

About the Cover

Two views of Burgers' equation

This month's cover portrays two ways of visualizing a solution to Burgers' equation, which is one of the topics Chi-Wang Shu discusses briefly in his article "Efficient algorithms for solving partial differential equations with discontinuous solutions" in this issue. Burgers' equation is the simplest model of many phenomena appearing in the solution of nonlinear hyperbolic partial differential equations, and exhibits in a particularly simple manner the way in which solutions become singular.

What this means is that solutions, even with very reasonable initial conditions such as the example

$$u(x, 0) = \cos x \quad (|x| \leq \pi)$$

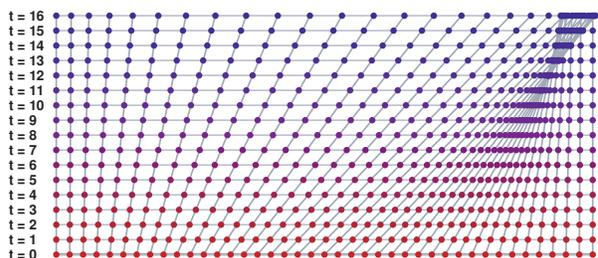
shown on the cover, cease to be honest functions at some point. This phenomenon is commonly called discontinuity and, as Shu mentions, it is ubiquitous in nonlinear PDEs.

There is much discussion of Burgers' equation on the Internet, but we haven't seen any that mentioned the very simple and intuitive interpretation in terms of traffic flow. We learned about it from Chapitre 2 of the book by the late Vladimir Arnold whose title in French is *Chapitres supplémentaires de la théorie des équations différentielles ordinaires*. (Author query: Has it been translated into English?) In this model, at initial time $t=0$ a collection of particles is assembled on the x -axis with velocity given by a function $u(x, 0)$ depending only on position. As time proceeds, the function $u(x, t)$ specifies the velocity of the particles at position x (but they are no longer the particles originally there), and it satisfies Burgers' equation

$$u_t + u u_x = 0.$$

If the initial function has a negative slope at any point, then of course the faster particles eventually catch up to the slower ones, bringing about a singularity of sorts, and at this time $u(x, t)$ ceases to be a function of x . The top image on the cover shows plots for various times as a graph in 3D, while the bottom one and the figure below show in a different way how the particles with different velocities eventually give rise to a caustic in the plot, where the particles accumulate to infinite density.

—Bill Casselman
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Philip Schaefer

It was my privilege to be one of Larry Payne's Ph.D. students when he was on the faculty at the University of Maryland. Twenty-five years later we began collaborating on overdetermined and nonstandard boundary value problems, growth and decay results, energy and pointwise bounds, and lower (upper) bounds for blow-up time in partial differential equations and systems. Much of this work was accomplished when Larry spent the winter months of his retirement in the milder climate of Tennessee. On my last visit to Cornell, we did not learn of the 9/11 tragedy until 10:45 a.m. because we were "hard at work". In 1987 we hosted the "Maximum Principles and Eigenvalue Problems in Partial Differential Equations" conference (*Pitman Research Notes* 175) at the University of Tennessee, which reunited Larry with Murray Protter and Hans Weinberger and with other of his collaborators such as Cornelius Horgan, Howard Levine, and Brian Straughan. A final conference with international participants which honored Larry's contributions to mathematics was held in Cagliari, Sardinia, in 2003. Without a doubt, Larry Payne was a "man for all seasons"—a gentleman and a scholar—a prolific researcher and dedicated teacher and mentor. He will be sadly missed by many people that he touched.

Simon Levin

When I was a graduate student at the University of Maryland, Larry Payne—then one of my professors and later to be a valued colleague—gave me a copy of a manuscript he was ready to submit for publication. He said that he knew that I was working on similar ideas for my thesis and that if I found anything in his paper that would compete with my results, I should let him know so that he could remove them from his manuscript. I was young and naive, but even so I recognized that this was a remarkable gesture from a remarkable person. Although there was nothing that needed removing, this incident had an indelible effect upon me as a lesson for how to deal with others, especially one's students, and I have tried to remember the lesson in my own mentoring. I learned too that this was not an isolated behavior from Larry, who was not only a great and generous mathematician but a model of how one should live one's life.

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