conference on Emerging Applications of Information Technology (EAIT 2012). The event will be comprised of Pre-Conference Tutorials, plenary sessions, invited lectures by eminent speakers of international repute, session papers and panel discussions. Detailed call for papers can be found at: `https://sites.google.com/site/csieait2012/cfp`.

Information: `http://sites.google.com/site/csieait2012`.

December 2012

* 3–7 **Combinatorial Commutative Algebra and Applications**, Mathematical Sciences Research Institute, Berkeley, California.

Description: This workshop on Combinatorial Commutative Algebra aims to bring together researchers studying toric algebra and degenerations, simplicial objects such as monomial ideals and Stanley-Reisner rings, and their connections to tropical geometry, algebraic statistics, Hilbert schemes, D-modules, and hypergeometric functions.

Information: `http://www.msri.org/web/msri/scientific/workshops/programmatic-workshops/show/-/event/Wm571`.

* 10–14 **AIM Workshop: Log minimal model program for moduli spaces**, American Institute of Mathematics, Palo Alto, California.

Description: This workshop, sponsored by AIM and the NSF, will be devoted to applications of the minimal model program (MMP) to the study of geometry of moduli spaces of algebraic varieties.

Information: `http://www.aimath.org/ARCC/workshops/logminmoduli.html`.

* 17–21 **AIM Workshop: Rational Catalan Combinatorics**, American Institute of Mathematics, Palo Alto, California.

Description: This workshop, sponsored by AIM and the NSF, will be devoted to understanding the interaction between new developments in algebra and combinatorics. In particular, it will focus on combinatorial objects counted by generalizations of Catalan numbers and their interaction with the representation theory of Cherednik algebras.

Information: `http://aimath.org/ARCC/workshops/rationalcatalan.html`.

April 2013

* 8–10 **Fourteenth International Conference on Numerical Combustion (NC13)**, Holiday Inn Riverwalk, San Antonio, Texas.

Description: Advances in computational algorithms, hardware, and software continue to have a revolutionary impact on the combustion sciences and permit the examination of scientific and engineering problems of increasing complexity. Detailed combustion simulations and models are now being considered as part of integrated system applications. The International Conference on Numerical Combustion will focus on the integration of theory, modeling, and numerical implementation in the study of basic combustion physics and technological applications. The distinct questions and challenges found in combustion and phase transitions arise from the multiplicity of length and time scales defined by the chemical, geometric, and flow ingredients. Physically descriptive, efficient, and accurate numerical modeling of complex phenomena and the design and implementation of complex, integrated simulation are the challenges to be addressed at this conference.

Information: `http://www.siam.org/meetings/nc13/`.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

June 2013

* 4–14 **Conference on Nonlinear Mathematical Physics: Twenty Years of JNMP**, The Sophus Lie Conference Center, Nordfjordeid, Norway.

Description: The conference is a celebration of the twentieth anniversary of the *Journal of Nonlinear Mathematical Physics* (JNMP), which is published jointly by World Scientific and Atlantis Press.

Aim: To bring together experts and young scientists in the area of Mathematical Physics that concern Nonlinear Problems in Physics and Mathematics.

Main topic: The main topic is centered around the scope of JNMP: continuous and discrete integrable systems including ultradiscrete systems, nonlinear differential and difference equations, applications of Lie transformation groups and Lie algebras, nonlocal transformations and symmetries, differential-geometric aspects of integrable systems, classical and quantum groups, super geometry and super integrable systems.

Information: `http://staff.www.ltu.se/~johfab/jnmp/index.html`.

July 2013

* 20–25 **European Meeting of Statisticians**, Eotvos Lorand University, Budapest, Hungary.

Description: The European Meeting of Statisticians is uniquely the broadest and most prestigious regular meeting of the profession in Europe, having long history and well established traditions. Two distinguishing features of the current occasion are worth being emphasized, however. Beyond providing a natural forum for exchange of ideas for European statisticians and probabilists, particular organisational effort has been made to represent both traditional and newly emerging ties of the European professionals with the whole World. Hence, we expect colleagues from India, China, South-East Asia, the Middle-East, North- and Latin-America to participate in greater than usual number. It is also the ambition of the organisers to stimulate the inseminating tie between probability and statistics by a balanced representation of intertwined topics of both disciplines. The year 2013 itself provides the framework as it brings a number of celebrating anniversaries of probability theory and statistics.

Information: `http://www.ems2013.eu`.

August 2013

* 3–11 **Groups St. Andrews 2013**, University of St. Andrews, St. Andrews, Fife, Scotland, UK.

Description: This conference, the ninth in the series of Groups St. Andrews conferences, will be organized along similar lines to previous events in this series. The conference aims to cover all aspects of group theory, and to be accessible to postgraduate students, postdoctoral fellows, and researchers in all areas of group theory.

Speakers: The principal speakers will each deliver a short lecture course: Emmanuel Breuillard (Université Paris-Sud 11), Martin Liebeck (Imperial College, University of London), Alan Reid (University of Texas), Karen Vogtmann (Cornell University). The one-hour speakers are: Inna Capdeboscq (University of Warwick), Radha Kessar (University of Aberdeen), Markus Lohrey (Universität Leipzig), Derek Robinson (University of Illinois at Urbana-Champaign), Christopher Voll (University of Bielefeld). There will be further plenary talks, the opportunity for contributed talks, and an extensive social programme. The webpage for expressions of interest is now open.

Information: `http://www.groupsstandrews.org/2013/index.shtml`.

* 22–23 **Connections for Women on Optimal Transport: Geometry and Dynamics**, Mathematical Sciences Research Institute, Berkeley, California.
Description: This two-day event aims to connect women graduate students and beginning researchers with more established female researchers who use optimal transportation in their work and can serve as professional contacts and potential role-models. As such, it will showcase a selection of lectures featuring female scientists, both established leaders and emerging researchers. These lectures will be interspersed with networking and social events such as lunch or tea-time discussions led by successful researchers about (a) the particular opportunities and challenges facing women in science, including practical topics such as work-life balance and choosing a mentor, and (b) promising new directions in optimal transportation and related topics. Junior participants will be paired with more senior researchers in mentoring groups, and all participants will be encouraged to stay for the Introductory Workshop the following week, where they will have the opportunity to propose a short research communication.
Information: http://www.msri.org/web/msri/scientific/workshops/all-workshops/show/-/event/Wm9225.

* 26–30 **Introductory Workshop on Optimal Transport: Geometry and Dynamics**, Mathematical Sciences Research Institute, Berkeley, California.
Description: The workshop is intended to give an overview of the research landscape surrounding optimal transportation, including its connections to geometry, design applications, and fully nonlinear partial differential equations. As such, it will feature some survey lectures or minicourses by distinguished visitors and/or a few of the organizers of the theme semester, amounting to a kind of summer school. These will be complemented by a sampling of research lectures and short presentations from a spectrum of invited guests and other participants, including some who attended the previous week's *Connections for Women* workshop.
Information: http://www.msri.org/web/msri/scientific/workshops/all-workshops/show/-/event/Wm9226.

September 2013

* 3–4 **Connections for Women: Mathematical General Relativity**, Mathematical Sciences Research Institute, Berkeley, California.
Description: Ever since the epic work of Yvonne Choquet-Bruhat on the well-posedness of Einstein's equations initiated the mathematical study of general relativity, women have played an important role in many areas of mathematical relativity. In this workshop, some of the leading women researchers in mathematical relativity present their work.
Information: http://www.msri.org/web/msri/scientific/workshops/programmatic-workshops/show/-/event/Wm9551.

* 9–13 **Introductory Workshop: Mathematical General Relativity**, Mathematical Sciences Research Institute, Berkeley, California.
Description: Mathematical relativity is a very widely ranging area of mathematical study, spanning differential geometry, elliptic and hyperbolic PDE, and dynamical systems. We introduce in this workshop some of the leading areas of current interest, with a special focus on those areas which are related to the geometry and physics of the initial data of general relativity, and those which primarily involve Riemannian geometry and elliptic PDE.
Information: http://www.msri.org/web/msri/scientific/workshops/programmatic-workshops/show/-/event/Wm9552.

October 2013

* 14–18 **Fluid Mechanics, Hamiltonian Dynamics, and Numerical Aspects of Optimal Transportation**, Mathematical Sciences Research Institute, Berkeley, California.
Description: The workshop will be devoted to emerging approaches to fluid mechanical, geophysical and kinetic theoretical flows based on optimal transportation. It will also explore numerical approaches to optimal transportation problems.
Information: http://www.msri.org/web/msri/scientific/workshops/all-workshops/show/-/event/Wm9227.

November 2013

* 18–22 **Evolution Problems in General Relativity**, Mathematical Sciences Research Institute, Berkeley, California.
Description: With cosmic censorship, the formation of black holes, and the stability of Kerr black holes as focus problems, the study of the evolution of solutions of Einstein's equations has made dramatic progress in recent years. In this workshop, we highlight some of this recent development, and examine the major areas in which future progress is likely.
Information: http://www.msri.org/web/msri/scientific/workshops/programmatic-workshops/show/-/event/Wm9554.

January 2014

* 23–24 **Connections for Women: Algebraic Topology**, Mathematical Sciences Research Institute, Berkeley, California.
Description: This two-day workshop will consist of short courses given by prominent female mathematicians in the field. These introductory courses will be appropriate for graduate students, post-docs, and researchers in related areas. The workshop will also include a panel discussion featuring successful women at various stages in their mathematical careers.
Information: http://www.msri.org/web/msri/scientific/workshops/programmatic-workshops/show/-/event/Wm9545

* 27–31 **Introductory Workshop: Algebraic Topology**, Mathematical Sciences Research Institute, Berkeley, California.
Description: Algebraic topology is a rich, vibrant field with close connections to many branches of mathematics. This workshop will describe the state of the field, focusing on major programs, open problems, exciting new tools, and cutting edge techniques.
Information: http://www.msri.org/web/msri/scientific/workshops/programmatic-workshops/show/-/event/Wm9546.

April 2014

* 7–11 **Reimagining the Foundations of Algebraic Topology**, Mathematical Sciences Research Institute, Berkeley, California.
Description: Recent innovations in higher category theory have unlocked the potential to reimagine the basic tools and constructions in algebraic topology. This workshop will explore the interplay between these higher and ∞-categorical techniques with classical algebraic topology, playing each off of the other and returning the field to conceptual, geometrical intuition.
Information: http://www.msri.org/web/msri/scientific/workshops/programmatic-workshops/show/-/event/Wm9550.

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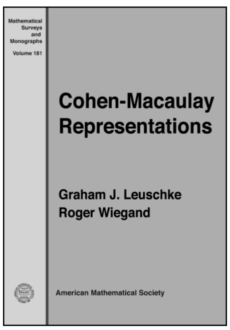
Algebra and Algebraic Geometry



Cohen-Macaulay Representations

Graham J. Leuschke, *Syracuse University, NY*, and **Roger Wiegand**, *University of Nebraska-Lincoln, NE*

This book is a comprehensive treatment of the representation theory of maximal Cohen-Macaulay (MCM) modules over local rings. This topic is at the intersection of commutative algebra, singularity theory, and representations of groups and algebras.

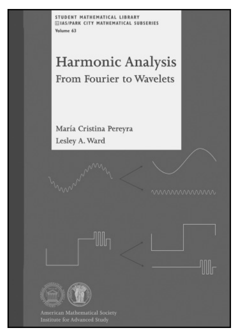
Two introductory chapters treat the Krull-Remak-Schmidt Theorem on uniqueness of direct-sum decompositions and its failure for modules over local rings. Chapters 3–10 study the central problem of classifying the rings with only finitely many indecomposable MCM modules up to isomorphism, i.e., rings of finite CM type. The fundamental material—ADE/simple singularities, the double branched cover, Auslander-Reiten theory, and the Brauer-Thrall conjectures—is covered clearly and completely. Much of the content has never before appeared in book form. Examples include the representation theory of Artinian pairs and Burban-Drozd's related construction in dimension two, an introduction to the McKay correspondence from the point of view of maximal Cohen-Macaulay modules, Auslander-Buchweitz's MCM approximation theory, and a careful treatment of nonzero characteristic. The remaining seven chapters present results on bounded and countable CM type and on the representation theory of totally reflexive modules.

Contents: The Krull-Remak-Schmidt theorem; Semigroups of modules; Dimension zero; Dimension one; Invariant theory; Kleinian singularities and finite CM type; Isoalted singularities and classification in dimension two; The double branched cover; Hypersurfaces with finite CM type; Ascent and descent; Auslander-Buchweitz theory; Totally reflexive modules; Auslander-Reiten theory; Countable Cohen-Macaulay type; The Brauer-Thrall conjectures; Finite CM type in higher dimensions; Bounded CM type; Basics and background; Ramification theory; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 181

June 2012, 367 pages, Hardcover, ISBN: 978-0-8218-7581-0, LC 2012002344, 2010 *Mathematics Subject Classification:* 13C14, 16G50; 13B40, 13C05, 13C60, 13H10, 16G10, 16G60, 16G70, **Individual member US$76.80**, **AMS members US$76.80**, List US$96, Order code SURV/181

Analysis



Harmonic Analysis

From Fourier to Wavelets

María Cristina Pereyra, *The University of New Mexico, Albuquerque, NM*, and **Lesley A. Ward**, *University of South Australia, Mawson Lakes Campus, Adelaide, Australia*

In the last 200 years, harmonic analysis has been one of the most influential bodies of mathematical ideas, having been exceptionally significant both in its theoretical implications and in its enormous range of applicability throughout mathematics, science, and engineering.

In this book, the authors convey the remarkable beauty and applicability of the ideas that have grown from Fourier theory. They present for an advanced undergraduate and beginning graduate student audience the basics of harmonic analysis, from Fourier's study of the heat equation, and the decomposition of functions into sums of cosines and sines (frequency analysis), to dyadic harmonic analysis, and the decomposition of functions into a Haar basis (time localization). While concentrating on the Fourier and Haar cases, the book touches on aspects of the world that lies between these two different ways of decomposing functions: time–frequency analysis (wavelets). Both finite and continuous perspectives are presented, allowing for the introduction of discrete Fourier and Haar transforms and fast algorithms, such as the Fast Fourier Transform (FFT) and its wavelet analogues.

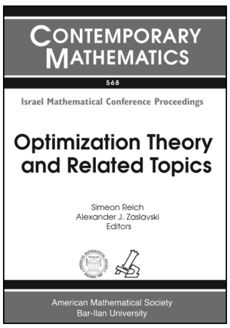
The approach combines rigorous proof, inviting motivation, and numerous applications. Over 250 exercises are included in the text. Each chapter ends with ideas for projects in harmonic analysis that students can work on independently.

This item will also be of interest to those working in applications.

Contents: Fourier series: Some motivation; Interlude: Analysis concepts; Pointwise convergence of Fourier series; Summability methods; Mean-square convergence of Fourier series; A tour of discrete Fourier and Haar analysis; The Fourier transform in paradise; Beyond paradise; From Fourier to wavelets, emphasizing Haar; Zooming properties of wavelets; Calculating with wavelets; The Hilbert transform; Useful tools; Bibliography; Index of names; Subject index.

Student Mathematical Library, Volume 63

June 2012, approximately 411 pages, Softcover, ISBN: 978-0-8218-7566-7, 2010 *Mathematics Subject Classification:* 42-01; 42-02, 42Axx, 42B25, 42C40, **Individual member US$46.40**, **AMS members US$46.40**, List US$58, Order code STML/63



Optimization Theory and Related Topics

Simeon Reich and **Alexander J. Zaslavski**, *Technion-Israel Institute of Technology, Haifa, Israel*, Editors

This volume contains the proceedings of the workshop on Optimization Theory and Related Topics, held in memory of Dan Butnariu, from January 11–14, 2010, in Haifa, Israel. An active researcher in various fields of applied mathematics, Butnariu published over 80 papers. His extensive bibliography is included in this volume.

The articles in this volume cover many different areas of Optimization Theory and its applications: maximal monotone operators, sensitivity estimates via Lyapunov functions, inverse Newton transforms, infinite-horizon Pontryagin principles, singular optimal control problems with state delays, descent methods for mixed variational inequalities, games on MV-algebras, ergodic convergence in subgradient optimization, applications to economics and technology planning, the exact penalty property in constrained optimization, nonsmooth inverse problems, Bregman distances, retraction methods in Banach spaces, and iterative methods for solving equilibrium problems.

This volume will be of interest to both graduate students and research mathematicians.

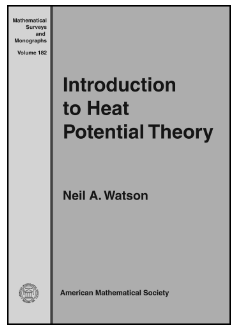
This item will also be of interest to those working in applications.

This book is co-published with Bar-Ilan University (Ramat-Gan, Israel).

Contents: Z. Artstein, Sensitivity estimates via Lyapunov functions and Lyapunov metrics; **H. H. Bauschke**, **X. Wang**, and **L. Yao**, On the maximal monotonicity of the sum of a maximal monotone linear relation and the subdifferential operator of a sublinear function; **A. Ben-Israel**, An inverse Newton transform; **J. Blot**, Infinite-horizon discrete-time Pontryagin principles via results of Michel; **S. D. Flåm**, On sharing of risk and resources; **M. Gabour** and **S. Reich**, The expected retraction method in Banach spaces; **V. Y. Glizer**, Solution of a singular optimal control problem with state delays: A cheap control approach; **I. Ioslovich**, **P.-O. Gutman**, and **A. Lichtsinder**, Robust reduction of dimension of a linear programming problem with uncertainties: Implication for robust production and technology planning; **I. V. Konnov**, Descent methods for mixed variational inequalities with non-smooth mappings; **T. Kroupa**, A generalized Möbius transform of games on MV-algebras and its application to a Cimmino-type algorithm for the core; **T. Larsson**, **M. Patriksson**, and **A.-B. Strömberg**, Ergodic convergence in subgradient optimization with application to simplicial decomposition of convex programs; **O. Mangoubi**, Strategic behavior in multiple-period financial markets; **D. Reem**, The Bregman distance without the Bregman function II; **S. Reich** and **S. Sabach**, Three strong convergence theorems regarding iterative methods for solving equilibrium problems in reflexive Banach spaces; **E. Resmerita**, Towards using coderivatives for convergence rates in regularization; **A. J. Zaslavski**, Existence of exact penalty in constrained optimization and the Mordukhovich basic subdifferential; **A. Zaslavski**, Weakly agreeable programs for the Robinson-Solow-Srinivasan (RSS) model.

Contemporary Mathematics, Volume 568

May 2012, 271 pages, Softcover, ISBN: 978-0-8218-6908-6, LC 2011051454, 2010 *Mathematics Subject Classification:* 34-XX, 39-XX, 46-XX, 47-XX, 49-XX, 52-XX, 65-XX, 90-XX, 91-XX, 93-XX, **Individual member US$77.60**, **AMS members US$77.60**, List US$97, Order code CONM/568



Introduction to Heat Potential Theory

Neil A. Watson, *University of Canterbury, Christchurch, New Zealand*

This book is the first to be devoted entirely to the potential theory of the heat equation, and thus deals with time dependent potential theory. Its purpose is to give a logical, mathematically precise introduction to a subject where previously many proofs were not written in detail, due to their similarity with those of the potential theory of Laplace's equation.

The approach to subtemperatures is a recent one, based on the Poisson integral representation of temperatures on a circular cylinder. Characterizations of subtemperatures in terms of heat balls and modified heat balls are proved, and thermal capacity is studied in detail. The generalized Dirichlet problem on arbitrary open sets is given a treatment that reflects its distinctive nature for an equation of parabolic type. Also included is some new material on caloric measure for arbitrary open sets.

Each chapter concludes with bibliographical notes and open questions. The reader should have a good background in the calculus of functions of several variables, in the limiting processes and inequalities of analysis, in measure theory, and in general topology for Chapter 9.

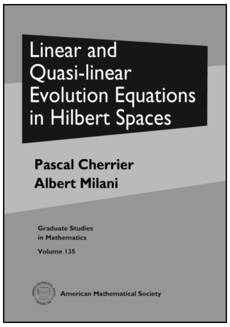
This item will also be of interest to those working in differential equations.

Contents: The heat operator, temperatures and mean values; The Poisson integral for a circular cylinder; Subtemperatures and the Dirichlet problem on convex domains of revolution; Temperatures on an infinite strip; Classes of subtemperatures on an infinite strip; Green functions and heat potentials; Polar sets and thermal capacity; The Dirichlet problem on arbitrary open sets; The thermal fine topology; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 182

May 2012, 266 pages, Hardcover, ISBN: 978-0-8218-4998-9, LC 2012004904, 2010 *Mathematics Subject Classification:* 31-02, 31B05, 31B20, 31B25, 31C05, 31C15, 35-02, 35K05, 31B15, **Individual member US$68.80**, **AMS members US$68.80**, List US$86, Order code SURV/182

Differential Equations



Linear and Quasi-linear Evolution Equations in Hilbert Spaces

Pascal Cherrier, *Université Pierre et Marie Curie, Paris, France*, and **Albert Milani**, *University of Wisconsin, Milwaukee, WI*

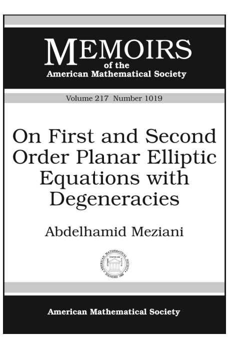
This book considers evolution equations of hyperbolic and parabolic type. These equations are studied from a common point of view, using elementary methods, such as that of energy estimates, which prove to be quite versatile. The authors emphasize the Cauchy problem and present a unified theory for the treatment of these equations. In particular, they provide local and global existence results, as well as strong well-posedness and asymptotic behavior results for the Cauchy problem for quasi-linear equations. Solutions of linear equations are constructed explicitly, using the Galerkin method; the linear theory is then applied to quasi-linear equations, by means of a linearization and fixed-point technique. The authors also compare hyperbolic and parabolic problems, both in terms of singular perturbations, on compact time intervals, and asymptotically, in terms of the diffusion phenomenon, with new results on decay estimates for strong solutions of homogeneous quasi-linear equations of each type.

This textbook presents a valuable introduction to topics in the theory of evolution equations, suitable for advanced graduate students. The exposition is largely self-contained. The initial chapter reviews the essential material from functional analysis. New ideas are introduced along with their context. Proofs are detailed and carefully presented. The book concludes with a chapter on applications of the theory to Maxwell's equations and von Karman's equations.

Contents: Functional framework; Linear equations; Quasi-linear equations; Global existence; Asymptotic behavior; Singular convergence; Maxwell's and von Karman's equations; List of function spaces; Bibliography; Index.

Graduate Studies in Mathematics, Volume 135

August 2012, approximately 378 pages, Hardcover, ISBN: 978-0-8218-7576-6, LC 2012002958, 2010 *Mathematics Subject Classification:* 35L15, 35L72, 35K15, 35K59, 35Q61, 35Q74, **Individual member US$60, AMS members US$60**, List US$75, Order code GSM/135



On First and Second Order Planar Elliptic Equations with Degeneracies

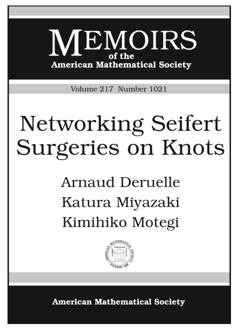
Abdelhamid Meziani, *Florida International University, Miami, FL*

Contents: Introduction; Preliminaries; Basic Solutions; Example; Asymptotic behavior of the basic solutions of $\mathcal{L}$; The kernels; The homogeneous equation $\mathcal{L}u = 0$; The nonhomogeneous equation $\mathcal{L}u = F$; The semilinear equation; The second order equation: Reduction; The homogeneous equation $Pu = 0$; The nonhomogeneous equation $Pu = F$; Normalization of a class of second order equations with a singularity; Bibliography.

Memoirs of the American Mathematical Society, Volume 217, Number 1019

April 2012, 77 pages, Softcover, ISBN: 978-0-8218-5312-2, LC 2011051781, 2010 *Mathematics Subject Classification:* 35J70; 35F05, 30C20, **Individual member US$36**, List US$60, Institutional member US$48, Order code MEMO/217/1019

Geometry and Topology



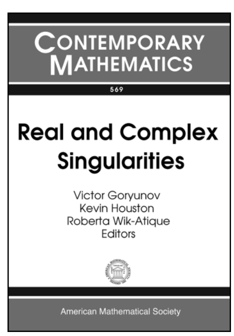
Networking Seifert Surgeries on Knots

Arnaud Deruelle, *Nihon University, Tokyo, Japan*, **Katura Miyazaki**, *Tokyo Denki University, Japan*, and **Kimihiko Motegi**, *Nihon University, Tokyo, Japan*

Contents: Introduction; Seiferters and Seifert Surgery Network; Classification of seiferters; Geometric aspects of seiferters; S-linear trees; Combinatorial structure of Seifert surgery network; Asymmetric seiferters and Seifert surgeries on knots without symmetry; Seifert surgeries on torus knots and graph knots; Paths from various known Seifert surgeries to those on torus knots; Bibliography.

Memoirs of the American Mathematical Society, Volume 217, Number 1021

April 2012, 130 pages, Softcover, ISBN: 978-0-8218-5333-7, LC 2011051783, 2010 *Mathematics Subject Classification:* 57M25, 57M50; 57N10, **Individual member US$42.60**, List US$71, Institutional member US$56.80, Order code MEMO/217/1021



Real and Complex Singularities

Victor Goryunov, *University of Liverpool, United Kingdom*, **Kevin Houston**, *University of Leeds, United Kingdom*, and **Roberta Wik-Atique**, *ICMC/USP-São Carlos, Brazil*, Editors

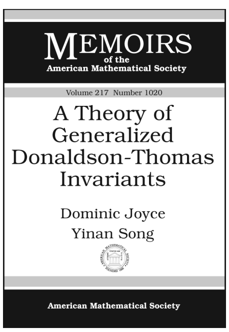
This volume is a collection of papers presented at the 11th International Workshop on Real and Complex Singularities, held July 26–30, 2010, in São Carlos, Brazil, in honor of David Mond's 60th birthday. This volume reflects the high level of the conference discussing the most recent results and applications of singularity theory. Articles in the first part cover pure singularity theory: invariants, classification theory, and Milnor fibres. Articles in the second part cover singularities in topology and differential geometry, as well as algebraic geometry and bifurcation theory: Artin-Greenberg function of a plane curve singularity, metric theory of singularities, symplectic singularities, cobordisms of fold maps, Goursat distributions, sections of analytic varieties, Vassiliev invariants, projections of hypersurfaces, and linearity of the Jacobian ideal.

This item will also be of interest to those working in algebra and algebraic geometry.

Contents: J. L. Cisneros-Molina, **J. Seade**, and **J. Snoussi**, Milnor fibrations and the concept of d-regularity for analytic map germs; **J. C. F. Costa**, **M. J. Saia**, and **C. H. Soares Júnior**, Bi-Lipschitz $\mathcal{G}$-triviality and Newton polyhedra, $\mathcal{G} = \mathcal{R}, \mathcal{C}, \mathcal{K}, \mathcal{R}_V, \mathcal{C}_V, \mathcal{K}_V$; **W. Domitrz** and **Ż. Trębska**, Symplectic S_μ singularities; **R. Araújo dos Santos**, **D. Dreibelbis**, and **N. Dutertre**, Topology of the real Milnor fiber for isolated singularities; **C. Maquera** and **W. T. Huaraca**, Compact 3-manifolds supporting some $\mathbb{R}^2$-actions; **M. Kasedou**, Timelike canal hypersurfaces of spacelike submanifolds in a de Sitter space; **D. Lehmann**, Residues in K-theory; **Y. Mizota** and **T. Nishimura**, Multicusps; **P. Mormul**, Small growth vectors of the compactifications of the contact systems on $J^r(1,1)$; **T. Ohmoto**, Vassiliev type invariants for generic mappings, revisited; **B. Oréfice** and **J. N. Tomazella**, Sections of analytic variety; **S. Saleh**, The Artin-Greenberg function of a plane curve singularity; **M. Shubladze**, Singularities with critical locus a complete intersection and transversal type A_1.

Contemporary Mathematics, Volume 569

June 2012, 202 pages, Softcover, ISBN: 978-0-8218-5359-7, LC 2011052531, 2010 *Mathematics Subject Classification:* 58Kxx, 57Rxx, 57Qxx, 32Sxx, 14Pxx, 37Cxx, **Individual member US$59.20**, **AMS members US$59.20**, List US$74, Order code CONM/569



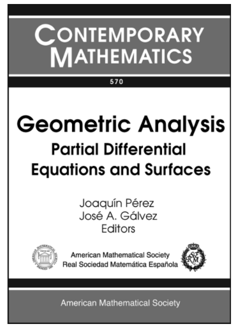
A Theory of Generalized Donaldson-Thomas Invariants

Dominic Joyce, *The Mathematical Institute, Oxford, United Kingdom*, and **Yinan Song**, *Budapest, Hungary*

Contents: Introduction; Constructible functions and stack functions; Background material from [**51, 52, 53, 54**]; Behrend functions and Donaldson-Thomas theory; Statements of main results; Examples, applications, and generalizations; Donaldson-Thomas theory for quivers with superpotentials; The proof of Theorem 5.3; The proofs of Theorems 5.4 and 5.5; The proof of Theorem 5.11; The proof of Theorem 5.14; The proofs of Theorems 5.22, 5.23 and 5.25; The proof of Theorem 5.27; Bibliography; Glossary of Notation; Index.

Memoirs of the American Mathematical Society, Volume 217, Number 1020

April 2012, 199 pages, Softcover, ISBN: 978-0-8218-5279-8, LC 2011051782, 2010 *Mathematics Subject Classification:* 14N35; 14J32, 14F05, 14J60, 14D23, **Individual member US$51.60**, List US$86, Institutional member US$68.80, Order code MEMO/217/1020



Geometric Analysis

Partial Differential Equations and Surfaces

Joaquín Pérez and **José A. Gálvez**, *Universidad de Granada, Spain*, Editors

This volume contains research and expository articles from the courses and talks given at the RSME Lluis A. Santaló Summer School, "Geometric Analysis", held June 28–July 2, 2010, in Granada, Spain.

The goal of the Summer School was to present some of the many advances currently taking place in the interaction between partial differential equations and differential geometry, with special emphasis on the theory of minimal surfaces.

This volume includes expository articles about the current state of specific problems involving curvature and partial differential equations, with interactions to neighboring fields such as probability. An introductory, mostly self-contained course on constant mean curvature surfaces in Lie groups equipped with a left invariant metric is provided.

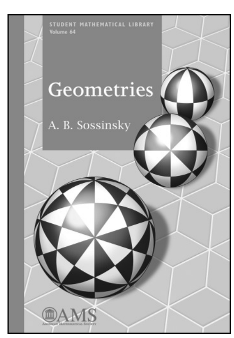
The volume will be of interest to researchers, post-docs, and advanced Ph.D. students in the interface between partial differential equations and differential geometry.

This item will also be of interest to those working in differential equations.

Contents: J. A. Gálvez and **P. Mira**, Geometric PDEs in the presence of isolated singularities; **W. H. Meeks III** and **J. Pérez**, Constant mean curvature surfaces in metric Lie groups; **R. W. Neel**, Stochastic methods for minimal surfaces; **F. Pacard**, The role of minimal surfaces in the study of the Allen-Cahn equation; **G. Tinaglia**, On curvature estimates for constant mean curvature surfaces.

Contemporary Mathematics, Volume 570

June 2012, approximately 191 pages, Softcover, ISBN: 978-0-8218-4992-7, 2010 *Mathematics Subject Classification:* 53A10; 35B33, 35B40, 35J20, 35J25, 35J96, 35J60, 49Q05, 53C42, 53C45, **Individual member US$59.20**, **AMS members US$59.20**, List US$74, Order code CONM/570



Geometries

A. B. Sossinsky, *Independent University of Moscow, Russia*

The book is an innovative modern exposition of geometry, or rather, of geometries; it is the first textbook in which Felix Klein's Erlangen Program (the action of transformation groups) is systematically used as the basis for defining various geometries. The course of study presented is dedicated to the proposition that all geometries are created equal—although some, of course, remain more equal than others. The author concentrates on several of the more distinguished and beautiful ones, which include what he terms "toy geometries", the geometries of Platonic bodies, discrete geometries, and classical continuous geometries.

The text is based on first-year semester course lectures delivered at the Independent University of Moscow in 2003 and 2006. It is by no means a formal algebraic or analytic treatment of geometric topics, but rather, a highly visual exposition containing upwards of 200 illustrations. The reader is expected to possess a familiarity with elementary Euclidean geometry, albeit those lacking this knowledge may refer to a compendium in Chapter 0. Per the author's predilection, the book contains very little regarding the axiomatic approach to geometry (save for a single chapter on the history of non-Euclidean geometry), but two Appendices provide a detailed treatment of Euclid's and Hilbert's axiomatics. Perhaps the most important aspect of this course is the problems, which appear at the end of each chapter and are supplemented with answers at the conclusion of the text. By analyzing and solving these problems, the reader will become capable of thinking and working geometrically, much more so than by simply learning the theory.

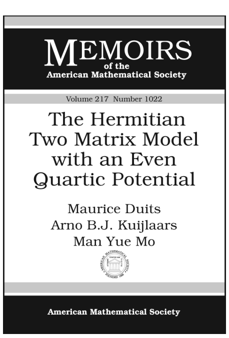
Ultimately, the author makes the distinction between concrete mathematical objects called "geometries" and the singular "geometry", which he understands as a way of thinking about mathematics. Although the book does not address branches of mathematics and mathematical physics such as Riemannian and Kähler manifolds or, say, differentiable manifolds and conformal field theories, the ideology of category language and transformation groups on which the book is based prepares the reader for the study of, and eventually, research in these important and rapidly developing areas of contemporary mathematics.

Contents: About Euclidean geometry; Toy geometries and main definitions; Abstract groups and group presentations; Finite subgroups of $SO(3)$ and the platonic bodies; Discrete subgroups of the isometry group of the plane and tilings; Reflection groups and Coxeter geometries; Spherical geometry; The Poincaré disk model of hyperbolic geometry; The Poincaré half-plane model; The Cayley-Klein model; Hyperbolic trigonometry and absolute constants; History of non-Euclidean geometry; Projective geometry; "Projective geometry is all geometry"; Finite geometries; The hierarchy of geometries; Morphisms of geometries; Excerpts from Euclid's "Elements"; Hilbert's axioms for plane geometry; Answers & hints; Bibliography; Index.

Student Mathematical Library, Volume 64

June 2012, 301 pages, Softcover, ISBN: 978-0-8218-7571-1, LC 2012002357, 2010 *Mathematics Subject Classification:* 51-01; 51-02, 01A20, 01A55, 18-01, **Individual member US$38.40**, **AMS members US$38.40**, List US$48, Order code STML/64

Mathematical Physics



The Hermitian Two Matrix Model with an Even Quartic Potential

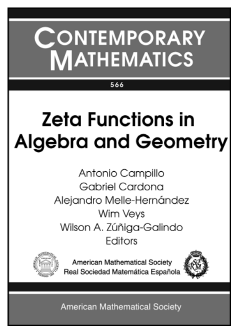
Maurice Duits, *California Institute of Technology, Pasadena, CA*, **Arno B.J. Kuijlaars**, *Katholieke Universiteit Leuven, Belgium*, and **Man Yue Mo**, *University of Bristol, United Kingdom*

Contents: Introduction and statement of results; Preliminaries and the proof of Lemma 1.2; Proof of Theorem 1.1; A Riemann surface; Pearcey integrals and the first transformation; Second transformation $X \mapsto U$; Opening of lenses; Global parametrix; Local parametrices and final transformation; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 217, Number 1022

April 2012, 105 pages, Softcover, ISBN: 978-0-8218-6928-4, LC 2011051784, 2010 *Mathematics Subject Classification:* 30E25, 60B20; 15B52, 30F10, 31A05, 42C05, 82B26, **Individual member US$42**, List US$70, Institutional member US$56, Order code MEMO/217/1022

Number Theory



Zeta Functions in Algebra and Geometry

Antonio Campillo, *Universidad de Valladolid, Spain*, **Gabriel Cardona**, *Universitat de les Illes Balears, Palma de Mallorca, Spain*, **Alejandro Melle-Hernández**, *Universidad Complutense de Madrid, Spain*, **Wim Veys**, *University of Leuven, Belgium*, and **Wilson A. Zúñiga-Galindo**, *Cinvestav I.P.N., Mexico City, Mexico*, Editors

The volume contains the proceedings of the "Second International Workshop on Zeta Functions in Algebra and Geometry", held May 3–7, 2010 at the Universitat de les Illes Balears, Palma de Mallorca, Spain.

Zeta functions can be naturally attached to several mathematical objects, including fields, groups, and algebras. The conference focused on the following topics: arithmetic and geometric aspects of local, topological, and motivic zeta functions, Poincaré series of valuations, zeta functions of groups, rings, and representations, prehomogeneous vector spaces and their zeta functions, and height zeta functions.

This item will also be of interest to those working in algebra and algebraic geometry.

Contents: *L-functions of varieties over finite fields and Artin L-functions:* **P. Bayer**, Computational aspects of Artin L-functions; **A. Frühbis-Krüger** and **S. Kadir**, Zeta-functions for families of Calabi-Yau n-folds with singularities; **A. Rojas-León**, Estimates for exponential sums with a large automorphism group; *Height zeta functions and arithmetic:* **D. Essouabri**, Height zeta functions on generalized projective toric varieties; **Yu. I. Manin**, Combinatorial cubic surfaces and reconstruction theorems; **S. Tanimoto** and **Y. Tschinkel**, Height zeta functions of equivariant compactifications of semi-direct products of algebraic groups; *Motivic zeta functions, Poincaré series, complex monodromy and knots:* **N. Budur**, Singularity invariants related to Milnor fibers: Survey; **C. Galindo** and **F. Monserrat**, Finite families of plane valuations: Value semigroup, graded algebra and Poincaré series; **E. Gorsky**, q, t-Catalan numbers and knot homology; **L. H. Halle** and **J. Nicaise**, Motivic zeta functions for degenerations of abelian varieties and Calabi-Yau varieties;

A. Némethi and **F. Román**, The lattices cohomology of $S^3_{-d}(K)$; *Zeta functions for groups and representations:* **N. Avni**, **B. Klopsch**, **U. Onn**, and **C. Voll**, Representation zeta functions of some compact p-adic analytic groups; **A. Shalev**, Applications of some zeta functions in group theory.

Contemporary Mathematics, Volume 566

May 2012, approximately 347 pages, Softcover, ISBN: 978-0-8218-6900-0, LC 2011050434, 2010 *Mathematics Subject Classification:* 11F66, 11G25, 11L07, 11G50, 14D10, 14E18, 14G40, 22E50, 32S40, 57M27, **Individual member US$86.40**, **AMS members US$86.40**, List US$108, Order code CONM/566

New AMS-Distributed Publications

Algebra and Algebraic Geometry

Champs de Hurwitz

J. Bertin and **M. Romagny**, *University Pierre et Marie Curie, Paris, France*

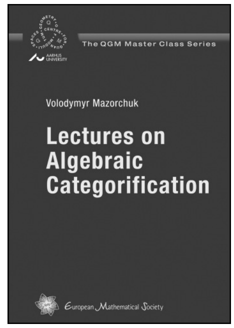
In this work, the authors give a thorough study of Hurwitz stacks and associated Hurwitz moduli spaces, both in the Galois and the non-Galois case, with particular attention to correspondances between these moduli spaces. They compare their construction to those proposed by Abramovich-Corti-Vistoli, Harris-Mumford, and Mochizuki-Wewers. They apply their results to revisit some classical examples, particularly the stacks of stable curves equipped with an arbitrary level structure, and the stacks of tamely ramified cyclic covers. In a second part they exhibit some tautological bundles and cohomology classes naturally living on Hurwitz stacks, and give some universal relations, in particular a higher analogue of the Riemann-Hurwitz formula, between these classes. Applications are given to the stack of cyclic covers of the projective line, with special attention to Cornalba-Harris type relations and to cyclic, in particular hyperelliptic Hodge integrals.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Classification des revêtements; Familles de G-courbes lisses et Théorème de Chevalley-Weil; Familles de G-courbes stables; Déformations des revêtements modérément ramifiés; Champs de Hurwitz; Graphes et revêtements; Structures de niveau sur les courbes stables; Revêtements cycliques; Classes tautologiques; Index; Bibliographie.

Mémoires de la Société Mathématique de France, Number 125/126

January 2012, 219 pages, Softcover, ISBN: 978-2-85629-333-1, 2010 *Mathematics Subject Classification:* 14H10, 14H30, 14H37, 14A20, 14C17, 11G20, 11G30, 14C40, **Individual member US$67.50**, List US$75, Order code SMFMEM/125/126



Lectures on Algebraic Categorification

Volodymyr Mazorchuk, *Uppsala University, Sweden*

The term "categorification" was introduced by Louis Crane in 1995 and refers to the process of replacing set-theoretic notions by the corresponding category-theoretic analogues.

This text mostly concentrates on algebraical aspects of the theory, presented in the historical perspective, but also contains several topological applications, in particular, an algebraic (or, more precisely, representation-theoretical) approach to categorification. It consists of fifteen sections corresponding to fifteen one-hour lectures given during a Master Class at Aarhus University, Denmark in October 2010. There are some exercises collected at the end of the text and a rather extensive list of references. Video recordings of all (but one) lectures are available from the Master Class website.

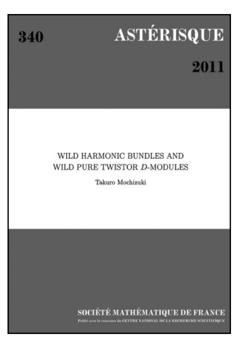
The book provides an introductory overview of the subject rather than a fully detailed monograph. The emphasis is made on definitions, examples and formulations of the results. Most proofs are either briefly outlined or omitted. However, complete proofs can be found by tracking references. It is assumed that the reader is familiar with the basics of category theory, representation theory, topology, and Lie algebra.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Basics: decategorification and categorification; Basics: from categorification of linear maps to 2-categories; Basics: 2-representations of finitary 2-categories; Category $\mathcal{O}$: definitions; Category $\mathcal{O}$: projective and shuffling functors; Category $\mathcal{O}$: twisting and completion; Category $\mathcal{O}$: grading and combinatorics; $\mathbb{S}_n$-categorification: Soergel bimodules, cells and Specht modules; $\mathbb{S}_n$-categorification: (induced) cell modules; Category $\mathcal{O}$: Koszul duality; $\mathfrak{sl}_2$-categorification: simple finite-dimensional modules; Application: categorification of the Jones polynomial; $\mathfrak{sl}_2$-categorification of Chuang and Rouquier; Application: blocks of $\mathbb{F}[\mathbb{S}_n]$ and Broué's conjecture; Applications of $\mathbb{S}_n$-categorifications; Exercises; Bibliography; Index.

The QGM Master Class Series, Volume 2

March 2012, 128 pages, Softcover, ISBN: 978-3-03719-108-8, 2010 *Mathematics Subject Classification:* 18-01, 18D05, 17B10, 17B55, 18A40, 18E30, 57M27, **Individual member US$28.80**, **AMS members US$28.80**, List US$36, Order code EMSQGM/2



Wild Harmonic Bundles and Wild Pure Twistor D-Modules

Takuro Mochizuki, *Kyoto University, Japan*

The author studies (i) the asymptotic behaviour of wild harmonic bundles, (ii) the relation between semisimple meromorphic flat connections and wild harmonic bundles, (iii) the relation between wild harmonic bundles and polarized wild pure twistor D-modules. As an application, he shows the hard

Lefschetz theorem for algebraic semisimple holonomic D-modules, conjectured by M. Kashiwara and also studies resolution of turning points for algebraic meromorphic flat bundles.

This item will also be of interest to those working in analysis.

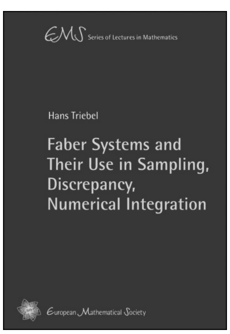
A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; *Part I. Good meromorphic ϱ-flat bundles:* Good formal property of a meromorphic ϱ-flat bundle; Stokes structure of a good ϱ-meromorphic flat bundle; Full Stokes data and Riemann-Hilbert-Birkhoff correspondence; L^2-cohomology of filtered λ-flat bundle on curves; Meromorphic variation of twistor structure; *Part II. Prolongation of wild harmonic bundle:* Prolongments $\mathcal{PE}^\lambda$ for unramifiedly good wild harmonic bundles; Some basic results in the curve case; Associated family of meromorphic λ-flat bundles; Smooth divisor case; Prolongation and reduction of variations of polarized pure twistor structures; Prolongation as $\mathcal{R}$-triple; *Part III. Kobayashi-Hitchin correspondence:* Preliminaries; Construction of an initial metric and preliminary correspondence; Preliminaries for the resolution of turning points; Kobayashi-Hitchin correspondence and some applications; *Part IV. Application to wild pure twistor D-modules:* Wild pure twistor D-modules; The Hard Lefschetz Theorem; Correspondences; *Part V. Appendix:* Preliminaries from analysis on multi-sectors; Acceptable bundles; Review on $\mathcal{R}$-modules, $\mathcal{R}$-triples and variants; Bibliography; Index.

Astérisque, Number 340

January 2012, 607 pages, Softcover, ISBN: 978-2-85629-332-4, 2010 *Mathematics Subject Classification:* 14J60, 32C38, 53C07, **Individual member US$121.50**, List US$135, Order code AST/340

Analysis



Faber Systems and Their Use in Sampling, Discrepancy, Numerical Integration

Hans Triebel, *Friedrich-Schiller University of Jena, Germany*

This book deals first with Haar bases, Faber bases and Faber frames for weighted function spaces on the real line and the plane. It extends results in the author's book, "Bases in Function Spaces, Sampling, Discrepancy, Numerical Integration" (EMS, 2010), from unweighted spaces (preferably in cubes) to weighted spaces.

The obtained assertions are used to study sampling and numerical integration in weighted spaces on the real line and weighted spaces with dominating mixed smoothness in the plane. A short chapter deals with the discrepancy for spaces on intervals.

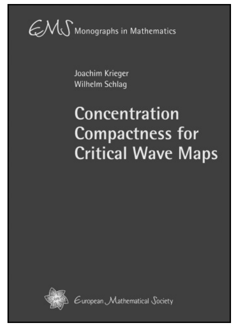
A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Introduction, definitions, basic assertions; Spaces on intervals; Spaces on the real line; Spaces on the plane; Bibliography; Symbols; Index.

EMS Series of Lectures in Mathematics, Volume 16

March 2012, 115 pages, Softcover, ISBN: 978-3-03719-107-1, 2010 *Mathematics Subject Classification:* 46-02, 46E35, 42C40, 42B35, 68Q17, 41A55, **Individual member US$28.80**, **AMS members US$28.80**, List US$36, Order code EMSSERLEC/16

Differential Equations



Concentration Compactness for Critical Wave Maps

Joachim Krieger, *EPFL, Lausanne, Switzerland*, and **Wilhelm Schlag**, *University of Chicago, IL*

Wave maps are the simplest wave equations taking their values in a Riemannian manifold (M, g). Their Lagrangian is the same as for the scalar equation, the only difference being that lengths are measured with respect to the metric g. By Noether's theorem, symmetries of the Lagrangian imply conservation laws for wave maps, such as conservation of energy.

In coordinates, wave maps are given by a system of semilinear wave equations. Over the past 20 years important methods have emerged which address the problem of local and global wellposedness of this system. Due to weak dispersive effects, wave maps defined on Minkowski spaces of low dimensions, such as $\mathbb{R}^{2+1}_{t,x}$, present particular technical difficulties. This class of wave maps has the additional important feature of being energy critical, which refers to the fact that the energy scales exactly like the equation.

Around 2000 Daniel Tataru and Terence Tao, building on earlier work of Klainerman-Machedon, proved that smooth data of small energy lead to global smooth solutions for wave maps from 2+1 dimensions into target manifolds satisfying some natural conditions. In contrast, for large data, singularities may occur in finite time for $M = \mathbb{S}^2$ as target. This monograph establishes that for $\mathbb{H}$ as target the wave map evolution of any smooth data exists globally as a smooth function.

While the authors restrict themselves to the hyperbolic plane as target the implementation of the concentration-compactness method, the most challenging piece of this exposition, yields more detailed information on the solution. This monograph will be of interest to experts in nonlinear dispersive equations, in particular to those working on geometric evolution equations.

This item will also be of interest to those working in geometry and topology.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Introduction and overview; The spaces $S[k]$ and $N[k]$; Hodge decomposition and null-structures; Bilinear estimates involving S and N spaces; Trilinear estimates; Quintilinear and higher nonlinearities; Some basic perturbative results; BMO, A_p, and weighted commutator estimates; The Bahouri–Gérard concentration compactness method; The proof of the main theorem; Appendix; References; Index.

EMS Monographs in Mathematics, Volume 5

February 2012, 490 pages, Hardcover, ISBN: 978-3-03719-106-4, 2010 *Mathematics Subject Classification:* 35L05, 35L52, 53Z05, **Individual member US$94.40**, **AMS members US$94.40**, List US$118, Order code EMSMONO/5

Classified Advertisements

Positions available, items for sale, services available, and more

MINNESOTA

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS (IMA)
Accepting Board of Governors Nominations

The Institute for Mathematics and its Applications (IMA) is accepting nominations for its Board of Governors. The board is the IMA's principal governing body, consisting of 15 distinguished members from academia, industry, and national laboratories. The board provides oversight and advice on matters of institute management, development, and institutional relationships. Board members play an active scientific role in planning and developing annual program themes as well as identifying lead program organizers. Members will serve a five-year term, beginning January 1, 2013. Details and an online application available at `http://www.ima.umn.edu/governance`. Applications due July 31, 2012. Please contact Fadil Santosa, director of the IMA, at `santosa@ima.umn.edu`, for further information.

000026

CHILE

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
Department of Mathematics

The Department of Mathematics invites applications for three tenure-track positions at the Assistant Professor level beginning either March or August 2013. Applicants should have a Ph.D. in Mathematics, proven research potential either in pure or applied mathematics, and a strong commitment to teaching and research. The regular teaching load for assistant professors consists of three one-semester courses per year, reduced to two during the first two years. The annual salary will be US$47,000 (calculated at the current exchange rate of 500 chilean pesos per dollar).

Please send a letter indicating your main research interests, potential collaborators in our Department (`www.mat.puc.cl`), detailed curriculum vitae, and three letters of recommendation to:

Monica Musso
Director
Pontificia Universidad Católica de Chile
Av. Vicuña Mackenna 4860
Santiago, Chile;
fax: (56-2) 552-5916;
email: `mmusso@mat.puc.cl`

For full consideration, complete application materials must arrive by June 30, 2012.

000023

NIGERIA

UNIVERSITY OF LAGOS, NIGERIA
Pastor Enoch A. Adeboye Professorial Chair of Mathematics

Applications are invited from suitably qualified candidates for a Professorial Chair in the University of Lagos which is a Tertiary Institution of Learning located in the West African sub-region. The conditions for eligibility for the position are as stated on our website.

Interested candidates are advised to visit the University website and the National Mathematical Center website at `www.unilag.edu.ng` and `www.nmcabuja.org` for further details on the method of aplciation. The closing date is 15th May, 2012.

000028

FOR SALE

JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS

200 complete volumes, *Journal of Mathematical Analysis and Applications*. Contact: `mathzsa@yahoo.com`.

000027

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2012 rate is $3.50 per word with a minimum two-line headline. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: June/July 2011 issue–April 28, 2011; August 2011 issue–May 27, 2010; September 2011 issue–June 28, 2011; October 2011 issue–July 28, 2011; November 2012 issue–August 30, 2012; December 2012 issue–October 1, 2012.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Rochester, New York

Rochester Institute of Technology

September 22–23, 2012

Saturday – Sunday

Meeting #1082

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June/July 2012
Program first available on AMS website: July 19, 2012
Program issue of electronic *Notices*: September 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: May 15, 2012
For abstracts: July 10, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Steve Gonek, University of Rochester, *Title to be announced*.

James Keener, University of Utah, *Title to be announced*.

Dusa McDuff, Barnard College, *Title to be announced*.

Peter Winkler, Dartmouth College, *Title to be announced*.

Special Sessions

Analytic Number Theory (Code: SS 5A), **Steve Gonek**, University of Rochester, and **Angel Kumchev**, Towson University.

Applied and Computational Mathematics (Code: SS 11A), **Ludwig Kohaupt**, Beuth University of Technology, and **Yan Wu**, Georgia Southern University.

Continuum Theory (Code: SS 3A), **Likin C. Simon Romero**, Rochester Institute of Technology.

Difference Equations and Applications (Code: SS 10A), **Michael Radin**, Rochester Polytechnic Institute.

Financial Mathematics (Code: SS 1A), **Tim Siu-Tang Leung**, Columbia University.

Frontiers in Applied and Industrial Mathematics (Code: SS 13A), **Kara L. Maki** and **David S. Ross**, Rochester Institute of Technology.

Geometric, Categorical and Combinatorial Methods in Representation Theory (Code: SS 12A), **David Hemmer** and **Yiqiang Li**, State University of New York at Buffalo.

Inverse Problems and Nonsmooth Optimization: Celebrating Zuhair Nashed's 75th Birthday (Code: SS 7A), **Patricia Clark**, **Baasansuren Jadama**, and **Akhtar A. Khan**, Rochester Institute of Technology, and **Hulin Wu**, University of Rochester.

Mathematical Image Processing (Code: SS 20A), **Nathan Cahill**, Rochester Institute of Technology, and **Lixin Shen** and **Yuesheng Xu**, Syracuse University.

Microlocal Analysis and Nonlinear Evolution Equations (Code: SS 2A), **Raluca Felea**, Rochester Institute of Technology, and **Dan-Andrei Geba**, University of Rochester.

Modern Relativity (Code: SS 6A), **Manuela Campanelli** and **Yosef Zlochower**, Rochester Institute of Technology.

New Advances in Graph Theory (Code: SS 9A), **Jobby Jacob**, Rochester Institute of Technology, and **Paul Wenger**, University of Colorado Denver.

Nonlinear Dynamics of Excitable Media (Code: SS 18A), **Elizabeth Cherry**, Rochester Institute of Technology.

Nonlinear Partial Differential Equations in the Physical and Biological Sciences (Code: SS 14A), **Tony Harkin**, Rochester Institute of Technology, and **Doug Wright**, Drexel University.

Operator Theory and Function Spaces (Code: SS 4A), **Gabriel T. Prajitura** and **Ruhan Zhao**, State University of New York at Brockport.

Permutations Patterns, Algorithms, and Enumerative Combinatorics (Code: SS 19A), **Howard Skogman** and **Rebecca Smith**, State University of New York at Brockport.

Probability and Statistical Physics (Code: SS 16A), **Wenbo Li**, University of Delaware, and **Carl Mueller** and **Shannon Starr**, University of Rochester.

Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs (Code: SS 8A),

Bernard Brooks, **Darren Narayan**, and **Tamas Wiandt**, Rochester Institute of Technology.

Symplectic and Contact Topology (Code: SS 15A), **Dusa McDuff**, Barnard College, and **Vera Vertesi**, Massachusetts Institute of Technology.

Wavelet and Frame Theoretic Methods in Harmonic Analysis and Partial Differential Equations in Memory of Daryl Geller (Code: SS 17A), **Alex Iosevich**, University of Rochester, and **Azita Mayeli**.

New Orleans, Louisiana

Tulane University

October 13–14, 2012

Saturday – Sunday

Meeting #1083

Southeastern Section
Associate secretary: Robert Daverman
Announcement issue of *Notices*: June/July 2012
Program first available on AMS website: September 6, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 3, 2012
For abstracts: August 28, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Anita Layton, Duke University, *Title to be announced.*

Lenhard Ng, Duke University, *Title to be announced.*

Henry K. Schenck, University of Illinois at Urbana-Champaign, *From approximation theory to algebraic geometry: The ubiquity of splines.*

Milen Yakimov, Louisiana State University, *Title to be announced.*

Special Sessions

Algebraic Combinatorics: Rook Theory and Applications (Code: SS 4A), **Mahir Bilen Can** and **Michael Joyce**, Tulane University, and **Jeff Remmel**, University of California at San Diego.

Approximation Theory, Geometric Modelling, and Algebraic Geometry (Code: SS 7A), **Henry Schenck**, University of Illinois at Urbana-Champaign.

Biological Fluid Dynamics: Modeling, Computations, and Applications (Code: SS 5A), **Anita T. Layton**, Duke University, and **Sarah D. Olson**, Worcester Polytechnic Institute.

Combinatorial Commutative Algebra (Code: SS 1A), **Chris Francisco**, Oklahoma State University, **Tai Huy Ha**, Tulane University, and **Adam Van Tuyl**, Lakehead University.

Diffusion Processes in Biology (Code: SS 2A), **Gustavo Didier**, Tulane University, and **Greg Forest**, University of North Carolina, Charlotte.

Interactions of Geometry and Topology in Low Dimensions (Code: SS 3A), **John Etnyre**, Georgia Tech, **Rafal Komendarczyk**, Tulane University, and **Lenhard Ng**, Duke University.

Quantum Groups and Noncommutative Algebraic Geometry (Code: SS 6A), **Kailash C. Misrah**, North Carolina State University, and **Milen Yakimov**, Louisiana State University.

Akron, Ohio

University of Akron

October 20–21, 2012

Saturday – Sunday

Meeting #1084

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: August 2012
Program first available on AMS website: September 27, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 10, 2012
For abstracts: September 4, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Tanya Christiansen, University of Missouri, *Title to be announced.*

Tim Cochran, Rice University, *Title to be announced.*

Ronald Solomon, Ohio State University, *Title to be announced.*

Ben Weinkove, University of California San Diego, *Title to be announced.*

Special Sessions

Commutative Algebra (Code: SS 8A), **Livia Hummel**, University of Indianapolis, and **Sean Sather-Wagstaff**, North Dakota State University.

Complex Analysis and its Broader Impacts (Code: SS 5A), **Mehmet Celik**, University of North Texas at Dallas, **Alexander Izzo**, Bowling Green State University, and **Sonmez Sahutoglu**, University of Toledo.

Complex Geometry and Partial Differential Equations (Code: SS 4A), **Gabor Szekelyhidi**, University of Notre

Dame, **Valentino Tosatti**, Columbia University, and **Ben Weinkove**, University of California San Diego.

Extremal Graph Theory (Code: SS 2A), **Arthur Busch**, University of Dayton, and **Michael Ferrara**, University of Colorado Denver.

Groups, Representations, and Characters (Code: SS 1A), **Mark Lewis**, Kent State University, **Adriana Nenciu**, Otterbein University, and **Ronald Solomon**, Ohio State University.

Harmonic Analysis and Convexity (Code: SS 7A), **Benjamin Jaye**, **Dmitry Ryabogin**, and **Artem Zvavitch**, Kent State University.

Noncommutative Ring Theory (Code: SS 6A), **S. K. Jain**, Ohio University, and **Greg Marks** and **Ashish Srivastava**, St. Louis University.

Spectral, Scattering, and Inverse Scattering Theory (Code: SS 3A), **Tanya Christiansen**, University of Missouri, and **Peter Hislop** and **Peter Perry**, University of Kentucky.

Tucson, Arizona

University of Arizona, Tucson

October 27–28, 2012

Saturday – Sunday

Meeting #1085

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2012
Program first available on AMS website: October 4, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 17, 2012
For abstracts: September 11, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael Hutchings, University of California Berkeley, *Title to be announced*.

Kenneth McLaughlin, University of Arizona, Tucson, *Title to be announced*.

Ken Ono, Emory University, *Adding and Counting* (Erdős Memorial Lecture).

Jacob Sterbenz, University of California San Diego, *Regularity of hypberbolic gauge field equations*.

Goufang Wei, University of California, Santa Barbara, *Title to be announced*.

Special Sessions

Dispersion in Heterogeneous and/or Random Environments (Code: SS 2A), **Rabi Bhattacharya**, Oregon State University, Corvallis, and **Edward Waymire**, University of Arizona, Tucson.

Geometric Analysis and Riemannian Geometry (Code: SS 4A), **David Glickenstein**, University of Arizona, **Guofang Wei**, University of California Santa Barbara, and **Andrea Young**, Ripon College.

Geometrical Methods in Mechanical and Dynamical Systems (Code: SS 3A), **Akif Ibragimov**, Texas Tech University, **Vakhtang Putkaradze**, Colorado State University, and **Magdalena Toda**, Texas Tech University.

Harmonic Maass Forms and q-Series (Code: SS 1A), **Ken Ono**, Emory University, **Amanda Folsom**, Yale University, and **Zachary Kent**, Emory University.

Inverse Problems and Wave Propagation (Code: SS 7A), **Leonid Kunyansky**, University of Arizona, Tucson.

Mathematical Physics: Spectral and Dynamical Properties of Quantum Systems (Code: SS 6A), **Bruno Nachtergaele**, University of California, Davis, **Robert Sims**, University of Arizona, and **Günter Stolz**, University of Alabama, Birmingham.

Mathematics of Optical Pulse Propagation: Modeling, Analysis, and Simulations (Code: SS 8A), **Jason Fleischer**, Princeton University, and **Moysey Brio**, **Karl Glasner**, and **Shankar Venkataramani**, University of Arizona.

Representations of Groups and Algebras (Code: SS 5A), **Klaus Lux** and **Pham Huu Tiep**, University of Arizona.

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday

Meeting #1086

Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: September 25, 2012

Oxford, Mississippi

University of Mississippi

March 1–3, 2013
Friday – Sunday

Meeting #1087
Southeastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: August 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Chestnut Hill, Massachusetts

Boston College

April 6–7, 2013
Saturday – Sunday

Meeting #1088
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: September 6, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boulder, Colorado

University of Colorado Boulder

April 13–14, 2013
Saturday – Sunday

Meeting #1089
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: September 12, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: February 19, 2013

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

Associative Rings and Their Modules (Code: SS 1A), **Greg Oman** and **Zak Mesyan**, University of Colorado, Colorado Springs.

Dynamics and Arithmetic Geometry (Code: SS 2A), **Su-ion Ih**, University of Colorado at Boulder, and **Thomas J. Tucker**, University of Rochester.

Extremal Graph Theory (Code: SS 3A), **Michael Ferrara**, University of Colorado Denver, **Stephen Hartke**, University of Nebraska-Lincoln, and **Michael Jacobson**, University of Colorado Denver.

Ames, Iowa

Iowa State University

April 27–28, 2013
Saturday – Sunday

Meeting #1090
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: April 2013
Issue of *Abstracts*: To be announced

Deadlines
For organizers: September 27, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

Generalizations of Nonnegative Matrices and Their Sign Patterns (Code: SS 3A), **Minerva Catral**, Xavier University, **Shaun Fallat**, University of Regina, and **Pauline van den Driessche**, University of Victoria.

Operator Algebras and Topological Dynamics (Code: SS 1A), **Benton L. Duncan**, North Dakota State University, and **Justin R. Peters**, Iowa State University.

Zero Forcing, Maximum Nullity/Minimum Rank, and Colin de Verdiere Graph Parameters (Code: SS 2A), **Leslie**

Hogben, Iowa State University and American Institute of Mathematics, and **Bryan Shader**, University of Wyoming.

Alba Iulia, Romania

June 27–30, 2013

Thursday – Sunday

Meeting #1091

First Joint International Meeting of the AMS and the Romanian Mathematical Society, in partnership with the "Simion Stoilow" Institute of Mathematics of the Romanian Academy.

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Sergiu Klainerman, Princeton University, *Title to be announced.*

George Lusztig, Massachusetts Institute of Technology, *Title to be announced.*

Stefan Papadima, Institute of Mathematics of the Romanian Academy of Sciences, *Title to be announced.*

Dan Timotin, Institute of Mathematics of the Romanian Academy of Sciences, *Title to be announced.*

Srinivasa Varadhan, New York University, *Title to be announced.*

Louisville, Kentucky

University of Louisville

October 5–6, 2013

Saturday – Sunday

Meeting #1092

Southeastern Section
Associate secretary: Robert Daverman
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 5, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Philadelphia, Pennsylvania

Temple university

October 12–13, 2013

Saturday – Sunday

Meeting #1093

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 12, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

St. Louis, Missouri

Washington University

October 18–20, 2013

Friday – Sunday

Meeting #1094

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 20, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Riverside, California

University of California Riverside

November 2–3, 2013

Saturday – Sunday

Meeting #1095

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: September 10, 2013

Baltimore, Maryland

Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15–18, 2014

Wednesday – Saturday
Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: To be announced
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lubbock, Texas

Texas Tech University

April 11–13, 2014

Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 18, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Tel Aviv, Israel

Bar-Ilan University, Ramat-Gan and Tel-Aviv University, Ramat-Aviv

June 16–19, 2014

Monday – Thursday
The 2nd Joint International Meeting between the AMS and the Israel Mathematical Union.
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Eau Claire, Wisconsin

University of Wisconsin-Eau Claire

September 20–21, 2014

Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: February 20, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: August 5, 2014

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday
Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the

winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015
Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Porto, Portugal

University of Porto

June 11–14, 2015

Thursday – Sunday
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday
Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday
Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday
Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: To be announced
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2017
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: `lapidus@math.ucr.edu`; telephone: 951-827-5910.

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: `benkart@math.wisc.edu`; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: `steve.weintraub@lehigh.edu`; telephone: 610-758-3717.

Southeastern Section: Robert J. Daverman, Department of Mathematics, University of Tennessee, Knoxville, TN 37996-6900, e-mail: `daverman@math.utk.edu`; telephone: 865-974-6900.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** `www.ams.org/meetings/`**.**

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 111 in the the January 2012 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit `http://www.ams.org/cgi-bin/abstracts/abstract.pl`. Questions about abstracts may be sent to `abs-info@ams.org`. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see `http://www.ams.org/meetings/` for the most up-to-date information on these conferences.)

June 10–June 30, 2012: MRC Research Communities, Snowbird, Utah. (Please see `http://www.ams.org/amsmtgs/mrc.html` for more information.)

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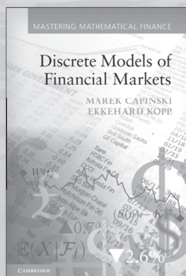
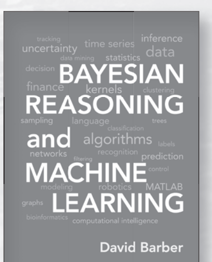
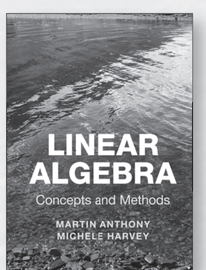
Discrete Models of Financial Markets
Marek Capinski *and* Ekkehard Kopp
Mastering Mathematical Finance
$80.00: Hb: 978-1-107-00263-0
$39.99: Pb: 978-0-521-17572-2: 192 pp.





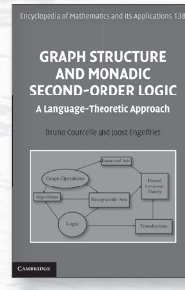
Bayesian Reasoning and Machine Learning
David Barber
$90.00: Hb: 978-0-521-51814-7: 728 pp.

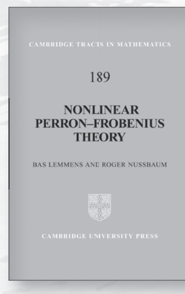
Linear Algebra
Concepts and Methods
Martin Anthony *and* Michele Harvey
$55.00: Pb: 978-0-521-27948-2: 536 pp.



Prices subject to change.

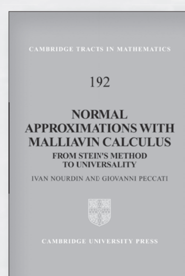
Graph Structure and Monadic Second-Order Logic
A Language-Theoretic Approach
Bruno Courcelle *and* Joost Engelfriet
Encyclopedia of Mathematics and its Applications
$160.00: Hb: 978-0-521-89833-1: 700 pp.





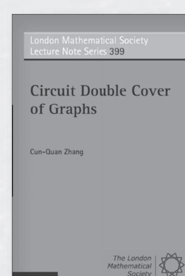
Nonlinear Perron–Frobenius Theory
Bas Lemmens *and* Roger Nussbaum
Cambridge Tracts in Mathematics
$99.00: Hb: 978-0-521-89881-2: 250 pp.

Geometric Analysis
Peter Li
Cambridge Studies in Advanced Mathematics
$75.00: Hb: 978-1-107-02064-1: 415 pp.



Normal Approximations with Malliavin Calculus
From Stein's Method to Universality
Ivan Nourdin *and* Giovanni Peccati
Cambridge Tracts in Mathematics
$80.00: Hb: 978-1-107-01777-1: 270 pp.

Circuit Double Cover of Graphs
Cun-Quan Zhang
London Mathematical Society Lecture Note Series
$70.00: Pb: 978-0-521-28235-2: 375 pp.



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The connection between mathematics and art goes back thousands of years. Mathematics has been used in the design of Gothic cathedrals, Rose windows, oriental rugs, mosaics and tilings. Geometric forms were fundamental to the cubists and many abstract expressionists, and award-winning sculptors have used topology as the basis for their pieces. Dutch artist M.C. Escher represented infinity, Möbius bands, tessellations, deformations, reflections, Platonic solids, spirals, symmetry, and the hyperbolic plane in his works.

Mathematicians and artists continue to create stunning works in all media and to explore the visualization of mathematics--origami, computer-generated landscapes, tesselations, fractals, anamorphic art, and more.

> A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.
>
> —G. H. Hardy, A Mathematician's Apology

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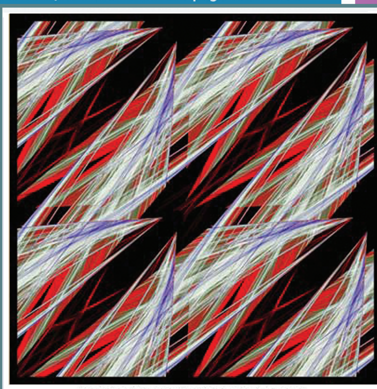
2009 Mathematical Art Exhibition

The Mathematical Art Exhibition held at the 20 Fathauer was the curator of the exhibition, and along with Nat Friedman and Reza Sarhangi. Mathematical Society and the Mathematical A (32), 2006;" Second Prize to Carlo Séquin, for No. 1, 2008." The Prize "for aesthetically pleas to the American Mathematical Society by an a mathematics expressed in a visual art form. T

Crocheted Lorenz Manifolds

Dr. Hinke Osinga and Professor Bernd Krausk (Engineering Mathematics, University of Bristol have turned the famous Lorenz equations into a beautiful real-life object, by crocheting computer-generated instructions of the Lorenz manifold.



Dear Bill,

Here's one of the e-postcards from the site.

Annette

"InHotPursuit," by Mike Field (University of Houston)

Mike Field : Realizations

An aspect of my art work that I particularly enjoy is that I write the software for all the programs I use and build the computers that run the software. In this sense, I like to feel that theory (mathematics), art (outcome), software (algorithms) and engineering (hardware) are integrated and interdependent and that no part survives without the others.

—Mike Field

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- Bridges: Mathematical Connections in Art, Music, and Science
- M.C. Escher: the Official Website
- Images and Mathematics, MathArchives
- The Institute for Figuring
- Kalendar, by Herwig Hauser
- The KnotPlot Site
- Mathematical Imagery by Jos Leys
- Mathematics Museum (Japan)
- Visual Mathematics Journal

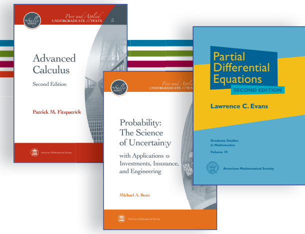
ARTICLES & RESOURCES

- Art & Music, MathArchives
- Geometry in Art & Architecture, by Paul Calter (Dartmouth College)
- *Harmony and Proportion*, by John Boyd-Brent
- International Society of the Arts, Mathematics and Architecture
- *Journal of Mathematics and the Arts*
- *Mathematics and Art*, the April 2003 Feature Column by Joe Malkevitch
- *Maths and Art: the whistlestop tour*, by Lewis Dartnell
- Mathematics and Art, (The theme for Mathematics Awareness Monthin 2003)
- Viewpoints: Mathematics and Art, by Annalisa Crannell (Franklin & Marshall College) and Marc Frantz (Indiana University)

www.ams.org/mathimagery

American Mathematical Society

TEXTBOOKS FROM THE AMS



The titles below are a sampling of our most timely and best-selling textbooks, which will appeal to students and professors alike. Textbooks from the AMS spark students' interest in modern mathematics and fuel their appreciation for research. While intended as classroom texts for undergraduate or graduate level courses, these titles also make excellent supplemental course reading and are ideal independent study resources.

To request an exam copy, visit: **ams.org/bookstore-exam-copy.**

GRADUATE

PARTIAL DIFFERENTIAL EQUATIONS

Second Edition

Lawrence C. Evans, *University of California, Berkeley, CA*

This book would be invaluable for a graduate student preparing to do research in PDEs; I wish I had a copy in graduate school.
—MAA Reviews

The definitive text on partial differential equations, with a new chapter on nonlinear wave equations and a near doubling of exercises

Graduate Studies in Mathematics, Volume 19; 2010; 749 pages; Hardcover; ISBN: 978-0-8218-4974-3; List US$93; AMS members US$74.40; Order code GSM/19.R

AN INTRODUCTION TO MEASURE THEORY

Terence Tao, *University of California, Los Angeles, CA*

An introduction to the foundation of modern real analysis, which focuses first on the concrete setting of Lebesgue measure and the Lebesgue integral before moving on to abstract measure and integration theory

Graduate Studies in Mathematics, Volume 126; 2011; 206 pages; Hardcover; ISBN: 978-0-8218-6919-2; List US$53; AMS members US$42.40; Order code GSM/126

ADVANCED MODERN ALGEBRA

Second Edition

Joseph J. Rotman, *University of Illinois at Urbana-Champaign, IL*

A graduate algebra text characterized by clarity and completeness of proofs, with new material on Galois theory and homological algebra

Graduate Studies in Mathematics, Volume 114; 2010; 1008 pages; Hardcover; ISBN: 978-0-8218-4741-1; List US$99; AMS members US$79.20; Order code GSM/114

UNDERGRADUATE

INTRODUCTION TO REPRESENTATION THEORY

Pavel Etingof, *Massachusetts Institute of Technology, Cambridge, MA*, **Oleg Golberg**, **Sebastian Hensel**, *Universität Bonn, Germany*, **Tiankai Liu**, *Massachusetts Institute of Technology, Cambridge, MA*, **Alex Schwendner**, *Two Sigma Investments, New York, NY*, **Dmitry Vaintrob**, *Harvard University, Cambridge, MA*, and **Elena Yudovina**, *University of Cambridge, United Kingdom*

with historical interludes by **Slava Gerovitch**, Massachusetts Institute of Technology, Cambridge, MA

The goal of this book is to give a "holistic" introduction to representation theory

Student Mathematical Library, Volume 59; 2011; 228 pages; Softcover; ISBN: 978-0-8218-5351-1; List US$42; AMS members US$33.60; Order code STML/59

AN INTRODUCTION TO COMPLEX ANALYSIS AND GEOMETRY

John P. D'Angelo, *University of Illinois, Urbana, IL*

Use of basic complex analysis to interweave elementary topics from analysis, geometry and algebra

Pure and Applied Undergraduate Texts, Volume 12; 2010; 163 pages; Hardcover; ISBN: 978-0-8218-5274-3; List US$57; AMS members US$45.60; Order code AMSTEXT/12

ADVANCED CALCULUS

INSTRUCTOR MANUAL *available*

Second Edition

Patrick M. Fitzpatrick, *University of Maryland, College Park, MD*

A rigorous presentation of the fundamental concepts of mathematical analysis, intended to encourage an appreciation of the subject's coherence

Pure and Applied Undergraduate Texts, Volume 5; 2006; 590 pages; Hardcover; ISBN: 978-0-8218-4791-6; List US$82; AMS members US$65.60; Order code AMSTEXT/5

UNDERGRADUATE GRADUATE

AN INTRODUCTORY COURSE ON MATHEMATICAL GAME THEORY

Applied Mathematics

Julio González-Díaz, *Universidade de Santiago de Compostela, Spain*, **Ignacio García-Jurado**, *Universidad de Coruña, Spain*, and **M. Gloria Fiestras-Janeiro**, *Universidade de Vigo, Spain*

A substantial treatment of game theory that includes the subtle and often overlooked topic of cooperative games

Graduate Studies in Mathematics, Volume 115; 2010; 324 pages; Hardcover; ISBN: 978-0-8218-5151-7; List US$62; AMS members US$49.60; Order code GSM/115

DIFFERENTIAL TOPOLOGY

Victor Guillemin, *Massachusetts Institute of Technology, Cambridge, MA*, and **Alan Pollack**

A frequently referenced introduction to differential topology that uses transversality to emphasize the subject matter's elegant intuitive content

AMS Chelsea Publishing, Volume 370; 1974; 222 pages; Hardcover; ISBN: 978-0-8218-5193-7; List US$40; AMS members US$36; Order code CHEL/370.H

PROBABILITY: THE SCIENCE OF UNCERTAINTY

INSTRUCTOR MANUAL *available*

with Applications to Investments, Insurance, and Engineering

Michael A. Bean

A review of the basic probability of distributions, emphasizing the many applications arising in investments, insurance, and engineering

Pure and Applied Undergraduate Texts, Volume 6; 2001; 448 pages; Hardcover; ISBN: 978-0-8218-4792-3; List US$72; AMS members US$57.60; Order code AMSTEXT/6

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