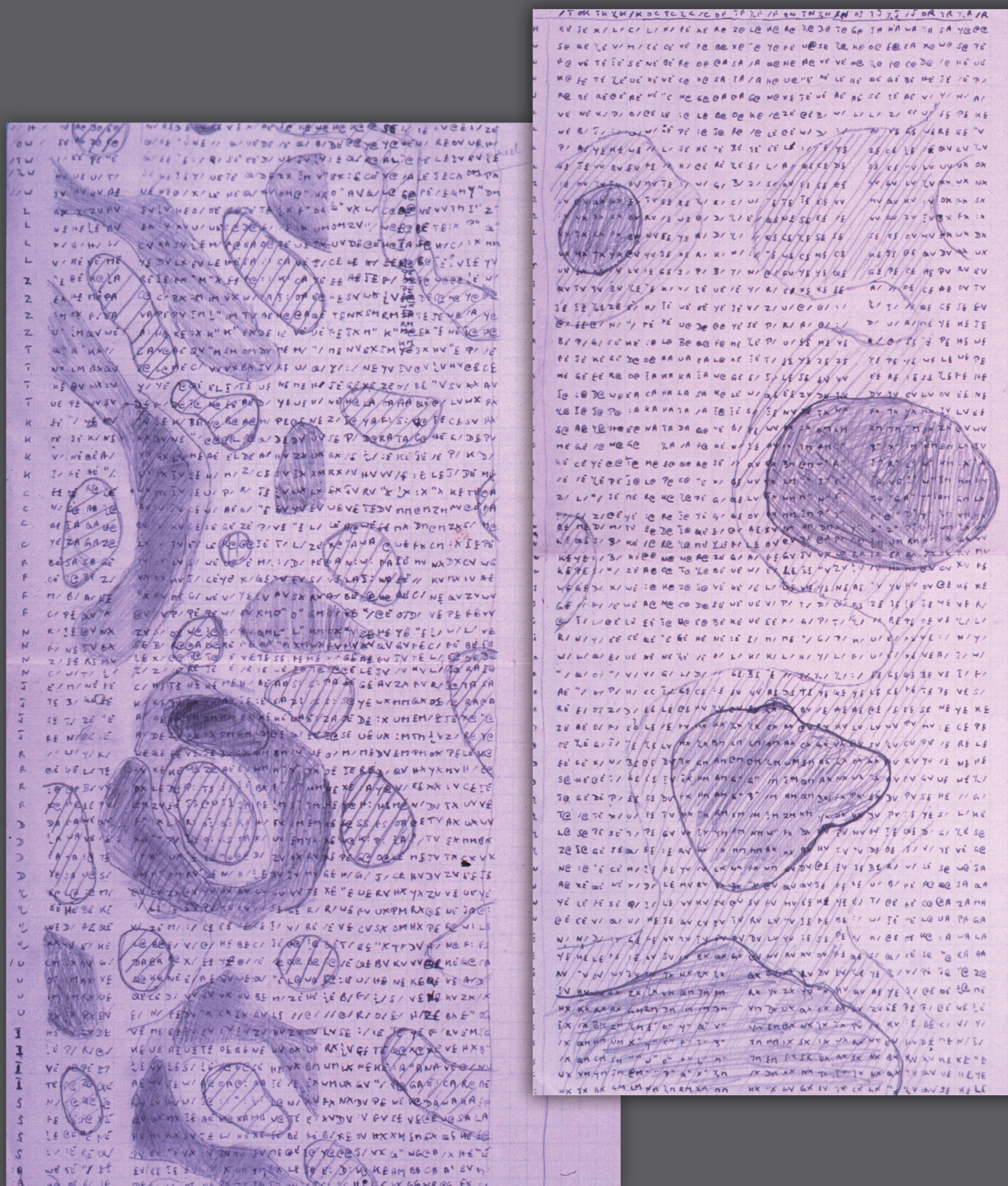
ISSN 0002-9920

Notices
of the American Mathematical Society

June/July 2012

Volume 59, Number 6



About the Cover: Alan Turing's morphogenesis plots *(See page 801)*

MBI Emphasis Year Mathematical Neuroscience 2012-2013

Organizers

Carmen Canavier (LSU), John Rinzel (NYU/CNS), Steven J. Schiff (Penn State), Eric Shea-Brown (Washington), Murray Sherman (Chicago)

Emphasis Year Workshops

October 1-5, 2012	Workshop 1: Mathematical Challenges in Neural Network Dynamics
December 10-14, 2012	Workshop 2: Cognitive Neuroscience
February 4-8, 2013	Workshop 3: Disease
March 18-22, 2013	Workshop 4: Rhythms and Oscillations
April 8-12, 2013	Workshop 5: Cellular and Subcellular
May 6-10, 2013	Workshop 6: Sensory Systems and Coding

Current Topic Workshops

August 27 - 31, 2012	2012 Workshop for Young Researchers in Mathematical Biology
October 15-18, 2012	Mathematical and Computational Challenges in Cilia- and Flagella-Induced Fluid Dynamics
November 13-16, 2012	Statistics of Time Warpings and Phase Variations
February 18-22, 2012	Mathematical Advances in Biomolecular/Biomedical Imaging and Visualization
June 24-28, 2013	Keyfitz Symposium on Mathematical Demography

MBI 10th Anniversary Meeting

September 19-21, 2012	Math Biology: Looking at the Future

mbi Mathematical Biosciences Institute

Limited support is available for MBI workshops. Please apply online at http://mbi.osu.edu

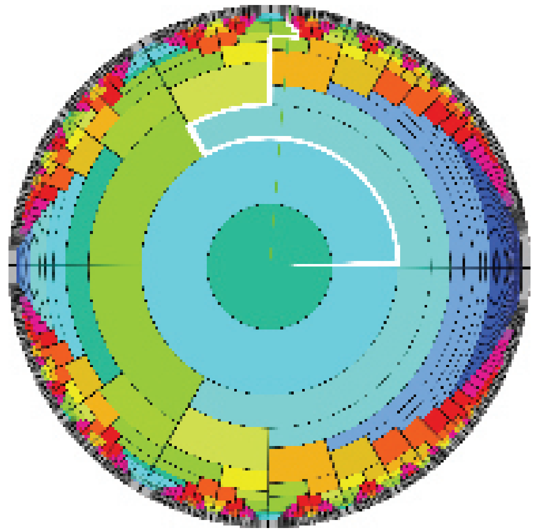
THE OHIO STATE UNIVERSITY

Notices
of the American Mathematical Society

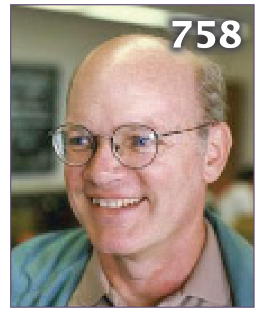
June/July 2012

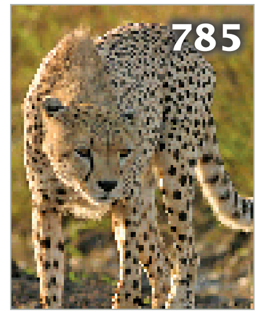
Communications

Commentary



746





June 2012 is the centenary of Alan Turing. We have two articles to celebrate that event. We also have an unusual article about Turing's contributions to the biological sciences. As a counterpoint, there are poignant articles about Farey series and about the Yang-Mills equations. In an effort to inform the readership of timely matters, we have two articles arguing the two sides of the Elsevier boycott. Finally, we remember the remarkable applied mathematician Jerrold Marsden.

—*Steven G. Krantz, Editor*

Features

Notices
of the American Mathematical Society

SUBSCRIPTION INFORMATION: Subscription prices for Volume 59 (2012) are US$531 list; US$424.80 institutional member; US$318.60 individual member; US$459 corporate member. (The subscription price for members is included in the annual dues.) A late charge of 10% of the subscription price will be imposed upon orders received from nonmembers after January 1 of the subscription year. Add for postage: Surface delivery outside the United States and India—US$27; in India—US$40; expedited delivery to destinations in North America—US$35; elsewhere—US$120. Subscriptions and orders for AMS publications should be addressed to the American Mathematical Society, P.O. Box 845904, Boston, MA 02284-5904 USA. All orders must be prepaid.

ADVERTISING: *Notices* publishes situations wanted and classified advertising, and display advertising for publishers and academic or scientific organizations. Advertising material or questions may be sent to `classads@ams.org` (classified ads) or `notices-ads@ams.org` (display ads).

SUBMISSIONS: Articles and letters may be sent to the editor by email at `notices@math.wustl.edu`, by fax at 314-935-6839, or by postal mail at Department of Mathematics, Washington University in St. Louis, Campus Box 1146, One Brookings Drive, St. Louis, MO 63130. Email is preferred. Correspondence with the managing editor may be sent to `notices@ams.org`. For more information, see the section "Reference and Book List".

NOTICES ON THE AMS WEBSITE: Supported by the AMS membership, most of this publication is freely available electronically through the AMS website, the Society's resource for delivering electronic products and services. Use the URL `http://www.ams.org/notices/` to access the *Notices* on the website.

[*Notices of the American Mathematical Society* (ISSN 0002-9920) is published monthly except bimonthly in June/July by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2294 USA, GST No. 12189 2046 RT****. Periodicals postage paid at Providence, RI, and additional mailing offices. POSTMASTER: Send address change notices to *Notices of the American Mathematical Society,* P.O. Box 6248, Providence, RI 02940-6248 USA.] Publication here of the Society's street address and the other information in brackets above is a technical requirement of the U.S. Postal Service. Tel: 401-455-4000, email: `notices@ams.org`.

Departments

From the AMS Secretary

Opinion

P_V: The Value of Publishing

The recent statement by a large number of prominent mathematicians and other scientists, prompted by a BLOG post from Tim Gowers,[1] inspired me to write down a few comments based on my longtime involvement in publishing and my background as a mathematician. (See also "Why publish mathematics"[2] and "Small independent publishers: Responsible, committed, and flourishing".[3])

As a mathematician and, subsequently, STM[4] publisher for nearly fifty years, I have experienced many changes in the industry. Major changes in publishing started in the mid-seventies when mergers and internal reorganization at larger publishing houses shifted responsibility for final decisions, like pricing, print runs, and the editing process, from editorial departments to financial management. Additionally, the development of $\TeX$, as well as the widespread availability of tools for producing and manipulating images, changed the relationship between the author and the publisher. At the same time, the Internet began to provide archiving and publication options. To be sure, those changes were gradual, but their effect has by now become obvious. I believe, however, that the current developments are more significant and consequential.

As a publisher I appreciate and endorse what I understand to be the broad goals of the protest. I do, however, want to emphasize the importance of the *value* that used to be—and still should be—*added* by publishers or other services that take their place.

When Gutenberg invented the printing press, publishers were the ones who knew how to produce multiple copies at "reasonable" cost; later the editorial function became important when academic societies became publishers. At the time when I became a publisher in 1964, marketing and distribution became more essential functions that were recognized by authors and customers alike. Springer Verlag opened an office in New York to respond to the more global needs. Some years later, direct marketing activities became a function of the editorial department in order to better provide targeted information and thereby optimize the dissemination of books and journals.

Typesetting of technical mathematical texts had become very expensive already by the 1970s, and the quality had gone down from the days of hand-setting. Book prices reflected these increased costs. Donald Knuth's monumental development of $\TeX$ and his making it publicly available revolutionized the production process for STM publishers. "Typeset" manuscripts replaced camera-ready submissions for publications such as Lecture Notes. Many publishers, driven by economic considerations, took advantage of this without applying classical standards of layout and typography. The important functions of editing and formatting easily became overlooked because manuscripts appeared to be already typeset. Hence, as a result of shifting responsibility, services benefitting authors (and also readers) were largely eliminated.

When the Internet initially provided the opportunity to upload and archive research papers, I had a friendly conversation with a mathematician who strongly championed an exclusive form of electronic publishing that would allow anyone to upload his/her papers to easily accessible servers. My question about quality control was answered by requiring a simple rule: "Whatever you upload, you can't retract. That will prevent any abuse." I knew then, and we all know now, that an established review and editing process is needed to achieve high standards of publication. Unfortunately, "author-pays" open access journals as well as some "reputable" publishers are no longer providing that service. To be very clear and explicit, the broadly accepted math archive[5] has become a standard that should be accepted by publishers. (At A K Peters, after some careful consideration, we accepted submissions to our journals that had been placed on the arXiv server but did not agree to the posting of the final edited and reformatted versions that were available in our printed and electronic journals.)

In my view, publishing should be a service that derives its justification from value added to the process. Publishers need the input and feedback from the scientific community in order to achieve this goal.

When I started in publishing, I "inherited" a group of advisors who were dedicated to the development of a book program that benefitted the community of their peers. They also understood the need for economic sustainability on the part of the publisher. Modest fees were paid to the advisors of the book program to compensate them for time and some administrative expenses.

One of my first encounters with an advisor reminds me of the current protest, although it happened on an individual basis and had an immediate and lasting effect. In 1965, on my first visit to the United States, I met Paul Halmos, one of the major advisors and editor of the series Ergebnisse der Mathematik, who became a friend and mentor, and whose understanding of the complex relationship between the publishing industry and the community that it served was at once practical and idea-driven (not idealistic). He opened our first meeting with a blunt statement: "If your company ever again prices a book as high as the recent *Homology* by Saunders Mac Lane, I will resign." The advice was heeded by the publisher and instilled a deep conviction about the importance of pricing that was later strengthened by experience when I learned more about the relationship between pricing and unit sales.

The situation for journals is quite different, and the current protest seems prompted by the lack of value added by the publisher and the expectation that the scientific community provide their expertise and services without compensation. In my experience, the very autonomous editorial boards of journals did not receive compensation

[1] http://gowers.wordpress.com/
[2] Notices of the AMS, *Volume 56, Number 7.*
[3] Logos, *14/2, Whurr Publishers, 2003.*
[4] *Scientific, technical, and medical.*

[5] http://arxiv.org/new/math.html

DOI: http://dx.doi.org/10.1090/noti867

other than for occasional administrative costs, mailing, travel, etc. Their rotating activities were considered a service to the community and did not include detailed editing or tracking, as that was handled by a very experienced staff at the publishing house. With the availability of typesetting programs such as TEX, growing expertise in image manipulation by the authors, and online reviewing tools, publishers have largely abandoned their professional obligation and rely on authors and editors to perform and control many of the publisher's responsibilities.

Hand in hand with this reduction in service there was a disproportional price increase, as well as the introduction of the business model of "bundling", which has been deplored by librarians and the scientific community alike. I have checked with a few librarians and found that "bundling" can be prevented or modified but requires tough bargaining and some clout. Journals that used to be considered a self-sustaining, modestly profitable and prestigious part of the publishing program, attracting potential book authors, have become major profit centers as a result of these pricing and bundling policies. (Journal programs were, of course, already a major economic attraction for the publisher due to the cyclic subscription model that generated working capital that would be used over the subscription period.)

Today, new opportunities for self-publishing are evolving rapidly and include tools that were previously exclusively offered by publishers. I strongly believe that these opportunities require procedures that help maintain a desirable level of quality control.

As a mathematician and (former) publisher I would hope to encourage the continued involvement of the community with responsible publishers to be sure that the following essential elements of the process not be destroyed:

- *Technical reviews* should be a joint responsibility of the publisher and the scientific community to insure quality.
- *Copyediting* is a *sine qua non*. Besides assuring correct spelling and grammar, professional copyediting, a responsibility of the publisher, improves readability by pointing out inconsistency and repetition and by improving the organizational structure of a text where needed.
- *Professional formatting and image handling* are essential functions that the publisher needs to provide.
- *Effective marketing and worldwide distribution* are essential to the widest possible dissemination of a text and should be a major goal of the publisher rather than simply profit maximization.

To navigate the proliferation of information, selectivity and brand recognition are and should remain valuable attributes of a publishing environment, and they can only be achieved through a constructive cooperation between the scientific community and the "publishers" of the future.

—Klaus Peters
Founder and former publisher, A K Peters, Ltd.
`klauspeters@gmail.com`

Letters to the Editor

More on Norrie Everitt

An obituary of Professor Norrie Everitt was published in the November 2011 issue of the *Notices*. With this letter I want to add to the description of Professor Everitt's work as an editor of influential texts in differential operator theory and also point out an important but perhaps little-known detail about his generosity.

Professor Everitt was the editor of the English translations of two Russian texts on differential operator theory, one by M. A. Naimark (*Linear Differential Operators*) and another by N. I. Akhieser and I. M. Glazman (*Theory of Linear Operators in Hilbert Space*). The latter, which was not mentioned in the obituary, was published in two volumes by Pitman Advanced Publishing Program in 1981 and appeared posthumously for the authors. Professor Everitt dedicated the edition to the memory of both authors.

Professor Everitt was a member of the AMS for fifty-one years. He repeatedly donated funds to the AMS in memory of my late husband, I. M. Glazman, designating the gifts for support of young mathematicians. Professor Everitt made the last in this series of gifts in February 2011.

—Polina Naiman
New Haven, Connecticut
pelya009@yahoo.com

(Received March 9, 2012)

Home Schooling Rethought

In "Mathematics and home schooling" (April 2012 *Notices*), Acker, Gray, Jalali, and Pascal make the case for state-mandated education, reviewing its history and concluding that the state has the responsibility for education and the legal power necessary to carry out that responsibility. We must be careful, though, for what we wish; the authors neglected to consider the effects of various groups wielding that power over time.

Consider seventeenth-century Massachusetts, the first example by the authors of state-mandated education. Let's not forget that Massachusetts also had a state religion (denomination) at the time; it is my understanding that the public education offered was thoroughly infused with the theology of their state denomination, a state of affairs that we would all find unacceptable today. This occurred because, when the state has the power to mandate, those with control of the state can wield that power as they choose.

Abuses of the power of state-mandated education did not end with the adoption of the Constitution. During the nineteenth and early twentieth centuries public schools in the United States dispensed a generic Protestant education (termed "nonsectarian" at the time), a situation wholly unfair to Catholics, secularists, and other non-Protestants. The Catholics responded to this problem with their own system of schools. Secularists, however, eventually took a different approach, and by the middle of the twentieth century were able to take control of public education. As a result, the Protestants who previously were in favor of the state enforcing a common education and creating a common religious culture by doing so found themselves on the outside looking in. They responded in the same manner as the Catholics of the previous century, forming their own schools, and many eventually helped revive the home school movement.

Now surely we are not so naïve as to think that those currently in power will always, from now until the end of time, remain in power. When, in the course of time, the current secularists find themselves replaced by the next group, what will they then think of state-mandated education? To many parents from a wide variety of political and religious perspectives, educating their children in their deeply held values is more important than maximizing their children's mathematical, scientific, or literary abilities. We may disagree, but who are we to tell those parents that they are wrong? Just how arrogant are we? At issue is who makes such decisions, and when the state has the power, whoever is in control—and it won't always be you—makes those decisions.

Perhaps it is time for us to realize that if we wish to live together happily in a pluralistic, multicultural society, then we must resist the temptation to impose our own ways of thinking on other people's children through the power of the government, no matter how right our ideas might be and no matter how important to society is our cause.

—Bryan Dawson
Union University
bdawson@uu.edu

(Received March 17, 2012)

More on Home Schooling

The April 2012 issue of the *Notices* carried the article "Mathematics and home schooling", by me and three co-authors, Kathleen Acker, Behzad Jalali, and Matthew Pascal. As several correspondents have pointed out, there are many successful products of home schooling. The time and effort on the part of the homeschooling parents to give their children individualized lessons has much to do with academic achievement. And there is ample evidence that in general parents' involvement in their children's education can be beneficial whether they are schooled in the home or in public or private schools. The difficulty comes in balancing the rights of the children and the rights of the parents in a situation in which there is little regulation and no systematic evaluation. Of course, producing students well-educated in mathematics is also a societal concern.

We regret that the citation listed in our paper on page 518, second column, under the heading of Gender Equity, as (Dwyer 1994) was not correct. The correct citation is (Yuracko 2008); it is listed as Kimberly Yuracko, "Education off the grid: Constitutional constraints on homeschooling", *California Law Review* **96** (2008), 123–180 in the reference section.

—*Mary Gray*
American University
mgray@american.edu

(Received April 15, 2012)

Lack of Author/Referee Communication

I read with interest Michael Cowling's article "*Scripta Manent*: The future of mathematical publishing" and in particular his comments about refereeing being a thankless task. I have been that referee reading various iterations of the same paper, submitted to different journals, seemingly with the author(s) barely reading my reports.

However, as an author I have also received acceptance reports which were two lines long. I have received rejections which were purely subjective with no actual corrections to the mathematics. In one case where, fortunately, a coauthor dealt with submission, on our third attempt the referee complained bitterly that we were ignoring his/her previous reports. However, the journals we had previously submitted to had failed to send us any feedback!

When I write a report, I am writing for both the author and the journal. However, I think in every case I have no real knowledge of what the author gets to see. The anecdotes above suggest that a lot of "editing" by the journal can occur. Furthermore, in all but a few cases, as a referee I never learn the ultimate outcome—was the paper accepted or not? Journals could do a lot more to facilitate communication between author and referee.

—*Matthew Daws*
School of Mathematics
University of Leeds, UK
matthew.daws@gmail.com

(Received March 26, 2012)

Sense-making Teaching

In his article, "A modest proposal" (*Notices*, February 2012), Alan Schoenfeld says that mathematics should be taught and learned as "an act of sense-making". As Schoenfeld makes painfully clear, the exact opposite often occurs in math classes, where many students actually learn to abandon common sense! While we can certainly express our shock and dismay about this phenomenon occurring in K–12 classrooms, we may need to look more closely at what is going on in our own college classrooms, especially our entry-level courses. How much sense-making is actually going on in these courses? Schoenfeld mentions that he was once assigned to teach pre-calculus, a "universally despised" low-level course. He discussed helping students understand why the arc length formula, $s=r\theta$, makes sense. I wonder how many pre-calculus instructors pause to help their students make sense of this formula. My guess is not many. First of all, most of us are driven to get through the syllabus, which typically boils down to a very long list of procedures to be learned—i.e., the "how-to" side of math, not the sense-making side. This goal often severely limits time for thinking, analyzing the source of formulas, or working through real problems.

A second obstacle to promoting sense-making is the difficulty of achieving this goal. Schoenfeld claims that "approaching instruction [as the sense-making endeavor] will make mathematics easier to learn." In my experience, I have found teaching this way to be a major challenge (although often very rewarding!). First, helping students see concepts and problems in a way that makes sense to them, not just to me, is one of the perennial challenges of teaching. Second, for short-term purposes, memorizing is often easier than understanding. We professors may think that our students have to understand what they are doing in order to answer our questions, yet it turns out that many students are just memorizing without sense-making. (Just ask them.) I believe that if we want our college students to be making sense of the mathematics that we are teaching them, we ourselves have to "stop to think" (to use Schoenfeld's words). We have to ask ourselves questions such as the following. What are the larger goals we have for our courses, beyond such objectives as "applying Descartes' rule of signs"? Why should mathematics be part of general education for all college students? What do we expect our non-STEM math students to know about mathematics and be able to do with mathematics after they graduate? How can we teach and assess our students in ways that more effectively promote sense-making? Yes, this is a major challenge. Yet I believe, along with Alan Schoenfeld, that "We owe our students no less."

—*Betsy Darken*
Department of Mathematics
University of Tennessee at Chattanooga
betsy-darken@utc.edu

(Received March 17, 2012)

Corrections

Michael G. Cowling, author of the article "The future of mathematical publishing" (*Notices*, April 2012 issue) was misidentified in the author footnote that accompanied the article. The footnote should have read: Michael G. Cowling is professor of pure mathematics at the University of New South Wales. His email address is m.cowling@unsw.edu.au.

The photo key (page 555, April 2012 *Notices*) for the montage of Joint Mathematics Meetings photos was incorrectly titled "2011 New Orleans, LA, Joint Mathematics Meetings Photo Key". It should have read "2012 Boston, MA, Joint Mathematics Meetings Photo Key".

The DOI (Document Object Identifier) for the *Doceamus* article (page 668, May 2012 *Notices*) was mistakenly listed as noti837. It should be noti836.

—*Sandy Frost*

A Farey Tail

Patrice Philippon

The Garden of Visibles and the Farey Graph

The *garden of visibles* in the lattice $\mathbf{Z}^2$ (i.e., of points visible from the origin; cf. [2, page 29]) arises when one plants a tree at each point with relatively prime coordinates. The rays issuing from these visible points and pointing away from the origin cover all the hidden points of $\mathbf{Z}^2$ (i.e., the points that are not visible from the origin because they lie behind a visible one). We complete these hidden rays to an infinite graph, which we dub the *hidden graph*, by linking each visible point to the lower and upper neighboring rays with vertical line segments (and of course declaring the intersection points to be vertices).

In Figure 1 the origin is represented by the blue dot, and we have sketched a sector of the hidden graph. The complete graph is obtained as the union of images of this sector under the transformations $(x, y) \mapsto (\pm x, y+nx)$, $n \in \mathbf{Z}$, plus eventually the two isolated half-lines $\{0\}\times[1, +\infty[$ and $\{0\}\times]-\infty, -1]$. We will call any vertex coinciding with a visible point an *extremal vertex* of the hidden graph. These are the main vertices from which all the rays (slanted edges) of the graph stem.

For a better view it is adequate to straighten the hidden graph through the map $[1, \infty[\times\mathbf{R} \to [1, \infty[\times\mathbf{R}$, $(x, y) \mapsto (x, \frac{y}{x})$, thus obtaining the *Farey tail*, where the radial edges become parallel to the first horizontal axis and the edges parallel to the second vertical axis keep the same direction.

Thus, the Farey tail is the graph whose extremal vertices are the points $(q, \frac{p}{q})$, where $\frac{p}{q}$ runs over the set of rational numbers (written in reduced form), and the edges are the horizontal half-lines $(u, \frac{p}{q})$, $u \geq q$, together with the vertical segments linking each extremal vertex to vertices on the

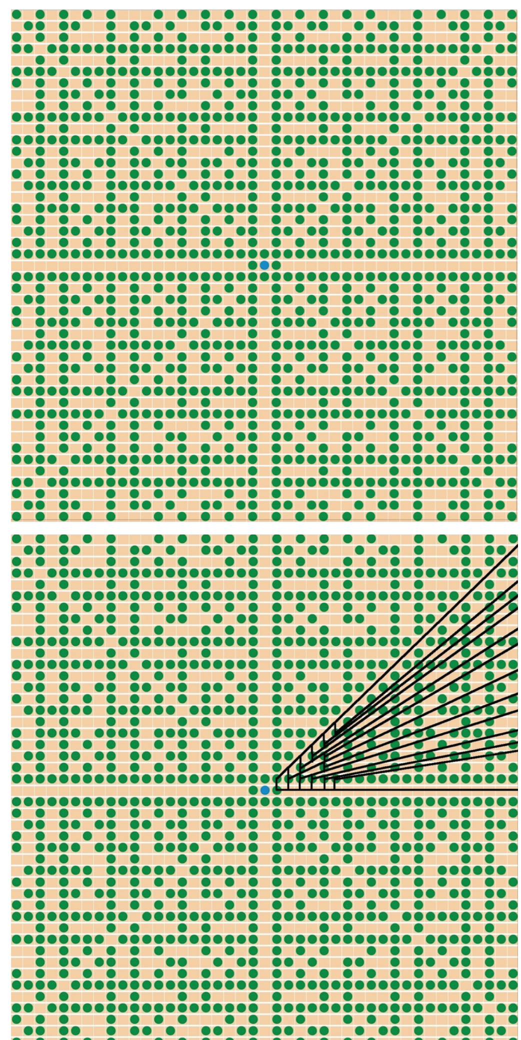


Figure 1. The garden of visibles and the hidden graph.

Patrice Philippon is Directeur de Recherche at CNRS, working at the Institut de Mathématiques de Jussieu in Paris. His email address is `pph@math.jussieu.fr`.

The author warmly thanks Dale Brownawell for his careful reading of this text and his help in straightening out many (linguistic or not) obscurities. He also thanks the editors of the Notices *for their encouragement and work in order to publish the article.*

DOI: http://dx.doi.org/10.1090/noti860

neighboring edges. The graph is periodic through vertical translation by 1 and it is symmetric with respect to the line of ordinate $\frac{1}{2}$. Figure 2 shows only the stripe $[1,\infty[\times[0,1]$, corresponding to the sector of the hidden graph already sketched in Figure 1.

Let's recall that for $n \in \mathbf{N}^*$ the *n-th Farey sequence*, denoted $\mathcal{F}_n$, is the sequence, in increasing order, of rational numbers between 0 and 1, the denominators of which are at most n: $\mathcal{F}_1 = 0, 1$; $\mathcal{F}_2 = 0, \frac{1}{2}, 1$; $\mathcal{F}_3 = 0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1 \ldots$; see [4, §§1 & 2] or [2, chap. 3], for example. Beware that in [4] the n-th Farey sequence, $n \in \mathbf{N}$, is extended to all rational numbers with denominator and absolute value of numerator at most n, furthermore including both "infinities" $\frac{-1}{0}$ and $\frac{1}{0}$ at the extremes: $\mathcal{F}'_0 = \frac{-1}{0}, \frac{0}{1}, \frac{1}{0}$; $\mathcal{F}'_1 = \frac{-1}{0}, \frac{-1}{1}, \frac{0}{1}, \frac{1}{1}, \frac{1}{0}$; $\mathcal{F}'_2 = \frac{-1}{0}, \frac{-2}{1}, \frac{-1}{1}, \frac{-1}{2}, \frac{0}{1}, \frac{1}{2}, \frac{1}{1}, \frac{2}{1}, \frac{1}{0} \ldots$. We recover the Farey sequence $\mathcal{F}_n$ on the Farey tail depicted in Figure 2, for example, as the sequence of ordinates of horizontal edges of the Farey graph encountered when going up the vertical line of abscissa n (or of any abscissa at least n and strictly less than $n+1$).

The *Farey comb* (Figure 3) is the horizontal contraction of the preceding graph through the transformation $(u, v) \longmapsto (\frac{u-1}{u}, v)$. As for the Farey tail, we show only the part of the graph in the square $[0,1[\times[0,1]$; the complete comb is the union of translates of this part by the points $(0, n)$, $n \in \mathbf{Z}$.

Finally, the *Farey eye* is obtained from the previous graph as an image under the exponential map $(s, t) \longmapsto s\mathrm{e}^{2\mathrm{i}\pi t}$ in the unit disc $D(0,1)$ of the complex plane. This transformation takes care of the periodicity of the Farey comb through entire vertical translations. In Figure 4 the Farey eye as just introduced is on the left; the right picture is its symmetric reflection (either through the origin or the vertical axis, since the eye is itself symmetric with respect to the horizontal axis).

To sum up, here are the transformations between the different representations of the Farey graph:[1]

$$
\begin{array}{ccccccc}
[1,+\infty[\times\mathbf{R} & \to & [1,+\infty[\times[0,1[& \to & [0,1]\times[0,1[& \to & D(0,1) \\
(x,y) & \mapsto & (x, \left\{\frac{y}{x}\right\}) & \mapsto & (\frac{x-1}{x}, \left\{\frac{y}{x}\right\}) & \mapsto & (\frac{x-1}{x}, 2\pi\left\{\frac{y}{x}\right\}) \\
 & & (u,v) & \mapsto\!\!< & (\frac{u-1}{u}, v) & \mapsto & (\frac{u-1}{u}, 2\pi v) \\
 & & & & (s,t) & \mapsto & (s, 2\pi t) \\
 & & & & & & (\rho,\theta)
\end{array}
$$

[1] *We denote by* $\{\star\}$ *the number in the interval* $[0,1[$ *congruent to* $\star$ *modulo* $\mathbf{Z}$. *For a visual summary of these transformations and more, visit:* `http://www.math.jussieu.fr/~pph/` *and double-click on the black screen there.*

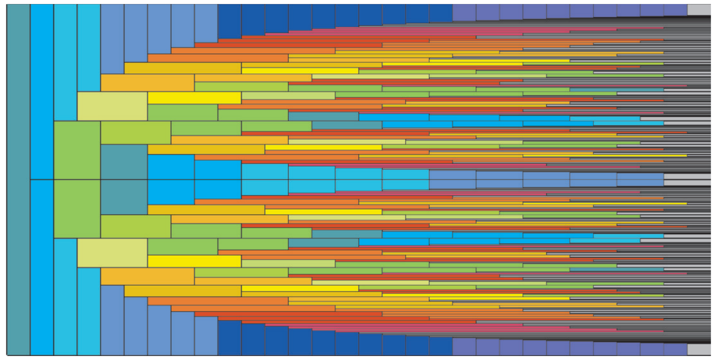


Figure 2. The Farey tail.

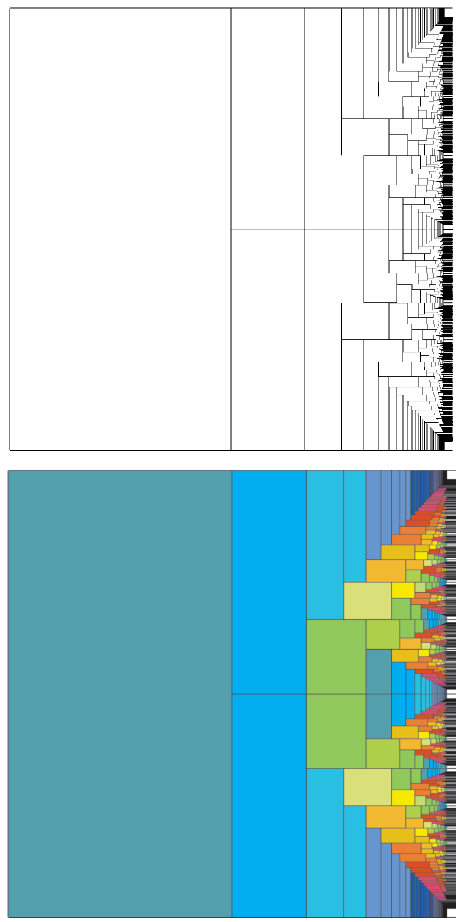
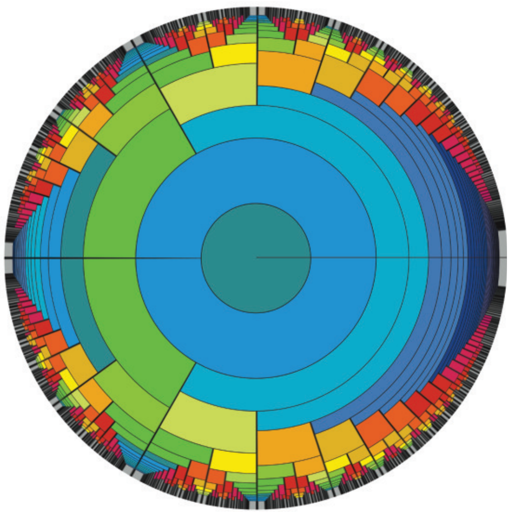


Figure 3. The Farey comb.

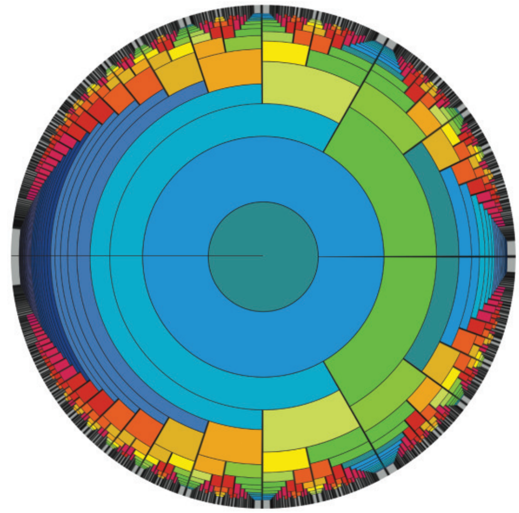
The first three systems of coordinates are Cartesian, and the last one is polar.[2]

As one might guess from Figures 2–4, the Farey graph is a genuine fractal object. Indeed, the fractal dimension (also called Mandelbrot or Kolmogorov dimension) of the Farey comb or of the Farey eye

[2] *The reader may be interested in checking that lines in the garden of visibles (except the vertical axis) are transformed into lines in the Farey comb.*



The Farey eye.



The symmetrical Farey eye.

Figure 4.

as subsets of $\mathbf{R}^2$ (that is, of the image graphs in the square or in the unit disc) is equal to 3/2, a true fraction.[3]

Paths and Best Approximations

Let's orient the edges of the Farey graph so that in the Farey tail, for example, one can move along the horizontal edges from the left to the right and on the vertical edges toward the vertex (dubbed extremal vertex in the previous section) of the unique horizontal edge stemming from this vertical edge and running to the right.

A path in the Farey tail can therefore be described by a sequence of fractions $\frac{p_0}{q_0}, \frac{p_1}{q_1}, \ldots$ corresponding to the different horizontal edges it takes. We note that, in fact, one can enter such a horizontal edge only from its extremal vertex, where the only two vertical edges oriented toward this horizontal edge end.

One-Sided Approximations

We introduce the following area form, expressed in the different representations of the Farey graph described in the previous section:

$$\frac{dx\wedge dy}{x} \quad\leftrightarrow\quad du\wedge dv \quad\leftrightarrow\quad \frac{ds\wedge dt}{(1-s)^2} \quad\leftrightarrow\quad \frac{d\rho\wedge d\theta}{2\pi(1-\rho)^2}.$$

We associate to a real number $\xi\in[0,1[$ the ray in the Farey eye, the angle of which is $2\pi\xi$, and we consider the path in the Farey graph minimizing the area between this ray and the path, computed with the area form above. Alternatively, it is the path in the Farey eye for which the radial edges cut a minimal angle on the unit circle, with the direction determined by ξ. If $\frac{p_0}{q_0}, \frac{p_1}{q_1}, \ldots$ is the sequence of fractions describing this path, the area is given by the formula (for the computation, one may prefer to go back to the Farey tail)

$$\sum_{k=0}^{\infty}(q_{k+1}-q_k)\left|\xi-\frac{p_k}{q_k}\right|. \tag{1}$$

Therefore, it is minimal when each irreducible fraction $\frac{p_k}{q_k}$ represents the rational number of denominator $\le q_k$ closest to ξ. Thus, the path under consideration is described by the sequence of fractions giving the *best approximations of type* 1 of ξ, that is, satisfying for all $k\in\mathbf{N}$:

$$\begin{aligned}&\forall(p,q)\in\mathbf{Z}\times\mathbf{N}^*,\\ &q\le q_k,\ \frac{p}{q}\ne\frac{p_k}{q_k},\\ &\left|\xi-\frac{p}{q}\right|>\left|\xi-\frac{p_k}{q_k}\right|.\end{aligned}\tag{2}$$

We remark that since two consecutive elements in the Farey sequence $\mathcal{F}_n$, $n\in\mathbf{N}$, $n>1$, cannot have the same denominator (cf. [2, Thm. 31]), condition (2) determines uniquely the sequence of best approximations $\frac{p_k}{q_k}$ with $q_k>1$. Furthermore, when $q_k=1$ there is ambiguity determining the best integral approximation only for the half integers $\xi\in\frac{1}{2}+\mathbf{N}$. Therefore, to each real number ξ (not a half integer) is associated a definite path in the Farey eye which then becomes an "approximoscope"; see Figure 5, where, following the white lightning, one reads the sequence of best approximations of type 1 of $\frac{9}{38}$: 0, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{13}$, $\frac{4}{17}$, $\frac{5}{21}$, $\frac{9}{38}$.

[3] *Computation of this fractal dimension can be found in: P. Philippon, Un œil et Farey,* `http://hal.archives-ouvertes.fr/hal-00488471` *(02/06/2010).*

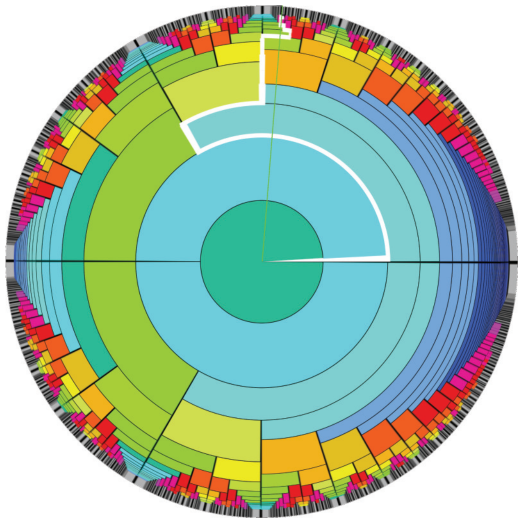


Figure 5. The approximoscope set at $\xi = \frac{9}{38}$ (green ray).

We note that the convergents of the regular continued fraction of ξ are some best approximations of type 1, in the sense of (2) (cf. [2, Thm. 181]), except possibly for the first one, since the first best approximation $\frac{p_0}{q_0}$ of ξ (with denominator $q_0 = 1$) is the integer closest to ξ, not its integer part (with the ambiguity already mentioned for half integers $\xi \in \frac{1}{2} + \mathbf{N}$). But these convergents of the regular continued fraction do not give all the best approximations of type 1. They give all the *best approximations of type* 0, in the stronger sense (cf. [2, Thm. 182]):

$$\begin{aligned} &\forall (p,q) \in \mathbf{Z} \times \mathbf{N}^*, \\ &q \le q_k, \frac{p}{q} \ne \frac{p_k}{q_k}, \\ &|q\xi - p| > |q_k\xi - p_k|. \end{aligned} \tag{3}$$

The quantity $|q\xi - p|$ can be visualized in the Farey comb as the absolute value of the slope of the line linking the point $(1, \xi)$ to the point $(1 - \frac{1}{q}, \frac{p}{q})$. Therefore, the approximations of type 0 are represented by the points $(1 - \frac{1}{q}, \frac{p}{q})$ such that the rhomboid defined by the equations $0 \le x < 1 - \frac{1}{q}$ and $|\xi - y| \le |q\xi - p|.(1 - x)$ does not contain any vertex of the Farey comb.

This leads to Klein's geometric interpretation, according to which these best approximations of type 0 can be detected as follows. From the left split the horizontal line of ordinate ξ in two and pull the extremity of each resulting string downwards for the lower string and diagonally left upwards for the upper string (see Figure 6, where $\xi = \frac{9}{38}$ is the ordinate of the green horizontal line). Suppose these rubber strings cannot cross the horizontal edges of the graph; then they shape polygonal lines spanned on the vertices of some horizontal edges of the graph. (In Figure 6 these two polygonal lines are drawn in black, while the white line is the lightning of best approximations of type 1 discussed previously, the image of which in the Farey eye has already been shown in Figure 5.) The corners of these polygonal lines correspond alternatively to the upper and lower best approximations of type 0 of ξ. Furthermore, between two corners, the number of horizontal edges touched at their vertices by the polygonal line is the corresponding partial quotient of the expansion in regular continued fraction[4] of ξ minus 1.

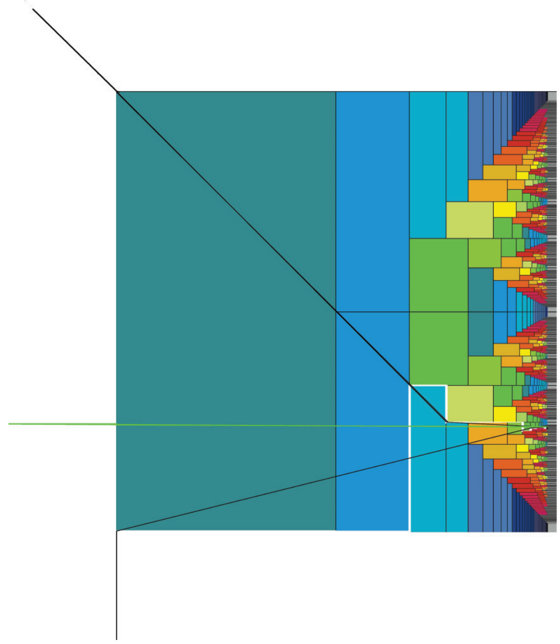


Figure 6. Farey comb with lightning and strings spanned at $\xi = \frac{9}{38}$.

Example 1. 1) The convergents of the regular continued fraction of $\frac{1}{4}$ are 0 and $\frac{1}{4}$, the best approximations of type 0, whereas the sequence of best approximations of type 1 is $0, \frac{1}{3}, \frac{1}{4}$.

2) In Figure 6 the upper and lower black lines are composed of two and three segments respectively. For the lower one, the first segment goes upwards to the vertex $(0, 0)$; the second segment from $(0, 0)$ to $(\frac{16}{17}, \frac{4}{17})$, touching the intermediate vertices $(\frac{4}{5}, \frac{1}{5})$, $(\frac{8}{9}, \frac{2}{9})$, $(\frac{12}{13}, \frac{3}{13})$; and the third lower segment goes directly from $(\frac{16}{17}, \frac{4}{17})$ to $(\frac{37}{38}, \frac{9}{38})$. The first segment in the upper line goes from

[4] *More on continued fraction expansions will come in the next section.* See *in particular the beginning of the subsection "Sequences of Approximations" to identify the intermediate vertices with the intermediate convergents, first in the garden of visibles, then in the Farey comb, thanks to the hint in footnote 2.*

the upper left corner down to $(\frac{3}{4}, \frac{1}{4})$, touching the intermediate vertices $(0,1)$, $(\frac{1}{2}, \frac{1}{2})$, $(\frac{2}{3}, \frac{1}{3})$; and the second segment goes from $(\frac{3}{4}, \frac{1}{4})$ to $(\frac{37}{38}, \frac{9}{38})$, touching the intermediate vertex $(\frac{20}{21}, \frac{5}{21})$. The sequence of corners is therefore $(0,0)$, $(\frac{3}{4}, \frac{1}{4})$, $(\frac{16}{17}, \frac{4}{17})$, and $(\frac{37}{38}, \frac{9}{38})$, giving the sequence of best approximations of type 0 of $\frac{9}{38}$: $0, \frac{1}{4}, \frac{4}{17}, \frac{9}{38}$. Counting the number of intermediate vertices on each segment, one gets the expansion in regular continued fractions: $\frac{9}{38} = [0,4,4,2]$ (the last segment of the lower string is ignored, since the number is reached from above, but for irrational numbers the sequence of segments continues indefinitely).

Two-Sided Approximations

In the same notation as above, starting from the pair of integers $\left(\frac{[\xi]}{1}, \frac{[\xi]+1}{1}\right)$ framing the real number ξ, one may consider the *Hurwitz chain associated to* ξ (see [4]), consisting of pairs $\left(\frac{p_k'}{q_k'}, \frac{p_k''}{q_k''}\right)$, $k \in \mathbf{N}$, of elements in the Farey sequence $\mathcal{F}_1, \mathcal{F}_2, \dots$, framing ξ (a pair can give the frame for several consecutive Farey sequences). Therefore $\frac{p_k'}{q_k'}$ and $\frac{p_k''}{q_k''}$ are consecutive elements in some Farey sequence $\mathcal{F}_n$. But the first Farey sequence containing an element (necessarily unique, because two consecutive elements in a Farey sequence have different denominators) strictly between these two fractions is the one containing the *mediant* $\frac{p_k'+p_k''}{q_k'+q_k''}$ (cf. [2, Thm. 29]). We conclude that the next pair $\left(\frac{p_{k+1}'}{q_{k+1}'}, \frac{p_{k+1}''}{q_{k+1}''}\right)$ in the Hurwitz chain is either $\left(\frac{p_k'}{q_k'}, \frac{p_k'+p_k''}{q_k'+q_k''}\right)$ or $\left(\frac{p_k'+p_k''}{q_k'+q_k''}, \frac{p_k''}{q_k''}\right)$, according to whether the mediant $\frac{p_k'+p_k''}{q_k'+q_k''}$ is larger or smaller than ξ. This way of producing the Hurwitz chain is also called the *Farey process*[5] in [7].

Example 2. The Hurwitz chain associated to $\frac{85}{38} = 2 + \frac{9}{38}$ is

$$\left(\tfrac{2}{1}, \tfrac{3}{1}\right) \ \left(\tfrac{2}{1}, \tfrac{5}{2}\right) \ \left(\tfrac{2}{1}, \tfrac{7}{3}\right) \ \left(\tfrac{2}{1}, \tfrac{9}{4}\right) \ \left(\tfrac{11}{5}, \tfrac{9}{4}\right)$$
$$\left(\tfrac{20}{9}, \tfrac{9}{4}\right) \ \left(\tfrac{29}{13}, \tfrac{9}{4}\right) \ \left(\tfrac{38}{17}, \tfrac{9}{4}\right) \ \left(\tfrac{38}{17}, \tfrac{47}{21}\right) \ \left(\tfrac{85}{38}\right).$$

When ξ is rational it coincides with one of the mediants $\frac{p_k'+p_k''}{q_k'+q_k''}$, for some k. Beyond this index k one may extend the Hurwitz chain keeping either the pair $\left(\frac{p_k'}{q_k'}, \frac{p_k'+p_k''}{q_k'+q_k''}\right)$ or the pair $\left(\frac{p_k'+p_k''}{q_k'+q_k''}, \frac{p_k''}{q_k''}\right)$. We prefer not to consider here the Hurwitz chain of rational numbers beyond the mediant it coincides with. Therefore, rational numbers are the numbers with a finite Hurwitz chain (the last pair reducing to the given rational).

Remark 1. When ξ does not belong to the interval $[0,1]$, our definition of the Hurwitz chain differs from that described in [1]; see also [4]. In these references, supposing $\xi > 0$, one starts from the pair $\left(\frac{0}{1}, \frac{1}{0}\right)$, and the Hurwitz chain begins with $\left(\frac{1}{1}, \frac{1}{0}\right)$ $\left(\frac{2}{1}, \frac{1}{0}\right) \dots \left(\frac{[\xi]}{1}, \frac{1}{0}\right)$ before the appearance of the pair $\left(\frac{[\xi]}{1}, \frac{[\xi]+1}{1}\right)$ of elements of $\mathcal{F}_1$ framing ξ.

The Hurwitz chain of ξ produces both sequences of lower and upper rational approximations of ξ. One deduces also the *Hurwitz sequence* built from the components of the pairs in the Hurwitz chain, not repeated and ordered by increasing denominators (this is the sequence of mediants appearing successively in the Farey process). The Hurwitz sequence $\left(\frac{p_k}{q_k}\right)_{k \in \mathbf{N}}$ of ξ starts with $\frac{p_0}{q_0} = \frac{[\xi]}{1}$ $\frac{p_1}{q_1} = \frac{[\xi]+1}{1}$ $\frac{p_2}{q_2} = \frac{2[\xi]+1}{2} \dots$. It contains all the "rational approximations" of ξ (in any reasonable sense)[6] and in particular the best approximations of type 0 and 1 already discussed. However, one still has to learn how to sort these best approximations out of the complete Hurwitz sequence.

Example 3. The Hurwitz sequence of $\frac{85}{38}$ is 2 3 $\frac{5}{2}$ $\frac{7}{3}$ $\frac{9}{4}$ $\frac{11}{5}$ $\frac{20}{9}$ $\frac{29}{13}$ $\frac{38}{17}$ $\frac{47}{21}$ $\frac{85}{38}$, and the best approximations of type 0 are 2 $\frac{9}{4}$ $\frac{38}{17}$ $\frac{85}{38}$.

The Hurwitz sequence $\left(\frac{p_k}{q_k}\right)_{k \in \mathbf{N}}$ of ξ is characterized by the sequence of signs encoding whether the corresponding approximation of ξ is below or above ξ. More precisely, one reconstructs step by step the Hurwitz sequence of ξ from the integer part of ξ and its *characteristic sequence* of signs $\varepsilon_{k+1} := \frac{p_k - q_k\xi}{|p_k - q_k\xi|}$, $k \geq 2$. We set $\varepsilon_1 = -1$ and $\varepsilon_2 = +1$ in order to complete the characteristic sequence in $\varepsilon_1\ \varepsilon_2\ \varepsilon_3 \dots$ (note the shift of index in comparison with the Hurwitz sequence, which starts with index 0). To recover the Hurwitz sequence from the characteristic sequence, one checks that the term $\frac{p_{k+1}}{q_{k+1}}$ is equal to the mediant of $\frac{p_k}{q_k}$ and $\frac{p_\ell}{q_\ell}$, where ℓ is the largest index $< k$ satisfying $\varepsilon_k \varepsilon_\ell = -1$ ($\frac{p_\ell}{q_\ell}$ is the surviving component in the pairs of the Hurwitz chain until the next change of sign), and that the corresponding pair in the Hurwitz chain is

[5] *As an exercise, show that $\frac{p_k'+p_k''}{q_k'+q_k''}$ is on the same side of ξ as $\frac{p_k'}{q_k'}$ (resp. $\frac{p_k''}{q_k''}$) if and only if $|q_k'\xi - p_k'| > |q_k''\xi - p_k''|$ (resp. $|q_k'\xi - p_k'| < |q_k''\xi - p_k''|$). Deduce that the sequence of best approximations of type 0 of ξ alternates under and above ξ.*

[6] *More precisely, a rational number $\frac{p}{q}$ is a (rational) approximation of ξ if there is no other rational number of denominator less than or equal to q between $\frac{p}{q}$ and ξ. Obviously, rational numbers that do not satisfy this property cannot qualify for the status of approximation. However, remember that, given an approximation, there may be other approximations closer to ξ and of smaller denominators but on the other side of ξ.*

deduced from the signs ε_ℓ, ε_k, and ε_{k+1}. Whenever ξ is a rational number and k is the first index such that $p_k - q_k\xi = 0$, one may set by convention $\varepsilon_{k+1} = \varepsilon_k$, and the characteristic sequence stops with $k+1$ terms.

Example 4. The characteristic sequence of $\frac{85}{38}$ is $- \ + \ + \ + \ + \ - \ - \ - \ - \ + \ +$.

Remarkably, one can also recover straightforwardly the expansion of ξ as a regular continued fraction from its characteristic sequence. Putting aside the first sign and, with our convention above, on the last sign in case ξ is a rational number, one checks that the length of the blocks of identical signs in the characteristic sequence are the *partial quotients* in the expansion of ξ as a regular continued fraction, except for the integer part of ξ, which starts equally the continued fraction and the Hurwitz sequence in our description. The approximations of type 0, that is, the *convergents* (also known as *approximants*) of this regular continued fraction, are the elements $\frac{p_k}{q_k}$ of the Hurwitz sequence positioned just before a change of sign (i.e., such that $\varepsilon_{k+1}\varepsilon_{k+2} = -1$); see [4, §§3 & 5].[7]

The Hurwitz sequence of ξ is represented on the Farey graph by both paths minimizing the surface that each of them cuts with the ray determined by ξ, but with the constraint that these paths never cross the ray. Therefore, there are two paths approaching the ray determined by ξ from one or the other side. The radial edges of each of these paths enumerate the sequences of best approximations of ξ from below and from above respectively. In Klein's interpretation, the vertices crossed by these two paths are all the vertices touched (corners and intermediate ones) by the two black strings in Figure 6.

Continued Fraction Expansions

A very good reference for this section is O. Perron [6]; we refer the reader to this monograph for most of the definitions and statements. We start with a quick tour of the zoo of continued fraction expansions of real numbers. We introduce the classical terminology for continued fractions, along with practical algorithms computing the complete quotients and partial fractions for several continued fraction expansions, selecting some remarkable sequences of approximations of a given real number.

[7] *This is because the approximations of type* 0 *alternate under and above* ξ*; see footnote 5 and also the next section.*

The Zoo of Continued Fraction Expansions

The "ordinary" expansion in continued fractions is well known; its convergents give the sequence of best approximations of type 0 of a real number. Continued fractions of the form

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \dots,}}} = [a_0, a_1, a_2, \dots] \tag{4}$$

with partial quotients a_1, a_2, ... positive integers and a_0 an integer, are called *regular*. Infinite, regular continued fractions are convergent (that is, their sequences of convergents converge!) and their limits are in bijection with the set of all irrational real numbers. To each rational number correspond exactly two regular continued fractions which are finite. The numbers of their partial quotients differ by 1: the longest has 1 as last partial quotient and the shortest not. Expansions in regular continued fractions of quadratic numbers are those that are *ultimately periodic* (Lagrange's theorem). *Purely periodic* expansions correspond to *reduced* real quadratic numbers, that is, real quadratic numbers larger than 1 such that their conjugates lay between -1 and 0 (Galois's theorem).

Here is the well-known algorithm computing the expansion in regular continued fractions of the real number ξ in the form (4), namely, $\xi = [a_0, a_1, a_2, \dots]$:

Algorithm[8] *RCF*:

```
-1) v_0 = ξ;
 k) a_k = [v_k], then v_{k+1} := 1/(v_k - a_k) > 1;
```

the convergents of which are all the approximations of ξ of type 0 ordered by strictly increasing denominators; see also [2, Chap. X] for example.

More generally, *semi-regular* continued fractions are those of the form[9]

$$a_0 + \cfrac{\varepsilon_1}{a_1 + \cfrac{\varepsilon_2}{a_2 + \cfrac{\varepsilon_3}{a_3 + \dots}}} = a_0 + \underline{\varepsilon_1|}\overline{a_1} + \underline{\varepsilon_2|}\overline{a_2} + \dots \tag{5}$$

where $\varepsilon_i \in \{\pm 1\}$ for $i \geq 1$, a_1, a_2, ... are positive integers satisfying $a_i + \varepsilon_{i+1} \geq 1$ for $i \geq 1$ and, if the continued fraction is infinite, $a_i + \varepsilon_{i+1} \geq 2$ for an infinity of i; whereas if the continued fraction is finite, one requires that its last partial quotient be > 1 (except when it coincides with the initial term). Semi-regular continued fractions are also

[8] *This algorithm and the others described in the sequel stop as soon as* $v_{k+1} = \infty$ *(see the definition of this parameter in each case); otherwise they continue indefinitely. They work on the principle of a* "`while`" *loop that must be repeated from* $k = 0$ *on indefinitely, unless one gets* $v_{k+1} = \infty$ *for some* k.

[9] *Observe that with the notation in* (4) *and* (5) *one has* $[a_0, a_1, a_2, \dots] = a_0 + \underline{1|}\overline{a_1} + \underline{1|}\overline{a_2} + \dots$.

convergent, and to each irrational real number ξ and sequence of signs $(\varepsilon_i)_{i\in\mathbf{N}^*}$ corresponds an expansion in semi-regular continued fractions having ε_i as partial numerators and ξ as limit. A rational number of denominator q has exactly q expansions in semi-regular continued fractions. Periodic semi-regular continued fractions are in bijection with the expansions of quadratic real numbers for prescribed periodic sequences of signs. Here is an algorithm computing the semi-regular expansion in continued fractions of a real number ξ with a given prescribed sequence of signs $\varepsilon_1, \varepsilon_2, \ldots$:

Algorithm[10] *SRCF*:

-1) $v_0 = \xi$;

k) $a_k = \begin{cases} \lfloor v_k \rfloor \text{ if } \varepsilon_{k+1} = +1 \\ \lceil v_k \rceil \text{ if } \varepsilon_{k+1} = -1 \end{cases}$, then $v_{k+1} := \frac{1}{|v_k - a_k|} > 1$.

In general, the sequence of convergents of a semi-regular continued fraction is not produced with increasing denominators; some sparse partial fractions $\frac{-1|}{|1}$ may alter this natural order.

Of course, prescribing the sequence of all positive signs, one recovers the expansion in regular continued fractions as one of the expansions in semi-regular continued fractions. On the opposite side, one may prescribe the sequence of all negative signs, possibly allowing a plus sign for the first one. The convergents of these continued fractions are all the approximations of the real numbers ξ from above when $\varepsilon_1 = -1$ and from below when $\varepsilon_1 = +1$, ordered by strictly increasing denominators.[11] Here are the algorithms specialized from *SRCF* computing these "negative" continued fractions:

Algorithm NCF-:

-1) $v_0 = \xi$;

k) $a_k = \lceil v_k \rceil$, then $v_{k+1} := \frac{1}{a_k - v_k} > 1$ and $\varepsilon_{k+1} = -1$;

the convergents of which are the approximations of ξ from above.

Algorithm NCF+:

-1) $v_0 = \xi$;

0) $a_0 = \lfloor v_0 \rfloor$, then $v_1 := \frac{1}{v_0 - a_0} > 1$ and $\varepsilon_1 = +1$;

k) $a_k = \lceil v_k \rceil$, then $v_{k+1} := \frac{1}{a_k - v_k} > 1$ and $\varepsilon_{k+1} = -1$;

the convergents of which are the approximations of ξ from below.

Continued fractions of the form (5) with $\varepsilon_i \in \{\pm 1\}$ and a_i positive integers are called *unitary*. If such a continued fraction does not contain the partial fraction $\frac{-1|}{|1}$ it converges.[12] J. Goldman [1, Thm. 6] shows that the convergents of a unitary continued fraction that does not contain any partial fraction equal to $\frac{-1|}{|1}$ all belong to the Hurwitz sequence of its limit. In particular, the Hurwitz sequence of a real number ξ identifies with the sequence of convergents of a unitary continued fraction (however, not semi-regular), which we call a *complete* continued fraction, such that $\frac{\varepsilon_i|}{|a_i} = \frac{1|}{|1}$ or $\frac{-1|}{|2}$, for $i \geq 1$ and $a_0 = [\xi]$. If the Hurwitz sequence is finite, the continued fraction is also finite; whereas if the Hurwitz sequence is infinite, then $\frac{1|}{|1}$ appears infinitely many times in the continued fraction. Reciprocally, the sequence of convergents of a continued fraction of this type is the Hurwitz sequence of its limit; cf. [1]. Here is how to compute the complete continued fraction of the real number ξ:

Algorithm CCF:

-1) $v_0 = \xi$;

0) $a_0 = \lfloor v_0 \rfloor$, then $v_1 := \frac{1}{v_0 - a_0} > 0$ and $\varepsilon_1 = +1$;

k) $a_k = \begin{cases} 1 \text{ if } \varepsilon_k = +1 \\ 2 \text{ if } \varepsilon_k = -1 \end{cases}$, then $v_{k+1} := \frac{1}{|v_k - a_k|}$ and ε_{k+1} is the sign of $v_k - a_k$;

the convergents of which form the complete Hurwitz sequence of ξ, ordered by strictly increasing denominators.

On another hand, given a sequence of rational numbers $(\frac{p_k}{q_k})_{k\in\mathbf{N}}$ ordered by strictly increasing denominators, this sequence can be obtained as the sequence of convergents of a unitary continued fraction (with no partial fraction $\frac{-1|}{|1}$) if and only if any two successive terms satisfy $|p_{k+1}q_k - p_k q_{k+1}| = 1$.

A semi-regular continued fraction is said to be *singular* if one has $a_i \geq 2$ and $a_i + \varepsilon_i \geq 2$ for all $i \in \mathbf{N}^*$. Every real number has an expansion in singular continued fractions, unique if it is not equivalent to $\frac{\sqrt{5}-1}{2}$ (under the action of $\mathrm{Sl}_2(\mathbf{Z})$). Again, the expansions of real quadratic numbers in singular continued fractions are those which are periodic; cf. [3, §4]. The following algorithm computes the expansion of the real number ξ in a singular continued fraction.

Algorithm SGCF:

-1) $v_0 = \xi$;

[10] *Here we denote by $\lceil \star \rceil$ the smallest integer larger than $\star$ and by $\lfloor \star \rfloor$ the usual integer part (also denoted by $[\star]$ elsewhere in this text, for example, in algorithm RCF).*

[11] *The reader can check that the daunting partial fraction $\frac{-1|}{|1}$ never occurs in these two "negative" continued fractions.*

[12] *Actually, it suffices that the occurrences of the partial fraction $\frac{-1|}{|1}$ are sparse enough (certainly no two consecutive), as in the case of semi-regular continued fractions already mentioned.*

k) $a_k = \left\lfloor \nu_k + \frac{3-\sqrt{5}}{2} \right\rfloor$, then $\nu_{k+1} := \frac{1}{|\nu_k - a_k|} \geq \frac{\sqrt{5}+1}{2}$ and ε_{k+1} is the sign of $\nu_k - a_k$;

the convergents of which, $\frac{p}{q}$, satisfy $\left|\xi - \frac{p}{q}\right| \leq \frac{\sqrt{5}-1}{2q^2}$ and are ordered by strictly increasing denominators.

Note that in algorithm *SGCF* the partial quotient a_k is the integer closest to $\nu_k + 1 - \frac{\sqrt{5}}{2}$, with the choice of the larger one if $\nu_k - \frac{\sqrt{5}}{2}$ is a half integer.

One may also expand a real number in a continued fraction along the lines of the usual algorithm *RCF*, but *selecting the closest integer* (rather than taking the integer part) at each step. One then obtains a semi-regular continued fraction satisfying, furthermore, $a_i \geq 2$ and $a_i + \varepsilon_{i+1} \geq 2$ for all $i \in \mathbf{N}^*$. For rational numbers this expansion is the shortest, while for an irrational, quadratic real number it is periodic; cf. [3, §2]. Here is the corresponding algorithm that computes the closest integer continued fraction expansion of ξ:

Algorithm CICF:

-1) $\nu_0 = \xi$;

k) $a_k = \lceil \nu_k - \frac{1}{2} \rceil$, then $\nu_{k+1} := \frac{1}{|\nu_k - a_k|} \geq 2$ and ε_{k+1} is the sign of $\nu_k - a_k$;

the convergents of which, $\frac{p}{q}$, satisfy $\left|\xi - \frac{p}{q}\right| \leq \frac{\sqrt{5}-1}{2q^2}$ and are ordered by strictly increasing denominators.

In algorithm *CICF* the partial quotient a_k is the integer closest to ν_k, with the choice of the smallest one when ν_k is a half integer.[13]

More generally, McKinney [5] has studied the expansions in continued fractions along the closest integer after shifting by a fixed real number λ, which he calls λ*-development*.

Any convergent $\frac{p}{q}$ of the expansions in singular continued fractions and along the closest integer continued fraction of a real number are best approximations of type 0 satisfying

$$\left|\xi - \frac{p}{q}\right| \leq \frac{\sqrt{5}-1}{2q^2} \quad . \tag{6}$$

The lists of convergents of these two continued fractions overlap, but also contain distinct approximations. However, the union of the two lists does not contain all the best approximations of type 0 satisfying (6). Furthermore, the rational numbers satisfying (6) are not necessarily best approximations of type 0 or even 1.

It is trickier to devise an algorithm selecting precisely all the rational approximations of ξ satisfying (6), ordered by strictly increasing denominators. First, one has to verify that the sequence $(\frac{p_k}{q_k})_{k\in\mathbf{N}}$ of these approximations satisfies $|p_{k+1}q_k - p_kq_{k+1}| = 1$ for all $k \in \mathbf{N}$, which is true but not obvious. In fact, for any real number $\varpi \in [\frac{1}{2}, \frac{2}{3}]$ the sequence of rational approximations of ξ satisfying

$$\left|\xi - \frac{p_k}{q_k}\right| \leq \frac{\varpi}{q_k^2} \tag{7}$$

also satisfies $|p_{k+1}q_k - p_kq_{k+1}| = 1$ and can therefore be obtained as the sequence of convergents of a unitary continued fraction. This may no longer be true for a positive number ϖ strictly smaller than $\frac{1}{2}$ or strictly larger than $\frac{2}{3}$.

For $\varpi \in [\frac{1}{2}, \frac{2}{3}]$ the following algorithm *DCF(ϖ)* produces the continued fraction that has the sequence of all rational approximations of ξ satisfying (7) as sequence of convergents. It does not seem to appear in the literature. Thus, checking that it indeed selects the asserted sequence of approximations is a real challenge proposed to the reader.

Algorithm DCF(ϖ):

-1) $\nu_0 = \xi$;

0) $a_0 = \begin{cases} \lfloor \nu_0 \rfloor \text{ if } \nu_0 - \lfloor \nu_0 \rfloor \leq \varpi \\ \lceil \nu_0 \rceil \text{ otherwise} \end{cases}$

$\nu_1 := \frac{1}{|\nu_0 - a_0|}$,
ε_1 is the sign of $\nu_0 - a_0$, $r_1 := 0$ and $e_1 := |\nu_0 - a_0|$;

k) $a_k = \begin{cases} \frac{3-\varepsilon_k}{2} \text{ if } e_k\left|\nu_k - \frac{3-\varepsilon_k}{2}\right|\left(\frac{3-\varepsilon_k}{2} + \varepsilon_k r_k\right) \leq \varpi \\ \left\lceil \nu_k - \frac{1}{2e_k}\left(1 - \sqrt{1 - 4\varpi e_k}\right)\right\rceil \text{ otherwise} \end{cases}$

$\nu_{k+1} := \frac{1}{|\nu_k - a_k|}$, ε_{k+1} is the sign of $\nu_k - a_k$, $r_{k+1} := \frac{1}{a_k + \varepsilon_k r_k}$ and $e_{k+1} := \frac{e_k}{\nu_{k+1} r_{k+1}}$;

the convergents of which, $\frac{p}{q}$, are all the approximations of ξ satisfying $\left|\xi - \frac{p}{q}\right| \leq \frac{\varpi}{q^2}$, ordered by strictly increasing denominators.

The reader may like to check that the quantities e_k introduced in the algorithm *DCF(ϖ)* satisfy $e_k(\nu_k + \varepsilon_k r_k) = 1$. Thus, eliminating e_k, the definition of a_k in the second case of the general step can be rewritten accordingly as

$$a_k = \left\lceil \frac{1}{2}\left(\nu_k - \varepsilon_k r_k + \sqrt{(\nu_k + \varepsilon_k r_k)^2 - 4\varpi(\nu_k + \varepsilon_k r_k)}\right)\right\rceil$$

and the condition in the first case reads

$$\left|\nu_k - \frac{3-\varepsilon_k}{2}\right|\left(\frac{3-\varepsilon_k}{2} + \varepsilon_k r_k\right) \leq \varpi(\nu_k + \varepsilon_k r_k).$$

Furthermore, if the condition in the first case is not satisfied, then $\nu_k + \varepsilon_k r_k \geq 4\varpi$ and the square root in the definition of a_k in the second case is a definite nonnegative real number.

---

[13] *With this choice the continued fraction ends as* $\lfloor \nu_k \rfloor + 1\rfloor 2$ *when* ν_k *is a half integer for some k(note that* ξ *is then a rational number). With the other choice the continued fraction would end as* $\lceil \nu_k \rceil + \overline{-1}\rfloor 2$, *but the penultimate convergent may not be an approximation of type* 0.

For $\varpi = \frac{1}{2}$ one obtains the so-called *diagonal* continued fraction, which is semi-regular. It can also be computed by *singularization*[14] of the regular continued fraction of the real number ξ. Its convergents $\frac{p}{q}$ are all the best approximations of ξ of type 0 satisfying

$$\left|\xi - \frac{p}{q}\right| \leq \frac{1}{2q^2}\ , \tag{8}$$

since it is well known that rational numbers satisfying (8) are best approximations of ξ of type 0 (cf. [2, Thm. 184]), and from any two consecutive best approximations of type 0, at least one satisfies (8) (cf. [2, Thm. 183]). The expansions in diagonal continued fraction of quadratic numbers are periodic; see [6].

Finally, we want to introduce a last algorithm, the convergents of which form the remarkable sequence of best approximations of type 1 of a given real number ξ, ordered by increasing denominators. The corresponding continued fraction is unitary of the form (5). Here is this algorithm.[15]As before, if ξ is a rational number, we stop the algorithm as soon as $a_k = \nu_k$, that is, $\nu_{k+1} = \infty$.

Algorithm BACF:

- `-1)` $\nu_0 = \xi$;
- `0)` a_0 `is the integer closest to` ν_0`,` `let` $\nu_1 := \frac{1}{|\nu_0 - a_0|} \geq 2$`,` ε_1 `the sign of` $\nu_0 - a_0$ `and` $r_1 := 0$;
- k`)` $a_k := 1 + \left\lfloor \frac{1}{2}(\nu_k - \varepsilon_k r_k)\right\rfloor$`, then` $\nu_{k+1} := \frac{1}{|\nu_k - a_k|} > 0$`,` ε_{k+1} `the sign of` $\nu_k - a_k$ `and` $r_{k+1} := \frac{1}{a_k + \varepsilon_k r_k}$`;`

the convergents of which are all the best approximations of ξ of type 1, ordered by strictly increasing denominators.

This expansion in unitary continued fractions of the real number ξ is ultimately periodic if and only if ξ is a quadratic number.[16]

Note that the quantity r_{k+1} introduced above is just the ratio of the denominators of the $(k-1)$-th and k-th convergents of the continued fractions $a_0 + \underline{\varepsilon_1|}\overline{a_1} + \underline{\varepsilon_2|}\overline{a_2} + \ldots$; that is, $r_{k+1} = \frac{q_{k-1}}{q_k} = \underline{1|}\overline{a_k} + \underline{\varepsilon_k|}\overline{a_{k-1}} + \cdots + \underline{\varepsilon_2|}\overline{a_1}$. But, although this parameter is a rational number, it does not need to be computed exactly. It suffices to compute it as a real number with the same precision as the parameter ν_k.

Sequences of Approximations

We explain in this subsection how the complete continued fraction expansion (algorithm *CCF* in the previous subsection) of a real number ξ is simply related to its expansion in regular continued fractions (algorithm *RCF* in the previous subsection).

Recall that the sequence of convergents of a continued fraction of the form (5) is given by $\begin{cases} p_0 = a_0 \\ q_0 = 1 \end{cases}$ and, with the convention $\begin{cases} p_{-1} = 1 \\ q_{-1} = 0 \end{cases}$,

$$\begin{cases} p_{k+1} = a_{k+1}p_k + \varepsilon_{k+1}p_{k-1} \\ q_{k+1} = a_{k+1}q_k + \varepsilon_{k+1}q_{k-1} \end{cases} \qquad \text{for } k \in \mathbf{N}.$$

Approximations of type 0 are the convergents of the expansion in regular continued fractions of a real number ξ. On another hand, the Hurwitz sequence of ξ is made of all the convergents $\left(\frac{p_k}{q_k}\right)_{k\in\mathbf{N}}$ of this expansion in regular continued fractions plus all the *intermediate convergents* (of denominators between q_k and q_{k+1}):

$$\frac{\lambda p_k + p_{k-1}}{\lambda q_k + q_{k-1}}\ , \quad \lambda = 1, \ldots, a_{k+1} - 1\ , \quad k \in \mathbf{N}^*.$$

The complete expansion in unitary continued fractions of a real number ξ is related in a simple way to the expansion in regular continued fractions. If $\xi = [c_0, c_1, c_2, \ldots]$ is the sequence of partial quotients of an irrational real number ξ, the complete expansion in unitary continued fractions is written

$$\xi = [\xi] + \underline{1|}\overline{1} + \overline{[\underbrace{\underline{1|}\overline{1} + \underline{-1|}\overline{2} + \cdots + \underline{-1|}\overline{2}}_{c_i \text{ term(s)}}]}_{i\geq 1}$$

where the overline means that each time the motif between brackets is reproduced, the integral index i must be incremented by 1 starting from 1 up to $+\infty$. Each successive group contains c_i terms, the first one being $\underline{1|}\overline{1}$ and the possible followers being $\underline{-1|}\overline{2}$. When $\xi = [c_0, c_1, \ldots, c_m]$ is a rational number there are m groups, and the last one contains only $c_m - 1$ terms so that the total length of the expansion is $c_1 + \cdots + c_m$ (leaving aside the initial integer part). For all integer i we set $\tilde{c}_i = c_1 + \cdots + c_i$. One has the following correspondence between the complete continued fraction expansion, Hurwitz sequence, and the characteristic sequence of ξ:

$$\begin{array}{cccccccc}
[\xi] + & \underline{1|}\overline{1} + [& \underline{1|}\overline{1} & + & \underline{-1|}\overline{2} & + \ldots + & \underline{-1|}\overline{2} & + \underline{-1|}\overline{2}]_{i\geq 1} \\
\frac{p_0}{q_0} & \frac{p_1}{q_1} & \Big[\frac{p_{\tilde{c}_{i-1}+2}}{q_{\tilde{c}_{i-1}+2}} & & \frac{p_{\tilde{c}_{i-1}+3}}{q_{\tilde{c}_{i-1}+3}} & \ldots & \frac{p_{\tilde{c}_i}}{q_{\tilde{c}_i}} & \frac{p_{\tilde{c}_i+1}}{q_{\tilde{c}_i+1}}\Big]_{i\geq 1} \\
- & + & [(-1)^{i-1} & & (-1)^{i-1} & \ldots & (-1)^{i-1} & (-1)^{i}]_{i\geq 1}.
\end{array}$$

Beware that, in the above picture, when $c_i = 1$ or $c_i = 2$ the partial fractions corresponding to

---

[14]*The singularization of a continued fraction* $a_0 + \underline{\varepsilon_1|}\overline{a_1} + \cdots + \underline{\varepsilon_{k+1}|}\overline{a_{k+1}} + \ldots$ *at* a_{k+1} *is the continued fraction* $a_0 + \underline{\varepsilon_1|}\overline{a_1} + \cdots + \underline{\varepsilon_{k-1}|}\overline{a_{k-1}} + \underline{\varepsilon_k|}\overline{a_k + 1} + \underline{-1|}\overline{a_{k+2} + 1} + \underline{\varepsilon_{k+3}|}\overline{a_{k+3}} + \ldots$. *Here, one has to singularize all the partial quotients* a_{k+1} *of the regular continued fraction such that the convergent* $\frac{p_k}{q_k}$ *satisfies* $\left|\xi - \frac{p_k}{q_k}\right| > \frac{1}{2q_k^2}$.

[15]*Computations establishing the previous statement can be found in: P. Philippon, Un œil et Farey,* `http://hal.archives-ouvertes.fr/hal-00488471` *(02/06/2010).*

[16]See *the reference in footnote 15 for details of the proof of this fact.*

the convergents $\frac{p_{\tilde{c}_i}}{q_{\tilde{c}_i}}$ and $\frac{p_{\tilde{c}_i+1}}{q_{\tilde{c}_i+1}}$ are not necessarily $\underline{-1}|\overline{2}$, because the first term of each group between brackets is always $\underline{1}|\overline{1}$ and the fraction $\frac{p_{\tilde{c}_i}}{q_{\tilde{c}_i}}$ may even stand before the beginning of that group when $c_i = 1$. The convergents of the expansion in regular continued fractions (i.e., the best approximations of type 0) are $\frac{p_0}{q_0}$ and $\frac{p_{\tilde{c}_i}}{q_{\tilde{c}_i}}$, $i \geq 1$.

Complete Quotients

The quantities ν_k introduced in the descriptions of the algorithms in "The zoo of continued fraction expansions" are related to the complete quotients of the continued fraction of the form (5); that is, for $k \in \mathbf{N}$,

$$\nu_k = a_k + \cfrac{\varepsilon_{k+1}}{a_{k+1} + \cfrac{\varepsilon_{k+2}}{a_{k+2} + \cfrac{\varepsilon_{k+3}}{a_{k+3} + \dots}}}.$$

Supposing that the fraction (5) converges towards a real number ξ and denoting by ν_k the k-th complete quotient as written above and by $\frac{p_k}{q_k}$ the k-th convergents, one then checks:

$$\xi = \frac{\nu_{k+1}p_k + \varepsilon_{k+1}p_{k-1}}{\nu_{k+1}q_k + \varepsilon_{k+1}q_{k-1}},$$
$$\nu_{k+1} = -\varepsilon_{k+1}\frac{q_{k-1}\xi - p_{k-1}}{q_k\xi - p_k},$$
$$\nu_{k+1} = \frac{\varepsilon_{k+1}}{\nu_k - a_k}$$

and

$$\nu_{k+1} = -\,\underline{\varepsilon_{k+1}}|\overline{a_k} + \underline{\varepsilon_k}|\overline{a_{k-1}} + \cdots + \underline{\varepsilon_2}|\overline{a_1} + \underline{\varepsilon_1}|\overline{a_0 - \xi}.$$

The complete quotients of a regular continued fraction satisfy $\nu_k > 1$ for any $k \in \mathbf{N}$. Therefore one can isolate the best approximations of type 0 among the convergents of the expansion in continued fractions along the closest integer or the complete expansion in unitary continued fractions, for example (or of any other expansion the sequence of convergents of which contains the sequence of best approximations of type 0 ordered by increasing denominators). Practically, one considers for these fractions the first product, $\nu_2 \dots \nu_{k_1+1}$ of modulus > 1, then the following products, $\nu_{k_{i-1}+2} \dots \nu_{k_i+1}$, again of modulus > 1, which give (with $k_0 = 0$) the indices k_i corresponding to the best approximations of type 0. Then, the complete and partial quotients of the expansion in regular continued fractions are $u_{i+1} := |\nu_{k_{i-1}+2} \dots \nu_{k_i+1}| = \left|\frac{q_{k_{i-1}}\xi - p_{k_{i-1}}}{q_{k_i}\xi - p_{k_i}}\right|$ and $a_{i+1} := [u_{i+1}]$.

Remark 2. If ξ is the limit of the continued fraction (5), its complete quotients are given by the action of the following elements of $\mathrm{Gl}_2(\mathbf{Z})$ on ξ ($k \in \mathbf{N}$):

$$\begin{aligned}\nu_{k+1}&\\ = [\varepsilon_{k+1}]ST^{-a_k}&[\varepsilon_k]ST^{-a_{k-1}}[\varepsilon_{k-1}]ST^{-a_{k-2}}\\ &\dots[\varepsilon_1]ST^{-a_0}(\xi),\end{aligned}$$

where $S = x \mapsto \frac{1}{x}$, $T : x \mapsto x + 1$, and $[\varepsilon] : x \mapsto \varepsilon x$.

Intermediate Convergents

For the best approximations of type 1, the usual way to look for them is to sort them from the intermediate convergents of the expansion in regular continued fractions. More precisely, if $\left(\frac{\tilde{p}_i}{\tilde{q}_i}\right)_{i\in\mathbf{N}}$ is the sequence of best approximations of type 0 of a real number $\xi = [c_0, c_1, c_2 \dots]$, the intermediate convergents between $\frac{\tilde{p}_{i-1}}{\tilde{q}_{i-1}}$ and $\frac{\tilde{p}_i}{\tilde{q}_i}$ are written

$$\frac{\lambda\tilde{p}_i + \tilde{p}_{i-1}}{\lambda\tilde{q}_i + \tilde{q}_{i-1}}, \quad \lambda = 1, \dots, c_{i+1} - 1. \tag{9}$$

Using the notation introduced in "Sequences of approximations", these intermediate convergents (9) are the elements of the Hurwitz sequence lying between $\frac{\tilde{p}_i}{\tilde{q}_i} = \frac{p_{\tilde{c}_i}}{q_{\tilde{c}_i}}$ and $\frac{\tilde{p}_{i+1}}{\tilde{q}_{i+1}} = \frac{p_{\tilde{c}_{i+1}}}{q_{\tilde{c}_{i+1}}}$ (recall that $\tilde{c}_i = c_1 + \cdots + c_i$). Indeed we check

$$\begin{aligned}\begin{pmatrix} p_{\tilde{c}_i+\lambda} \\ \tilde{q}_{c_i+\lambda}\end{pmatrix} &= \lambda\begin{pmatrix} p_{\tilde{c}_i} \\ q_{\tilde{c}_i}\end{pmatrix} + \begin{pmatrix} p_{\tilde{c}_{i-1}} \\ q_{\tilde{c}_{i-1}}\end{pmatrix}\\ &= \lambda\begin{pmatrix} \tilde{p}_i \\ \tilde{q}_i\end{pmatrix} + \begin{pmatrix} \tilde{p}_{i-1} \\ \tilde{q}_{i-1}\end{pmatrix}.\end{aligned}$$

Then, the best approximations of type 1 (that are not of type 0) read, for $\lambda, i \in \mathbf{N}^*$:

$$\frac{\lambda\tilde{p}_i + \tilde{p}_{i-1}}{\lambda\tilde{q}_i + \tilde{q}_{i-1}}$$

with $c_{i+1} < 2\lambda \leq 2c_{i+1} - 2$ or $2\lambda = c_{i+1}$ and $[c_{i+1}, c_i, \dots, c_2, c_1] > [c_{i+1}, c_{i+2}, c_{i+3}, \dots]$; see [6, Satz 22, p. 60]. Indeed, the condition to be satisfied for a best approximation of type 1 is $\left|\xi - \frac{\lambda\tilde{p}_i + \tilde{p}_{i-1}}{\lambda\tilde{q}_i + \tilde{q}_{i-1}}\right| < \left|\xi - \frac{\tilde{p}_i}{\tilde{q}_i}\right|$, which is equivalent to $2\lambda > u_{i+1} - r_{i+1}$, where $u_{i+1} := -\frac{\tilde{q}_{i-1}\xi - \tilde{p}_{i-1}}{\tilde{q}_i\xi - \tilde{p}_i}$ and $r_{i+1} := \frac{\tilde{q}_{i-1}}{\tilde{q}_i}$. Since $c_{i+1} = [u_{i+1}]$ this gives $2\lambda \geq c_{i+1}$ with equality if and only if $u_{i+1} - [u_{i+1}] < r_{i+1}$, which is the condition stated because $u_{i+1} = [c_{i+1}, c_{i+2}, \dots]$ and $[u_{i+1}] + r_{i+1} = [c_{i+1}, c_i, \dots, c_2, c_1]$.

Compare this approach with the algorithm *BACF* presented at the end of "The zoo of continued fraction expansions".

Some Expansion in Continued Fractions, the Convergents of Which Are Best Approximations of Types 0 and 1

The algorithms described above are efficient and easy to program (with PARI, for example). We give here the expansion in continued fractions of type

0 (ordinary) and of type 1 for some particular numbers. An overline indicates a sequence that must be repeated indefinitely with the possible increment of the parameter shown in subscript. Small points indicate that the expansion continues without any apparent motif. Finally, we describe precisely the structure of the expansion in continued fractions of best approximations of type 1.

Remark 3. We have marked in red the partial fractions corresponding to convergents giving the best approximations of type 0.

$$\frac{1}{4} = [0,4] = 0 + \frac{1|}{|3} + \frac{1|}{|1}$$

$$\frac{85}{38} = [2,4,4,2] = 2 + \frac{1|}{|3} + \frac{1|}{|1} + \frac{-1|}{|4} + \frac{1|}{|1} + \frac{-1|}{|2} + \frac{1|}{|1}$$

$$\frac{92}{13} = [7,13] = 7 + \frac{1|}{|7} + \frac{1|}{|1} + \frac{-1|}{|2} + \frac{-1|}{|2} + \frac{-1|}{|2} + \frac{-1|}{|2} + \frac{-1|}{|2}$$

$$\sqrt{2} = [1,\overline{2}] = 1 + \frac{1|}{|2} + [\,\overline{\frac{1|}{|1} + \frac{1|}{|1} + \frac{-1|}{|3}}\,]$$

$$\sqrt{3} = [1,\overline{1,2}] = 2 + \frac{-1|}{|2} + [\,\overline{\frac{1|}{|1} + \frac{-1|}{|2} + \frac{1|}{|1}}\,]$$

$$\sqrt{5} = [2,\overline{4}] = 2 + \frac{1|}{|3} + [\,\overline{\frac{1|}{|1} + \frac{-1|}{|3} + \frac{1|}{|1} + \frac{-1|}{|2} + \frac{-1|}{|4}}\,]$$

$$\sqrt{6} = [2,\overline{2,4}] = 2 + \frac{1|}{|2} + [\,\overline{\frac{1|}{|2} + \frac{1|}{|1} + \frac{-1|}{|2} + \frac{-1|}{|3}}\,]$$

$$\sqrt{7} = [2,\overline{1,1,1,4}] = 3 + \frac{-1|}{|2} + [\,\overline{\frac{1|}{|1} + \frac{1|}{|2} + \frac{1|}{|1} + \frac{-1|}{|2} + \frac{-1|}{|2} + \frac{1|}{|1}}\,]$$

$$\frac{1+\sqrt{5}}{2} = [\overline{1}] = 2 + \frac{-1|}{|2} + [\,\overline{\frac{1|}{|1}}\,]$$

$$e = [2,\overline{1,2n,1}]_{n\in\mathbb{N}^*} = 3 + \frac{-1|}{|2} + \frac{1|}{|1} + \frac{-1|}{|2} + [\,\overline{\frac{1|}{|1} + \frac{1|}{|n} + \frac{1|}{|1} + \underbrace{\frac{-1|}{|2} + \cdots + \frac{-1|}{|2} + \frac{-1|}{|2}}_{n}}\,]_{n\geq 2}$$

$$\sqrt[q]{e} = [\overline{1, q-1+2qn, 1}]_{n\in\mathbb{N}} = 1 + \frac{1|}{|\left[\frac{q+1}{2}\right]} + \frac{1|}{|1} + \underbrace{\frac{-1|}{|2} + \cdots + \frac{-1|}{|2} + \frac{-1|}{|2}}_{\left[\frac{q-2}{2}\right]} + [\,\overline{\frac{1|}{|1} + \frac{1|}{|\left[\frac{q}{2}\right] + qn} + \frac{1|}{|1} + \underbrace{\frac{-1|}{|2} + \cdots + \frac{-1|}{|2} + \frac{-1|}{|2}}_{\left[\frac{q-1}{2}\right]+qn}}\,]_{n\in\mathbb{N}^*},$$

$$q \geq 3 (\geq 6)$$

$$\sqrt{e} = [\overline{1,1+4n,1}]_{n\in\mathbb{N}} = 2 + \frac{-1|}{|2} + [\,\overline{\frac{1|}{|1} + \frac{1|}{|1+2n} + \frac{1|}{|1} + \underbrace{\frac{-1|}{|2} + \cdots + \frac{-1|}{|2} + \frac{-1|}{|2}}_{0+2n}}\,]_{n\in\mathbb{N}^*}$$

$$\sqrt[3]{e} = [\overline{1,2+6n,1}]_{n\in\mathbb{N}} = 1 + \frac{1|}{|2} + \frac{1|}{|1} + [\,\overline{\frac{1|}{|1} + \frac{1|}{|1+3n} + \frac{1|}{|1} + \underbrace{\frac{-1|}{|2} + \cdots + \frac{-1|}{|2} + \frac{-1|}{|2}}_{1+3n}}\,]_{n\in\mathbb{N}^*}$$

$$\sqrt[4]{e} = [\overline{1,3+8n,1}]_{n\in\mathbb{N}} = 1 + \frac{1|}{|2} + \frac{1|}{|1} + \frac{-1|}{|2} + [\,\overline{\frac{1|}{|1} + \frac{1|}{|2+4n} + \frac{1|}{|1} + \underbrace{\frac{-1|}{|2} + \cdots + \frac{-1|}{|2} + \frac{-1|}{|2}}_{1+4n}}\,]_{n\in\mathbb{N}^*}$$

$$\sqrt[5]{e} = [\overline{1,4+10n,1}]_{n\in\mathbb{N}} = 1 + \frac{1|}{|3} + \frac{1|}{|1} + \frac{-1|}{|2} + [\,\overline{\frac{1|}{|1} + \frac{1|}{|2+5n} + \frac{1|}{|1} + \underbrace{\frac{-1|}{|2} + \cdots + \frac{-1|}{|2} + \frac{-1|}{|2}}_{2+5n}}\,]_{n\in\mathbb{N}^*}$$

$$\pi = [3,7,15,1,\dots]$$
$$= 3 + \frac{1|}{|4} + \frac{1|}{|1} + \frac{-1|}{|2} + \frac{-1|}{|2} + \frac{-1|}{|9} + \frac{1|}{|1} + \frac{-1|}{|2} + \frac{-1|}{|2} + \frac{-1|}{|2} + \frac{-1|}{|2} + \frac{-1|}{|2} + \frac{-1|}{|2} + \frac{-1|}{|2} + \frac{1|}{|146} + \frac{1|}{|1} + \frac{-1|}{|2} + \frac{-1|}{|2} + \dots$$

The first formulas, giving the expansions in regular continued fractions, are known; see [6] for example. Proving the equalities with the expansions in unitary continued fractions giving the best approximations of type 1 can be done as follows. One converts the latter expansion in the former with the help of the transformation (elimination of superfluous convergents, here $\frac{p_i}{q_i}$) taking the expansion:

$$\dots + \frac{c_i|}{|a_i} + \frac{c_{i+1}|}{|a_{i+1}} + \frac{c_{i+2}|}{|a_{i+2}} + \frac{c_{i+3}|}{|a_{i+3}} + \dots$$

to the transformed one:

$$\dots + \frac{a_{i+1}c_i|}{|a_i a_{i+1} + c_{i+1}} + \frac{-c_{i+1}c_{i+2}|}{|a_{i+1}a_{i+2} + c_{i+2}} + \frac{a_{i+1}c_{i+3}|}{|a_{i+3}} + \dots.$$

Furthermore, when the first approximation does not coincide with the integer part of the common limit number, we shorten the regular continued fraction from its initial term with the transformation:

$$a_0 + \frac{1|}{|1} + \frac{c_2|}{|a_2} + \dots = (a_0 + 1) + \frac{-c_2|}{|a_2 + c_2} + \dots.$$

We then observe that the expansions deduced with these transformations coincide.

Combined with the results on intermediate convergents recalled in "Intermediate convergents", this method allows us to deduce the expansion in continued fractions of best approximations of type 1 from the expansion in regular continued fractions. More precisely, if $\xi = [c_0, c_1, c_2, \dots]$ is the sequence of partial quotients of the expansion in regular continued fractions, we first set

$$c_0' := \begin{cases} 1 & \text{if } \xi > [\xi] + \frac{1}{2} \\ 0 & \text{if } \xi < [\xi] + \frac{1}{2} \end{cases} \quad \text{and}$$

$$c_i' := \begin{cases} c_{i+1} & \text{if } \xi > [\xi] + \frac{1}{2} \\ c_i & \text{if } \xi < [\xi] + \frac{1}{2} \end{cases} \quad \text{for } i \geq 1,$$

then $\lambda_0 = 0$ and for $i \geq 1$

$$\lambda_i := \begin{cases} \frac{c_i'}{2} & \text{if } c_i' \text{ is even and } [c_i', c_{i-1}', \dots, c_2, c_1] > [c_i', c_{i+1}', c_{i+2}', \dots], \\ \left[\frac{c_i'}{2}\right] + 1 & \text{otherwise} \end{cases}$$

and finally

$$a_0 := \begin{cases} [\xi]+1 & \text{if } \xi > [\xi]+\frac{1}{2} \\ [\xi] & \text{if } \xi < [\xi]+\frac{1}{2} \end{cases} \quad \text{and}$$

$$\underline{\varepsilon_i|}\overline{a_i} := \begin{cases} \underline{-1|}\overline{\lambda_i+1} & \text{if } c'_{i-1} > \lambda_{i-1} \\ \underline{1|}\overline{\lambda_i} & \text{if } c'_{i-1} = \lambda_{i-1} \end{cases}, \quad i \geq 1.$$

Then the expansion in unitary continued fractions giving the best approximations of type 1 reads:
(10)

$$\xi = a_0 + [\overline{\underbrace{\underline{\varepsilon_i|}\overline{a_i} + \underline{1|}\overline{1} + \underline{-1|}\overline{2} + \cdots + \underline{-1|}\overline{2}}_{c'_i-\lambda_i+1 \text{ term(s)}}}]_{i\geq 1}.$$

We know exactly which elements of the Hurwitz sequence are the best approximations of type 1 of ξ. With the notations of "Intermediate convergents" and "Sequences of Approximations", those are a_0 and the fractions $\frac{p_{c'_0+\cdots+c'_{i-1}+\lambda}}{q_{c'_0+\cdots+c'_{i-1}+\lambda}}$ with $\lambda = \lambda_i, \ldots, c'_i$ and $i \geq 1$. Therefore it suffices to eliminate the superfluous convergents between $\frac{p_{c'_0+\cdots+c'_{i-1}}}{q_{c'_0+\cdots+c'_{i-1}}}$ and $\frac{p_{c'_0+\cdots+c'_{i-1}+\lambda_i}}{q_{c'_0+\cdots+c'_{i-1}+\lambda_i}}$. One checks in the example presented above that the expansions in continued fractions of best approximations of type 1 do have the structure described in formula (10). The delicate point consists of checking the condition in the definition of λ_i when c'_i is even; this is what the algorithm *BACF* described at the end of "The zoo of continued fraction expansions" does automatically.

References

1. J. Goldman, Hurwitz sequences, the Farey process and general continued fractions, *Adv. in Math.*, 72/2, 1988, 239–260.
2. G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers*, 5th edition, Oxford Univ. Press, 1979.
3. A. Hurwitz, Über eine besondere Art der Kettenbruch-Entwicklung reeller Grössen, *Acta Math.*, 12, 1889, 367–405; and *Oeuvres*, tome II, pp. 84–128.
4. A. Hurwitz, Über die angenäherte Darstellung der Zahlen durch rationale Brüche, *Math. Ann.*, 14, 1894, 417–436; and *Oeuvres*, tome II, pp. 137–156.
5. T. E. McKinney, Concerning a certain type of continued fractions depending on a variable parameter, *Amer. J. Math.*, 29/3, 1907, 213–278.
6. O. Perron, *Die Lehre von den Kettenbrüchen*, 2nd edition, Chelsea Publ. Company, New York, 1930.
7. I. Richards, Continued fractions without tears, *Math. Magazine*, 54/4, 1981, 163–171.

Remembering Jerry Marsden (1942–2010)

Tudor Ratiu and Alan Weinstein, Coordinating Editors

Jerrold Eldon Marsden, known to all his friends and colleagues as Jerry, was born in Ocean Falls, British Columbia, on August 17, 1942. He passed away at home in Pasadena, California, on September 21, 2010.

Looking at the long and wide-ranging list (numbering 367 in MathSciNet as of January 13, 2011) of published books and articles by Jerrold E. (or J. E.) Marsden, someone who did not know Jerry could be forgiven for thinking that "Marsden" was the pseudonym of a collaborative group consisting of several pure and applied mathematicians. (This may have already been written about others, such as Halmos or Lang, but it applies to Jerry as much as to anyone.) In fact, as the testimonials below illustrate in many ways, Jerry, our dear friend and colleague, was a distinctive individual as well as a great scientist.

Photo by George Bergman.

Jerry Marsden, Berkeley, 1997.

Jerry's first publication, written when he was an undergraduate at the University of Toronto and appearing in the *Canadian Mathematical Bulletin* in 1965 [22], was a note concerning products of involutions in Desarguesian projective planes. It was reviewed in *Mathematical Reviews* by none other than H. S. M. Coxeter. This was followed by two papers in 1966 with Mary Beattie and Richard Sharpe [4], [5], one on finite projective planes and one on a categorical approach to separation axioms in topology. Already in this earliest work, one can see the interest in symmetry which guided Jerry's view of mathematics and mechanics.

Jerry began his graduate work at Princeton in 1965, where his interests in analysis and mechanics were stimulated by contact with Ralph Abraham, Gustave Choquet, and Arthur Wightman (among others). Another result was the beginning of Jerry's long career in book writing and editing. He assisted Abraham in the writing of *Foundations of Mechanics* [1], Choquet with the three-volume *Lectures on Analysis* [15], and Wightman with his Princeton *Lectures on Statistical Mechanics* [38]. In 1968 he completed his Ph.D. thesis under Wightman's direction on *Hamiltonian One Parameter Groups and Generalized Hamiltonian Mechanics* and came to Berkeley as a Lecturer in Mathematics. He was appointed as an assistant professor in 1969 and remained at Berkeley until 1995. From 1988 until he left, he was also a professor of electrical engineering and computer science.

Jerry's thesis was published as two back-to-back articles in the *Archive of Rational Mechanics and Analysis* [23], [24]. This work on flows "generated" by nonsmooth Hamiltonians was motivated in part by problems of Hamiltonian dynamics in infinite dimensions, the subject of much of Jerry's work throughout his life. Rereading this work today, one also sees ideas which have reappeared recently in the theory of continuous Hamiltonian dynamics (see [41]).

The next decade was marked by the choice of many of the subjects to which Jerry would devote the rest of his life. The first was his major work with David Ebin [18], [19], applying infinite-dimensional

Tudor Ratiu is chaired professor of geometric analysis in the Department of Mathematics at the École Polytechnique Fédérale de Lausanne and director of the Bernoulli Center. His email address is `tudor.ratiu@epfl.ch`.

Alan Weinstein is professor of the Graduate School in the Department of Mathematics at the University of California, Berkeley. His email address is `alanw@math.berkeley.edu`.

DOI: http://dx.doi.org/10.1090/noti852

geometric analysis to prove uniqueness and short-time existence of solutions to the Euler equations of motion for ideal, homogeneous, incompressible fluids. This paper remains to this day the model of global analysis techniques in the study of fluid motion; its methods have been extended to incompressible inhomogeneous Euler equations, to the averaged Euler equations, and to the Camassa-Holm equation in arbitrary dimensions. Around the same time as the work on fluids, Jerry started a ten-year-long collaboration with Arthur Fischer (and later others) on the Hamiltonian structure of Einstein's evolution equations. For this work he received two prizes in the Relativity Essay Contest (with Fischer) in 1973 (1st prize) and in 1976 (2nd prize). The second, significantly enlarged to the extent of being encyclopedic, edition of *Foundations of Mechanics* appeared in 1978 [2]; it set the tone for what later became a major field of research: geometric mechanics. The book *Hopf Bifurcation and Its Applications* [30] started Jerry's lifelong work in dynamical systems and bifurcation theory, to which he later added considerations of symmetry. Toward the end of this decade, Jerry also became interested in elasticity, a subject on which he would work in the years to come in collaboration with Tom Hughes, Juan Simo, Michael Ortiz, and other engineers. His 1983 book with Hughes [28] was the first to link deep differential geometric ideas to nonlinear elasticity and is still today the source and inspiration of much work in this area.

In the 1980s, Jerry added many items to his long list of research subjects. Work with Weinstein on the Hamiltonian formulation of the Maxwell-Vlasov equations of plasma dynamics led to a long collaboration with Holm, Ratiu, Weinstein, and others in which methods of Poisson geometry were used to study the stability of motions in all sorts of continuum systems by the so-called "energy-Casimir method", due originally to V. I. Arnol'd. Other topics included classical field theory, geometric phases, and control theory (with Bloch and others).

Towards the middle of the 1980s, Jerry became more and more interested in structure-preserving algorithms and engineering problems. As his work on control theory began to move to the center of his attention, his research shifted gradually toward engineering mathematics. One result of this change of focus was his move to the Control and Dynamical Systems Program at Caltech in 1995. During the last fifteen years of his life, Jerry added yet more topics to his list of interests, including nonholonomic mechanics, structure-preserving algorithms, variational calculus for mechanical systems (both continuous and discrete), mission design for spacecraft, and Dirac structures. At the same time, he worked on foundational problems in mechanics, all related to reduction, from all possible points of view: Hamiltonian, Lagrangian, Dirac. Stabilization problems and control continued to be a major area of research, as well as dynamical systems, geometric mechanics, global analysis, elasticity, and fluid mechanics. Jerry never dropped a subject, he only added to his list of interests. As a result, he saw bridges between very different areas. For example, he would apply ideas from relativity to elasticity, thereby arriving at remarkable foundational results. Or he would use geometric mechanics ideas in numerical algorithms. Control theory and dynamical systems techniques would find their way into spacecraft mission design, asteroid orbit analysis, or molecular motion. Discrete differential geometry or locomotion problems would be treated using geometric mechanics and Dirac structures.

For his pioneering work, Jerry won several major prizes, such as the Norbert Wiener Prize, the John von Neumann Prize, the Humboldt Prize, the Max Planck Research Award, and the Thomas Caughey Award. He was a member of both the Canadian and the London Royal Societies and the American Academy of Arts and Sciences, and he held an honorary degree from the University of Surrey. Although Jerry certainly appreciated this recognition, his life was enriched as much by the personal interaction which he had with so many colleagues and students, to whose careers he made extraordinary contributions. The pieces below show how others' lives were enriched by his.

First lecture.

A list of Jerry Marsden's coauthors may be found at `http://ams.rice.edu/mathscinet/search/author.html?mrauthid=120260` and a list of his Ph.D. students at `http://genealogy.math.ndsu.nodak.edu/id.php?id=28380`.

Anthony M. Bloch

In 1986, when I was a postdoc at the University of Michigan, I was fortunate enough to meet Jerry at a conference in Boulder. Jerry, who was in the audience, came up after my talk to introduce himself. I was surprised and flattered to learn that

Anthony M. Bloch is Alexander Ziwet Collegiate professor of mathematics at the University of Michigan. His email address is abloch@umich.edu.

he had read a couple of my papers, one on infinite-dimensional Hamiltonian systems and another on coupled flexible and rigid body motion. This was the beginning of a wonderful collaboration and friendship which has enriched both my academic and personal life over the past twenty-five years.

A couple of years later, Jerry invited me to visit him and Phil Holmes at Cornell University. This turned out to be a great experience for me both professionally and personally, and I continued to see much of Jerry after that. We met at Berkeley and Ohio State and later at Caltech and Michigan, as well as at various conferences around the world. I first met Jerry's wife, Barbara, at Cornell, and I spent many happy hours in subsequent years with them both. I also met Tudor Ratiu, and this in turn led to many wonderful collaborations with both Jerry and Tudor.

High school science fair.

Jerry had a panoramic view of mathematics, and one could talk to him about almost any aspect of mathematics, physics, and mathematical engineering. Indeed, he introduced me to a great cross-section of the mathematical community involved in mechanics and geometry. Jerry had a wonderful way of interacting with people both mathematically and personally.

Jerry's generosity was both personal and mathematical, ranging from hosting me at his house when I visited to helping my graduate students. One thing I remember in particular in Ithaca was Jerry offering to videotape my then three-year-old son; we have some wonderful video from that time, which also features Jerry, Tudor, and Krishna (P. S. Krishnaprasad).

Despite doing so many things, Jerry took great care in everything, and this included meticulous care in writing. In all our collaborations he held me and anyone else involved to the highest standards. This was true in particular with the book [7], which we wrote on nonholonomic systems. Writing a book involves a lot of things, and Jerry was a marvel at all of them. We are currently doing a second edition, and I will very much miss his input.

Jerry's view of mathematics influenced a great deal of research in geometric control theory. This includes the work of Grizzle and Marcus on symmetry and reduction for nonlinear control systems, and the work of Roger Brockett, Peter Crouch, and Arjan van der Schaft on Lagrangian and Hamiltonian control systems (see e.g., [13]). The work of Grizzle and Marcus in turn influenced the work of Jerry and his student Gloria Alvarez Sanchez on Poisson control systems. This work, together with earlier work of Krishnaprasad, inspired our joint work on controlled Lagrangian systems (see e.g., [11]). This work used the geometry of mechanical systems, and specifically the mechanical connection, to construct a controlled system that remained Lagrangian. Its stability could then be analyzed by the so-called energy-Casimir or energy momentum method which Jerry did so much to pioneer. A related project which I very much enjoyed was our work with P. S. Krishnaprasad and Tudor Ratiu on dissipation-induced instabilities (see [10]). We showed there that if the energy momentum method failed, the smallest amount of dissipation would give instability. This work used a beautiful combination of classical mechanics and modern geometry, which was typical of Jerry.

One of the last projects I worked on with Jerry (together with Arieh Iserles, Tudor Ratiu, and Vasile Brînzănescu) was the integrability of a new class of flows on symmetric matrices (see [8]). I remember well the moment when we were able to prove the involution of the integrals. Jerry's thrill at this moment matched mine, and I shall never forget it.

Jerry and I were also working with Dmitry Zenkov and Jared Maruskin on a paper on transpositional relations in mechanics; the three of us are in the process of completing this, sadly, without Jerry's participation.

It is impossible to say how much I will miss Jerry's presence in my life.

Hernan Cendra

Jerrold Eldon Marsden's papers, conferences, and books will remain among the most influential applied mathematical (therefore, mathematical) works written in the second half of the twentieth century and part of the twenty-first century. Their combination of mathematics with models of reality from an astonishing variety of fields is unique. Just one example of Jerry's fundamental works (in collaboration with Alan Weinstein) is Marsden-Weinstein reduction theory. Historically rooted in mechanics and inspired by pioneering works of Smale and of Arnold, it is both a powerful tool and a source of inspiration to discover the role of symmetry in an increasing number of

Hernan Cendra is professor of mathematics at the Universidad Nacional del Sur. His email address is `hcendra@uns.edu.ar`.

fields, driven by new questions arising in models of reality or of a pure mathematical nature. The latter include highly nontrivial and very interesting generalizations of the theory, like its extension to singular cases and the development of the Ortega-Ratiu optimal momentum mapping.

Many people working in engineering, physics, celestial mechanics, robotics, numerical analysis, and other fields have realized that reading some of Marsden's papers or books opens a new window for them. The unmistakable Marsden style can be easily appreciated in those works, every one of them connected to some important idea, motivating, not artificially complicated, and making evident the relevance of some fundamental mathematics coming from differential geometry, Poisson geometry, symplectic geometry, Riemannian geometry, or Lie groups. I know an increasing number of mathematicians, especially geometers, who have discovered that some questions and ideas from their own research are deeply rooted in such long-standing sciences as mechanics or electromagnetism, a connection made crystal clear in the geometrically inspired work of Jerry and some of his students.

One of Jerry's well-known books, *Foundations of Mechanics* in collaboration with Ralph Abraham, remains one of the most important books on mechanics written in the twentieth century. Jerry's undergraduate books are also written in the unmistakable Marsden's style and have played an important role in the teaching of calculus to many generations of students in several parts of the world.

It is impossible to imagine Jerry's way of doing mathematics, with the participation of so many students and collaborators from all around the world and coming from such a diversity of fields and cultures, without thinking about the qualities which made this possible: his ability to create lively collaboration among people, human sensibility, understanding, clarity, and wide and deep mathematical knowledge. He was always kind to everybody and had time for coffee and a talk about different topics.

In the fall of 1984 I was in Berkeley as a visiting scholar, working on integration in finite terms. In a course taught by Jerry, I asked him about the inverse problem in the calculus of variations. He mentioned a well-known book about this topic, and then he suggested that I take a look at a paper by Seliger and Whitham on fluid mechanics, as well as at his paper with Weinstein on coadjoint orbits and Clebsch variables. I hardly knew at that time what a coadjoint orbit was, but I somehow perceived that that book and those articles were of an entirely different nature! This is how I started a long collaboration with Jerry, which was a great opportunity for me. Once a year for fifteen years, through 2010, I visited Caltech, generously invited by Jerry. Tudor was also there, and we worked for a month. Those were great days. While we were writing our long article on reduction by stages [14], I imagine that we all had in mind some kind of solvability of the equations of mechanics, in the spirit of differential Galois theory, but we never really talked about it. As I start studying this kind of question now, the memory of Jerry saying, "I won't do that unless I have a good example!" comes to my mind.

I also remember with pleasure the many occasions when he invited me and Tudor and colleagues for a party at his home in Pasadena, with his kind and joyful wife, Barbara.

With the highest respect and appreciation, I would like to say that it seems to me that for the benefit of future generations, the momentum of his mathematical life and creations is conserved, that is, alive.

Alexandre Chorin

In the late 1970s I had the pleasure of writing a book with Jerry, *A Mathematical Introduction to Fluid Mechanics* [16], which became quite successful. The book is different in style and approach from the way either one of us usually wrote: it is a genuine compromise between different styles and points of view, and it is the effort to bridge the differences that made the book accessible to many readers. I greatly admired Jerry's thoroughness and hard work, the depth of his insights, and his willingness to see other points of view and to explain his own as many times as needed. The book was meant to be quite introductory, but to my surprise it is also widely cited by researchers.

Our other collaboration was a paper on product formulas (also with Tom Hughes and Marge McCracken), which included a discussion of a formula Jerry originally named after me but that I had not thought of, certainly not in the general form Jerry put it into. This is also quite widely cited, and I have felt that Jerry was very generous in the assignment of credit.

In the course of these collaborations, I came to admire Jerry not only as an exceptional mathematician but also as a person. He was consistently open-minded, generous, patient, and fair. I was very sorry when he left Berkeley. It was a great loss for us, for the obvious academic reasons, but also because his departure left our department poorer in human terms.

Alexandre Chorin is professor of mathematics at the University of California, Berkeley. His email address is chorin@math.berkeley.edu.

Michael Dellnitz

Being in Toronto while writing these words makes it particularly hard for me. Here, as well as in nearby Waterloo, I spent many days together with Jerry when he was director of the Fields Institute. I also remember the Workshop on Geometry, Dynamics and Mechanics in honor of his sixtieth birthday at the Fields Institute as if it were yesterday. It is terrible that we cannot celebrate Jerry's seventieth birthday in his presence.

The first time I met Jerry was during his visit to Hamburg in 1989 as a Fellow of the Alexander von Humboldt Foundation. I still remember being very excited that I had been given the opportunity to meet this outstanding mathematician. All the more, I appreciated his patience and the attention he paid by listening to the report on my Ph.D. thesis. In fact, based on what he heard, this was the first time for me that Jerry suggested investigating an important mathematical problem: the classification of Hamiltonian Hopf bifurcations in the presence of symmetries. A couple of years later, this work (performed together with Jerry and Ian Melbourne) eventually led to the equivariant version of the celebrated Darboux Theorem. It took me a while to understand that Jerry was aware of this potential right from the beginning.

Jerry (at right) with Barbara Marsden and Michael Dellnitz.

Those of us who have experienced Jerry in discussions know how his brilliant ideas and suggestions came about. Typically they started with the statement, "That is (very) interesting," followed quite rapidly by a suggestion like, "I believe that it is worthwhile to consider." I don't know any other mathematician who had so many great ideas and such an unerring feeling for important developments in mathematics. Many colleagues, postdocs, and Ph.D. students have profited from this expertise. Thus, without Jerry, we would certainly not have made such significant progress in mathematics over the last decades.

Allow me to close with a paragraph of even more personal comments. Over the last twenty years, for me Jerry has not just been an admirable mathematician; both he and his wonderful wife, Barbara, also became close friends of mine. I will never forget the exciting rounds of golf that we played in Pasadena, the boat trip to the Channel Islands, or the very enjoyable visits to their house. Jerry, I miss you as an outstanding mathematician, but not less as an outstanding friend.

Michael Delinitz is professor of mathematics at Universität Paderborn. His email address is `dellnitz@math.uni-paderborn.de`.

Mathieu Desbrun

As a young researcher in computer science and computational geometry, I was not familiar with Jerry's books: my lack of training in mathematics made them seem unattainable and/or nearly irrelevant to my interests. My first encounter with Jerry irreversibly challenged these preconceived notions. Imagine my surprise when "the" Jerrold E. Marsden, accomplished scientist and prolific author, came to see me after one of my talks, with his warmth and caring nature evident in his signature smile. He proceeded to convey to me that my approach bore resemblance to exterior calculus and geometric mechanics. When confronted by my blank stare, he patiently spent the next five minutes recommending books with undecipherable titles and convincing me that under the "mathematical mumbo-jumbo", I may find key concepts at the core of my research goals. It took me a while, but enthused by his encouragements, I soldiered through the books and found what Jerry promised, and more.

Since then, Jerry has had such an immense influence on both my students and me that it would be difficult to describe without sounding hyperbolic. As a collaborator, he had an uncanny ability to see beyond mathematical minutiae, yet he could extract the mathematical essence of our confusing questions in a matter of seconds. He could explain deep, abstract concepts with a single figure or a simple example: his mastery of science bordered on artistry, and we would relish each of his geometric insights, as well as his detailed explanations of the roots of this principle or that equation. Consequently, a 30-minute meeting with Jerry usually ended with renewed enthusiasm and a sense that his elucidative comments saved us months of desperate pondering. It also had the unexpected effect of making us fearlessly build intellectual bridges across fields: geometry being at the roots of so many topics, Jerry's guidance would often lead us down paths we would not have otherwise dared approach.

Jerry was a mentor, a role model, a source of inspiration, and a great supporter, all rolled up into one singular character. Having such a colleague

Mathieu Desbrun is professor of computing and mathematical sciences at the California Institute of Technology. His email address is `mathieu@caltech.edu`.

was precious and wonderfully humbling. I think I speak on behalf of many people when I say that Caltech will be forever indebted to Jerry. I know I am.

Marty Golubitsky and Barbara Keyfitz

One benefit of academic life is the chance to meet, interact with, learn from, and become friends with a large number of fascinating and gifted people. Jerry Marsden was extraordinary even in this rather unusual group — he worked with and mentored an uncommon number of people and at all levels. We had the chance to see Jerry in action in a number of ways, and we shall always be grateful to have had this opportunity.

Marty's introduction to Jerry was, like that of many, through one of his books (in this case the one published by Ralph Abraham on Hamiltonian mechanics but based on course notes taken by Jerry). Barbara and Jerry had known each other even longer, as students at the University of Toronto, where Jerry, even as an undergraduate, was a wonderful mentor and study-mate. Our friendship continued over the next decade through repeated contacts at meetings. Jerry was delighted that his two friends—Barbara and Marty—decided to marry each other.

Our first substantive introduction to the Marsden sphere occurred in spring 1982. Jerry arranged for us to spend six months at Berkeley. In his typical way, Jerry not only did the inviting but also found us a house to rent (we were coming with two young children: Elizabeth, aged three, and Alex, aged six months). Our visit to Berkeley was, in addition to the opportunity to get to know and to work with Jerry, a chance over time to meet many in his Berkeley orbit.

What impressed us then and what continued to impress us throughout Jerry's career was the unusual number of people who worked with Jerry and the unusual number of different projects (research, book writing, and eventually editorial) which he managed to carry out. To work like this, one had to be a clear thinker—it also helped to be a clear expositor. Marty once asked Jerry how he was able to get so much done. Jerry's response: if you are having trouble sleeping, get up and work until you are ready to go back to sleep. At one meeting, the organizers decided to have a prize for best lecture based on a participant poll. Jerry, in his low-key and clear way, won hands down.

Marty Golubitsky is Distinguished Professor of Natural and Mathematics Sciences and director of the Mathematical Biosciences Institute at The Ohio State University. His email address is mg@mbi.osu.edu.

Barbara Keyfitz is Dr. Charles Saltzer Professor of Mathematics at The Ohio State University. Her email address is bkeyfitz@math.ohio-state.edu.

Lecturing to his dog Poco.

There were other experiences that have made wonderful memories. Marty spent a summer month in 1988 together with Jerry and Alison, then 12, traveling around China (arranged by Li Kaitai) attending meetings, giving lectures, and sightseeing—from Beijing to Xian to Chengdu, by boat from Chongching to Wuhan through the now nonexistent Three Gorges, and to Shanghai. Jerry chronicled the trip with then-new technology and created eight hours of videotape. It was a delightful trip with a few memorable moments. One occurred when we were separated on our trip from Xian to Chengdu; Marty was on an early flight and Jerry and Alison on a flight an hour or so later. Marty arrived in Chengdu and was taken to the university guesthouse, but Jerry and Alison did not appear that evening. The next morning, Jerry related that their plane had some mechanical difficulties after takeoff. Jerry had been approached by the stewardess, who asked, "How do I explain to the passengers in English that the door is improperly closed and that we have to return to Xian without getting them upset?"

As is well known, in the early 1990s, Jerry helped create a new mathematics institute in Canada, the Fields Institute, and became its first director. At its inception, the institute was located in Waterloo. Jerry arranged for us both to visit in spring 1993. It was an exciting time; there were many visitors and lots of new contacts. At Jerry's invitation, Barbara was able to bring a postdoc from Houston and to embark on a new project. Both of us realized during this visit the wonderful potential that institutes have. Jerry's incredible energy, vision, political acumen, and reputation deserve much credit for making the Fields Institute happen.

Two other endeavors continue this theme. The first is Jerry's editorial work with Springer on two text series and then on the *Journal of Nonlinear Dynamics*. The second is Jerry's work with Ivar Ekeland as scientific co-chair of ICIAM 2011, which took place in Jerry's hometown of Vancouver in July 2011. We both served on the steering committee for this meeting. Despite his illness, Jerry was a part of every phone call of the committee. He did his share of appointing committees and balancing speaker lists. The meeting included a symposium arranged to honor Jerry's many research contributions and to recognize his enormous influence on a generation of scholars. Ivar spoke truly when he said, "Jerry was the heart and soul of the meeting."

We are proud to be part of some of Jerry's legacies—Fields and ICIAM 2011. We will miss him.

Mark Gotay

Calgary, in the fall of 1979, was an exciting venue mathematically. There were symplecticians (J. Śniatycki, W. Tulczyjew, and myself) and several people in related fields (such as dynamical systems and relativity) on the faculty then. But what made that semester *really* special was that Jerry Marsden was in residence on a Killam Fellowship.

Jerry was coming off a *decennium mirabile* of work in mathematical physics; the second edition of *Foundations of Mechanics* surveys the state of the art in 1978. Notable interests of Jerry's included, among many others, the dynamical structure of general relativity, the Dirac theory of constraints, geometric aspects of classical field theory, and momentum maps. It was the right place and the right time for us all, and we spent an intense autumn exploring these topics. This was the beginning for me of a life-long collaboration with Jerry on the mathematical aspects of classical field theory.

I sketch the state of the main thrust of our investigations as of 2010: for a given Lagrangian field theory, we have shown how to construct a *multisymplectic* or "covariant Hamiltonian" formulation of it, which encodes, in a clear and concise way, the interrelations between the dynamics, the initial value constraints, and the gauge ambiguity of such a theory. In a nutshell, this amounts to writing the evolution equations for the dynamic fields ψ and their conjugate momenta ρ in the "adjoint form"

$$\frac{d}{d\lambda}\begin{pmatrix}\psi\\ \rho\end{pmatrix} = \mathbb{J}\cdot\sum_i \left[D\Phi^i(\psi(\lambda),\rho(\lambda))\right]^* \alpha_i(\lambda),$$

where λ is a slicing parameter ("time"), the adjoint is taken relative to an L^2-inner product on the symplectic space of Cauchy data, $\mathbb{J}$ is a calibrated almost complex structure, the components of the "energy-momentum mapping" Φ comprise the totality of first-class constraints, and the "atlas fields" α are twisted combinations of generators of the gauge group and nondynamic fields. Other joint interests focused on stress-energy-momentum tensors and general covariance.

Having met Jerry towards the end of my grad student years, I was always impressed by his approach to both mathematics and mentoring. He was an inspiration: he taught us wisdom, enthusiasm, and persistence. (The proof that crashed was never a cause for disappointment; rather it provided an opportunity to learn something interesting—a new way forward.) With Jerry, one was welcomed to the "symplectic group" wholeheartedly, and he was always available to listen patiently and criticize gently. He was also well known to be imperturbable: once while working on my lana'i, he calmly completed a calculation even as floodwaters threatened to inundate the house!

It is sad that Jerry is no longer around to chart the course and take the helm, and most of all to enjoy as a friend. I miss him immeasurably.

Mark Gotay is formerly professor of mathematics at the University of Hawaii and currently assistant director of the Pacific Institute for the Mathematical Sciences at the University of British Columbia. His email address is `gotay@pims.math.ca`.

Jim Isenberg

I had been sitting in Jerry Marsden's office for several weeks before I really got a chance to speak with him. No, it was not that he was too busy to spend any time with me. I had arrived in Berkeley in late May of 1980 to begin my postdoctoral position with Jerry, and he was away for the first half of that summer. When I asked the Berkeley math department staff about an office for me, they said that there was nothing available until at least September. A few days later, when I spoke to Jerry and mentioned what had happened, he said that I should just use his office for the rest of the summer. At the time I was very surprised by his generous offer. As time went on and I got to know Jerry, I became less and less surprised by his wonderful generosity.

A couple of years earlier, I had met Jerry briefly at a meeting in Calgary. (This Calgary meeting was

Jim Isenberg is professor of mathematics at University of Oregon. His email address is `isenberg@uoregon.edu`.

effectively the birthplace of the "GIMMSY" program, which uses a multisymplectic approach to study the dynamics of general relativity and other classical field theories whose gauge transformations include the spacetime diffeomorphisms.) I was a physics graduate student, working on gravitational physics, and I was interested in learning much more of the mathematics of general relativity. At the time, Jerry was among the world's leaders in mathematical relativity, having done beautiful work (with Arthur Fischer and Vincent Moncrief) on the stratified structure of the space of solutions of the Einstein constraint equations, among other important and exciting work on the dynamical nature of Einstein's equations. So when I applied for a Chaim Weizmann postdoctoral fellowship (which, back in those days, could be done with any chosen advisor at one of forty chosen U.S. institutions), I asked Jerry to be my designated advisor. I am very happy that he agreed to do it.

Besides his remarkably broad and deep mathematical knowledge, there are two things that made Jerry a great advisor and collaborator. The first is the completely free, honest, unpretentious, and noncompetitive nature of mathematical discussions with him. We spent countless hours talking about a wide range of mathematical projects, with a very wide range of other colleagues, and during these sessions it was okay (and encouraged!) to ask any question, seek any clarification, and raise any issue relevant to the mathematics under discussion. This may sound like a recipe for wasting a lot of time, but I can attest that it was (and is!) just the opposite. There was no worrying about "dumb questions", and there was always a delightful and productive give-and-take. This feature made Jerry a wonderful collaborator as well as advisor. The other thing that made Jerry a great advisor was his zeal and skill in helping to find good positions for his advisees. Many advisors believe that their job is done once they have written a decent letter of recommendation. For Jerry that was the beginning of a long and carefully planned campaign involving calls to strategic colleagues, advice on writing grant proposals, and long hours coaching the preparation of a collection of audience-targeted job talks.

My relationship with Jerry continued long after those wonderful postdoc years and was not free of rough spots. However, both for me and for many others I know, he was a tremendous advisor and collaborator whose wonderful influence will long be felt.

P. S. Krishnaprasad

Meeting Jerry Marsden in the pages of the first edition of *Foundations of Mechanics* (FoM) was a pivotal moment in my life. At that time I was trying to understand the geometry of rational functions under the guidance of my thesis advisor, Roger Brockett. Taking a Hamiltonian point of view and learning the Arnold theorem on cylindrical decompositions from Abraham-Marsden led me eventually to construct a foliation of the space Rat (p, q) of real rational functions of McMillan degree $(p + q)$ and Cauchy index $(p - q)$. Thus, flows on rational functions were my entry into Hamiltonian mechanics.

Jerry and I met in person at a celebration of the centennial of Case Institute of Technology in 1980. Following his wide-ranging lecture [25] on bifurcations, chaos, and control in infinite dimensions, Jerry responded over coffee to my questions on symplectic structures. He then arranged to send me a draft of his CBMS lecture notes. By then I had become a fan of the second edition of FoM. Thus began my appreciation of the enormous range of Jerry's interests and contributions. In the years immediately following our first meeting at Case, I became interested in nonlinear control theory from a geometric viewpoint. It was inevitable that I should be drawn to the gyroscope as a model problem, "an emissary from the six-dimensional symplectic world to our three-dimensional one," in the words of Manin [21]. I worked out the Poisson structure and stability theory of a rigid body controlled by spinning reaction wheels. Jerry reacted enthusiastically to this work and invited me to Berkeley. This was the beginning of a most stimulating collaboration.

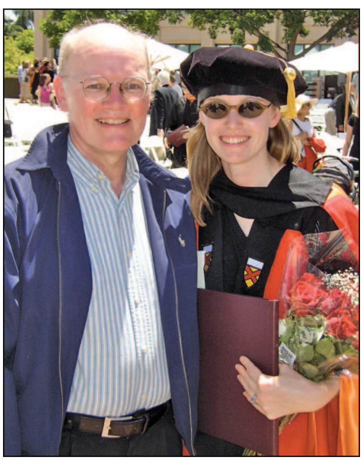
While the flowering of symplectic geometry in the twentieth century was in part inspired by the problems of celestial mechanics, already in the work of Routh [42] one recognizes a new class of many-body problems with links to modern-day preoccupations of engineers concerned with the design of mechanisms, spacecraft with articulated rigid and flexible components, robots, and such. These newer problems proved to be a fertile field for geometric thinking, including symmetry principles, reduction, conservation laws, stability, bifurcations, and control [29]. I had the special privilege of engaging Jerry's interest in these problems and collaborating with him for well over a decade. For me the high points of collaboration included visits to Berkeley, periods of serene immersion in the beauty of momentum maps,

P. S. Krishnaprasad is professor of mathematics in the Department of Electrical and Computer Engineering at the University of Maryland. His email address is krishna@umd.edu.

Poisson brackets, Casimir functions and energy methods; meeting Alan Weinstein, Tudor Ratiu and Darryl Holm; participating in workshops; and visits to the Mathematical Institutes at Berkeley, Minnesota, and Cornell, Fields Institute (then at Waterloo), and Oberwolfach. At Cornell we spoke at length about geometric phases and associated optimal control problems. The role of connections in this context was a precursor to a full-bore attack on problems of nonholonomic mechanics and newer bundlelike notions of momentum maps adapted to this setting [9]. Perhaps this and related work on dissipation and feedback control system design persuaded skeptical practitioners of the value of geometric methods in mechanics.

As a teacher, collaborator, and friend, Jerry remained close and offered a personal example through the generosity of his spirit and the obvious delight he took in exploring the natural and the engineered worlds. For me, he lives on in his work and in the perspectives he shared.

Alison Marsden



Alison Marsden's Ph.D. graduation (Stanford, June 2004).

I have always felt lucky to be the daughter of Jerry Marsden. Not only was he a brilliant mathematician but he was, as many of his colleagues also know, kind, generous, and a gentle soul.

A wonderful and caring father, he encouraged me in the most gentle of ways, always pointing out things to explore in nature and the world around us. He encouraged my interest in math and science, not by pushing me towards success, but by allowing his own love and curiosity to rub off on me. When I was a young child, he would take me up to his study and make me hand-drawn workbooks where we would do multiplication using pieces of toast and other silly pictures. He would take me in the backyard to check our rain gauge, and then I would be assigned to do a coloring project to graph the weather of the day. When we would go on walks, he would tell me things like "Look at this line of ants here. How do you think they know to stay in a line?" In my schoolwork, he encouraged me to take my time, never to rush, and to always make sure I understood things. I came to understand that this was the way he approached his work, and this became an important lesson in life. He took time to contemplate things, to wonder about them, and to think about them deeply. He developed this trait at a very early age. A 1961 article on his high school science fair project notes, "He spent over half an hour every night for five months counting cosmic rays with his homemade Geiger counter," and I imagine him as a teenager outside in the dark wondering about the universe.

Alison Marsden is assistant professor of mechanical engineering at the University of California, San Diego. Her email address is amarsden@ucsd.edu.

With Alison in Hamelin, Germany.

I have always admired my dad's ability to maintain a calm outlook on life amid the hectic pace that we all too often find ourselves caught up in. This admiration has never been greater than it is today, now that I find myself as a mother of two children and an assistant professor of mechanical engineering. While my dad clearly influenced my choice to become an engineer and enter academia, we also had many other fun times together. He taught me how to ski, showed me the best strategy in Monopoly, and told silly bedtime stories that he would make up on the spot. He loved science fiction, contests, and had a keen sense of humor. He was legendary in the family for being absent-minded and once left the car at the grocery store, walked home, and then thought it was stolen.

Because of his worldwide reputation in academia, people have always asked me, "Is it hard being the daughter of Jerry Marsden?" On the contrary, I have always felt lucky to have him as a guide, mentor, role model, loving father, and a grandfather to my children. He will be greatly missed by family, friends, students, and colleagues as the years go on.

Richard Montgomery

Jerry was my advisor at UC Berkeley from 1982 to 1986 towards the end of the heyday of his collaborations with Alan Weinstein on symplectic reduction and its applications. He was an inspired and devoted advisor.

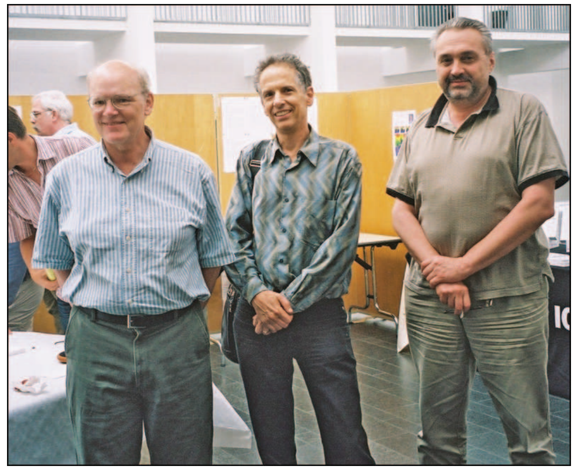
Jerry had a buoyant optimism about each new research project, coupled with a near-mystic belief in overarching theories within which all techniques or disciplines could be placed. His fervor for grand theories was strongly tempered with an insistence that any theory worth its salt must be tested on examples (and later on, applications) from the very beginning. He urged his students to build up an intimate acquaintance with examples from mechanics.

Here are some maxims he passed on. Be generous with credit. Write so that at least you yourself can read and understand what you wrote five years later. Talk to people in other fields. So, if you want to work in mathematical biology or mathematical physics, talk to practicing biologists or physicists.

He made strong, daily efforts towards mentoring and towards nurturing research collaborations at all levels, from the undergraduate, through post-doctoral, all the way to people like me, tenured and past mid-career. I learned of the following example after his death, just a week ago as I write, from one of my main collaborators, Misha Zhitomirskii of Technion (Haifa). Misha wrote me: "I never met Prof. Marsden, but exchanged an email message with him. In 1998 I asked him about the possibility to spend my sabbatical year at his university, without much hope of a positive answer. His answer was better than positive: he advised that I contact you. I do not think I would have asked you about this first sabbatical if Prof. Marsden did not advise me to do so: our collaboration before 1998 was not that productive and I was afraid I could not collaborate with you at all. Therefore Prof. Marsden changed my life very much for the better."

Michael Ortiz

I was a graduate student in engineering at UC Berkeley while Jerry was there, and his influence was tremendous. He, perhaps more than anyone else in the department, was responsible for getting engineering students interested in advanced



Jerry with Alan Weinstein and Tudor Ratiu in Lausanne, 2008.

mathematics and then fostering mathematical education among those students. His influence over students was tremendous, and he very much "schooled" them in mathematics, in the sense that the students would always carry with them a certain "Marsdenian" way of thinking throughout their subsequent careers.

As Jerry's colleague later at Caltech, I would also add that he was extremely supportive of junior faculty and that he worked hard and cared seriously for their advancement.

Tudor S. Ratiu

Jerry's disappearance marks the end of an era. The applied mathematics branch anchored in geometry, which he helped create and develop, has suddenly lost its leader; Jerry died when he was very productive and had many projects going, doing work till the last days of his life. He gave ideas to many people, he influenced other sciences—pure and applied—with his projects and leadership, he provided guidance to scores of young scientists. He leaves a vacuum that will be impossible to fill in a large and very active field of research.

For me personally this loss is even harder, because Jerry was also one of my closest friends and someone who truly changed my life. This is the story I would like to tell, since it speaks volumes about Jerry's influence on others. It relates to trust, encouragement, and friendship.

My contact with Jerry predates our personal meeting in 1975 at Berkeley. It took place in the summer of 1972 at the end of my third year as an undergraduate at the University of Timişoara in Romania during an extremely difficult time in my life when all seemed lost. I will not go into the details of the horrible political

Richard Montgomery is professor of mathematics at the University of California, Santa Cruz. His email address is `rmont@math.ucsc.edu`.

Michael Ortiz is professor of mathematics at the California Institute of Technology. His email address is `ortiz@aero.caltech.edu`.

July 2007, Rivaz, Switzerland. Left to right: Edriss Titi, Tudor Ratiu, Jerry Marsden, Mac Hyman, Darryl Holm, and Roberto Camassa.

troubles, interrogations, and detentions to which I was subjected at the time by the Communist dictatorship. It suffices to say that during that summer I was informed that my studies would be stopped and that I would never be able to have a graduate education.

During the same period, I was struggling with the first edition of *Foundations of Mechanics*, trying to learn at the same time the abstract setting and putting things in the context of the very heavy classical theoretical mechanics course I had just taken that year. Of course, I was lost and did not understand several things, so I decided to write to Jerry for clarifications. I did not really expect an answer for two reasons. First, at that time any letter leaving Romania was opened, read, and filed with the secret police as a negative point for the sender. Often the letter would not even leave the country. Second, I really had my doubts that Jerry would bother answering someone from some obscure place in Eastern Europe. But I was lucky: the letter did go through, and an answer came. In it, Jerry not only clarified the points I raised but also promised to send me his most recent work, which then started to come at regular intervals; every time he had more than one paper, a large envelope would arrive at my house containing his last preprints. So I am probably one of the few who read the preprint of the famous reduction paper he wrote with Alan Weinstein [37].

Jerry's answer and subsequent letters completely changed my life. In hindsight, it was a major bifurcation point, as he would say, an event that influenced everything that happened to me later. First, I set as my goal to really understand the *Foundations of Mechanics* book. Second, I informed the mathematics department at my university that my BA diploma work would be in the area known today as geometric mechanics. Needless to say, I had a very hard time, since no one knew what I was talking about. Third, I decided to risk all in order to come to Berkeley to study and work in this area. This may seem unrealistic, but coming to this conclusion was not so difficult. After all, I knew already that I would not be allowed to study for a Ph.D. and that I would be sent after graduation to work somewhere totally cut off from mathematics. This actually happened, and I worked for one year punching computer cards. My own (and my family's) political situation was horrible, with frequent interrogations and detentions so that, in some sense, there was no additional risk, short of prolonged jail time. In any case, I could not continue to work as a mathematician, and this pushed me over the brink. But during all this time, I continued to receive short encouraging notes from Jerry, along with various preprints that I was trying to understand. These were among the very few rays of hope during those days. By 1975 it became increasingly clear that a crisis point was coming, and, sure enough, by the end of June I was expelled from Romania. To be fair, I was given a choice: leave the country in 48 hours with no right to ever return or go to an extermination camp.

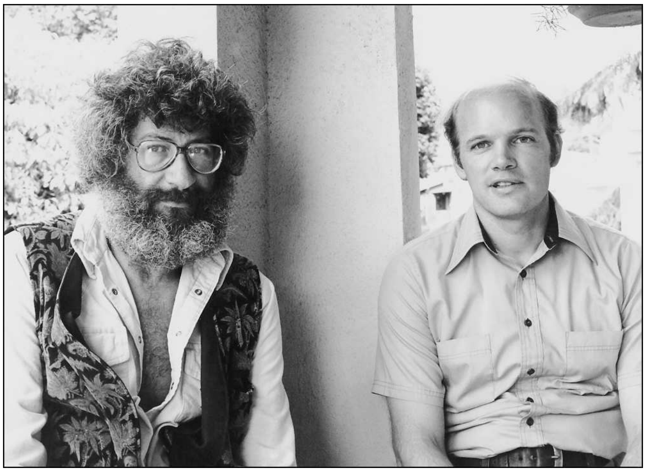
I met Jerry personally for the first time in the fall of 1975. He took me aside and told me that during my first year he expected me to finish all requirements of the Berkeley Ph.D. program, with the exception of the thesis. Till then, there was no point in discussing with me any research project, since I would not be able to work on it. I followed his advice to the letter and in June of 1976 showed up again in his office to inform him that all of this was done. The first serious discussion with him left a lasting impression on me. He began to talk to me about various broad areas of mathematics, trying to gauge my personal taste and, at the same time, presenting to me the major questions in every field in order to arouse my interest. I learned during that half hour more mathematics than I could read in months. Then he told me to study his recent book on the Hopf bifurcation and gave me some of his notes in fluid dynamics. I was supposed to come back periodically and tell him about my progress as well as ask questions on points I did not understand.

During one of the early meetings, I noticed on his desk the *Foundations of Mechanics* book, totally cut up with some pages glued on sheets with various notes scribbled on them. I asked him what this was, since I was so interested in this book; it was, ultimately, the cause of my presence in Berkeley. He informed me that he and Ralph Abraham just started work on the second, vastly enlarged, edition. I told him that I would like to discuss certain parts of the book where I thought the exposition could be improved. His reaction was totally unexpected. He picked up the first stack of papers, handed them to me, and said that these were the first sections of the book, that I should now read also this material, and that he expected me to come back with a list of serious comments. I was stunned and protested that I could not take the only existing copy and that I really was not prepared to suggest improvements. He just smiled and said that he trusted me not to lose the manuscript and that he was sure that I would do much more than just read the text. It is hard to explain what this meant to me, what his trust in my abilities as a mathematician did to my self-confidence. At the next meeting I presented a list of discussion topics concerning the text. We went point by point in the typical systematic style of Jerry, and at the end he handed me another pile of papers, this time copied, but gave me different instructions: now I was supposed to change the text, add or cut whatever I liked. Again, this took me by surprise, because I never thought to be allowed to actually intervene in the text. But I felt more relaxed, since this was, after all, a copy and he could just throw it away if he disagreed with my changes. The contrary happened at the next meeting. Again, we went point by point through the manuscript, and he showed me why some of my writing was clear and why other parts were not up to his standards. This marked the beginning of my schooling in mathematics writing, and I could not have had a more talented teacher. At the end of the meeting, he handed me a third stack of papers, again with different instructions. This time we went through the entire text together beforehand, and I noticed that some parts were sketchy, hardly containing some ideas. My job was now to actually write about these subjects and present a proposal. Again, I was taken by surprise and told him that I simply did not know enough to be able to contribute in a significant way. He just looked at me and said that he was sure I was wrong. "Let's see what you can really do" were his parting words. There could not have been a stronger encouragement possible, and I plunged into reading entire shelves of the Berkeley Mathematics Library in order to be worthy of this trust.

As time progressed, Jerry gave me more and more independence, while at the same time the frequency of the meetings increased. They became the highlight of every week. We would go through what I wrote and later through the new text he or Ralph produced and discuss everything; at the end, we would speculate on various open problems in geometric mechanics. The amount of mathematics I learned in these sessions was enormous. But he taught me much more: Why does one ask a certain question? Why is it important? Does it bear on some other field of mathematics or some other science? How does one recognize a good question? What is fashion and what is fundamental? So I learned from him geometric mechanics and all the other mathematical areas connected to it in the best possible way: Jerry gave me the opportunity to learn the subject by doing it. When the book was published in 1978, we celebrated with a tremendous sense of achievement. Soon thereafter, I got my Ph.D., with Jerry as principal and Steve Smale as secondary advisor. I followed Jerry's advice and specialized in integrable systems. Three years later we were again meeting almost daily during my summer vacation, which I always spent in Berkeley, to understand nonlinear stability in Poisson systems, a project that also involved Darryl Holm and Alan Weinstein. By that time we were already close friends, had just finished a book on infinite-dimensional manifolds [3], were working on another one on geometric mechanics [31], and were in constant contact that was never interrupted, no matter on what continent each one of us happened to be.


Jerry and Alison with Hans Duistermaat.

The day I was told that Jerry passed away was one of the saddest of my life, comparable only to those when a close family member died. The sense of loss was tremendous. The entire day I could only remember scenes of various joint events with Jerry. I suddenly realized that with the exception of my marriage, no single serious decision in my life was taken without discussing it first with Jerry. He was always present: on the phone, through email, or personally, ready to talk. He always had time for me, something I never understood how he managed to do, because I am in a constant time crisis. He always listened, gave advice and examples from his own life, trying to really understand the problem that was bothering



With Ralph Abraham.

me, be it mathematical research, academic administration, or just personal. I miss these discussions very much.

Thank you. Jerry, for the lifelong warm friendship you gave me.

Geneviève Raugel

In 1985, all I knew about Jerrold Marsden was his 1983 paper with M. Golubitsky on the Morse Lemma [27] and his 1976 book *The Hopf Bifurcation and its Applications* [30], co-authored with M. McCracken, in which I had studied center manifolds and Hopf bifurcations in infinite dimensions. At that time, it was one of the rare books dealing with bifurcation theory and the extension of finite-dimensional techniques to the infinite-dimensional setting, which was essential for applications to partial differential equations. Even today this book contains elegant results that are hard to find elsewhere.

Having worked in numerical analysis and related bifurcation questions, I intended to explore more theoretical problems in bifurcation theory. I therefore asked Jerrold Marsden, without much hope, whether I could visit him for six months in 1986. I was very fortunate that he accepted me as a postdoc.

So, one day in the middle of August 1986, I entered Jerrold Marsden's office. After having assigned me Tudor Ratiu's office, he said, "For your entertainment during your stay, I was thinking about linearization of Hamiltonian systems along given trajectories." Then followed a long explanation on the blackboard, full of terms like Lie-Poisson brackets, symplectic connections, adjoint representations …. With a trembling voice, I confessed that I did not understand a single word. Marsden did not look disappointed. He kept quite cool, gave me a pile of various lecture notes and papers, and told me with his typical smile, "You have plenty of time (two or three months) to learn all the needed stuff." Leaving his office, I rushed to the nearest bookstore and bought the book *Foundations of Mechanics*, by Abraham and Marsden [2].

Six wonderful months followed, during which Jerrold Marsden introduced me into the fascinating "symplectic" world of Marsden-Weinstein symplectic reduction, Clebsch variables, momentum maps, the energy-Casimir stability method, etc. Many years have elapsed, but I still see Jerrold Marsden, with his legendary mug of already cold coffee with milk and sugar, either in his hands or on the edge of his desk, talking about Einstein's equations or stability questions in Hamiltonian mechanics. The geometric interpretation of the Euler equations, in Ebin and Marsden's masterpiece "Groups of diffeomorphisms and the motion of an incompressible fluid", [18] was a real revelation for me.

In the pleasant working atmosphere in Berkeley, Jerrold Marsden gave me the great opportunity to meet mathematicians like Alan Weinstein, Tudor Ratiu, and Darryl Holm. This resulted in a long collaboration and friendship with Jerry (as Jerrold had now become for me) and Tudor.

Gradually, I became aware that Jerry was not only a great mathematician and a living encyclopedia but also a generous, patient, fair, and easily approachable person. And my admiration grew! Jerry taught me to be generous in giving credit to others. He also advised me to give a chance to a priori less gifted students and to accept some of them as graduate students.

Jerry had a passion for understanding, whether it was mathematics, mechanics, weather phenomena, astronomy, physics, etc. I remember Jerry explaining with enthusiasm how he measured the diameter of the moon at different hours of the night, when he was a student, in order to prove that the perceived difference in size was just an optical illusion.

I never met anyone as efficient, fast, and well organized as Jerry. He answered email messages, and more generally all requests, almost immediately. He could follow a talk, ask deep questions, and, at the same time, carefully proofread one of his manuscripts. Once, when I had complimented him on this ability, he smiled and replied, "Life is so short…" (was it a premonition?) "and we have so much to discover in mathematics." That did not prevent him from devoting his time generously to

Geneviève Raugel is research director at the CNRS and University Paris-Sud. Her email address is `genevieve.raugel@math.u-psud.fr`.

students, postdocs, coworkers, and colleagues. If you wanted to discuss something with him, he always found a slot in his tight schedule. I remember that on a three-day meeting with Jerry and Tudor at Cornell, during which we hardly found any time for eating, Jerry took us to visit a former postdoc of his, who had been hospitalized for a few days. After hard work, there was also room for recreation. I look back with emotion to the nice dinners I had with Jerry and his lively wife, Barbara, at their home in Pasadena and elsewhere. Among other recollections, I vividly remember Jerry showing us the impressive cliffs in the sunset light at La Jolla during a conference in San Diego in January 1987.

Jerry was a true friend. Even if I had not been in touch with him for months, I knew that I could always rely on him. The last time I met Jerry was in June 2006 at Lausanne. We worked with Tudor on Euler equations for thin spherical shells, and Jerry had a decisive idea. In these too short days, as usual, Jerry gave me friendly advice and enthusiastically talked to me about mathematics—this time about Lagrangian coherent structures. All these happy and bright days are engraved forever in my memory.

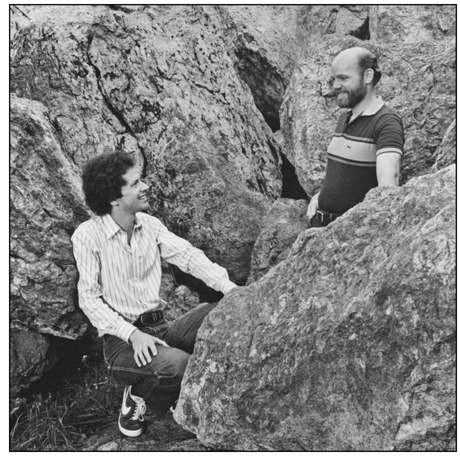
Jürgen Scheurle

I would like to express here my fond remembrance of Jerry as an outstanding scholar and a very nice human being. I am one of his many friends and colleagues all over the world who were deeply shocked by his early death.

I first got to know Jerry personally during my visit to Berkeley in 1982. This was the beginning of a close friendship and collaboration. At that time I had just finished my Habilitation thesis "Bifurcation of quasiperiodic solutions for reversible dynamical systems"; cf. [43]. We expanded this work by studying the codimension two bifurcation to tori of B. Langford, G. Iooss, and J. Guckenheimer. This led to our first joint paper [45]. The paper includes a computable criterion for the existence of stable tori carrying quasi-periodic flow. This was successfully applied to the Brusselator, a PDE model in reaction diffusion theory, as well as to various fluid flow problems.

Later, we continued to visit each other quite frequently in Berkeley and at other places in the USA, where I worked and lived for some time, and later in Germany. Our wives, Barbara and Karin, became good friends too. The children loved Jerry. He always took them seriously, even at a very young age.

Jürgen Scheurle is professor of mathematics at the Technische Universität München. His email address is scheurle@mathematik.tu-muenchen.de.



With Alan Weinstein.

Professionally, I more and more became a member of Jerry's applied mathematics and geometric mechanics school. We continued our joint work by developing a quite general theory concerning the construction and smoothness of invariant manifolds based on the deformation method, with special emphasis on optimal smoothness results [33]. In particular, we gave a new proof of the center manifold theorem, which has created a lot of interest. Together with P. Holmes, we subsequently embarked on the ambitious project of studying the exponentially small splitting of separatrices in rapidly forced dynamical systems. For instance, such systems come up in KAM theory as well as in the analysis of degenerate bifurcation problems. The usual transformation methods for showing transversal intersection of separatrices do not work here. Indeed, the failure of the usual theory is precisely the reason the proofs in H. Poincaré's famous 1890 paper on nonintegrability of the three-body problem are not complete. As a kind of prototype model, we studied the so-called rapidly forced pendulum equation containing the same essential difficulty; namely, the splitting of the separatrices is of exponentially small order with respect to the forcing period. We were successful in this project and rigorously derived exponentially small upper as well as lower bounds for the splitting distance and thus proved the transversal intersection of the separatrices in nontrivial cases [20], [44], [46]. Our method, inspired by classical work of Perron together with the complex embedding of trajectories, used a new expansion in which we showed that each term is of exponentially small order and in which we were able to control the error. In the meanwhile, quite a number of papers have been written about our methods and results, extending them in various ways, but our original technique appears to remain as a fundamental advance.

While I was holding a professor position in Hamburg, Jerry was there on an Alexander von Humboldt Research Prize in the first half of 1991. At that time he wrote one of his trend-setting books, *Lectures in Mechanics* [26]. In the course

Photo courtesy of Jürgen Scheurle.

Bavarian Alps, 1999. Left to right: Jürgen Scheurle, Jerry Marsden, Barbara Marsden.

of writing this book, he gave a series of lectures on the subject in Hamburg. Attending these lectures, I got introduced to the area of geometric mechanics. We started to work on our paper on the double spherical pendulum and Lagrangian reduction [34]. The Lagrangian reduction methods we subsequently developed [35], [32] turned out to be particularly important in nonholonomic mechanics and control theory; a number of people, including A. Bloch and P. S. Krishnaprasad, have adapted our ideas to solve some basic problems about nonholonomic constraints.

In Hamburg, Jerry and I also started a project on the subject of "pattern evocation", which is a theoretical-numerical method for detecting discrete symmetries of solutions in mechanical systems with symmetries. If one visualizes solutions naively, then one might see "chaos", but in the right rotating frame, very interesting patterns emerge. We showed how one can construct these rotating frames using ideas of connections and geometric phases [36].

Jerry's frequent presence and scholarly activities in Germany were actually very beneficial to many people. Especially, he helped several junior people from Germany to start and to establish a scientific career. He always invited a large number of junior people to the Oberwolfach conferences on Dynamical Systems and Geometric Mechanics which he regularly organized, together with changing coorganizers, starting with K. Kirchgässner. In 2000 the very prestigious German Max Planck Research Award was granted to him in order to honor him for his great contributions to the applied mathematics and mechanics community.

It has been a great pleasure to have such a close personal and scientific relationship with Jerry for so many years. I am very grateful to him indeed. All of us will miss Jerry very much, inside and outside of Germany.

Alan Weinstein

Jerry came to Berkeley as a lecturer in 1968 and was appointed as an assistant professor the following year. I joined the department in 1969, and we quickly found that we had many mutual interests, with our first joint publication [49] appearing in 1970.

Soon thereafter, Jerry and I attended Steve Smale's special topics course on the topology of the *n*-body problem. In those lectures Smale introduced the energy-momentum map (see [47], [48]) for the lift to phase space of a group action on configuration space, and he gave a unified geometric explanation of many reduction constructions in classical mechanics, including "fixing the center of mass" and Jacobi's elimination of the nodes. Combining this idea with Lie, Kostant, and Souriau's notion of momentum map for general Hamiltonian actions of groups, we arrived at our construction of reduced symplectic manifolds [37]. For each of us, according to Google Scholar, this is by a good margin our most cited paper. (There were 839 Google citations on 1/24/11, the most for me and the most among all of Jerry's papers (but not books).) This idea of reduction (also found by Meyer [39] in a slightly different setting) has had immense ramifications in our own work, as well as throughout geometric mechanics (and related areas like representation theory and algebraic geometry).

Through the 1970s Jerry continued working on general relativity and spaces of metrics, fluid dynamics, and the analysis of general nonlinear evolution equations, and he began a collaboration with Phil Holmes on bifurcation theory. At the same time, my own interests drifted toward microlocal analysis. Then, in the early 1980s, Jerry and I once again were inspired by attending a seminar together, this time the "Dynamics Seminar" organized at Lawrence Berkeley National Laboratory by Allan Kaufman, a plasma physicist. The subject was the discovery by Phil Morrison, a student of the Princeton physicist John Greene, of a Hamiltonian formulation of the Maxwell-Vlasov plasma evolution equations in terms of a noncanonical Poisson bracket on a space of charge densities and electromagnetic fields [40]. Morrison had found his Poisson bracket by trial and error and had verified by two months of calculation that it satisfied the Jacobi identity.

Jerry and I felt that there should be a geometric explanation of Morrison's bracket. After six months of work, we found that it was obtained by reduction of the canonical bracket on the cotangent bundle of the product of the group of phase space symplectomorphisms and the space of electromagnetic potentials. This six months of work reduced Morrison's two-month verification to five minutes. By itself that was not a clear improvement, but we did find along the way that Morrison's bracket (which did not quite coincide with ours) did not actually satisfy the Jacobi identity. More important, our geometric approach led to Hamiltonian structures for a wide variety of evolution equations in continuum mechanics to which we could apply a method developed by Arnold to establish stability of stationary solutions. This led to many years of collaboration with Darryl Holm, Tudor Ratiu, and others. For me, the work with Jerry on fluids and plasmas stimulated an interest in the geometry of Poisson structures themselves.

By the end of the 1980s our research interests diverged again, but each of us was now and then inspired by the work of the other. For example, Jerry's work on discrete reduction led me to an interpretation in terms of Lie algebroids and groupoids, which was then picked up by his Caltech student Melvin Leok. More significantly for me, I can say in retrospect that a large fraction of my research in the past twenty-five years has its origin in the interest in Poisson structures and their applications that arose in our joint work.

Hiroaki Yoshimura

I first met Jerry Marsden at Caltech in July 1997, when I was interested in nonholonomic Lagrangian mechanics in conjunction with *nonenergic systems*, a term coined by Birkhoff [6]. As known, electric circuits can be understood as *interconnected systems* through nonenergic multiports. Until then it was known that interconnection could be represented by the *Dirac structures* developed by Courant and Weinstein [17]. However, I could not explain this well to Jerry in the Lagrangian context at that time. Later, I had a chance to spend a sabbatical year starting in September 2002 at Caltech as a visiting faculty member, sponsored by Jerry. We started to explore Lagrangian systems in the context of Dirac structures, circuits, and nonholonomic systems. We talked enthusiastically two or three days a week and noticed that circuits are a typical degenerate Lagrangian system. Then we developed a notion of *implicit Lagrangian systems* [50] in the context of Dirac structures. These systems are triples (X, L, D) which satisfy

$$(X, \mathbf{d}E_L|_{TP}) \in D_{\Delta_Q},$$

where D_{Δ_Q} is a Dirac structure on T^*Q induced from a distribution Δ_Q on a manifold Q; L is a Lagrangian on TQ, possibly degenerate; $E_L = \langle p, v\rangle - L(q, v)$ is the generalized energy; $X : TQ \oplus T^*Q \to TT^*Q$ is a partial vector field;

Hiroaki Yoshimura is professor of applied mathematics and mechanics at Waseda University. His email address is yoshimura@waseda.jp.

Jerry with 2003–4 graduate students (Shane Ross, Anil Hirani, Melvin Leok, and Sergiy Vasktylkevych) and their families.

and P is the image of Δ_Q under the Legendre transformation. It follows that

$$p = \frac{\partial L}{\partial v}, \quad v = \dot{q} \in \Delta_Q(q), \quad \text{and} \quad \dot{p} - \frac{\partial L}{\partial q} \in \Delta_Q^{\circ}(q).$$

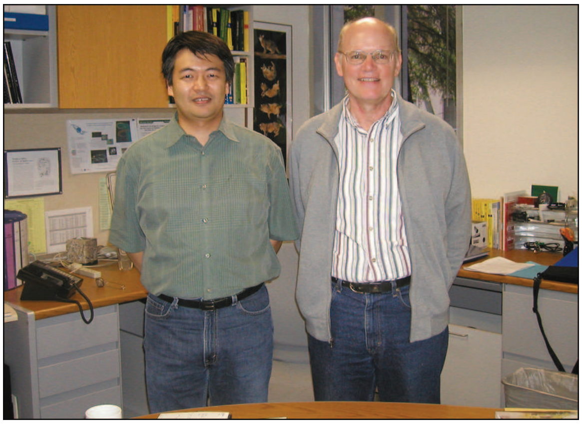
For unconstrained cases we can derive *implicit Euler-Lagrange equations*, consistent with the Hamilton-Pontryagin principle:

$$\delta \int_a^b L(q, v) + \langle p, \dot{q} - v\rangle \, dt = 0.$$

We had a chance to present our idea at the Alanfest held in Vienna, August 2003, in honor of Alan Weinstein's sixtieth birthday. Jerry gave a lecture on our idea. In Vienna I spent ten days with him, mostly from morning to night, which was one of the greatest times in my life.

During my sabbatical I learned a lot of things in mathematics and mechanics directly from Jerry. After that we continued our collaboration to establish Dirac reduction theories under the assumption that a Lie group G acts freely and properly on Q; namely, *Lie–Dirac reduction* [51] for the case $Q = G$ as well as *Dirac cotangent bundle reduction* [52] in the case where there is an associated principal bundle $Q \to Q/G$. These Lagrange-Dirac reduction theories succeed in accommodating Lagrangian, Hamiltonian, and a variational view of reduction simultaneously.

Near the end of Jerry's life, we started to develop the general theory of Dirac reduction in the context of *Dirac anchored vector bundles* with Hernan Cendra and Tudor Ratiu. We also began to consider the field theoretic analogue of Dirac structures called *multi-Dirac structures* with Joris Vankerschaver. It was March 2010 that I saw Jerry last at Caltech to discuss the project on interconnection of distinct Dirac structures and associated Lagrange-Dirac systems with his Ph.D.



With Hiraoi Yoshimura.

student, Henry Jacobs. Until his final days, we continued to discuss by email, and he never lost his enthusiasm for research. I received the news of his passing the day before I would talk on our new theory at a conference in Greece. Many things remain unfinished, which are left to us to complete.

Jerry, thank you very much.

References

[1] R. Abraham, *Foundations of Mechanics.* With the assistance of Jerrold E. Marsden. Four appendices: one by the author, two by Al Kelley, the fourth, a translation of an article by A. N. Kolmogorov. W. A. Benjamin, Inc., New York-Amsterdam, 1967.

[2] R. Abraham and J. E. Marsden, *Foundations of Mechanics*, second edition, revised and enlarged. With the assistance of Tudor Ratiu and Richard Cushman. Benjamin/Cummings Publishing Co., Inc., Advanced Book Program, Reading, Mass., 1978.

[3] R. Abraham, J. E. Marsden, and T. S. Ratiu, *Manifolds, Tensor Analysis, and Applications*, Global Analysis Pure and Applied: Series B, 2. Addison-Wesley Publishing Co., Reading, Mass., 1983. Second edition, Applied Mathematical Sciences, 75, Springer-Verlag, New York, 1988.

[4] M. Beattie, J. E. Marsden, and R. Sharpe, A universal factorization theorem in topology, *Canad. Math. Bull.* **9** (1966), 201–207.

[5] ______, Order in finite affine planes, *Canad. Math. Bull.* **9** (1966), 407–411.

[6] G. D. Birkhoff, *Dynamical Systems*, volume 9 of Colloquium Publications, Amer. Math. Soc., Providence, RI, 1927.

[7] A. M. Bloch, with the collaboration of J. Baillieul, P. Crouch and J. Marsden, *Nonholonomic Mechanics and Control*, Springer, New York, 2003.

[8] A. M. Bloch, V. Brinzanescu, A. Iserles, J. E. Marsden, and T. S. Ratiu, A class of integrable flows on the space of symmetric matrices, *Communications in Mathematical Physics* **290** (2009), 399–435.

[9] A. M. Bloch, P. S. Krishnaprasad, J. E. Marsden, and R. M. Murray, Nonholonomic mechanical systems with symmetry, *Arch. Rational Mech. Anal.* **136** (1996), 21–99.

[10] A. M. Bloch, P. S. Krishnaprasad, J. E. Marsden, and T. S. Ratiu, Dissipation induced instabilities, *Ann. Inst. H. Poincaré, Analyse Nonlineaire* **11** (1994), 37–90.

[11] A. M. Bloch, N. Leonard, and J. E. Marsden, Controlled Lagrangians and the stabilization of mechanical systems. I: The first matching theorem, *IEEE Trans. Automat. Control* **45** (2000), 2253–2270.

[12] A. M. Bloch and J. E. Marsden, Controlling homoclinic orbits, *Theoretical and Computational Fluid Dynamics* **1** (1989), 179–190.

[13] R. W. Brockett, Control theory and analytical mechanics, in *1976 Ames Research Center (NASA) Conference on Geometric Control Theory* (R. Hermann and C. Martin, eds.), Lie Groups: History, Frontiers, and Applications **VII**, Math. Sci. Press, Brookline, Massachusetts, 1977, 1–46.

[14] H. Cendra, J. E. Marsden, and T. S. Ratiu, Lagrangian reduction by stages, *Mem. Amer. Math. Soc.* **152** (2001), no. 722, x+108 pp.

[15] G. Choquet, *Lectures on Analysis* (three volumes), edited by J. Marsden, T. Lance, and S. Gelbart. W. A. Benjamin, Inc., New York-Amsterdam, 1969.

[16] A. J. Chorin and J. E. Marsden, *A Mathematical Introduction to Fluid Mechanics*, Springer-Verlag, New York-Heidelberg, 1979.

[17] T. Courant and A. Weinstein, Beyond Poisson structures, in *Actions hamiltoniennes de groupes. Troisième théoerème de Lie (Lyon, 1986)*, volume 27 of Travaux en Cours, pp. 39–49, Hermann, Paris, 1988.

[18] D. G. Ebin and J. E. Marsden, Groups of diffeomorphisms and the motion of an incompressible fluid, *Ann. of Math.* (2) **92** (1970), 102–163.

[19] ______, On the motion of incompressible fluids, *Actes Du Congres Int.* **2** (1970), 211–214.

[20] P. J. Holmes, J. E. Marsden, and J. Scheurle, Exponentially small splittings of separatrices with applications to KAM theory and degenerate bifurcations, Contemporary Mathematics, Amer. Math. Soc., Providence, RI, 81, 1988, pp. 213–244.

[21] Yu. I. Manin, *Mathematics and Physics*, Progress in Physics, Birkhäuser, Basel, 1981.

[22] J. E. Marsden, A theorem on harmonic homologies, *Canad. Math. Bull.* **8** (1965), 375–377.

[23] ______, Generalized Hamiltonian mechanics, *Arch. Rational Mech. Anal.* **28** (1968), 323–361.

[24] ______, Hamiltonian one parameter groups, *Arch. Rational Mech. Anal.* **28** (1968), 362–396.

[25] ______, Four applications of nonlinear analysis to physics and engineering, in P. J. Hilton and G. S. Young (eds.), *New Directions in Applied Mathematics*, pp. 85–108, Springer-Verlag, New York, 1982.

[26] ______, *Lectures on Mechanics*, London Mathematical Society Lecture Note Series, Vol. 174, Cambridge University Press, 1992.

[27] M. Golubitsky and J. E. Marsden, The Morse lemma in infinite dimensions via singularity theory, *SIAM J. Math. Anal.* **14** (1983), 1037–1044.

[28] J. E. Marsden and T. Hughes, *Mathematical Foundations of Elasticity*, Prentice Hall, 1983; reprinted by Dover Publications, 1994.

[29] J. E. Marsden, P. S. Krishnaprasad, and J. C. Simo (eds.), *Dynamics and Control of Multibody Systems*, Contemporary Mathematics, 97, Amer. Math. Soc., Providence, RI, 1989.

[30] J. E. Marsden and M. McCracken, *The Hopf Bifurcation and Its Applications*, with contributions by P. Chernoff, G. Childs, S. Chow, J. R. Dorroh, J. Guckenheimer, L. Howard, N. Kopell, O. Lanford, J. Mallet-Paret, G. Oster, O. Ruiz, S. Schecter, D. Schmidt, and S. Smale, Applied Mathematical Sciences, 19, Springer-Verlag, 1976.

[31] J. E. Marsden and T. S. Ratiu, *Introduction to Mechanics and Symmetry*, Texts in Applied Mathematics, 17, Springer-Verlag, New York, 1994. Second edition, 1999.

[32] J. E. Marsden, T. S. Ratiu, J. Scheurle, Reduction theory and the Lagrange-Routh equations, *J. Math. Phys.* **41** (2000), 3379–3429.

[33] J. E. Marsden and J. Scheurle, The construction and smoothness of invariant manifolds by the deformation method, *SIAM J. Math. Anal.* **18** (1987), 1261–1274.

[34] ———, Lagrangian reduction and the double spherical pendulum, *ZAMP* **44** (1993), 17–43.

[35] ———, The reduced Euler-Lagrange equations, *Fields Inst. Commun.* **1** (1993), 139–164.

[36] ———, Pattern evocation and geometric phases in mechanical systems with symmetry, *Dynamics and Stability of Systems* **10** (1995), 315–338.

[37] J. E. Marsden and A. Weinstein, Reduction of symplectic manifolds with symmetry, *Rep. Math. Phys.* **5** (1974), 121–130.

[38] J. E. Marsden and A. S. Wightman, *Lectures on Statistical Mechanics*, Parts I and II, Princeton University Press, 1967.

[39] K. R. Meyer, Symmetries and integrals in mechanics, in *Dynamical Systems*, M. Peixoto, ed., Academic Press, New York, 1973, pp. 259-273.

[40] P. J. Morrison, The Maxwell-Vlasov equations as a continuous Hamiltonian system, *Phys. Lett. A* **80** (1980), 383–386.

[41] S. Müller and Y. G. Oh, The group of Hamiltonian homeomorphisms and C^0 symplectic topology, *J. Symplectic Geom.* **5** (2007), 167–219.

[42] E. J. Routh, *Dynamics of Systems of Rigid Bodies, Elementary and Advanced Parts*, Dover Publishing, New York, 1960 (original edition, 1905).

[43] J. Scheurle, Bifurcation of quasiperiodic solutions from equilibrium points of reversible dynamical systems, *Arch. Rat. Mech. Anal.* **97** (1987), 103–139.

[44] ———, Chaos in a rapidly forced pendulum equation, *Dynamics and Control of Multibody Systems*, (Brunswick, ME, 1988), Contemporary Mathematics, 97, Amer. Math. Soc., Providence, RI, 1989, pp. 411–419.

[45] J. Scheurle and J. E. Marsden, Bifurcation to quasiperiodic tori in the interaction of steady state and Hopf bifurcations, *SIAM J. Math. Anal.* **15** (1984), 1055–1074.

[46] J. Scheurle, J. E. Marsden, and P. J. Holmes, Exponentially small estimates for separatrix splittings, *Proc. Conf. Beyond all Orders*, H. Segur and S. Tanveer, eds., Birkhäuser, Boston, 1991, pp. 187-195.

[47] S. Smale, Topology and mechanics. I, *Invent. Math.* **10** (1970), 305–331.

[48] ———, Topology and mechanics. II. The planar n-body problem, *Invent. Math.* **11** (1970), 45–64.

[49] A. Weinstein and J. E. Marsden, A comparison theorem for Hamiltonian vector fields, *Proc. Amer. Math. Soc.* **26** (1970), 629–631.

[50] H. Yoshimura and J. E. Marsden, Dirac structures in Lagrangian mechanics. Part I: Implicit Lagrangian systems, Part II: Variational structures, *J. Geom. Phys.* **57** (2006), 133–156; 209–250.

[51] ———, Reduction of Dirac structures and the Hamilton-Pontryagin principle, *Reports on Mathematical Physics* **60** (2007), 381–426.

[52] ———, Dirac cotangent bundle reduction, *J. Geom. Mech.* **1** (2009), 87–158.

[53] D. V. Zenkov, A. M. Bloch, and J. E. Marsden, The energy momentum method for the stability of nonholonomic systems, *Dynam. Stability of Systems* **13** (1998), 123–166.

Incomputability after Alan Turing

S. Barry Cooper

The year 2012 marks the 100th anniversary of the birth of Alan Turing. The following two articles were inspired by the work of Alan Turing. For information about the centenary celebration of Turing's life and work, visit the website `http://www.mathcomp.leeds.ac.uk/turing2012/`.

Living in a Computable World

Those of us old enough may remember being fascinated by George Gamow's popular books on mathematics and science—with the most famous being *One Two Three…Infinity*. Gamow got us to imagine living on the surface of a two-dimensional balloon with only two-dimensional experience of the surface. And then he got us to understand how we might detect its three-dimensional curved character via purely two-dimensional observations. In Figure 1 is his picture from page 103 of the 1961 edition.

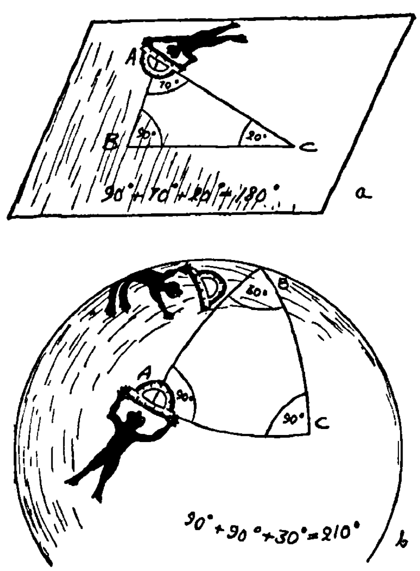


Figure 1. Two-dimensional scientists of the flat and curved "surface worlds" check the Euclidean theorem about the sum of the angles in a triangle. ***From [14].***

S. Barry Cooper is professor of mathematics at the University of Leeds and chair of the Turing Centenary Advisory Committee. His email address is `pmt6sbc@maths.leeds.ac.uk`.

Preparation of this article completed while the author was a visiting fellow at the Isaac Newton Institute for the Mathematical Sciences, in the program "Semantics and Syntax: A Legacy of Alan Turing".

DOI: http://dx.doi.org/10.1090/noti866

Algorithms, as a way of traversing our four dimensions, have been with us for literally thousands of years. They provide recipes for the control and understanding of every aspect of everyday life. Nowadays, they appear as computer programs. Algorithms, or computer programs, can be thought of as a kind of causal dimension all their own. Then the questions arise: Is there a causal dimension that is not algorithmic? Does it *matter* if there is?

Notice that Gamow's example showed that on one hand it was tricky to live in two dimensions and find evidence of a third. But it *did* matter precisely because we *could* find that evidence. Of course, if we took the mathematical model presented by the picture, the missing dimension becomes clear to us—we have an overview. But observe that while the mathematical overview gives us a better understanding of the nature of curved space, it does not tell us that the model is relevant to our world. We *still* need to look at the triangle from within the two-dimensional world to match up reality and mathematics and to be able to apply the full power of the model.

Back in the 1930s, people such as Kurt Gödel, Stephen Kleene, Alonzo Church, and Alan Turing did build mathematical models of the computable dimension of causal relations. This enabled Church and Turing to get outside this dimension and to use their models to explore the new dimension of incomputability.

A specially important part of what the twenty-four-year-old Alan Turing did was to base his investigation of the extent of the computable on a new *machine-like* model. The Turing machine was to make him famous in a way no one could have foreseen. He had the idea of using Gödel's coding trick to turn Turing machine programs into data the machine could compute from, and hence was born the *universal* Turing machine, able to take a code for *any* other given machine as input data and to compute exactly like it. The universal machine *stored* programs and so gave us an understanding of the modern stored-program computer before anyone had even built a real one.

This caused all sorts of problems. As with Gamow's example, it was easy to get the mathematical overview. The problem was to match it up with reality. And this was a problem with a practical aspect. Even a toy avatar of the abstract machine was hard to make. The engineers eventually came up with clever solutions to the problem: the EDVAC in Pennsylvania, the Manchester "Baby", Maurice Wilkes's EDSAC, and the Pilot ACE growing out of Turing's own attempts to build a computer at the National Physical Laboratory (NPL). But to this day, there are engineers who find it hard to excuse (or even understand) Turing's reputation as the "inventor" of the computer. The impact that John von Neumann's 1945 EDVAC report had on the history of computing has been better acknowledged than has the impact of Turing's work. Nevertheless, in his Hixon Symposium lecture in Pasadena in 1948 [37], von Neumann gave Turing his due.

More importantly, the computer changed every aspect of our lives and strengthened our experience of living in a computable world. Incomputability became a mathematical oddity, a playground for researchers who were not too concerned about real-world significance but liked doing hard mathematics with a distinct feel of reality. Of *course* it felt like reality. It was real numbers connected by computable relations—a bit like a well-behaved scientific world of information structured by computable causal relations.

A Short History of Incomputability

Computability has always been with us. The universe is full of it: natural laws whose computability enables us to survive in the world, animal and human behavior guided by biological and learned algorithms, computable natural constants such as π and *e*. The algorithmic content gives the mathematics of nature its infinitary character and opens the door to incomputability. Richard Feynman, no less, may have decided [13] "It is really true, somehow, that the physical world is representable in a discretized way, and…we are going to have to change the laws of physics," but real numbers persist in the mathematics of the real world.

There have ever been doubts about our ability to make sense of causality. Adding deities may reassure but produces its own uncertainties. Questioning the scope of predictable causation certainly goes back to the eleventh century and Al-Ghazali's *The Incoherence of the Philosophers* and is traceable through Hume and Berkeley, arriving, for instance, at the modern interest in emergent phenomena. According to the *Oxford English Dictionary* (1971 edition), the first recorded use of the word "incomputable" goes back to 1606, some forty years even before "computable". The term acquired its precise meaning only in the 1930s, with the formulation of a number of different models of what it means for a function over the natural numbers to be computable. As we have already mentioned, it was these that enabled Church and Turing to get their examples of incomputable objects. The key observation, captured in what we now know as the *Church-Turing Thesis*, is that

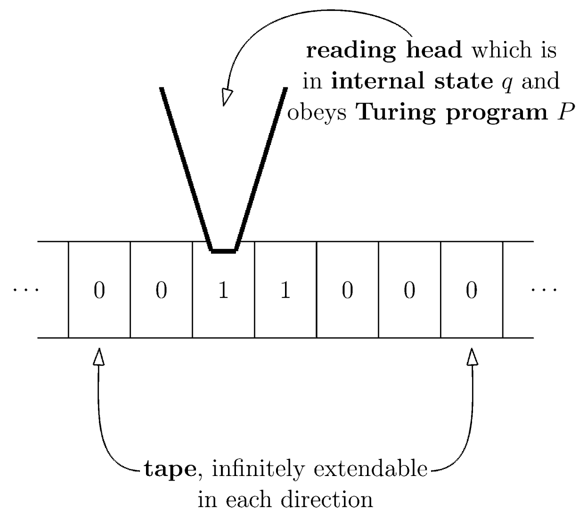


Figure 2. Turing machine from S. B. Cooper, *Computability Theory*, Chapman & Hall/CRC, 2004.

there is a robust intuitive notion of computability to which all our different formalisms converge. It was Turing's carefully argued 1936 paper, based on the Turing machine model (see Figure 2), that convinced Gödel of the validity of such a thesis.

As Gödel's friend Hao Wang recounts [38, p. 96]:

> Over the years G habitually credited A. M. Turing's paper of 1936 as the definitive work in capturing the intuitive concept [of computability], and did not mention Church or E. Post in this connection. He must have felt that Turing was the only one who gave persuasive arguments to show the adequacy of the precise concept.…In particular, he had probably been aware of the arguments offered by Church for his 'thesis' and decided that they were inadequate. It is clear that G and Turing (1912–1954) had great admiration for each other,…

Within mathematics, Turing's paper was a blow to David Hilbert's view, famously expressed on September 8, 1931, in a Königsberg address (quoting from John Dawson's 1997 biography of Gödel) that:

> For the mathematician there is no Ignorabimus, and, in my opinion, not at all for natural science either… The true reason why [no one] has succeeded in finding an unsolvable problem is, in my opinion, that there is no unsolvable problem. In contrast to the foolish Ignorabimus, our credo avers:
>
> We must know, We shall know.

The universality of Turing's machine meant it had to implement a lot of faulty programs. Some programs would lead to computations that never stopped, and one could not tell in general which inputs led to a proper computation. The Halting Problem for the Universal Turing Machine (UTM) turns out to be unsolvable. The set of inputs that lead to a terminating computation of the UTM is *computably enumerable*: you can progressively set in motion an array of all possible computations and observe which ones output a result. This enables you to enumerate the inputs on which the machine halts, but this set is *not computable*, since you can *never be certain* that a computation in progress will not one day succeed.

More dramatically, all sorts of mathematical theories are capable of "talking about" our UTM. Turing used natural numbers to code the activities of the machine, using the trick Gödel had earlier used to enable Peano arithmetic to talk about itself. Turing's discovery was that *any* reasonably strong mathematical theory was *undecidable,* that is, had an incomputable set of theorems. In particular, Turing had a proof of what became known as *Church's Theorem*, telling us that there is no computer program for testing a statement in natural language for logical validity. Since then, a huge number of undecidable theories have been found.

The drawback to this powerful technique for proving the existence of many natural incomputable sets is that there are no other good techniques for proving incomputability. Moreover, John Myhill showed in 1955 that the known natural examples of incomputable objects tended to be all the same—computationally, just notational translations of each other. This would not have mattered if it were not for the fact that the so-called Turing universe of incomputable sets turned out to have a very rich and mathematically challenging structure, and if this were to be embodied in the real world to any extent, most of the computational character of its embodiment would be hidden from us. If we managed to solve a problem, all well and good. If we failed, we might never know whether that bit of the world had an incomputable character different from that of the halting problem or might be computable via some program that has eluded us so far.

This simple difficulty with *recognizing* mathematical incomputability was explained by results that told us such recognition was itself a highly incomputable problem. Here were all the ingredients for a parting of the ways between the mathematics and real-world concerns. Turing himself made his last great contribution to the logic of computability theory in his amazing 1939 paper, based on his work with Alonzo Church in Princeton. Incomputability later played a very important role in mathematics and computer science, encountered in a range of undecidability results. Wilfried Sieg's *Normal forms for puzzles: A variant of Turing's Thesis*, in [7], is an excellent commentary on Turing's continuing interest in the decidability problems for games, groups, etc. Of course, the most celebrated undecidability result was the negative solution to Hilbert's Tenth Problem by Martin Davis, Yuri Matiyasevich, Julia Robinson, and Hilary Putnam. Who would have thought that just one existential quantifier added to everyday high school arithmetic would give rise to a problem unsolvable by any computer program?

After 1939 Turing's work was more obviously rooted in reality than the earlier overarching abstraction. The mathematicians had the halting problem and its variants, mathematically comprehensive and canonical, a little too grandiose for the everyday ad hoc world, while the endless complications of everyday existence could not be classified. The theory was useless.

For the recursion-theoretic period of mathematics, with its isolation and loss of sense of mission in the wider world, see recent papers by Robert I. Soare (for example [29]). Turing's Manchester work on artificial intelligence, connectionist models, and morphogenesis contained inspired anticipations of the shape of things to come.

Mathematical Steps towards an Incomputable Reality

The unsolvable problems of the 1930s may deliver examples of incomputable objects, but their mathematical abstraction appears far from the embodied mathematics of a Newton or an Einstein. At the same time, the deep and intractable problems from the real world are hard to subject to logical analysis. What makes Turing's work so important is the way it draws out the computability-theoretic core of very different real-world mysteries. He had a knack of getting inside structures and imaging their constructive cores in new ways. Typically, Turing does not *apply* mathematics; he *builds* it within the context he is exploring. For Einstein [10, p. 54]:

> When we say that we understand a group of natural phenomena, we mean

that we have found a constructive theory which embraces them.

Turing takes this literally. Mathematics for Turing often comes mentally embodied. The, for many, repellent abstraction of Turing's 1939 paper is all about how Gödel's incompleteness theorem plays out in practice. We notice that the true sentence unprovable in Gödel's theory for arithmetic is easily described, while enlarging our theory by adding the sentence gives us a larger theory which has a similarly described unprovable sentence. For Turing, this was the seed for a computably iterated process of enlargement. So, using Kleene's computable ordinals, one could extend the process transfinitely. Later, logicians such as Solomon Feferman and Michael Rathjen would enhance Turing's ladder into the incomputable and proof-theoretically reach levels far beyond those Turing had achieved. But in a sense, Turing had found out what he wanted to know. Tucked away amongst the mountains of abstraction is his characteristically candid take on what he had done (Turing 1939, pp.134–5):

> Mathematical reasoning may be regarded…as the exercise of a combination of…intuition and ingenuity.… In pre-Gödel times it was thought by some that all the intuitive judgements of mathematics could be replaced by a finite number of…rules. The necessity for intuition would then be entirely eliminated. In our discussions, however, we have gone to the opposite extreme and eliminated not intuition but ingenuity, and this in spite of the fact that our aim has been in much the same direction.

The outcome is that mathematically we have a brilliant analysis of how we may constructively navigate our way through a phase transition but storing up a level of incomputability arising from the ad hoc nature of the route. There is a degree of arbitrariness in a climber's choice of hand and foot holds, which gives the climb more than algorithmic interest. This ties in with an irreversibility of computation noted by people such as Prigogine in quite different contexts.

There is a special interest for the mathematician in Turing's analysis of an incomputable route to computable outcomes. It is an analysis that fits nicely with our experience of creative derivation of theorems, followed by the uncovering of algorithmic approaches to them. We rely on the memetic character of our proofs, fit to circulate the community like a virus. A basic rule for lifting small truths to bigger ones, a logical counterpart of the more visceral causality of nature, is mathematical induction. For proof theorists, induction plays a key role. They categorize theorems according to the level of complexity of the induction used in the proof. Most theorems turn out to be proof-theoretically very simple. Now that we have a proof of Fermat's Last Theorem, the logician Angus MacIntyre has been able to outline it within first-order arithmetic. This view of the proof involves simple incremental accretions of truth. The discovery of the proof was something very different, as is our understanding of it.

We are beginning to see a pattern—literally: simple rules, unbounded iteration, emergent forms—defined at the edge of computability. This is just what Turing later observed in nature and mathematically tried to capture.

Another hugely important mathematical tool to take on our explorations of the incomputable is the oracle Turing machine, also tucked away on one page of Turing's 1939 paper. The idea was to allow the machine to compute relative to a given real which may or may not be computable. If one looked at the function computed *using* this oracle, one could frame the function as being computed *from* the oracular real as argument. It is then a small step to summarize what the machine does as computing one real from another. It delivers us a computational model within which to fit basic computable laws of nature, namely, most of what underlies our knowledge of how the world works. Harking back to the computable numbers of 1936, the oracle machine computes a real number but *from another* real number. We can allow the machine to compute relative to different oracles. Then we can acknowledge the higher type nature of the computational process by calling the mappings computed by oracle machines *Turing functionals*. These, acting over the reals, give us the *Turing universe*.

In fact, Turing was not notably interested in the mathematical development of his oracle machines despite his preoccupation with computers that interacted. It was left to another seminal figure, Emil Post, to gather together equivalence classes of reals—or *degrees of unsolvability*—that were computable from each other. Then, using an ordering induced by the ordering of reals via Turing functionals, Post obtained a structure that has become known simply as the *Turing degrees*.

Three key things we observe about this structure are: firstly, that it is very complex; secondly, that if we take some scientific domain described in terms of real numbers and computable laws over them, then it is embeddable in the Turing universe, so that the structure of the corresponding restriction of the Turing universe tells us something about the causal structure of the real world (the causal third dimension of the Gamow-like two-dimensional person we met earlier); thirdly, we can view this model as a terrain in which computation can be hosted but in which information takes a leading role, structuring the form of the computation—a *reembodiment* of computation.

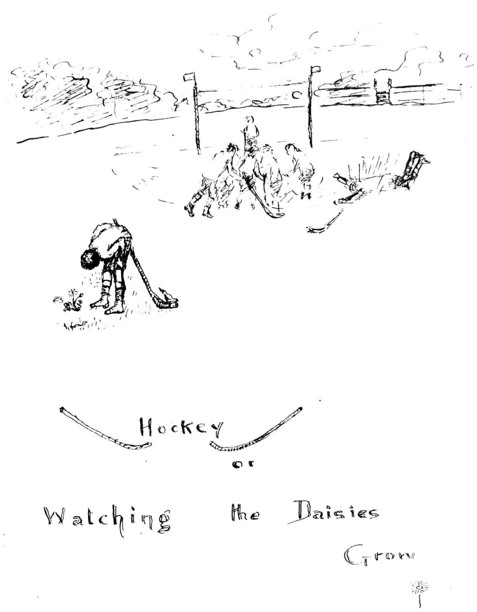


Figure 3. Drawing of Alan Turing by his mother, at his preparatory school, Hazelhurst, Sussex, 1923. ***Image courtesy of Sherborne School.***

Messages from the Real World

Turing's final years in Manchester saw both personal eclipse and the sowing of the seeds of a later renaissance—a growing renown and increasing scientific impact. Today, his fame appears as a mixed blessing: too many webpages and articles in periodicals with misleading information about what Turing did and did not do. On the other hand, having mathematicians such as Turing and Gödel in the public eye is good for basic science. As far as the science itself goes, Turing's work has been powerfully influential in a piecemeal way, with a number of different fields laying claim to particular bits of Turing. In the October 2004 *Notices*, Lenore Blum wrote a nice description of the dichotomy between "two traditions of computation" (in "Computing over the reals: Where Turing meets Newton", pp. 1024–1034):

> The two major traditions of the theory of computation have, for the most part, run a parallel nonintersecting course. On the one hand, we have numerical analysis and scientific computation; on the other hand, we have the tradition of computation theory arising from logic and computer science.

Turing's 1948 "Rounding-off errors in matrix processes" was influential in the former and the 1936 Turing machine paper in the latter.

Now there is a growing appreciation of the coherence of approach represented by these different contributions. On the one hand, we have a computational world over which we have control; on the other, we must live with approximations and errors. As we move up the informational-type structure from discrete to continuous data, we lose the sure footholds but identify emergent controls at higher levels. Turing had earlier introduced Bayesian code breaking methods at Bletchley Park in another adjustment to the realities of extracting form from a complex world.

Turing's late great contributions traced the computational content of phase transitions in the real world from two different vantage points. Turing had been interested in the emergence of form in nature from his school days; see his mother's sketch (Figure 3) of Alan "Watching the Daisies Grow". A short piece by Peter Saunders in the forthcoming *Alan Turing—His Work and Impact* ([7], edited by Cooper and van Leeuwen) discusses Turing's motivation and background reading in getting interested in morphogenesis. Saunders writes:

> The obvious question to ask about "The Chemical Basis of Morphogenesis" is why Turing took up the problem at all. Pattern formation, interesting though it may be to biologists, does not look like the sort of fundamental problem that Turing characteristically chose to devote his time and effort to. The answer is simply that he saw it not as a mere puzzle but as a way of addressing what he considered to be a crucial issue in biology. As he said to his student Robin Gandy, his aim was to "defeat the argument from design."

It seems Turing wanted to add a bit more computational convergence to Darwin's theory and not leave an opportunity for God to tidy up. In the same volume, Philip Maini outlines the way in which Turing produced complex outcomes from simple algorithmic ingredients:

> Alan Turing's paper, "The chemical basis of morphogenesis" [35] has been hugely influential in a number of areas. In this paper, Turing proposed that biological pattern formation arises in response to a chemical pre-pattern which, in turn, is set up by a process which is now known as *diffusion-driven instability* [see Figure 4]. The genius of this work was that he considered a system which was stable in the absence of diffusion and then showed that the addition of diffusion, which is naturally stabilizing, actually caused an instability. Thus it was the integration of the parts that was as crucial to the understanding of embryological development as the parts themselves—patterns emerged or self-organized as a result of the individual parts interacting. To see how far ahead of his time he was, one has to note that it is only now in the post-genomic era of systems biology that the majority of the scientific community has arrived at the conclusion he came to some 60 years ago.

Things have moved on since 1952, but the basic approach retains a powerful influence on the field, hardly noticed by logicians and computer scientists as a group.

More importantly for us, Turing's examples point to general principles underlying emergent phenomena. Turing would have been a teenager at the time the "British emergentists" such as C. D. Broad, Samuel Alexander, and C. Lloyd Morgan were at their height. Broad and Turing overlapped at Cambridge. As a group, they were very prescient in seeking out examples of the complex arising from simple rules in highly connected environments. Unfortunately, some of their examples of emergence taken from chemistry turned out to be explainable in terms of quantum mechanics. But Turing's differential equations gave us a new sense both of the character and the origins of emergence and indicated how emergent form might be captured mathematically.

Turing's equations might well have computable solutions, but they pointed to the principle of emergent patterns on animal coats, etc., corresponding to definable relations over basic mathematical structure. If one could define a relation in nature, it had a robustness, a tangible presence that one might expect to find observable, just as Turing had brought us to expect observed emergent phenomena to have descriptions. These descriptions, if at least as complicated as that giving us the halting problem, could be expected to lead to incomputable sets. Somewhere between the tidy abstraction of the universal Turing machine and the mysteries of emergence in nature we have the fractal family. Well known as analogues of emergent phenomena, they also have their own well-defined mathematics. For various reasons, the most informative of these is the Mandelbrot set (detail Figure 5). As Roger Penrose puts it in his 1994 book *The Emperor's New Mind* [26]:

> Now we witnessed…a certain extraordinarily complicated looking set, namely the Mandelbrot set. Although the rules which provide its definition are surprisingly simple, the set itself exhibits an endless variety of highly elaborate structures.

With its definition based on a well-known simple equation over the complex numbers, the quantifier form of the complement of the Mandelbrot set can be reduced to something similar to that of the halting set of the UTM. Using this, one can simulate it on a computer screen, getting the fascinating range of images we recognize so easily. It has what the halting problem does not: the visual embodiment of a natural phenomenon. This enables us to appreciate the higher-order intricacy we encounter as we travel deeper and deeper into this endlessly surprising mathematical object. It is harder to be sure of one's notion of computability in this context. But for the computable analysts, the computability of the Mandelbrot set is still a challenging open problem.

Compared to other levels of the real world that Turing was drawn to, emergence seems relatively straightforwardly mathematical. One has an objective view of the whole picture, basic rules, and emergence of a surprising character—the surprise is one of the criteria for emergence, though there is no proper definition. Mathematically, we tend to look for quantifiers or nonlinearity in the description based on the basic operations, a kind of association of true emergence with halting problem-like incomputability. At the quantum level, which Turing was always very interested in but did not live long enough to get a grip on, the problem of pinning down the basic causality is not so easy. We are looking down from above and are never sure we have the whole picture. And the phase transition from quantum ambiguity to the familiar classical world seems to involve not just definability but a transition from a structure in which there are some prohibitions on simultaneous definition of entities—something familiar to model theorists—which disappear as one passes from particle physics to other scientific areas. There is experimental evidence that such breakdowns of definability occur in human mentality. The problem we as observers have in this context is not one of viewing from above but of being trapped inside, although modern neuroscience is augmenting this inner view with a huge amount of useful information.

In his final years Turing approached brain functionality from two directions: mathematically modelling the physical connectivity of the brain

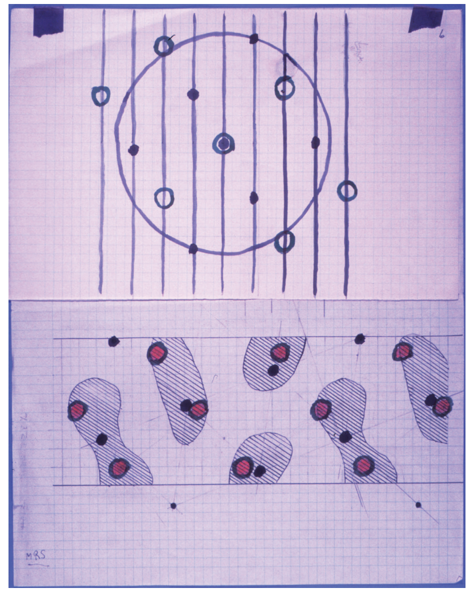


Figure 4. Colored diagrams showing patterns of dappling and calculations, made by Turing in connection with work on morphogenesis. ***Courtesy of P. N. Furbank.***

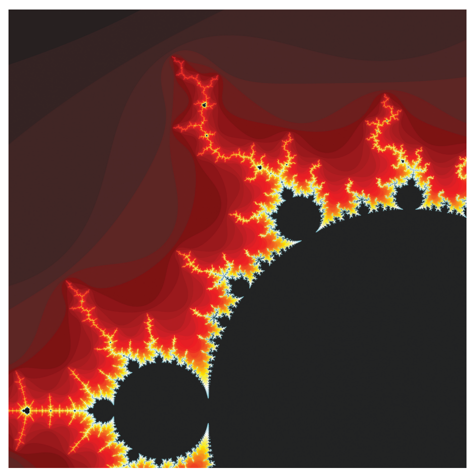


Figure 5. Mandelbrot set. ***Courtesy of Niall Douglas.***

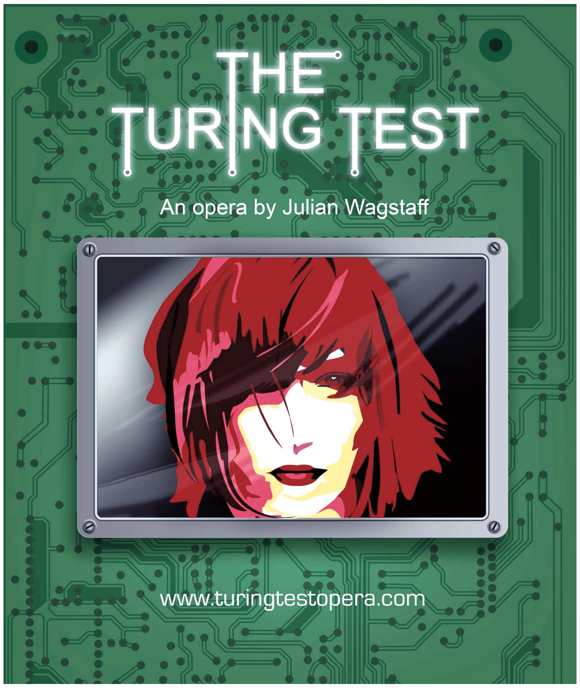


Figure 6. Turing Test image by Joe Smith and Pete Rix. ***Image courtesy of the composer, Julian Wagstaff.***

and via his much better known discussion of intelligent thought. The latter is of special interest for mathematicians, and not just for its relevance to incomputability. Around the same time as Turing was approaching "intuition and ingenuity" via his hierarchical analysis of the limits of Gödel's theorem, Jacques Hadamard was covering very similar ground from a more sociological viewpoint. Given that the mathematical product is presented algorithmically via a proof, the associated mathematical thinking becomes a good case study for clarifying the "intuition versus ingenuity" dichotomy. A principal source for Hadamard's 1945 book *The Psychology of Invention in the Mathematical Field* [16] were lectures of Henri Poincaré to the Société de Psychologie in Paris in the early part of the twentieth century. Here is Hadamard's account of an example of apparent nonalgorithmic thinking:

> At first Poincaré attacked [a problem] vainly for a fortnight, attempting to prove there could not be any such function…[quoting Poincaré]: "Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it…I did not verify the idea…I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience sake, I verified the result at my leisure."

Many writers focus on the surprise and dissociation from consciously rational thought. What is also striking is the connection with Turing's 1939 paper—the "perfect certainty" that Poincaré experienced. Did Poincaré have all the details of the proof immediately sorted in his mind? Unlikely. What we understand from Turing's analysis is that there is a process of definition, with a range of proofs emerging. Poincaré extracted one of these proofs on his return to Caen.

As well as the connection back to Turing's 1939 paper, there is the connection forward to his 1952 paper and the final work defining emergence of form in nature. Since 1954 neuroscience is one of the areas in which emergence has taken centerstage for many researchers. Turing's connection between definability in terms of both logical structure and physical context is a remarkable anticipation of current thinking.

At the bottom of the process is the basic physical functionality, and Turing made his own groundbreaking contribution to connectionist models of the brain in his unpublished 1948 National Physical Laboratory report *Intelligent Machinery* [36]. Called unorganized machines by Turing, they were preempted by the more famous neural nets of McCulloch and Pitts. See Christof Teuscher's book *Turing's Connectionism* [30]. These are Teuscher's comments on the history in his commentary on *Intelligent Machinery* in *Alan Turing—His Work and Impact* [31]:

> In his work, Turing makes no reference to McCulloch and Pitts' 1943 paper, nor do they mention Turing's work [on unorganised machines]. It is an open question how much their work influenced each other, yet, we have to assume that they were at least aware of each other's ideas. We hypothesize that both bad timing and the fact that Turing's neurons are simpler and more abstract contributed to his work being largely ignored.

Connectionist models have come a long way since Turing's time. Their physical emulation of the brain does bring dividends. Paul Smolensky, for instance, talks in his 1988 paper "On the proper treatment of connectionism" [28] of a possible challenge to "the strong construal of Church's Thesis as the claim that the class of well-defined computations is exhausted by those of Turing machines."

Of course, it was the celebrated 1950 *Mind* paper "Computing machinery and intelligence" [34] that became one of Turing's three most cited papers. The Turing Test for intelligence has entered the popular culture, with an opera (see Figure 6) and students walking around with T-shirts proclaiming "I failed the Turing Test".

So what has all this to do with the mathematics of incomputability? Turing's focus on the logical structure of computation has had huge influence on modern thinking. Nevertheless, since his time at Bletchley Park, Turing was involved in many ways with *embodied* computation. Implicit in this respect for embodiment is a recognition of a difference in the way humans (and intelligent machines in general) interact with—are *embedded* in—*information*. This interaction is seen as a necessary attribute of intelligence. People involved with taking artificial intelligence forward have had to take on board this extended, embodied, physical, information-respecting model of the future. We give some brief excerpts from Rodney Brooks's contribution ("The Case for Embodied Intelligence") [4] to *Alan Turing—His Work and Impact*:

> For me Alan Turing's 1948 paper Intelligent Machinery was more important than his 1950 paper Computing Machinery and Intelligence....For me, the critical, and new, insights in Intelligent Machinery were two fold. First, Turing made the distinction between embodied and disembodied intelligence.... Modern researchers are now seriously investigating the embodied approach to intelligence and have rediscovered the importance of interaction with people as the basis for intelligence. My own work for the last twenty-five years has been based on these two ideas.

Figure 7. Alan Turing in 1928, age 16. ***Image courtesy of Sherborne School.***

Turing Points the Way Past the Turing Barrier

Alan Turing's work was incomplete. For Turing the end came too early via an uneasy confluence of algorithm (the UK law of the time) and incomputability, a bizarre piece of history that nobody could have invented for a man who had served mathematics and science—and his country—so well. Some have questioned the description "computability theory" for the subject Turing cofounded with other luminaries of the period—Gödel, Post, Church, Kleene—because it deals primarily with the incomputable.

Turing (Figure 7) was a mathematician of his time who worked from within the world, trying to give mathematical substance to physical and mental processes. He gave us a basic model of what we understand to be computation. He observed computation as an organic whole, discovering incomputability as a definability-theoretic extension of Gödel's incompleteness theorem. He encouraged us to see the universe as something that does compute and to engage with its features, tracing embodied analogues of his halting problem within biology and neuroscience. He loved the truth and was open to doubts about his doubts. He thought a machine could not be intelligent if it was expected to be infallible. He saw something different about an embedded computer and was drawn to the computational mysteries of quantum theory.

Turing did not live long enough to appreciate Stephen Kleene's investigations of higher-type computability but would surely have made the connection between the mathematics of incomputability, definability, and the computations that arise from them. He did not see the mathematical theory of randomness flourish or the yearly award of a prize in his name for a subject he played a founding role in. He never saw the employment of thousands who talked about "Turing" machines and a "Turing" test. The "incomputable reality" (as *Nature* [6] recently described it) is still dangerous to inhabit. But we do have a lot to celebrate in 2012.

Bibliography

1. S. Alexander, *Space, Time, and Deity*, Vol. 2, 1927.
2. L. Blum, F. Cucker, M. Shub, and S. Smale, *Complexity and Real Computation,* Springer, 1997.
3. C. D. Broad, *The Mind and Its Place in Nature*, Kegan-Paul, London, 1923.
4. R. Brooks, The case for embodied intelligence, in *Alan Turing—His Work and Impact* (S. B. Cooper and J. van Leeuwen, eds.), Elsevier, 2012.

5. S. B. Cooper, *Computability Theory*, Chapman & Hall/CRC Press, Boca Raton, FL, New York, London, 2004.
6. ______, Turing centenary: The incomputable reality, *Nature*, **482**:465, 2012.
7. S. B. Cooper and J. van Leeuwen (eds.), *Alan Turing—His Work and Impact*, Elsevier, 2012.
8. A. R. Damasio, *The Feeling of What Happens: Body and Emotion in the Making of Consciousness*, Harcourt Brace, 1999.
9. M. Davis, *The Universal Computer: The Road from Leibniz to Turing*, A K Peters/CRC Press, 2011.
10. A. Einstein, *Out of My Later Years*, volume 48, Philosophical Library, 1950.
11. S. Feferman, Transfinite recursive progressions of axiomatic theories, *J. Symbolic Logic,* **27**:259–316, 1962.
12. S. Feferman, Turing in the Land of O(z), in *The Universal Turing Machine: A Half-Century Survey* (R. Herken, ed.), Oxford University Press, New York, 1988, pp. 113–147.
13. R. P. Feynman, Simulating physics with computers, *Int. J. Theoretical Physics*, **21**:467–488, 1981/82.
14. G. Gamow, *One, Two, Three…Infinity* (1947, revised 1961), Viking Press (copyright renewed by Barbara Gamow, 1974), Dover Publications.
15. R. O. Gandy, The confluence of ideas in 1936, in *The Universal Turing Machine: A Half-Century Survey* (R. Herken, ed.), Oxford University Press, New York, 1988, pp. 51–102.
16. J. Hadamard, *The Psychology of Invention in the Mathematical Field*, Princeton Univ. Press, Princeton, 1945.
17. W. Hasker, *The Emergent Self,* Cornell University Press, Ithaca, London, 1999.
18. A. Hodges, *Alan Turing: The Enigma*, Vintage, London, Melbourne, Johannesburg, 1992.
19. J. Kim, *Physicalism, or Something Near Enough*, Princeton University Press, Princeton, Oxford, 2005.
20. S. C. Kleene, Recursive functionals and quantifiers of finite types. I, *Trans. Amer. Math. Soc.*, **91**:1–52, 1959.
21. S. C. Kleene, Recursive functionals and quantifiers of finite types. II, *Trans. Amer. Math. Soc.*, **108**:106–142, 1963.
22. W. McCulloch and W. Pitts, A logical calculus of the ideas immanent in nervous activity, *Bull. Math. Biophys.*, **5**:115–133, 1943.
23 B. P. McLaughlin, The rise and fall of British emergentism, in *Emergence or Reduction?—Essays on the Prospects of Nonreductive Physicalism* (A. Beckermann, H. Flohr, J. Kim, eds.), de Gruyter, Berlin, 1992, pp. 49–93.
24. J. Myhill, Creative sets, *Z. Math. Logik Grundlagen Math.*, **1**:97–108, 1955.
25. P. Odifreddi, *Classical Recursion Theory*, North-Holland, Amsterdam, New York, Oxford, 1989.
26. R. Penrose, *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics*, Oxford University Press, Oxford, New York, Melbourne, 2002.
27. E. L. Post, Degrees of recursive unsolvability: Preliminary report (abstract), *Bull. Amer. Math. Soc.*, **54**:641–642, 1948.
28. P. Smolensky, On the proper treatment of connectionism, *Behavioral and Brain Sciences*, **11**:1–74, 1988.
29. R. I. Soare, Turing computability and information content*, Philos. Trans. Royal Soc. London*, *Series A*, to appear.
30. C. Teuscher, *Turing's Connectionism, An Investigation of Neural Network Architectures*, Springer-Verlag, London, 2002.
31. ______, A modern perspective on Turing's unorganized machines, in *Alan Turing—His Work and Impact* (S. B. Cooper and J. van Leeuwen, eds.), Elsevier, 2012.
32. A. M. Turing, On computable numbers with an application to the Entscheidungsproblem, *Proc. London Math. Soc.* (3), 42:230–265, 1936. A correction, **43**:544–546, 1937.
33. ______, Systems of logic based on ordinals, *Proc. London Math. Soc.* (3), **45**:161–228, 1939.
34 ______, Computing machinery and intelligence, *Mind*, **59**:433–460, 1950.
35. ______, The chemical basis of morphogenesis, *Philos. Trans. Royal Soc. London. Series B*, *Biological Sciences*, **237**(641):37–72, 1952.
36. ______, Intelligent machinery, in D. C. Ince, (ed.), *Collected Works of A. M. Turing—Mechanical Intelligence*, Elsevier Science Publishers, 1992.
37. J. von Neumann, The general and logical theory of automata, in: L. A. Jeffress (ed.), *Cerebral Mechanisms in Behaviour: The Hixon Symposium*, September 1948, Pasadena, Wiley & Sons, New York, 1951.
38. H. Wang, *Reflections on Kurt Gödel*, MIT Press, Cambridge, MA, 1987.

Why Are There No 3-Headed Monsters? Mathematical Modeling in Biology

J. D. Murray

Alan Turing's crucial intelligence work in the Second World War is well known. His contribution to the interdisciplinary field of mathematics and the biological sciences is less so. Turing published only one paper related to biology, "The chemical basis of morphogenesis", in 1952 [64], which has been seminal in several areas of spatial patterning modeling in development, ecology, and other biological areas since its rediscovery in the 1960s. He did not apply his model to any specific biological situation.

Basically Turing showed how, in a system of reacting chemicals where the chemicals can also diffuse, the system can generate a steady-state heterogeneous spatial pattern of chemical concentrations. He called these chemicals morphogens. He hypothesized that these morphogenetic prepatterns could cue cell differentiation and result in observed spatial patterns. His model is encapsulated in the coupled system of reaction diffusion equations, of the general form

$$\frac{\partial u}{\partial t} = \gamma f(u,v) + D_u \nabla^2 u, \quad \frac{\partial v}{\partial t} = \gamma g(u,v) + D_v \nabla^2 v, \tag{1}$$

where the functions $f(u,v)$ and $g(u,v)$ denote the reaction kinetics associated with the chemicals u, called the activator, and v, called the inhibitor, with D_u and D_v the diffusion coefficients of u and v respectively. The parameter γ, which arises when the system is written in this nondimensional form, is an important measure of scale, as we shall see below. A stability analysis of the steady states of the kinetics shows that to generate spatial patterns in u and v, it is necessary, among other things, that the inhibitor have a higher diffusion rate than the activator, that is $D_v > D_u$; see, for example, [44], [46]. A review article [35] is specifically devoted to Turing's theory.

To get an intuitive idea of how the reaction diffusion patterning works, consider the following, albeit unrealistic scenario, of a field of dry grass in which there is a large number of grasshoppers. If grasshoppers get warm, they can generate a lot of moisture by sweating. Now suppose the grass is set alight at several random points and a flame front starts to propagate from each. We can think of the grasshopper as an inhibitor and the fire as an activator. If there were no moisture to quench the flames, the fires would simply spread over the whole field, which would result in a uniform charred area. Suppose, however, that when the grasshoppers get warm enough they can generate enough moisture to dampen the grass so that when the flames reach such premoistened areas the grass will not burn. The scenario for a heterogeneous spatial pattern of charred and uncharred grass patches is then the following. The fires start to spread; these represent one of the "reactants", the activator, with a fire "diffusion" coefficient, which quantifies how fast the fire spreads. The grasshoppers, which constitute the inhibitor "reactant", feel the flame fronts coming and move quickly ahead of them. The grasshoppers then sweat profusely and generate enough moisture to prevent the fires spreading into the moistened area. In this way the charred areas

---

J. D. Murray is professor emeritus of mathematical biology, University of Oxford; senior scholar in applied and computational mathematics, Princeton University; and emeritus professor of applied mathematics, University of Washington. His email address is `james.murray@maths.ox.ac.uk` *and* `jdmurray@princeton.edu`.

DOI: http://dx.doi.org/10.1090/noti865

are restricted to finite domains that depend on the "diffusion" coefficients of the reactants—fire and grasshoppers—and the various "reaction" parameters. If the grasshoppers and flame fronts "diffused" at the same speed, no such spatial pattern could evolve.

Closely related to Turing's theoretical work, and done independently, is the important experimental work of Boris Belousov. Although the exact date is not known, it was in the early 1950s that he showed how a group of three reacting chemicals could spontaneously oscillate between a colorless and a yellow solution. His paper was twice rejected for publication, and he was eventually persuaded to publish it in an obscure journal of abstracts in 1959. This seminal work was rediscovered by Anatol Zhabotinsky in 1961 [69], and this classic groundbreaking reaction, known as the Belousov-Zhabotinsky reaction, has been widely studied experimentally (see, for example, [52]) and mathematically (see, for example, [44]).

A vast number of papers refer to such oscillating systems and to reactions that exhibit oscillatory behavior and generate complex spatial patterns: see, for example, the early work in 1972 [66] that shows how spiral waves of reactants arose. The books [44], [46] discuss the reaction diffusion systems in detail. There are several webpages that visually demonstrate these oscillating reactions and their spatial patterning.

There are several classic examples of the mathematical models. One very early one appears in the little-known paper (in French) [6], written by Bernoulli in 1760, when smallpox was rampant. He proposed a differential equation model to quantify how cowpox inoculation affects the spread of smallpox. As well as providing an interesting and practical mathematical model, the article gives some interesting data on the devastating child mortality at the time. He used the model to assess the practical advantages of a vaccination control program. One wonders if he knew of the ancient Chinese custom where children were made to inhale a powder made from the crusts of skin lesions of people recovering from smallpox. A few researchers in the first half of the twentieth century were getting involved in mathematical modeling associated with population interaction and epidemic models, but they seem not to have known about Bernoulli's remarkable paper [15], [26], [27], [32], [65]. A major interdisciplinary work, albeit not specifically mathematical but heuristically so, is D'Arcy Thompson's [63] monumental book *On Growth and Form*, first published in 1917, which discusses among other things concepts associated with morphological patterns. The rediscovery of Turing's 1952 paper in the late 1960s had a major influence in the development of the field of mathematical biology, surprisingly more than the experimentally justified model of Hodgkin and Huxley published in 1952 [22] on nerve conduction and excitation, which was awarded a Nobel Prize.

Considerable interest in the study of pattern formation had been developing by the time the remarkable work of Geoffroy Saint-Hilaire appeared in 1836 [55]. He was particularly interested in teratology and was probably the first to introduce the important concept of a developmental constraint, something we shall talk about below. His emphasis on the parallels between the study of form in physical systems and of biological form was an early indication of later papers in the field.

Until around 1970 mathematical involvement in the biomedical sciences was restricted to just a few researchers who developed predator-prey models, basic epidemic models and others associated with problems in body mechanics, facilitated diffusion in tissues [37], and others. The use of mathematics in biology really started to blossom from the mid-1970s, with several thousand researchers now actively involved and mathematical modeling being used in practically every field in the biomedical sciences.

Much of the research in the twentieth century in the applications of mathematical modeling in biology was in biological pattern formation, a field about which there was no general acceptance but a lot of controversy. With the burgeoning of genetic studies, the belief that genetics would solve all these developmental problems has certainly not been borne out. There was, of course, considerable research in the ecological and epidemiological sciences, where the use of mathematical models had a long history.

In the past twenty to thirty years, genetic studies have spawned exciting, important, and genuinely interdisciplinary research involving theoreticians and experimentalists, with the common aim of elucidating the underlying mechanisms involved in developmental biology and medicine. However, most of the mechanisms are essentially still unknown.

This interdisciplinary field is nowadays referred to as mathematical biology, theoretical biology, or systems biology. An increasing number of books provide surveys of some of the early work in the field as well as perspectives on the field's remarkable growth since the 1970s; examples include [37], [44], [46] and the definitive text [25] on the relatively recent growth area of modeling in physiology. There are many other books with author-contributed chapters, often associated with conference proceedings, that give a picture of how the field developed, such as [7], [11], [23], [29], [31], [33]. The book [36] is specifically devoted to reaction-diffusion theory. Research on theoretical models in ecology and epidemiology has also been growing; see, for example, the 1992 review [30]. With the ever-increasing number of people getting involved in the field, the number of genuinely practical examples has grown, as has the number of essentially mathematics papers where the primary interest is in the mathematics, with scant connection to any real biology. By the mid-1980s it was becoming more widely acknowledged that any real contribution to the biological sciences from modeling must be genuinely interdisciplinary and hence related to real biology. The best research helps our understanding of real biological problems by developing models for specific biological situations and providing predictions that are confirmed (or contradicted) by experiment; a specific example is discussed below.

A significant influence on the development of mathematical biology came from the seminal experimental work on the importance of chemical gradients in embryonic development [67], which introduced the concept of "positional information". The experiments showed that cells react to a chemical concentration associated with where they are in a chemical gradient. This work stimulated a huge amount of experimental and theoretical work, often controversial, that is still going on. For a review, see the book [68] on the author's views on the principles of development.

Since the late 1990s there has been an ever-increasing number of truly interdisciplinary studies covering a remarkably wide spectrum of topics. One example is research on wound healing [14], [16], [56], [57]. Another is on tumor growth, such as brain tumors (see [47] for a short review), briefly described below, which is being used medically to quantify the efficacy of individual patient treatment scenarios prior to treatment (see [46] for a survey).

Many of these cancer studies involve modeling the phenomena across many different scales. One example is the seminal work [17]. The suggestion here was that tumor cells create an environment that allows certain mutations to be selected and hence the evolution of mutant cell populations to occur within the body; this is called somatic evolution. The authors analyzed somatic evolution in this context and showed a number of evolutionary pathways in ductal carcinoma *in situ*. The experimental colleagues suggested that different mutant clones would emerge in a well-defined temporal sequence, while the mathematical simulations showed that this was highly unlikely. The simulations predicted that hypoxia (lack of oxygen) should promote emergence of varying-sized nodules of a mutant clone of a certain type surrounded by other cell types and that over time these nodules would grow and merge. This stimulated the experimental colleagues to carry out experiments, which confirmed this prediction. Another study highlights cellular adaptions of cancer cells in colorectal cancer [24]. In the following section we briefly discuss one application of reaction-diffusion theory. We shall point out some of the limitations of Turing-type reaction-diffusion mechanisms that necessitated a new, and more experimentally verifiable, approach to biological pattern formation, known as the mechanical theory of biological pattern formation [40], [41], [50].

How the Leopard Gets Its Spots

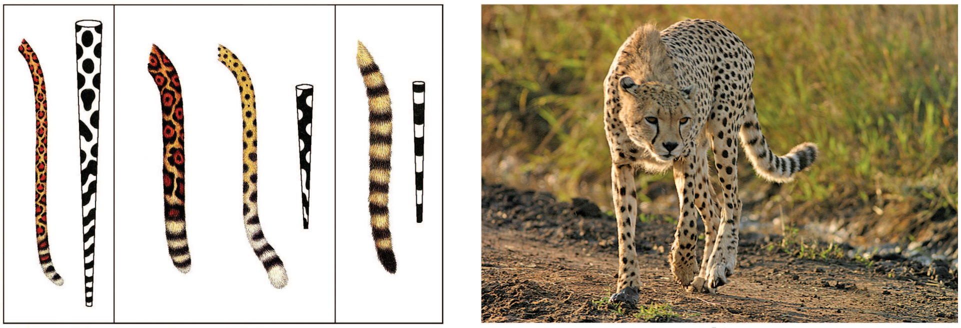
A specific experimental reaction-diffusion mechanism proposed in 1975 [62] was used to study how animal coat patterns might be formed [38], [39], [43], [44]. The reaction terms used in (1) were

$$\begin{aligned} f(u,v) &= a - u - h(u,v), \\ g(u,v) &= \alpha(b - v) - h(u,v), \\ h(u,v) &= \frac{\rho u v}{1 + u + K u^2}, \end{aligned} \tag{2}$$

where a, b, α, ρ, and K are constants and assigned so that the solutions exhibited steady-state spatially heterogeneous solutions. They were kept fixed for all the calculations. *Only* the scale and geometry of the domain were varied. The resulting patterns in Figures 1 and 2 are colored dark and light in regions where the concentration of one of the morphogens is greater than or less than the concentration in the homogeneous steady state. Even with such limitations on the parameters, the wealth of possible patterns is remarkable. For a given domain size and geometry each set of initial conditions gave a similar but *unique* pattern, a fact reflected in nature.

It was shown that a single prepatterning mechanism was capable of generating the typical geometry of mammalian coat patterns, from the mouse to the badger to the giraffe to the elephant and almost everything in between, with the end pattern governed simply by the size and shape of the embryo at the time the pattern formation process was initiated. In solving these reaction-diffusion systems, the domain size and shape are crucial. If, given a particular mechanism, one tries to simulate a solution in a very small domain, one finds that it is not possible to obtain steady-state spatial patterns; a minimum size is needed to drive any sustainable spatial pattern. Or, if the domain is long and thin, only stripes can be generated.

Suppose the reaction-diffusion domain is a rectangle. The rectangle must have a minimum size for a simulation to exhibit spatial heterogeneity. As the size of the rectangle is increased, a series of



(a) (b)

Figure 1. (a) Examples of a developmental constraint. Spotted animals can have striped tails but not the other way round. From left to right are the typical tail of the leopard, of the cheetah, and of the genet, together with the solutions from a reaction-diffusion system which can generate steady-state spatial patterns. The geometry and scale when the pattern mechanism is activated play crucial roles in the resulting coat patterns. Dark regions represent areas of high morphogen concentration. (Tail art work reproduced from [43] with permission of Patricia Wynne.) (b) A cheetah (*Acinonyx jubatus*), which is an example of the developmental constraint described in (a). (Photograph courtesy of Professor Andrew Dobson.)

(a) (b)

(c) (d)

Figure 2. (a) These show the result of numerical simulations of the reaction-diffusion model analysis [38], [39], [43], [44] for the generation of coat markings on animals; it is the same mechanism used in Figure 1. The model parameters were also the same; only the scale parameter was varied. The domain sizes have been reduced to fit in a single figure, but in the simulations there was a scale difference of 1,000 between the smallest and the largest figure. (b) An example of the first bifurcation: Valais Blackneck goat. (Photograph by B. S. Thurner Hof, Wikimedia Commons.) (c) Belted Galloway cows. (d) Giraffes (*Giraffa camelopardis*) in the Serengeti. (Photograph courtesy of Professor Andrew Dobson.)

increasingly complex spatial patterns emerge. The concept behind the model is that the simulated spatial patterns, solutions of a reaction-diffusion mechanism, reflect the final morphogen melanin landscape observed on animal coats. With this scenario, the cells react to a given level in morphogen concentration, thus producing melanin (or rather becoming melanocytes, cells that produce melanin). In Figures 1(a) and 2(a), the black regions represent high levels of melanin concentration. It should be emphasized that this model is a hypothetical one that has not been verified experimentally but rather circumstantially. The main purpose is to show how scale and shape play major roles in animal coat patterns, as they must in other developmental processes. Such an approach has also been used to study butterfly wing patterns [38], [39], [44], [49]. Importantly, the work [49] presents experimental confirmation of the theoretical predictions.

The solutions of the reaction-diffusion system (1) and (2) in domains shown in Figure 1(a) were first computed as an example of how the geometry constrains the possible pattern modes. When the domain is very narrow, only simple, essentially one-dimensional modes can exist. Two-dimensional patterns require the domain to have enough two dimensionality. Consider a tapering cylinder as in Figure 1(a). If the radius at one end is large enough, two-dimensional patterns can exist on the surface. So, a tapering cylinder can exhibit a gradation from a two-dimensional pattern to simple stripes, as illustrated in Figure1(a).

This shows that the conical domain mandates that it is not possible to have a tail with spots at its tip and stripes at its base, but only the converse; Figure 1(a) shows some examples of specific animal tails. This is a genuine example of a *developmental constraint*. The cheetah, a photo of which is shown in Figure 1(b), is a prime example of this, as are other spotted animals. If the threshold level of the morphogen is changed, a different but related pattern can develop. Such mechanisms can thus form different but globally similar patterns and could be the explanation for the different types of spatial patterns observed on different species of the same animal genre, such as the spots of the cheetah in Figure 1(b) and the spots of the giraffe in Figure 2(d).

The interpretation of Figure 2 is that if the animal embryo is too small when the patterning mechanism is activated, as in the mouse, or too large, as in the hippopotamus and elephant, then no clear pattern will be observed and these animals are essentially uniform in color.

In between there is a progressively more complex pattern as the size increases. The first two bifurcations are illustrated in Figures 2(a) and 2(b), with the larger animals still showing coat pattern but looking progressively uniform in color, as shown in Figure 2(a).

There have been numerous developments and an increased understanding of how patterns on animals—fish and butterflies, for example—are formed with the addition and combination of other pattern-forming mechanisms, such as chemotaxis, whereby there is movement of cells up chemical gradients. Some examples of the numerous review articles and books are [5], which discusses somite formation; [53], which discusses fish stripes; [28], which discusses evolving fish patterns and other patterned species [10], [34], [35], [59].

It is pointed out in [4] that an understanding of the diversity of animal coat patterns requires an understanding of both the mechanisms that create them and importantly their adaptive value. Among other things, this paper discusses the advantages of specific patterns in different environments. The authors use a reaction-diffusion model, but their conclusions are general and do not rely on any specific reaction-diffusion models, as is essentially the case with the above. They convincingly show how different markings relate to specific natural environments for the specific fields.

The areas of application of reaction-diffusion models are now legion. One particular simple application, which has turned out to be surprisingly practical, was initially proposed in 1995 [9] and pertains to the growth of gliomablastoma brain tumors. The model is given by

$$\frac{\partial c}{\partial t} = \nabla . D(x) \nabla c + \rho c. \tag{3}$$

Here $c(\mathbf{x}, t)$ is the cancer cell density, measured in cells/mm^3, at position $\mathbf{x}$ in the brain at time t measured in months; $D(\mathbf{x})$ is the cell diffusion (invasion), measured in mm^2/month, which quantifies the invasiveness of the cancer cells at position $\mathbf{x}$ in the brain, since it varies depending on position; and ρ is the net proliferation rate (/month) of the cancer cells, which gives the cell turnover time as $\log 2/\rho$ (months). It is possible with this model and patient brain scans to quantify the efficacy of various treatment scenarios for individual patients prior to treatment [60], [61]. These tumors are always fatal: see [46] for a full discussion and [47] for a recent brief review.

Mechanical Theory of Biological Pattern Formation in Morphogenesis

Because of the paucity of experimental verification, the limitations of reaction-diffusion theory gave rise to a totally new theory of biological pattern formation (exceptions are ecology and epidemiology; see, for example, [18], [29] for reviews). The theory was based on extant biological facts about cells and

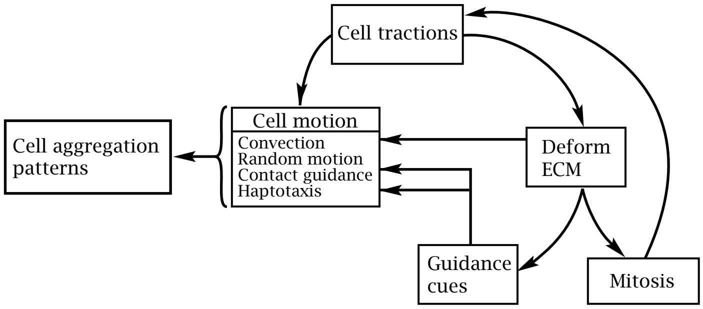


Figure 3. Mechanical theory patterning scenario. Here ECM denotes the extracellular matrix through which the cells move, mitosis is cell division, and haptotaxis is cell movement directed by the deformed biological matrix.

the extracellular tissue matrix, namely, the Oster-Murray mechanical theory of pattern formation [40], [41], [50], [51]. A short introduction is given in [42] with a full survey in [44], [46]. The class of models captures the key interactions between the mechanical forces generated by the cells and their extracellular matrix milieu. Together they give rise to developmental processes that could be experimentally verified. The model shows that a purely mechanical version of the theory could be responsible for certain observed patterns and how they are actually formed in development. Such mechanical models are based on basic mechanical concepts and do not specify the type of cells and matrix involved but instead consider only possible mechanical interactions between the various components.

The basic model hinges on two key experimentally determined properties of mesenchymal cells *in vivo*: (i) cells migrate within a tissue substratum made up of fibrous extracellular matrix, the ECM; (ii) cells can generate large traction forces, thereby deforming the matrix. The basic mechanism models the mechanical interaction between the motile cells and the elastic substratum within which they move. Mesenchymal cells move by exerting forces on their surroundings, consisting of the elastic fibrous ECM and the surface of other cells. They use their cellular protrusions, which stretch out from the cell in all directions, gripping whatever is available and pulling. Due to the heterogeneity in matrix and cell densities, cell traction tension lines form between the cell clusters. These tension lines correspond to aligned matrix fibers along which cells actively move, thereby defining cellular highways between the clusters. One of the major roles of the modeling and its analysis was to indicate what features are essential for biological pattern formation.

The models consist of three nonlinear partial differential equations; the cell conservation equation; another for the cell-extracellular tissue interaction, which incorporates cell movement, that is, mechanotaxis or haptotaxis; and the third equation, which quantifies the cell-matrix mechanical interaction. The models pose numerous challenging mathematical problems, both analytical and numerical, as well as biological modeling problems, many of which have not yet been investigated in any depth. Although the model system is analytically formidable, its conceptual framework is quite clear, as illustrated in Figure 3. The parameters in the model equations are all quantifiable from experiment.

Several factors affect the movement of embryonic mesenchymal cells. Among these are: (i) convection, whereby cells may be passively carried along on a deforming substratum; (ii) diffusion, where the cells move randomly but generally down a cell density gradient; (iii) contact guidance, in which the substratum on which the cells crawl suggests a preferred direction; (iv) contact inhibition by the cells, whereby a high density of neighboring cells inhibits motion; (v) haptotaxis, where the cells move up an adhesive gradient; (vi) chemotaxis, whereby a chemical gradient can direct cell motion both up and down a concentration gradient; (vii) galvanotaxis, where movement from the field generated by electric potentials, which are known to exist in embryos, provides a preferred direction of motion. These effects are all well documented from experiment. The analysis of the field equations incorporating only (i)–(v) showed how regular spatial aggregates of cells come about.

Evolution and Morphogenetic Rules in Cartilage Formation in the Vertebrate Limb: Why Are There No 3-Headed Monsters?

One major application of this mechanical theory was to limb development, the results of which were also put into an evolutionary context [51]. Since the limb is one of the most morphologically diversified of the vertebrate organs and one of the more easily studied developmental systems, it is not surprising it is so important in both embryology and evolutionary biology, where there is a rich fossil record documenting the evolution of limb diversification.

Although morphogenesis appears deterministic on a macroscopic scale, on a microscopic scale cellular activities during the formation of the limb involve considerable randomness. Order emerges as an average outcome with some high probability. It was shown that some morphogenetic events are extremely unlikely, such as trifurcations from a single chondrogenic condensation. Mathematically, of course, they are not strictly forbidden by the pattern formation process but are highly unlikely, since they correspond to a delicate choice of

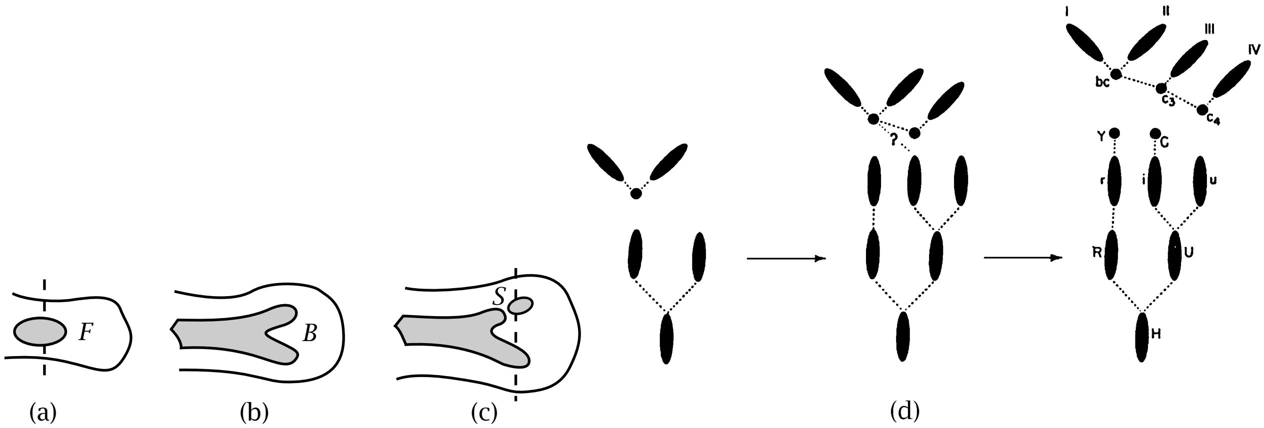


Figure 4. Morphogenetic rules. The three basic cell condensations: (a) a single condensation, *F*; (b) a branching bifurcation, *B*; (c) a segmental condensation, *S*. Complicated patterns can be built from a combination of these basic bifurcations, as in (d), which is one example of the complex morphogenesis and digitations of the forelimb of the salamander *Ambystoma mecicanum*, which was obtained experimentally [44], [58].

conditions and parameter tuning. This is another experimentally verified example of a *developmental constraint*. The "morphogenetic rules" for limb cartilage patterning are summarized in Figure 4.

A variety of limb buds were treated with the mitotic inhibitor colchicines [3]. This chemical reduces the dimensions of the limb by reducing cell proliferation. It was predicted from the mathematical model that such a reduction in tissue size reduces the number of bifurcation events. Figure 5 quantitatively shows the consequence of the inhibitor on limb cartilage growth.

Using the basic rules of cartilage pattern formation in [3], a series of comparative studies [58] were carried out with amphibians, reptiles, birds, and mammals, which confirmed the mathematical predictions, or hypothesis, that tetrapod limb development consists of iterations of the processes of focal condensation, segmentation, and branching. Furthermore, it was shown that the patterns of precartilage cell condensation display several striking regularities in the formation of the limb pattern. The experimental results supported the theoretical conclusion that branching, segmentation, and *de novo* condensation events are reflections of the basic cellular properties of cartilage-forming tissue.

We thus see the possibility of evolution moving backward. It is clearly possible when we consider evolution of form as simply variations in mechanical (or rather mechanistic) parameters. Figure 5 is an unequivocal example where this has happened solely through changing the morphogenetic processes.

The study of these theoretical mechanical models for pattern formation showed that there are considerable restrictions as to the possible patterns of chondrogenesis (as well as other developmental aspects). From the morphogenetic laws it is highly unlikely that a trifurcation is possible, that is, a branching of *one* element into *three* elements. There are numerous examples of two-headed snakes and other reptiles, conjoined twins, and so on. Although there is sometimes an appearance of a 1-to-3 splitting, the theory suggests that all branchings are initially binary. This is because a trifurcation is possible only under a very narrow set of parameter values and conditions. Including asymmetries makes it even more unlikely. This notion of the unlikelihood of trifurcations is the reason we do not see any three-

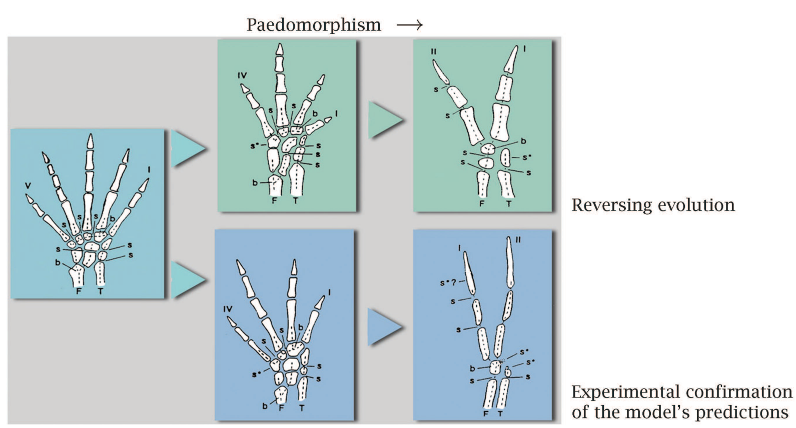


Figure 5. The effect of treating the foot of the salamander *Ambystoma mexicanum* with the mitotic inhibitor colchicine is to reduce the number of skeletal elements. The effect of the inhibitor is to reduce the cell number in the limb and hence the size (after [1], [2]).

(a)

(b)

Figure 6. Examples of branching in humans. (a) The skeleton (19th century) of a *Dicephalus*, a young boy. (b) An example (19th century) of a *Tricephalus*.

headed monsters. Figure 6 shows two examples of human branching.

Very few three-headed monsters have been reported, and of these the veracity is usually highly questionable. If we come back to the limited bifurcations suggested by the morphogenetic laws above, and specifically that a trifurcation is highly unlikely, we can see how a three-headed monster can arise, namely, via a bifurcation of the body axis, such as we see in the skeleton in Figure 6(a), followed by a further bifurcation of one of the branches, as seems clear in the example in Figure 6(b). See [46] for a very brief history of writings on monsters.

Teratology highlights some of the most fundamental questions in evolution: namely, why do we not get certain forms in nature? The developmental process embodies various systems of constraint that bias the evolution of the system. Teratologies, among other things, provide an excellent source of information on the potential of developmental processes. They also suggest which monstrosities are possible and which are not. It is interesting that specific morphologies are found in quite different species, suggesting a certain common developmental process for part of their development.

Concluding Discussion

This has been a very short and personal choice from the vast literature associated with the application of mathematical models in the biomedical sciences. In the 1980s, with most of the research conclusions speculative, there was a decrease in the new applications of reaction-diffusion models, since demonstrating the existence of specific morphogens was proving elusive. This resulted in the new mechanochemical theory of biological pattern and form discussed briefly above, which was based on experimental data on real cells and the forces they could exert in the generation of pattern and form. From the mid-1990s on, the practical use of reaction-diffusion models again increased, with the biological applications becoming much more verifiable experimentally, as has research and developments of the Murray-Oster mechanochemical theory of pattern formation as more experimental data and confirmation of modeling predictions have been found.

As seen in many of the papers referred to here, models and their biological predictions encouragingly have been a major stimulant for guiding critical experiments, which have resulted in significant discoveries. This, of course, should be the aim of any mathematical biology modeling, namely, to stimulate in any way whatsoever any endeavor that results in furthering our understanding of biology. Although with the major developments in the past twenty years we now know a lot more about pattern development, most mechanisms involved in development are still not fully understood. We do not know, for example, the complete mechanisms of how cartilage patterns in developing limbs are formed or the specialized structures in the skin, such as feathers, scales, glands, and hairs, or the myriad of widely observed patterns. Many of the rich spectrum of spatial patterns observed in development evolve from a homogeneous mass of cells that are orchestrated by genes that initiate and control the pattern formation mechanisms; genes themselves are not involved in the actual physical process of pattern generation. The basic philosophy behind practical modeling in biology is to try to incorporate the physicochemical events, which from observation and experiment appear to be going on during development, within a model mechanistic framework that can then be studied mathematically and, importantly, the results related back to the biology. These morphogenetic models provide the embryologist with possible scenarios as to how and when pattern is laid down, how elements in the embryo might be created, and what constraints on possible patterns are imposed by different models. Many of the references in this article have greatly increased our biological understanding.

Both the mechanochemical models and reaction-diffusion models have been fruitfully applied to a vast range of biological problems, not only in morphogenesis but elsewhere, such as feather primordia arrangement, wound healing mentioned above, wound scarring, shell and mollusk patterns, and many others [46]. It is almost certain that both mechanisms are involved in development, and although they are in a sense competing theories, in fact the mechanisms complement each other. Perhaps the most fundamental difference between the theories is that the elements involved directly in the mechanical theory are all real biological quantities: namely, cells, tissue, and the forces generated by the cells. *All* quantities involved are measurable. In the end, however, the key aspect of these mechanisms is their ability to predict the subsequent pattern and form. The final arbiter of a model's correctness and usefulness is how consistent it appears in the light of subsequent experiments and observations.

The explosion in biochemical techniques over the last several decades has led to a still larger increase in our biological knowledge but has partially eclipsed the study of the intermediate mechanisms that translate gene influence into chemicals, into gradients, and into pattern and form. As a result, there is much still to be done in this area, both experimentally and theoretically.

We have clearly only scratched the surface of a huge, important, and ever-expanding interdisciplinary world. Biology, in its broadest sense, is clearly *the* science of the foreseeable future. What is clear is that the application of mathematical modeling in the biological, medical, ecological, psychological, and social sciences is going to play an increasingly important role in future major discoveries and epidemiological and population control strategies. There is an ever-increasing number of areas where theoretical modeling is important, such as social behavior, adaption to habitat changes, climate change, and so on. In the case of zebras, for example, in [54] it is shown, by unraveling how species adapt to specific environmental changes such as land use, why of two types of zebra in the same environment the Grevy zebra (*Equus grevyi*) is nearing extinction, while another, the plains zebra (*Equus burchelli*), has adapted its behavior to survive. Behavioral ecology is another important expanding area of research. How bird flocks, schools of fish, and so on reach community decisions is another exciting, relatively new area; see [48] for research on fish community decisions and [8] for locusts.

Mathematical biology now has active researchers, numbering in the thousands, in practically all of the biomedical sciences. Mathematical modeling in the social sciences is another growth area of the future. One example of this involvement is the theoretical model developed for a major study on marital interaction and divorce prediction. The basic model and its practical application are based on a model proposed in [12], developed and used in a major study of seven hundred newly married couples (see [20], [21] and for a survey [46]). The prediction of the future of marital stability proved surprisingly accurate, with an accuracy of 94 percent. Its use in marital therapy is proving highly successful.

Any mathematical or theoretical biological research must have genuine interdisciplinary content. There is no way mathematical modeling can solve major biological problems on its own. On the other hand, it is highly unlikely that even a reasonably complete understanding could come solely from experiment.

References

[1] P. Alberch, Developmental constraints in evolutionary processes, in J. T. Bonner, editor, *Evolution and Development*, Dahlem Conference Report, volume 20, Springer-Verlag, Berlin-Heidelberg-New York, 1982, 313–332.

[2] ———, The logic of monsters: Evidence for internal constraint in development and evolution, *Geo. Bios., Mémoire Spéciale* **12** (1988), 21–57.

[3] P. Alberch and E. Gale, Size dependency during the development of the amphibian foot, Colchicine induced digital loss and reduction, *J. Embryol. Exp. Morphol.* **76** (1983), 177–197.

[4] William L. Allen, Innes C. Cuthill, Nicholas E. Scott-Samuel, and Roland Baddeley, Why the leopard got its spots: Relating pattern development to ecology in felids, *Proc. R. Soc. B* **278** (2011), 1373–1380.

[5] R. E. Baker, S. Schnell, and P. K. Maini, A clock and wavefront mechanism for somite formation, *Dev. Biol.* **293** (2006), 116–126.

[6] D. Bernoulli, Essai d'une nouvelle analyse de la mortalité causée par la petite vérole, et des avantages de l'inoculation pour la prévenir, *Histoire de l'Acad. Roy. Sci. (Paris) avec Mém. des Math. et Phys. and Mém.* (1760), 1–45.

[7] S. Brenner, J. D. Murray, and L. Wolpert (editors), *Theories of Biological Pattern Formation*, Proceedings of the Royal Society meeting of that name at the Royal Society, London, 1981.

[8] J. Buhl, D. J. T. Sumpter, I. C. Couzin, J. Hale, E. Despland, E. Miller, and S. J. Simpson, From disorder to order in marching locusts, *Science* **213** (2006), 1402–1406.

[9] P. K. Burgess, P. M. Kulesa, J. D. Murray, and E. C. Alvord Jr., The interaction of growth rates and diffusion coefficients in a three-dimensional mathematical model of gliomas, *J. Neuropathol Exp. Neurol* **56** (1997), 704–713.

[10] S. Camazine, J.-L. Deneubourg, J. Franks, J. Sneyd, Guy Theraulaz, and Eric Bonabeau, *Self-Organization in Biological Systems*, Princeton University Press, 2001.

[11] M. A. J. Chaplain, G. D. Singh, and J. C. Maclachlan, *On Growth and Form Spatio-temporal Pattern Formation in Biology*, John Wiley and Son, Ltd., 1999.

[12] J. Cook, R. Tyson, K. A. J. White, R. Rushe, J. Gottman, and J. D. Murray, Mathematics of marital conflict: Qualitative dynamic mathematical modelling of marital interaction, *J. Family Psychology* **9** (1995), 110–130.

[13] G. C. Cruywagen, D. E. Woodward, P. Tracqui, G. T. Bartoo, J. D. Murray, and E. C. Alvord Jr.,

The modeling of diffusive tumors, *J. Biol. Systems* **3** (1995), 937–945.

[14] P. D. Dale, P. K. Maini, and J. A. Sherratt, Mathematical modelling of corneal epithelial wound healing, *Math. Biosci.* **124** (1994), 127–147.

[15] R. A. Fisher, *The Genetical Theory of Natural Selection*, 1930 (reprint: Dover, New York, 1958).

[16] E. A. Gaffney, P. K. Maini, J. A. Sherratt, and S. Tuft, The mathematical modelling of cell kinetics in corneal epithelial wound healing, *J. Theor. Biol.* **197** (1999), 15–40

[17] R. A. Gatenby, K. Smallbone, P. K. Maini, F. Rose, J. Averill, R. B. Nagle, L. Worrall, and R. J. Gillies, Cellular adaptations to hypoxia and acidosis during somatic evolution of breast cancer, *Brit. J. Cancer* **97** (2007), 646–653.

[18] B. T. Grenfell, O. G. Pybus, J. R. Gog, J. L. N. Wood, J. M. Daly, J. A. Mumford, and E. C. Holmes, Unifying the epidemiological and evolutionary dynamics of pathogens, *Science* **303** (2004), 327–332.

[20] J. M. Gottman, J. D. Murray, C. Swanson, R. Tyson, and K. R. Swanson, *The Mathematics of Marriage: Dynamic Nonlinear Models*, MIT Press, Cambridge, MA, 2002.

[21] J. M. Gottman, C. Swanson, and J. D. Murray, The mathematics of marital conflict: Dynamic mathematical nonlinear modelling of newlywed marital interaction, *J. Family Psychol.* **13** (1999), 1–17.

[22] A. L. Hodgkin and A. F. Huxley, A quantitative description of membrane current and its application to conduction and excitation in nerve, *J. Physiol.* (Lond.) **117** (1952), 500–544.

[23] W. Jäger and J. D. Murray (editors), *Modeling of Patterns in Space and Time*, Proceedings of a workshop of that name in Heidelberg, 1983, Springer-Verlag, Heidelberg, 1994.

[24] M. D. Johnston, C. M. Edwards, W. F. Bodmer, P. K. Maini, and S. J. Chapman, *Mathematical modeling of cell population dynamics in the colonic crypt and in colorectal cancer*, *PNAS* **104** (2007), 4008–4013.

[25] J. Keener and J. Sneyd, *Mathematical Physiology*, Springer-Verlag, 1998. (Second Edition in 2 volumes, Springer-Verlag, 2008.)

[26] W. O. Kermack and A. G. Mckendrick, Contributions to the mathematical theory of epidemics, *Proc. R. Soc. Lond.* A **138** (1932), 55–83.

[27] A. Kolmogoroff, I. Petrovsky, and N. Piscounoff, Étude de l'équation de la diffusion avec croissance de la quantité de matière et son application à un problème biologique, *Moscow University Math. Bull.* **1** (1937), 1–25.

[28] S. Kondo, M. Iwashita, and M. Yamaguchi, How animals get their skin patterns: Fish pigment pattern as a live Turing wave, *Inst. J. Dev. Biol.* **53** (2009), 851–856.

[29] Simon A. Levin (editor), *Frontiers in Mathematical Biology*, Springer-Verlag, Berlin, 1992.

[30] ———, The problem of pattern and scale in ecology *Ecology* **73** (6) (1992), 1943–1967.

[31] M. A. Lewis, M. A. J. Chaplain, J. P. Keener, and P. K. Maini (editors), *Mathematical Biology*, IAS/Park City Mathematics Series **14**, Amer. Math. Soc., 2009.

[32] A. J. Lotka, *Elements of Physical Biology*, Williams and Wilkins, Baltimore, 1925.

[33] Philip K. Maini and Hans G. Othmer (editors), *Mathematical Models for Biological Pattern Formation*, Mathematics and its Applications, IMA Vol. 121, Springer, New York, 2000.

[34] Philip K. Maini, How the mouse got its stripes, *Proc. Nat. Acad. Sci.* **100** (2003), 9656–9657.

[35] ———, Using mathematical models to help understand biological pattern formation, *C. R. Biologies* **327** (2004), 225–234.

[36] H. Meinhardt, *Models of Biological Pattern Formation*, Academic Press, London, 1982.

[37] J. D. Murray, *Nonlinear Differential Equation Models in Biology*, Clarendon Press, Oxford, 1977.

[38] ———, A pre-pattern formation mechanism for animal coat markings, *J. Theor. Biol.* **88** (1981), 161–199.

[39] ———, On pattern formation mechanisms for lepidopteran wing patterns and mammalian coat markings, *Phil. Trans. Roy. Soc. (Lond.) B* **295** (1981), 473–496.

[40] J. D. Murray, G. F. Oster, and A. K. Harris, A mechanical model for mesenchymal morphogenesis. *J. Math. Biol.* **17** (1983), 125–129.

[41] J. D. Murray and G. F. Oster, Generation of biological pattern and form, *IMA J. Maths. Appl. in Medic. & Biol.* **1** (1984), 51–75.

[42] J. D. Murray and P. K. Maini, A new approach to the generation of pattern and form in embryology, *Science Progress* **70** (1986), 539–553.

[43] J. D. Murray, Mammalian coat patterns: How the leopard gets its spots, *Scientific American* **256** (1988), 80–87.

[44] ———, *Mathematical Biology*, Springer-Verlag, Heidelberg, 1989.

[45] ———, Turing's theory of morphogenesis—its influence on modeling biological pattern and form, *Bull. Math. Biol.* **52** (1990), 119–152.

[46] ———, *Mathematical Biology. I: An Introduction*, Springer, New York, 2002; *Mathematical Biology. II: Spatial Models and Biomedical Applications* (3rd edition in 2 volumes), Springer, New York, 2003.

[47] ———, On the Growth of Brain Tumours: Enhancing Imaging Techniques, Highlighting Limitations of Current Imaging, Quantifying Therapy Efficacy and Estimating Patient Life Expectancy, in: *Advances in Artificial Life, ECAL 2011: Proceedings of the Eleventh European Conference on the Synthesis and Simulation of Living Systems* (eds. Tom Lenaerts, Mario Giacobini, Hugues Bersini, Paul Bourgigne, Marca Dorigo, René Doursat), MIT Press, 2011, pp. 23–26. (MIT Press online open-access proceedings volume: `http://mitpress.mit.edu/catalog/item/default.asp?ttype=2&tid=12760`.)

[48] B. Nabet, N. Leonard, I. D. Couzin, and S. A. Levin, Dynamics of decision making in animal group motion, *Journal of Nonlinear Science* **19** (4) (2009), 399–345.

[49] N. Frederick Nijhout, Philip K. Maini, Anotida Madzvamuse, Andrew J. Wathen, and Toshio Sekimura, Pigmentation pattern formation in butterflies: Experiments and models, *C. R. Biologies* **326** (2003), 717–727.

[50] G. F. Oster, J. D. Murray, and A. K. Harris, Mechanical aspects of mesenchymal morphogenesis, *J. Embryol. Exp. Morph.* **78** (1983), 83–125.

[51] G. F. Oster, N. Shubin, J. D. Murray, and P. Alberch, Evolution and morphogenetic rules: The shape of

the vertebrate limb in ontogeny and phylogeny, *Evolution* **42** (1988), 862–884.

[52] Q. Ouyang and H. L. Swinney, Transition from a uniform state to hexagonal and striped Turing patterns, *Nature* **352** (1991), 610–612.

[53] K. J. Painter, P. K. Maini, and H. G. Othmer, Stripe formation in juvenile *Pomacanthus* explained by a generalized Turing mechanism with chemotaxis, *Proc. Nat. Acad. Sci.* **96** (1999), 5549–5554.

[54] D. I. Rubenstein, Ecology, social behavior, and conservation in zebras, in: *Advances in the Study Behavior: Behavioral Ecology of Tropical Animals* (Macedo, R. ed.), Vol. 42, Elsevier Press, Oxford, UK, 2010, pp. 231–258.

[55] Geoffroy Saint-Hilaire, *Traité de Tératologie*, Vols. 1–3, Baillière, Paris, 1836.

[56] J. A. Sherratt, *Mathematical Models of Wound Healing*, D. Phil. Thesis, University of Oxford, 1991.

[57] J. A. Sherratt and J. D. Murray, Epidermal wound healing: The clinical implications of a simple mathematical model, *Cell Transplantation* **1** (1992), 365–371.

[58] N. Shubin and P. Alberch, *A morphogenetic approach to the origin and basic organization of the tetrapod limb*, in M. Hecht, B. Wallace, and W. Steere, editors, *Evolutionary Biology*, Volume 20, 319–387. Plenum, New York, 1986.

[59] N. Suzuki, M. Hirata, and S. Kondo, Traveling stripes on the skin of a mutant mouse, *Proc. Nat. Acad. Sci. USA* **100** (2003), 9680–9685.

[60] K. R. Swanson, E. C. Alvord Jr., and J. D. Murray, Virtual brain tumors (gliomas) enhance the reality of medical imaging and highlight inadequacies of current therapy, *Br. J. Cancer* **86** (2002a), 14–18.61.

[61] ______, Quantifying the efficacy of chemotherapy of brain tumors with homogeneous and heterogeneous drug delivery, *Acta Biotheoretica* **50** (2002b), 223–237.

[62] D. Thomas, Artificial enzyme membranes, transport, memory, and oscillatory phenomena, in: *Analysis and Control of Immobilized Enzyme Systems*, D. Thomas and J.-P. Kernevez (editors), Springer-Verlag, Berlin-Heidelberg-New York, 1975, 115–150.

[63] D'Arcy W. Thompson, *On Growth and Form*, Cambridge University Press, 1917. (2nd edition, 1942.)

[64] A. M. Turing, The chemical basis of morphogenesis, *Phil. Trans. Roy. Soc.* **B237** (1952), 37–72.

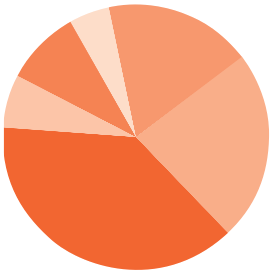
[65] V. Volterra, Variazionie fluttuazioni del numero d'individui in specie animali conviventi (Variations and fluctuations of a number of individuals in animal species living together, translation by R. N. Chapman. In: *Animal Ecology*, McGraw Hill, New York, 1931, pp. 409–448; *Mem. Acad. Lincei.* **2** (1926), 31–113.

[66] A. T. Winfree, Spiral waves of chemical activity, *Science* **175** (1972), 634–36.

[67] L. Wolpert, Positional information and the spatial pattern of cellular differentiation, *J. Theor. Biol.* **25** (1969), 1–47.

[68] ______, *Principles of Development*, Oxford University Press, 2006.

[69] A. M. Zhabotinsky, Periodical oxidation of malonic acid in solution (a study of the Belousov reaction kinetics), *Biofizika* **9** (1964), 306–11.

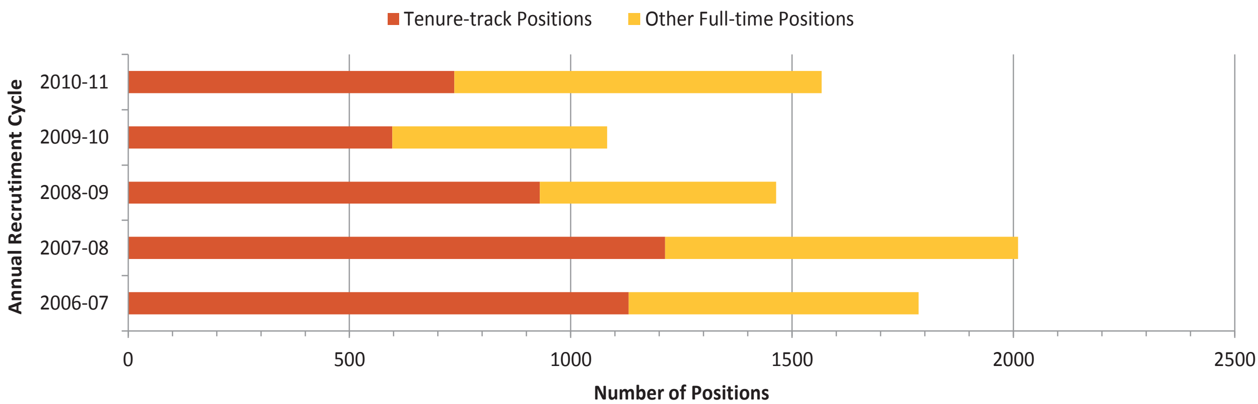


Report on 2010–2011 Academic Recruitment and Hiring

James W. Maxwell and Colleen Rose

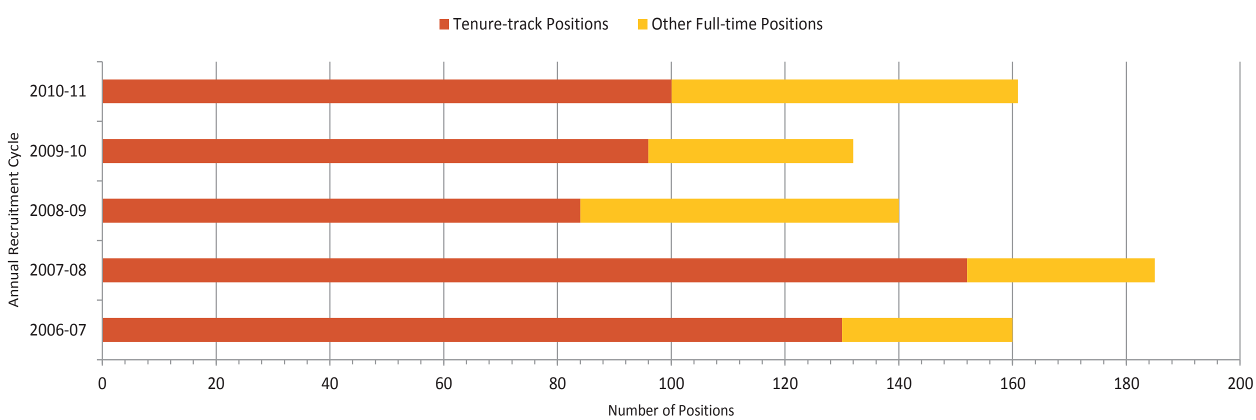
The number of full-time positions under recruitment in mathematics departments increased during the 2010–2011 academic recruitment cycle. The total number of positions under recruitment by all mathematics departments combined was 1,567. This number is up 45% from the 2009–2010 total and up 7% from the 2008–2009 total. (Note: Throughout this report, the term tenure-track encompasses positions that come with tenure as well as those which provide the option of earning tenure at some point after appointment.)

Figure R.1: Positions Under Recruitment in All Mathematics Departments Combined



The doctoral statistics and biostatistics departments, Group IV, also saw an increase (22%) in the level of recruitment, though not as precipitous an increase as for the mathematics departments. Group IV consists of 93 departments and so constitutes a much smaller employment pool than the approximately 1,400 mathematics departments. In 2010 there were approximately 1,860 full-time faculty in Group IV compared to approximately 23,023 full-time faculty in all mathematics departments combined.

Figure R.2: Positions Under Recruitment in Doctoral Statistics and Biostatistics Departments Combined



James W. Maxwell is AMS associate executive director for special projects. Colleen A. Rose is AMS survey analyst.

Positions Under Recruitment

The percentage increase in positions under recruitment varied significantly among the various reporting groups within the combined mathematics departments. There was a 34% increase for the doctoral mathematics departments combined, a 63% increase for masters mathematics departments (Group M) and a 51% increase for bachelors mathematics departments (Group B). Similarly, the increases in tenure-track positions under recruitment for these same department groupings were 20%, 27%, and 25%, respectively.

Figure R.3: Positions Under Recruitment in All Mathematics Departments by Highest Mathematical Sciences Degree Offered

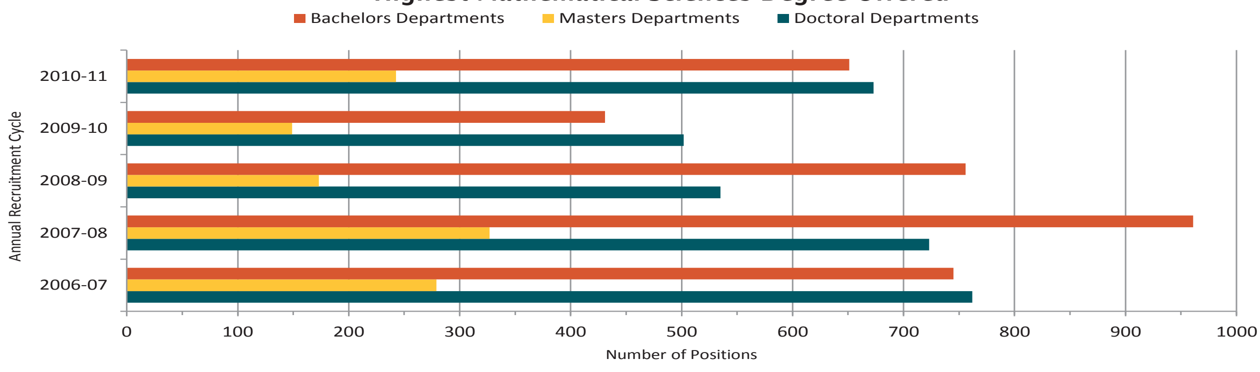
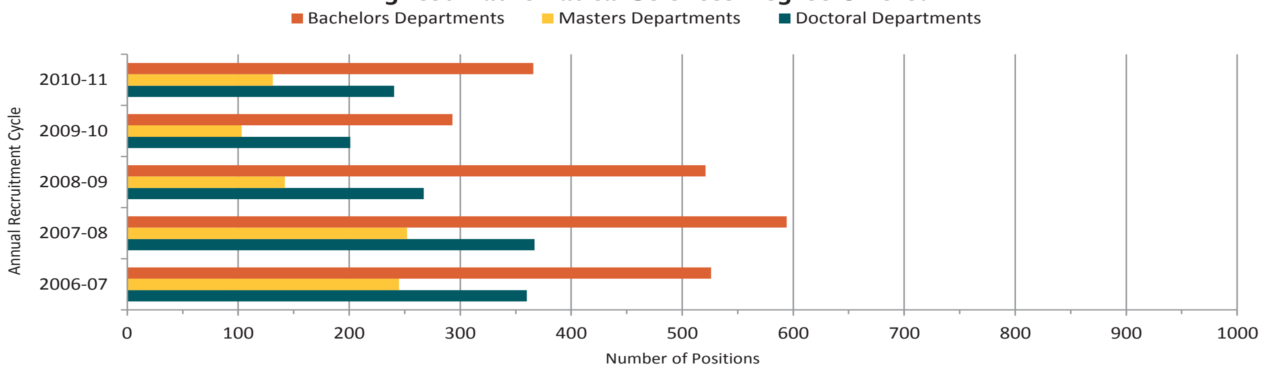


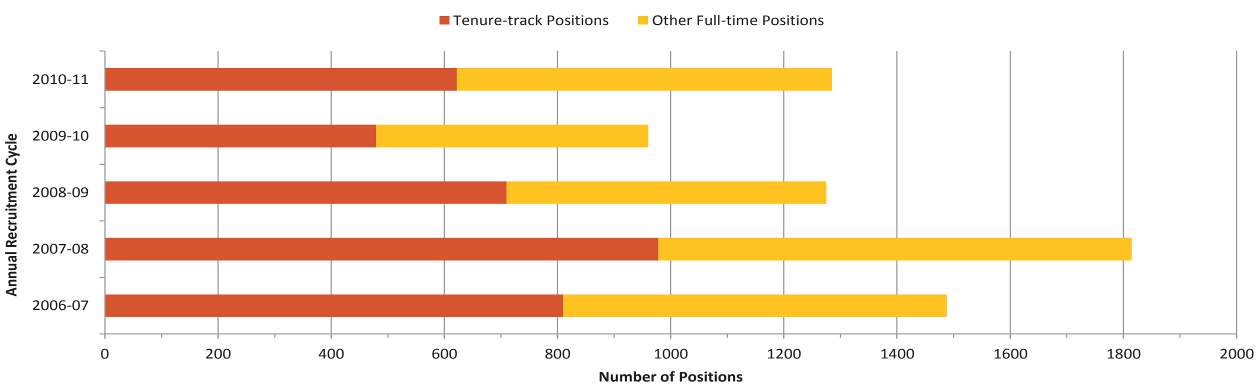
Figure R.4: Tenure-track Positions Under Recruitment in All Mathematics Departments by Highest Mathematical Sciences Degree Offered



Positions Filled

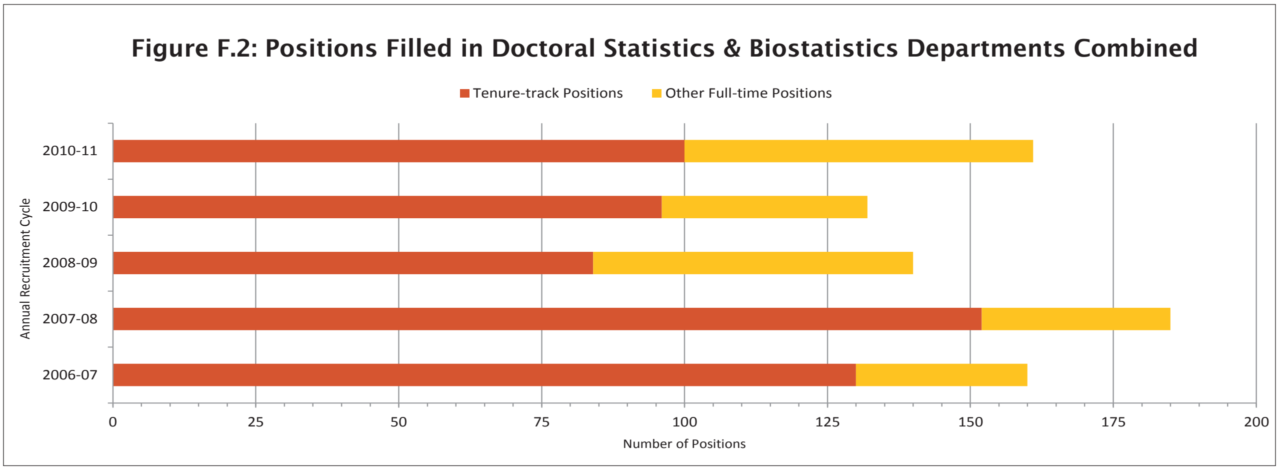
A total of 1,290 positions were filled during the 2010–2011 academic cycle for employment beginning in fall 2011 by all mathematics departments combined. This total is up 34% from the 2009–2010 total and up 1% from the 2008–2009 total.

Figure F.1: Positions Filled in All Mathematics Departments Combined

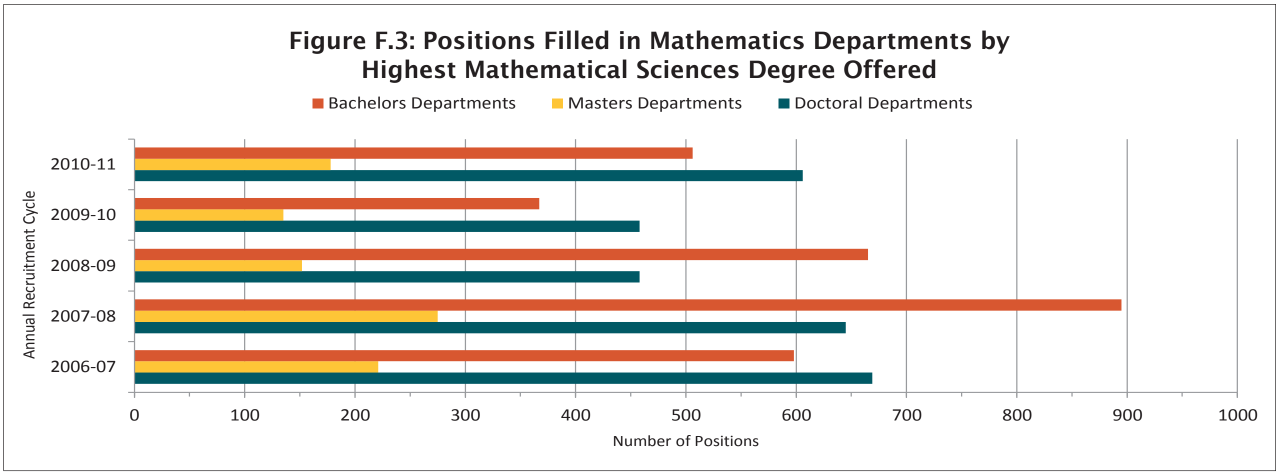


Positions Filled

The situtation for doctoral statistics departments and biostatistics departments combined was much the same, as demonstrated by the accompanying figure. The total of filled positions, is up 21% from the 2009-2010 total and up 15% from the 2008-2009 total.



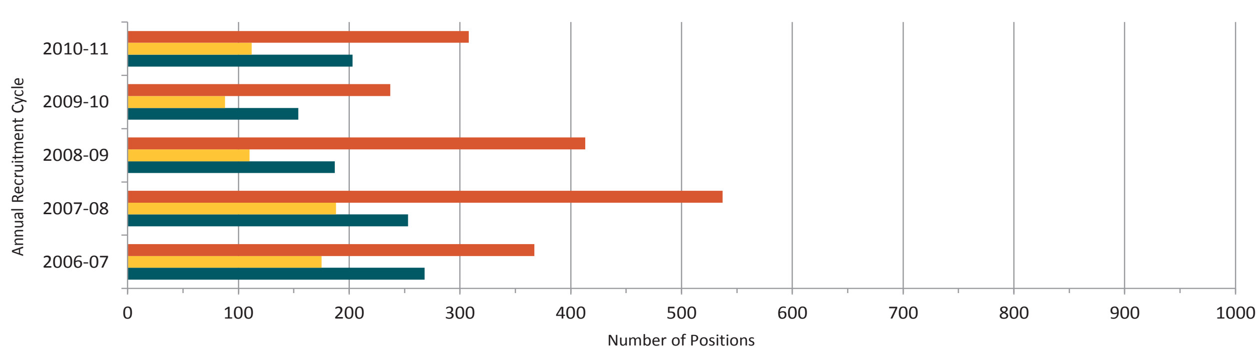
The increase in positions filled for fall 2011 also varied widely among the various reporting groups within all mathematics departments. For the doctoral mathematics departments combined, the number of positions filled was 606, an increase of 32% from the fall 2010 and fall 2009 counts. For Group M the count was 178, up 32% from fall 2010 and up 17% from the fall 2009 count. For Group B the count was 506, up 38% from fall 2010 and down 24% from fall 2009.



Positions Filled

The total tenure-track positions filled during the 2010–2011 academic cycle by all mathematics departments combined was 623, up 30% from the 2009–2010 total of 479. This total is down 12% from the 2008–2009 figure of 710 and down 36% from the 2007–2008 total of 978 (the ten year high).

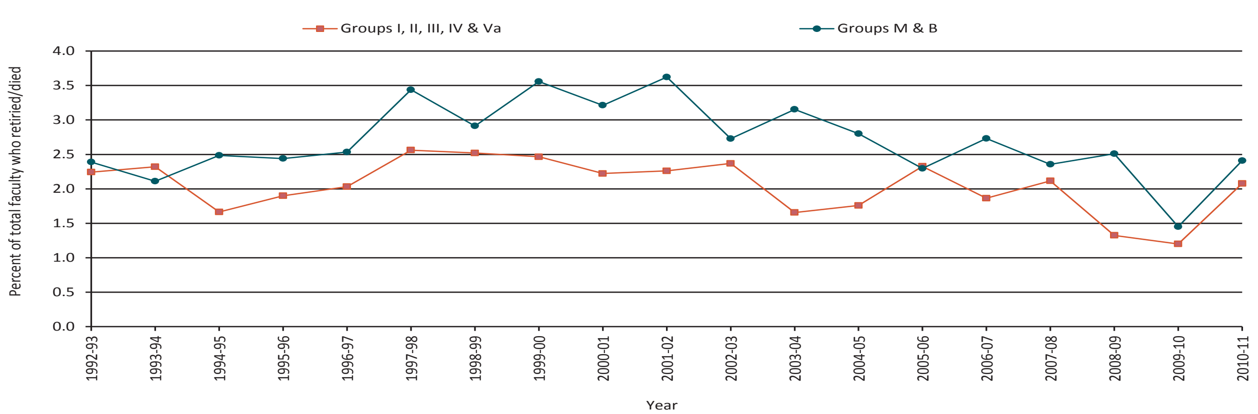
Figure F.4: Tenure-track Positions Filled in Mathematics Departments by Highest Mathematical Sciences Degree Offered



Faculty Attrition

Figure A.1 shows the trends in attrition from deaths and retirements among the full-time faculty for the academic years 1994–1995 through 2010–2011. In the late 1990s attrition leveled off, then began dropping after 2000, reaching the smallest rate of attrition in 2009–2010.

Figure A.1: Percentage Full-time Faculty Died/Retired*



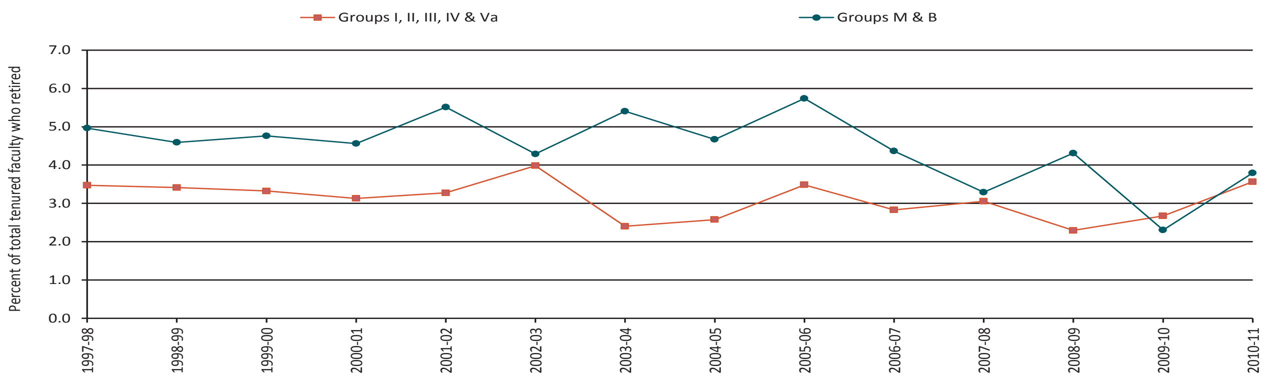
* The percentage of full-time faculty who died or retired is the number of faculty who died or retired at some point during the academic year (September 1 through August 31) divided by the number of full-time faculty at the start of the academic year.

- Group Va reported the smallest rate of attrition, 0.2%
- Group I Pub reported the highest rate of attrition, 2.5%.
- Group II reported the largest percentage increase in attrition at 2.4% up from 1.1% last year.

Faculty Attrition

Figure A.2 shows an alternative way of looking at the trends in annual faculty retirements compared to that offered in Figure A.1. It seems highly likely that the vast majority of individuals who are reported by their department as retiring are, in fact, members of the tenured faculty. Given that, it makes sense to look at the ratio of those retiring during an academic year to the total tenured faculty at the start of that year, as is done in A.2. Data collected for the first time during the 2011 Annual Survey confirmed that over 90% of those retiring where tenured. For 2011, the percentage of tenured faculty that retired from Groups I–Va was 3.3; among Groups M & B this percentage was 3.4.

Figure A.2: Retired Faculty as a Percentage of Tenured Faculty



* Each percentage in this figure is the number of full-time faculty that retired at some point during the academic year (September 1 through August 31) divided by the number of full-time tenured faculty at the start of the academic year.

Other Information

The interested reader may view additional details on the results of this survey and prior year trends by visiting the AMS website at `www.ams.org/annual-survey`. Survey results for the doctoral departments in statistics and biostatistics are available there.

Survey Response Rates

Faculty Recruitment & Hiring Response Rates

Department*	Number	Percent
Group I (Public)	19 of 25	76
Group I (Private)	15 of 23	65
Group II	47 of 56	84
Group III	65 of 81	80
Group IV (Statistics)	37 of 58	64
Group IV (Biostatistics)	19 of 35	54
Group Va	17 of 20	85
Group M	99 of 179	55
Group B	440 of 1013	43

* Departments and programs that do not formally "house" faculty and their salaries are excluded from this survey.

Group Descriptions

Group I is composed of 48 departments with scores in the 3.00–5.00 range. Group I Public and Group I Private are Group I departments at public institutions and private institutions, respectively.

Group II is composed of 56 departments with scores in the 2.00–2.99 range.

Group III contains the remaining U.S. departments reporting a doctoral program, including a number of departments not included in the 1995 ranking of program faculty.

Group IV contains U.S. departments (or programs) of statistics, biostatistics, and biometrics reporting a doctoral program.

Group V contains U.S. departments (or programs) in applied mathematics/applied science, operations research, and management science that report a doctoral program.

Group Va is applied mathematics/applied science; Group Vb, which was no longer surveyed as of 1998–99, was operations research and management science.

Group M contains U.S. departments granting a master's degree as the highest graduate degree.

Group B contains U.S. departments granting a baccalaureate degree only.

Listings of the actual departments which compose these groups are available on the AMS website at `www.ams.org/annual-survey/groups_des`.

About the Cover

Alan Turing's Morphogenesis Plots

The cover displays two remarkable pictures drawn by Alan Turing (who was born 100 years ago), presumably derived from computer output based on the earliest known computations of pattern generation, sometime between 1952 and 1954, following his theory of morphogenesis. The only publications I am aware of that discuss these are by Jonathan Swinton:

"Watching the daisies grow: Turing and Fibonacci phyllotaxis" in the book *Alan Turing: Life and Legacy of a Great Thinker*, available at

`http://www.swintons.net`

"Turing, morphogenesis, and Fibonacci phyllotaxis: Life in pictures" to appear soon in a book edited by S. Barry Cooper.

The numbers laid out on the grid are expressed in "digits" 0–31 of the teleprinter code, in which / is 0 and \ is 32, used on the Mark 1 computer, explained at

`http://www.computer50.org/kgill/mark1/RobertTau/node4.html`

Swinton tells us: "In general each square contains two digits from the base 32 code, most significant digit on the right. It's clear that the contouring is based on that most significant digit… The exact algorithm is unknown. Almost certainly they are reaction-diffusion systems, mathematically similar or probably identical to those described in the manuscript *Outline of the Development of the Daisy*. Turing always thought of these systems in Fourier space, where the cylindrical geometry restricts wavevectors to being on a set of parallel lines. Being near to to a given symmetry restricts to discrete points on those lines, and being near to a Turing instability makes only the points near the critical wavenumber circle be important. It's likely that the code that generated these Figures computed the evolution of the magnitude of a small number of these Fourier coefficients, and also computed the resulting pattern in real space. There are code fragments I think were used to do this, but I can't tie them definitively to the Figures."

We obtained the photographs from King's College Library, Cambridge, where they are catalogued as AMT/K/3/7-8. You can see these and others in the series at

`http://www.turingarchive.org/browse.php/K/3`

We wish to thank the archivist at King's College, Patricia McGuire, for her great help.

—*Bill Casselman*
Graphics Editor
(notices-covers@ams.

Yang-Mills and Beyond

Samuel L. Marateck

As recently as 2010, three groups received the J. J. Sakurai Prize for showing how spontaneous symmetry breaking explains the mechanism for generating vector boson masses, thus enabling the solution of one of the puzzles of the standard model of elementary particles. Since the Yang-Mills field strength incorporating a non-abelian feature is not only one of the cornerstones of the standard model, which is the basis of the ongoing search for the Higgs boson, but also since Yang-Mills theory influenced the development of algebraic and differential geometry (e.g., see Donaldson [1] and [2], and Dragomir et al. [3]), perhaps it is not out of order to explore the development of the formula for the field strength, F. In modern notation it is given in the form $F = dB + B \wedge B$, where B is the gauge field. Although Yang-Mills (YM) gauge theory can be done using fiber-bundle theory—see, for instance, the review articles of Daniel and Viallet [4] and Marateck [5]—it is historically useful to analyze the way it developed using classical gauge theory.

In their seminal paper [6], Yang and Mills invented the non-abelian field strength[1] to satisfy certain criteria but did not explain how it could be derived. In the penultimate section here we show how the Yang-Mills field strength derives from Yang's gauge transformation. The preceding sections place Yang-Mills theory in historical perspective and cover material relating to the field strength. We begin in the next section by reminding the reader what gauge invariance is, starting with its use in electromagnetism, then showing phase-invariance and gauge-invariance use in quantum electrodynamics, thus laying the groundwork for Yang's derivation of the YM field transformation. We then review the history of gauge theory in the beginning of the quantum mechanics era.

In their 1954 paper [6], when Yang and Mills discuss the phase factor-gauge transformation relationship, they cite Pauli's review paper [7]. It is interesting that, although Pauli in that paper presents the electromagnetic field strength in terms of a commutator, for whatever reason Yang and Mills did not use the commutator approach to obtain the YM non-abelian field strength; they obtained it by generalizing the electromagnetic field strength and presumably found the result by trial and error. In the section on Yang-Mills field strength we present the derivation of this field strength using the commutator approach. In the following section Yang's field transformation is derived in a slightly different way than Yang did, and it is indicated that this transformation reduces to the gauge transformation for electromagnetism when the theory is considered abelian. In the penultimate section we show that the YM field strength derives from the electromagnetic field strength and that the commutation part is dictated by the field transformation. The final section indicates how Yang-Mills theory was combined with spontaneous symmetry breaking, the Goldstone theorem, and subsequent work to contribute to the standard model of particle physics.

Samuel L. Marateck is senior lecturer at the Courant Institute of Mathematical Sciences, New York University. His email address is `marateck@courant.nyu.edu`.

[1] *Their intent was to describe the strong interactions.*

DOI: http://dx.doi.org/10.1090/noti851

Gauge Theory

In order to appreciate gauge invariance in quantum mechanics, let's back up and investigate gauge invariance in electromagnetism. There a gauge transformation is a transformation of $\mathbf{A}_\mu$, the four-vector potential, that leaves the **B** and **E** fields unchanged and hence Maxwell's equations invariant. The nonhomogeneous Maxwell equations[2] can be expressed as

$$d * F = J, \tag{1}$$

where J is the current and $*$ is the Hodge star operator; and the homogeneous ones[3] can be expressed as

$$dF = 0, \tag{2}$$

where[4] $F = dA$. Here A, a one-form, is a connection on a fiber bundle and the field strength F, a two-form, is its curvature. Since $d^2 = 0$, then $dJ = 0$. This is the continuity equation.[5] It is evident that F and thus Maxwell's equations are invariant under the following transformation:

$$A \to A + d\alpha(x). \tag{3}$$

Because α is a function of x, (3) is an example of a local gauge transformation. The invariance of Maxwell's equations under (3) is called *gauge invariance* or *gauge symmetry.* In electromagnetism, gauge invariance facilitates problem solving. In quantum electrodynamics, it's much more useful, as will be seen when abelian gauge invariance is discussed below.

The Lagrangian density that produces the Dirac equation is

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi, \tag{4}$$

where $\partial\!\!\!/ = \gamma^\mu \partial_\mu$. Here **c**, the speed of light, and $\hbar$, Plank's constant divided by 2π, are set to one. A global phase transformation, e.g., $\psi' = e^{-iq\alpha}\psi$, is one in which the phase α is constant. It corresponds to a rotation in the complex plane. Since, in quantum mechanics, a phase transformation does not alter the physical reality, it is an example of a symmetry and can be used to reveal an underlying physical principle. One can show—although we will not—using the Euler-Lagrange equations for a Lagrangian with a global phase transformation, the existence of a conserved current. This is an example of Noether's theorem that a conserved current is associated with a continuous symmetry of the Lagrangian.

[2] *In covariant form it is* $\partial^\mu F_{\mu\nu} = j_\nu$.

[3] *In covariant form it is* $\epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta} = 0$, *where* $\epsilon^{\alpha\beta\gamma\delta}$ *is the Levi-Civita symbol.*

[4] *In covariant form it is* $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

[5] *In covariant form it is* $\partial^\nu j_\nu = 0$ *or* $\nabla \cdot \vec{\mathbf{J}} + \partial\rho/\partial t = 0$.

Let's now see the effect that a local gauge transformation has on the Lagrangian density in (4). Substituting the local phase transformation

$$\psi \to \psi' = e^{-iq\alpha(x)}\psi \tag{5}$$

into (4) spoils the invariance of the Lagrangian and has it depend on the choice of phase, that is,

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + q\partial\!\!\!/\alpha(x) - m)\psi. \tag{6}$$

In order to regain the invariance, substitute the covariant form $A \to A + \partial_\mu\alpha(x)$ of the gauge transformation in (3) and $\partial_\mu \to \partial_\mu + iqA_\mu$ (operating on ψ) into equation (6), getting $\mathcal{L} = \bar{\psi}(i\gamma^\mu(\partial_\mu + iqA_\mu) - m)\psi$. Letting $D_\mu = \partial_\mu + iqA_\mu$ and adding to the Lagrangian the electromagnetic energy density[6] $-(\frac{1}{2})F \wedge *F$ produce the gauge invariant form $\mathcal{L} = \bar{\psi}(iD\!\!\!\!/ - m)\psi - (\frac{1}{2})F \wedge *F$, where $D\!\!\!\!/$ is $\gamma^\mu(\partial_\mu + iqA_\mu)$. Moreover, if J^μ is set equal to $i\bar{\psi}q\gamma^\mu\psi$, the Lagrangian density can be written as $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - J \wedge A - (\frac{1}{2})F \wedge *F$. The second and third terms yield the nonhomogeneous Maxwell equations; the homogeneous ones are satisfied trivially, because the definition of F satisfies them automatically [8]. Note that F, and therefore $F \wedge *F$, is invariant under the gauge transformation in (3). If there were a term in the Lagrangian for the mass μ of the photon, because of the Klein-Gordon equation it would take the form $\frac{1}{2}\mu^2 A_\nu A^\nu$, but this term is not gauge invariant. So gauge invariance dictates that the mass of the photon must be zero. Okun [9] comments[7] that, although a tiny photon mass would destroy the gauge invariance of quantum electrodynamics (QED), it would not affect the renormalizability of QED and the resulting excellent agreement of it with experiment; the really important principle in QED is the conservation of charge. The article [9] cites Okun's previous work on this. Gauge invariance in which the phase, here α, is a function of x is called *gauge invariance of the second kind* and the Lagrangian is said to exhibit local symmetry. Global gauge invariance is called *gauge invariance of the first kind.*

When $\mathbf{D}_\mu$ operates on ψ it takes the same form in the primed system that it does in the unprimed system, i.e.,

$$D'_\mu\psi' = e^{-iq\alpha(x)}D_\mu\psi. \tag{7}$$

For that reason it is called the *covariant derivative.* As we will see in a later section, Yang

[6] *In covariant form it is* $-(\frac{1}{4})F_{\mu\nu}F^{\mu\nu}$.

[7] *Okun wrote the paper in 1998 for the celebration of the 100th anniversary of Fock's birth. It appeared in English (see* `http://www.itep.ru/theor/persons/lab180/okun/fock.pdf`*) in the UNESCO-sponsored* Quantum Theory in Honour of Vladimir A. Fock *(1998). Among other things, it relates the beginning of the collaboration between Jackson and Okun* [16]. *Okun revised it in 2010* [9] *and added an appendix.*

used this idea to derive the field transformation for the non-abelian case. Note that $\partial_\mu \psi' = (-iq\partial_\mu \alpha\psi + \partial_\mu \psi)e^{-iq\alpha(x)}$ and D'_μ, which equals $\partial_\mu + iq(A_\mu + \partial_\mu \alpha)$, operates on $e^{-iq\alpha(x)}\psi$, so (7) is obvious. Gauge invariance determines the type of interaction; here, this is manifested by the inclusion of the vector potential in the covariant derivative. This is called the *gauge principle*, and $\mathbf{A}_\mu$ is called the *gauge field* or *gauge potential*. The phase factors $e^{-iq\alpha(x)}$ form the gauge group $U(1)$ of unitary 1×1 matrices. Since the elements of the group commute, it is an abelian group. Associated with this gauge group is the gauge field $\mathbf{A}_\mu$. The quantum of the gauge field is the gauge boson, which in the case of the electromagnetic field is the photon. A boson has integral spin and therefore obeys Bose-Einstein statistics.

The introduction of gauge invariance has promoted the four-vector potential $\mathbf{A}_\mu$ from being a mathematical construct in electromagnetism—as J. D. Jackson related to Okun [9], the three-vector version of (3) was known to Maxwell—to causing the shift in the interference pattern in the Aharonov-Bohm solenoid effect [10]. Moreover, gauge invariance determines interactions and thus the observable interaction forces. We have already discussed its use in electromagnetic theory: as the generalized four-vector $\mathbf{A}_\mu$ in the covariant derivative, $\mathbf{A}_\mu$ becomes the gauge field that mediates the electromagnetic interaction, i.e., the photon. In the theory of the weak interactions, $\mathbf{A}$ represents the intermediate vector bosons $W^\pm$ and Z^0 fields, and in the strong interaction, $\mathbf{A}$ represents the colored gluon fields. This application is not limited to physics. In mathematics $\mathbf{A}_\mu$ is understood as the connection on fiber-bundles in differential geometry.

Now that we have seen the mathematical basis of gauge theory, we can appreciate its historical development. As we discussed in [5], modern gauge theory has its roots in Weyl's 1918 paper [11], in which he tried to combine electromagnetism and gravity by requiring the theory to be invariant under a local scale change of the metric $g_{\mu\nu} \to g_{\mu\nu}e^{\alpha(x)}$, where x is a 4-vector. This was criticized by Einstein, because lengths and time intervals for a parallel transported vector would be path dependent in contradiction to the sharpness of spectral lines. It is in this paper, however, that Weyl introduced the German term for gauge invariance, *Eichinvarianz*. The first appearance of "gauge invariance" in English was in Weyl's 1929 English version [12] of his famous 1929 paper [13]. At the 1985 International Symposium on Particle Physics in the 1950s: Pions to Quarks [14], Yang noted that if in Einstein's objection to Weyl's 1918 paper you replaced scale with phase, substituted electrons for clocks, and had a magnetic flux within the circuit, you would reproduce the Aharonov-Bohm effect. In 1926, Fock [15] showed that for a quantum theory of charged particles interacting with the electromagnetic field, invariance under a gauge transformation of the potentials (3) required multiplication of the wave function by the now well-known phase factor,[8] which in present-day notation is given by (5). Fock, however, at that time did not give a name to this principle.

By 1929 Maxwell's equations had been combined with quantum mechanics to produce the start of quantum electrodynamics. Weyl in [13] turned from trying to unify electromagnetism and gravity to following (without attribution) Fock's suggestion [15] and introduced as the phase factor an exponential in which the phase $\lambda(x)$ is preceded by the imaginary unit i, i.e., $e^{i\lambda(x)}$. This multiplies the wave function in the wave equations. In [12] he states it quite succinctly:

> …the laws are invariant under the simultaneous substitution of $e^{i\lambda}\psi$ for ψ and $\varphi_p - \frac{\partial\lambda}{\partial x_p}$ for φ_p, where λ is an arbitrary function of position in space and time.

Here φ_p are the four potentials. So the change of scale in Weyl's 1918 paper becomes a change of phase. It was here that Weyl suggested that gauge invariance be used as a symmetry principle to derive the electromagnetic interactions. Of course, by that time, electromagnetic theory was already well established. The importance of gauge invariance, however, was in its application to quantum field theory—it was instrumental in charting the way field theory would develop. As Jackson and Okun [16] so eloquently put it:

> Historically, of course, Weyl's 1929 papers were a watershed. They enshrined as fundamental the modern principle of gauge invariance in which the existence of the 4-vector potentials (and field strengths) follow from the requirement that the matter equation be invariant under gauge transformation such as [our (5)] of the matter fields. This principle is the touchstone of the theory of gauge fields, so dominant in theoretical physics in the second half of the 20th century.

It is interesting that in 1938 Oskar Klein gave a paper [17] at a conference[9] in Warsaw, Poland, in which he presented an expression for the

[8] *Fock used $e^{ip/\hbar}$, where $p = p_1 - e/cf$ and f is an arbitrary function of space-time coordinates.*

[9] *The participants included some of the great physicists of the era. Among the attendees were N. Bohr, L. Brillouin, L. de Broglie, C. Darwin, A. Eddington, R. Fowler, G. Gamow, S. Goudsmit, O. Klein, H. Kramers, L. de Kronig, P. Langevin, C. Moeller, J. von Neumann, F. Perrin, L. Rosenfeld, and E. Wigner.*

electromagnetic field strength coming from the contribution of the charged vector boson that looks similar to the non-abelian Yang-Mills field strength but actually is not [18]. When questioned by Moeller at the end of his talk, Klein in effect generalized his SU(2) theory to one that was like an $\mathrm{SU}(2) \times U(1)$ gauge theory, thus anticipating the electroweak part of the standard model. Klein never referred to this work in any of his subsequent publications, and unfortunately it was forgotten. Pauli, in letters written to Abraham Pais in 1953, using dimensional reduction derives what he calls the "analogue of the field-strengths", which is the same form as the non-abelian Yang-Mills field strength. Pauli did not pursue this further. These letters, Klein's paper, and Weyl's 1929 article, along with his 1918 article and Fock's, and other key articles appear in translation in a work by O'Raifeartaigh [19] with his comments. Yang, on page 19 of his selected papers [20], cites Weyl's gauge theory results as reported by Pauli [7] as a source for YM gauge theory, although Yang didn't find out until much later that these were Weyl's results. Moreover, Pauli's article did not explicitly mention Weyl's geometric interpretation. It was only long after Yang and Mills published their article that Yang realized the connection between their work and geometry. In fact, on page 74 of his selected papers, Yang says:

> What Mills and I were doing in 1954 was generalizing Maxwell's theory. We knew of no geometrical meaning of Maxwell's theory, and we were not looking in that direction.

Independently of Yang and Mills and slightly after them, Shaw [21] and Utiyama [22] separately developed non-abelian gauge theories similar to the YM one. Utiyama's theory also included a gauge theory for gravity and electromagnetism. Part of Shaw's thesis and Utiyama's paper are also included in O'Raifeartaigh's book [19].

Yang-Mills Field Strength

Pauli, in equation 22a of Part I of his 1941 review article [7], gives the electromagnetic field strength in terms of a commutator.[10] In present-day usage it is

$$[D_\mu, D_\nu] = i\epsilon F_{\mu\nu}, \tag{8}$$

where $\mathbf{D}_\mu$ is the covariant derivative $\partial_\mu - i\epsilon A_\mu$. Mathematically the $F_{\mu\nu}$ in (8) corresponds to the curvature or field strength. In their 1954 paper [4] Yang and Mills do not mention this relation, although they do cite Pauli's 1941 article [7]. They use

$$\psi = S\psi', \tag{9}$$

where ψ is a wave function and S is a local isotopic spin rotation represented by an SU(2) matrix, to obtain the gauge transformation in equation (3) of their paper:

$$B'_\mu = S^{-1}B_\mu S + iS^{-1}(\partial_\mu S)/\epsilon, \tag{10}$$

where B_μ is the gauge field. They[11] then define the field strength as

$$F_{\mu\nu} = (\partial_\nu B_\mu - \partial_\mu B_\nu) + i\epsilon(B_\mu B_\nu - B_\nu B_\mu). \tag{11}$$

The first part corresponds to a curl; the second part is a commutator. They didn't know at that time that this corresponds to Cartan's second structural equation, which in differential geometry notation is $\Omega = \mathbf{dA} + [\mathbf{A}, \mathbf{A}]$, where A is a connection on a principal fiber bundle.

On page 19 of his selected papers [20], Yang states:

> Starting from [$\psi = S\psi'$ and $S(\partial_\mu - i\epsilon B'_\mu)\psi' = (\partial_\mu - i\epsilon B_\mu)\psi$] it was easy to get [our (10)]. Then I tried to define the field strength $F_{\mu\nu}$ by $F_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu$ which was a "natural" generalization of electromagnetism.

Yang returned to this work when he collaborated with Mills when they shared an office at Brookhaven. They published their results in their 1954 paper [6]. There they introduce (11) (their equation (4)) by saying:

> In analogy to the procedure of obtaining gauge invariant field strengths in the electromagnetic case, we define now
>
> $$F_{\mu\nu} = (\partial_\nu B_\mu - \partial_\mu B_\nu) + i\epsilon(B_\mu B_\nu - B_\nu B_\mu). \tag{4}$$
>
> One easily shows from [$B'_\mu = S^{-1}B_\mu S + iS^{-1}(\partial_\mu S)/\epsilon$] that
>
> $$F'_{\mu\nu} = S^{-1}F_{\mu\nu}S \tag{5}$$
>
> under an isotopic gauge transformation. Other simple functions of B than (4) do not lead to such a simple transformation property.

Later we will demonstrate that substituting (10) into the electromagnetic field strength $F'_{\mu\nu} = \partial_\nu B'_\mu - \partial_\mu B'_\nu$ dictates adding a nonelectromagnetic term, i.e., the non-abelian term, so that their (5), the covariant transformation, is satisfied.

[10] *It is presented in Pauli's equation (22a):* $D_i D_k - D_k D_i = -i\epsilon f_{ik}$, *where* $D_k = (\partial/\partial x_k) - i\epsilon\phi_k$, ϕ_k *is the electromagnetic potential,* ϵ *is the charge, and* $f_{ik} = (\partial\phi_k/\partial x_i) - (\partial\phi_i/\partial x_k)$ *is the field strength.*

[11] *Yang had earlier started studying this problem as a graduate student at the University of Chicago and derived (10). When he returned to this problem as a visitor at Brookhaven, he, in collaboration with Mills, obtained (as we will explain) the field strength. See page 17 in Yang's selected papers* [20].

Using the YM covariant derivative $(\partial_\mu - i\epsilon B_\mu)$, let's see how the YM field strength is obtained from the commutator

$$[D_\mu, D_\nu] = (\partial_\mu - i\epsilon B_\mu)(\partial_\nu - i\epsilon B_\nu) - (\partial_\nu - i\epsilon B_\nu)(\partial_\mu - i\epsilon B_\mu) \tag{12}$$

operating on the wave function ψ. Note that $-\partial_\mu(B_\nu\psi) = -(\partial_\mu B_\nu)\psi - B_\nu\partial_\mu\psi$ and $\partial_\nu(B_\mu\psi) = (\partial_\nu B_\mu)\psi + B_\mu\partial_\nu\psi$. So we get a needed $-B_\nu\partial_\mu$ and a $B_\mu\partial_\nu$ term to cancel $B_\nu\partial_\mu$ and $-B_\mu\partial_\nu$, respectively. Therefore the right-hand side of (12) becomes

$$i\epsilon(\partial_\nu B_\mu - \partial_\mu B_\nu) - \epsilon^2[B_\mu, B_\nu]. \tag{13}$$

So $[D_\mu, D_\nu] = i\epsilon F_{\mu\nu}$, which is (8). The failure of the covariant derivatives to commute is caused by a nonvanishing $F_{\mu\nu}$ and demonstrates the presence of curvature. We will show that (8) is related to $F'_{\mu\nu} = S^{-1}F_{\mu\nu}S$ when we discuss finding the field strength below.

The Field Transformation

We present a detailed derivation of the gauge transformation by using the transformation

$$\psi' = S\psi \tag{14}$$

instead of the traditional $\psi = S\psi'$, i.e., the one Yang and Mills used. In order to obtain the gauge transformation of the Yang and Mills paper,

$$B'_\mu = S^{-1}B_\mu S + iS^{-1}(\partial_\mu S)/\epsilon \tag{15}$$

requires you to use[12] $\partial_\mu S^{-1} = -S^{-1}(\partial_\mu S)S^{-1}$. The approach indicated by (14) is marginally more straightforward, since it doesn't require differentiating the inverse of a matrix.

The covariant derivative, $D_\mu = \partial_\mu - i\epsilon B_\mu$, transforms the same way as ψ does:

$$D'\psi' = SD\psi, \tag{16}$$

as we showed in (7): $D'_\mu\psi' = e^{-iq\alpha(x)}D_\mu\psi$ in the electromagnetic case. The left-hand side of (16) becomes

$$(\partial_\mu - i\epsilon B'_\mu)S\psi = (\partial_\mu S)\psi + S\partial_\mu\psi - i\epsilon B'_\mu S\psi. \tag{17}$$

But (17) equals $S\partial_\mu\psi - i\epsilon SB_\mu\psi$. Cancelling $S\partial_\mu\psi$ on both sides, we eventually get

$$B'_\mu = SB_\mu S^{-1} - i(\partial_\mu S)S^{-1}/\epsilon. \tag{18}$$

In differential geometry terms, this is expressed as $B \to SBS^{-1} - i/\epsilon(dS)S^{-1}$. Equation (18) reduces to (3) for the abelian case (where B and S commute) and where $S = e^{+i\epsilon\alpha(x)}$.

We will use[13] $S = e^{i\alpha(x)\cdot\sigma}$ to simplify (18), where σ are the Pauli spin matrices. For α infinitesimal, $S = 1 + i\alpha\cdot\sigma$ produces

$$B'_\mu = (1 + i\alpha\cdot\sigma)B_\mu(1 - i\alpha\cdot\sigma) - i(1/\epsilon)\partial_\mu(1 + i\alpha\cdot\sigma)(1 - i\alpha\cdot\sigma). \tag{19}$$

Remembering that $(a\cdot\sigma)(b\cdot\sigma) = a\cdot b + i\sigma\cdot(a\times b)$, setting $B_\mu = \sigma\cdot b_\mu$, and since α is infinitesimal, dropping terms of order α^2, results in

$$b'_\mu = b_\mu + 2(b_\mu\times\alpha) + (1/\epsilon)\partial_\mu\alpha, \tag{20}$$

which is equation (10) in the YM paper [6].

Finding the Field Strength

As opposed to the electromagnetic case where the transformation of $F_{\mu\nu}$ is gauge invariant, we will show at the end of this section that, in the nonabelian case, the field strength $\mathbf{F}_{\mu\nu}$ transformation is gauge covariant:

$$F'_{\mu\nu} = S^{-1}F_{\mu\nu}S. \tag{21}$$

But first let us find an expression for the field strength. Let's start our quest by examining (21) in the primed system,

$$F'_{\mu\nu} = \partial_\nu B'_\mu - \partial_\mu B'_\nu, \tag{22}$$

and express it in terms of the nonprimed system fields. To do this we calculate $\partial_\nu B'_\mu$ from the expression $B'_\mu = S^{-1}B_\mu S + iS^{-1}(\partial_\mu S)/\epsilon$, (10), obtaining

(23)
$$\partial_\nu B'_\mu = -S^{-1}(\partial_\nu S)S^{-1}B_\mu S + S^{-1}(\partial_\nu B_\mu)S + S^{-1}B_\mu\partial_\nu S + i/\epsilon[-S^{-1}(\partial_\nu S)S^{-1}\partial_\mu S + S^{-1}\partial_\nu\partial_\mu S].$$

So

(24)
$$\begin{aligned}\partial_\nu B'_\mu - \partial_\mu B'_\nu &= -S^{-1}[(\partial_\nu S)S^{-1}B_\mu - (\partial_\mu S)S^{-1}B_\nu]S \\ &\quad + S^{-1}[\partial_\nu B_\mu - \partial_\mu B_\nu]S + S^{-1}[B_\mu\partial_\nu - B_\nu\partial_\mu]S \\ &\quad + i/\epsilon[-S^{-1}(\partial_\nu S)S^{-1}\partial_\mu S + S^{-1}(\partial_\mu S)S^{-1}\partial_\nu S].\end{aligned}$$

We see that the $+S^{-1}[\partial_\nu B_\mu - \partial_\mu B_\nu]S$ term satisfies (21) if the field strength had only the electromagnetic-like contribution. The other terms must either represent the transformed nonelectromagnetic-like part of $\mathbf{F}_{\mu\nu}$ or be cancelled by adding the nonelectromagnetic terms to (22). Since S is used only for the transformation, it should not appear in the expression for $\mathbf{F}_{\mu\nu}$.

The i/ϵ term in (24) dictates that a term multiplied by $i\epsilon$ be added to (22). Since $S^{-1}(\partial_\mu S)$ and $S^{-1}\partial_\nu S$ appear in the expressions for $\mathbf{B}'_\mu$ and $\mathbf{B}'_\nu$, respectively, the product of $S^{-1}(\partial_\mu S)$ and $S^{-1}\partial_\nu S$ that appears in the last term of (24) suggests that we should start our quest to eliminate extra terms here by adding $i\epsilon B'_\mu B'_\nu$ to the equation. This product gives

$$i\epsilon S^{-1}B_\mu B_\nu S - i/\epsilon S^{-1}(\partial_\mu S)S^{-1}\partial_\nu S - S^{-1}B_\mu\partial_\nu S - S^{-1}(\partial_\mu S)S^{-1}B_\nu S. \tag{25}$$

All but the first term (which represents the transformation of $i\epsilon B_\mu B_\nu$) cancel components of the extraneous terms in (24). And the full expression,

---

[12] *The following can be obtained by differentiating $S^{-1}S = I$.*

[13] *In this section our S is the inverse of the Yang-Mills S.*

$i\epsilon(B'_\mu B'_\nu - B'_\nu B'_\mu)$, cancels all of the extraneous terms except the transformation of $i\epsilon(B_\mu B_\nu - B_\nu B_\mu)$. After performing the cancellation, we get

(26)
$$\partial_\nu B'_\mu - \partial_\mu B'_\nu + i\epsilon(B'_\mu B'_\nu - B'_\nu B'_\mu) = S^{-1}[\partial_\nu B_\mu - \partial_\mu B_\nu + i\epsilon(B_\mu B_\nu - B_\nu B_\mu)]S,$$

which satisfies (21).

The YM Lagrangian density consists of

$$-(\frac{1}{4})Tr(F^{\mu\nu}F_{\mu\nu}) \tag{27}$$

(similar to the kinetic part of the electromagnetic Lagrangian) plus a Dirac Lagrangian for a fermion doublet, giving

(28)
$$\mathcal{L} = -(\frac{1}{4})Tr(F^{\mu\nu}F_{\mu\nu}) - \bar{\psi}\gamma^\mu(\partial_\mu - i\epsilon B_\mu)\psi - m\bar{\psi}\psi.$$

The Euler-Lagrange equations used to obtain the field equations are

$$\partial_\mu(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}) - \frac{\partial\mathcal{L}}{\partial\phi} = 0 \tag{29}$$

for each independent field ϕ. Setting ϕ to $\bar{\psi}$ we get

$$\gamma^\mu(\partial_\mu - i\epsilon B_\mu)\psi + m\psi = 0, \tag{30}$$

and setting ϕ to $\mathbf{B}$, we get

$$\partial^\mu \mathbf{F}_{\mu\nu} - i\epsilon[\mathbf{B}^\mu, \mathbf{F}_{\mu\nu}] = -i\epsilon\bar{\psi}\gamma_\nu\psi = -\mathbf{J}_\nu. \tag{31}$$

As we have seen, in differential geometry terms, the field strength takes the form

$$F = dB + B \wedge B. \tag{32}$$

The Bs do not commute. Therefore, as opposed to the electromagnetic case, here the field strength has a $B \wedge B$ contribution. This represents the interaction of the quanta of the **B** field, which is due to the isospin they carry, and for the nonneutral ones, their charge. The Euler-Lagrange equations obtained from the kinetic part of the Lagrangian are

$$d_B F = 0, \tag{33}$$

the Bianchi identity, and

$$d_B * F = 0, \tag{34}$$

where d_B is the covariant exterior derivative.

We conclude this section by showing how the covariant transformation in (21) is related to the commutator in (8). We shall start with the transformation

$$\psi' = S^{-1}\psi. \tag{35}$$

This means that the covariant derivative transforms as

$$D'_\mu\psi' = S^{-1}D_\mu\psi, \tag{36}$$

and repeated operations of D_μ will transform in the same way. So we can write

$$[D'_\mu, D'_\nu]\psi' = S^{-1}[D_\mu, D_\nu]\psi \tag{37}$$

and obtain

$$F'_{\mu\nu}S^{-1} = S^{-1}F_{\mu\nu}. \tag{38}$$

Operating on the right by S gives us the required result,

$$F'_{\mu\nu} = S^{-1}F_{\mu\nu}S. \tag{39}$$

Concluding Remarks

It was proven in the 1960s that if the non-abelian Lagrangian would have an explicitly added mass term that was not gauge invariant, the theory would not be renormalizable and therefore unable to produce finite results. See, for instance, [23] and [24]. Therefore the gauge particles predicted by the original YM theory must have zero mass and infinite range. This contradicts the fact that the weak and strong interactions are short range. To solve this problem, physicists took over the spontaneous symmetry breaking (SSB) technique from superconductivity. In this approach, symmetry breaking is not imposed explicitly but arises from the theory itself. SSB occurs when the Lagrangian has a symmetry and the ground state, known in quantum field language as the *vacuum*, does not. If the vacuum is unique, however, it must share the symmetry of the Langrangian. Therefore the vacuum must be degenerate for SSB to occur. The continuous symmetry seen in the Lagrangian for the Mexican hat potential diagram is exemplified by the infinite number of degenerate minima on the orbit at the bottom of the hat. These degenerate vacua are related to each other by the symmetry operation of O(2) rotation. As soon as one vacuum state is chosen, the symmetry is broken. As early as 1937 Landau saw the possibility of SSB in second-order phase transitions [25]. (See Michel's talk on page 377 of [14].) After World War II, Ginzburg and Landau [26] devised a model of SSB to describe superconductors. Witten [27] describes this model and *inter alia* the standard model; one can see from this the Landau-Ginzburg model relationship to the standard model. In the west, Nambu [28], [29] and Anderson [30] saw that SSB of the electromagnetic field occurs in superconductors and made the analogy to particle physics. In fact, in [30], Anderson ends with "…the Goldstone zero-mass difficulty is not a serious one, because we can probably cancel it off against an equal Yang-Mills zero-mass problem."

It had been shown by Nambu [31], generalized and announced as a theorem and proven by Goldstone [32], and proven more rigorously by Goldstone, Salam, and Weinberg [33], that for every continuous symmetry that is spontaneously broken, a zero-mass, zero-spin boson appears in the theory. These are called Goldstone or Nambu-Goldstone (NG) bosons. One of the criteria for the theorem to apply is that the gauge used be manifestly Lorentz covariant. One choice is to use the Coulomb gauge as the local gauge so that, as we will see, the Goldstone bosons disappear.

In the Mexican hat potential problem, fluctuations along the degenerate vacua orbit produce a Goldstone boson. An example of Goldstone's theorem in ferromagnetism is SSB creating Bloch spin waves; in superfluids, an example is the Landau phonon. In elementary particles, the pion (its mass is[14] 135 MeV for the π^0 and 139.6 MeV for the π^+ and π^-; by comparison, the electron's mass is 0.5 MeV) is considered a Goldstone boson of spontaneous broken *approximate* symmetry; also called spontaneous broken chiral symmetry. According to Weinberg [34], for a while this view of the pion as being a form of the Goldstone boson dampened the enthusiasm to make the Goldstone bosons vanish. Even so, physicists were still faced with the problem that there is only one zero mass boson, the photon. So zero-mass particle production had to be explained away.

Now physicists were faced with two problems: zero-mass intermediate bosons (IB) and the Nambu-Goldstone zero-mass bosons that accompanied SSB. At the February 2010 American Physical Society meeting, three groups were awarded the J. J. Sakurai Prize for their work in solving this problem. The groups and their epic papers are: Englert and Brout[15] [35]; Higgs [36] (who did not attend); and Guralnick, Hagen, and Kibble [37]. Essentially, what is done is to make the gauge transformation local and have the "IB eat the NG bosons". The zero-mass IB have two polarizations. The NG bosons contribute a longitudinal polarization to the IB, making them massive. This technique is called the *Higgs mechanism*, and it is thought that it is responsible for giving particles their mass. Another massive particle (the Higgs boson) is also produced. The search for the Higgs boson is ongoing at the CERN Large Hadron Collider (LHC). The CMS (Compact Muon Solenoid) collaboration at the LHC has posted their results at [41]. They have a hint of evidence that the Higgs mass is around 124 GeV, while the ATLAS collaboration, which posts its results at [42], feels the mass is between 115 and 127 GeV. Much more data has to be taken before the two groups zero in on a value they can have confidence in.

As we have seen, the conserved current of quantum electrodynamics is associated with the U(1) gauge group, which has the photon as its gauge particle. Yang and Mills, because their theory incorporated SU(2) symmetry with $e^{-i\alpha(x)\cdot\sigma}$ as an isotopic spin rotation, had the insight to predict the existence of three gauge particles, the three vector bosons associated with SU(2). The Higgs mechanism, along with the YM non-abelian gauge theory and these mathematical groups, is incorporated into the electroweak force by the Glashow-Salam-Weinberg (GSW) $\mathrm{SU}(2)\times U(1)$ theory [38], [39], [40]. The electroweak interaction is mediated by four gauge bosons: the three massive ones, $W^\pm$ and Z^0, and the photon. The theory predicts the masses of the $W^\pm$ and Z^0: The mass of the W is approximately 80.4 GeV and that of the Z^0 is approximately 91.2 GeV. They were experimentally detected in 1983. G. 't Hooft [43], helped by techniques developed by Veltman, showed in 1971 that YM theories are renormalizable. This gave prominence to the GSW theory. In fact, the number of references to Weinberg's electroweak paper [40] surged from 1 in 1970 to 64 in 1972 [44].

In [34] Weinberg says:

> The importance of the renormalizability of the electroweak theory was not so much that the infinities could be removed by renormalization, but rather that the theory has the potentiality of describing weak and electromagnetic interactions at energies greater than 300 GeV, and perhaps all the way up to the Planck scale.[16]

[14] *An electron volt is the energy gained by an electron accelerating through one volt. A MeV is* 10^6 *ev. A GeV is* 10^3 *MeV. Masses are also measured in MeV.*

[15] *Unfortunately, Robert Brout passed away on 3 May 2011.*

[16] *The Planck scale is an energy scale around* 1.22×10^{19} *GeV.*

References

1. S. K. Donaldson, Yang Mills theory and geometry, http://www2.imperial.ac.uk/~skdona/YMILLS.PDF (2005).
2. ———, Anti-self-dual Yang-Mills connections over complex algebraic surfaces and stable vector bundles, *Proc. Lond. Math. Soc.* **50** (1985), 1–26.
3. S. Dragomir, T. Ichiyama and H. Urakawa, Yang-Mills theory and conjugate connections, *Differential Geom. Appl.* **18** (2003), no. 2, 229–238.
4. M. Daniel and C. M. Viallet, The geometrical setting of gauge theories of the Yang-Mills type, *Rev. Mod. Phys.* **52** (1980), 175–197.
5. S. L. Marateck, The differential geometry and physical basis for the applications of Feynman diagrams, *Notices Amer. Math. Soc.* **53** (2006), 744–752.
6. C. N. Yang and R. L. Mills, Conservation of isotopic spin gauge invariance, *Phys. Rev.* **96** (1954), 191–195.
7. W. Pauli, Relativistic field theories of elementary particles, *Rev. Mod. Phys.* **13** (1941), 203–232.
8. J. D. Jackson, *Classical Electrodynamics*, 3rd ed., John Wiley and Sons, 1998, p. 600.
9. L. B. Okun, V. A. Fock and gauge symmetry, *Physics-Uspekhi* **53** (2010), 835–837.
10. Y. Aharonov and D. Bohm, Significance of electromagnetic potentials in the quantum theory, *Phys. Rev.* **115** (1959), 485–491.

11. Hermann Weyl, *Gravitation und Elektrizität*, Sitzungsber. Preuss. Akad. Wiss., Berlin, 1918, p. 465.
12. ______, Gravitation and the electron, *Proc. Natl. Acad. Sci. U.S.A.* **15** (1929), 323–334.
13. ______, Elektron und Gravitation, *Zeit. f. Physik* **56** (1929), 330–352.
14. L. Brown, M. Dresden and L. Hoddeson, *Pions to Quarks*, Cambridge University Press, 1985, p. 45.
15. V. Fock, On the invariant form of the wave and motion equations for a charged point mass, *Zeit. f. Physik* **39** (1926), 226.
16. J. D. Jackson and L. B. Okun, Historical roots of gauge invariance, *Rev. Mod. Phys.* **73** (2001), 663–680.
17. O. Klein, On the theory of charged fields, *1938 Conference on New Theories in Physics*, Warsaw, Poland, 1938.
18. D. Gross, Oskar Klein and gauge theory, arXiv: hep-th/9411233v1, PUPT-1508, Princeton University, 1994.
19. L. O'Raifeartaigh, *The Dawning of Gauge Theory,* Princeton University Press, 1997.
20. C. N. Yang, *Selected Papers (1945–1980) with Commentary*, World Scientific, Hackensack, NJ, 2005, p. 45.
21. Ronald Shaw, Invariance under general isotopic spin transformations, Ph.D. dissertation, Cambridge University, 1955.
22. Ryoyu Utiyama, Invariant theoretical interpretation of interaction, *Phys. Rev.* **101** (1956), 1597–1607.
23. A. Komar and A. Salam, Renormalization problem for vector meson theories, *Nucl. Phys.* **21** (1960), 624–630.
24. S. Kamefuchi, L. O'Raifeartaigh and A. Salam, Change of variables and equivalence theories in quantum field theories, *Nucl. Phys.* **28** (1961), 529.
25. L. D. Landau, Theory of phase transitions. Part II, *Phys. Z. Sowjetunion* **11** (1937), 545–555.
26. V. Ginzburg and L. Landau, On the theory of superconductivity, *JETP* **20** (1950), 1064–1082.
27. E. Witten, From superconductors and four-manifolds to weak interactions, *Bull. of AMS (N.S.)* **44** (2007), 361–391.
28. Y. Nambu, Axial vector current conservation in weak interactions, *Phys. Rev. Lett.* **4** (1960), 380.
29. Y. Nambu and G. Jona-Lasino, Dynamical model elementary particles based on an analogy with superconductivity. I, *Phys. Rev.* **122** (1961), 345–358.
30. P. M. Anderson, Plasmons gauge invariance, and mass, *Phys. Rev.* **130** (1963), 439–442.
31. Y. Nambu, Quasi-particles and gauge invariance in the theory of superconductivity, *Phys. Rev.* **117** (1960), 648–663.
32. J. Goldstone, Field theories with superconductor solutions (Italian summary), *Nuovo Cimento* **19** (1961), 154–164.
33. J. Goldstone, A. Salam and S. Weinberg, Broken symmetries, *Phys. Rev.* **127** (1962), 965–970.
34. S. Weinberg, in *50 Years of Yang-Mills Theory*, G. 't Hooft, editor, World Scientific, Hackensack, NJ, 2005, p. 105.
35. F. Englert and R. Brout, Broken symmetry and the mass of gauge vector mesons, *Phys. Rev. Lett.* **13** (1964), 321–323.
36. P. Higgs, Broken symmetry and the masses of gauge bosons, *Phys. Rev. Lett.* **13** (1964), 508–509.
37. G. Guralnik, C. Hagen and T. Kibble, Global conservation laws and massless particles, *Phys. Rev. Lett.* **13** (1964), 585–587.
38. S. L. Glashow, Partial symmetries of weak interactions, *Nucl. Phys.* **22** (1961), 579–588.
39. A. Salam and J. Ward, Electromagnetic and weak interactions, *Phys. Rev. Lett.* **13** (1961), 168–171.
40. S. Weinberg, A model of leptons, *Phys. Rev. Lett.* **19** (1967), 1264–1266.
41. CMS Collaboration results at `https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsHIG`.
42. ATLAS Collaboration results at `https://twiki.cern.ch/twiki/bin/view/AtlasPublic/`.
43. G. 't Hooft, Renormalizable Lagrangians for massive Yang-Mills fields, *Nucl. Phys.* **B35** (1971), 167–188.
44. L. Hoddeson, L. Brown, M. Riordan, and M. Dresden, *The Rise of the Standard Model*, Cambridge University Press, 1997, p. 16.

Do the Math

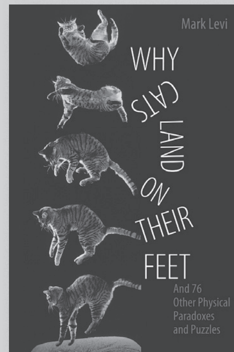
X and the City

Modeling Aspects of Urban Life

John A. Adam

"In *X and the City*, John Adam proves himself to be a genial and endlessly curious companion as he takes us on a stroll through that fascinating place where reality meets the mathematical imagination. How many squirrels live in Central Park? Should you walk or run in the rain? Anyone who's ever pondered puzzles like these will find this book to be a treat."
—Steven Strogatz, Cornell University

Cloth $29.95 978-0-691-15464-0



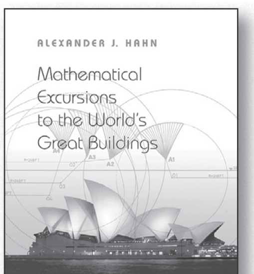
Why Cats Land on Their Feet

And 76 Other Physical Paradoxes and Puzzles

Mark Levi

"*Why Cats Land on Their Feet* is a collection of fun physical puzzles that will be attractive to a wide spectrum of readers, from teachers to those looking simply for good reading and educational entertainment. Levi's sense of humor and friendly, informal style add to the pleasure of the book."
—Paul J. Nahin, author of *Number-Crunching*

Paper with French folds $19.95 978-0-691-14854-0

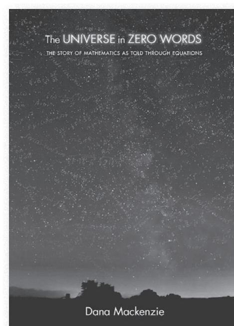


Mathematical Excursions to the World's Great Buildings

Alexander J. Hahn

"Readers who enjoy connecting mathematics to real-world applications will find this book intriguing, as will anyone who wants to learn more about the forces and mathematics behind the construction of the world's great buildings."
—Michael Huber, author of *Mythematics*

Cloth $49.50 978-0-691-14520-4



Dana Mackenzie, Winner of the 2012 Joint Policy Board for Mathematics Communications Award

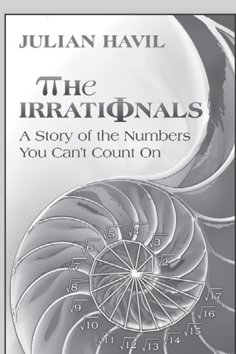
The Universe in Zero Words

The Story of Mathematics as Told through Equations

Dana Mackenzie

"Demanding very little prior mathematical knowledge, this is one of the best popular histories of mathematics in recent years. Dana Mackenzie's prose is lively and easy to read, and his mix of historical background and personal biographies of the main characters is engaging."
—Eli Maor, author of *The Pythagorean Theorem*

Cloth $27.95 978-0-691-15282-0



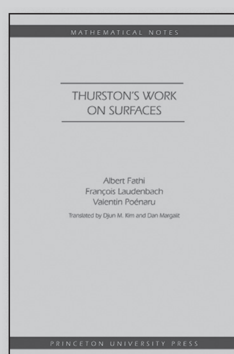
The Irrationals

A Story of the Numbers You Can't Count On

Julian Havil

"Readers will be swept away by Havil's command of the subject and his wonderful writing style. *The Irrationals* is a lot of fun."
—Robert Gross, coauthor of *Fearless Symmetry*

Cloth $29.95 978-0-691-14342-2



Thurston's Work on Surfaces

Albert Fathi, François Laudenbach & Valentin Poénaru

Translated by Djun M. Kim & Dan Margalit

This book provides a detailed exposition of William Thurston's work on surface homeomorphisms, available here for the first time in English.

Paper $60.00 978-0-691-14735-2

PRINCETON UNIVERSITY PRESS

See our E-Books at
press.princeton.edu

WHAT IS...

a Biholomorphic Mapping?

Eric Bedford

Let $\mathbf{C}^n = \mathbf{C} \times \cdots \times \mathbf{C}$ denote complex Euclidean space, and let $D_1, D_2 \subset \mathbf{C}^n$ be domains. A mapping $f(z_1, \ldots, z_n) = (f_1, \ldots, f_n) : D_1 \to D_2$ is *holomorphic* if each of the coordinate functions f_j is holomorphic. If f is one-to-one and onto, then there is an inverse function $f^{-1} : D_2 \to D_1$, and this may be shown to be holomorphic. In this case, we say that f is *biholomorphic*. Biholomorphic maps preserve many of the basic properties of analytic function theory and serve as the isomorphisms in the category of analytic objects.

Let us first consider the case $n = 1$. The derivative of f at a point z_0 is a complex number $f'(z_0)$. For a biholomorphic mapping, the derivative is nonzero, and if we write $f'(z_0) = re^{i\theta}$, $r > 0$, then, infinitesimally at z_0, f looks like the translation $z_0 \mapsto f(z_0)$, followed by the rotation by angle θ about $f(z_0)$ and a dilation by the factor r centered at $f(z_0)$. Such a map is called *conformal* since it preserves angles at the infinitesimal level. In other words, if γ_1 and γ_2 are two smooth curves both passing through z_0 and crossing with an angle of α with respect to each other, then the image curves $f(\gamma_1)$ and $f(\gamma_2)$ will both pass through $f(z_0)$ and cross each other with the same angle α. In dimension 1, biholomorphic maps are conformal.

A domain D in the plane $\mathbf{C}$ is *simply connected* if its complement in the Riemann sphere is connected. A topological fact is that a simply connected domain in the plane $\mathbf{C}$ is topologically equivalent (homeomorphic) to the disk $\Delta = \{|z| < 1\}$, but this is less than obvious, since simply connected domains can have very complicated boundaries.

Riemann Mapping Theorem (RMT). *Suppose that $D \subset \mathbf{C}$ is simply connected and $D \neq \mathbf{C}$. Then there is a biholomorphic map $\varphi : D \to \Delta$.*

There are many consequences to the RMT. In particular, there are a lot of conformal maps: for every proper, simply connected domain, there is a conformal map to the disk. On the other hand, $\mathbf{C}$ is not biholomorphically equivalent to Δ because a holomorphic map $\psi : \mathbf{C} \to \Delta$ is necessarily bounded, so by the Liouville Theorem ψ is constant. Thus, by the RMT, there are only two classes of simply connected domains in $\mathbf{C}$ which are inequivalent up to conformal equivalence, namely the class of Δ and the class of $\mathbf{C}$. In fact, if $f : \mathbf{C} \to D$ is biholomorphic, then $D = \mathbf{C}$ and f has the form $z \mapsto \alpha z + \beta$ with $\alpha, \beta \in \mathbf{C}$.

The case $n > 1$ is different. Many important one-dimensional results do not hold for $n > 1$, and we find new, and sometimes unexpected, phenomena. First, biholomorphic maps are not necessarily conformal (consider $(z_1, z_2) \mapsto (z_1, 2z_2)$). Further, if $n > 1$, we find "rigidity" results, and distinct domains are "typically" inequivalent. On the other hand, for unbounded domains, we find larger spaces of holomorphic mappings.

Biholomorphic self-maps of a domain D are called *automorphisms*, and the automorphism group $\mathrm{Aut}(D)$ is a biholomorphic invariant. However, for a "typical" domain $D \subset \mathbf{C}^n$, $n > 1$, the automorphism group $\mathrm{Aut}(D)$ consists only of the identity transformation. There is no single domain in $\mathbf{C}^n$, $n > 1$, which plays the special role that Δ plays in $\mathbf{C}$. Leading competitors are

Eric Bedford is professor of mathematics at Indiana University in Bloomington. His email address is `bedford@indiana.edu`.

DOI: http://dx.doi.org/10.1090/noti864

the polydisk $\Delta^n := \{\max(|z_1|, \ldots, |z_n|) < 1\}$ and the ball $\mathbf{B}^n := \{|z_1|^2 + \cdots + |z_n|^2 < 1\}$. An exercise in several complex variables is to determine the automorphism groups of Δ^n and $\mathbf{B}^n$. These groups are not isomorphic, so the bidisk is not biholomorphically equivalent to the ball. Thus there is no RMT in dimension > 1.

One obstruction for the existence of holomorphic mappings for $n > 1$ is given by rigidity phenomena. For instance, suppose that U is a domain with $U \cap \partial\mathbf{B}^n \neq \emptyset$. If $f : U \to f(U)$ is biholomorphic and if $f(U \cap \partial\mathbf{B}^n) \subset \partial\mathbf{B}^n$, then f coincides with an automorphism of $\mathbf{B}^n$. This was shown by H. Poincaré for $n = 2$ and by H. Alexander for $n > 2$. More generally, if $D_1, D_2 \subset \mathbf{C}^n$ are simply connected domains, bounded, with real analytic, strictly convex boundaries, and if $f : U \to f(U)$ is a biholomorphism taking $U \cap \partial D_1 \neq \emptyset$ to $f(U) \cap \partial D_2$, then f extends to a biholomorphic map $\hat{f} : D_1 \to D_2$. S. Webster showed that if D_1 and D_2 are both defined by polynomials, then f must be algebraic. Such rigidity phenomena give another obstruction to formulating an RMT for $n > 1$.

There are also natural questions about the boundary behavior of a biholomorphism. For instance, if $D_1, D_2 \subset \mathbf{C}^n$ are bounded with real analytic boundaries, does a biholomorphism $f : D_1 \to D_2$ extend continuously (or holomorphically) to $\overline{D_1}$? While the affirmative answer has been proved for $n = 1$ and 2, the question remains open for $n > 2$.

It is natural to ask: What are the domains D for which $\mathrm{Aut}(D)$ is "large" in some sense? H. Cartan showed that, if D is bounded, then $\mathrm{Aut}(D)$ is a Lie group. If $p \in D$, then we define $\mathrm{Aut}_p(D) = \{f \in \mathrm{Aut}(D) : f(p) = p\}$. If D is bounded, then by the Cauchy estimates, there is a bound (independent of f) on $\partial^\alpha f(p)$ for any derivative $\partial^\alpha = \partial^{(\alpha_1, \ldots, \alpha_n)} = (\partial/\partial z_1)^{\alpha_1} \cdots (\partial/\partial z_n)^{\alpha_n}$. Applying this bound simply to the differential $df^n(p) = (df(p))^n$ for all $n \in \mathbf{Z}$, we conclude that $df(p) \in U(n)$ is (essentially) an element of the unitary group. Thus $f \mapsto \rho(f) := df(p)$ gives a representation $\rho : \mathrm{Aut}_p(D) \to U(n)$. Next suppose that $Df(p) = I$ is the identity. Setting $p = 0$, we may assume that $f(z) = z + h(z) + O(|z|^{m+1})$, where $h(z)$ is homogeneous of degree $m \geq 2$. Iterating this, we see that $f^j(z) = z + jh(z) + O(|z|^{m+1})$. If $|\alpha| = m$, then we have $|\partial^\alpha f^j(p)| = |j\partial^\alpha h(p)|$. Since this must hold for all j, we may apply the Cauchy estimates to conclude that $h = 0$. Thus the representation ρ is injective.

It follows from the injectivity of ρ that $\mathrm{Aut}_p(D)$ is compact if D is bounded. Thus $\mathrm{Aut}(D)$ can be noncompact (a natural sense of being "large") only if the orbit $O(p) = \{f(p) : f \in \mathrm{Aut}(D)\}$ is noncompact. Let us give some smooth, bounded domains with noncompact automorphism groups. Start with coordinates $(z, w) = (z_1, \ldots, z_n, w)$ on $\mathbf{C}^n \times \mathbf{C}$ and write $w = u + iv$. Any domain of the form $D = \{(z, w) : v + \psi(z, \bar{z}) < 0\}$ is invariant under real translations $(z, w) \mapsto (z, w + s)$ for all $s \in \mathbf{R}$, so $\mathrm{Aut}(D)$ is not compact. Given weights $\delta_1, \ldots, \delta_n > 0$, we define the (weighted) Cayley transform $\Phi : (z, w) \mapsto (z^*, w^*)$ by $w = (1 - iw^*/4)(1 + iw^*/4)^{-1}$, $z_j = z_j^*(1 + iw^*/4)^{-2\delta_j}$. This defines a biholomorphic map of any domain $D \subset \{v < 0\}$ to its image $\Phi(D) \subset \mathbf{C}^{n+1}$. The weights of monomials are given by $\mathrm{wt}(z^J) = \mathrm{wt}(z_1^{j_1} \cdots z_n^{j_n}) = \sum_\ell \delta_\ell j_\ell$, $\mathrm{wt}(\bar{z}^J) = \mathrm{wt}(z^J)$, and $\mathrm{wt}(z^J \bar{z}^K) = \mathrm{wt}(z^J) + \mathrm{wt}(\bar{z}^K)$. Now let us suppose that $\psi = \sum_{J,K} a_{J,K} z^J \bar{z}^K$ is a nonnegative polynomial which is homogeneous and "balanced" with respect to this selection of weights in the following sense: $\mathrm{wt}(J) = \mathrm{wt}(K) = 1/2$ for all (J, K) such that $a_{J,K} \neq 0$. It follows that the Cayley transform takes $D = \{\psi(z, \bar{z}) + v < 0\}$ to the smoothly bounded domain $\Phi(D) = \{\sum_{J,K} a_{J,K} z^J \bar{z}^K + |w|^2 < 1\}$. Since there is great freedom in choosing weights and coefficients, this gives a large number of smoothly bounded domains with noncompact automorphism groups. As natural as these domains seem, it is not known whether all smoothly bounded domains with noncompact automorphism groups are biholomorphically equivalent to some $\Phi(D)$.

Up to this point, we have focused our attention on bounded domains. The situation is somehow the opposite for unbounded domains: there are many automorphisms of $\mathbf{C}^n$ when $n > 1$, and the structure of the group $\mathrm{Aut}(\mathbf{C}^n)$ is not well understood. Perhaps the most striking difference is the existence of *Fatou-Bieberbach domains:* such domains are strict subsets of $\mathbf{C}^n$ which are biholomorphically equivalent to $\mathbf{C}^n$. These domains arise naturally from considerations of complex dynamics, as we will show here. For parameters a and b, we consider the automorphism $f(z_1, z_2) = (z_1^3 + az_1 - bz_2, z_1)$, which is a cubic mapping of $\mathbf{C}^2$ with a cubic inverse. These maps belong to the family of Hénon mappings. These have been studied as dynamical systems $f : \mathbf{C}^2 \to \mathbf{C}^2$ and have been found to have rich dynamics, although their dynamical behaviors are still only partially understood. The map f is odd in the sense that $f(-z) = -f(z)$. The fixed points, by definition, satisfy $f(z) = z$ and are three points: $(0, 0)$ and $\pm P$. For generic a and b, the differential $Df(\pm P)$ at $\pm P$ has distinct eigenvalues $|\lambda_1| > |\lambda_2|$. We may choose a, b such that $1 > |\lambda_1| > |\lambda_2|$. We define the basin of P as $B(P) := \{z \in \mathbf{C}^2 : \lim_{n\to\infty} f^n z = P\}$, which are all the points which approach P in forward time. Since the mapping is odd, the other basin is given by $B(-P) = -B(P)$. To examine this basin in more detail, we may uniformize it. That is, we find a biholomorphic map $h : \mathbf{C}^2 \to B(P)$.

Now we treat f as a dynamical system. We set $L(z) = (\lambda_1 z_1, \lambda_2 z_2)$. After an affine change of coordinates, we may suppose that $P = 0$ and that $f(z) = L(z) + \tilde{f}(z)$, where $\tilde{f}(z) = O(|z|^2)$ denotes terms of degree 2 and higher. We will find h as a solution to the functional equation

$$(*) \qquad L \circ h = h \circ f.$$

Thus h conjugates the dynamical system $(f, B(P))$ to the linear dynamical system $(L, \mathbf{C}^2)$. We wish to show the existence of the uniform limit $h(z) := \lim_{n\to\infty} L^{-n} \circ f^n(z) = \mathrm{id} + \tilde{h}(z)$ in a neighborhood U of the origin. Such a limit clearly must satisfy $(*)$. Technically, we may show the existence of the limit directly if we further restrict a, b so that $|\lambda_2| > |\lambda_1|^2$. Now that h is defined on U, we may extend it to $f^{-1}(U)$ using the identity $h = L^{-1} \circ h \circ f$ on U. It follows that h extends holomorphically to a map $h : \mathbf{C}^2 \to B(P) = \bigcup_{n\geq 0} f^{-n}U$. Using the property that $B(P)$ is the basin, we conclude that h is biholomorphic, there are no other fixed points in $B(P)$, and $B(P)$ is disjoint from $B(-P) = -B(P)$. Thus both $B(P)$ and $-B(P)$ are disjoint Fatou-Bieberbach domains, and if we think of $B(P)$ as "large", then we must also think of the complement $\mathbf{C}^2 - B(P)$ as "large".

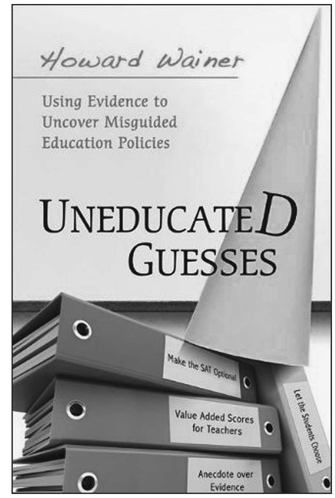
Now that we have the existence of Fatou-Bieberbach domains, we may also consider their increasing limits. That is, suppose that $f_j : \mathbf{C}^2_j \to f(\mathbf{C}^2_j) \subset \mathbf{C}^2_{j+1}$, $j = 1, 2, 3, \ldots$, is a sequence of biholomorphic mappings to Fatou-Bieberbach domains in $\mathbf{C}^2$. Now consider the direct limit M of this family, which is the complex manifold given as $M := \bigsqcup_j \mathbf{C}^2_j / \sim$, the disjoint union of infinitely many copies of $\mathbf{C}^2$ with the identification $z_j \sim z_{j+1}$ if $z_j \in \mathbf{C}^2_j$, $z_{j+1} \in \mathbf{C}^2_{j+1}$, and $f_j(z_j) = z_{j+1}$. There are instances where the limit M is biholomorphically equivalent to $\mathbf{C}^2$ and examples where it is not.

Further Reading

K. Fritzsche and H. Grauert, *From Holomorphic Functions to Complex Manifolds*, Graduate Texts in Mathematics, Vol. 213, Springer-Verlag, New York, 2002.

Uneducated Guesses: Using Evidence to Uncover Misguided Education Policies

Reviewed by John Ewing



Uneducated Guesses: Using Evidence to Uncover Misguided Education Policies
by Howard Wainer
Princeton University Press, 2011
US$24.95, 175 pages
ISBN-13: 978-0-691-14928-8

> *Uneducated Guesses* challenges everything our policymakers thought they knew about education and education reform.…In this explosive book, Howard Wainer uses statistical evidence to show why some of the most widely held beliefs in education today…are wrong.

So begins the description on the dust jacket, which promises that the author "exposes today's educational policies to the light of empirical evidence and offers solutions for fairer and more viable policies in the future." It claims this is an exposé that "no one who is concerned about seeing our children achieve their full potential can afford to ignore…" I wasn't looking forward to a book that claimed to solve all our educational problems.

It turned out, however, that the dust jacket was not merely hyperbole—it was just plain wrong. Rather than a book of policy, this was a collection of vignettes about the use of tests in education, mixed with occasional asides on elementary statistics, homely philosophy, and pleasant (if occasionally biting) wit. Not many books on statistics use references from the Bible in one chapter (Judges 12:4-6) and Indiana Jones in another, quote Hume and Kant alongside Feynman and Satchel Paige, or blend stories about Jaime Escalante and John Stuart Mill. This is a book with surprising charm, occasional insight, and many good stories, although not many "solutions for fairer and more viable policies." Ironically, the author *does* expose the way current educational policy is made, but not in the way the dust jacket suggests—more on this later.

Howard Wainer was for many years the principal research scientist at Educational Testing Service, and so it is not surprising that much of the material, especially in the first half of the book, concerns the use of standardized tests.

> *Question*: If you make submitting SAT scores optional for college admission, what happens?
>
> *Answer*: Those with high scores continue to submit SAT information; those with low scores do not. As a result, the value of SAT scores diminishes for screening applicants.
>
> *Question*: If instead of using a common entrance exam (an "aptitude test") we substitute scores on tests in a variety of subjects ("achievement tests"), what happens?
>
> *Answer*: It becomes more difficult to use tests to compare one applicant against another. One cannot easily judge whether being outstanding in history is as valuable as excelling in mathematics.
>
> *Question*: If we need to allocate a scarce resource (say, seats in an AP Calculus class) among a group of

John Ewing is president of Math for America. His email address is `jewing@mathforamerica.org`.

DOI: http://dx.doi.org/10.1090/noti874

> students, what is the most efficient way to do it?
>
> *Answer*: Use scores on a standardized aptitude test (PSAT), which turn out to be a good predictor of scores on standardized AP exams at the end of the course. (The surprise is that success in AP Psychology is better predicted than success in AP Calculus.)

None of these answers is a surprise, and the real surprise is that sensible people would have thought otherwise. In fact, this is the true import of the account. College admissions officers apparently thought that making SAT scores optional merely improved the admissions process without distorting it. A major reputable organization (the National Association for College Admission Counseling) advocated using sets of achievement tests in place of a standard aptitude test without realizing that this made it difficult to compare applicants. The public may be hoodwinked into believing a particular teacher has an exceptional gift when teaching advanced placement courses, when in fact the real gift turns out to be selecting the right students before the course begins. The author scoffs at such foolishness and counters with data, graphs, and elementary statistics.

Some of the vignettes are prosaic. Chapter 10 on ranking colleges says very little of substance about the rankings, commenting that the *US News and World Report* ranking "does seem to reflect generally the consensus of opinion about the quality of colleges and universities." The author includes a short discourse on the treatment of missing data in preference ranking, but there isn't much anyone could disagree with here. Some of the vignettes are slightly tedious. Chapter 8 tells the story of a third-grade teacher who had sixteen of her twenty-five students with a perfect score on a standardized mathematics test. She was accused of cheating, and indeed there was the possibility that a teacher's aide had inadvertently helped students. A young Ph.D. with only a course in "measurement" under his belt affirmed the accusation. Professional statisticians showed that the case was weak. A modestly interesting story, but too long in the telling.

All the vignettes are told with alacrity, wit, and fervor. The lessons on statistics are mostly presented deftly so that someone with little training in mathematics can easily comprehend them. The prose is attractive and clear.

The one chapter that touches on a current, sensitive, and hotly debated issue was disappointing. Chapter 9 deals with teacher evaluation using value-added modeling, and unlike the rest of the book, readers who are new to the ideas of statistics will find this hard to digest. It is one of the few places where equations appear. The author begins to explain the value-added model:

> The model itself begins simply by representing a student's test score in the first year, Y_1, as the sum of the district's average for that grade, subject and year, say μ_1, and the incremental contribution of the teacher, say θ_1, and systematic and unsystematic errors, say ϵ_1. When these pieces are put together we obtain a simple equation for the first year,
>
> $$y_1 = \mu_1 + \theta_1 + \epsilon_1,$$
>
> or
>
> $$\text{Student's score}(1) = \text{district average }(1) + \text{teacher effect }(1) + \text{ error }(1).$$

The expression for "value-added," he notes, is "statistically convenient" because there are "fewer parameters."

This is an explanation that is insufficiently precise for the mathematically sophisticated and overwhelming for everyone else. More troubling, however, is his analysis of the efficacy of the model. Again, the unsophisticated reader will be confused and likely will come away with the notion that with a bit more effort one could overcome these "technical" problems. Near the end of the chapter, the author confirms this when he writes: "Value-added assessment may yet help us in this task, but there are many challenges yet to overcome before these models are likely to help us …" We just need to work harder, he seems to say. The fact that the model often reflects noise (and little else) is never mentioned.

This brings me to the most troubling aspect of this otherwise charming book: The author is a true believer in "data-driven education". This belief, shared by policymakers and politicians alike, has become the driving force behind education reform, and it is deeply troubling.

The book begins and ends with a discourse on epistemology. How do we find out about the world? What is the basis of our beliefs? What information should guide our actions? How do we judge whether those actions are the right ones? These are fundamental questions in any area of human endeavor, but they are especially important in education, where understanding our goals, how we might achieve them, and whether we have succeeded are crucial to preparing future generations.

On the first page of the introduction, the author begins to lay out his answers to these questions with a famous quote from Richard Feynman, made in the Messenger Lecturers at Cornell University in 1964, attempting to explain the scientific method:

> In general we look for a new law by the following process. First we guess it. Then we compute the consequences of the guess to see what would be implied if this law that we guessed is right. Then we compare the result of the computation to nature, with experiment or experience, compare it directly with observations, to see if it works. If it disagrees with experiment it is wrong. In that simple statement is the key to science. *It does not make any difference how beautiful your guess is. It does not make any difference how smart you are, who made the guess, or what his name is—if it disagrees with experiment it is wrong. That is all there is to it.* [Feynman, R. P., 1965. *The Character of Physical Law*, p. 156, Cambridge: MIT Press]

So far, so good (it's hard to disagree with Richard Feynman). But then the author engages in a rhetorical slight-of-hand when he writes:

> It was clear that Feynman placed evidence in an exalted position. It vetoed all else.

Notice how in Wainer's sentence the phrase "experiment or experience" is narrowed to the single word "evidence". Soon he replaces the word "evidence" with "data" and then restricts data even further:

> Evidence of success in contemporary education encompasses many things, but principal among them are test scores. When scores are high, we congratulate all involved. When they are low, we look to make changes. [p. 4]

As the book proceeds, this point is driven home in chapter after chapter. Evidence means test scores, success means higher scores, "misguided policies" means anything that is not supported by higher scores. If evidence truly "encompasses many things," those many things are missing from this account altogether.

The American philosopher Abraham Maslow is often quoted as saying: "If the only tool you have is a hammer, you tend to see every problem as a nail." Statisticians see evidence as data, because their discipline is the tool that is associated with data. For modern researchers in education, their hammer is statistics and their nails are test scores—nothing else matters, no other evidence is valued.

But education is much more than tests, and educational evidence is much more than scores. Education is about critical thinking, about developing taste, empathy, and values. Education is about learning to carry out complicated analyses of complex problems over extended periods of time. Education is about learning to learn, so that whatever limited accomplishments one has in school are amplified throughout one's life. Education is much, much more, and little of this is measured by test scores. Evidence requires looking at students over many years, not one, and it includes information about the ongoing accomplishments of students, teachers, and administrators. Evidence even includes anecdotes (which the author dismisses as unworthy), because examples and case studies are derived from experience too. Evidence is not "data related to a claim" [p. 148]; evidence is "experience related to a claim." This is much, much larger than a nail.

The epilogue contains a clear and unambiguous description of the author's vision of data-driven education reform, which begins with an attack on the "experts".

> Experience has taught us a great deal about what kinds of optimization methods work in complex systems and what kinds do not. An almost surefire path to failure is to convene a blue-ribbon committee with a title like "Education 2020" whose mandate is to ponder existing problems and come out with recommendations for the future system. It doesn't work because even all-stars aren't that smart. [p. 157]

While I share his skepticism of committees, this seems to dismiss education professionals altogether. To emphasize that we should avoid "expert opinion" in crafting education, the author cites another famous quote of Feynman from the same 1965 Messenger Lecture: "Science is the belief in the ignorance of the experts." But he has taken this out of context and consequently changed its meaning. The full quote has a quite different message:

> We have many studies in teaching, for example, in which people make observations, make lists, do statistics, and so on, but these do not thereby become established science, established knowledge. They are merely an imitative form of

> science…The result of this pseudoscientific imitation is to produce experts, which many of you are. [But] you teachers, who are really teaching children at the bottom of the heap, can maybe doubt the experts. As a matter of fact, I can also define science another way: Science is the belief in the ignorance of experts. [Feynman, 1965]

A master teacher with years of experience who is an acknowledged craftsman in the profession surely is more likely to solve problems and shape the future of education than an economist, statistician, or politician. Richard Feynman seems to agree.

The epilogue goes on to explain how education policy should be made.

> What does work [in dealing with complex systems] is the implementation of constant experimentation, in which small changes are made to the process. Then the effects of the changes are assessed. If the process improves, the size of the changes is increased and the outcome monitored. If the process gets worse, the changes are reversed and another variable is manipulated. Gradually, the entire complex process moves toward optimization. [p. 157]

"Constant experimentation"—with the goal of higher test scores! When they go up, do a little more experimenting on the same variable; when they go down, try manipulating another variable. The results are predictable: The education system becomes corrupted; teachers find ways to manipulate scores, either by focusing their students' experience only on the tests or by cheating; administrators and politicians fixate on scores as the only evidence of value—the ultimate goal of all education; and the true educational professionals become disgusted and find something more rewarding to do with their lives. Surely this is not optimal.

This otherwise charming book inadvertently tells us a great deal about the nature of data-driven education. The author's goal is to help people think clearly about applying statistical thinking to education reform, and to some extent he succeeds. But statistics is only one tool among many, expertise can be found in disciplines other than statistics, and evidence is not synonymous with data. Some things are not measured by data and are not well suited to statistical analysis. The beauty of a great painting, the magnificence of a great symphony, the power of great poetry cannot be captured by data. Similarly, many prosaic parts of life are not aptly judged by data and statistics alone. Great and inspired education is among them.

Exhibit Review: Mathématiques, un dépaysement soudain

Mathématiques, un dépaysement soudain
Exhibit at the Fondation Cartier, Paris
October 21, 2011–March 18, 2012

A Little More Mad Ophelia, Please

Nathalie Sinclair

With the likes of David Lynch and Patti Smith as collaborators and the Fondation Cartier's elegant centre for contemporary art as the setting, the exhibition Mathématiques, un dépaysement soudain was irresistible, even given the six-hour train ride I'd have to endure to get from Torino to Paris. Before reaching the Fondation Cartier, I happened upon a much smaller, less ambitious show at the city hall of the fifth *arrondissement*, also devoted to the theme of mathematics meets art. It contained the usual array of images of fractals and circle packings, as well as architectural and geometric sculptures that seem to feature in public exhibitions devoted to convincing audiences that mathematics is much more like art than most people think. I knew the Fondation Cartier show would be different. It was. It was certainly grander and glitzier. It featured the crème de la crème of the European mathematics community, such as Misha Gromov and Sir Michael Atiyah. It had many more visitors than the humbler show in the fifth. It had a few breathtaking pieces, but it did not live up to its goals. These, according to the text written by one of the commissionaires of the exhibition, mathematician Jean-Pierre Bourguignon, were mainly didactic—to use the creativity of artists to help elucidate, for the public, the objects and practices of mathematicians, and especially *creative* mathematicians.

Exhibit Information and Pictures

For information about the exhibit, as well as pictures of the displays, go to the webpage `http://tinyurl.com/fond-cart-math`. Please note that the exhibit closed in March 2012.

The show consists of three main rooms: two upstairs and one down. While there are some interesting bits in each room, there is little overall coherence. Geographically, one feels a little lost moving through the space, and while the exhibition's title clearly showcases mathematics, one feels jostled by appearances of physics, neuroscience, and biology. This is not to say that specific exhibits fail to provoke or delight. The impressionable and mathematically inclined viewer would certainly find it exciting, but there is no argument or narrative that leads the viewer to an interesting destination. If there is a theme, it might be the cult of the individual elite mathematician, with age being the main axis of diversity.

The "Great Books" Room

The visit begins with The Library of Mysteries, where images of book covers and selected quotations from these books are projected on the wall. The display is accompanied by, though not necessarily related to, a David Lynch film entitled *Universe Coming from Zero* projected on the ceiling, featuring a growing set of concentric circles made up of increasingly bigger objects (from nothing, going to protons and neutrons, eventually passing through humans and mountains, and ending up with stars and galaxies), accompanied by a soundtrack. I remember being much more struck by Eames's film *Powers of 10*, which communicates a similar sense of dimension but puts the viewer at the centre of the action in the dazzling voyage. In Lynch's soundtrack, Patti Smith's entrancing

Nathalie Sinclair is professor of mathematics education at Simon Fraser University in Canada. Her email address is `nathsinc@sfu.ca`.

DOI: http://dx.doi.org/10.1090/noti862

voice can sometimes be heard reciting things, very slowly. (I wondered whether she was having a hard time pronouncing the word "exponentially"!)

As one listens to the movie, one learns that the mysteries of the universe are four: the nature of the laws of physics, the mystery of life, the role of the brain, and the structure of mathematics related to the three others. These mysteries were picked by the eminent Russian mathematician Misha Gromov. Notice that mathematics is the umbrella mystery under which questions of nature, life, and thought are subsumed. To this Platonic view of mathematics, I prefer A. N. Whitehead's (1929, p. 31) take on the situation:

> Even now there is a very wavering grasp of the true position of mathematics as an element in the history of thought. I will not go so far as to say that to construct a history of thought without profound study of the mathematical ideas of successive epochs is like omitting Hamlet from the play which is named after him. That would be claiming too much. But it is certainly analogous to cutting out the part of Ophelia. This simile is singularly exact. For Ophelia is quite essential to the play, she is very charming—and a little mad. Let us grant that the pursuit of mathematics is a divine madness of the human spirit, a refuge from the goading urgency of contingent happenings.

A little less ponderous Hamlet and a little more mad Ophelia in this exhibition would have been nice.

As I sat on the floor watching the film unroll, I wondered how much the sequence of book covers (about twenty-five of them) felt like mathematics class to the adolescents sprawled on the floor and the seats in the room: a long list of facts whose interconnections are not often explained. Maybe the public would come to better appreciate mathematics after seeing titles by the likes of Descartes, Riemann, and Poincaré appearing alongside authors such as Plato, Darwin, and Einstein. The connection between the books, of course, is that they are books Gromov thinks speak to the mysteries of the universe. His list would make great fodder for the canon wars of literature: no female or non-Western worlds—the leap from Euclid to Descartes rather glaringly omits Arabic scholars.

There wasn't much to engage in either artistically or mathematically in this room, whose design seemed meant to convey the idea of zero. The outer walls could indeed be taken as gesturing to the perimeter of a circle, and one passed through an opening in the wall to enter a vaguely ovoid projection chamber. But the evocation of zero through the shape of its written digit (ah, Arabic mathematics at last!) wasn't nearly as inspired as one might wish. In the next room, the visitor finds a much simpler, more persuasive attempt at doing this, which I will describe shortly. I did wonder, also, what zero had to do with the mysteries evoked by the library of books. One could go a long way toward developing the mysteriousness of that number, as Brian Rotman does through words in *Signifying Nothing: The Semiotics of Zero*.

Geometry to the Rescue

After the Library of Mysteries, one moves across the hallway of the main entry to the second stop on the visit, the Room of the Four Mysteries. Here one encounters several visual and interactive wall installations and a large concave projection screen sitting atop a colorful, IKEA-like tripod. Although one has already learned about the four mysteries, there's little in the room to help orient attention to how these mysteries are being expressed. It would have been a little too simple to have one display for each mystery, of course, and maybe walking into the room was supposed to evoke the kind of *dépaysement* for which the exhibition is named (the word *dépaysement* refers to the sometimes disturbing feeling one gets when stepping outside of one's usual reference points). I was with my six-year-old daughter, who quickly gravitated toward the colorful magnetic tiles on the wall that visitors could try to fit together. She spent a good half hour there, eventually joining forces with a couple of young university students. I would come and check on her every once in a while and heard some interesting discussions about whether or not it was worth looking for patterns to help guide the placing of the tiles. The fifteen-year age difference didn't seem to bother anyone.

The tiles display was one of the two installations here that offered the visitor a genuine chance to engage in mathematical activity, to think about pattern and structure while satisfying an aesthetic urge to make things fit and grow. The other consisted of two workspaces—a touch screen calculator and a blackboard—where visitors could try their hand at producing the number 2011 by using natural numbers (in order, starting at one and no skipping) and any combination of operations. This simple puzzle attracted many students, especially to the blackboard. I saw one young girl listing the results of 2!, 3!, 4!,…. Another was listing the results of 2^3, 3^3, 4^5, 5^6,…. Both were having a lot of difficulty doing the calculations (but didn't seem interested in using the calculator); both were engaged in a very systematic effort in which the gathering of ingredients seemed more important than a brute attempt at adding and multiplying numbers. A young man was proposing that instead of just considering 2^3, one might also calculate $(2^3)^4$. The instructions included a challenge to find the "best" method. According to these instructions, this meant finding the shortest way. It's not clear how this aesthetic imposition (why not use the fewest number of operations or a repeating pattern or…?) influenced the work of the museum-goers, but many persevered under its order.

On a large concave screen, a series of visual mathematical concepts and results were shown: Penrose tilings, the Pythagorean theorem, Ulam's spiral, and their like. I wasn't sure why the screen was dome-shaped, since most of the visualizations were drawn from plane geometry. We've seen these same visualizations before (and better, interacted with them) on the Web and in many kinds of mathematics software. They are still appealing to look at and probably intended to communicate something about the structure of Gromov's fourth mystery, but little about the mode of presentation, the materials used, or the engagement with the audience made it seem like an encounter between mathematics and contemporary art.

Over on the farthest wall another film about the micro- and macrocosm was projected, in which an empty blackboard is quickly filled with ghostly mathematical writing produced by no visible hand. The narrator (the ghost?) informs us that, in physics, the very idea of nothing has radically changed. First, a simple square is traced in chalk on the blackboard which can be thought of as containing "nothing". We are told that this is how we used to think about nothing. Then another square is traced, this time adding a bunch of squiggles representing neutrons and organisms. Now (he goes on to say) nothing is the potential for something. This all sounds like grand narrative—genesis and the big bang proceed similarly—but the argument must leave many visitors perplexed. Nonetheless, I was drawn past the hyperbolic language by the simple and quite lovely visual diagramming and its suggestion of a physico-mathematical epistemology: the chalk leading the thoughts carrying the idea.

Off to the side, a small room contained a big sphere. If you walked around to the opposite side, you could see what were supposed to be young robotic creatures. (This is what the catalog calls them; the uninitiated might mistake them for plastic lampshades.) Positioned as explorers of objects in their environment, they ostensibly embody Gromov's theoretical concept of ergosystems. But when I was there, nothing moved—another broken science museum exhibit?—and the deeper purpose of the display remained opaque.

The Human Touch

What I have just described appears in the exhibit's upstairs portion, which is somewhat bewildering and disjoint, despite the alleged continuity of the Gromov narrative. Moving downstairs, the visitor finds three subrooms. In the first and largest room, eight short videos of mathematicians talking about various aspects of their work and passion were being projected. These were produced by Raymond Depardon and Claudine Nougaret, who manage to capture very personal and engaging mathematical portraits of Sir Michael Atiyah, Jean-Pierre Bourguignon, Carolina Canales González and Giancarlo Luccini, Alain Connes, Nichola El Karoui, Misha Gromov, Cédric Villani, and Don Zagier. Zagier enthuses about the beauty, creativity, and satisfaction of mathematics and provides a persuasive example, involving a sequence of integers, of the kind of misleading simplicity that delights so many mathematicians. He also evokes a somewhat "lone ranger" sense of the mathematician engaged in the "battle" between the individual and nature using only the "human mind" and "no apparatus". This mathematical mythos, though compelling, alas undervalues the social and material nature of work in the discipline. The young Villani emphasizes the importance of communication in mathematics and, in contrast to the lone ranger Zagier, works in tandem with his blackboard ("*le tableau noir est toujours là*"). He also gives a nice example of his own sense of wonder at discovering simple variations of the Pythagorean theorem. Atiyah evokes the social nature of mathematics by talking about the quickness with which contemporary mathematicians can exchange ideas, compared with the "archaic" mode of the written word. He also makes explicit analogies between the work of mathematicians and that of artists—some of the only in the show!—and quotes Weyl's insistence that in mathematics, one inevitably chooses beauty over truth.

These short performances were mostly effective at capturing the experience of mathematicians, including perhaps the element of Ophelian madness so pleasingly noted by Whitehead. None of the mathematicians talks about Gromov's themes (nature, the brain, or life) nor of the economical and political concerns that have led to the highly publicized departure from the discipline of at least two great mathematicians (including Grothendieck, whose writing provides the title and maybe the inspiration for the exhibit) nor of the pain and anxiety summoned for a large part of the population by the very idea of mathematics. Despite these omissions, taken as a whole, the videos may still succeed in challenging the image of mathematicians many of their viewers bring.

In the middle room there was a large "mural fresco" showing the thematic constellations of Poincaré's work. But it was hardly a fresco in the usual artistic sense: instead, it was a bunch of points on the wall (labeled with topics that Poincaré studied) connected by lines (showing how these topics relate).

Across the room was a beautiful film by Jean-Michel Alberola entitled *La main de Cédric Villani*, whose purpose was to show how mathematics writing, both rigorous and impulsive, is a gestural choreography. The film shows a closeup of Villani presenting Cercignani's conjecture on a blackboard. In the previous room he had already established this perennial tool of the mathematical lecture room as a kind of right arm, an extension of himself he could use to communicate his ideas. (In fact, despite the growing use of computers by mathematicians—and not just the experimental mathematics community—this digital tool seems to remain here in the closet.) The film begins with a large shot of the blackboard, with Villani pacing back and forth in front of it. Then it zooms in as he raises his chalk and makes a few marks, as if warming up. Then he lets loose on a dazzling array of points, lines, curves, all in a rhythm of anticipation. He looked like a conductor goading his mathematical objects along. The viewer watches the performance but feels that the substantial practice and repetition involved in the dance of the chalk is somehow overshadowed by a sense of immediacy, persuasiveness, and seeming newness. I was reminded of Gilles Châtelet's (1993) assertion of the intimate link between gestures and diagrams in mathematics and, especially, the gesture as the locus of mathematical inventiveness. This small film seemed to be the most compelling example to me of mathematics and art and of mathematics as art.

In the last large room stood a comparatively lonely aluminum sculpture by Hiroshi Sugimoto of a surface of revolution of constant negative curvature. Apparently, the extreme tip—the gesture to a point at infinity—is so small that the artist required modern robotics to fashion it.

On Art and Mathematics

There are many conferences, books, courses, and classroom activities that try, in various ways, to explore or forge connections between mathematics and art. In many educational settings, art is used as a motivational context in which to attract the attention of learners so that they might compare some ratios (Alberti's perspective drawing) or calculate some areas (Mondrian's geometric abstraction), just to name some popular examples. One consequence of these well-meaning approaches is that they endorse the belief that mathematics itself is an aesthetically sterile domain or at least one whose potentialities are realised only through engagement with external domains of interest. The mathematicians videotaped by Depardon and Nougaret insist otherwise, and the situations shown in the second room provide at least some visual insight into the compelling patterns and structures that mathematicians work with. But I had hoped that this meeting of art with mathematics would have more provocatively, subtly, and perhaps even uncomfortably transformed the viewer's way of thinking of mathematics.

I wonder whether the framing of the exhibit—at least the top floor—in terms of Gromov's four mysteries started things off on the wrong foot. Art tends to be good when it evokes mysteries for the viewer or nudges the viewer toward mysteries otherwise overlooked, but when it earnestly points them out, the viewer is left with little more than a fact.

References

Gilles Châtelet, *Les Enjeux du mobile: Mathématiques, physique, philosophie,* Seuil, Paris, 1993.

Brian Rotman, *Signifying Nothing: The Semiotics of Zero*, St. Martin's Press, New York, 1987.

A. Whitehead, *Science and the Modern World*, Macmillan, New York, 1926.

Sudden Disorientation in a Paris Museum

Michael Harris

At least one French journalist is convinced that the message of the exhibition that opened last October in Paris at the Fondation Cartier for Contemporary Art is that Alexander Grothendieck has now been "rehabilitated". Maybe she reached this conclusion because the exhibition, a collaborative effort involving (among others) nine artists, eight mathematicians, and a Large Hadron Collider—more on them later—is entitled *Mathématiques, un dépaysement soudain*, a quotation from Grothendieck's unclassifiable and (so far) unpublishable 900-page memoir *Récoltes et Sémailles.* Who was the first to realize Grothendieck was in need of rehabilitation? It wasn't a mathematician: though those who knew him continue to regret his decision to abandon his position at the center of algebraic geometry in the early 1970s on political grounds, his influence has only grown in the intervening years, and he is now regularly listed as one of the greatest mathematicians of the twentieth century, occasionally as the greatest of all.[1]

That Grothendieck might need rehabilitation and that his time has now come sounds like an idea hatched by the French public relations industry, known in France as *communications* or just *com*, always alert to the question of what is appropriate to believe about any subject of importance—and

Michael Harris is professor of mathematics at UFR de Mathématiques, Université Paris-Diderot Paris 7. His email address is harris@math.jussieu.fr.

DOI: http://dx.doi.org/10.1090/noti861

[1] *For example, at this site:* http://blog.tanyakhovanova.com/?p=218.

in France mathematics is such a subject. By turning his back on prestige, going so far as to refuse the Crafoord Prize in 1988, Grothendieck broke with acceptable public opinion, expressing ideas potentially subversive to the social order. But now he can be forgiven.

If Grothendieck's ideas are no longer dangerous, it's not only because his public statements over the last twenty years or so have become increasingly bizarre, culminating with his insistence in 2010 that all copies of his work be removed from libraries and destroyed. Ideas like Grothendieck's have in any case lost their relevance to opinion makers.[2] The evolution was symbolized by the election of French President Nicolas Sarkozy in 2007 on a platform of *argent décomplexé*, relaxed money. Sarkozy's supporters called upon the rich not to be ashamed of their wealth, and the president himself was notorious for his fascination with symbols of affluence: yachts, expensive restaurants, and especially the Rolex. Jacques Séguéla, *com* champion closely associated with the (opposition) French Socialist Party, was perplexed when the press kept writing about the Rolex: "Si à 50 ans, on n'a pas une Rolex, on a raté sa vie" [If you don't have a Rolex by the time you're 50, you've wasted your life].

Nowadays *dépaysement* is a commodity French travel agents market to busy professionals looking for novel vacation experiences, the prepackaged unfamiliarity of an unfamiliar sun, an unfamiliar landscape, a (slightly) unfamiliar cuisine, comparable to the English "change of scene" rather than to *A Beautiful Elsewhere*, the Cartier exhibition's official English title. But the word literally refers to the state of not being in one's hometown, and its alternative meaning of "disorientation" is by far the better translation in reference to Grothendieck. Imagine Club Med offering a one-way ticket to the middle of a war zone in a foreign country where you are at constant risk of deportation and death. That profoundly disorienting experience, still on offer in many parts of the world, was Grothendieck's as a teenager during the Second World War. The experience one takes home from *A Beautiful Elsewhere* is not of comparable intensity.

The Fondation Cartier is the creation of Rolex's rival, the French jeweler and watchmaker Cartier. I'm no expert in the semiotics of luxury timekeeping and can't tell you where Rolex stands relative to Cartier—official purveyor in times past to such kings as Carlos I of Portugal, Peter I of Serbia, Fouad I of Egypt, and Zog I of Albania—on the scale of prestige vs. vulgarity. What I do know is that if I were Cartier, I would be jealous of the lineup Rolex has assembled, both for its gravitas (Hans Magnus Enzensberger! Toni Morrison!) and for its hipness (Brian Eno!! Kate Valk!!) in its annual Mentor and Protegé Initiative. The luxury industry (Espace Cardin and LVMH in Paris, Fondazione Prada in Venice) is well represented among the branded art exhibition spaces that have proliferated in recent decades, alongside insurance (the Generali Foundation in Vienna), banking, shopping (Selfridge's), and *com* itself (the Saatchi Gallery in London).[3]

Damien Hirst, hardly the most high-minded of the Young British Artists, once said, …"I'm not Charles Saatchi's barrel-organ monkey.…He only recognises art with his wallet…he believes he can affect art values with buying power, and he still believes he can do it."[4] Years before the Fondation Cartier moved into its Jean Nouvel-designed exhibition space on Boulevard Raspail—of which Eric Hazan wrote that it "at least has the merit of having preserved Chateaubriand's cedar tree"—Alain-Dominique Perrin, then, as now, president of the Fondation Cartier, wrote candidly about the goals of art sponsorship: "Patronage of the arts is not only a formidable public relations [i.e., *communications*] tool, it's much more than that; it's a tool to seduce public opinion." The strategic goal, wrote Perrin, is to "neutralize criticism".[5]

Echoing situationist Guy Debord but from the other side of the barricades, so to speak, Perrin added, "The efficacy of this PR strategy is not limited to creating the event…patronage is…a medium that makes use of the other media." Media coverage of *A Beautiful Elsewhere* has indeed been massive, including an entire special issue of the monthly popular science magazine *Sciences et Avenir* (with the Cartier logo on the front cover), an ad campaign that plastered every corner of the French capital with billboards, and of course a few brief articles in the daily and weekly press.

I ought to stress that I'm not opposed to private philanthropy or even corporate sponsorship per se. I have benefited from the former both personally and as an organizer of conferences, and in any case there's no way to work these days as a mathematician, much less as an artist, without coming to some arrangement with private funding sources. I ran across Perrin's remarks in a 1994 conversation between radical conceptual artist Hans Haacke and sociologist Pierre Bourdieu. Haacke's projects in the 1980s included a collage (*Cowboy with Cigarette*) in the style of Picasso as a reaction to Philip

[2] *Or so it seemed when the exhibition was planned, before the surprising events of 2011.*

[3] *For banking: the Deutsche Bank collection in Frankfurt, New York, and around the world, and the Bank Austria Kunstforum in Vienna. To this list we might add the former collaboration between Philip Morris and the Whitney Museum of American Art in New York.*

[4] `http://www.guardian.co.uk/uk/2003/nov/27/arts.artsnews`. *This year Saatchi himself observed (*Guardian, *2 December 2011) that "being an art buyer these days is comprehensively and indisputably vulgar."*

[5] *Quoted in P. Bourdieu and H. Haacke,* Libre-échange, *Paris: Seuil (1994), pp. 26–27, 37.*

Morris sponsorship of a 1989 exhibit on early cubism and an exploration, modeled on jewelers' window displays, of Cartier's links with apartheid South Africa. But Haacke himself has works in the Generali Foundation collection, and who can blame him? The IHES is no less brilliant a center of research since the creation a few years ago of the AXA Chair for Mathematics.[6] But any occupant of the chair has to know that, as far as the insurance company is concerned, he or she is now wearing the AXA jersey.[7]

No such branding accompanied the unveiling in October of a plaque at the École Normale Supérieure thanking the Fondation Jean-Luc Lagardère for the renovation of the Département des Mathématiques et Applications. There is no mention of the event on the Lagardère or EADS websites. Nor did the weekly magazine *M—Le Monde*'s answer to the *New York Times*' *T*—refer to the ENS in its cover story, published in October, on "the dream life of [the late Jean-Luc's son] Arnaud Lagardère," who "reigns over arms and media, aviation and publishing." This is a bit strange, since ENS is in the center of Paris and is much better known than the IHES. Grothendieck may be indirectly responsible for this discretion. Although the Lagardère conglomerate is mainly active in publishing and media, it is "a major shareholder in EADS…the leading aeronautics, space and defence group in Europe and the second largest in the world…and exercises joint control over the company."[8] Grothendieck, the "great thinker, unknown outside theoretical cliques," is mentioned several times in the exhibition catalogue—astrophysicist Michel Cassé, one of the exhibition's three curators,[9] even dedicates his catalogue contribution to Grothendieck—but there's not a word to explain his absence from the community of researchers. His resignation in 1970 from the IHES is mentioned cryptically in the introduction to the special issue of *Sciences et Avenir* on the Cartier exhibition. You'll have to turn to the *Notices of the AMS* to learn that his departure was precipitated by his "conflict with the founder and director of the IHES…over military funding for the institute."[10]

Visitors arriving at *A Beautiful Elsewhere* are first directed to the Library of Mysteries, fruit of a collaboration of filmmaker David Lynch and punk rock icon Patti Smith with geometer Misha Gromov. The laws of physics, life, the human brain, and mathematical structure are the mysteries in question. Perfectly innocent when Gromov listed them a few years ago in a popular book about mathematicians entitled *Les déchiffreurs*, in the hands of the Cartier exhibition's curators these mysteries acquire the metaphysical urgency of the "Mysteries of Isis" to which Tamino is promised after his successful passage through the "Temple of Tests" in Mozart's *Magic Flute*. And one must indeed walk through a colonnade in order to enter the library, on one of whose walls a selection of books, chosen by Gromov for the light they attempt to shed on the four mysteries, thunder down from the zenith against the background of a handheld impending storm in a recognizably Lynchian night.[11] David Foster Wallace wrote that

> AN ACADEMIC DEFINITION of Lynchian might be that the term "refers to a particular kind of irony where the very macabre and the very mundane combine in such a way as to reveal the former's perpetual containment within the latter".[12]

Irony being altogether absent at the Fondation Cartier and in the exhibition catalogue, it would be better to say that the books in the library are framed by a *parody* of the Lynchian night. Other images are occasionally projected on the wall: when a white sheep appears against a neutral background, Patti Smith's voice recites "Baa baa black sheep" ("Yes, Sir" is translated "Oui monsieur"). Later in the cycle, her face materializes, swaying on the library ceiling ("in the shape of a zero"), blurring and fading as she sings an excerpt from Swinburne's *Loch Torridon*:

> All above us, the livelong night,
> Shadow, kindled with sense of light;
> All around us, the brief night long,
> Silence, laden with sense of song.

The next space is called the Room of the Four Mysteries and features one exhibit for each mystery on Gromov's list, plus a few bonus items. A collage by Beatriz Milhazes entitled *O Paraiso* (Paradise) represents the Mystery of Life as a kind of Club Med travel poster to a tropically chaotic world of fluid dynamics and diffusion reactions,

[6] *A move that brings no material advantage to its holder but instead allows the IHES to use his or her salary to invite additional visitors.*

[7] *See* `www.axa-research.org/sites/dev/files/u/video/axa_institutionnel_rework.flv`.

[8] *Information from* `www.lagardere.com`.

[9] *Along with IHES director Jean-Pierre Bourguignon and Hervé Chandes, director general of the Fondation Cartier.*

[10] *From Allyn Jackson's article Comme Appelé du Néant—As If Summoned from the Void: The Life of Alexandre Grothendieck, part 2,* Notices of the AMS, *November 2004, p. 1199.*

[11] *Alternating with a nervous blood red and a steady sky blue. The library includes works by Poincaré, Helmholtz, Heraclitus, Archimedes, Darwin, Galileo, and many others, including Grothendieck's* Récoltes et Sémailles. *Edifying excerpts are projected helpfully onto the wall, translated into French and English.*

[12] *In "David Lynch keeps his head",* Premiere, *September 1996.*

featuring a jaguar, a red parrot, a peacock, fire, and an enormous wave, each tagged by the relevant equation. Lynch offers a high-contrast handheld brooding film of the glowing hands of Bruno Mansoulié, a physicist at CERN, drawing Feynman diagrams, punctuated by occasional real-time interruptions by an instrument panel at the Large Hadron Collider (the very small) or the Planck satellite (the very large): the *Laws of Physics*. When Mansoulié has finished his lecture, Patti Smith's off-camera voice recites Gromov's text on the four mysteries; the "Mother Courage of Rock" (as Luc Sante called her recently in the *New York Review of Books*) adds poetry as her own choice for fifth on the list. The Mystery of the Brain is displayed in the form of "Artificial Curiosity", a "tribe of young robotic creatures" modeling Gromov's concept of an ergosystem. The creatures are meant to interact with spectators and learn in the process, "an experiment" (the press packet informs us) "that will allow the […] scientists [from INRIA and the Université de Bordeaux] to advance even further in their revolutionary research program." This mystery, unlike the first two mentioned, is actually quite entertaining[13]—the ninth graders visiting with their math class told me it was what they liked best—but its only obvious connection with contemporary art is the plastic head designed by Lynch, reminiscent of the skull of the baby in *Eraserhead*, topping each of the artificially curious robots.

The week the exhibition opened was a special one, with a six-page spread on Grothendieck in the French edition of *GQ* and recent Fields Medalist Cédric Villani identifying himself as "the Lady Gaga of mathematics" in the weekly middlebrow culture magazine *Télérama*. The interview mentioned Villani's participation in the Cartier exhibition, but like most of the press coverage, had very little to say about what was on display. For this you have to read the blogs, where comments like this one are typical:

> En effet, une expo très décevante! Artistiquement rien de plus que décorative et mathématiquement totalement superficielle, une imposture qui cultive le mysticisme autour des maths.…Ne perdez pas votre temps à y aller.

Mathematicians were divided between those so put off by the *com* style of the exhibition's promotion that they threw away their complementary tickets and those who thought that any widely publicized event that brings mathematics to the attention of the general public deserves the benefit of the doubt. What I haven't heard from French colleagues who have been to the show are thoughts about the relations, if there are any, between art and mathematics. No one seems to have noticed what a paradox it is to hear mathematicians claim without hesitation that beauty is the object of their work—not that it's so easy to attribute a precise meaning to this claim, and in a technical sense it's pretty clear that Gromov, Lynch, and Smith were aiming at the *sublime*—in an institution "for contemporary art" where that sort of talk is generally considered to be beside the point. *Notices* readers don't need to be told that the word "art" in the contemporary world is extraordinarily inclusive, but it seems to me that what it designates needs at a minimum to be capable of being incorporated in some sort of dialogue with traditional and historical uses of the word. If such a dialogue is under way at the Fondation Cartier, I was unable to detect it, and I am tempted to define *com* as precisely the form of dialogue in which opinions travel in one direction only.

The exhibition continues downstairs with a sculpture of a surface of constant negative curvature by Hiroshi Sugimoto, culminating at its apex in what is supposed to represent a singularity at infinity, spectacular but somehow pointless, enormous and yet much too small for the room in which it is displayed. Jean-Michel Alberola's contributions are especially unconvincing: a mural representing a conceptual map of Poincaré's work and yet another film of hands writing equations, belonging this time to Villani.

If you look elsewhere than in *A Beautiful Elsewhere* you can easily find evidence that Alberola is in fact an interesting artist, like the others participating in this cross-cultural experiment, and you are likely to wonder about a quite different mystery: how the collaboration of so many undeniably talented people, artists and mathematicians alike, gave rise to such an exercise in futility. Could it be as simple as this, that the relations between mathematics and the arts (such as they are) do not develop in interesting directions when *com* is the catalyst? My thoughts returned to Grothendieck, whose story is an extraordinary gift from mathematics to world culture that remains to be unwrapped. I used to think that David Lynch would be just the right artist to find the images to go along with words like these:

> Peu à peu au cours de la réflexion se révèle ce qui, dans ma vie, a été comme le "noyau dur", le centre redoutable de ce mystère, comme le coeur même de "l'énigme du Mal" : la violence qu'on peut appeler "gratuite", ou "sans cause", la violence pour le seul plaisir,

[13]*As is the infographic display in the same room that projects a sampling of the Mystery of Mathematical Structure (Penrose tilings, Euclidean geometry, Ulam's spiral of prime numbers, calculations in the symbolism of traditional Chinese and Japanese mathematics) in brilliant colors at dizzying speed. But when art meets mathematics, why does the result resemble nothing so much as high-tech advertising?*

> dirait-on, de blesser, de nuire ou de dévaster—une violence qui jamais ne dit son nom, feutrée souvent, sous des airs d'ingénuité innocente et affable, et d'autant plus efficace à toucher et à ravager—la "griffe dans le velours", délicate, vive et sans merci.[14]

But now I'm not so sure. It's a long way from Club Med to Club Silencio, the iconic theater of guilty conscience that marks the tremulous passage between two worlds (or two mysteries, if you prefer) in Lynch's *Mulholland Drive*. Or maybe not such a long way: only a half-hour metro trip from the Fondation Cartier to Lynch's new Paris nightclub, also called Club Silencio. All of the artists represented in *A Beautiful Elsewhere* have worked with the Fondation Cartier previously, in some cases more than once. Those who don't live in town presumably have their reasons to come to Paris. Lynch, "based off and on…for the last four years" in Paris, according to a recent *Guardian* interview, is an Officier de la Légion d'Honneur; Smith was named Commandeur de l'ordre des arts et lettres, in part for her appreciation for Rimbaud. Nobody seems to have been inconvenienced by Cartier's *dépaysement*; it's even mentioned as a footnote to the *Guardian* article, which focuses on Lynch's new CD, *Crazy Clown Time*, and on his enthusiasm for transcendental meditation:

> "Légion d'Honneur! Légion d'Honneur!" Grothendieck was shouting from the back of the auditorium, waving a paper facsimile of the Légion d'Honneur cross. …Grothendieck then mounted the podium and began speaking against NATO support for the conference.[15]

The final basement room is devoted to a 32-minute documentary, entitled *Au bonheur des maths* (The Joy of Math?), by Raymond Depardon and Claudine Nougaret, consisting of eight 4-minute interviews, each devoted to one (or in one case two) of the participating mathematicians: Sir Michael Atiyah, Jean-Pierre Bourguignon, Alain Connes, Nicole El Karaoui, Carolina Canales Gonzales and Giancarlo Lucchini, Misha Gromov, Cédric Villani, Don Zagier. The mathematicians, mostly shown in extreme closeup against neutral backgrounds, bookshelves, or blackboards, say what's on their minds with the authenticity one expected from Patti Smith, the humor one expected from Takeshi Kitano (whose contribution to the Room of the Four Mysteries is not even worth mentioning), the sensitivity to the uncanny one expected from David Lynch. I was particularly impressed by Villani's segment—he displays a real sense of dramatic timing in explaining how he rediscovered the triangles of his adolescence after two decades of mathematical research—and by Gromov's paradoxical observation that mathematical thinking and biological evolution move in opposite directions. But the speakers were uniformly thoughtful, articulate, and appealing; the film, in which the presence of the artists is reduced to a bare minimum, almost redeems the exhibition.

It's probably pointless to ask Pedro Almodóvar to film Grothendieck's life. If you've seen Almodóvar's *Talk to Her,* you'll remember the scene where Caetano Veloso delivers an indescribably beautiful rendition of a Mexican folk song in an improbably beautiful private garden to a select group of impossibly beautiful "beautiful people". If I've learned anything from the exhibit at Fondation Cartier, it's that such scenes take place in real life as well. But I learned that indirectly by reading an article published in *Le Monde*'s magazine *M.* On the Friday following the opening, Patti Smith read Swinburne's *Loch Torridon*, accompanied by David Lynch on electric piano, before a select group of guests sitting on the floor of the Fondation Cartier—probably in the basement room where the Depardon-Nougaret interviews are projected during the day. *M*'s reporter "had the impression of attending a proof *in situ* of the theorem [sic] on two parallel lines that never meet." Attending were actress Isabelle Huppert, actor Vincent Lindon, filmmaker Agnès Varda, and a scattering of local celebrities, but whether or not any of the mathematical stars of *Un dépaysement soudain* was considered beautiful enough to number among the two-hundred guests and to join them for the after party at Club Silencio, none was beautiful enough to merit mention in *Le Monde*.

A few years ago I saw another film by Depardon and Nougaret in that same basement room. Entitled *Donner la parole*, translated as "Hear them speak", practically the same length as *Bonheur des maths*, the film consisted of monologues by people from literally all over the world, describing in their own languages their cultures and ways of life, all threatened with extinction. I hope it was a coincidence.

Acknowledgment

My thanks to Nathalie Sinclair and Vladimir Tasić for their insightful comments on a first draft of this article, to Gaël Octavia for playing the devil's advocate, and to Jean-Michel Kantor for more reasons than I can mention.

[14] Récoltes et Sémailles*, p. 923.*

[15] *From Allyn Jackson's 2004* Notices *article, already cited. The scene was the 1972 Antwerp summer school on modular functions, and Grothendieck was interrupting Jean-Pierre Serre, who had recently been named to the Légion d'Honneur. It was one of his last appearances at a mathematics conference.*

Remembering Yahya Ould Hamidoune

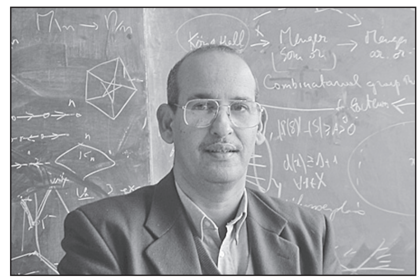
Toka Diagana

Yahya Ould Hamidoune, one of the most famous Mauritanian mathematicians, passed away in Paris, France, on March 11, 2011. Yahya was born in 1947 in Atar, Mauritania, to a family of very famous scholars and poets. After completing elementary and middle schools in Mauritania, Yahya attended high school and college in Cairo, Egypt, while his father was serving there as a diplomat. Next he went to France to pursue graduate studies. He received his Ph.D. degree in mathematics in 1978 from the University of Paris VI under the supervision of Michel Las Vergnas. He then joined the CNRS and was promoted to the rank of Research Fellow in 1981.

Yahya authored about one hundred papers, mainly on abstract theory of connectedness, enumerative combinatorics, the topology of Cayley graphs, Erdős-Heilbronn conjecture, Erdős conjectures, subsequences of a zero-sum, the Diderrich conjecture, theorems of addition, the Waring problem over finite fields, and the problem of Frobenius. His most significant result is undoubtedly his proof of the Erdős-Heilbronn conjecture, which he obtained in 1992 along with Dias da Silva. Additionally, he recently solved a question raised by Terence Tao. That new result is called Hamidoune's Freiman-Kneser theorem for nonabelian groups.

In 2001 Yahya was awarded the Prix Chinguitt for his work "Recent results in additive number theory". The Prix Chinguitt is the most important prize that the Mauritanian government awards yearly to recognize the merits of either a citizen of Mauritania or of another country for major research achievements.

Apart from his research activities, Yahya had various ambitions for Mauritania, which he so loved. Among other things, he particularly wanted to see his country emerge among the nations in which scientific research is an essential tradition. Such an ambition led him in 2002 to organize a large meeting in Nouakchott, Mauritania, which attracted nearly all the Mauritanian scientists from around the world. The meeting was, without any doubt, a great success. Unfortunately, a repeat of that meeting has not yet occurred, mostly due to the political instability into which Mauritania has fallen in recent years.



Photograph courtesy of Michel Las Vergnas.

Yahya Ould Hamidoune

Despite his busy schedule, Yahya was a tireless advocate when it came to assisting in the improvement of education in Mauritania. In particular, he was involved in numerous worthwhile projects. For instance, a few years ago he worked tirelessly with Mohameden Ould Ahmedou (Giessen University, Germany) on a project that consisted of founding preparatory classes for the Grandes Écoles of Engineering. More recently, Yahya, along with Mohameden Ould Ahmedou and Alain Plagne (École Polytechnique, France), had undertaken another major project related to education in Mauritania. Their project was to provide opportunities for bright young students from Mauritania to pursue their studies at the École Polytechnique of Paris. Yahya was deeply committed to this project, but unfortunately, it will now be done without him. Clearly, Mauritania has lost one of its best ambassadors in the person of Yahya Ould Hamidoune.

Acknowledgment: The author expresses his thanks to Profs. Mohameden Ould Ahmedou, Ahmedou Ould Haouba, Saleh Ould Moulaye Ahmed, and Alain Plagne for careful reading of this note and insightful comments.

Toka Diagana is professor of mathematics at Howard University. His email address is tdiagana@howard.edu.

DOI: http://dx.doi.org/10.1090/noti859

Mathematicians Take a Stand

Douglas N. Arnold and Henry Cohn

In an effort to keep *Notices* readers apprised of timely matters, we offer here articles presenting the two sides of the Elsevier boycott issue. Douglas Arnold and Henry Cohn write in favor of the boycott, while David Clark and Laura Hassink write on behalf of Elsevier. These two articles represent the views of the authors, and not those of the editors or the American Mathematical Society. This editor (SGK) is in fact the editor of an Elsevier journal. He takes no side in this discussion.

—Steven G. Krantz

Mathematicians care deeply about the mathematical literature. We devote much of our lives to learning from it, expanding it, and guaranteeing its quality. We depend on it for our livelihoods, and our contributions to it will be our intellectual legacy.

It has long been anticipated that technological advances will make the literature more affordable and accessible. Sadly, this potential is not being fully realized. The prices libraries pay for journals have been growing with no end in sight, even as the costs of publication and distribution have gone down, and many libraries are unable to maintain their subscriptions.[1]

The normal market mechanisms we count on to keep prices in check have failed for a variety of reasons. For example, mathematicians have a professional obligation to follow the relevant literature, which leads to inelastic pricing. This situation is particularly perverse because we provide the content, editorial services, and peer review free of charge, implicitly subsidized by our institutions. The journal publishers then turn to the same institutions and demand prices that seem unjustifiable.

Although the detailed situation is complex, the fundamental cause of this sad state of affairs is not hard to find. While libraries are being forced to cut acquisitions, a small number of commercial publishers have been making breathtaking profits year after year. The largest of these, Elsevier, made an adjusted operating profit of $1.12 billion in 2010 on $3.14 billion in revenue, for a profit margin of 36 percent, up from 35 percent in 2009 and 33 percent in 2008.[2] Adding insult to injury, Elsevier has aggressively pushed bundling arrangements that result in libraries paying for journals they do not want and that obscure the actual costs.[3] They have fought transparency of pricing, going so far as to seek a court injunction in an unsuccessful attempt to stop a state university from revealing the terms of their subscription contract. They have imposed unacceptable restrictions on dissemination by authors. And, while their best journals make important contributions to the mathematical literature, Elsevier also publishes many weaker journals, some of which have been caught in major lapses of peer review or ethical standards. These scandals have done harm to the integrity and reputation of mathematics.

This situation has been extensively analyzed many times before, including in the *Notices*. There have been some high-profile actions, such as mass resignations of entire Elsevier editorial boards over pricing concerns: the *Journal of Logic Programming* in 1999, the *Journal of Algorithms* in 2003, and *Topology* in 2006. These boards have done a valuable service for the community by founding replacement journals, but there has been little relief from the overall trend. As Timothy Gowers wrote in his blog in January, "It might seem inexplicable that this situation has been allowed to continue. After all, mathematicians (and other scientists) have been complaining about it for a long time. Why can't we just tell Elsevier that we no longer

Douglas N. Arnold is McKnight Presidential Professor of Mathematics at the University of Minnesota. His email address is `arnold@umn.edu`.

Henry Cohn is principal researcher at Microsoft Research New England and adjunct professor of mathematics at MIT. His email address is `cohn@microsoft.com`.

[1] *For example, MIT's spending on serials increased by 426 percent over the period 1986–2009, while the number of serials purchased decreased by 16 percent, and the Consumer Price Index increased by only 96 percent.*

DOI: http://dx.doi.org/10.1090/noti857

[2] *Reed Elsevier Annual Report 2010, SEC form 20-F (based on data from p. 25 and average exchange rate from p. 6).*

[3] *T. Bergstrom, Librarians and the terrible fix: economics of the Big Deal,* Serials ***23*** *(2010), 77–82.*

Table 1: Summary information for six journals.

Journal	Publisher	Metrics	Price	$/art.	$/page	$/cite
Annals of Mathematics	Princeton	3.7/A*	$447	5.39	0.12	0.06
SIAM J. Appl. Math.	SIAM	1.8/A*	$642	5.95	0.27	0.13
Journal of the AMS	AMS	3.6/A*	$300	9.09	0.24	0.13
Advances in Mathematics	Elsevier	1.6/A*	$3,899	11.53	0.35	0.90
Journal of Algebra	Elsevier	0.7/A*	$6,944	13.89	0.75	1.22
Journal of Number Theory	Elsevier	0.6/B	$2,745	17.49	1.12	1.91

Metrics are the 2010 5-year impact factor from *Journal Citation Reports* and the 2010 rating by the Australian Research Council (based on expert opinion). A* = top-rated, B = "solid, though not outstanding".

Elsevier prices are the amounts actually paid by the University of Minnesota for electronic-only institutional subscriptions in 2012. The lowest prices we could find on the Elsevier website as of February 29 were $3,555.20, $5,203, and $2,226.40. The *Annals* price is again the actual amount paid by UMN, which is slightly greater than the $435 list price. The SIAM and AMS prices are the list prices, although UMN paid less because of institutional membership.

Columns 5–7 normalize by the most recent data available: the numbers of articles and pages published in 2011 and the number of citations to the journal made in 2010 (as reported in *Journal Citation Reports*).

wish to publish with them?" Gowers then revealed that he had been quietly boycotting Elsevier for years, and he suggested it would be valuable to create a website where like-minded researchers could *publicly* declare their unwillingness to contribute to Elsevier journals.

Within days, Tyler Neylon responded to this need by creating `http://thecostofknowledge.com`. More than 2,000 people signed on in the first week, and participation has grown steadily since then, to over 8,000 as of early March. Each participant chooses whether to refrain from publishing papers in, refereeing for, or editing Elsevier journals. The boycott is ongoing, and it holds the promise of sparking real change. *We urge you to consider adding your voice.*

The boycott is a true grassroots movement. No individual or group is in charge, beyond Gowers's symbolic position as the first boycotter. However, a group of thirty-four mathematicians[4] (including Gowers and the authors of the present article) issued their best attempt at a consensus statement of purpose for the boycott. It is available online,[5] and we highly recommend it for reading. For reasons of space, we will not cover every aspect of that statement here.

Before we proceed, we must address two pressing questions about the boycott. First, why is a boycott appropriate? After all, Elsevier employs many reasonable and thoughtful people, and many mathematicians volunteer their services, helping to produce journals of real value. Isn't a boycott overly confrontational? Could one not take a more collaborative approach? Unfortunately, such approaches have been tried time and again without success. Fifteen years of reasoned discussions have failed to sway Elsevier.[6] Elsevier's leadership seems to be driven only by their fiduciary responsibility to maximize profit for their shareholders. The one hope we see for change is to demonstrate that their business depends on us and that we will not cooperate with them unless they earn our respect and goodwill.

Second, why is the focus solely on Elsevier? Some of the problems we discuss are common among large commercial publishers, and indeed we hope the boycott will help spur changes in the whole industry. But we must start somewhere, and we believe it is more effective to focus on one publisher whose behavior has been particularly

[4]*Scott Aaronson, Douglas N. Arnold, Artur Avila, John Baez, Folkmar Bornemann, Danny Calegari, Henry Cohn, Ingrid Daubechies, Jordan Ellenberg, Matthew Emerton, Marie Farge, David Gabai, Timothy Gowers, Ben Green, Martin Grötschel, Michael Harris, Frédéric Hélein, Rob Kirby, Vincent Lafforgue, Gregory F. Lawler, Randall J. LeVeque, László Lovász, Peter J. Olver, Olof Sisask, Terence Tao, Richard Taylor, Bernard Teissier, Burt Totaro, Lloyd N. Trefethen, Takashi Tsuboi, Marie-France Vignéras, Wendelin Werner, Amie Wilkinson, and Günter M. Ziegler.*

[5]*See* `http://umn.edu/~arnold/sop.pdf` *or the March 2012* London Mathematical Society Newsletter.

[6]*R. Kirby, Comparative prices of math journals, 1997,* `http://math.berkeley.edu/~kirby/journals.html`; *J. Birman, Scientific publishing: A mathematician's viewpoint,* Notices of the AMS ***47*** *(2000), 770–774; R. Kirby, Fleeced?,* Notices of the AMS ***51*** *(2004), 181; W. Neumann, What we can do about journal pricing, 2005,* `http://www.math.columbia.edu/~neumann/journal.html`; *D. N. Arnold, Integrity under attack: The state of scholarly publishing,* SIAM News ***42*** *(2009), 2–3; P. Olver, Journals in flux,* Notices of the AMS ***58*** *(2011), 1124–1126.*

Table 2: Historical prices per page in constant 2012 dollars.

Journal	1994	1997	2000	2003	2006	2009	2010	2011
Annals of Mathematics	0.19	0.20	0.14	0.15	0.13	0.13	0.09	0.10
SIAM J. Appl. Math.	0.20	0.24	0.23	0.25	0.27	0.24	0.18	0.27
Journal of the AMS	0.22	0.26	0.27	0.29	0.30	0.27	0.25	0.24
Advances in Mathematics	0.65	0.74	0.95	1.01	0.55	0.61	0.44	0.33
Journal of Algebra	0.36	0.43	0.50	0.73	0.60	0.77	0.92	0.66
Journal of Number Theory	0.57	0.67	0.98	1.01	1.04	0.86	0.95	1.05

Prices are from the AMS journal price survey (`http://www.ams.org/membership/mem-journal-survey`), adjusted using the Consumer Price Index.

egregious than to directly confront an entire industry at once. Many of the successful boycotts in history took the same tack.

Journal Pricing

Table 1 exhibits prices for three of Elsevier's mathematics journals: *Advances in Mathematics,* the *Journal of Algebra*, and the *Journal of Number Theory*. For comparison, the table includes three more affordable journals.

The *Annals of Mathematics*, published by the Princeton math department and IAS, provides exceptional quality at a rock-bottom price that just covers costs. The other two are highly regarded journals published by the Society for Industrial and Applied Mathematics (SIAM) and by the American Mathematical Society (AMS). Both of these organizations make a profit on their journal publishing operations, which helps to subsidize their other activities. For example, in 2011 SIAM's journal publication costs, including overhead, were 89 percent of their subscription revenues, resulting in an 11 percent profit margin.

Elsevier's recent pricing changes, apparently in response to the boycott, have at times led to multiple conflicting prices on their website. We have listed the prices actually paid by the University of Minnesota in 2012, but the notes after the table indicate the lowest prices we found offered on the Web. We made no attempt to select the highest-priced Elsevier journals, and in fact *Advances in Mathematics* is among the most affordable. For comparison, the 2011 prices per page of the thirty-six Elsevier journals listed in the AMS journal price survey ranged from \$0.33 (*Advances in Mathematics*) to \$4.05 (*Mathematical Social Sciences*), with a mean of \$1.35 and a median of \$0.96.

As shown in the table, the prices of the SIAM and AMS journals are within a factor of two of that of the *Annals*, with differences depending on whether one normalizes the raw journal price by number of articles, pages, or citations. But the Elsevier prices are a different story. The price per page of the *Journal of Algebra*, for example, is triple that of the society journals and six times that of the *Annals*, and the *Journal of Number Theory* is 50 percent more expensive yet.

Moreover, as demonstrated in Table 2, this problem has grown over time. The inflation-adjusted prices per page of the *Journals of Algebra* and *Number Theory* increased by more than 80 percent between 1994 and 2011, compared with much smaller increases for the society journals and a decrease for the *Annals*. It is noteworthy that the recent prices of *Advances in Mathematics*, while still high, have come closer to the prices of the society journals. This supports our belief that Elsevier could offer substantially lower prices and still make a reasonable profit.

We do not mean to suggest that publishing is cheap in the electronic age. True, electronic distribution is very cheap: the arXiv requires just \$7 per submission, or 1.4 cents per download, in funding.[7] But journal publishing involves significant additional costs, such as IT infrastructure, administrative support, oversight, sales, copyediting, typesetting, archiving, etc. Many of these costs scale roughly with the number of published pages, and some of them benefit from economies of scale (so large publishers like Elsevier should, if anything, achieve lower costs).

Of course, journals are not all the same. A low-circulation journal may need to command a higher price per page to stay afloat. The community might find some such journals too expensive to support, but one viewed as worthy of support might reasonably charge a higher price until more libraries subscribe. Another journal might have extraordinary expenses, for example from translation. But these factors do not apply to the cases we have considered or to many other Elsevier journals.

We see no good reason to pay much more for Elsevier journals than for journals that earn mathematical societies a tidy profit. Even the price

[7] `http://arXiv.org/help/support/faq`.

targets for mathematics journals that Elsevier announced in response to the boycott[8]—$0.50 to $0.60 per page—would leave their journals costing twice as much as the comparison journals in Table 1. Elsevier's prices have become far out of proportion and have a way to go to return to reasonable.

What's the Big Deal about Bundling?

Bundling refers to grouping together collections of journals and selling access as a single product, discounted from list price. Elsevier commonly negotiates bundles that include all the journals to which the library has recently subscribed. The bundles may also include access to nearly all of Elsevier's roughly 2,000 journals. Librarians have termed enormous bundles "the big deal".

While there is nothing wrong with offering quantity discounts,[9] it is the way in which Elsevier and other large publishers have implemented bundling that is objectionable. They have turned it into a powerful tool for subverting the market forces that would keep prices in check. The then director of Harvard's library summarized it thus: "Elsevier is among a handful of journal publishers whose commercial bundling practices are squeezing library budgets. Their licensing programs require libraries to maintain large, fixed levels of expenditure, without the ability to cancel unneeded subscriptions."[10]

Let us see how this works. While Elsevier has gone to great lengths to keep the details of their bundle contracts secret, some have come to light, thanks to open records laws.[11] Judging by the contracts we have seen and librarians we have consulted, it works essentially as follows.

The university commits to subscribing to the journals it currently receives at a negotiated total price that is typically around the same as they were previously paying and to continuing to subscribe to them for a period of three to five years with annual price increases. Elsevier has called this the "Complete Collection", and it is a large expense. For example, for the University of Minnesota in 2006 it came to $1.8 million (about 18 percent of their total serials budget), and for the University of Michigan in 2007 it was $1.9 million. In both cases, 5 percent yearly price increases were built into the contracts, although the actual rate of inflation for the contract periods was only about 2 percent per year. Cancellation of titles in the Complete Collection is restricted, which makes it difficult or impossible to cut back on the expenditure.

For an additional fee Elsevier offers their "Freedom Collection", which adds deeply discounted access to nearly all of the Elsevier journals to which the library had *not* chosen to subscribe. This option cost the University of Michigan about $19,000 more in 2007, inflating 5 percent a year thereafter. The University of Minnesota elected against it.

Although prices increase quickly inside the bundle, list prices can increase even more quickly, so a university that decides not to renew its bundle may face a steep price increase to hold onto the journals it wants. Because of bundling, ever larger portions of library budgets are locked into Elsevier contracts, budgetary pressures force the cancellation of titles from smaller publishers, and funds for new subscriptions disappear. Furthermore, bundling leads to a lack of clarity on pricing. The discounts on the additional journals in the Freedom Collection can sound impressive, but it is the pricing of the primary subscriptions that drains library budgets.

The constraints imposed by bundles have led some universities to conclude that even paying exorbitant prices for the journals they choose is a better deal. Harvard, MIT, the University of Minnesota, and others have now gone this route. However, many academic libraries remain tied to the big deal.

Price disclosure is necessary for a well-functioning market with competitive pricing, so the lack of transparency in bundling contracts is particularly troubling. As an Elsevier vice president wrote in support of Elsevier's 2009 lawsuit enjoining Washington State University from revealing the prices of their subscriptions, "Elsevier representatives apply pricing formulae and methods which are not generally known (to our competitors or potential customers)" and "disclosure could disadvantage Elsevier in that, if its pricing to customer X was known to customers Y and Z, the latter could demand the same pricing".[12] Elsevier may indeed profit from keeping Y and Z in the dark, but the academic community values sunlight. Without transparency of subscription contracts and costs, the community will remain skeptical of Elsevier's pricing, whatever changes they make to list prices.

Posting Policies

Gowers's suggestion of an Elsevier boycott struck a chord in many researchers. Besides pricing and bundling, there are other issues that have contributed to so many researchers' readiness to abandon Elsevier. One of these is Elsevier's policies concerning

[8] *D. Clark and L. Hassink, A letter to the mathematics community, February 27, 2012,* `http://www.elsevier.com/wps/find/P11.cws_home/lettertothecommunity`.

[9] *For example, Mathematical Sciences Publishers offers a bundle of six mathematics journals at a 31 percent discount, bringing their price down to $0.08 per page.*

[10] `http://hul.harvard.edu/news/2004_0101.html`.

[11] *T. Bergstrom, P. Courant, and R. P. McAfee,* Big Deal Contract Project.

[12] `http://www.econ.ucsb.edu/~tedb/Journals/WSUCourtCase/ElsevierStatementbySalesChief.pdf`.

dissemination. Thanks to the Internet, authors have additional ways of disseminating their work besides the printed journal and journal website. For example, it has become common practice for authors to post a finalized version of their manuscript to a repository such as the arXiv for open dissemination, as allowed by many publishers.[13] Elsevier's actions suggest that they view this development primarily as a threat to their profits, not as an opportunity to advance mathematics or increase their authors' readership. In short, their interests are not aligned with ours.

Elsevier's policies are complex and difficult to understand. In the words of the scholarly communications officer at Duke University, "It seems clear that the intent of these statements, policies and contracts is not to clarify the authors' obligations so much as it is to confuse and intimidate them."[14] Their posting policy[15] specifically prohibits posting an "accepted author manuscript"—the author's own version of a manuscript that has been accepted for publication—on an email list, a subject repository, or even the author's own institutional repository *if* the institution has a posting mandate. The last is not a typo: if your institution mandates posting the accepted author manuscript in its repository, then Elsevier stipulates that you may not, although they permit such posting when there is no mandate!

Fortunately, since hearing complaints from the boycotters about their posting policy, Elsevier has introduced an exception to allow posting to the arXiv. However, that is not enough. There are other noncommercial subject repositories that are important to segments of the community (Optimization Online, the Cryptology ePrint Archive, etc.), and more will undoubtedly be created in the future. Elsevier should allow authors to post accepted manuscripts to any such repository, as well as to university repositories, regardless of whether there is a posting mandate. Furthermore, this right should be guaranteed by the publishing agreement, not just by a posting policy that is subject to change at any time.

Ethics and Peer Review

Another source of frustration with Elsevier is their history of lapses in peer review and ethics. The case of the journal *Chaos, Solitons & Fractals* (CS&F) has become widely known. This journal published 273 papers by its own editor in chief over eighteen years, 57 of them in a single year. Suspicions that these papers were not subject to peer review are corroborated by the editor's declaration that "senior people are above this childish, vain practice of peer review."[16] Elsevier owes the community an explanation for this and other fiascos. Was there no oversight in place? Have changes been made so this will not happen again? What about the other papers in CS&F? Are there records of peer review? Will any papers that were not peer reviewed be retracted or otherwise flagged? The current situation leaves the literature in a bad state and compromises the position of authors who submitted manuscripts for peer review in good faith. If Elsevier wants to place this issue behind them, they need to deal with it thoroughly, forthrightly, and transparently.

In another notorious case, for five years Elsevier "published a series of sponsored article compilation publications, on behalf of pharmaceutical clients, that were made to look like journals and lacked the proper disclosures."[17]

There are other incidents in which peer review has failed at Elsevier journals, sometimes in spectacular fashion.[18] For many of us, these call into question Elsevier's ability to meet the standards of quality and ethics we require if we are to collaborate with them.

Initial Responses to the Boycott

On February 27, Elsevier publicly withdrew its support for the Research Works Act, which would have prohibited open access mandates for government-funded research. The bill was declared dead by its sponsors in Congress on the very same day. This victory confirmed the boycott's success in delivering a message where we were never able to get through before.

Further confirmation came that day in an open letter from Elsevier senior vice presidents David Clark and Laura Hassink to the mathematics community.[8] Besides reporting the about-face on the Research Works Act, they announced the target price for "core mathematics titles" that we discussed above. They also stated, correctly, that it would be necessary to address concerns about "large discounted agreements" (bundling) and said that this will come.

[13] *K. Fowler, Do mathematicians get the author rights they want?,* Notices of the AMS ***59*** *(2012), 436–438.*

[14] *K. Smith, What a mess!, S*cholarly Communications @ Duke, *July 7, 2011,* http://blogs.library.duke.edu/scholcomm/2011/07/07/what-a-mess/.

[15] http://www.elsevier.com/wps/find/authorsview.authors/postingpolicy, *accessed March 3, 2012.*

[16] *C. Whyte, El Naschie questions journalist in* Nature *libel trial, updated November 16, 2011,* http://www.newscientist.com/article/dn21169.

[17] *Statement from Michael Hansen, CEO of Elsevier's Health Sciences Division, regarding Australia-based sponsored journal practices between 2000 and 2005, May 7, 2009,* http://www.elsevier.com/wps/find/authored_newsitem.cws_home/companynews05_01203.

[18] http://umn.edu/~arnold/reasons.html.

Finally, Clark and Hassink announced that free access has been granted to the archives of fourteen core mathematics journals for the years from 1995 through four years before the present day. Access to back issues is indeed critical, and we strongly believe that all research papers should be made freely available long before copyright expires. The shorter the delay the better, of course, but we consider four years a defensible choice, compatible with the subscription model for journal publishing. The AMS's experience with a five-year window shows that such a move is financially viable. We hope that Elsevier's announcement is just the first step and that expansion to the full set of mathematical journals and the period before 1995 will be announced soon.[19] We also hope that this is not just a temporary measure. A binding commitment not to revoke access in the future would be reassuring on that point.

Moving Forward

While the mathematical literature itself is a treasure, the current system of scholarly publishing is badly broken. Elsevier is the largest and, in our view, the most egregious example of what is wrong. We hope many readers will agree with us that by choosing to withdraw our cooperation from Elsevier, we are sending a valuable message to them and to the scholarly publishing industry more broadly. Please consider joining the movement at `http://thecostofknowledge.com`.

What is our vision for the future? The mathematical community needs a period of experimentation and healthy competition, in which a variety of publishing models can flourish and develop. Possibilities include various approaches to open access publishing,[20] refereed journals tightly integrated with the arXiv or similar servers, increased reliance on nonprofit publishers, hybrid models in which community-owned journals subcontract their operations to commercial publishers, commercially owned journals with reasonable prices and policies, etc. It is too early to predict the mix of models that will emerge as the most successful. However, any publisher that wants to be part of this mix must convince the community that they oversee peer review with integrity, that they aid dissemination rather than hinder it, and that they work to make high-quality mathematical literature widely available at a reasonable price.

Let's work together to foster good practices and build better models. The future of mathematics publishing is in our hands.

[19] *All three journals discussed here began publishing in the 1960s. The issues before 1995 are currently available from Elsevier online but remain behind their paywall.*

[20] *For example, based on publication charges or on sponsoring consortia such as SCOAP³* (`http://scoap3.org`).

Elsevier's Response to the Mathematics Community

Laura Hassink and David Clark

During the last months we have spoken to many people in the community about the move by Timothy Gowers and some colleagues to declare their wish not to work with Elsevier and the subsequent boycott movement.

At the end of this article we summarize how we are responding to the feedback from the community and the very specific steps that we are taking. But we would first like to address the concerns raised and some of the arguments.

We are the leading journal publisher in scientific publishing and so will attract criticism that is directed at publishing as a whole. While we may disagree with much that has been said, we do recognize that Elsevier has not done a good job communicating what we do and how we support both the peer-review process and the dissemination of work. In particular, we have left authors, editors, reviewers, and board members with the impression that we are focussed on restricting access rather than making their research as widely available as possible.

Helping editors, authors, reviewers, and board members to work easily on journals is central to us. We have systems in place to make it easier for editors to run large journals, some of which are dealing with thousands of submissions each year—something which most smaller publishers are not equipped to do. There will, of course, be people

Laura Hassink is Senior Vice President, Publishing, Elsevier. Her email address is `l.hassink@elsevier.com`.

David Clark is Senior Vice President, Publishing, Elsevier. His email address is `david.clark@elsevier.com`.

DOI: http://dx.doi.org/10.1090/noti853

Finally, Clark and Hassink announced that free access has been granted to the archives of fourteen core mathematics journals for the years from 1995 through four years before the present day. Access to back issues is indeed critical, and we strongly believe that all research papers should be made freely available long before copyright expires. The shorter the delay the better, of course, but we consider four years a defensible choice, compatible with the subscription model for journal publishing. The AMS's experience with a five-year window shows that such a move is financially viable. We hope that Elsevier's announcement is just the first step and that expansion to the full set of mathematical journals and the period before 1995 will be announced soon.[19] We also hope that this is not just a temporary measure. A binding commitment not to revoke access in the future would be reassuring on that point.

Moving Forward

While the mathematical literature itself is a treasure, the current system of scholarly publishing is badly broken. Elsevier is the largest and, in our view, the most egregious example of what is wrong. We hope many readers will agree with us that by choosing to withdraw our cooperation from Elsevier, we are sending a valuable message to them and to the scholarly publishing industry more broadly. Please consider joining the movement at `http://thecostofknowledge.com`.

What is our vision for the future? The mathematical community needs a period of experimentation and healthy competition, in which a variety of publishing models can flourish and develop. Possibilities include various approaches to open access publishing,[20] refereed journals tightly integrated with the arXiv or similar servers, increased reliance on nonprofit publishers, hybrid models in which community-owned journals subcontract their operations to commercial publishers, commercially owned journals with reasonable prices and policies, etc. It is too early to predict the mix of models that will emerge as the most successful. However, any publisher that wants to be part of this mix must convince the community that they oversee peer review with integrity, that they aid dissemination rather than hinder it, and that they work to make high-quality mathematical literature widely available at a reasonable price.

Let's work together to foster good practices and build better models. The future of mathematics publishing is in our hands.

[19] *All three journals discussed here began publishing in the 1960s. The issues before 1995 are currently available from Elsevier online but remain behind their paywall.*

[20] *For example, based on publication charges or on sponsoring consortia such as SCOAP³* (`http://scoap3.org`).

Elsevier's Response to the Mathematics Community

Laura Hassink and David Clark

During the last months we have spoken to many people in the community about the move by Timothy Gowers and some colleagues to declare their wish not to work with Elsevier and the subsequent boycott movement.

At the end of this article we summarize how we are responding to the feedback from the community and the very specific steps that we are taking. But we would first like to address the concerns raised and some of the arguments.

We are the leading journal publisher in scientific publishing and so will attract criticism that is directed at publishing as a whole. While we may disagree with much that has been said, we do recognize that Elsevier has not done a good job communicating what we do and how we support both the peer-review process and the dissemination of work. In particular, we have left authors, editors, reviewers, and board members with the impression that we are focussed on restricting access rather than making their research as widely available as possible.

Helping editors, authors, reviewers, and board members to work easily on journals is central to us. We have systems in place to make it easier for editors to run large journals, some of which are dealing with thousands of submissions each year—something which most smaller publishers are not equipped to do. There will, of course, be people

Laura Hassink is Senior Vice President, Publishing, Elsevier. Her email address is `l.hassink@elsevier.com`.

David Clark is Senior Vice President, Publishing, Elsevier. His email address is `david.clark@elsevier.com`.

DOI: http://dx.doi.org/10.1090/noti853

who argue against the involvement of privately owned organizations in academic publishing, but we believe that a mixed economy brings benefits to mathematics.

Professor Gowers's protest is specifically concerned with three issues: the pricing of journals; the practice of offering journals in large "bundles"; and, in particular, Elsevier's support, along with others, of a set of legislation, including the Research Works Act, in the United States.

Pricing

Mathematics journals published by Elsevier tend to be large, with a great many articles published each year. On a price-per-article or price-per-page level, our prices are typically, but not always, lower than those of other mathematics publishers. Our average price increase over the last eight years has been in the lowest quartile of the sector. The Cost of Knowledge statement selects ten (of thirty-eight) Elsevier journals to quote an average 2007 price per page of \$1.30 and compares this to prices per page ranging from \$0.13 to \$1.21 for selected journals from other publishers. However, the average price per page for all thirty-eight Elsevier journals in the AMS dataset for that year is \$0.76 per page, with several below \$0.50 per page and as low as \$0.35 per page. This is below the average for all mathematics journals in the AMS dataset. The document mentions that seven of the top ten most expensive journals are from Elsevier but does not show that the average price per page for those seven Elsevier journals is \$0.61.

That said, these figures are five years old, and in recent years we have made moves to reduce or freeze the prices of a number of our mathematics titles, recognizing that this field is not well funded and the articles are used intensively rather than frequently.

Journals such as the *Journal of Algebra, Topology and its Applications* and the *Journal of Number Theory* have all seen price reductions in recent years. **Our target is for all of our core mathematics titles to be at or below US\$11 per article[1] (equivalent to \$0.50–\$0.60 cents per normal typeset page) by next year**, placing us below most university presses, some societies, and all other commercial competitors. That will lead to a number of our titles seeing further and significant price reductions in their next volumes.

"Bundling"

Most journals are subscribed to as part of large deals or national consortia agreements, and so universities receive access to many more journal titles than they individually subscribe to and thus pay less than the list price described above. Although such packages are offered by virtually all publishers, many mathematicians have expressed dislike for such policies.

To describe this concisely, such agreements involve universities maintaining a core holding of journals and then, depending upon the size of the institution, having the option to subscribe to subcollections, such as in mathematics, or to all of our titles at a discount of the normal journal subscription list price. These collections can be as low as 2.5 percent of the catalogue value of the collection, which is one of the reasons why they have been so popular. A similar arrangement allows national consortia of universities to share electronic access to all their subscribed journals amongst themselves, without each university needing to hold an individual subscription.

We therefore disagree with the term "bundling", as it is not mandatory for a customer to enter into such a large deal. Libraries can decide what they want to subscribe to, whether that is individual titles or individual titles within a collection, or to join a national consortium. We do recognize the wish for more choice and flexibility, especially within departments, and we are currently experimenting with new ways of doing this. But switching off such schemes would, in our view, have a detrimental impact on access to the research literature. Because of the introduction of such large agreements, coupled with the simultaneous organization of universities and libraries into large consortia, access to journal content has never been better. However, clearly there are still areas and occasions where access is not at the level where it should be, and we are determined to address this issue.

Access

Later in this response we describe the major new steps that we are taking to ensure substantially wider free access to the mathematics articles that Elsevier publishes, but we'd like to flag two other issues that we think are misunderstood.

First, all of our titles, including our mathematics titles, are available in the poorest countries. We are a founding partner in **Research4Life**, a public/private partnership providing journal content to researchers in the developing world. More than 2,000 Elsevier journals and 6,000 Elsevier e-books are available through Research4Life.

Second, authors in mathematics can post their author manuscripts, including corrections made in peer review and editing, on the arXiv. We have been allowing this for years. Elsevier supplies metadata directly to the arXiv and has for many years. There seems to be some misunderstanding on this point, as Elsevier's policies are no different from other major mathematics publishers in this regard, and we are happy to be able to correct this point. See our article posting

[1] *This is calculated by dividing the institutional list price of a journal by the number of articles puclished.*

policies (`http://www.elsevier.com/wps/find/authorsview.authors/postingpolicy`) for more information.

U.S. Legislation

We are conscious that much of what triggered Professor Gowers's original posting was the support that Elsevier, along with others, gave to proposed U.S. legislation concerned with state mandates for publishing final versions of articles.

Almost all publishers are uncomfortable about laws determining what's published and under what conditions, but the critical feedback on this issue has been very sobering for many of us and has led to much reflection within the company.

That is why Elsevier announced that we are withdrawing our support for the Research Works Act.

Quality of Journals

The discussion about publishing and large subscription agreements has also highlighted concerns about the quality of particular journals. In the specific case of *Chaos, Solitons and Fractals*, this is a journal which has been rebuilt, with a new set of editors—whose work and commitment we would like to acknowledge—and much involvement from in-house Elsevier staff. We think that we did the right thing to seek to rebuild this journal. This journal has changed and informed the way in which we work on many journals, from our basic editorial contracts with editors to the use of much more formal editorial processes, electronic peer review, clearer statements on ethical issues, and the introduction of additional staff in support of journals.

We have put a great deal of effort in recent years into developing our support for journals, including significant editorial changes. Publishing judgment can go wrong and that will happen in any organization. But we learn from our mistakes and make considerable efforts to address and resolve them.

We do, however, need to be more open about the actions that we have taken when things go wrong. Specifically, we need to develop a better forum for listening to the mathematics community, to hearing specific criticisms, and to jointly developing policies and to hearing critical feedback.

How Elsevier Is Responding to Feedback from the Community

In this article we have responded to some of the concerns raised, but our goal is to do what it takes to ensure that the leading mathematics journals that we publish are as valuable and respected—and contribute as much back to the community—as any journal published by a society or university press. We are therefore taking the following steps now:

1. To make clear our commitment to wider access, we have made the archives of over forty core mathematics journals open for free, from four years after publication back to 1995, the year when we started publishing digitally;
2. On subscription pricing, we will target a price of US$10–US$11 per article (equivalent to $0.50–$0.60 cent a page) for our core mathematics journals, below that of most of our competitors.
3. On "bundling", we are open to engaging with the mathematics community on how the system could work better, with greater flexibility and choice, and especially for small institutions without access to wider institutional resources. As a first step we are defining a smaller subcollection of core mathematics titles.
4. We will create an advisory scientific council for mathematics to ensure that we are working in tandem with the mathematics community to address feedback and to give greater control and transparency to the community.
5. We have announced that we are withdrawing our support for the Research Works Act.

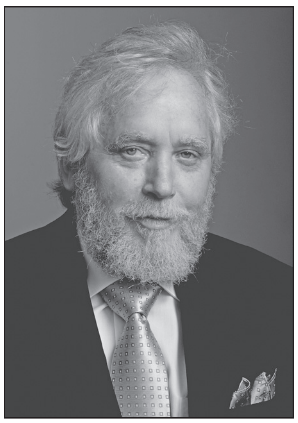
We do not regard this as the end of our discussion with mathematicians, but rather as the continuation of our efforts.

We are very open to talking with anyone in the community about what we do and how we do it. Some fair criticism has come to us, which we will address.

More generally, we seek out and welcome the views of any concerned member of the mathematics community.

Szemerédi Receives 2012 Abel Prize

The Norwegian Academy of Science and Letters has awarded the Abel Prize for 2012 to Endre Szemerédi of the Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, and Rutgers University "for his fundamental contributions to discrete mathematics and theoretical computer science, and in recognition of the profound and lasting impact of these contributions on additive number theory and ergodic theory." The Abel Prize recognizes contributions of extraordinary depth and influence to the mathematical sciences and has been awarded annually since 2003. It carries a cash award of 6,000,000 Norwegian kroner (approximately US$1 million). Endre Szemerédi received the Abel Prize at an award ceremony in Oslo, Norway, on May 22, 2012.



Courtesy of the Norwegian Academy of Sciences. Photo by Knut Falch.

Endre Szemerédi

Citation

Discrete mathematics is the study of structures such as graphs, sequences, permutations, and geometric configurations. The mathematics of such structures forms the foundation of theoretical computer science and information theory. For instance, communication networks such as the Internet can be described and analyzed using the tools of graph theory, and the design of efficient computational algorithms relies crucially on insights from discrete mathematics. The combinatorics of discrete structures is also a major component of many areas of pure mathematics, including number theory, probability, algebra, geometry, and analysis.

Endre Szemerédi has revolutionized discrete mathematics by introducing ingenious and novel techniques and by solving many fundamental problems. His work has brought combinatorics to the center stage of mathematics by revealing its deep connections to such fields as additive number theory, ergodic theory, theoretical computer science, and incidence geometry.

In 1975 Endre Szemerédi first attracted the attention of many mathematicians with his solution of the famous Erdős-Turán conjecture, showing that in any set of integers with positive density, there are arbitrarily long arithmetic progressions. This was a surprise, since even the case of progressions of lengths 3 or 4 had earlier required substantial effort by Klaus Roth and by Szemerédi himself, respectively.

A bigger surprise lay ahead. Szemerédi's proof was a masterpiece of combinatorial reasoning and was immediately recognized to be of exceptional depth and importance. A key step in the proof, now known as the Szemerédi Regularity Lemma, is a structural classification of large graphs. Over time, this lemma has become a central tool of both graph theory and theoretical computer science, leading to the solution of major problems in property testing and giving rise to the theory of graph limits.

Still other surprises lay in wait. Beyond its impact on discrete mathematics and additive number theory, Szemerédi's theorem inspired Hillel Furstenberg to develop ergodic theory in new directions. Furstenberg gave a new proof of Szemerédi's theorem by establishing the multiple recurrence theorem in ergodic theory, thereby unexpectedly linking questions in discrete mathematics to the theory of dynamical systems. This fundamental connection led to many further developments, such as the Green-Tao theorem asserting that there are arbitrarily long arithmetic progressions of prime numbers.

Szemerédi has made many additional deep, important, and influential contributions to both discrete mathematics and theoretical computer science. Examples in discrete mathematics include the Szemerédi-Trotter theorem, the Ajtai-Komlós-Szemerédi semirandom method, the Erdős-Szemerédi sum-product theorem, and the Balog-Szemerédi-Gowers lemma. Examples in theoretical computer science include the Ajtai-Komlós-Szemerédi sorting network, the Fredman-Komlós-Szemerédi hashing scheme, and the Paul-Pippenger-Szemerédi-Trotter theorem separating deterministic and nondeterministic linear time. Szemerédi's approach to mathematics exemplifies

DOI: http://dx.doi.org/10.1090/noti855

the strong Hungarian problem-solving tradition. Yet the theoretical impact of his work has been a game changer.

Biographical Sketch

Endre Szemerédi was born in Budapest, Hungary, in 1940. He received his Ph.D. from Moscow State University in 1970 under the direction of Israel M. Gelfand. He has held visiting positions at Stanford University (1974), McGill University (1980), the University of South Carolina (1981–1983), and the University of Chicago (1985–1986). He was the Fairchild Distinguished Scholar at the California Institute of Technology in 1987–1988. He is also a recipient of the Aisenstadt Chair at the Centre de Recherches Mathématiques, University of Montreal. In 2008 he was the Eisenbud Professor at the Mathematical Sciences Research Institute, Berkeley. He has received many awards and honors for his contributions to mathematics and computer science. In 2008 he was awarded the Leroy P. Steele Prize for Seminal Contribution to Research by the AMS. In the same year he received the Rolf Schock Prize in Mathematics from the Royal Swedish Academy of Sciences. Other prizes include the Grünwald Prize (1967, 1968), the Rényi Prize (1973), the Pólya Prize for Achievement in Applied Mathematics of the Society for Industrial and Applied Mathematics (SIAM) (1975), and the Prize of the Hungarian Academy of Sciences (1979). He has been a member of the Hungarian Academy of Sciences since 1987 and of the U.S. National Academy of Sciences since 2010.

About the Prize

The Niels Henrik Abel Memorial Fund was established in 2002 to award the Abel Prize for outstanding scientific work in the field of mathematics. The prize is awarded by the Norwegian Academy of Science and Letters, and the choice of Abel Laureate is based on the recommendation of the Abel Committee, which consists of five internationally recognized mathematicians.

Previous recipients of the Abel Prize are: Jean-Pierre Serre (2003), Michael Atiyah and I. M. Singer (2004), Peter Lax (2005), Lennart Carleson (2006), S. R. S. Varadhan (2007), John G. Thompson and Jacques Tits (2008), Mikhail L. Gromov (2009), John Tate (2010), and John Milnor (2011).

—From announcements of the Norwegian Academy of Science and Letters

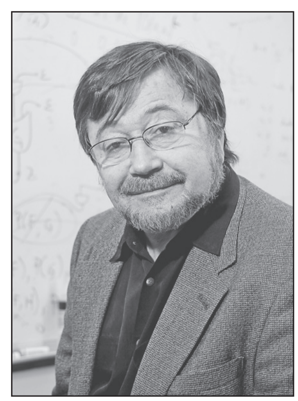
Pearl Receives 2011 Turing Award

The Association for Computing Machinery (ACM) has announced that Judea Pearl of the University of California, Los Angeles, has been named the recipient of the 2011 ACM A. M. Turing Award "for fundamental contributions to artificial intelligence through the development of a calculus for probabilistic and causal reasoning." The award carries a cash prize of US$250,000 and will be presented to Pearl at the annual ACM Awards Banquet on June 16, 2012, in San Francisco, California.

Citation

Judea Pearl's work has transformed artificial intelligence (AI) by creating a representational and computational foundation for the processing of information under uncertainty. Pearl's work went beyond both the logic-based theoretical orientation of AI and its rule-based technology for expert systems. He identified uncertainty as a core problem faced by intelligent systems and developed an algorithmic interpretation of probability theory as an effective foundation for the representation and acquisition of knowledge.

Focusing on conditional independence as an organizing principle for capturing structural aspects of probability distributions, Pearl showed how graph theory can be used to characterize conditional independence and invented message-passing algorithms that exploit graphical structure to perform probabilistic reasoning effectively. This breakthrough has had major impact on a wide variety of fields where the restriction to simplified models had severely limited the scope of probabilistic methods; examples include natural language processing, speech processing, computer vision, robotics, computational biology, and error-control coding.



Reed Hutchinson, UCLA Photographic Services.

Judea Pearl

Equally significant is Pearl's work on causal reasoning, where he developed a graph-based calculus of interventions that makes it possible to derive

DOI: http://dx.doi.org/10.1090/noti854

causal knowledge from the combined effects of actions and observations. This work has been transformative within AI and computer science and has had major impact on allied disciplines of economics, philosophy, psychology, sociology, and statistics.

Biographical Sketch

Judea Pearl is a professor of computer science at the University of California, Los Angeles, where he was director of the Cognitive Systems Laboratory. Before joining UCLA in 1970, he was at RCA Research Laboratories working on superconductive parametric and storage devices. Previously, he was engaged in advanced memory systems at Electronic Memories, Inc. Pearl is a graduate of the Technion, the Israel Institute of Technology, with a B.S. degree in electrical engineering. In 1965 he received a master's degree in physics from Rutgers University and in the same year was awarded a Ph.D. degree in electrical engineering from the Polytechnic Institute of Brooklyn.

Among his many awards, Pearl is the recipient of the 2012 Harvey Prize in Science and Technology from the Technion and the 2008 Benjamin Franklin Medal in Computers and Cognitive Science from the Franklin Institute. He was presented with the 2003 Allen Newell Award from ACM and the AAAI (Association for the Advancement of Artificial Intelligence). His groundbreaking book on causality, *Causality: Models, Reasoning, and Inference*, won the 2001 Lakatos Award from the London School of Economics and Political Science "for an outstanding significant contribution to the philosophy of science."

Pearl is a member of the National Academy of Engineering and a Fellow of AAAI and the Institute for Electrical and Electronic Engineers (IEEE). He is president of the Daniel Pearl Foundation, named after his son.

About the Award

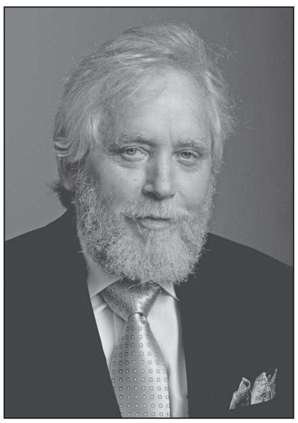
The A. M. Turing Award was named for Alan M. Turing, the British mathematician who articulated the mathematical foundation and limits of computing and who was a key contributor to the Allied cryptanalysis of the German Enigma cipher during World War II. Since its inception in 1966, the Turing Award has honored the computer scientists and engineers who created the systems and underlying theoretical foundations that have propelled the information technology industry.

The award is given to an individual selected for contributions of a technical nature made to the computing community. The contributions should be of lasting and major technical importance to the computer field. Financial support for the Turing Award is provided by the Intel Corporation and Google Inc.

—ACM announcement

Szemerédi Receives 2012 Abel Prize

The Norwegian Academy of Science and Letters has awarded the Abel Prize for 2012 to Endre Szemerédi of the Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, and Rutgers University "for his fundamental contributions to discrete mathematics and theoretical computer science, and in recognition of the profound and lasting impact of these contributions on additive number theory and ergodic theory." The Abel Prize recognizes contributions of extraordinary depth and influence to the mathematical sciences and has been awarded annually since 2003. It carries a cash award of 6,000,000 Norwegian kroner (approximately US$1 million). Endre Szemerédi received the Abel Prize at an award ceremony in Oslo, Norway, on May 22, 2012.



Courtesy of the Norwegian Academy of Sciences. Photo by Knut Falch.

Endre Szemerédi

Citation

Discrete mathematics is the study of structures such as graphs, sequences, permutations, and geometric configurations. The mathematics of such structures forms the foundation of theoretical computer science and information theory. For instance, communication networks such as the Internet can be described and analyzed using the tools of graph theory, and the design of efficient computational algorithms relies crucially on insights from discrete mathematics. The combinatorics of discrete structures is also a major component of many areas of pure mathematics, including number theory, probability, algebra, geometry, and analysis.

Endre Szemerédi has revolutionized discrete mathematics by introducing ingenious and novel techniques and by solving many fundamental problems. His work has brought combinatorics to the center stage of mathematics by revealing its deep connections to such fields as additive number theory, ergodic theory, theoretical computer science, and incidence geometry.

In 1975 Endre Szemerédi first attracted the attention of many mathematicians with his solution of the famous Erdős-Turán conjecture, showing that in any set of integers with positive density, there are arbitrarily long arithmetic progressions. This was a surprise, since even the case of progressions of lengths 3 or 4 had earlier required substantial effort by Klaus Roth and by Szemerédi himself, respectively.

A bigger surprise lay ahead. Szemerédi's proof was a masterpiece of combinatorial reasoning and was immediately recognized to be of exceptional depth and importance. A key step in the proof, now known as the Szemerédi Regularity Lemma, is a structural classification of large graphs. Over time, this lemma has become a central tool of both graph theory and theoretical computer science, leading to the solution of major problems in property testing and giving rise to the theory of graph limits.

Still other surprises lay in wait. Beyond its impact on discrete mathematics and additive number theory, Szemerédi's theorem inspired Hillel Furstenberg to develop ergodic theory in new directions. Furstenberg gave a new proof of Szemerédi's theorem by establishing the multiple recurrence theorem in ergodic theory, thereby unexpectedly linking questions in discrete mathematics to the theory of dynamical systems. This fundamental connection led to many further developments, such as the Green-Tao theorem asserting that there are arbitrarily long arithmetic progressions of prime numbers.

Szemerédi has made many additional deep, important, and influential contributions to both discrete mathematics and theoretical computer science. Examples in discrete mathematics include the Szemerédi-Trotter theorem, the Ajtai-Komlós-Szemerédi semirandom method, the Erdős-Szemerédi sum-product theorem, and the Balog-Szemerédi-Gowers lemma. Examples in theoretical computer science include the Ajtai-Komlós-Szemerédi sorting network, the Fredman-Komlós-Szemerédi hashing scheme, and the Paul-Pippenger-Szemerédi-Trotter theorem separating deterministic and nondeterministic linear time. Szemerédi's approach to mathematics exemplifies

DOI: http://dx.doi.org/10.1090/noti855

the strong Hungarian problem-solving tradition. Yet the theoretical impact of his work has been a game changer.

Biographical Sketch

Endre Szemerédi was born in Budapest, Hungary, in 1940. He received his Ph.D. from Moscow State University in 1970 under the direction of Israel M. Gelfand. He has held visiting positions at Stanford University (1974), McGill University (1980), the University of South Carolina (1981–1983), and the University of Chicago (1985–1986). He was the Fairchild Distinguished Scholar at the California Institute of Technology in 1987–1988. He is also a recipient of the Aisenstadt Chair at the Centre de Recherches Mathématiques, University of Montreal. In 2008 he was the Eisenbud Professor at the Mathematical Sciences Research Institute, Berkeley. He has received many awards and honors for his contributions to mathematics and computer science. In 2008 he was awarded the Leroy P. Steele Prize for Seminal Contribution to Research by the AMS. In the same year he received the Rolf Schock Prize in Mathematics from the Royal Swedish Academy of Sciences. Other prizes include the Grünwald Prize (1967, 1968), the Rényi Prize (1973), the Pólya Prize for Achievement in Applied Mathematics of the Society for Industrial and Applied Mathematics (SIAM) (1975), and the Prize of the Hungarian Academy of Sciences (1979). He has been a member of the Hungarian Academy of Sciences since 1987 and of the U.S. National Academy of Sciences since 2010.

About the Prize

The Niels Henrik Abel Memorial Fund was established in 2002 to award the Abel Prize for outstanding scientific work in the field of mathematics. The prize is awarded by the Norwegian Academy of Science and Letters, and the choice of Abel Laureate is based on the recommendation of the Abel Committee, which consists of five internationally recognized mathematicians.

Previous recipients of the Abel Prize are: Jean-Pierre Serre (2003), Michael Atiyah and I. M. Singer (2004), Peter Lax (2005), Lennart Carleson (2006), S. R. S. Varadhan (2007), John G. Thompson and Jacques Tits (2008), Mikhail L. Gromov (2009), John Tate (2010), and John Milnor (2011).

—From announcements of the Norwegian Academy of Science and Letters

---

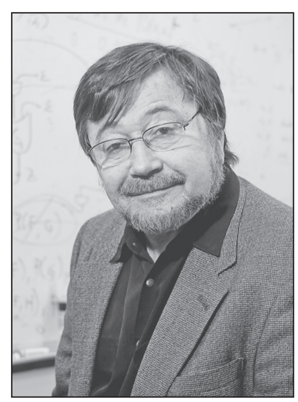
Pearl Receives 2011 Turing Award

The Association for Computing Machinery (ACM) has announced that Judea Pearl of the University of California, Los Angeles, has been named the recipient of the 2011 ACM A. M. Turing Award "for fundamental contributions to artificial intelligence through the development of a calculus for probabilistic and causal reasoning." The award carries a cash prize of US$250,000 and will be presented to Pearl at the annual ACM Awards Banquet on June 16, 2012, in San Francisco, California.

Citation

Judea Pearl's work has transformed artificial intelligence (AI) by creating a representational and computational foundation for the processing of information under uncertainty. Pearl's work went beyond both the logic-based theoretical orientation of AI and its rule-based technology for expert systems. He identified uncertainty as a core problem faced by intelligent systems and developed an algorithmic interpretation of probability theory as an effective foundation for the representation and acquisition of knowledge.

Focusing on conditional independence as an organizing principle for capturing structural aspects of probability distributions, Pearl showed how graph theory can be used to characterize conditional independence and invented message-passing algorithms that exploit graphical structure to perform probabilistic reasoning effectively. This breakthrough has had major impact on a wide variety of fields where the restriction to simplified models had severely limited the scope of probabilistic methods; examples include natural language processing, speech processing, computer vision, robotics, computational biology, and error-control coding.


Reed Hutchinson, UCLA Photographic Services.

Judea Pearl

Equally significant is Pearl's work on causal reasoning, where he developed a graph-based calculus of interventions that makes it possible to derive

DOI: http://dx.doi.org/10.1090/noti854

causal knowledge from the combined effects of actions and observations. This work has been transformative within AI and computer science and has had major impact on allied disciplines of economics, philosophy, psychology, sociology, and statistics.

Biographical Sketch

Judea Pearl is a professor of computer science at the University of California, Los Angeles, where he was director of the Cognitive Systems Laboratory. Before joining UCLA in 1970, he was at RCA Research Laboratories working on superconductive parametric and storage devices. Previously, he was engaged in advanced memory systems at Electronic Memories, Inc. Pearl is a graduate of the Technion, the Israel Institute of Technology, with a B.S. degree in electrical engineering. In 1965 he received a master's degree in physics from Rutgers University and in the same year was awarded a Ph.D. degree in electrical engineering from the Polytechnic Institute of Brooklyn.

Among his many awards, Pearl is the recipient of the 2012 Harvey Prize in Science and Technology from the Technion and the 2008 Benjamin Franklin Medal in Computers and Cognitive Science from the Franklin Institute. He was presented with the 2003 Allen Newell Award from ACM and the AAAI (Association for the Advancement of Artificial Intelligence). His groundbreaking book on causality, *Causality: Models, Reasoning, and Inference*, won the 2001 Lakatos Award from the London School of Economics and Political Science "for an outstanding significant contribution to the philosophy of science."

Pearl is a member of the National Academy of Engineering and a Fellow of AAAI and the Institute for Electrical and Electronic Engineers (IEEE). He is president of the Daniel Pearl Foundation, named after his son.

About the Award

The A. M. Turing Award was named for Alan M. Turing, the British mathematician who articulated the mathematical foundation and limits of computing and who was a key contributor to the Allied cryptanalysis of the German Enigma cipher during World War II. Since its inception in 1966, the Turing Award has honored the computer scientists and engineers who created the systems and underlying theoretical foundations that have propelled the information technology industry.

The award is given to an individual selected for contributions of a technical nature made to the computing community. The contributions should be of lasting and major technical importance to the computer field. Financial support for the Turing Award is provided by the Intel Corporation and Google Inc.

—*ACM announcement*

Yet the theoretical impact of his work has been a game changer.

Biographical Sketch

Endre Szemerédi was born in Budapest, Hungary, in 1940. He received his Ph.D. from Moscow State University in 1970 under the direction of Israel M. Gelfand. He has held visiting positions at Stanford University (1974), McGill University (1980), the University of South Carolina (1981–1983), and the University of Chicago (1985–1986). He was the Fairchild Distinguished Scholar at the California Institute of Technology in 1987–1988. He is also a recipient of the Aisenstadt Chair at the Centre de Recherches Mathématiques, University of Montreal. In 2008 he was the Eisenbud Professor at the Mathematical Sciences Research Institute, Berkeley. He has received many awards and honors for his contributions to mathematics and computer science. In 2008 he was awarded the Leroy P. Steele Prize for Seminal Contribution to Research by the AMS. In the same year he received the Rolf Schock Prize in Mathematics from the Royal Swedish Academy of Sciences. Other prizes include the Grünwald Prize (1967, 1968), the Rényi Prize (1973), the Pólya Prize for Achievement in Applied Mathematics of the Society for Industrial and Applied Mathematics (SIAM) (1975), and the Prize of the Hungarian Academy of Sciences (1979). He has been a member of the Hungarian Academy of Sciences since 1987 and of the U.S. National Academy of Sciences since 2010.

About the Prize

The Niels Henrik Abel Memorial Fund was established in 2002 to award the Abel Prize for outstanding scientific work in the field of mathematics. The prize is awarded by the Norwegian Academy of Science and Letters, and the choice of Abel Laureate is based on the recommendation of the Abel Committee, which consists of five internationally recognized mathematicians.

Previous recipients of the Abel Prize are: Jean-Pierre Serre (2003), Michael Atiyah and I. M. Singer (2004), Peter Lax (2005), Lennart Carleson (2006), S. R. S. Varadhan (2007), John G. Thompson and Jacques Tits (2008), Mikhail L. Gromov (2009), John Tate (2010), and John Milnor (2011).

—From announcements of the Norwegian Academy of Science and Letters

---

Pearl Receives 2011 Turing Award

The Association for Computing Machinery (ACM) has announced that Judea Pearl of the University of California, Los Angeles, has been named the recipient of the 2011 ACM A. M. Turing Award "for fundamental contributions to artificial intelligence through the development of a calculus for probabilistic and causal reasoning." The award carries a cash prize of US$250,000 and will be presented to Pearl at the annual ACM Awards Banquet on June 16, 2012, in San Francisco, California.

Citation

Judea Pearl's work has transformed artificial intelligence (AI) by creating a representational and computational foundation for the processing of information under uncertainty. Pearl's work went beyond both the logic-based theoretical orientation of AI and its rule-based technology for expert systems. He identified uncertainty as a core problem faced by intelligent systems and developed an algorithmic interpretation of probability theory as an effective foundation for the representation and acquisition of knowledge.

Focusing on conditional independence as an organizing principle for capturing structural aspects of probability distributions, Pearl showed how graph theory can be used to characterize conditional independence and invented message-passing algorithms that exploit graphical structure to perform probabilistic reasoning effectively. This breakthrough has had major impact on a wide variety of fields where the restriction to simplified models had severely limited the scope of probabilistic methods; examples include natural language processing, speech processing, computer vision, robotics, computational biology, and error-control coding.

Reed Hutchinson, UCLA Photographic Services.

Judea Pearl

Equally significant is Pearl's work on causal reasoning, where he developed a graph-based calculus of interventions that makes it possible to derive

DOI: http://dx.doi.org/10.1090/noti854

causal knowledge from the combined effects of actions and observations. This work has been transformative within AI and computer science and has had major impact on allied disciplines of economics, philosophy, psychology, sociology, and statistics.

Biographical Sketch

Judea Pearl is a professor of computer science at the University of California, Los Angeles, where he was director of the Cognitive Systems Laboratory. Before joining UCLA in 1970, he was at RCA Research Laboratories working on superconductive parametric and storage devices. Previously, he was engaged in advanced memory systems at Electronic Memories, Inc. Pearl is a graduate of the Technion, the Israel Institute of Technology, with a B.S. degree in electrical engineering. In 1965 he received a master's degree in physics from Rutgers University and in the same year was awarded a Ph.D. degree in electrical engineering from the Polytechnic Institute of Brooklyn.

Among his many awards, Pearl is the recipient of the 2012 Harvey Prize in Science and Technology from the Technion and the 2008 Benjamin Franklin Medal in Computers and Cognitive Science from the Franklin Institute. He was presented with the 2003 Allen Newell Award from ACM and the AAAI (Association for the Advancement of Artificial Intelligence). His groundbreaking book on causality, *Causality: Models, Reasoning, and Inference*, won the 2001 Lakatos Award from the London School of Economics and Political Science "for an outstanding significant contribution to the philosophy of science."

Pearl is a member of the National Academy of Engineering and a Fellow of AAAI and the Institute for Electrical and Electronic Engineers (IEEE). He is president of the Daniel Pearl Foundation, named after his son.

About the Award

The A. M. Turing Award was named for Alan M. Turing, the British mathematician who articulated the mathematical foundation and limits of computing and who was a key contributor to the Allied cryptanalysis of the German Enigma cipher during World War II. Since its inception in 1966, the Turing Award has honored the computer scientists and engineers who created the systems and underlying theoretical foundations that have propelled the information technology industry.

The award is given to an individual selected for contributions of a technical nature made to the computing community. The contributions should be of lasting and major technical importance to the computer field. Financial support for the Turing Award is provided by the Intel Corporation and Google Inc.

—ACM announcement

What Should Students Get from Calculus? (And How Can We Provide It?)

Frank Quinn

In the September 2011 *Doceamus* column [1], Keith Stroyan takes on this question and reports success with extended exploration of applications. Experiences with a different population of students have led me in rather different directions on practical levels, but with important commonalities.

I back up a bit for perspective. The actual title of Stroyan's article is "Why do so many students take calculus?" The honest answer to this is "because it is required in the curriculum"; the real problem is that traditional calculus courses do not serve students particularly well. Stroyan actually addresses a variation: "We've got them here; what is the best use we can make of this opportunity?" His answer is that calculus gives quick access to rich and varied applications, and he exploits this by reducing emphasis on lectures and the nitty-gritty of calculus and instead exploring applications in some detail. I believe this is a good answer for students who are not in technical programs. Nearly all of my students are in science and engineering, however, so a more mission-oriented version of the question is appropriate. Namely, "What do these students need, and what are the most valuable things they get?" My main concern is with course design. Scientists and engineers do still need a foundation in calculus, but I see calculus as a setting rather than a goal, and even for this group I don't think "knowledge of calculus" is among the most valuable outcomes. The next three sections describe other important goals.

Frank Quinn is professor of mathematics at Virginia Tech. His email address is `quinn@math.vt.edu`.

DOI: http://dx.doi.org/10.1090/noti858

Complex Rules and Accuracy

It is a vital skill for science and engineering to be able to work accurately with complex, rule-based systems. This skill is also transferrable to many more domains than any specific content. But this is a skill that my students certainly don't have when they get here.

Most high-school programs have de-emphasized rule skills in favor of "understanding" and working intuitively. If you can "see" the problem, it should be easy. Calculator use has replaced a lot of rule-based work and attendant skills. AP calculus is a partial exception, but it is test-driven with greatly simplified rules used mechanically on short, routine problems. Low skill levels give mediocre results even on simplified problems, but this has been compensated for by generous partial and extra credit and by curved grades.

Given all this, I feel that helping my students develop disciplined rule skills is the most important service I can provide. An implication for course design—again for students in science and engineering—is that corners should not be cut. Do all the standard techniques of integration, with the full set of elementary functions, to provide enough complexity to require careful and systematic use.

Finally, I think this is the time to get real about getting things right. High-tech employers don't give "A"s for work that is 90% correct. I expect students to get up to speed, rather than reducing expectations to their comfort zone. Their poor preparation makes this a serious challenge, but an important one, and most of them rise to it.

Abstract and Symbolic Work

Technical challenges in science and engineering are getting more difficult, while dealing with numbers is getting easier. A consequence is that work on an abstract and symbolic level—even if only to organize numerical work—is increasingly important. But these skills are also declining. Some of my students have trouble with any problem whose answer is not a number: they can handle circles of radius 3, but simple problems with circles of radius "r" are foreign territory.

Again, I feel my students are better served if I can help develop these skills. My examples and problems usually have symbolic parameters, and I emphasize what these reveal about scaling, optimization, and error analysis. I usually use exact arithmetic. This preserves structure (π and $\sqrt{2}$ don't disappear into decimals) and is halfway to symbolic work. Again this is a challenge, and quite a few students need remediation before it is accessible, but if so they can hardly expect to succeed in a science and engineering curriculum without it.

Applications

Applications provide opportunities for students to exercise their skills and see the methods in action. However, applications do not have to be physical problems, and in fact I find most physical applications unsatisfactory.

- It is a good idea to plug in numbers from time to time, but it destroys a lot of functionality and does not exercise the most important skills.

- Most of these students have specific interests. Applications that address their interests will be done in more depth in other courses. Applications that don't address their interests often don't engage them.

- Superficial applications are usually little more than vocabulary (replace "velocity" with the first derivative, etc.). These are worth mentioning, but as testable material they are not a good use of their time.

On the other hand, working a bit more abstractly and symbolically opens up mathematical topics to explore. These are, in effect, applications that are both mission-related and quickly accessible because techniques and terminology are already in place.

Resource Constraints

Unfortunately, one more version of the title question must be addressed: "If we do figure out what students need, can we afford to provide it?" I individually, and my department at Virginia Tech collectively, have tried many things that improved learning but had to be abandoned because they required unsustainable levels of faculty overtime. These include group projects along the lines described in [1]. My suggestions here are also problematic. They don't directly cost more, but increasing expectations increases failure rates unless individual help is provided, and appropriate help would definitely be over-budget.

We should remember that per-student resource levels were established at a time when we only lectured and gave tests. In many colleges they have declined substantially below this level, and huge classes taught by adjuncts are increasingly common. In this climate any innovation that costs more is a dead end. Real impact in first- and second-year courses will require innovations whose resource requirements are competitive with huge classes taught by adjuncts. So far, an educational approach to innovation has been taken: "discipline stifles creativity, so let a thousand flowers bloom." Unsurprisingly, we have reaped education-quality outcomes: the thousand flowers bloomed and wilted, and very few students are better off. Maybe it is time to get real about getting it right, perhaps with a science and engineering approach: "no discipline, no results."

The sticking point is that, as far as I can see, the only way to both innovate *and* reduce costs is to give up traditional classrooms. Nothing else has enough value. This could work, though: at Virginia Tech we now have more than 10,000 students per semester taking lower-level computer-based courses. Unit costs are enough below the huge-class-with-adjuncts cutoff that Stroyan's group projects, or the individual help that I feel is so important, could be done within current budgets. There is still a big challenge: developing high-quality courseware and tests that would provide a good environment for these other innovations. Most courseware follows the classroom model, and it really should be clear by now that this is a poor model. We need materials much better adapted to individual use. Real success will also require sophisticated adjustments in the content. My belief is that this is a job for mathematicians, not educators.[1]

Summary

Stroyan suggests a kinder, gentler calculus with extended projects on physical applications. I propose a more rigorous course with fewer physical applications. How can I see these as basically similar?

Both of us are concerned that traditional calculus courses do not serve students particularly well. We both feel—for rather different reasons—that calculus is a good setting and that the real problem is the traditional format. In particular, calculus is not the main learning goal even in a calculus course. We both believe that better goals should reflect student needs; the differences in specific approaches reflect different student populations. We have both concluded that serving students well will require activity—again different in detail—outside traditional classroom settings.

A final similarity is that wide implementation of either approach is seriously limited by resource constraints. They might be seen as examples of enrichments and student-specific variations that would be possible with high-quality computer-based courses.

References

[1] Keith Stroyan, *Doceamus:* Why do so many students take calculus?, *Notices Amer. Math. Soc.* **58** (2011), no. 8, 1122–1123.

[1] *See the essays at* `http://www.math.vt.edu/people/quinn/education/` *for extensive discussion of these issues.*

SCRIPTA MANENT

...written words endure

Shifting Editorial Boards

Susan Hezlet

With the launch of the Elsevier boycott, the example of the new *Journal of Topology* has come up several times. Some scientists are proposing that other journal boards resign and move to new publishers. It is not that simple. In recent years the London Mathematical Society (LMS) has taken on three journal projects whose different stories illustrate the problems along with the benefits to be gained if you get it right. What follows is a personal account of the journal moves.

Case 1. Compositio Mathematica

The journal, founded by Brouwer in the 1930s, is owned by a Dutch foundation, Compositio Mathematica. It was published for many years by Kluwer (now a part of Springer), but in reaction to increasingly higher prices, the foundation looked for an alternative cooperation with a learned society, and agreement was reached with the LMS. The LMS negotiated a separate agreement with Cambridge University Press that they would print, host online, and sell the journal.

Kluwer did not own the journal and handed over the subscription data along with archives for

Susan Hezlet is the publisher at the London Mathematical Society. Her email address is `hezlet@lms.ac.uk`.

DOI: http://dx.doi.org/10.1090/noti856

and it really should be clear by now that this is a poor model. We need materials much better adapted to individual use. Real success will also require sophisticated adjustments in the content. My belief is that this is a job for mathematicians, not educators.[1]

Summary

Stroyan suggests a kinder, gentler calculus with extended projects on physical applications. I propose a more rigorous course with fewer physical applications. How can I see these as basically similar?

Both of us are concerned that traditional calculus courses do not serve students particularly well. We both feel—for rather different reasons—that calculus is a good setting and that the real problem is the traditional format. In particular, calculus is not the main learning goal even in a calculus course. We both believe that better goals should reflect student needs; the differences in specific approaches reflect different student populations. We have both concluded that serving students well will require activity—again different in detail—outside traditional classroom settings.

A final similarity is that wide implementation of either approach is seriously limited by resource constraints. They might be seen as examples of enrichments and student-specific variations that would be possible with high-quality computer-based courses.

References

[1] Keith Stroyan, *Doceamus:* Why do so many students take calculus?, *Notices Amer. Math. Soc.* **58** (2011), no. 8, 1122–1123.

[1]*See the essays at* `http://www.math.vt.edu/people/quinn/education/` *for extensive discussion of these issues.*

SCRIPTA MANENT

...written words endure

Shifting Editorial Boards

Susan Hezlet

With the launch of the Elsevier boycott, the example of the new *Journal of Topology* has come up several times. Some scientists are proposing that other journal boards resign and move to new publishers. It is not that simple. In recent years the London Mathematical Society (LMS) has taken on three journal projects whose different stories illustrate the problems along with the benefits to be gained if you get it right. What follows is a personal account of the journal moves.

Case 1. Compositio Mathematica

The journal, founded by Brouwer in the 1930s, is owned by a Dutch foundation, Compositio Mathematica. It was published for many years by Kluwer (now a part of Springer), but in reaction to increasingly higher prices, the foundation looked for an alternative cooperation with a learned society, and agreement was reached with the LMS. The LMS negotiated a separate agreement with Cambridge University Press that they would print, host online, and sell the journal.

Kluwer did not own the journal and handed over the subscription data along with archives for

Susan Hezlet is the publisher at the London Mathematical Society. Her email address is `hezlet@lms.ac.uk`.

DOI: http://dx.doi.org/10.1090/noti856

a small fee. At the time, the foundation received none of the journal profits, so the LMS had to bear any financial risk in the move. However, because the foundation had ownership and rights, the risks were minor, and the LMS was very proud to be chosen above a number of strong contenders.

Together with the foundation, we redesigned the journal format, increased the content of the journal to remove an inherited backlog of papers, and dropped the price by a third. The drop in price had no effect on the subscriptions, which actually fell during the move. We even received some negative reactions, including a claim by a well-known topologist that the journal had increased in price! He had not checked the increase in page size or the number of pages per volume. The lesson we learned was that libraries do not reward good behavior through subscriptions, even though the mathematical community later acknowledged and appreciated the change.

Since then the journal price increases have been very modest, and the journal has grown in size. A few years ago the French project NUMDAM offered to retrodigitize the early volumes and make them freely available. The foundation also requires us to provide a five-year moving wall so that all the content older than five years is free. This is not something we do on our own journals; however, *Compositio Mathematica* makes a healthy profit, and the LMS is happy to comply. The journal revenue has not grown at the same rate as for our core journals, but this is not necessarily due to the broader access policy; it could be due to many factors. The "healthy profit" is fed back to LMS society activities and to the foundation, which now supports meetings, and most notably the European Math Society prizes to ten young researchers (a total of 50,000 euros), which will be awarded this summer.

In recent years, publishers have provided librarians with their institution's annual usage statistics by journal title, and, knowing how much they have paid for the annual subscription, they calculate the "cost-per-download". A worrying recent trend is that this data is given as the primary reason for cancellation. Mathematicians do not necessarily read the older *Compositio* articles via the "virtual library corridor" of access because the articles are freely available. This reduces the usage statistic that the librarian relies on to calculate the cost-per-download. Math journals generally have low usage in comparison with the "big sciences", but their historic content is read almost as frequently as the newer material; this helps our LMS journals, particularly the *Proceedings*, to be well used. For *Compositio*, even if we could put all the usage of the free articles back into the librarians' data, they would reasonably argue that they are not paying for free articles, so it should not count towards the calculation of the cost-per-download.

Case 2. The Journal of Topology

Once upon a time there was a distinguished board of an Elsevier journal who were unhappy with the large price increases and found they were unable to persuade Elsevier to make the changes they wanted. After many years of unsuccessful negotiations, the board approached the LMS, and we agreed to launch a brand new journal. We took legal advice and were very careful to make it clear that the *Journal of Topology* is a new journal, wholly owned by the London Mathematical Society, which does not lay claim to any benefit that Elsevier may have given to the community through the publication of its journal *Topology*. It is easy for us to do this, because we have shown that the major benefit comes from the community, not the publisher. A journal is not just an editorial board; it requires authors, referees, and readers to support its existence. We received the support of excellent authors, and the whole community moved behind the launch of the new journal.

From the business side, of course, we had no initial subscriber list to work with, and we agreed to a contract with Oxford University Press (OUP) to print, host, and sell the journal on our behalf. They also sell our three core journals, and that is why we decided to make a special discount to any library that takes all four journals; so, yes, we are also guilty of "bundling"! Growing a brand new journal from scratch is not easy. If you put it into large-scale library consortia deals, it has no basic subscriber list, and its sole income comes from the premium which does not grow as the journal grows. OUP added the journal to consortia deals for three years, but we asked them to remove the journal when we found the net income per library was £19.

Now the hard fact is that the journal is not covering all its costs, and unless more libraries take subscriptions, we cannot reduce the price per page without losing even more money. We know from the *Compositio* example that libraries take little notice of price, but it is very difficult to persuade a library to pay for a new journal outside of their bundle deals. Despite the support that the new journal has received from authors, referees, and editors, we also need readers to persuade their libraries to support these projects. I find it depressing that some of our best-endowed libraries in universities whose mathematicians are clamoring for change do not subscribe to the journal. (You know who you are, and if you don't, why not check with your librarian!)

While authors appreciate the benefits of being published, many topologists would say that they rarely bother to read the published article when they have already read the math arXiv version. However, published mathematics papers have a long life, and the good ones are both cited and read for many years. I showed some evidence for this at a meeting held last year at MSRI, Berkeley.[1] It may be that, while the arXiv version is read more frequently in the first few years, the published version takes over in later years as the cited version of record becomes better known. As with the five-year moving wall, it is too early for us to have decent data from which to draw conclusions, but we should be careful about accepting short-term solutions when we[2] have a responsibility for the long-term preservation and cataloguing of the mathematics literature.

It is inevitable that journals whose readership relies heavily on the arXiv have lower usage and unfavorable "cost-per-download" data. Of course, not all math journals suffer equally; we know that barely 50 percent of the papers published in our general LMS journals are to be found on the arXiv, but for *Compositio* and the *Journal of Topology*, the figure is closer to 90 percent. There is very little we can do about this; discouraging authors from placing their preacceptance versions on the arXiv would clearly be unacceptable to the topology community! However, until the *Journal of Topology* builds up its back volumes, there are relatively few published articles to be read, and the cost-per-download is high in comparison with journals that publish a large number of papers each year or have built up historic archives.

Finally, despite the large amount of unpaid work contributed to journal publishing, journals are not cost free and some money has to go into the system somewhere. If mathematicians don't support a journal through subscriptions, the alternative is paid open access. Most mathematicians have no access to grant funding on the scale of "big science" and prefer the subscription model to open access fees.

Case 3. Mathematika

This journal was founded by Davenport in the 1950s and is owned by the maths department at University College London (UCL). The department was responsible for the choice of editorial board and the publication of the issues and sales—they did everything! Despite an uncertain production schedule and no attempt at marketing, the journal retained many traditional subscriptions because its price was so low that it kept below the radar of many library cuts. UCL asked us if we could take on the management of the journal, from providing an article management service to finding a large publisher to host and sell the journal to libraries in all the ways they expect, i.e., pay-per-view, traditional subscriptions, and consortia agreements. We agreed to a contract with Cambridge University Press, and both the LMS and CUP took the financial risk that moving the journal would lead to cancellations, as happened with *Compositio Mathematica*.

The managing editor, Alex Sobolev, and the editorial board put in extra hours to solicit good papers for the "relaunch". In several cases they volunteered papers that would have found homes in more distinguished journals to help the journal get back on its feet. This has worked; the journal is still small but receives sufficient papers to put out regular issues, and it is slowly growing.

We recognized the value of the early volumes; some very good papers were published in the early years and are only available in print. We retrodigitized the archive, and this is available to subscribers who hold a current subscription as an incentive to keep the subscriptions alive. Why not make it freely available? Because it is the value of the archive as well as the new research being published that puts the journal on a secure financial footing; i.e., the income we receive covers the costs and gives a small return to the LMS. As in the other cases, the "small return" gets put back into supporting mathematical activities.

[1] `http://www.msri.org/attachments/workshops/587/MSRIfinalreport.pdf.`

[2] *In writing this statement, it struck me that libraries used to be the repositories, but many now require publishers to grant "perpetual access" licenses, and the burden of responsibility has shifted with the digital age. It's something we should consider when dreaming up new publishing models: will our mathematical grandchildren thank us?*

Mathematics People

2012–2013 AMS Centennial Fellowship Awarded

Photo by Riza Falk.

Karin Melnick

The AMS has awarded its Centennial Fellowship for 2012–2013 to Karin Melnick of the University of Maryland. The fellowship carries a stipend of US$80,000, an expense allowance of US$8,000, and a complimentary Society membership for one year.

Karin Melnick was born and raised in Marin County, California. She attended Reed College in Portland, Oregon, and completed her Ph.D. at the University of Chicago in 2006 under the direction of Benson Farb. With a Postdoctoral Research Fellowship from the National Science Foundation, she went to Yale University as a Gibbs Assistant Professor. She received a Junior Research Fellowship from the Erwin Schrödinger Institute in spring 2009. She has been an assistant professor at the University of Maryland since fall 2009.

Melnick's research is on differential-geometric aspects of rigidity. She studies the relationship between algebraic or dynamical properties of the group of automorphisms of a geometric structure on a manifold M and the geometry and topology of M. Her work has focused on manifolds with Lorentzian metrics, conformal pseudo-Riemannian structures, and parabolic geometries in general.

Please note: Information about the competition for the 2013–2014 AMS Centennial Fellowships will be published in the Mathematics Opportunities section of an upcoming issue of the *Notices*.

—Allyn Jackson

Aaronson Receives Waterman Award

Scott Aaronson of the Massachusetts Institute of Technology has been selected as a recipient of the 2012 Alan T. Waterman Award of the National Science Foundation (NSF). He is a theoretical computational scientist whose research interests focus on the limitations of quantum computers and computational complexity theory more generally. His research addresses a variety of topics, including the information content of quantum states, the physical resources needed for quantum computers to surpass classical computers, and the barriers to solving computer science's vexing P versus NP question, that is, whether every problem whose solution can be quickly verified by a computer can also be quickly solved by a computer.

The Waterman Award is given annually to one or more outstanding researchers under the age of thirty-five in any field of science or engineering supported by the NSF. The prize consists of a medal and a US$1 million grant to be used over a five-year period for further advanced study in the awardee's field.

—From an NSF announcement

Kalai Awarded Rothschild Prize

Gil Kalai of the Einstein Institute of Mathematics, The Hebrew University of Jerusalem, has been named a recipient of the 2012 Rothschild Prize for his groundbreaking work in combinatorics, convexity, and probability theory. According to the prize citation, his work on discrete harmonic analysis has had a major influence in the field of computational complexity and on other areas of theoretical computer science.

The Rothschild Prizes are awarded in two-year cycles in recognition of original and outstanding published work in the disciplines of agriculture, chemical sciences, engineering, humanities, Jewish studies, life sciences, mathematics, physical sciences, and social sciences. A prize is awarded for each discipline once in four years. The prize carries a cash award of US$50,000 and is intended to support, encourage, and advance the sciences and humanities in Israel.

—From a Hebrew University announcement

2012 Balaguer Prize Awarded

The Ferran Sunyer i Balaguer Foundation has awarded the 2012 Ferran Sunyer i Balaguer Prize to Angel Cano, Universidad Nacional Autónoma de Mexico, Juan Pablo Navarrete, Universidad Autónoma de Yucatan, and Jose Seade, Universidad Nacional Autónoma de Mexico, for their joint monograph *Complex Kleinian Groups*.

The prize citation reads in part: "Kleinian groups were introduced by Henri Poincaré in the 1880s as the monodromy groups of certain second-order differential equations. These are discrete groups of automorphisms of the complex projective line CP^1; they can be regarded also as groups of isometries of the real hyperbolic 3-space. These groups have played, for decades, a major role in several fields of mathematics, as, for example, in the theory of Riemann surfaces, in holomorphic dynamics, and in the geometrization conjecture for 3-manifolds. The monograph by the prize recipients lays the foundations of the theory of complex Kleinian groups."

The Ferran Sunyer i Balaguer Foundation of the Institut d'Estudis Catalans (IEC) awards this international prize every year to honor the memory of Ferran Sunyer i Balaguer (1912–1967), a self-taught Catalan mathematician who gained international recognition for his research in mathematical analysis despite the serious physical disabilities with which he was born. The prize carries a cash award of 15,000 euros (approximately US$20,000); the winning monographs are published by Birkhäuser Verlag.

—From a Ferran Sunyer i Balaguer Foundation announcement

Prizes of the Canadian Mathematical Society

The Canadian Mathematical Society (CMS) has awarded a number of prizes for 2012.

Gregory Smith of Queen's University has been awarded the 2012 Coxeter-James Prize for young mathematicians who have made outstanding contributions to mathematical research. His research centers on combinatorial varieties, the fundamental objects at the interface between algebra, combinatorics, and geometry. Combinatorial varieties account for a large number of the important geometric objects that arise in commutative algebra, representation theory, and mathematical physics, and their explicit nature makes them a good testing ground for general theories and conjectures, as well as computational experimentation.

Roland Speicher of Queen's University, Kingston, Ontario, Canada, has been awarded the 2012 Jeffery-Williams Prize for Research Excellence for his research in free probability. The prize citation reads in part: "Through his focus on the combinatorial side of free probability, Speicher has helped reveal key links between several areas of mathematics. These linkages have, in turn, led to the resolution of several mathematical problems that have long been in question." The Jeffery-Williams Prize is awarded annually to an individual who has made outstanding mathematical research contributions.

Ailana Fraser of the University of British Columbia has been awarded the 2012 Krieger-Nelson Prize by the CMS for her work in the study of geometrical analysis. According to the prize citation, she has made important contributions to the theory of minimal surfaces, including existence and Morse index estimates. She has also found striking applications to Riemannian geometry and to extremal eigenvalue questions for surfaces. The Krieger-Nelson Prize recognizes female mathematicians who have made outstanding contributions to mathematical research.

—From a CMS announcement

Sloan Fellowships Awarded

The Alfred P. Sloan Foundation has announced the names of the recipients of the 2012 Sloan Research Fellowships. Each year the foundation awards fellowships in the fields of mathematics, chemistry, computational and evolutionary molecular biology, computer science, economics, neuroscience, physics, and ocean sciences. Grants of US$50,000 for a two-year period are administered by each fellow's institution. Once chosen, fellows are free to pursue whatever lines of inquiry most interest them, and they are permitted to employ fellowship funds in a wide variety of ways to further their research aims.

Following are the names and institutions of the 2012 awardees in mathematics: Anar Akhmedov, University of Minnesota; Margaret Beck, Boston University; Grigoriy Blekherman, Georgia Institute of Technology; James Bremer, University of California, Davis; Pierre Germain, New York University; Alireza Salehi Golsefidy, University of California, San Diego; Florian Herzig, University of Toronto; Vera Mikyoung Hur, University of Illinois, Urbana-Champaign; Matthew Kahle, Ohio State University; Joel Kamnitzer, University of Toronto; Young-Heon Kim, University of British Columbia; Tong Liu, Purdue University; Yi Ni, California Institute of Technology; Benjamin Recht, University of Wisconsin, Madison; Sebastien Roch, University of California, Los Angeles; Marcus Roper, University of California, Los Angeles; Karl Schwede, Pennsylvania State University; Allan Sly, University of California, Berkeley; Valentino Tosatti, Columbia University; Rachel A. Ward, University of Texas, Austin.

—From a Sloan Foundation announcement

Keyfitz Awarded Kovalevsky Lectureship

Barbara Keyfitz of the Ohio State University has been chosen as the AWM-SIAM Sonia Kovalevsky Lecturer for 2012 by the Association for Women in Mathematics (AWM). She will deliver the AWM-SIAM Kovalevsky Lecture, titled "The Role of Characteristics in Conservation Laws", at the 2012 annual meeting of the Society for Industrial and Applied Mathematics (SIAM) in Minneapolis, Minnesota. She was honored in recognition of her pioneering and seminal contributions to the field of hyperbolic conservation laws. The Sonia Kovalevsky Lectureship honors significant contributions of women to applied or computational mathematics.

—From AWM-SIAM announcements

Cornuejols Awarded John von Neumann Theory Prize

GERARD P. CORNUEJOLS of Carnegie Mellon University has been awarded the 2011 John von Neumann Theory Prize, the highest prize given in the field of operations research and management science, for his fundamental and broad contributions to discrete optimization, including his deep research on balanced and ideal matrices, perfect graphs, and cutting planes for mixed-integer optimization. The award, which is presented by the Institute for Operations Research and the Management Sciences (INFORMS), carries a cash prize of US$5,000.

—From an INFORMS announcement

Rollo Davidson Prizes Awarded

The Rollo Davidson Trust has awarded the 2012 Rollo Davidson Prize jointly to VINCENT BEFFARA (École Normale Supérieure, Lyon) and HUGO DUMINIL-COPIN (University of Geneva) for their work on disordered systems in two dimensions, and in particular for their solution of the problem of the critical point of the random cluster model. The Rollo Davidson Trust was founded in 1975 and awards an annual prize to young mathematicians working in the field of probability.

—From a Rollo Davidson Trust announcement

Hertz Fellowships Awarded

Two young mathematicians are among fifteen graduate students chosen to receive 2012 Fannie and John Hertz Foundation Fellowships. BRIAN LAWRENCE of the California Institute of Technology and YUN WILLIAM YU of Indiana University, Bloomington, will receive support of more than US$250,000 each for up to five years of graduate work. Fellows have the freedom to innovate in their doctoral studies without university or research restrictions.

—From a Hertz Foundation announcement

MacLean Awarded 2012 PIMS Education Prize

MARK MACLEAN of the University of British Columbia has been awarded the 2012 PIMS Education Prize of the Pacific Institute for the Mathematical Sciences. The prize recognizes individuals who have played a major role in encouraging activities that have enhanced public awareness and appreciation of mathematics, as well as those who foster communication among various groups concerned with mathematical education at all levels.

According to the prize citation, Mark MacLean is an outstanding instructor making a major impact on teaching in mathematics at UBC and beyond. He was one of the original members of the UBC Science One (first-year science) program and has contributed greatly to its success, with a personal dedication to enhancing the learning environment. In addition to his excellent teaching, MacLean has taken leadership roles in course development, instructor supervision, tutorial centre management, and TA [teaching assistant] training. He has also chaired UBC committees on scholarships and teaching awards. He is also a leader in Aboriginal education.

—From a PIMS announcement

Putnam Prizes Awarded

The winners of the seventy-second William Lowell Putnam Mathematical Competition have been announced. The Putnam Competition is administered by the Mathematical Association of America (MAA) and consists of an examination containing mathematical problems that are designed to test both originality and technical competence. Prizes are awarded to both individuals and teams.

The five highest ranking individuals, listed in alphabetical order, were SAMUEL S. ELDER, California Institute of Technology; BRIAN R. LAWRENCE, California Institute of Technology; SEOK HYEONG LEE, Stanford University; XIAOSHENG MU, Yale University; and EVAN M. O'DORNEY, Harvard University. Each received a cash award of US$2,500.

Institutions with at least three registered participants obtain a team ranking in the competition based on the rankings of three designated individual participants. The five top-ranked teams (with team members listed in alphabetical order) were: Harvard University (ERIC K. LARSON, EVAN M. O'DORNEY, and ALEX (LIN) ZHAI); Carnegie-Mellon University (MICHAEL T. DRUGGAN, ALBERT GU, and ARCHIT U. KULKARNI); California Institute of Technology (ZARATHUSTRA E. BRADY, SAMUEL S. ELDER, and BRIAN R. LAWRENCE); Stanford University (SEOK HYEONG LEE, GYUJIN OH, and LYUBOSLAV N. PANCHEV); and the Massachusetts Institute of Technology (VLAD FIROIU, COLIN P. SANDON, and JACOB N. STEINHARDT). The first-place team receives an award of US$25,000, and each member of the team receives US$1,000. The awards for second place are US$20,000 and US$800; for third place, US$15,000 and US$600; for fourth place, US$10,000 and US$400; and for fifth place, US$5,000 and US$200. The Elizabeth Lowell Putnam Prize, which goes to the outstanding woman in the competition, was awarded to FEI SONG of the University of Virginia. She received a cash award of US$1,000.

—From a Putnam announcement

Guggenheim Fellowships Awarded

The John Simon Guggenheim Memorial Foundation has announced the names of 180 artists, scholars, and scientists from the United States, Canada, and the United Kingdom who were selected as Guggenheim Fellows for 2012. Guggenheim Fellows are appointed on the basis of distinguished achievement in the past and exceptional promise for future accomplishment. The mathematicians selected to receive the 2012 fellowships are Stavros Garoufalidis, Georgia Institute of Technology; Nets Katz, University of Indiana; and Alexander Kiselev, University of Wisconsin-Madison. In addition, Andrea Henderson of the University of California, Irvine, was awarded a fellowship for her project, Algebraic Art, an exploration of the aesthetic implications of mathematical accounts of symbolism and formal coherence.

—From a Guggenheim Foundation news release

Intel Science Talent Search Winners Announced

Two students who work in the mathematical sciences have received scholarship awards in the 2012 Intel Science Talent Search. Fengning (David) Ding, an eighteen-year-old student from Albany, California, was awarded fourth place for his project, which classified the irreducible finite-dimensional representations of infinitesimal Cherdnik algebras. This mathematics work reveals important symmetries in representation theory and is related to conservation laws. He received a prize of US$40,000.

Anirudh Prabhu, a seventeen-year-old student from West Lafayette, Indiana, was awarded seventh place and a US$25,000 prize for his project, in which he investigated odd perfect numbers, assigning the first nontrivial analytic lower bound for them, a bound in terms of the number of distinct prime divisors. His research suggests that odd perfect numbers do not exist.

—From an Intel Corporation announcement

NSF Graduate Research Fellowships

The National Science Foundation (NSF) has awarded a number of Graduate Research Fellowships for fiscal year 2012. Further awards may be announced later in the year. This program supports students pursuing doctoral study in all areas of science and engineering and provides a stipend of US$30,000 per year for a maximum of three years of full-time graduate study. Following are the names of the awardees in the mathematical sciences selected so far in 2012, followed by their undergraduate institutions (in parentheses) and the institutions at which they plan to pursue graduate work.

Lauren Bandklayder (New York University), New York University; Ashley Bell (Colorado School of Mines), Colorado School of Mines; Eva Belmont (Harvard University), Massachusetts Institute of Technology; Mario Bencomo (Rice University), Rice University; John Binder (Massachusetts Institute of Technology), Massachusetts Institute of Technology; Emily Butler (University of North Carolina, Chapel Hill), University of North Carolina, Chapel Hill; Frederick Campbell (Rice University), Rice University; Suzanne Carter (University of Iowa), University of Texas, Austin; Zev Chonoles (Brown University), University of Pennsylvania; Giulia DeSalvo (University of California, Berkeley), Stanford University; Benjamin Dozie (Harvard University), Stanford University; Sylvester Eriksson-Bique (New York University), New York University; Laure Flapan (Yale University), University of California, Berkeley; Benjamin Fogelson (University of California, Davis), University of California, Davis; Alexander Furger (Princeton University), Prince-ton University; John Goes (University of Minnesota, Twin Cities), University of Minnesota, Twin Cities; Ryan Goh (University of Minnesota, Twin Cities), University of Minnesota, Twin Cities; Sherry Gong (Cambridge University), Massachusetts Institute of Technology; Paul Grigas (Massachusetts Institute of Technology), Massachusetts Institute of Technology; Kelvin Gu (Duke University), Duke University; Boaz Haberman (University of California, Berkeley), University of California, Berkeley; Nathan Harman (University of Massachusetts, Amherst), University of California, Berkeley; Kyle Hasenstab (University of California, Los Angeles), University of California, Los Angeles; William Karr (University of Illinois, Urbana-Champaign), University of Illinois, Urbana-Champaign; Christopher Kinson (University of Illinois), University of Illinois; Isabel Kloumann (Cornell University), Cornell University; Brian Lawrence (California Institute of Technology), Harvard University; Oleg Lazarev (Princeton University), Stanford University; Margaret-Rose Leung (Oregon State University), Cornell University; Danika Lindsay (California State University, Channel Islands), Brown University; Xue Liu (Princeton University), Massachusetts Institute of Technology; Nicole Looper (Dartmouth College), University of Michigan, Ann Arbor; Juan Lopez Arriaza (University of California, Santa Cruz), University of California, Santa Cruz; John Lopez (Brigham Young University), Massachusetts Institute of Technology; Nicholas Lowman (North Carolina State University), North Carolina State University; Eileen Martin (University of Texas, Austin), New York University; Michele Meisner (North Carolina State University), North Carolina State University; Victor Minden (Tufts University), University of Maryland, College Park; Hilary Monaco (Massachusetts Institute of Technology), Massachusetts Institute of Technology; Connor Mooney (Columbia University), Columbia University; Robin Neumayer (University of South Carolina, Columbia), University of Texas, Austin; Derek Olson (University of Minnesota, Twin Cities), University of Minnesota, Twin Cities; Suchandan Pal (University of Michigan, Ann Arbor), University of Michigan, Ann Arbor; Aleksandr Pankov (University of California, San Francisco), University of

California, San Francisco; Joseph Paulson (University of Maryland), University of Maryland; Cristian Potter (East Carolina University), Purdue University; Suzette Puente (University of California, Berkeley), University of California, Berkeley; Evan Randles (Cornell University), Cornell University; Robert Richardson (University of California, Santa Cruz), University of California, Santa Cruz; Larry Rolen (Emory University), Emory University; David Rolnick (Massachusetts Institute of Technology), Massachusetts Institute of Technology; Krishanu Sankar (Massachusetts Institute of Technology), Harvard University; Barret Schloerke, Rice University; Benjamin Schweinhart (Princeton University), Princeton University; Ayon Sen (California Institute of Technology), California Institute of Technology; Emily Sergel (University of California, San Diego), University of California, San Diego; Jay Shah (University of Chicago), Harvard University; Alexander Shapiro (University of California, Berkeley), University of California, Berkeley; Yiwei She (University of Chicago), University of Chicago; Kyler Siegel (Stanford University), Stanford University; Anthony Simms (Rice University), Rice University; Slater Stich (University of California, Berkeley), University of California, Berkeley; Anandh Swaminathan (California Institute of Technology), California Institute of Technology; Daniel Thompson (Yale University), Massachusetts Institute of Technology; Linda Tran (University of California, Berkeley), University of California, Berkeley; Rachel Vishnepolsky (University of Chicago), University of Chicago; Olivia Walch (University of Michigan), University of Michigan; Robert Walker (University of Michigan, Ann Arbor), University of Michigan, Ann Arbor; Brian Williams (University of Florida), University of Pennsylvania; Cameron Williams (University of Houston), University of Texas, Austin; Dylan Wilson (University of Washington), Harvard University; Scott Yang (University of Texas, Austin), University of Texas, Austin; Qiaochu Yuan (Massachusetts Institute of Technology), University of California, Berkeley; Yi Zeng (Massachusetts Institute of Technology), Massachusetts Institute of Technology; Charles Zheng (Texas A&M University), Texas A&M University.

—From an NSF announcement

SIAM Fellows Elected

The Society for Industrial and Applied Mathematics (SIAM) has elected thirty-five new fellows for 2012. Their names and institutions follow.

Tamer Basar, University of Illinois, Urbana-Champaign; Michele Benzi, Emory University; Anthony Bloch, University of Michigan; Pavel Bochev, Sandia National Laboratories; Richard Brualdi, University of Wisconsin, Madison; Gui-Qiang Chen, University of Oxford; G. Bard Ermentrout, University of Pittsburgh; Richard Falk, Rutgers University; Lisa Fauci, Tulane University; David Ferguson, Applied Mathematical Analysis; M. Gregory Forest, University of North Carolina, Chapel Hill; Susan Friedlander, University of Southern California; Irene M. Gamba, University of Texas, Austin; Walter Gautschi, Purdue University; Donald Goldfarb, Columbia University; Sven Hammarling, University of Manchester; Pavol Hell, Simon Fraser University; Bruce Hendrickson, Sandia National Laboratories, University of New Mexico; Kirk Jordan, IBM T. J. Watson Research Center; Michael Jordan, University of California, Berkeley; James Keener, University of Utah; Naomi Leonard, Princeton University; Philip Maini, University of Oxford; Geoffrey McFadden, National Institute of Standards and Technology; Edward Ott, University of Maryland, College Park; Tamar Schlick, Courant Institute of Mathematical Sciences, New York University; David Shmoys, Cornell University; Mary Silber, Northwestern University; Barry Smith, Argonne National Laboratory; Tao Tang, Hong Kong Baptist University; Edriss Titi, Weizmann Institute of Science; Robert Vanderbei, Princeton University; Richard Varga, Kent State University; Jan Willems, K.U. Leuven; and Thaleia Zariphopoulou, University of Oxford, University of Texas, Austin.

AAAS Elects New Members

The American Academy of Arts and Sciences (AAAS) has chosen 220 new members and 17 foreign honorary members for 2012. Following are the names and affiliations of the new members who work in the mathematical sciences: Bonnie Berger, Massachusetts Institute of Technology; Joan S. Birman, Barnard College, Columbia University; Russel E. Caflisch, University of California, Los Angeles; Ngù Bao Chu, University of Chicago; Bjorn M. Poonen, Massachusetts Institute of Technology; Steven H. Strogatz, Cornell University; Richard L. Taylor, Institute for Advanced Study. Louis Boutet de Monvel, Universitè Pierre et Marie Curie, Paris, France, was elected an honorary foreign member.

—From an AAAS announcement

Royal Society Elections

The Royal Society has elected its new fellows for 2012. The new fellows whose work involves the mathematical sciences include: Alan Bundy, University of Edinburgh; Alasdair Houston, University of Bristol; Christopher Hull, Imperial College London; Dominic Joyce, University of Oxford and Lincoln College; Richard Kerswell, University of Bristol; Chandrashekhar Khare, University of California, Los Angeles; John McNamara, University of Bristol; and Mathukumalli Vidyasagar, University of Texas, Dallas.

—From a Royal Society announcement

Mathematics Opportunities

NSF CAREER Awards

The National Science Foundation (NSF) solicits proposals for the Faculty Early Career Development (CAREER) Awards. These awards support junior faculty members who exemplify the role of teacher-scholars through outstanding research, excellent education, and the integration of education and research within the context of the mission of their organizations. In addition, award recipients are eligible to be selected for Presidential Early Career Awards for Scientists and Engineers (PECASE). The deadline for submission of proposals in the mathematical sciences is **July 25, 2012**. For more information, see `http://www.nsf.gov/pubs/2011/nsf11690/nsf11690.htm`.

—From an NSF announcement

Call for Entries for Balaguer Prize

The Ferran Sunyer i Balaguer Foundation invites entries for the 2013 Ferran Sunyer i Balaguer Prize. The prize will be awarded for a mathematical monograph of an expository nature presenting the latest developments in an active area of research in mathematics. The prize consists of 15,000 euros (approximately US$19,800) and publication of the winning monograph in Birkhäuser Verlag's series Progress in Mathematics. The deadline for submission is **December 3, 2012**. For more information, see the website `http://ffsb.iec.cat`.

—From a Ferran Sunyer i Balaguer Foundation announcement

Fulbright Postdoctoral Fellowships in Israel

The Fulbright Israel Program, managed by the United States-Israel Educational Foundation (USIEF), will award postdoctoral fellowships for research in Israel during the academic years 2013–2014 and 2014–2015. Eight fellowships for at least two academic years will be awarded to American postdoctoral researchers, who must be hosted by an accredited Israeli institution of higher education. The amount of the stipend is US$20,000 per academic year, which is intended to complement stipends provided by the host institutions. The deadline for applications is **August 1, 2012**. For more details, see the website `http://j.mp/IsFuyC`; application materials are available at `http://j.mp/HXbXgo`.

—From a USIEF announcement

Call for Nominations for Popov Prize

The Interdisciplinary Mathematics Institute (IMI) invites nominations for the 2013 Vasil A. Popov Prize for outstanding research contributions in fields related to the work of Popov in approximation theory. Nominees must have received their Ph.D. within the past six years. The prize recipient will be asked to deliver a plenary lecture at the Fourteenth International Conference in Approximation Theory, to be held in April 2013 in San Antonio, Texas. The deadline for nominations is **November 15, 2012**. For more details, see the website `http://imi.cas.sc.edu/popov-prize-call-nominations/`.

—From an IMI announcement

Mathematical Sciences Research Institute Fall Workshops, 2012, Berkeley, CA

With funding from the National Science Foundation (NSF) and the National Security Agency (NSA), the Mathematical Sciences Research Institute (MSRI) will hold four workshops in Commutative Algebra and Cluster Algebras during the fall of 2012. Established researchers, postdoctoral fellows, and graduate students are invited to apply for funding. It is the policy of MSRI to actively seek to achieve diversity in its workshops. Thus, a strong effort is made to remove barriers that hinder equal opportunity, particularly for those groups that have been historically underrepresented in the mathematical sciences. The workshops to be held are as follows:

August 22–24, 2012: Connections for Women: Joint Workshop on Commutative Algebra and Cluster Algebras. `http://www.msri.org/web/msri/scientific/workshops/show/-/event/Wm556`

August 27–September 7, 2012: Joint Introductory Workshop in Cluster Algebras and Commutative Algebra.

http://www.msri.org/web/msri/scientific/workshops/show/-/event/Wm557

October 29–November 2, 2012: Cluster Algebras in Combinatorics, Algebra, and Geometry.
http://www.msri.org/web/msri/scientific/workshops/show/-/event/Wm570

December 3–7, 2012: Combinatorial Commutative Algebra and Applications.
http://www.msri.org/web/msri/scientific/workshops/show/-/event/Wm571

Call for Nominations: CRM-Fields-PIMS Prize

The Centre de recherches mathématiques (CRM), the Fields Institute, and the Pacific Institute for the Mathematical Sciences (PIMS) invite nominations for the joint CRM-Fields-PIMS Prize, awarded in recognition of exceptional research achievement in the mathematical sciences. The candidate's research should have been conducted primarily in Canada or in affiliation with a Canadian university.

The prize was established as the CRM-Fields Prize in 1994. Renamed in 2005, the 2006 and later prizes have been awarded jointly by all three institutes. Previous recipients are H. S. M. Coxeter, George A. Elliott, James Arthur, Robert Moody, Stephen A. Cook, Israel Michael Sigal, William T. Tutte, John Friedlander, John McKay, Edwin Perkins, Donald Dawson, David Boyd, Nicole Tomczak-Jaegermann, Joel Feldman, Allan Borodin, Martin Barlow, Gordon Slade, Mark Lewis, and Stevo Todorcevic.

The selection committee formed by the three institutes will select a recipient for the 2013 prize on the basis of outstanding contributions to the advancement of the mathematical sciences, with excellence in research as the main selection criterion.

A monetary prize will be awarded, and the recipient will be asked to present a lecture at each of CRM, the Fields Institute, and PIMS.

Nominations should be submitted by **November 1, 2012**, by at least two sponsors of recognized stature and should include the following elements: three supporting letters, curriculum vitae, list of publications, and up to four preprints. Nominations will remain active for two years. During any academic year, at most one prize will be awarded.

—Clare Kiernan
Pacific Institute for the Mathematical Sciences
The University of British Columbia

Inside the AMS

AMS–AAAS Mass Media Fellow Chosen

Evelyn Lamb has been awarded the 2012 American Mathematical Society's AAAS Mass Media fellowship. Evelyn is a Ph.D. student in mathematics at Rice University and will work at *Scientific American* for ten weeks this summer.

The Mass Media Science & Engineering Fellowship program is organized by the American Association for the Advancement of Science (AAAS). It is a very competitive program designed to improve public understanding of science and technology by placing advanced science, mathematics, and engineering students in newsrooms nationwide. Fellows work with media professionals to improve their communication skills and increase their understanding of the editorial process by which events and ideas become news.


Evelyn Lamb

The program is available to college or university students (in their senior year, or in any graduate or postgraduate level) in the natural, physical, health, engineering, computer, or social sciences or mathematics who have outstanding written and oral communication skills and a strong interest in learning about the media. The program has supported over five hundred fellows over thirty years.

For more information on the AAAS Mass Media Science & Engineering Fellowship Program, visit the website http://www.aaas.org/programs/education/MassMedia/.

—AMS Washington office

Erdős Memorial Lecture

The Erdős Memorial Lecture is an annual invited address named for the prolific mathematician Paul Erdős (1913–1996). The lectures are supported by a fund created by Andrew Beal, a Dallas banker and mathematics enthusiast. The Beal Prize Fund, now US$100,000, is being held by the AMS until it is awarded for a correct solution to the Beal Conjecture (see www.math.unt.edu/~mauldin/beal.html). At Mr. Beal's request, the interest from the fund is used to support the Erdős Memorial Lecture.

The Erdős Memorial Lecturer for 2012 will be Ken Ono of Emory University. He will deliver the Erdős Lecture on October 27, 2012, at the Fall Western Section Meeting at the University of Arizona.

—AMS announcement

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.wustl.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Information for *Notices* Authors

The *Notices* welcomes unsolicited articles for consideration for publication, as well as proposals for such articles. The following provides general guidelines for writing *Notices* articles and preparing them for submission. Contact information for *Notices* editors and staff may be found on the *Notices* website, `http://www.ams.org/notices`.

***Notices* readership**. The *Notices* publishes articles that have broad appeal for a diverse audience with many different types of readers: graduate students, academic mathematicians, industrial mathematicians, researchers in mathematically based fields, and amateur enthusiasts. The paper edition of the *Notices* is sent to the approximately 33,000 members of the AMS, most of whom are professional mathematicians; about 25,000 of them reside in North America. Because the *Notices* is accessible for free over the Internet, the number of readers is much larger than the AMS membership. All readers may be assumed to be interested in mathematics research, but they are not all active researchers.

Notices Feature Articles

Topics. The *Notices* seeks exceptional articles that report on major new developments in mathematics or that describe episodes from mathematics history that have connection to current research in the field. We also welcome articles discussing aspects of the mathematics profession, such as grant programs, the job market, professional opportunities for mathematicians, publishing, electronic communications, etc. We are also interested in articles about mathematics education at all levels. We publish reviews of books, films, plays, software, and mathematical tools.

Reaching the audience. Our goal is to educate the readership about new developments in mathematics and in the mathematics profession, as well as other matters of interest to the working mathematician. Each article is expected to have a large target audience of readers, perhaps 5,000 of the 33,000 subscribers. Authors must therefore write their articles for nonexperts rather than for experts or would-be experts. In particular, the mathematics articles in the *Notices* are expository. A *Notices* article should have an introduction that anyone

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

can understand, and almost all readers should be able to understand the key points of the article.

Structure of articles. Most feature articles, including those on mathematics, are expected to be of long-term value and should be written as such. Ideally each article should put its topic in a context, providing some history and other orientation for the reader, and, as necessary, relating the subject matter to things that readers are likely to understand. In most cases, articles should progress to dealing with contemporary matters, not giving only historical material. The articles that are received the best by readers tend to relate different areas of mathematics to each other.

By design the *Notices* is partly magazine and partly journal, and authors' expository styles should take this into account. For example, many readers want to understand the mathematics articles without undue effort and without consulting other sources.

Format and length. Mathematics feature articles in the *Notices* are normally six to nine pages, sometimes a little longer. Shorter articles are more likely to be read fully than are longer articles. The first page is 400 or 500 words, and subsequent pages are about 800 words. From this one should subtract an allowance for figures, photos, and other illustrations and an appropriate allowance for any displayed equations and bibliography. The *Notices* is especially interested in the creative use of graphics and color and encourages illustrations. Articles on professional topics are typically 3 to 5 pages, as are book reviews.

Editorial process. The *Notices* aims to publish exceptionally well-written articles that appeal to a broad audience of mathematicians. Highly technical, specialized articles with a great deal of notation, insider jargon, and a long list of references are not suitable for the *Notices*. Some articles will be rejected by the editors without any external review. Other articles will be carefully refereed, and then a detailed editorial process will be used to bring the article up to the *Notices* standard. There will be considerable give and take between the author(s) and the editor, and it may take several drafts to get the article right.

The "WHAT IS...?" Column

Nearly every issue of the *Notices* carries an installment of the "WHAT IS...?" column. The purpose of the column is to provide brief, nontechnical descriptions of mathematical objects in use in current research. The target audience for the columns is first-year graduate students.

Each "WHAT IS...?" column provides an expository description of a single mathematical object being used in contemporary research. Thus "WHAT IS *M*-Theory?" would be too broad, but "WHAT IS a Brane?" would be appropriate; ideally "WHAT IS a Brane?" would give a flavor of what *M*-theory is.

The writing should be nontechnical and informal. Narrative description conveying main ideas should be favored over notation-heavy precision.

There is a strict limit of two *Notices* pages (1,400 words with no picture or 1,200 words with one picture). A list of "Further Reading" should contain no more than three references. Inquiries and comments about the "WHAT IS...?" column are welcome and may be sent to `notices-whatis@ams.org`.

Upcoming Deadlines

June 30, 2012: Nominations for SASTRA Ramanujan Prize. For more information, email `sastraprize@math.ufl.edu`, or see the website `http://www.math.ufl.edu/sastra-prize/nominations-2011.html`.

July 2, 2012: Full proposals for NSF Integrative Graduate Education and Research Training (IGERT) program. See `http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=12759`.

July 10, 2012: Full proposals for NSF Research Networks in the Mathematical Sciences. See `http://www.nsf.gov/pubs/2010/nsf10584/nsf10584.htm?WT.mc_id=USNSF_25&WT.mc_ev=click`.

July 25, 2012: Proposals for NSF Career Awards. See "Mathematics Opportunities" in this issue.

August 1, 2012: Applications for Fulbright Israel Program. See "Mathematics Opportunities" in this issue.

August 1, 2012: Applications for National Academies Research Associateship Programs. See `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email `rap@nas.edu`.

August 14, 2012: Full proposals for NSF Scholarships in Science, Technology, Engineering, and Mathematics (S-STEM) program. See `http://www.nsf.gov/pubs/2012/nsf12529/nsf12529.htm`.

September 15, 2012: Applications for spring 2013 semester of Math in Moscow. See `http://www.mccme.ru/mathinmoscow` or write to: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: `mim@mccme.ru`. Information and application forms for the AMS scholarships are available on the AMS website at `http://www.ams.org/programs/travel-grants/mimoscow` or by writing to: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2294; email `student-serv@ams.org`.

October 1, 2012: Applications for AWM Travel Grants. See `http://www.awm-math.org/travelgrants.html#standard` or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030; telephone 703-934-0163; `awm@awm-math.org`.

October 17, 2012: Proposals for NSF Postdoctoral Research Fellowships. See `http://www.nsf.gov/pubs/2008/nsf08582/nsf08582.htm`.

November 1, 2012: Nominations for CRM-Fields-PIMS Prize. See "Mathematics Opportunities" in this issue.

November 1, 2012: Applications for National Academies Research Associateship Programs. See `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC

20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

November 15, 2012: Nominations for 2013 Vasil A. Popov Prize. See "Mathematics Opportunities" in this issue.

December 3, 2012: Entries for Ferran Sunyer i Balaguer Prize. See "Mathematics Opportunities" in this issue.

Book List

The Book List highlights recent books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. Suggestions for books to include on the list may be sent to notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

The Adventure of Reason: Interplay between Philosophy of Mathematics and Mathematical Logic, 1900–1940, by Paolo Mancosu. Oxford University Press, January 2011. ISBN-13: 978-01995-465-34.

The Autonomy of Mathematical Knowledge: Hilbert's Program Revisited, by Curtis Franks. Cambridge University Press, December 2010. ISBN-13: 978-05211-838-95.

The Beginning of Infinity: Explanations That Transform the World, by David Deutsch. Viking Adult, July 2011. ISBN-13: 978-06700-227-55. (Reviewed April 2012.)

The Best Writing on Mathematics: 2010, edited by Mircea Pitici. Princeton University Press, December 2010. ISBN-13: 978-06911-484-10. (Reviewed November 2011.)

**Bibliography of Raymond Clare Archibald* by Scott Guthery. Docent Press, April 2012. ISBN-13: 978-0983700425.

The Blind Spot: Science and the Crisis of Uncertainty, by William Byers. Princeton University Press, April 2011. ISBN-13: 978-06911-468-43.

**Calculating Curves: The Mathematics, History, and Aesthetic Appeal of T. H. Gronwall's Nomographic Work,* by Thomas Hakon Gronwall, with contributions by Ron Doerfler and Alan Gluchoff, translation by Paul Hamburg, and bibliography by Scott Guthery. Docent Press, April 2012. ISBN-13: 978-09837-004-32.

The Calculus of Selfishness, by Karl Sigmund. Princeton University Press, January 2010. ISBN-13: 978-06911-427-53. (Reviewed January 2012.)

Chasing Shadows: Mathematics, Astronomy, and the Early History of Eclipse Reckoning, by Clemency Montelle. Johns Hopkins University Press, April 2011. ISBN-13: 978-08018-969-10. (Reviewed March 2012.)

**Classic Problems of Probability*, by Prakash Gorroochurn. Wiley, May 2012. ISBN: 978-1-1180-6325-5.

The Crest of the Peacock: Non-European Roots of Mathematics, by George Gheverghese Joseph. Third edition. Princeton University Press, October 2010. ISBN: 978-0-691-13526-7.

Cycles of Time: An Extraordinary New View of the Universe, by Roger Penrose. Knopf, May 2011. ISBN-13: 978-03072-659-06.

Divine Machines: Leibniz and the Sciences of Life, by Justin E. H. Smith. Princeton University Press, May 2011. ISBN-13: 978-06911-417-87.

An Early History of Recursive Functions and Computability from Gödel to Turing, by Rod Adams. Docent Press, May 2011. ISBN-13: 978-09837-004-01.

**Elliptic Tales: Curves, Counting, and Number Theory,* by Avner Ash and Robert Gross. Princeton University Press, March 2012. ISBN-13: 978-06911-511-99.

Emmy Noether's Wonderful Theorem, by Dwight E. Neuenschwander. Johns Hopkins University Press, November 2010. ISBN-13: 978-08018-969-41.

The Evolution of Logic, by W. D. Hart. Cambridge University Press, August 2010. ISBN-13: 978-0-521-74772-1

Excursions in the History of Mathematics, by Israel Kleiner. Birkhäuser, 2012. ISBN-13: 978-08176-826-75.

Experimental and Computational Mathematics: Selected Writings, by Jonathan Borwein and Peter Borwein. PSIpress, 2011. ISBN-13: 978-19356-380-56.

Fascinating Mathematical People: Interviews and Memoirs, edited by Donald J. Albers and Gerald L. Alexanderson. Princeton University Press, October 2011. ISBN-13: 978-06911-482-98.

Gösta Mittag-Leffler: A Man of Conviction, by Arild Stubhaug (translated by Tiina Nunnally). Springer, November 2010. ISBN-13: 978-36421-167-11.

Gottfried Wilhelm Leibniz: The Polymath Who Brought Us Calculus, by M. B. W. Tent. A K Peters/CRC Press, October 2011. ISBN-13: 978-14398-922-20.

The History and Development of Nomography, by H. A. Evesham. Docent Press, December 2010. ISBN-13: 978-14564-796-26.

In Pursuit of the Unknown: 17 Equations That Changed the World, by Ian Stewart. Basic Books, March 2012. ISBN-13: 978-04650-297-30.

In Service to Mathematics: The Life and Work of Mina Rees, by Amy Shell-Gellasch. Docent Press, December 2010. ISBN-13: 978-0-9837004-1-8.

The Information: A History, a Theory, a Flood, by James Gleick. Pantheon, March 2011. ISBN-13: 978-03754-237-27.

**The Irrationals: A Story of the Numbers You Can't Count On*, by Julian Havil. Princeton University Press, June 2012. ISBN-13: 978-0691143422.

Knots Unravelled: From String to Mathematics, by Meike Akveld and Andrew Jobbings. Arbelos, October 2011. ISBN-13: 978-09555-477-20.

Late Style: Yuri I. Manin Looking Back on a Life in Mathematics. A DVD documentary by Agnes Handwerk and Harrie Willems. Springer, March 2012. ISBN NTSC: 978-3-642-24482-7; ISBN PAL: 978-3-642-24522-0.

Lost in a Cave: Applying Graph Theory to Cave Exploration, by Richard L. Breisch. National Speleological Society, January 2012. ISBN-13: 978-1-879961-43-2.

The Lost Millennium: History's Timetables Under Siege, by Florin Diacu. Johns Hopkins University Press (second edition), November 2011. ISBN-13: 978-14214-028-88.

Magical Mathematics: The Mathematical Ideas That Animate Great Magic Tricks, by Persi Diaconis and Ron Graham. Princeton University Press, November 2011. ISBN13: 978-06911-516-49.

The Man of Numbers: Fibonacci's Arithmetic Revolution, by Keith Devlin. Walker and Company, July 2011. ISBN-13: 978-08027-781-23. (Reviewed May 2012.)

A Mathematical Nature Walk, by John A. Adam. Princeton University

Press, October 2011 (paperback edition). ISBN-13: 978-06911-526-53.

**The Mathematical Writings of Évariste Galois*, edited by Peter M. Neumann. European Mathematical Society, October 2011. ISBN-13: 978-3-03719-104-0.

**A Mathematician Comes of Age*, by Steven G. Krantz. Mathematical Association of America, December 2011. ISBN-13: 978-08838-557-82.

Mathematics and Reality, by Mary Leng. Oxford University Press, June 2010. ISBN-13: 978-01992-807-97.

**Mathematics in Victorian Britain,* by Raymond Flood, Adrian Rice, and Robin Wilson. Oxford University Press, October 2011. ISBN-13: 978-019-960139-4.

The Mathematics of Life, by Ian Stewart. Basic Books, June 2011. ISBN-13: 978-04650-223-80. (Reviewed December 2011.)

Mathematics, Religion and Ethics: An Epistemological Study, by Salilesh Mukhopadhyay. Feasible Solution LLC, September 2010. ISBN-13: 978-1-4507-3558-2.

NIST Handbook of Mathematical Functions, Cambridge University Press, edited by Frank W. J. Olver, Daniel W. Lozier, Ronald F. Boisvert, and Charles W. Clark. Cambridge University Press, May 2010. ISBN-13: 978-05211-922-55 (hardback plus CD-ROM); ISBN-13: 978-05211-406-38 (paperback plus CD-ROM). (Reviewed September 2011.)

The Noether Theorems: Invariance and Conservation Laws in the Twentieth Century, by Yvette Kosmann-Schwarzbach. Springer, December 2010. ISBN-13: 978-03878-786-76.

Numbers: A Very Short Introduction, by Peter M. Higgins. Oxford University Press, February 2011. ISBN-13: 978-0-19-958405-5. (Reviewed January 2012.)

One, Two, Three: Absolutely Elementary Mathematics (hardcover), David Berlinski. Pantheon, May 2011. ISBN-13: 978-03754-233-38.

The Perfect Swarm: The Science of Complexity in Everyday Life, by Len Fisher. Basic Books, March 2011 (paperback). ISBN-13: 978-04650-202-49.

The Philosophy of Mathematical Practice, Paolo Mancosu, editor. Oxford University Press, December 2011. ISBN-13: 978-01996-401-02. (Reviewed March 2012.)

The Pleasures of Statistics: The Autobiography of Frederick Mosteller, edited by Stephen E. Fienberg, David C. Hoaglin, and Judith M. Tanur. Springer, January 2010. ISBN-13: 978-03877-795-53.

Pricing the Future: Finance, Physics, and the 300-Year Journey to the Black-Scholes Equation, by George G. Szpiro. Basic Books, November 2011. ISBN-13: 978-04650-224-89.

The Proof Is in the Pudding: A Look at the Changing Nature of Mathematical Proof, by Steven G. Krantz. Springer, May 2011. ISBN-13: 978-03874-890-87.

Riot at the Calc Exam and Other Mathematically Bent Stories, by Colin Adams. AMS, July 2009. ISBN-13: 978-08218-481-73.

Roads to Infinity: The Mathematics of Truth and Proof, by John C. Stillwell. A K Peters/CRC Press, July 2010. ISBN-13: 978-15688-146-67.

Scientific Reflections: Selected Multidisciplinary Works, by Richard Crandall. PSIpress, 2011. ISBN-13: 978-19356-380-87.

Six Gems of Geometry, by Thomas Reale. PSIpress, 2010. ISBN-13: 978-19356-380-25.

**Sources in the Development of Mathematics: Series and Products from the Fifteenth to the Twenty-first Century*, by Ranjan Roy. Cambridge University Press, June 2011. ISBN-13: 978-05211-147-07.

The Strangest Man, by Graham Farmelo. Basic Books, August 2009. ISBN-13: 978-04650-182-77. (Reviewed October 2011.)

Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving, by Sanjoy Mahajan. MIT Press, March 2010. ISBN-13: 978-0-262-51429-3. (Reviewed August 2011.)

Taking Sudoku Seriously: The Math behind the World's Most Popular Pencil Puzzle, by Jason Rosenhouse and Laura Taalman. Oxford University Press, January 2012. ISBN-13: 978-01997-565-68.

The Theory That Would Not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy, by Sharon Bertsch McGrayne. Yale University Press, April 2011. ISBN-13: 978-03001-696-90. (Reviewed May 2012.)

Top Secret Rosies: The Female Computers of World War II. Video documentary, produced and directed by LeAnn Erickson. September 2010. Website: `http://www.topsecretrosies.com`. (Reviewed February 2012.)

**Turbulent Times in Mathematics: The Life of J.C. Fields and the History of the Fields Medal*, by Elaine McKinnon Riehm and Frances Hoffman. AMS, November 2011. ISBN-13 978-08218-691-47.

**The Universe in Zero Words: The Story of Mathematics As Told through Equations*, by Dana Mackenzie. Princeton University Press, April 2012. ISBN-13: 978-06911-528-20.

Viewpoints: Mathematical Perspective and Fractal Geometry in Art, by Marc Frantz and Annalisa Crannell. Princeton University Press, August 2011. ISBN-13: 978-06911-259-23.

Vilim Feller, istaknuti hrvatsko-americki matematicar/William Feller, Distinguished Croatian-American Mathematician, by Darko Zubrinic. Bilingual Croatian-English edition, Graphis, 2011. ISBN-13: 978-953-279-016-0.

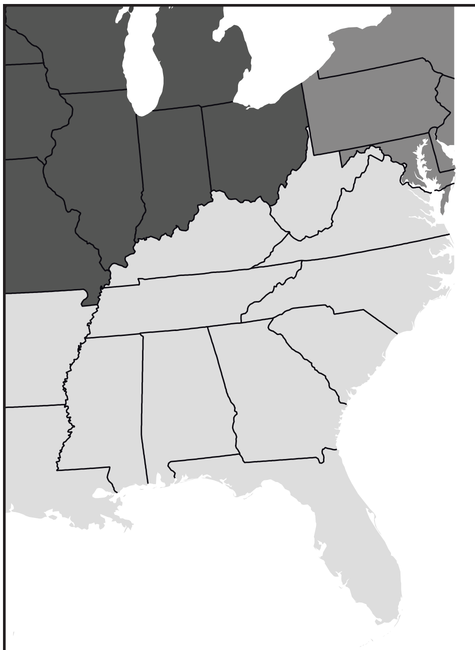
Visual Thinking in Mathematics, by Marcus Giaquinto. Oxford University Press, July 2011. ISBN-13: 978-01995-755-34.

What's Luck Got to Do with It? The History, Mathematics and Psychology of the Gambler's Illusion, by Joseph Mazur. Princeton University Press, July 2010. ISBN-13: 978-0-691-13890-9. (Reviewed February 2012.)

**Who's #1?: The Science of Rating and Ranking*, by Amy N. Langville and Carl D. Meyer. Princeton University Press, February 2012. ISBN-13: 978-06911-542-20.

Why Beliefs Matter: Reflections on the Nature of Science, by E. Brian Davies. Oxford University Press, June 2010. ISBN-13: 978-01995-862-02. (Reviewed April 2012.)

**Why Cats Land on Their Feet (and 76 Other Physical Paradoxes and Puzzles)*, by Mark Levi. Princeton University Press, May 2012. ISBN-13: 978-0691148540.



ASSOCIATE SECRETARY

Southeastern Section

Position

The American Mathematical Society is seeking applications and nominations of candidates for the post of Associate Secretary of the Southeastern Section. The states comprising that section are highlighted on the map to the left.

An Associate Secretary is an officer of the Society and is appointed by the Council to a two-year term, ordinarily beginning on 01 February. In this case the term would begin 01 February 2013 and end 31 January 2015. Reappointments are possible and desirable. All necessary expenses incurred by an Associate Secretary in performance of duties for the Society are reimbursed, including travel and communications as well as part-time support for secretarial help.

Duties

The primary responsibility of an Associate Secretary is to oversee scientific meetings of the Society in the section. Once every four years an Associate Secretary has primary responsibility for the Society's program at the January Joint Mathematics Meetings. An Associate Secretary is a member of the Secretariat, a committee consisting of the four Associate Secretaries and the Secretary, which approves all applications for membership in the Society and approves all sites and dates of meetings of the Society. Occasionally an Associate Secretary is in charge of an international joint meeting. Associate Secretaries are the principal contact between the Society and its members in the various sections. They are invited to all Council meetings and have a vote on the Council on a rotating basis.

Applications

An Associate Secretary is appointed by the Council upon recommendation by the Executive Committee and Board of Trustees. Applications—including a brief CV and names of three references—should be sent to:

Robert J. Daverman, Secretary, American Mathematical Society
238 Ayres Hall, University of Tennessee
Knoxville TN 37996-1320

email: daverman@math.utk.edu

Applications received by 15 September 2012 will be assured full consideration.

AMS EXEMPLARY PROGRAM PRIZE

At its meeting in January 2004, the AMS Council approved the establishment of a new award called the AMS Award for an Exemplary Program or Achievement in a Mathematics Department. It is to be presented annually to a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. Examples might include a department that runs a notable minority outreach program, a department that has instituted an unusually effective industrial mathematics internship program, a department that has promoted mathematics so successfully that a large fraction of its university's undergraduate population majors in mathematics, or a department that has made some form of innovation in its research support to faculty and/or graduate students, or which has created a special and innovative environment for some aspect of mathematics research.

The prize amount is $5,000. All departments in North America that offer at least a bachelor's degree in the mathematical sciences are eligible.

The Prize Selection Committee requests nominations for this award, which will be announced in Spring 2013. Letters of nomination may be submitted by one or more individuals. Nomination of the writer's own institution is permitted. The letter should describe the specific program(s) for which the department is being nominated as well as the achievements that make the program(s) an outstanding success, and may include any ancillary documents which support the success of the program(s). The letter should not exceed two pages, with supporting documentation not to exceed an additional three pages.

All nominations should be submitted to the AMS Secretary, Robert J. Daverman, American Mathematical Society, 238 Ayres Hall, University of Tennessee, Knoxville, TN 37996-1320. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations when appropriate. The nominations will be forwarded by the Secretary to the Prize Selection Committee, which will make the final decision on the award.

Deadline for nominations is September 14, 2012.

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at `http://www.ams.org/cgi-bin/mathcal-submit.pl`. The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at `http://www.ams.org/mathcal/`.

June 2012

1–22 **Financial Time Series Analysis: High-Dimensionality, Non-stationarity and the Financial Crisis**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Jan. 2012, p. 103)

1–July 31 **Geometric and Analytic Techniques in Calculus of Variations and Partial Differential Equations**, Centro di Ricerca Matematica "Ennio De Giorgi", Piazza dei Cavalieri 3, 56100 Pisa, Italy. (May 2012, p. 713)

3–30 **Clay Mathematics Institute 2012 Summer School "The Resolution of Singular Algebraic Varieties"**, Obergurgl, Tyrolean Alps, Austria. (Jan. 2012, p. 103)

4–6 **International meeting on "Statistical Analysis: Theory and Applications" (JIASTA2012)**, Mohamed First University, Faculty of Science, Oujda, Morocco. (Apr. 2012, p. 589)

4–6 **Workshop on Parameter Estimation for Dynamical Systems**, Eurandom, Eindhoven, The Netherlands. (Apr. 2012, p. 590)

4–8 **AIM Workshop: Cohomology bounds and growth rates**, American Institute of Mathematics, Palo Alto, California. (May 2012, p. 713)

4–8 **BIOCOMP2012 - Mathematical Modeling and Computational Topics in Biosciences**, Hotel Lloyd's Baia, Vietri sul Mare, Italy. (Jan. 2012, p. 103)

* 4–8 **Geometry of Eigenvalues and Eigenfunctions**, Centre de Recherches Mathématiques, Montréal, Canada.
Description: Geometric spectral theory focuses on the properties of eigenvalues and eigenfunctions of the Laplacian and other differential operators defined on geometric objects, such as Riemannian manifolds and Euclidean domains. Many problems in the field are motivated by questions originating in the study of real life phenomena: quantum-mechanical effects, vibration of membranes and plates, oscillations of fluids, etc.
Information: `http://www.crm.umontreal.ca/2012/Eigenvalues12/index_e.php`.

4–8 **Probability, Control and Finance**, Columbia University, New York, New York. (Jan. 2012, p. 104)

4–8 **Workshop on Geometry of Eigenvalues and Eigenfunctions**, Centre de Recherches Mathématiques, Université de Montréal, Pavillon André-Aisenstadt, 2920, Chemin de la tour, 5th floor, Montréal (Québec), H3T 1J4 Canada. (Apr. 2012, p. 590)

6–8 **Joint Conference of the Belgian, Royal Spanish, and Luxembourg Mathematical Societies**, University of Liege, Belgium. (May 2012, p. 713)

6–9 **Banach Spaces Workshop**, University of Birmingham, Birmingham, United Kingdom. (May 2012, p. 713)

6–9 **EUROMECH Colloquium 535 Similarity and Symmetry Methods in Solid Mechanics**, Varna, Bulgaria. (Dec. 2011, p. 1617)

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to `notices@ams.org` or `mathcal@ams.org`.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: `http://www.ams.org/`.

7–9 **Quantum invariants of 3-manifolds**, Institut de Recherche Mathématiquée Avancé, University of Strasbourg, France. (Nov. 2011, p. 1494)

8–10 **The International Conference on the Frontier of Computational and Applied Mathematics: Tony Chan's 60th Birthday Conference**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Feb. 2012, p. 339)

8–13 **38th International Conference "Applications of Mathematics in Engineering and Economics" AMEE'12**, Leisure House of the Technical University of Sofia, Sozopol, Bulgaria. (Jan. 2012, p. 104)

8–13 **XIVth International Conference on Geometry, Integrability and Quantization**, Sts. Constantine and Elena resort (near Varna, Bulgaria). (Dec. 2011, p. 1617)

10–16 **Probability and Analysis**, Mathematical Research and Conference Center of the Polish Academy of Sciences, Bedlewo, Poland. (Mar. 2012, p. 451)

11–13 **"Games and Strategy in Paris", held on the occasion of Sylvain Sorin's 60th birthday**, Institut Henri Poincaré, Paris, France. (May 2012, p. 713)

11–14 **Ninth Advanced Course in Operator Theory and Complex Analysis**, Faculty of Mathematics, University of Seville, Seville, Spain. (May 2012, p. 713)

11–14 **Operator Theory, Analysis and Mathematical Physics**, Centre de Recerca Matemàtica, Bellaterra, Barcelona. (May 2012, p. 713)

11–15 **Arrangements in Pyrenees: School and Workshop on "Hyperplane arrangements and related topics"**, Universite de Pau et des Pays de l'Adour, Pau, France. (Feb. 2012, p. 339)

11–15 **International Workshop on Complex Analysis and its Applications**, Walchand College of Engineering, Sangli 416415, India. (Apr. 2012, p. 590)

11–15 **Mathematical Problems in Industry (MPI) Workshop 2012**, University of Delaware, Newark, Delaware. (May 2012, p. 713)

11–15 **NSF/CBMS Conference: Finite Element Exterior Calculus (FEEC)**, ICERM, Providence, Rhode Island. (Mar. 2012, p. 451)

* 11–15 **SIDE 10 International Conference: Symmetries and Integrability of Difference Equations**, Xikou, Ningbo, near Shanghai, China.
Description: SIDE 10 is the tenth in a series of biennial conferences devoted to Symmetries and Integrability of Difference Equations and related topics: ordinary and partial difference equations, analytic difference equations, orthogonal polynomials and special functions, symmetries and reductions, difference geometry, integrable discrete systems on graphs, integrable dynamical mappings, discrete Painlevé equations, singularity confinement, algebraic entropy, complexity and growth of multivalued mapping, representations of affine Weyl groups, quantum mappings and quantum field theory on the space-time lattice.
Information: `http://www2.nbu.edu.cn/lousenyue/`.

11–22 **Mathematical Modeling on Ecology and Epidemiology**, University of Wyoming, Laramie, Wyoming. (Jan. 2012, p. 104)

12–14 **Ninth Edition of the Advanced Course in Operator Theory and Complex Analysis**, Sevilla, Spain. (May 2012, p. 714)

12–15 **5th Chaotic Modeling and Simulation International Conference (CHAOS2012)**, Athens, Greece. (Nov. 2011, p. 1494)

12–15 **"The Incomputable" — A workshop of the 6-month Isaac Newton Institute programme — "Semantics and Syntax: A Legacy of Alan Turing" (SAS)**, Kavli Royal Society International Centre, Chicheley Hall, Newport Pagnell MK16 9JJ, United Kingdom. (Aug. 2011, p. 1013)

13–15 **USENIX ATC '12: 2012 USENIX Annual Technical Conference**, Sheraton Boston Hotel in Boston, Massachusetts. (Mar. 2012, p. 452)

13–16 **SIAM Conference on Nonlinear Waves and Coherent Structures (NW12)**, The University of Washington, Seattle, Washington. (Sept. 2011, p. 1186)

17–22 **BIOMATH 2012: International Conference on Mathematical Methods and Models in Biosciences**, Bulgarian Academy of Sciences, Sofia, Bulgaria. (Dec. 2011, p. 1617)

18–22 **AIM Workshop: Dynamics of the Weil-Petersson geodesic flow**, American Institute of Mathematics, Palo Alto, California. (Sept. 2011, p. 1186)

18–22 **Conference on Geometry and Quantization of Moduli Spaces (2012 VBAC Conference)**, Centre de Recerca Matemàtica, Bellaterra, Barcelona. (May 2012, p. 714)

* 18–22 **Mini-courses in Mathematical Analysis 2012**, University of Padova, Padova, Italy.
Description: Following a long-standing tradition, the University of Padova is organizing the meeting "Mini-courses in Mathematical Analysis 2012". The meeting will take place at "Torre Archimede", a new building of the University of Padova in the city center. The program consists of four lecture courses delivered by invited speakers and a limited number of short communications. The meeting aims to introduce the participants to important current research fields in Mathematical Analysis. The meeting is particularly addressed not only to graduate students, postdocs and young researchers but also to well-established experts in Mathematical Analysis. In association with ISAAC - International Society for Analysis, its Applications and Computation.
Information: `http://minicourses.dmsa.unipd.it/`.

18–22 **NSF/CBMS Regional Conference in the Mathematical Sciences: Hodge Theory, Complex Geometry, and Representation Theory**, Texas Christian University, Fort Worth, Texas. (Dec. 2011, p. 1617)

* 18–22 **Workshop on analytic torsion and its applications**, Department of Mathematics, Université Paris-Sud 91405, Orsay, France.
Description: The workshop, organized by Jean-Michel Bismut and Werner Müller, is devoted to analytic torsion, in holomorphic and real form. Special emphasis will be given to asymptotic torsion and its connection with torsion groups, torsion of hyperbolic manifolds, stratified spaces, the hypoelliptic Laplacian, and the trace formula.
Speakers: C. Aldana, N. Bergeron, J. Brüning, X. Dai, G. Freixas, S. Goette, U. Ludwig, X. Ma, V. Maillot, F. Nier, J. Pfaff, J. Porti, M. Spreafico, B. Vertman, K. Yoshikawa, W. Zhang.
Information: `http://www.math.u-psud.fr/torsion/index-eng.php`.

18–23 **Turing Centenary Conference (CiE 2012): How the World Computes**, University of Cambridge, Cambridge, United Kingdom. (Aug. 2011, p. 1013)

18–29 **Noncommutative Algebraic Geometry**, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2011, p. 1617)

18–29 **St. Petersburg School in Probability and Statistical Physics**, Chebyshev Laboratory, St. Petersburg State University, Russia. (Mar. 2012, p. 452)

18–August 10 **SummerICERM: Geometry and Dynamics**, Institute for Computational and Experimental Research in Mathematics (ICERM), 121 South Main Street, Providence, Rhode Island. (Feb. 2012, p. 339)

18–August 15 **Random Matrix Theory and its Applications II**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Jan. 2012, p. 104)

19–22 **Financial Engineering Summer School 2012**, Madrid Stock Exchange, Madrid, Spain. (Mar. 2012, p. 452)

19–24 **FRG Workshop and Conference on Characters, Liftings, and Types**, American University, Washington, District of Columbia. (Feb. 2012, p. 339)

20–24 **International Conference on Applied Analysis and Algebra (ICAAA2012)**, Davutpasa Campus, Yildiz Technical University, Istanbul, Turkey. (Mar. 2012, p. 452)

22–23 **2nd International Conference of Information Systems, Computer Engineering & Application (ICISCEA 2012)**, Kathmandu, Nepal. (Apr. 2012, p. 590)

* 22–26 **VI International Summer School on Geometry, Mechanics and Control**, La Cristalera, Miraflores de la Sierra, Madrid, Spain.
Description: The school is oriented to young researchers, Ph.D. students and postdoctoral fellows in Mathematics, Physics and Engineering. This Summer School is a post-graduate course of Consejo Superior de Investigaciones Científicas (CSIC) on science and physical technologies within the activities of the Instituto de Ciencias Matemáticas (ICMAT) and the Geometry, Mechanics and Control Network (`http://www.gmcnetwork.org`).
Speakers: D. Holm (Imperial College London, UK), Geometric Mechanics for Pedestrians: Lessons from Fermat, Hamilton, Lie and Poincaré; A. Iserles (University of Cambridge, UK), Lie Group Methods for ODEs; J. Montaldi (University of Manchester, UK), Dynamics in Symmetric Hamiltonian Systems .
Deadlines: For scholarships: April 30, 2012. Registration deadline: May 15, 2012.
Information: `http://gmcnetwork.org/?q=activity-detaill/665`.

* 25–27 **Workshop on Geometry of Interfaces and Capillarity**, Granada, Spain.
Description: The objective of the workshop is to bring together researchers from mathematics and physics working on geometric problems involving interfaces, capillarity, surfaces with constant mean curvature and wetting phenomena.
Invited Speakers: Martin Brinkmann (MPI for Dynamics and Self-Organization, Göttingen), Rustum Choksi (McGill University), Robert Finn (Stanford University), Reinhard Lipowsky (MPI of Colloids and Interfaces, Potsdam), John McCuan (Georgia Institute of Technology), Bennett Palmer (Idaho State University), Ralf Seemann (Universität des Saarlandes), Henry Wente (University of Toledo).
Contact: Rafael López (Granada); email: `rcamino@ugr.es`.
Information: `http://www.ugr.es/~surfaces/workshop-interfaces/`.

25–28 **5th Podlasie Conference on Mathematics**, Bialystok University of Technology, Bialystok, Poland. (Feb. 2012, p. 339)

* 25–28 **Nonlinear Partial Differential Equations: Theory and Applications to Complex Systems. An International Conference in honor of Hiroshi Matano on the occasion of his 60th birthday**, Institut des Hautes Etudes Scientifiques (IHES), Le Bois-Marie 35, route de Chartres, 91440 Bures-sur-Yvette, France.
Description: This conference, which is co-organized by the IHES and the CNRS Japan-France International Laboratory ReaDiLab; `http://www.math.u-psud.fr/~readilab`, has the purpose to develop a better mathematical understanding of complex phenomena arising in biology, biomimetic systems, and medicine. More precisely it will bear on the modeling, the mathematical analysis, and the numerical simulations of reaction-diffusion processes involved in those domains. It will essentially focus on the following themes: propagation in inhomogeneous media, formation of spatio-temporal patterns, oscillatory instabilities and application to genetic processes, bacterial infections, micro-bacterial colonies and population dynamics in ecology, interface and spike dynamics, and blow-up in finite time.
Information: `http://www.ihes.fr`.

25–29 **AIM Workshop: Hypergeometric motives**, International Centre for Theoretical Physics, Trieste, Italy. (Nov. 2011, p. 1494)

25–29 **3rd European Seminar on Computing (ESCO 2012)**, Pilsen, Czech Republic. (Nov. 2011, p. 1494)

25–29 **Conference on Differential and Difference Equations and Applications 2012 (CDDEA2012)**, Terchova (close to Zilina), Slovak Republic. (Apr. 2012, p. 590)

* 25–29 **ESF-EMS-ERCOM Conference: Perspectives in Discrete Mathematics**, Centre de Recerca Matemàtica, Bellaterra, Barcelona, Spain.
Description: The scope of this conference is Discrete Mathematics in a wide sense: graph theory, combinatorics, discrete geometry, computer science, and applications. One of the main goals is to stress connections with other areas, in particular: analysis, geometry, number theory, physics and probability. We ultimately aim to provide a panoramic view of deep mathematical developments in the area of discrete mathematics.
Information: `http://www.crm.cat/ESFdiscretemathematics`.

25–29 **European Seminar on Computing (ESCO 2012)**, Pilsen, Czech Republic. (May 2012, p. 714)

25–29 **The Fourteenth International Conference on "Hyperbolic Problems: Theory, Numerics, Applications" (HYP2012)**, Università di Padova, Italy. (Mar. 2012, p. 452)

25–30 **IVth Workshop on Coverings, Selections, and Games in Topology**, Department of Mathematics, Seconda Università di Napoli, Caserta, Italy. (Jan. 2012, p. 104)

* 25–30 **The Fifteenth International Conference on Fibonacci Numbers and Their Applications**, Eszterházy Károly College, Eger, Hungary.
Description: The purpose of the conference is to bring together people from all branches of mathematics and science with interests in recurrence sequences, their applications and generalizations, and other special number sequences.
Information: `http://fib15.ektf.hu`.

25–July 6 **Seminaire de Mathematiques Superieures 2012: Probabilistic Combinatorics**, University of Montreal, Montreal, Canada. (Apr. 2012, p. 590)

26–28 **The 11th International Workshop on Dynamical Systems and Applications**, Cankaya University, Ankara, Turkey. (Apr. 2012, p. 590)

26–30 **International Conference "Probability Theory and its Applications" in Commemoration of the Centennial of Boris Vladimirovich Gnedenko**, Moscow State University, Moscow, Russia. (May 2012, p. 714)

27–29 **Differential Geometry Days. In honour of Luis A. Cordero**, Departamento de Xeometría e Topoloxía, Facultade de Matemàticas, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain.

27–29 **International Conference on Special Functions and Their Applications and Symposium on Life and Works of Ramanujan (XIth Annual Conference of Society of Special Function and Their Applications)**, Department of Applied Mathematics and Humanities, S.V. National Institute of Technology, Surat 395 007, Gujarat, India. (Apr. 2012, p. 590)

27–29 **Low dimensional conformal structures and their groups**, Institute of Mathematics, Gdansk University, Gdansk, Poland. (Mar. 2012, p. 453)

July 2012

1–5 **9th AIMS Conference on Dynamical Systems, Differential Equations and Applications**, Orlando, Florida. (Sept. 2011, p. 1186)

1–5 **Mathematical Modeling of Microbiological Systems (M3S)**, Marburg, Germany. (May 2012, p. 714)

1–7 **Workshop on geometric structures on manifolds and their applications**, Castle Rauischholzhausen near Marburg, Germany. (Nov. 2011, p. 1494)

1–16 **Special Case for Interpolation by Using Bézier Curve Numerically**, Al-Muthana University, College of Science, Department of Mathematics & Computer Applications, Muthana, Iraq. (Feb. 2012, p. 339)

1–21 **IAS/PCMI Summer 2012: Geometric Group Theory**, IAS/Park City Mathematics Institute, Park City, Utah. (Apr. 2012, p. 590)

2–4 **Superluminal Physics & Instantaneous Physics – as new trends in research (electronic conference)**, University of New Mexico, 200 College Road, Gallup, New Mexico. (Mar. 2012, p. 453)

2–6 **Algebra and Topology: Methods, Computation, and Science (ATMCS 2012)**, International Centre for Mathematical Sciences, Edinburgh, United Kingdom. (Apr. 2012, p. 590)

2–6 **12th International Conference on p-Adic Functional Analysis**, University of Manitoba, Winnipeg, Canada. (Dec. 2011, p. 1618)

* 2–6 **Exploratory Conference on the Mathematics of Biodiversity**, Centre de Recerca Matemàtica, Bellaterra, Barcelona, Spain.
Description: What is diversity? How do we measure it? This event will bring together life scientists and mathematicians to advance our understanding of diversity and its measurement. 1. Develop diversity measures that are sensitive to species similarity. 2. Diversity of partitioned communities: the mathematics of within-group and between-group diversity, including when inter-species similarity is taken into account. 3. What are the implications of a mathematically rigorous theory of diversity and differentiation for conservation biology, ecology, evolutionary theory, epidemiology and population genetics? 4. What can mathematicians learn from recent advances in diversity measurement?
Information: `http://www.crm.cat/wkbiodiversity`.

* 2–6 **Manifolds of Metrics and Probabilistics Methods in Geometry and Analysis**, Centre de Recherches Mathématiques, Montréal, Canada.
Description: This workshop is devoted to random fields in geometry and physics that model the intuitive notion of a "random surface". The focus is on the geometric properties or morphology of random surfaces. The simplest random surfaces are defined by graphs of Gaussian random functions of various kinds, which have been used in physics to model water waves, speckle patterns in laser light, the mass distribution of the early universe or vacua of string theory. The study of their geometry is sometimes called statistical topography. The interesting geometric properties range from connectivity properties and curvatures of contour lines to the distribution and correlation of peak points to the structure of excursion sets. The "roughness" of the random surfaces corresponds to the Sobolev class of the random functions and ranges from spherical harmonics of fixed but large degree to the GFF (Gaussian free field), which is a random distribution rather than a function.
Information: `http://www.crm.umontreal.ca/2012/Metrics12/index_e.php`.

2–6 **Model Theory in Algebra, Analysis and Arithmetic**, Cetraro, Italy. (Jan. 2012, p. 104)

* 2–6 **Workshop Periods and Motives: A modern perspective on renormalization**, Instituto de Ciencias Matemàticas, Madrid, Spain.
Aim: This workshop aims to bring together leading experts in algebraic geometry, number theory, combinatorics and quantum field theory to report on recent developments in the study of motives and periods with an emphasis on the connections to physics. Young researchers can apply for financial support to attend the workshop at the website below (deadline May 15, 2012).
Speakers: J. Ayoub (Zurich), P. Belkale (NC Chapel Hill), S. Bloch (Chicago), C. Bogner (HU Berlin), P. Brosnan (Maryland), P. Cartier (IHES), O. Ceyhan (Amsterdam), D. Doryn (HU Berlin), C. Duhr (ETH Zurich), H. Gangl (Durham), A. Goncharov (Yale), D. Kreimer (HU Berlin) (tbc), M. Marcolli (Caltech), S. Paycha (Potsdam), L. Schneps (Paris 6), O. Schnetz (HU Berlin), M. Spradlin (Brown), G. Tabuada (MIT), A. Volovich (Brown), K. Yeats (Simon Fraser). There will be a special young researchers session.
Sponsors: ICMAT Severo Ochoa Program and CNRS GDR Renormalisation.
Information: `http://www.icmat.es/congresos/periods-and-motives/`.

2–7 **24th Conference in Operator Theory**, West University, Timisoara, Romania. (Nov. 2011, p. 1495)

* 2–7 **BIOMAT-2012. Self-organization and collective dynamics in the live sciences: swarms, biofilms, traffic**, Facultad de Ciencias (Universidad de Granada), Granada, Spain.
Description: The aim of the BIOMAT school is to combine long seminars with courses of several specialists in the following topics. Programme: Bellomo, Nicola (Politecnico di Torino, Italy): On the mathematical theory of the dynamics of swarms viewed as complex systems; Bressan, Alberto (Penn State University, USA): Modeling and optimization for traffic flow; Degond, Pierre (Universit de Toulouse, France): Mathematical models of collective dynamics in life sciences; Hemelrijk, Charlotte K. (University of Groningen, The Netherlands): The self-organisation of fish schools and bird flocks; Klapper, Isaac (Montana State University, USA): Microbial communities; Schadschneider, Andreas (University of Cologne, Germany): From ant trails to pedestrian dynamics: empirical studies and cellular automata models. During the school a meeting of "young researchers modelling biological processes" is being organized. The call to participate in this meeting is open until May 31.
Information: `http://www.ugr.es/local/kinetic/biomat/`.

2–13 **Summer School on Algebraic and Enumerative Combinatorics**, Centro de Estudos Camilianos, S. Miguel de Seide, Guimarães, Portugal. (May 2012, p. 714)

2–20 **RTG Summer School on Inverse Problems & Partial Differential Equations**, University of Washington, Seattle, Washington. (Mar. 2012, p. 453)

3–5 **2nd International Conference on Mathematical Applications in Engineering 2012, ICMAE'12**, Kuala Lumpur, Malaysia. (Dec. 2011, p. 1618)

3–6 **International Symposium on Asymptotic Methods in Stochastics, in Honour of Miklós Csörgő's work on the occasion of his 80th birthday**, Carleton University, Ottawa, Canada. (Apr. 2012, p. 590)

4–6 **The 2012 International Conference of Applied and Engineering Mathematics**, Imperial College London, London, United Kingdom. (Jan. 2012, p. 104)

4–6 **Numerical Software: Design, Analysis and Verification**, Universidad de Cantabria, Santander, Spain. (Mar. 2012, p. 453)

4–6 **Workshop: Statistical Inference in Complex/High-Dimensional Problems**, University of Vienna, Vienna, Austria. (Jan. 2012, p. 104)

5–10 **Mathematical Physics in Bahia: Algebraic Analysis, Quantization and Representations**, Instituto de Matematica, UFBA, Salvador, Brazil. (May 2012, p. 714)

6–7 **National Conference on Mathematical and Computational Sciences**, Adikavi Nannaya University, Rajahmundry, Andhra Pradesh, India. (May 2012, p. 714)

8–11 **Trends in Set Theory**, Stefan Banach International Mathematical Center, Warsaw, Poland. (Dec. 2011, p. 1618)

8–15 **International Conference on Wavelets and Applications**, Euler International Mathematical Institute, St. Petersburg, Russia. (Jan. 2012, p. 105)

9–11 **15th Galway Topology Colloquium**, University of Oxford, Oxford, United Kingdom. (Apr. 2012, p. 591)

9–11 **Third Workshop on Mathematical Cryptology**, International Centre for Mathematical Meetings (CIEM), Castro Urdiales (Cantabria), Spain. (Jan. 2012, p. 105)

* 9–12 **Calculus of Variations and PDEs**, Szczawnica, Poland.
Description: This is a satellite conference of the 6th European Congress of Mathematics (Krakow 2012), organized by the Institute of Mathematics of the Polish Academy of Sciences, Jagiellonian University and University of Warsaw.
Topics: Calculus of variations, partial differential equations with emphasis on PDEs of elliptic and parabolic types, modern methods in nonlinear PDEs, new challenges arising in the study of mathematical models, PDEs with variable exponent, partial differential inclusions, geometric function theory, function spaces in PDEs, Fourier analysis and geometric measure theory. There will be plenary lectures as well as short presentations and poster sessions.
Information: `http://sets.impan.pl/~calcvarpde/`.

9–13 **Additive Combinatorics in Paris 2012—Combinatoire Additive à Paris 2012**, Institut Henri Poincaré, Paris, France. (Jan. 2012, p. 105)

9–13 **Algorithmic Number Theory Symposium (ANTS-X)**, University of California, San Diego, California. (Apr. 2012, p. 591)

9–13 **EVEQ 2012: International Summer School on Evolution Equations**, Prague, Czech Republic. (Jan. 2012, p. 105)

9–15 **The 10th International Conference on Fixed Point Theory and its Applications (ICFPTA-2012)**, Faculty of Mathematics and Computer Science, Babes, Bolyai University, Cluj-Napoca, Romania. (Jun/Jul. 2011, p. 862)

9–20 **Mathematical General Relativity**, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2011, p. 1618)

9–27 **2012 Summer School on Geometry and Data**, Washington State University & University of Idaho, held in Moscow, Idaho. (May 2012, p. 714)

9–27 **Graduate Summer School: Deep Learning, Feature Learning**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Feb. 2012, p. 339)

10–14 **3Quantum: Algebra, Geometry, Information**, Tallinn University of Technology, Tallinn, Estonia. (Feb. 2012, p. 340)

11–13 **Third International Conference on Symbolic Computation and Cryptography**, International Centre for Mathematical Meetings (CIEM), Castro Urdiales (Cantabria), Spain. (Jan. 2012, p. 105)

* 11–13 **Workshop on Nanomaths - Nanomaths 2012**, Centre de Recerca Matemàtica, Bellaterra, Barcelona, Spain.
Description: Nanotechnology is a rapidly growing and exciting research area that is constantly issuing new challenges. Research in this field is dominated by experiment and computation, whilst the associated theory required to advance development is lagging behind. The purpose of the Nanomath 2012 Workshop is to bring together mathematicians, chemists, physicists and engineers to explore the potential for mathematicians and experimental scientists to gain a better insight into the workings of the nanoworld.
Information: `http://www.crm.cat/wknanomaths`.

* 12–13 **Conference: Dynamics of Memory: What is the evidence?**, Centre de Recerca Matemàtica, Bellaterra, Barcelona, Spain.
Aim: The aim of this conference is to discuss experimental approaches that evaluate and assess theoretical mechanistic explanations for how memories are stored, retrieved and maintained in the brain. Recent technological and theoretical developments make this the right time to bring together theoreticians and experimentalists to discuss the dynamics of memory. This meeting should be of interest to systems neuroscientists, both those using experimental as well as theoretical approaches.
Information: `http://www.crm.cat/cdynamics`.

12–19 **International Summer School on Fundamental Algorithms and Computable Modeling for High-Performance and Multi-scale Scientific/Engineering Computing**, Nankai University, School of Mathematical Sciences, No. 94 Weijin Road, Nankai District, Tianjin 300071, People's Republic of China. (May 2012, p. 715)

15–20 **XXII Brazilian Algebra Meeting**, Salvador, in the State of Bahia, Brazil. (Apr. 2012, p. 591)

16–18 **Recent Developments in Statistical Multiscale Methods**, University of Goettingen, Germany. (Apr. 2012, p. 591)

* 16–19 **III Jaen Conference on Approximation, Computer Aided Geometric Design, Numerical Methods and Applications**, Ubeda, Jaen, Spain.
Description: The conference is a new activity of the Jaen Approximation Project. It has organized ten editions of the Ubeda Meeting on Approximation and two editions of the Jaen Conference on Approximation. It has also issued the *Jaen Journal on Approximation* since 2009. The objective of these conferences is to provide a useful and nice forum for researchers in the subjects to meet and discuss.
Speakers: The Conference will feature seven invited speakers (Andrei Martínez Finkelshtein, Erik Koelink, Jzsef Szabados, Larry L. Schumaker, Manfred v. Golitschek, Marie-Laurence Mazure, Martin Buhmann). We invite you to contribute a talk or a poster. We provide the possibility of organizing mini-symposia on subjects of current interest.
Information: `http://jja.ujaen.es`.

* 16–19 **Mixed Integer Programming 2012**, University of California, Davis, California.
Description: The upcoming workshop in Mixed Integer Programming (MIP 2012) will be the ninth in a series of annual workshops held in North America designed to bring the integer programming community together to discuss very recent developments in the field. The workshop is especially focused on providing opportunities for junior researchers to present their most recent work. The workshop series consists of a single track of invited talks.
Confirmed speakers: Shabbir Ahmed, Gennadiy Averkov, Sam Burer, Philipp Christophel, Jesús A. DeLoera, Alberto Del Pia, Friedrich Eisenbrand, Ricardo Fukasawa, Vineet Goyal, Marcos Goycoolea, Yongpei Guan, Volker Kaibel, Kiavash Kianfar, Mustafa Kilinc, Fatma Kilinc-Karzan, David Morton, Ted Ralphs, Edward Rothberg, Siqian Shen, Renata Sotirov, Dan Steffy, Alejandro Toriello, Christian Wagner.
Aim: The workshop is designed to provide ample time for discussion and interaction between the participants, as one of its aims is to facilitate research collaboration.
Information: `http://www.math.ucdavis.edu/mip2012`.

16–20 **Applications of Graph Spectra in Computer Science**, Centre de Recerca Matemàtica, Bellaterra, Barcelona.

* 16–20 **A Conference on Partial Differential Equations Analytic and Geometric Aspects, in honor of Michael Taylor's 65th Birthday**, University of North Carolina, Chapel Hill, North Carolina.
Description: The aim of the Conference is to bring together experts in the fields of microlocal and geometric analysis, which dramatically impact the study of partial differential equations, geometry, topology and spectral theory. The Conference will reflect the fundamental contributions to the analysis of partial differential equations made by Michael Taylor throughout his career, in particular helping to develop the topics of microlocal and geometric analysis in the study thereof. Such techniques have led to a surge in activity

relating to dispersive equations on manifolds, spectral theory and mathematical physics, touching such areas as general relativity, fluid mechanics, geometric optics and inverse problems.
Information: http://www.math.psu.edu/mazzucat/uncconf2012/index.html.

16–21 **West Coast Algebraic Topology Summer School**, Stanford University, Stanford, California. (Apr. 2012, p. 591)

16–December 21 **Topological Dynamics in the Physical and Biological Sciences**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (April 2011, p. 629)

16–18 **SING8: 8th Spain-Italy-Netherlands Meeting on Game Theory**, Institute of Economics, Hungarian Academy of Sciences and Corvinus University, Budapest, Hungary. (Nov. 2011, p. 1495)

16–20 **CRM Conference on Applications of Graph Spectra in Computer Science**, CRM, Barcelona, Spain. (Mar. 2012, p. 454)

16–20 **23rd International Workshop on Operator Theory and its Applications (IWOTA 2012)**, University of New South Wales, Sydney, Australia. (May 2012, p. 715)

16–28 **XV Summer Diffiety School**, Pomorski Park Naukowo-Technologiczny, Gdynia, Poland. (May 2012, p. 715)

17–27 **The 5th Mathematical Society of Japan Seasonal Institute, 2012 International Summer School and Conference on Schubert Calculus**, Osaka City University, Osaka, Japan. (Jan. 2012, p. 105)

18–August 1 **Summer School on Discrete Morse Theory and Commutative Algebra**, Institut Mittag-Leffler, Stockholm, Sweden. (May 2012, p. 715)

* 19–22 **XX Oporto Meeting on Geometry, Topology and Physics**, Mathematics Building, Faculty of Sciences, University of Oporto, Oporto, Portugal.
Description: The main theme of this year's special anniversary meeting will be Noncommutative Geometry and Conformal Field Theory with mini-courses given by our main speakers: Jurgen Fuchs (Karlstad), Characters and Coends in Conformal Field Theory; Terry Gannon (Alberta); TBA Yasuyuki Kawahigashi (Tokyo), Superconformal Field Theory and Operator Algebras; Roberto Longo (Rome), Operator Algebras and Boundary Quantum Field Theory. The talks are at the advanced graduate or postdoctoral level. Interested participants are invited to submit relevant titles and abstracts.
Organizers: David Evans (Cardiff) Chairman, Miguel Costa (Oporto), Carlos Herdeiro (Aveiro), Marco Mackaay (Algarve), Jose Mourao (IST, Lisbon), Roger Picken (IST, Lisbon), Paulo Pinto (IST, Lisbon), Joao Nuno Tavares (Oporto).
Registration deadline: April 30, 2012, Recently Oporto was awarded the prestigious title of Best European Destination 2012 by European Consumers Choice.
Information: http://cmup.fc.up.pt/cmup/omgtp/2012/.

* 21–22 **Differential Equations Workshop (DEW)**, Fairfield Inn & Suites, San Jose Airport (near Santa Clara University), 1755 North First Street, San Jose, California.
Description: Too many of today's seminars are too theoretical and quite frankly too academic. If you're one of today's active problem solvers, you just do! So, just DEW! DEW or Differential Equations Workshop gives you hands-on, very practical and quick analytical methods and ways for solving linear and nonlinear ordinary differential equations (ODEs), partial differential equations (PDEs), and integral equations. We find general, particular and other kinds of solutions to differential equations such as Black-Scholes, Vlasov-Maxwell, Poisson's, Schrödinger's, Ricci, KdV, and many more as well as variations of these encountered in practice and research. This series of workshops is for bio-engineers, chemists/chemical engineers, engineers in general, CFAs, computer engineers/scientists, financial engineers, options analysts/traders, physicists, applied mathematicians and others who must or are expected to deal with these and other differential equations in their practice and/or research.
Information: http://deworkshop.org.

22–27 **Vibration and structural acoustics measurement and analysis**, Universidade do Porto, Porto, Portugal. (Oct. 2011, p. 1325)

23–27 **Algebraic topology: applications and new directions, Stanford Symposium 2012**, Stanford University, Palo Alto, California. (Oct. 2011, p. 1325)

* 23–27 **Functional Itó Calculus and Malliavin Calculus for Lévy Processes**, Centre de Recerca Matemàtica, Bellaterra, Barcelona.
Description: The first Barcelona Summer School on Stochastic Analysis is mainly addressed to Ph.D. students and to young researchers in Stochastic Analysis. The objective is to give an overview of the current state of Malliavin calculus for Lévy processes and to introduce the newly developed functional Itô calculus. A comparison between the two approaches with applications to financial mathematics will be presented.
Information: http://www.crm.cat/acstochasticanalysis.

* 23–27 **Spectral Invariants on Non-compact and Singular Spaces**, Centre de Recherches Mathématiques, Montréal, Canada.
Description: The development and use of spectral invariants on non-compact and singular spaces is a very active area of research with many promising avenues. The purpose of the workshop is to stimulate the exchange of ideas between experts dealing with spectral invariants in different, but related geometric contexts with the hope this will lead to new discoveries.
Information: http://www.crm.math.ca/2012/Invariants12/index_e.php.

* 23–27 **Strathmore University International Mathematics Research Meeting**, Strathmore University, Nairobi, Kenya.
Description: The meeting will focus on five areas of mathematics research and application, namely pure mathematics, mathematical biology, mathematical finance, statistical modeling, and mathematics education. It will be organized into five 4-day parallel workshops of invited and contributed talks in each of the areas of focus listed above. Each workshop will include graduate students and postdoctoral fellows, and a poster session. The meeting will be crowned on the last day with a one-day symposium that will include plenary talks by leading researchers in each of the listed areas, and an expert panel to discuss current problems and future directions in mathematics research and applications in Africa. The symposium will be open to participants from industry.
Information: http://www.strathmore.edu/maths.

23–August 3 **Model Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2011, p. 1618)

23–August 3 **Poisson 2012: Poisson Geometry in Mathematics and Physics (Summer School and Conference)**, Utrecht University, Utrecht, The Netherlands. (Mar. 2012, p. 454)

23–August 17 **Spectral Theory of Relativistic Operators**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Sept. 2011, p. 1187)

* 25 **PIMS Marsden Memorial Lecture Series: Richard Montgomery presents "An Octahedral Gem Hidden in Newton's Three Body Problem"**, Fields Institute, 222 College Toronto, Ontario M5T 3J1, Canada.
Description: The three-body problem of Newton has singularities and symmetries. Singularities arise from collisions. McGehee [1974] showed how to eliminate the triple collision singularity through a blow-up procedure. Levi-Civita [1921] showed how to eliminate the binary collision singularities through a regularization procedure. Lagrange [1772], followed by a host of others, showed how to eliminate symmetries through a reduction procedure. Richard Montgomery will sketch how a systematic and democratic application of all

three procedures leads to a regularized reduced three body problem which consists of a complete but non-Hamiltonian vector field on a manifold with boundary.
Information: `http://www.pims.math.ca/scientific-event/120725-pmmlaoghntbp`.

* 25–28 **27th Annual Summer Topology and its Applications Conference**, Minnesota State University, Mankato, Minneosta.
Description: The 27th Summer Topology and its Applications Conference (SumTopo 2102) is being organized by Minnesota State University and will be held July 25-28 in Mankato, MN. It will feature 8-10 plenary speakers and 2 workshop presenters as well as special sessions in asymmetric topology, continuum theory, dynamical systems, geometric topology, and set-theoretic topology. The event is dedicated to Ralph Kopperman on the occasion of his 70th birthday.
Information: `http://cset.mnsu.edu/mathstat/sumtopo/`.

27–August 5 **Summer School and Workshop on Cohomology and Support in Representation Theory and Related Topics**, University of Washington, Seattle, Washington. (Mar. 2012, p. 454)

* 29–August 2 **AIM Workshop: Frame theory intersects geometry**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to outstanding problems that are in the intersection of frame theory and geometry.
Information: `http://www.aimath.org/ARCC/workshops/frametheorygeom.html`.

29–August 3 **XVIII EBT: 18th Brazilian Topology Meeting**, Circuito das Aguas, in the State of Sao Paulo, Brazil. (Apr. 2012, p. 591)

30–August 3 **The 24th International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC.12)**, Nagoya University, Nagoya, Japan. (Nov. 2011, p. 1495)

30–August 3 **Iwasawa 2012**, Heidelberg University, Heidelberg, Germany. (Feb. 2011, p. 336)

30–August 3 **Variational Methods for Evolving Objects**, Room 3-309, Faculty of Science Building #3, Hokkaido University, Sapporo, Japan. (Mar. 2012, p. 454)

30–August 11 **Workshop and Conference on Holomorphic Curves and Low-Dimensional Topology**, Stanford University, Palo Alto, California. (Jan. 2012, p. 105)

August 2012

1–15 **Artificial General Intelligence Summer School 2012**, Reykjavik University, Reykjavik, Iceland. (May 2012, p. 715)

* 3–5 **2012 The Fourth IEEE International Symposium on IT in Medicine and Education Call for Papers: IEEE (ITME 2012)**, Hokkaido, Japan.
Description: You are invited to submit a full paper for review before May 1, 2012. All papers accepted will be included in IEEE Xplore and indexed by Ei Compendex and ISTP. Prospective authors are encouraged to submit a full paper for review before May 1, 2012, in PDF or Word format. Each paper should be written in English. Regular papers are allowed to 5 pages. Extra pages will incur additional charges. Please submit your papers by using the online submission system.
Information: `http://www.itme.sdnu.edu.cn`; `http://itme.xmu.edu.cn/login.asp`; email: `itme@vip.163.com`.

6–8 **The Sixth Global Conference on Power Control and Optimization PCO 2012**, Monte Carlo Resort and Casino, Las Vegas, Nevada. (Oct. 2011, p. 1187)

6–10 **Geometric Structures and Representation Varieties NSF Research Network Senior Retreat**, University of Illinois Urbana-Champaign, Urbana, Illinois. (Apr. 2012, p. 591)

6–10 **ICERM Topical Workshop: Bridging Scales in Computational Polymer Chemistry**, Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island. (Apr. 2012, p. 591)

6–11 **XVII International Congress on Mathematical Physics (ICMP12)**, Aalborg Kongress og Kultur Center, Europa Plads 4, 9000 Aalborg, Denmark. (Aug. 2011, p. 1013)

8–10 **21st USENIX Security Symposium (USENIX Security '12)**, Hyatt Regency Bellevue in Bellevue, Washington. (Mar. 2012, p. 454)

* 13–14 **International Conference on E-Education & Learning Technologies (ICEELT 2012)**, Hotel Changi Village, Singapore.
Description: (ICEELT 2012) is designed to provide a common platform to the experts and delegates to share their experiences, research ideas and discuss various related issues and challenges.
Information: `http://www.iceelt.com`.

13–17 **2012 CBMS-NSF Conference: Mathematics of the Social and Behavioral Sciences**, West Chester University, West Chester, Pennsylvania. (Apr. 2012, p. 592)

13–26 **Meeting the Challenges of High Dimension: Statistical Methodology, Theory, and Applications**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Jan. 2012, p. 105)

* 15–16 **International Conference on Information Integration and Computing Applications (ICIICA 2012)**, Singapore, Singapore.
Description: The ICIICA 2012 mission is to provide an effective and established international forum for discussion and dissemination of recent advances and innovations in use of technology in education.
Information: `http://www.iciica.com`.

* 16–19 **Set Optimization Meets Finance–International Mini-Conference on Set-Valued Variational Analysis and Optimization with Applications in Finance**, Lutherstadt Wittenberg, Germany.
Description: The mini-conference Set Optimization Meets Finance is devoted to set-valued optimization, its applications in finance (e.g. markets with transaction costs) and the underlying convex and variational analysis for vector- and set-valued functions. It is scheduled just before and closely located to the 21st International Symposium on Mathematical Programming (ISMP).
Information: `http://www.set-optimization.org`.

* 18 **National Seminar on Mathematical Modeling in Science and Technology**, The Technological Institute of Textile & Sciences, Birla Colony, Bhiwani-127021, India.
Description: Mathematical Modeling is a full-blown tool used in all sciences and discipline. Applied Mathematics is concerned with mathematical methods that are used in science, engineering, business, medicine and industry. Mathematics is an integral part of most branches of engineering. India is celebrating 2012 as "National Mathematical Year" marking the 125th birth anniversary of eminent mathematician S. Ramanujan. The seminar will comprise lectures by eminent scientists and academicians. The conference will include technical and poster sessions covering the given topic.
Information: `http://titsbhiwani.ac.in`.

20–24 **AIM Workshop: Invariants in convex geometry and Banach space theory**, American Institute of Mathematics, Palo Alto, California. (Aug. 2011, p. 1013)

20–24 **Spectral Theory and Differential Equations**, B. Verkin Institute for Low Temperature Physics & Engineering, V. Karazin Kharkiv National University, Kharkiv, Ukraine. (Dec. 2011, p. 1618)

20–24 **The Fourth Geometry Meeting dedicated to the centenary of A. D. Alexandrov**, The Euler International Mathematical Institute, Saint-Petersburg, Russia.

20–December 21 **Cluster Algebras Program**, Mathematical Sciences Research Institute, Berkeley, California. (Oct. 2011, p. 1325)

* 21–24 **Infinite Dimensional Lie Theory: Algebra, Geometry and Combinatorics**, Centre de Recherches Mathématiques, Montréal, Canada.
Description: Infinite dimensional Lie algebras have been studied in physical contexts for over 60 years. The advent of Kac-Moody theory in the late 1960s and its connections with conformal geometry has led to an explosion of mathematical interest in the field, which now includes applications and links to modular forms, singularity theory, soliton equations, Galois cohomology, crystal bases, integrable systems, and quantum groups, among many other topics.
Information: `http://www.crm.umontreal.ca/2012/Infinite12/index_e.php`.

22–24 **Berlin PUM Workshop 2012**, Humboldt University, Berlin, Germany. (May 2012, p. 716)

22–24 **Connections for Women: Discrete Lattice Models in Mathematics, Physics, and Computing**, Mathematical Sciences Research Institute, Berkeley, California. (Dec. 2011, p. 1619)

22–25 **The 20th Conference on Applied and Industrial Mathematics CAIM 2012**, Tiraspol State University, Chisinau, Republic of Moldova. (May 2012, p. 716)

23–26 **International Congress in Honour of Professor H. M. Srivastava**, The Auditorium at the Campus of Uludag University, Bursa, Turkey. (May 2012, p. 716)

* 24–30 **XIème Colloque Franco-Roumain de Mathématiques Appliquées**, Faculty of Mathematics and Computer Science of the University of Bucharest, Str. Academiei nr. 14, 010014, Bucharest, Romania.
Description: The meeting is organized by the Faculty of Mathematics and Computer Science of the University of Bucharest and Simion Stoilow Institute of Mathematics of the Romanian Academy. Special sessions: Geophysics and fluid mechanics; mathematical and numerical models in solid mechanics; statistics and fractional processes; numerical probabilities; random matrices and free probabilities; spectral methods and applications in mathematical physics; analysis and analysis of partial differential equations; control of partial differential equations; image treatment.
Information: `http://fmi.unibuc.ro/CFR2012/index.html`.

* 25–27 **The 37th Sapporo Symposium on Partial Differential Equations**, Room 203, Faculty of Science Building #5, Hokkaido University, Sapporo, Hokkaido, Japan.
Description: The Sapporo Symposium on Partial Differential Equations has been held annually to present the latest developments on partial differential equations (PDE).
Aim: The aim of the symposium is to help boost interaction and in-depth discussion among researchers working on different branches of PDE.
Financial Support: Limited amount of financial support for local expenses is available to non-resident visitors who are interested in the meeting. Please make inquiry to `cri@math.sci.hokudai.ac.jp` by April 30, 2012.
Organizers: S. Jimbo (Hokkaido University), H. Takaoka (Hokkaido University). Program Committee: S. Jimbo (Hokkaido University), H. Takaoka (Hokkaido University), Y. Tonegawa (Hokkaido University), T. Sakajo (Hokkaido University), G. Nakamura (Hokkaido University), Y. Giga (The University of Tokyo), T. Ozawa (Waseda University), K. Tsutaya (Hirosaki University).
Contact: email: `cri@math.sci.hokudai.ac.jp`.
Information: `http://www.math.sci.hokudai.ac.jp/sympo/sapporo/program120825_en.html`.

* 27–30 **The 43rd Annual Iranian Mathematics Conference**, University of Tabriz, Tabriz, Iran.
Description: We are the host of the 43rd Annual Iranian Mathematics Conference at the University of Tabriz, Tabriz, Iran. The goal of this conference is to survey recent advances on a wide range of mathematics. The conference will provide a forum for mathematicians and scientists worldwide to present their latest results and a means to discuss their recent research with each other.
Information: `http://imc43.tabrizu.ac.ir`.

27–30 **PADGE 2012: Conference on Pure and Applied Differential Geometry**, Department of Mathematics, K. U. Leuven, Belgium. (Mar. 2012, p. 455)

27–31 **8th International Symposium on Geometric Function Theory and Applications**, Ohrid, Republic of Macedonia. (Feb. 2012, p. 340)

* 27–31 **Geometry and Algebra of PDEs 2012**, University of Tromso, Tromso, Norway.
Description: This meeting aims to discuss recent advances in the area of differential equations and relative areas of geometry and algebra. We plan to bring together researchers with various scientific interests and to stimulate mathematical discussions and productive cooperation.
Topics: Geometric methods of investigation of PDEs and finding its solutions; Compatibility, solvability and integrability of differential equations; Differential-geometric structures and equivalence problem; Lie groups, algebraic pseudogroups, representations and differential invariants; Homological and symbolic methods in PDEs and super-symmetric theories; Applications to mathematical physics and numerical methods.
Information: `http://www.math.uit.no/pdes2012/index.html`.

27–September 7 **Joint Introductory Workshop: Cluster Algebras and Commutative Algebra**, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2011, p. 1013)

* 28–September 1 **The 8th William Rowan Hamilton Geometry and Topology Conference**, The Hamilton Mathematics Institute, Trinity College, Dublin, Ireland.
Description: The workshop will consist of a two-day mini-course, August 28-29, followed by a three-day lecture series, Thursday, August 30st through Saturday, September 1th, 2012.
Topic: For the workshop is measure and asymptotics in group theory and low-dimensional geometry.
Funding and Sponsors: The workshop is co-sponsored by Boston College, the HMI and the NSF. Funding is available for graduate students and junior researchers wishing to attend the mini-course and workshop. A limited amount of funding is also available for both junior and senior researchers wishing to attend the workshop. If you would like to request travel support you can do so on the workshop website at `http://www.hamilton.tcd.ie/events/gt/gt2012/register.htm`.
Information: `http://www.hamilton.tcd.ie/events/gt/gt2012.htm`.

29–September 2 **Semigroups and Applications**, Department of Mathematics, Uppsala University, Uppsala, Sweden. (Mar. 2012, p. 455)

* 31–September 1 **Second workshop on algebra and its applications**, University of Mohaghegh Ardabili, Ardabil, Iran.
Description: We would like to invite you to participate, present a paper in this workshop, whose main topics are all areas of algebra and applications of algebra. The workshop will take place in the Faculty of Mathematical Sciences of University of Mohaghegh Ardabili, which is located in the main campus of the University. After the conference, selected papers will be published in the special issue of *Journal of Hyperstructures*. There will also be a summer school with the title "Combinatorial Commutative Algebra and its Applications" after the workshop for roughly two weeks. For more information please refer to Summer School.
Information: `http://www.uma.ac.ir/index.php?sid=25&slc_lang=en`.

September 2012

1–3 **13th International Pure Mathematics Conference**, Quaid-i-Azam University, Islamabad, Pakistan. (Apr. 2012, p. 592)

* 3–5 **Workshop on Combinatorics**, University of Lisbon, Lisbon, Portugal.
Description: The "Workshop on Combinatorics" will be held at IIIUL, http://www.ciul.ul.pt/index_en.htm, in Lisbon, Portugal, from September 3 to 5, 2012. This meeting consists of two mini-courses, supplemented by contributed talks and posters and it is mostly addressed to graduate students and researchers interested in exploring and extending their knowledge on topics related to Matroid Theory and Triangulations of Polytopes.
Invited Speakers: Henry Crapo: Topics on Matroid Theory, Francisco Santos: Triangulations of Polytopes.
Registration fees: Registration until June 30, 2012: 50 euro (students: 40 euro). Registration after June 30, 2012: 70 euro (students: 60 euro).
Deadline: For submission of contributed talks: July 15, 2012.
Information: If you have any further questions, please contact us at http://worklis2012.fc.ul.pt.

3–7 **Dynamical Systems: 100 years after Poincaré**, University of Oviedo, Gijn, Spain. (May 2012, p. 716)

3–7 **International Conference on Differential-Difference Equations and Special Functions**, University of Patras, Patras, Greece. (Aug. 2011, p. 1014)

3–7 **1st International Conference on Mathematical Sciences and Applications (IECMSA)**, Prishtine University, Prishtine, Kosovo. (Mar. 2012, p. 455)

* 3–7 **Third Iberoamerican Meeting on Geometry, Mechanics and Control**, University of Salamanca, Salamanca, Spain.
Description: The Iberoamerican Meetings on Geometry, Mechanics and Control are held every two years and are intended for young and senior researchers with interest in differential geometry, mechanics and control theory. These subject matters are highly topical, interacting in a common area of Mathematics, Physics, Mechanics, and Engineering.
Plenary speakers: Henrique Bursztyn (IMPA, Brazil), Marco Castrillón (UCM and ICMAT, Spain), Gonzalo Contreras (CIMAT, Mexico), Rui L. Fernandes (IST, Portugal), Mark J. Gotay (PIMS, Canada), Janusz Grabowski (IMPAN, Poland), Darryl Holm (Imperial College, United Kingdom), Sonia Martínez (UCSD, USA)–to be confirmed, Juan-Pablo Ortega (CNRS, France), Edith Padrón (ULL, Spain), Narcisco Roman-Roy (UPC, Spain) and Carlos Tomei (PUC-Rio, Brazil). There are also 15 invited talks and a poster session.
Information: http://fundacion.usal.es/3imgmc.

3–8 **XVII Geometrical Seminar**, Zlatibor, Serbia Hotel "Ratko Mitrović" (Apr. 2012, p. 592)

4–9 **MADEA 2012 International Conference on Mathematical Analysis, Differential Equations and Their Applications**, Mersin University, Mersin, Turkey. (Apr. 2012, p. 592)

5–7 **Complex patterns in wave functions: Drums, graphs, and disorder**, Kavli Royal Society International Centre, Chicheley Hall, Chicheley, Newport Pagnell, Buckinghamshire MK16 9JJ, UK. (May 2012, p. 716)

5–7 **ICERM Semester Program: Computational Challenges in Probability**, ICERM, Providence, Rhode Island. (Jan. 2012, p. 106)

5–9 **Lie Algebras and Applications**, Uppsala University, Uppsala, Sweden. (Mar. 2012, p. 455)

7–12 **Workshop on Stochastic and PDE Methods in Financial Mathematics**, Yerevan State University, Yerevan, Armenia. (May 2012, p. 716)

* 9–12 **2012 Federated Conference on Computer Science and Information Systems (FedCSIS)**, Wroclaw, Poland.
Description: We would like to cordially invite you to consider contributing a paper to the FedCSIS 2012.
Information: For all matters relating to this conference please visit: http://www.fedcsis.org.

* 9–14 **26th Large Installation System Administration Conference (LISA '12)**, Sheraton San Diego Hotel and Marina, 1380 Harbor Island Drive, San Diego, California.
Description: Join us for the most in-depth, practical system administration training available. The annual LISA conference is the meeting place of choice for system and network administrators and engineers; it is the crossroads of Web operations, DevOps, enterprise computing, educational computing, and research computing. The conference serves as a venue for a lively, diverse, and rich mix of technologists of all specialties and levels of expertise. LISA is the place to teach and learn new skills, debate current issues, and meet industry gurus, colleagues, and friends. The 6-day event offers training by industry leaders; invited talks; the latest research through refereed papers, practice and experience reports, posters, and workshops; answers to your toughest questions; and that all-important face-to-face time with experts in the community.
Information: http://static.usenix.org/events/lisa12/index.html.

10–12 **3rd IMA Conference on Numerical Linear Algebra and Optimisation**, University of Birmingham, United Kingdom. (Feb. 2012, p. 340)

10–14 **6th International Conference on Stochastic Analysis and its Applications**, The Mathematical Research and Conference Center of the Institute of Mathematics of the Polish Academy of Sciences, Bedlewo, Poland. (May 2012, p. 716)

* 10–15 **International Conference on Nonlinear PDE & Free Surface and Interface Problems Workshop**, Oxford, United Kingdom.
Description: The main objective of the conference is to bring together scientists with interests in the analysis of nonlinear partial differential equations (PDE) and their applications to present recent developments and explore new connections between nonlinear PDE and other areas in mathematics and related fields in the sciences. Held at the Maths Institute, Oxford, there will be 12 lectures and 8 mini-symposium sessions over 4 days (Monday to Thursday). The workshop will be held at St. Anne's College, Oxford, and consist of three sessions, each three hours in duration (Friday and Saturday).
Theme: The theoretical and numerical aspects of nonlinear hyperbolic and dispersive free boundary and interface problems.
Speakers: Luigi Ambrosio, Constantine Dafermos, Isabelle Gallagher, Martin Hairer, Fanghua Lin, Pierre-Louis Lions, Felix Otto, Frank Pacard, Richard Schoen, Gigliola Staffilani, Andrew Majda, Eitan Tadmor.
Information: http://www.maths.ox.ac.uk/groups/oxpde/events.

10–December 14 **Materials Defects: Mathematics, Computation and Engineering**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Oct. 2011, p. 1325)

11–14 **Workshop on Operator Theory and Operator Algebras 2012–WOAT 2012**, Instituto Superior Técnico, Universidade Técnica de Lisboa, Portugal. (May 2012, p. 717)

12–14 **Algebra Geometry and Mathematical Physics – 8th workshop – Brno 2012**, Faculty of Mechanical Engineering, Brno University of Technology, Czech Republic. (Mar. 2012, p. 455)

17–19 **IMA Conference on Mathematics of Medical Devices and Surgical Procedures**, University of London, United Kingdom. (Apr. 2012, p. 592)

17–21 **ICERM Workshop: Bayesian Nonparametrics**, ICERM, Providence, Rhode Island. (Jan. 2012, p. 106)

17–21 **Summer School on New Trends in Harmonic Analysis, Fractional Operator Theory, and Image Analysis**, Inzell, Germany. (Apr. 2012, p. 592)

* 18–21 **Dynamical systems on random graphs**, Cantabria, Spain.
Description: This conference aims to bring together mathematical physicists, experts in dynamical systems and neuroscientists, all with research interests in graph theory, networks, complex systems, continuous dynamical systems on random graphs and heterogeneous media, and their applications. The conference will comprise 4 sessions, each devoted to a different topic: random graphs and spectral theory, point models (e.g., neural networks or coupled ODEs), spatial models (e.g., quantum graphs or detailed neuron models) and complex networks. We plan to allow a significant amount of time for interdisciplinary discussion.
Speakers: Gustavo Deco (Universitat PompeuFabra, Barcelona, Spain), Krešimir Josić (University of Houston, USA), Bojan Mohar (Simon Fraser University, Canada), Uzy Smilansky (Weizmann Institute, Israel, and Cardiff University, UK).
Information: `http://www.uni-ulm.de/dsrg2012`.

* 19–21 **School on Singular Analysis**, University of Hanover, Germany.
Description: The school is aimed at doctoral students and offers introductory and advanced courses on the analysis of singular spaces. What the courses have in common is the use of pseudodifferential techniques to approach problems ranging from spectral theory to global analysis and nonlinear PDEs. Partial support for doctoral students is available.
Information: `http://www.math-conf.uni-hannover.de/ana12/`.

19–25 **ICNAAM2012: 3rd Symposium on Semigroups of Linear Operators and Applications**, Kypriotis Hotels and Conference Center, Kos, Greece. (Mar. 2012, p. 455)

20–22 **Lie and Klein; the Erlangen program and its impact on mathematics and physics**, Institut de Recherche Mathématiqué Avancé, University of Strasbourg, France. (Nov. 2011, p. 1495)

20–October 20 **ERC research period on Diophantine geometry**, Centro di Ricerca Matematica "Ennio De Giorgi", Scuola Normale Superiore, Piazza dei Cavalieri 3, 56100 Pisa, Italy. (May 2012, p. 717)

22–23 **AMS Eastern Section Meeting**, Rochester Institute of Technology, Rochester, New York. (Sept. 2011, p. 1187)

24–27 **Eighth National Congress on Finite Element Method**, School of Mathematical Sciences, Nankai University, 94 Weijin Road, Nankai District, Tianjin 300071, China. (May 2012, p. 717)

24–27 **56th Annual Meeting of the Australian Mathematical Society**, University of Ballarat, Mt. Helen Campus, Victoria, Australia. (Apr. 2012, p. 592)

24–28 **Mathematics and Physics of Moduli Spaces**, Heidelberg University, Germany. (May 2012, p. 717)

* 26–29 **Tensors and Their Geometry in High Dimensions**, University of California, Berkeley, Berkeley, California.
Description: Berkeley RTG on Representation Theory, Geometry, and Combinatorics will host a workshop on "Tensors and Their Geometry in High Dimensions", featuring Andrew Snowden, Jan Draisma, and Giorgio Ottaviani giving mini-courses. There will also be opportunities for some of the other participants to give related talks.
Information: For details visit: `http://math.berkeley.edu/~oeding/RTG/index.html`. There are no registration fees; however participants are kindly asked to register online. There may be some funding for young researchers.

October 2012

1–5 **New trends in Dynamical Systems**, Salou, Catalonia, Spain. (Feb. 2012, p. 340)

1–April 30 **Semester on Curves, Codes, Cryptography**, Sabanci University, Istanbul, Turkey. (Feb. 2012, p. 340)

2–4 **SIAM Conference on Mathematics for Industry: Challenges and Frontiers (MI12)**, The Curtis, A Doubletree by Hilton, Denver, Colorado. (Aug. 2011, p. 1014)

3–6 **International Conference on Applied and Computational Mathematics (ICACM)**, Middle East Technical University (METU), Ankara, Turkey. (Oct. 2011, p. 1325)

* 5–7 **Yamabe Memorial Symposium (50th anniversary)**, University of Minnesota, Minneapolis, Minnesota.
Theme: "Geometry and Analysis".
Speakers: The 8 confirmed speakers are: Huai-Dong Cao, Jean-Pierre Demailly, Benson Farb, Robert Hardt, Misha Kapovich, Conan Leung, Natasa Sesum, and Ben Weinkove.
Financial support: Has been provided by NSF to defray expenses for a number of participants. Highest preference will be given to younger scientists (grad students, postdocs, young faculty), but some funds will be reserved for more senior mathematicians. Applications from women and underrepresented minorities are especially encouraged. For more details and an application form, please consult our website.
Information: `http://www.math.umn.edu/yamabe/2012/`.

8–12 **ICERM Workshop: Uncertainty Quantification**, ICERM, Providence, Rhode Island. (Jan. 2012, p. 106)

9–11 **Algerian-Turkish International Days on Mathematics 2012, "ATIM'2012"**, Badji Mokhtar Annaba University, Annaba, Algeria. (Apr. 2012, p. 593)

13–14 **AMS Southeastern Section Meeting**, Tulane University, New Orleans, Louisiana. (Aug. 2011, p. 1014)

* 18 **2nd Annual Symposium on Large-Scale Inference**, AFI Silver Theatre and Cultural Center, Silver Spring, Maryland.
Description: Social & Scientific Systems, Inc. hosts its 2nd Annual Symposium on Large-Scale Inference.
Keynote Speaker: Dr. Carl N. Morris, Professor of Statistics, Harvard University.
Information: Please RSVP to: `http://LargeData@s-3.com`.

* 18–21 **First International Conference on Analysis and Applied Mathematics**, Gumushane University, Gumushane, Turkey.
Description: We are proud to announce the First International Conference on Analysis and Applied Mathematics. The aim of this conference is to bring together mathematicians working in the area of analysis and applied mathematics to share new trends of applications of math. In mathematics, the developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why we plan to found a journal to provide a forum for researchers and scientists to communicate their recent developments and to present their original results in various fields of analysis and applied mathematics.
Information: `http://icaam2012.gumushane.edu.tr`.

20–21 **AMS Central Section Meeting**, University of Akron, Akron, Ohio. (Aug. 2011, p. 1014)

22–26 **AIM Workshop: Lipschitz metric on Teichmüller space**, American Institute of Mathematics, Palo Alto, California. (May 2012, p. 717)

24–26 **International Conference in Modeling Health Advances 2012**, San Francisco, California. (Mar. 2012, p. 456)

24–26 **International Conference in Number Theory and Applications 2012 (ICNA 2012)**, Department of Mathematics, Faculty of Science, Kasetsart University, Bangkok, Thailand. (Apr. 2012, p. 593)

27–28 **AMS Western Section Meeting**, University of Arizona, Tucson, Arizona. (Aug. 2011, p. 1014)

29–31 **The 17th edition of the Symposium on Solid and Physical Modeling**, University of Burgundy, Dijon, France. (May 2012, p. 717)

29–November 2 **Cluster Algebras in Combinatorics, Algebra, and Geometry**, Mathematical Sciences Research Institute, Berkeley, California. (May 2012, p. 717)

29–November 2 **ICERM Workshop: Monte Carlo Methods in the Physical and Biological Sciences**, ICERM, Providence, Rhode Island. (Jan. 2012, p. 106)

November 2012

1–2 **Central and Eastern European Software Engineering Conference in Russia 2012**, Digital October Center, Moscow, Russia. (Apr. 2012, p. 593)

1–3 **The 13th International Conference of Mathematics and its Applications ICMA2012**, Department of Mathematics, "Politehnica" University of Timisoara, City of Timisoara, Romania. (May 2012, p. 717)

* 7–9 **International Conference on Advancement of Science and Technology (iCAST 2012)**, Kuantan, Pahang, Malaysia.
Description: iCAST2012 aims to provide an international forum for researchers to present and discuss recent advances and new findings in mathematics and its applications. Moreover, it should accelerate the growth of mathematics and its applications and their benefits to the community at large. The theme of the conference is Contemporary Mathematics, Mathematical Physics and Their Applications.
Information: `http://iium.edu.my/icast/2012/`.

9–10 **Blackwell Tapia Conference 2012**, Institute for Computational and Experimental Research in Mathematics (ICERM), 121 South Main Street, Providence, Rhode Island. (Feb. 2012, p. 340)

* 12–March 3, 2013 **A fixed point theorem for Meir-Keeler contractions and its applications to integral equations in ordered modular function spaces**, Semnan University, Semnan, Iran.
Description: The Banach contraction mapping principle is one of the pivotal results of analysis. Generalization of the above principle has been a heavily investigated branch of research. In particular, Meir and Keeler present the following fixed point theorem: Theorem: Let (X, d) be a complete metric space and $T : X \to X$ an operator. Suppose that for every $\epsilon > 0$ there exists $\delta > 0$ such that for $x, y \in X$, $\epsilon < d(x, y) < \epsilon + \delta$, then $d(Tx, Ty) < \epsilon$. Then, T admits a unique fixed point z and for any $x \in X$, the sequence $T^n x$ converges to z. On the other hand, it is well known that one of the standard proofs of Banach's fixed point theorem is based on Cantor's theorem in complete metric spaces. In this paper we present a fixed point theorem of Meir-Keeler type in ordered modular space. As an application we study the existence and uniqueness of a solution for an integral equation of Lipschitz type in a Musielak-Orlicz space.
Information: `http://Semnan.ac.ir`.

16–18 **Special Functions, Partial Differential Equations and Harmonic Analysis, a conference in honor of Calixto P. Calderón**, Roosevelt University, 425 Wabash Ave, Chicago, Illinois. (May 2012, p. 717)

17 **Info-Metrics and Nonparametric Inference**, University of California Riverside, Riverside, California. (Mar. 2012, p. 456)

* 19–20 **International Seminar on History of Mathematics and Celebration of National Mathematics Year 2012 to Commemorate 125th Birth Year of Srinivasa Ramanujan**, Department of Mathematics & Statistics, Ramjas College, Delhi University, North Campus, Delhi 110007, India.
Focus: The Seminar will cover all aspects of the history of mathematics and, in particular, the following areas: Life and Achievements of Srinivasa Ramanujan; History of Mathematics in Educational Curricula; Mathematics and Indigenous Cultures of the World; Historical Aspects of Modern Mathematics. The academic sessions will consist of invited talks, panel discussions and paper presentations. The Ramjas Seminar shall be followed by an International Conference on History of Mathematics at MD University, Rohtak, on November 21–24, 2012, along with the Annual Meeting of ISHM. Details are available at `http://www.indianshm.com`.
Organizers: Indian Society for History of Mathematics and Ramjas College, Delhi.
Deadline/Registration: Abstracts pertaining to the above are invited by October 1, 2012. Registered participants will be provided with free Conference Kit and meals at the Seminar venue. Shared accommodations for the outstation participants will be arranged in the University guest-houses. Those interested in accommodations in city hotels on payment may request them. Cultural programs and sightseeing tours for the participants could be arranged to nearby tourist places and historical monuments.
Contacts: Dr. Man Mohan, Convenor ISHM 2012, Ramjas College, Delhi, India; email: `manmohan@indianshm.com`; `http://www.indianshm.com`.

* 28–30 **ICERM Workshop: Performance Analysis of Monte Carlo Methods**, ICERM, Providence, Rhode Island.
Description: Monte Carlo methods have become increasingly important in Engineering and the Sciences. These application areas have posed challenges and opportunities in the analysis of modern Monte Carlo algorithms. The workshop's main focus is on: a) the mathematical techniques and aspects that have been key in the analysis of these algorithms, and b) the identification of techniques that are likely to play a role in future analysis.
Information: `http://icerm.brown.edu/sp-f12-w4`.

29–December 1 **2012 Third International Conference on Emerging Applications of Information Technology (EAIT 2012)**, Indian Statistical Institute, Kolkata, India. (May 2012, p. 717)

December 2012

3–7 **Combinatorial Commutative Algebra and Applications**, Mathematical Sciences Research Institute, Berkeley, California. (May 2012, p. 718)

10–14 **AIM Workshop: Log minimal model program for moduli spaces**, American Institute of Mathematics, Palo Alto, California. (May 2012, p. 718)

10–14 **36ACCMCC—The 36th Australasian Conference on Combinatorial Mathematics and Combinatorial Computing**, University of New South Wales, Sydney, Australia. (Feb. 2012, p. 340)

* 10–14 **Reproducibility in Computational and Experimental Mathematics**, Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island.
Description: The purpose of this workshop is to discuss aspects of reproducibility most relevant to the mathematical sciences among researchers from pure and applied mathematics from academics and other settings, together with interested parties from funding agencies, national laboratories, professional societies, and publishers. This will be a working workshop, with relatively few talks and dedicated time for breakout group discussions on the current state of the art and the tools, policies, and infrastructure that are needed to improve the situation. The groups will be charged with developing guides to current best practices and/or white papers on desirable advances.
Information: `http://icerm.brown.edu/tw12-5-rcem`.

* 10–21 **VI-MSS Event: Winter School and Conference on Computational Aspects of Neural Engineering**, Bangalore, India.
Description: We are pleased to announce the first joint IMI-ICERM Winter School on Computational Aspects of Neural Engineering. The course is directed at graduate students, postdoctoral fellows, and

other researchers from the physical sciences (e.g., physics, mathematics, computer science, engineering) and the life sciences (e.g., neuroscience, biology, physiology). The course will offer participants the opportunity to learn about the foundations of neural engineering and brain-computer interfacing, and develop their skills in computational analysis of neural data for the control of external devices.
Topics: Will range from primers on neuroscience, signal processing, and machine learning to brain-computer interfacing based on multi neuronal activity, electrocorticography (ECoG), and electroencephalography (EEG). The course will consist of 3 hours of lectures each morning, followed by a 3-hour MATLAB-based computer laboratory in the afternoon. Participants will pair up for these laboratories, and an effort will be made to pair someone from the life sciences with someone from the physical sciences. All classes and laboratories will be held on the campus of the Indian Institute of Science (IISc). This program is part of the IISc Mathematics Initiative (IMI) at the Indian Institute of Science and the VIMSS program at ICERM.
Information: `http://icerm.brown.edu/vi-mss`.

* 15–16 **2nd International Conference on Mathematical Sciences and Applications**, India International Centre, 40, Max Mueller Marg, Lodhi Estate, New Delhi-110 003, India.
Description: A premier forum for the presentation of new advances and research results in all areas of Mathematical Sciences and Applications. ICMSA-2012 will bring together leading researchers, engineers and scientists in the domain of interest from around the world. Leading mathematicians around the world shall deliver keynote addresses and chair sessions.
Topics: Of interest for submission include, but are not limited to: Algebra, algebraic topology, advanced calculus, advanced numerical methods, artificial neural networks, calculus and trigonometry, complex analysis, computational fluid dynamics, etc.
Directions: Nearest Metro Stations: "Khan Market" & "Jor Bagh" Map: IIC at Google Maps.
Information: `http://www.journalshub.com`.

16–22 **Commutative rings, integer-valued polynomials and polynomial functions**, Graz University of Technology, Graz, Austria. (Nov. 2011, p. 1495)

17–20 **9th IMA International Conference on Mathematics in Signal Processing**, Austin Court, Birmingham, United Kingdom. (Nov. 2011, p. 1495)

17–21 **AIM Workshop: Rational Catalan Combinatorics**, American Institute of Mathematics, Palo Alto, California. (May 2012, p. 718)

17–21 **International Conference on the Theory, Methods and Applications of Nonlinear Equations**, Department of Mathematics, Texas A&M University-Kingsville, Kingsville, Texas. (Feb. 2012, p. 340)

* 21–23 **6th International Conference of IMBIC on "Mathematical Sciences for Advancement of Science and Technology" (MSAST 2012)**, Salt Lake City, Kolkata, West Bengal, India.
Description: The main objective of the conference is to bring specialized topics in mathematics, statistics, computer science, information technology, bioinformatics and closely related interdisciplinary areas to the forefront. Original full papers are invited. All papers are to be screened and accepted papers will be published in the proceedings, except for a few full selective papers of high quality which may be published in the highly acclaimed series of monographs of IMBIC in Volume 2 (2013).
Contact: All correspondence in respect to the conference is to be addressed to Dr. Avishek Adhikari, Convenor MSAST 2012 & Secretary, IMBIC; email: `msast.paper@gmail.com`; `http://www.isical.ac.in/~avishek_r/`.
Information: `http://imbic.org/forthcoming.html`.

22–24 **The International Congress on Science and Technology**, Allahabad, U.P., India. (Apr. 2012, p. 593)

27–30 **Eighth International Triennial Calcutta Symposium on Probability and Statistics**, Department of Statistics, Calcutta University, Kolkata, West Bengal, India. (Nov. 2011, p. 1495)

* 31–January 11 **Recent Advances in Operator Theory and Operator Algebras**, Indian Statistical Institute, Bangalore, India.
Description: Focusing on the general theme of operator theory, operator algebras and function theory. There will be a one-week workshop December 31, 2012–January 5, 2013, followed by a conference January 7, 2013–January 11, 2013. The purpose of the workshop is to bring experts and students as well as researchers together to discuss the most recent developments in certain topics in those areas. The conference will concentrate on recent developments in Operator Theory, Operator Algebras and related fields.
Information: `http://www.isibang.ac.in/~jay/rota.html`.

January 2013

* 1–9 **VI-MSS Event: Workshop and Conference on Limit Theorems in Probability**, Bangalore, India.
Description: Ever since Jakob Bernoulli proved the law of large numbers for Bernoulli random variables in 1713, the subject of limit theorems has been a driving force for the development of probability theory as a whole. The elucidation of different flavours of laws of large number, central limit theorems and laws of iterated logarithm, their extensions to Markov chains or sums of weakly dependent or stationary processes, limit theorems for Banach space valued random variables, etc., have given rise to a rich theory as well as the basic tools for tackling any problem involving randomness. Today, 300 years after the landmark result of Bernoulli, it is fruitful to look back at the way in which the search for limit theorems has shaped the subject. It is also fruitful to consider how the emphasis has evolved over time from simple limit theorems to getting bounds on the rates of convergence or obtaining inequalities, which are of more immediate relevance in applications to nite samples. The current workshop and conference will focus on some of these topics, and also more broadly on issues of current interest in probability theory. The workshop will consist of five short courses on a variety of topics, aimed at the level of graduate students but also of potential interest to researchers in probability and related fields. The conference following the workshop will have lectures on recent developments in various relevant fields of probability.
Information: `http://icerm.brown.edu/vi-mss`.

6–8 **ACM-SIAM Symposium on Discrete Algorithms (SODA13)**, Astor Crowne Plaza Hotel, New Orleans, Louisiana. (Apr. 2012, p. 593)

7–12 **Iwasawa Theory, Representations, and the p-adic Langlands Program**, University of Münster, Münster, Germany. (Jan. 2012, p. 106)

* 14–18 **AIM Workshop: Modeling problems related to our environment**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will bring together mathematicians, graduate students, and industry and public agency representatives to work on mathematical modeling problems related to the planet Earth.
Information: `http://www.aimath.org/ARCC/workshops/modelenvironment.html`.

14–May 24 **Noncommutative Algebraic Geometry and Representation Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Oct. 2011, p. 1325)

24–25 **Connections for Women: Noncommutative Algebraic Geometry and Representation Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2011, p. 1014)

28–February 1 **Introductory Workshop: Noncommutative Algebraic Geometry and Representation Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2011, p. 1014)

28–May 3 **ICERM Semester Program: Automorphic Forms, Combinatorial Representation Theory and Multiple Dirichlet Series**, ICERM, Providence, Rhode Island. (Jan. 2012, p. 106)

February 2013

* 4–8 **AIM Workshop: Stochastics in geophysical fluid dynamics**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the mathematical foundations, physical underpinnings and applications of large scale stochastic models for climate and weather.
Information: `http://www.aimath.org/ARCC/workshops/stochasticfluid.html`.

* 4–8 **The Second Biennial International Group Theory Conference**, Dogus University, Istanbul, Turkey.
Description: We would like to announce herewith the second of the Biennial International Group Theory Conferences hosted every other year by the countries Malaysia, Turkey and Iran successively. The conference aims to bring together leading mathematicians and active researchers working on the theory of groups in order to exchange ideas, present new results and identify the key problems in the field. This series of conferences is considered as an important means to bring together young group theorists and graduate students with the leading experts of the world and help them to get acquainted with relevant and actual theories, problems and methods of their research area.
Information: Contact: `2big@dogus.edu.tr`; `http://istanbulgroup2013.dogus.edu.tr`.

* 11–15 **ICERM Workshop: Sage Days: Multiple Dirichlet Series, Combinatorics, and Representation Theory**, ICERM, Providence, Rhode Island.
Description: This workshop will bring together experienced Sage and Sage-Combinat developers and experts of multiple Dirichlet series and computational algebraic combinatorics. Like every workshop in the Sage Days series, it will welcome whoever wants to discover Sage, learn more about it, or contribute to it. This workshop will focus on Sage training and on the design and planning of new computational features of central interest for the semester, around Weyl groups, Hecke algebras and their representations, crystals, posets, combinatorial data visualization, etc. The workshop will consist of mathematical presentations, presentations on Sage and coding sprints. The mathematical presentations will include talks introducing the relevant mathematics for the entire audience and more advanced talks for interested participants. The Sage presentations will begin with introductory tutorials and progress to more advanced topics, including software development in Sage.
Information: `http://icerm.brown.edu/sp-s13-w1`.

March 2013

* 1–3 **2013 Spring Southeastern Section Meeting**, University of Mississippi, Oxford, Mississippi.
Information: `http://www.ams.org/meetings/sectional/sectional.html`.

* 4–8 **ICERM Workshop: Whittaker Functions, Schubert Calculus and Crystals**, ICERM, Providence, Rhode Island.
Description: Schubert calculus is the modern approach to classical problems in enumerative algebraic geometry, specifically on flag varieties and their many generalizations. Crystals are combinatorial tools based on quantum groups which arise in the study of representations of Lie algebras. Whittaker functions are special functions on Lie groups or p-adic groups, for example $GL(n,F)$, where F might be the real or complex numbers, or a p-adic field. The area of intersection between these three topics is combinatorial representation theory. Common tools such as Demazure operators, the Bruhat partial order, and Macdonald polynomials appear in all three areas. Some connections between these three areas are quite new. This workshop will explore these connections.
Information: `http://icerm.brown.edu/sp-s13-w2`.

11–14 **Interactions between Analysis and Geometry**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Oct. 2011, p. 1325)

* 20–22 **17th International Conference on Discrete Geometry for Computer Imagery (DGCI 2013)**, Escuela Tecnica Superior de Ingenieria Informatica, Univ. de Sevilla, Seville, Spain.
Aim: Of the DGCI conference is to gather researchers in discrete models, discrete geometry and topology, with applications in image analysis and image synthesis.
Topics: Of interest include (but are not limited to): Models for discrete geometry, discrete and combinatorial topology, geometric transforms, discrete shape representation, recognition and analysis, discrete tomography, morphological analysis, discrete modelling and visualization, discrete and combinatorial tools for image segmentation and analysis. Publication of the conference proceedings will be in LNCS by Springer-Verlag.
Submissions: Must be in electronic form (PDF or PS) and should be uploaded through the DGCI 2013 website. Submissions will be peer-reviewed by at least 2 qualified reviewers.
Important Dates: Full paper submission: July 2, 2012. Full paper notification: November 5, 2012. Camera-ready submission: December 5, 2012.
Information: `http://dgci2013.us.es/`.

* 25–29 **AIM Workshop: Mathematical problems arising from biochemical reaction networks**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the mathematical analysis of biochemical reaction networks arising in systems biology, in particular, in classes of nonlinear dynamical systems that arise from biologically relevant networks of chemical reactions, parametrized by rate constants that are usually unknown and difficult to measure.
Information: `http://www.aimath.org/ARCC/workshops/biochemnet.html`.

April 2013

* 6–7 **2013 Spring Eastern Sectional Meeting**, Boston College, Chestnut Hill, Massachusetts.
Information: `http://www.ams.org/meetings/sectional/sectional.html`.

8–10 **Fourteenth International Conference on Numerical Combustion (NC13)**, Holiday Inn Riverwalk, San Antonio, Texas.

8–12 **Interactions between Noncommutative Algebra, Representation Theory, and Algebraic Geometry**, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2011, p. 1014)

* 13–14 **2013 Spring Western Section Meeting**, University of Colorado Boulder, Boulder, Colorado.
Information: `http://www.ams.org/meetings/sectional/sectional.html`.

13–14 **3rd IIMA International Conference on Advanced Data Analysis, Business Analytics and Intelligence**, Indian Institute of Management, Ahmedabad, India. (Apr. 2012, p. 593)

* 15–19 **ICERM Workshop: Combinatorics, Multiple Dirichlet Series and Analytic Number Theory**, ICERM, Providence, Rhode Island.
Description: Recent years have seen a flurry of activity in the field of Weyl group multiple Dirichlet series. Surprising and unexpected connections between these multiple Dirichlet series and several different fields of mathematics have emerged. This workshop will survey recent results and set the stage for future developments which further interrelate analytic number theory, automorphic forms and combinatorial representation theory. Particular focus will be given

to applications of Weyl group multiple Dirichlet series to the following areas: Average value and nonvanishing results for families of L-functions, Periods of automorphic forms, Connections between characters sums over function fields and characters of affine root systems, Metaplectic Casselman-Shalika formulae and deformations of the Weyl character formula.

Information: `http://icerm.brown.edu/sp-s13-w3`.

* 27–28 **2013 Spring Central Section Meeting**, Iowa State University, Ames, Iowa.

Information: `http://www.ams.org/meetings/sectional/sectional.html`.

May 2013

19–23 **SIAM Conference on Applications of Dynamical Systems (DS13)**, Snowbird Ski and Summer Resort, Snowbird, Utah. (Oct. 2011, p. 1326)

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

June 2013

* 10–14 **Computational Methods and Function Theory 2013**, Shantou University, Shantou, Guangdong, China.

Description: The general theme of the meeting concerns various aspects of interaction of complex variables and scientific computation, including related topics from function theory, approximation theory and numerical analysis. Another important aspect of the CMFT meetings, previously held in Valparaiso 1989, Penang 1994, Nicosia 1997, Aveiro 2001, Joensuu 2005 and Ankara 2009, is to promote the creation and maintenance of contacts with scientists from diverse cultures.

International Organizing Committee: Ilpo Laine (University of Eastern Finland); Stephan Ruscheweyh (Universität Würzburg, Germany); Edward B. Saff (Vanderbilt University, USA); Yuefei Wang (AMSS, Chinese Academy of Sciences, China); Hasi Wulan (Shantou University, China).

Information: email: `ptli@stu.edu.cn`; `http://math.stu.edu.cn/cmft/index.asp`.

July 2013

* 1–5 **Erdös Centennial**, Budapest, Hungary.

Description: The Hungarian Academy of Sciences, the Alfréd Rényi Mathematical Institute of the Hungarian Academy of Sciences, the Eötvös Loránd University and the János Bolyai Mathematical Society announce that a conference dedicated to the 100th anniversary of Paul Erdös will be held in Budapest, Hungary.

Topics: Include all basic fields that Paul Erdös contributed to: Number theory, analysis, combinatorics, probability theory and set theory, among others. The emphasis will be on the recent developments in these areas, initiated or inspired by his mathematical legacy.

Information: `http://www.renyi.hu/conferences/erdos100/index.html`.

* 14–19 **The sixth International Congress of Chinese Mathematicians (ICCM)**, Opening ceremony on July 14 in the Big Hall of the Grand Hotel, Taipei, Taiwan.

Description: ICCM is a triennial event that brings together Chinese and overseas mathematicians to discuss the latest research developments in pure and applied mathematics. During the opening of the ICCM 2013, you will witness the presentation of the Morningside Medals, the most prestigious awards for Chinese mathematicians. During the conference, there will be at least fifteen plenary talks, several Morningside talks by international speakers, and over one hundred 45-minutes talks. The talks cover a full range of subjects in mathematical sciences, from number theory, geometry, differential equations to statistics and bio-mathematics. The first Congress was held in December 1998 in Beijing. The second Congress took place in Taipei 2001, then in Hong Kong, in Hangzhou, and 2010 in Beijing again. After twelve years, the ICCM 2013 will return to Taipei. We expect that there will be about 1,000 participants and nearly 200 presentations on a broad spectrum of mathematical sciences.

Lectures and Talks: From July 15 to July 19 on the campus of National Taiwan University, Taipei, Taiwan.

Information: `http://www.tims.ntu.edu.tw/ICCM2013/`.

September 2013

* 9–December 6 **ICERM Semester Program on "Low-Dimensional Topology, Geometry, and Dynamics"**, Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island.

Description: The program focuses on the recent impact of computation and experiment on the study of the pure mathematics sides of topology, geometry, and dynamics. Specific areas include 3-dimensional topology, the study of locally symmetric spaces, low-dimensional dynamics, and geometric group theory. Included are areas where computation has not yet had an impact, but might do so in the near future.

Information: `http://icerm.brown.edu/sp-f13`.

October 2013

* 5–6 **2013 Fall Southeastern Section Meeting**, University of Louisville, Louisville, Kentucky.

Information: `http://www.ams.org/meetings/sectional/sectional.html`.

* 12–13 **2013 Eastern Sectional Meeting**, Temple University, Philadelphia, Pennsylvania.

Information: `http://www.ams.org/meetings/sectional/sectional.html`.

* 18–20 **2013 Fall Central Section Meeting**, Washington University, St. Louis, Missouri.

Information: `http://www.ams.org/meetings/sectional/sectional.html`.

November 2013

* 2–3 **2013 Western Fall Section Meeting**, University of California Riverside, Riverside, California.

Information: `http://www.ams.org/meetings/sectional/sectional.html`.

May 2014

* 12–16 **Model Theory in Geometry and Arithmetic**, Mathematical Sciences Research Institute, Berkeley, California.

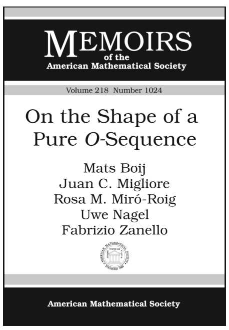
Description: The workshop will feature talks in a range of topics where model theory interacts with other parts of mathematics, especially number theory and arithmetic geometry, including: motivic integration, algebraic dynamics, diophantine geometry, and valued fields.

Information: `http://www.msri.org/web/msri/scientific/workshops/programmatic-workshops/show/-/event/Wm9547`.

New Publications Offered by the AMS

To subscribe to email notification of new AMS publications, please go to http://www.ams.org/bookstore-email.

Algebra and Algebraic Geometry



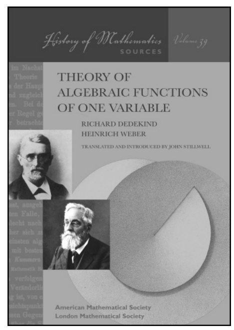
On the Shape of a Pure *O*-Sequence

Mats Boij, *Royal Institute of Technology, Stockholm, Sweded*, **Juan C. Migliore**, *University of Notre Dame, IN*, **Rosa M Miró-Roig**, *University of Barcelona, Spain*, **Uwe Nagel**, *University of Kentucky, Lexington, KY*, and **Fabrizio Zanello**, *Michigan Technological University, Houghton, MI*

Contents: Introduction; Definitions and preliminary results; Differentiability and unimodality; The interval conjecture for pure *O*-sequences; Enumerating pure *O*-sequences; Monomial Artinian level algebras of type two in three variables; Failure of the WLP and the SLP; Remarks on pure *f*-vectors; Some open or open-ended problems; Appendix A. Collection of definitions and notation; Bibliography.

Memoirs of the American Mathematical Society, Volume 218, Number 1024

June 2012, 78 pages, Softcover, ISBN: 978-0-8218-6910-9, LC 2012007082, 2010 *Mathematics Subject Classification:* 13D40, 05E40, 06A07, 13E10, 13H10; 05A16, 05B35, 14M05, 13F20, **Individual member US$36**, List US$60, Institutional member US$48, Order code MEMO/218/1024



Theory of Algebraic Functions of One Variable

Richard Dedekind and **Heinrich Weber**
Translated and introduced by John Stillwell

This book is the first English translation of the classic long paper *Theorie der algebraischen Functionen einer Veränderlichen (Theory of algebraic functions of one variable)*, published by Dedekind and Weber in 1882. The translation has been enriched by a Translator's Introduction that includes historical background, and also by extensive commentary embedded in the translation itself.

The translation, introduction, and commentary provide the first easy access to this important paper for a wide mathematical audience: students, historians of mathematics, and professional mathematicians.

Why is the Dedekind-Weber paper important? In the 1850s, Riemann initiated a revolution in algebraic geometry by interpreting algebraic curves as surfaces covering the sphere. He obtained deep and striking results in pure algebra by intuitive arguments about surfaces and their topology. However, Riemann's arguments were not rigorous, and they remained in limbo until 1882, when Dedekind and Weber put them on a sound foundation.

The key to this breakthrough was to develop the theory of algebraic functions in analogy with Dedekind's theory of algebraic numbers, where the concept of ideal plays a central role. By introducing such concepts into the theory of algebraic curves, Dedekind and Weber paved the way for modern algebraic geometry.

This volume is one of an informal sequence of works within the History of Mathematics series. Volumes in this subset, "Sources", are classical mathematical works that served as cornerstones for modern mathematical thought.

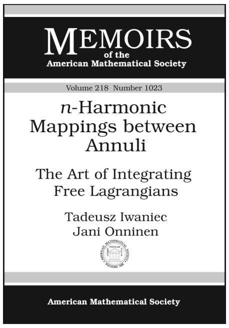
This item will also be of interest to those working in number theory and analysis.

Co-published with the London Mathematical Society beginning with Volume 4. Members of the LMS may order directly from the AMS at the AMS member price. The LMS is registered with the Charity Commissioners.

Contents: Translator's introduction; Theory of algebraic functions of one variable; Introduction; Part I; Part II; Bibliography; Index.

History of Mathematics, Volume 39

October 2012, approximately 157 pages, Softcover, ISBN: 978-0-8218-8330-3, 2010 *Mathematics Subject Classification:* 01-02, 01A55, **AMS members US$39.20**, List US$49, Order code HMATH/39



n-Harmonic Mappings between Annuli

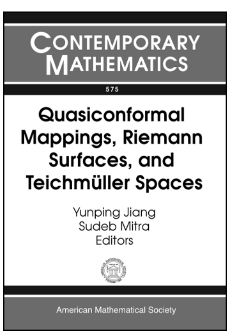
The Art of Integrating Free Lagrangians

Tadeusz Iwaniec, *Syracuse University, NY, and University of Helsinki, Finland*, and **Jani Onninen**, *Syracuse University, NY*

Contents: Introduction and overview; *Part 1. Principal Radial n-Harmonics:* Nonexistence of n-Harmonic homeomorphisms; Generalized n-Harmonic mappings; Notation; Radial n-harmonics; Vector calculus on annuli; Free Lagrangians; Some estimates of free Lagrangians; Proof of Theorem 1.15; *Part 2. The n-Harmonic Energy:* Harmonic mappings between planar annuli, Proof of Theorem 1.8; Contracting Pair, Mod $\mathbb{A}^* \leqslant$ Mod $\mathbb{A}$; Expanding Pair, Mod $\mathbb{A}^* >$ Mod $\mathbb{A}$; The uniqueness; Above the upper Nitsche bound, $n \geqslant 4$; Quasiconformal mappings between annuli; Bibliography.

Memoirs of the American Mathematical Society, Volume 218, Number 1023

June 2012, 105 pages, Softcover, ISBN: 978-0-8218-5357-3, LC 2012007086, 2010 *Mathematics Subject Classification:* 30C65, 30C75, 35J20, **Individual member US$42**, List US$70, Institutional member US$56, Order code MEMO/218/1023



Quasiconformal Mappings, Riemann Surfaces, and Teichmüller Spaces

Yunping Jiang and **Sudeb Mitra**, *Queens College, CUNY, Flushing, NY, and Graduate Center of the City University of New York, NY*, Editors

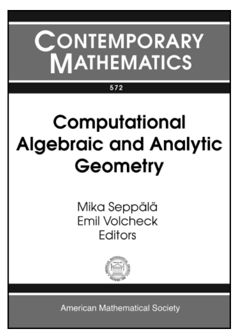
This volume contains the proceedings of the AMS Special Session on Quasiconformal Mappings, Riemann Surfaces, and Teichmüller Spaces, held in honor of Clifford J. Earle, from October 2–3, 2010, in Syracuse, New York.

This volume includes a wide range of papers on Teichmüller theory and related areas. It provides a broad survey of the present state of research and the applications of quasiconformal mappings, Riemann surfaces, complex dynamical systems, Teichmüller theory, and geometric function theory. The papers in this volume reflect the directions of research in different aspects of these fields and also give the reader an idea of how Teichmüller theory intersects with other areas of mathematics.

Contents: W. Abikoff, Some remarks on singly degenerate Kleinian groups; **O. Antolín-Camarena** and **S. Koch**, On a theorem of Kas and Schlessinger; **A. Basmajian**, Conformally scattered sets in the unit circle; **J. P. Bowman**, Finiteness conditions on translation surfaces; **C. J. Earle** and **A. Marden**, Holomorphic plumbing coordinates; **A. Fletcher** and **R. Fryer**, On Böttcher coordinates and quasiregular maps; **E. Fujikawa**, Discontinuity of asymptotic Teichmüller modular group; **F. P. Gardiner** and **Z. Wang**, Extremal annuli on the sphere; **J. Gilman** and **L. Keen**, Lifting free subgroups of $PSL(2,\mathbb{R})$ to free groups; **G. González-Diez** and **D. Torres-Teigell**, An introduction to Beauville surfaces via uniformization; **W. J. Harvey** and **A. Lloyd-Philipps**, Symmetry and moduli spaces for Riemann surfaces; **J. Hu** and **O. Muzician**, Conformally natural extensions of continuous circle maps: I. The case when the pushforward measure has no atom; **X. Huang** and **J. Liu**, Normal and quasinormal families of quasiregular mappings; **Y. Jiang**, Symmetric invariant measures; **Y. Jiang** and **S. Mitra**, Douady-Earle section, holomorphic motions, and some applications; **Y. Komori**, Cook-hats and crowns; **I. Kra**, On cohomology of Kleinian groups V: *b*-groups; **Z. Li** and **Y. Qi**, Fundamental inequalities of Reich-Strebel and triangles in a Teichmüller space; **K. Matsuzaki**, The Petersson series vanishes at infinity; **Y. Shen**, On fiber spaces over Teichmüller spaces; **H. Shiga**, On the number of holomorphic families of Riemann surfaces; **Y. Shinomiya**, Veech groups of flat structures on Riemann surfaces; **S. A. Wolpert**, On families of holomorphic differentials on degenerating annuli; **G. Yao**, Transformations of spheres without the injectivity assumption.

Contemporary Mathematics, Volume 575

August 2012, approximately 379 pages, Softcover, ISBN: 978-0-8218-5340-5, 2010 *Mathematics Subject Classification:* 30Fxx, 30C62, 30C65, 30C75, 30F40, 32G15, 32G13, 37F30, 30F10, 30F15, **AMS members US$95.20**, List US$119, Order code CONM/575



Computational Algebraic and Analytic Geometry

Mika Seppälä, *Florida State University, Tallahassee, FL, and University of Helsinki, Finland*, and **Emil Volcheck**, Editors

This volume contains the proceedings of three AMS Special Sessions on Computational Algebraic and Analytic Geometry for Low-Dimensional Varieties held January 8, 2007, in New Orleans, LA; January 6, 2009, in Washington, DC; and January 6, 2011, in New Orleans, LA.

Algebraic, analytic, and geometric methods are used to study algebraic curves and Riemann surfaces from a variety of points of view. The object of the study is the same. The methods are different. The fact that a multitude of methods, stemming from very different mathematical cultures, can be used to study the same objects makes this area both fascinating and challenging.

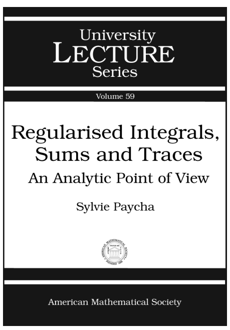
This item will also be of interest to those working in geometry and topology and applications.

Contents: A. Arnold and **K.-D. Semmler**, Large hyperbolic polygons and hyperelliptic Riemann surfaces; **G. Bartolini**, **A. F. Costa**, and **M. Izquierdo**, On isolated strata of pentagonal Riemann surfaces in the branch locus of moduli spaces; **E. Bujalance**, **F. J. Cirre**, and **M. D. E. Conder**, Finite group actions of large order on compact bordered surfaces; **I. Coskun**, Surfaces of low degree containing a canonical curve; **E. Fortuna**, **P. Gianni**, and **B. Trager**, Ideals of curves given by points; **D. Glass**, Non-genera of curves with automorphisms in characteristic p; **R. A. Hidalgo** and **M. Seppälä**, Numerical Schottky uniformizations of certain cyclic L-gonal curves; **E. Hironaka**, Generalized lantern relations and planar line arrangements; **K. S. Kedlaya**, Effective p-adic cohomology for cyclic cubic threefolds; **K. Magaard**, **S. Shpectorov**, and **G. Wang**, Generating sets of affine groups of low genus; **L. X. C. Ngô**, **J. R. Sendra**, and **F. Winkler**, Classification of algebraic ODEs with respect to rational solvability; **C. T. Sass**, **K. Stephenson**, and **G. B. Williams**, Circle packings on conformal and affine tori; **J. Schicho** and **D. Sevilla**, Effective radical parametrization of trigonal curves.

Contemporary Mathematics, Volume 572

July 2012, approximately 235 pages, Softcover, ISBN: 978-0-8218-6869-0, LC 2012009188, 2010 *Mathematics Subject Classification:* 14Hxx, 30Fxx, 68Wxx, **AMS members US$68.80**, List US$86, Order code CONM/572

Analysis



Regularised Integrals, Sums and Traces

An Analytic Point of View

Sylvie Paycha, *Universität Potsdam, Germany, and Université Blaise Pascal, Aubière, France*

"Regularization techniques" is the common name for a variety of methods used to make sense of divergent series, divergent integrals, or traces of linear operators in infinite-dimensional spaces. Such methods are often indispensable in problems of number theory, geometry, quantum field theory, and other areas of mathematics and theoretical physics. However arbitrary and noncanonical they might seem at first glance, regularized sums, integrals, and traces often contain canonical concepts, and the main purpose of this book is to illustrate and explain this.

This book provides a unified and self-contained mathematical treatment of various regularization techniques. The author shows how to derive regularized sums, integrals, and traces from certain canonical building blocks of the original divergent object. In the process of putting together these "building blocks", one encounters many problems and ambiguities caused by various so-called anomalies, which are investigated and explained in detail. Nevertheless, it turns out that the corresponding canonical sums, integrals, sums, and traces are well behaved, thus making the regularization procedure possible and manageable.

This new unified outlook on regularization techniques in various fields of mathematics and in quantum field theory can serve as an introduction for anyone from a beginning mathematician interested in the subject to an experienced physicist who wants to gain a unified outlook on techniques he/she uses on a daily basis.

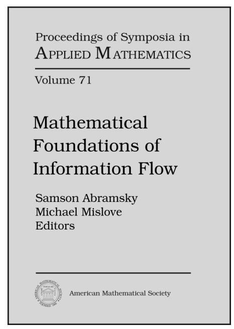
This item will also be of interest to those working in mathematical physics.

Contents: The Gamma function extended to nonpositive integer points; The canonical integral and noncommutative residue on symbols; The cut-off regularised integral; The noncommutative residue as a complex residue; The canonical sum on noninteger order classical symbols; Traces on pseudodifferential operators; Weighted traces; Logarithmic residues; Anomalies of regularised determinants; Bibliography; Index.

University Lecture Series, Volume 59

August 2012, approximately 192 pages, Softcover, ISBN: 978-0-8218-5367-2, LC 2012010029, 2010 *Mathematics Subject Classification:* 11M06, 40A10, 47L80, 81T50, 65B15, **AMS members US$34.40**, List US$43, Order code ULECT/59

Applications



Mathematical Foundations of Information Flow

Samson Abramsky, *University of Oxford, United Kingdom*, and **Michael Mislove**, *Tulane University, New Orleans, LA*, Editors

This volume is based on the 2008 Clifford Lectures on Information Flow in Physics, Geometry and Logic and Computation, held March 12–15, 2008, at Tulane University in New Orleans, Louisiana.

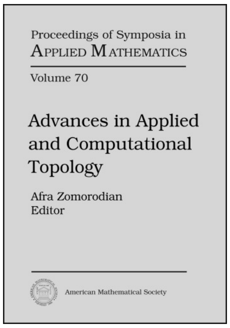
The varying perspectives of the researchers are evident in the topics represented in the volume, including mathematics, computer science, quantum physics and classical and quantum information. A number of the articles address fundamental questions in quantum information and related topics in quantum physics, using abstract categorical and domain-theoretic models for quantum physics to reason about such systems and to model spacetime.

Readers can expect to gain added insight into the notion of information flow and how it can be understood in many settings. They also can learn about new approaches to modeling quantum mechanics that provide simpler and more accessible explanations of quantum phenomena, which don't require the arcane aspects of Hilbert spaces and the cumbersome notation of bras and kets.

Contents: S. Abramsky and **C. Heunen**, H*-algebras and nonunital Frobenius algebras: First steps in infinite-dimensional categorical quantum mechanics; **H. Barnum**, **J. Barrett**, **M. Leifer**, and **A. Wilce**, Teleportation in general probabilistic theories; **A. Brandenburger**, **A. Friedenberg**, and **H. J. Keisler**, Fixed points in epistemic game theory; **B. Coecke** and **B. Edwards**, Spekken's toy theory as a category of processes; **P. Hines** and **P. Scott**, Categorical traces from single-photon linear optics; **K. H. Hofmann** and **M. Mislove**, Compact affine monoids, harmonic analysis and information theory; **K. Martin**, The scope of a quantum channel; **K. Martin** and **P. Panangaden**, Spacetime geometry from causal structure and a measurement; **D. Pavlovic**, Geometry of abstraction in quantum computation.

Proceedings of Symposia in Applied Mathematics, Volume 71

July 2012, approximately 270 pages, Hardcover, ISBN: 978-0-8218-4923-1, 2010 *Mathematics Subject Classification:* 18D35, 22A15, 78A15, 81P10, 81P45, 83C99, 91A10, **AMS members US$61.60**, List US$77, Order code PSAPM/71



Advances in Applied and Computational Topology

Afra Zomorodian, *The D. E. Shaw Group, New York, NY*, Editor

What is the shape of data? How do we describe flows? Can we count by integrating? How do we plan with uncertainty? What is the most compact representation? These questions, while unrelated, become similar when recast into a computational setting. Our input is a set of finite, discrete, noisy samples that describes an abstract space. Our goal is to compute qualitative features of the unknown space. It turns out that topology is sufficiently tolerant to provide us with robust tools.

This volume is based on lectures delivered at the 2011 AMS Short Course on Computational Topology, held January 4–5, 2011 in New Orleans, Louisiana.

The aim of the volume is to provide a broad introduction to recent techniques from applied and computational topology. Afra Zomorodian focuses on topological data analysis via efficient construction of combinatorial structures and recent theories of persistence. Marian Mrozek analyzes asymptotic behavior of dynamical systems via efficient computation of cubical homology. Justin Curry, Robert Ghrist, and Michael Robinson present Euler Calculus, an integral calculus based on the Euler characteristic, and apply it to sensor and network data aggregation. Michael Erdmann explores the relationship of topology, planning, and probability with the strategy complex. Jeff Erickson surveys algorithms and hardness results for topological optimization problems.

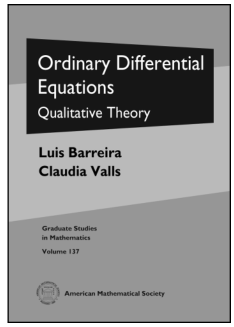
This item will also be of interest to those working in geometry and topology.

Contents: A. Zomorodian, Topological data analysis; **M. Mrozek**, Topological dynamics: Rigorous numerics via cubical homology; **J. Curry**, **R. Ghrist**, and **M. Robinson**, Euler calculus with applications to signals and sensing; **M. Erdmann**, On the topology of discrete planning with uncertainty; **J. Erickson**, Combinatorial optimization of cycles and bases; Index.

Proceedings of Symposia in Applied Mathematics, Volume 70

July 2012, 232 pages, Hardcover, ISBN: 978-0-8218-5327-6, LC 2012008031, 2010 *Mathematics Subject Classification:* 55N35, 55U05, 55-04, 37B30, 37M99, 37D45, 55N30, 53C65, 68T37, 68T40, 68W05, 68Q25, **AMS members US$48**, List US$60, Order code PSAPM/70

Differential Equations



Ordinary Differential Equations

Qualitative Theory

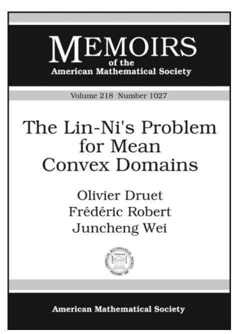
Luis Barreira and **Claudia Valls**, *Instituto Superior Técnico, Lisbon, Portugal*

This textbook provides a comprehensive introduction to the qualitative theory of ordinary differential equations. It includes a discussion of the existence and uniqueness of solutions, phase portraits, linear equations, stability theory, hyperbolicity and equations in the plane. The emphasis is primarily on results and methods that allow one to analyze qualitative properties of the solutions without solving the equations explicitly. The text includes numerous examples that illustrate in detail the new concepts and results as well as exercises at the end of each chapter. The book is also intended to serve as a bridge to important topics that are often left out of a course on ordinary differential equations. In particular, it provides brief introductions to bifurcation theory, center manifolds, normal forms and Hamiltonian systems.

Contents: *Basic concepts and linear equations:* Ordinary differential equations; Linear equations and conjugacies; *Stability of hyperbolicity:* Stability and Lyapunov functions; Hyperbolicity and topological conjugacies; Existence of invariant manifolds; *Equations in the plane:* Index theory; Poincaré-Bendixson theory; *Further topics:* Bifurcations and center manifolds; Hamiltonian systems; Bibliography; Index.

Graduate Studies in Mathematics, Volume 137

July 2012, 248 pages, Hardcover, ISBN: 978-0-8218-8749-3, LC 2012010848, 2010 *Mathematics Subject Classification:* 34-01, 34Cxx, 34Dxx, 37Gxx, 37Jxx, **AMS members US$51.20**, List US$64, Order code GSM/137



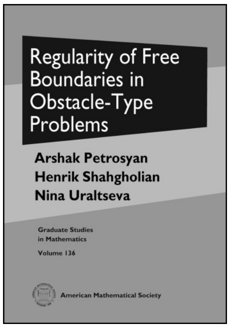
The Lin-Ni's Problem for Mean Convex Domains

Olivier Druet, *École Normale Supérieure de Lyon, France*, **Frédéric Robert**, *Université Henri Poincaré Nancy, Vandoeuvre-lès-Nancy, France*, and **Juncheng Wei**, *Chinese University of Hong Kong, Shatin, Hong Kong*

Contents: Introduction; L^∞-bounded solutions; Smooth domains and extensions of solutions to elliptic equations; Exhaustion of the concentration points; A first upper-estimate; A sharp upper-estimate; Asymptotic estimates in $C^1(\Omega)$; Convergence to singular harmonic functions; Estimates of the interior blow-up rates; Estimates of the boundary blow-up rates; Proof of Theorems 1 and 2; Appendix A. Construction and estimates on the Green's function; Appendix B. Projection of the test functions; Bibliography.

Memoirs of the American Mathematical Society, Volume 218, Number 1027

June 2012, 105 pages, Softcover, ISBN: 978-0-8218-6909-3, LC 2012007214, 2010 *Mathematics Subject Classification:* 35J20, 35J60, **Individual member US$42**, List US$70, Institutional member US$56, Order code MEMO/218/1027



Regularity of Free Boundaries in Obstacle-Type Problems

Arshak Petrosyan, *Purdue University, West Lafayette, IN*, **Henrik Shahgholian**, *Royal Institute of Technology, Stockholm, Sweden*, and **Nina Uraltseva**, *St. Petersburg University, Russia*

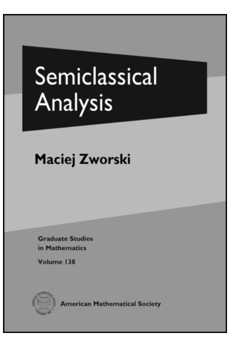
The regularity theory of free boundaries flourished during the late 1970s and early 1980s and had a major impact in several areas of mathematics, mathematical physics, and industrial mathematics, as well as in applications. Since then the theory continued to evolve. Numerous new ideas, techniques, and methods have been developed, and challenging new problems in applications have arisen. The main intention of the authors of this book is to give a coherent introduction to the study of the regularity properties of free boundaries for a particular type of problems, known as obstacle-type problems. The emphasis is on the methods developed in the past two decades. The topics include optimal regularity, nondegeneracy, rescalings and blowups, classification of global solutions, several types of monotonicity formulas, Lipschitz, C^1, as well as higher regularity of the free boundary, structure of the singular set, touch of the free and fixed boundaries, and more.

The book is based on lecture notes for the courses and mini-courses given by the authors at various locations and should be accessible to advanced graduate students and researchers in analysis and partial differential equations.

Contents: Introduction; Model problems; Optimal regularity of solutions; Preliminary analysis of the free boundary; Regularity of the free boundary: first results; Global solutions; Regularity of the free boundary: uniform results; The singular set; Touch with the fixed boundary; The thin obstacle problem; Bibliography; Notation; Index.

Graduate Studies in Mathematics, Volume 136

August 2012, approximately 225 pages, Hardcover, ISBN: 978-0-8218-8794-3, LC 2012010200, 2010 *Mathematics Subject Classification:* 35R35, **AMS members US$43.20**, List US$54, Order code GSM/136



Semiclassical Analysis

Maciej Zworski, *University of California, Berkeley, CA*

This book is an excellent, comprehensive introduction to semiclassical analysis. I believe it will become a standard reference for the subject.

—Alejandro Uribe, University of Michigan

Semiclassical analysis provides PDE techniques based on the *classical-quantum* (particle-wave) correspondence. These techniques include such well-known tools as geometric optics and the Wentzel–Kramers–Brillouin approximation. Examples of problems studied in this subject are high energy eigenvalue asymptotics and effective dynamics for solutions of evolution equations. From the mathematical point of view, semiclassical analysis is a branch of *microlocal analysis* which, broadly speaking, applies *harmonic analysis* and *symplectic geometry* to the study of linear and nonlinear PDE. The book is intended to be a graduate level text introducing readers to semiclassical and microlocal methods in PDE. It is augmented in later chapters with many specialized advanced topics which provide a link to current research literature.

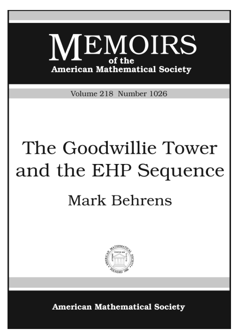
This item will also be of interest to those working in analysis.

Contents: Introduction; *Basic theory:* Symplectic geometry and analysis; Fourier transform, stationary phase; Semiclassical quantization; *Applications to partial differential equations:* Semiclassical defect measures; Eigenvalues and eigenfunctions; Estimates for solutions of PDE; *Advanced theory and applications:* More on the symbol calculus; Changing variables; Fourier integral operators; Quantum and classical dynamics; Normal forms; The FBI transform; *Semiclassical analysis on manifolds:* Manifolds; Quantum ergodicity; *Appendices:* Notation; Differential forms; Functional analysis; Fredholm theory; Bibliography; Index.

Graduate Studies in Mathematics, Volume 138

August 2012, approximately 429 pages, Hardcover, ISBN: 978-0-8218-8320-4, LC 2012010649, 2010 *Mathematics Subject Classification:* 35Q40, 81Q20, 35S05, 35S30, 35P20, 81S10, **AMS members US$60**, List US$75, Order code GSM/138

Geometry and Topology



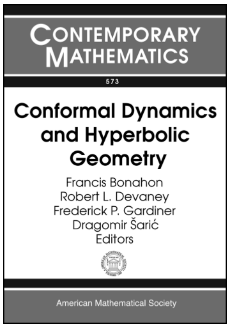
The Goodwillie Tower and the EHP Sequence

Mark Behrens, *Massachusetts Institute of Technology, Cambridge, MA*

Contents: Introduction; Dyer-Lashof operations and the identity functor; The Goodwillie tower of the EHP sequence; Goodwillie filtration and the P map; Goodwillie differentials and Hopf invariants; EHPSS differentials; Calculations in the 2-primary Toda range; Appendix A. Transfinite spectral sequences associated to towers; Bibliography.

Memoirs of the American Mathematical Society, Volume 218, Number 1026

June 2012, 90 pages, Softcover, ISBN: 978-0-8218-6902-4, LC 2012007213, 2010 *Mathematics Subject Classification:* 55Q40; 55Q15, 55Q25, 55S12, **Individual member US$40.20**, List US$67, Institutional member US$53.60, Order code MEMO/218/1026



Conformal Dynamics and Hyperbolic Geometry

Francis Bonahon, *University of Southern California, Los Angeles, CA*, **Robert L. Devaney**, *Boston University, MA*, **Frederick P. Gardiner**, *Brooklyn College, CUNY, New York, NY, and Graduate School and University Center of CUNY, New York, NY*, and **Dragomir Šarić**, *Queens College, CUNY, Flushing, NY, and Graduate School and University Center of CUNY, New York, NY*, Editors

This volume contains the proceedings of the Conference on Conformal Dynamics and Hyperbolic Geometry, held October 21–23, 2010, in honor of Linda Keen's 70th birthday.

This volume provides a valuable introduction to problems in conformal and hyperbolic geometry and one dimensional, conformal dynamics. It includes a classic expository article by John Milnor on the structure of hyperbolic components of the parameter space for dynamical systems arising from the iteration of polynomial maps in the complex plane. In addition there are foundational results concerning Teichmüller theory, the geometry of Fuchsian and Kleinian groups, domain convergence properties for the Poincaré metric, elaboration of the theory of the universal solenoid, the geometry of dynamical systems acting on a circle, and realization of Thompson's group as a mapping class group for a uniformly asymptotically affine circle endomorphism.

The portion of the volume dealing with complex dynamics will appeal to a diverse group of mathematicians. Recently many researchers working in a wide range of topics, including topology, algebraic geometry, complex analysis, and dynamical systems, have become involved in aspects of this field.

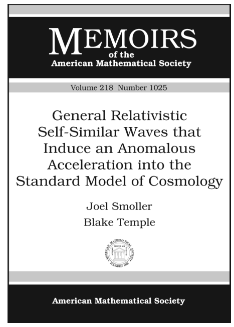
This item will also be of interest to those working in analysis.

Contents: **M. Beck**, **Y. Jiang**, and **S. Mitra**, Normal families and holomorphic motions over infinite dimensional parameter spaces; **R. Chamanara** and **D. Šarić**, Elementary moves and the modular group of the compact solenoid; **L. DeMarco**, Combinatorics and topology of the shift locus; **R. L. Devaney**, Dynamics of $z^n + \lambda/z^n$; Why the case $n = 2$ is crazy; **C. J. Earle** and **A. Marden**, On holomorphic families of Riemann surfaces; **F. P. Gardiner** and **Y. Jiang**, Circle endomorphisms, dual circles and Thompson's group; **J. Hu**, **F. G. Jimenez**, and **O. Muzician**, Rational maps with half symmetries, Julia sets, and multibrot sets in parameter planes; **N. Lakic** and **G. Markowsky**, The rate of convergence of the hyperbolic density on sequences of domains; **S. Maloni**, The asymptotic directions of pleating rays in the Maskit embedding; **J. Milnor** and **A. Poirier**, Hyperbolic components with an appendix by A. Poirier; **C. Wolf**, On barycenter entropy for rational maps; **S. Yuan**, Parameter plane of a family of meromorphic functions with two asymptotic values.

Contemporary Mathematics, Volume 573

August 2012, 256 pages, Softcover, ISBN: 978-0-8218-5348-1, LC 2012011231, 2010 *Mathematics Subject Classification:* 30Cxx, 32Gxx, 37Dxx, 37Fxx, **AMS members US$68.80**, List US$86, Order code CONM/573

Mathematical Physics



General Relativistic Self-Similar Waves that Induce an Anomalous Acceleration into the Standard Model of Cosmology

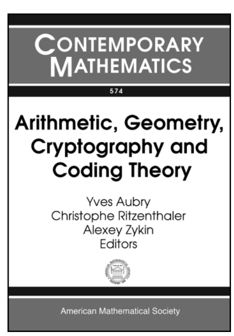
Joel Smoller, *University of Michigan, Ann Arbor, MI*, and **Blake Temple**, *University of California, Davis, CA*

Contents: Introduction; Self-similar coordinates for the $k = 0$ FRW spacetime; The expanding wave equations; Canonical co-moving coordinates and comparison with the $k \neq 0$ FRW spacetimes; Leading order corrections to the standard model induced by the expanding waves; A foliation of the expanding wave spacetimes into flat spacelike hypersurfaces with modified scale factor $R(t) = t^a$; Expanding wave corrections to the standard model in approximate co-moving coordinates; Redshift vs luminosity relations and the anomalous acceleration; Appendix: The mirror problem; Concluding remarks; Bibliography.

Memoirs of the American Mathematical Society, Volume 218, Number 1025

June 2012, 69 pages, Softcover, ISBN: 978-0-8218-5358-0, LC 2012007080, 2010 *Mathematics Subject Classification:* 34A05, 76L05, 83F05, 85A40, **Individual member US$34.80**, List US$58, Institutional member US$46.40, Order code MEMO/218/1025

Number Theory



Arithmetic, Geometry, Cryptography and Coding Theory

Yves Aubry, *Université du Sud Toulon-Var, La Garde Cedex, France*, **Christophe Ritzenthaler**, *Institut de Mathématiques de Luminy, Marseille, France*, and **Alexey Zykin**, *State University–Higher School of Economics, Moscow, Russia, and Institute for Information Transmission Problems, Moscow, Russia*, Editors

This volume contains the proceedings of the 13th AGC^{2}T conference, held March 14–18, 2011, in Marseille, France, together with the proceedings of the 2011 Geocrypt conference, held June 19–24, 2011, in Bastia, France.

The original research articles contained in this volume cover various topics ranging from algebraic number theory to Diophantine geometry, curves and abelian varieties over finite fields and applications to codes, boolean functions or cryptography.

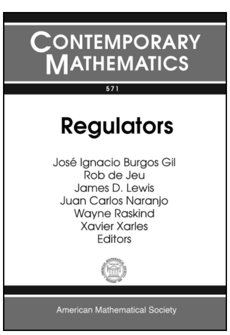
The international conference AGC^{2}T, which is held every two years in Marseille, France, has been a major event in the area of applied arithmetic geometry for more than 25 years.

This item will also be of interest to those working in applications.

Contents: C. Arène and **R. Cosset**, Construction of a $\Bbbk$-complete addition law on Jacobians of hyperelliptic curves of genus two; **R. Blache**, Number of points in an Artin-Schreier covering; **E. Férard**, **R. Oyono**, and **F. Rodier**, Some more functions that are not APN infinitely often. The case of Gold and Kasami exponents; **S. Fukasawa**, **M. Homma**, and **S. J. Kim**, Rational curves with many rational points over a finite field; **S. R. Ghorpade** and **S. Ram**, Enumeration of splitting subspaces over finite fields; **S. Haloui** and **V. Singh**, The characteristic polynomials of abelian varieties of dimension 4 over finite fields; **E. W. Howe**, New bounds on the maximum number of points on genus-4 curves over small finite fields; **G. M. Kyureghyan**, **F. Özbudak**, and **A. Pott**, Some planar maps and related function fields; **E. Leducq**, New families of APN functions in characteristic 3 or 5; **P. Lisoněk**, Identities for Kloosterman sums and modular curves; **A. Ostafe** and **I. Shparlinski**, Degree growth, linear independence and periods of a class of rational dynamical systems; **E. Rökaeus**, Computer search for curves with many points among abelian covers of genus 2 curves; **S. Rybakov**, The groups of points on abelian surfaces over finite fields; **B. Smith**, Computing low-degree isogenies in genus 2 with the Dolgachev-Lehavi method; **Yu. G. Zarhin**, Hodge classes on certain hyperelliptic prymians.

Contemporary Mathematics, Volume 574

August 2012, approximately 188 pages, Softcover, ISBN: 978-0-8218-7572-8, 2010 *Mathematics Subject Classification:* 11G10, 11G20, 11M38, 11R42, 11T06, 11T71, 14G10, 14G15, 14G50, 14Q05, **AMS members US$59.20**, List US$74, Order code CONM/574



Regulators

José Ignacio Burgos Gil, *ICMAT, Madrid, Spain*, **Rob de Jeu**, *VU University Amsterdam, The Netherlands*, **James D. Lewis**, *University of Alberta, Edmonton, AB, Canada*, **Juan Carlos Naranjo**, *University of Barcelona, Spain*, **Wayne Raskind**, *Arizona State University, Tempe, AZ*, and **Xavier Xarles**, *Universitat Autònoma de Barcelona, Catalunya, Spain*, Editors

This volume contains the proceedings of the Regulators III Conference, held from July 12 to July 22, 2010, in Barcelona, Spain.

Regulators can be thought of as realizations from motivic cohomology, which is very difficult to compute, to more computable theories such as Hodge, Betti, l-adic, and Deligne cohomology. It is a very intricate subject that thrives on its interaction with algebraic K-theory, arithmetic geometry, number theory, motivic cohomology, Hodge theory and mathematical physics.

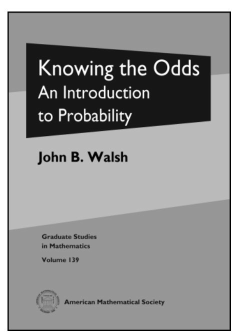
The articles in this volume are a reflection of the various approaches to this subject, such as results on motivic cohomology, descriptions of regulators, a revisiting of a number of fundamental conjectures (such as new results pertaining to the Hodge and standard conjectures), and more.

Contents: M. Asakura, Quintic surface of p-adic local fields with infinite p-primary torsion in the Chow group of 0-cycles; **A. Beilinson**, A remark on primitive cycles and Fourier-Radon transform; **A. Beilinson**, Remarks on Grothendieck's standard conjectures; **A. Besser**, On the derivative of a normal function associated with a Deligne cohomology class; **J.-L. Colliot-Thélène**, Quelque cas d'annulation du troisième groupe de cohomologie non ramifiée; **F. Déglise**, Coniveau filtration and mixed motives; **F. Déglise**, Around the Gysin triangle I; **C. Deninger**, Regulators, entropy and infinite determinants; **M. Felisatti** and **F. Neumann**, Secondary theories for étale groupoids; **T. Geisser**, Finite generation conjectures for motivic cohomology theories over finite fields; **D. Hébert**, Le foncteur de filtration par le poids; **R. Joshua**, K-theory and G-theory of DG stacks; **A. Del Padrone** and **C. Pedrini**, Derived categories of coherent sheaves and motives of K3 surfaces; **W. Raskind**, Serre-Tate parameters and jacobian inversion for rigid Calabi-Yau 3-folds; **A. Rosenschon** and **V. Srinivas**, An example concerning specialization of torsion subgroups of Chow groups; **J. Wildeshaus**, Motivic intersection complex.

Contemporary Mathematics, Volume 571

June 2012, 276 pages, Softcover, ISBN: 978-0-8218-5322-1, LC 2012005136, 2010 *Mathematics Subject Classification:* 14Cxx, 14Dxx, 14Fxx, 14Gxx, 19Dxx, 19Exx, 19Fxx, 11Gxx, 11Sxx, 11Mxx, **AMS members US$77.60**, List US$97, Order code CONM/571

Probability and Statistics



Knowing the Odds

An Introduction to Probability

John B. Walsh, *University of British Columbia, Vancouver, BC, Canada*

John Walsh, one of the great masters of the subject, has written a superb book on probability. It covers at a leisurely pace all the important topics that students need to know, and provides excellent examples. I regret his book was not available when I taught such a course myself, a few years ago.

—Ioannis Karatzas, Columbia University

In this wonderful book, John Walsh presents a panoramic view of Probability Theory, starting from basic facts on mean, median and mode, continuing with an excellent account of Markov chains and martingales, and culminating with Brownian motion. Throughout, the author's personal style is apparent; he manages to combine rigor with an emphasis on the key ideas so the reader never loses sight of the forest by being surrounded by too many trees. As noted in the preface, "To teach a course with pleasure, one should learn at the same time." Indeed, almost all instructors will learn something *new from the book (e.g. the potential-theoretic proof of Skorokhod embedding) and at the same time, it is attractive and approachable for students.*

—***Yuval Peres, Microsoft***

With many examples in each section that enhance the presentation, this book is a welcome addition to the collection of books that serve the needs of advanced undergraduate as well as first year graduate students. The pace is leisurely which makes it more attractive as a text.

—***Srinivasa Varadhan, Courant Institute, New York***

This book covers in a leisurely manner all the standard material that one would want in a full year probability course with a slant towards applications in financial analysis at the graduate or senior undergraduate honors level. It contains a fair amount of measure theory and real analysis built in but it introduces sigma-fields, measure theory, and expectation in an especially elementary and intuitive way. A large variety of examples and exercises in each chapter enrich the presentation in the text.

This item will also be of interest to those working in applications.

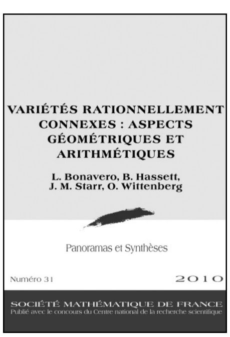
Contents: Probability spaces; Random variables; Expectations II: The general case; Convergence; Laws of large numbers; Convergence in distribution and the CLT; Markov chains and random walks; Conditional expectations; Discrete-parameter Martingales; Brownian motion; Bibliography; Index.

Graduate Studies in Mathematics, Volume 139

September 2012, approximately 452 pages, Hardcover, ISBN: 978-0-8218-8532-1, 2010 *Mathematics Subject Classification:* 60-01, **AMS members US$60**, List US$75, Order code GSM/139

New AMS-Distributed Publications

Algebra and Algebraic Geometry



Variétés Rationnellement Connexes: Aspects Géométriques et Arithmétiques

L. Bonavero, *Université de Grenoble 1, Saint-Martin d'Hères, France*, **B. Hassett**, *Rice University, Houston, TX*, **J. M. Starr**, *Stony Brook University, NY*, and **O. Wittenberg**, *École Normale Supérieure, Paris, France*

Over the last twenty years, rationally connected varieties have played an important role in the classification program of higher dimensional varieties. Over the last ten years, a number of their arithmetic properties have been discovered. It is the goal of this volume to report on many of these advances, as well as on a number of open questions.

This volume gathers the contributions of the four speakers at the CNRS/SMF workshop "Etats de la Recherche", which was organized by J.-L. Colliot-Thélène, O. Debarre, and A. Höring in Strasbourg in May 2008.

L. Bonavero discusses the fundamental geometric properties of rationally connected varieties and also offers an opening on modern birational classification techniques. O. Wittenberg surveys the arithmetic properties of rationally connected varieties, mostly over local fields and over finite fields (deformation techniques and cohomological techniques). B. Hassett reports on the weak approximation property for families of rationally connected varieties over a complex curve.

The emerging notion of simply rationally connected variety is at the heart of J. Starr's contribution. Starr's paper starts with a study of sections of families of such varieties over a complex surface and culminates with a partly simplified proof of the theorem by de A. J. Jong, J. Starr, and X. He: Serre's Conjecture II for principal homogeneous spaces holds over function fields in two variables over the complex field.

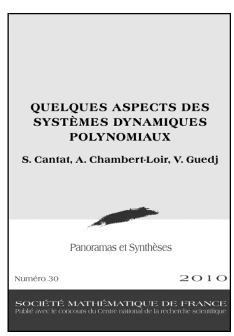
This item will also be of interest to those working in number theory.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: J.-L. Colliot-Thélène, Introduction; **O. Wittenberg**, La connexité rationnelle en arithmétique; **L. Bonavero**, Variétés rationnellement connexes sur un corps algébriquement clos; **J. M. Starr**, Rational points of rationally simply connected varieties; **B. Hassett**, Weak approximation and rationally connected varieties over function fields of curves.

Panoramas et Synthèses, Number 31

February 2012, 221 pages, Softcover, ISBN: 978-2-85629-339-3, 2010 *Mathematics Subject Classification:* 11G25, 12G05, 14C15, 14D05, 14D22, 14E05, 14E30, 14G05, 14J40, 14J45, 14M20, 14M22, **Individual member US$54**, List US$60, Order code PASY/31



Quelques Aspects des Systèmes Dynamiques Polynomiaux

S. Cantat and **A. Chambert-Loir**, *Université de Rennes I, France*, and **V. Guedj**, *Université Aix-Marseille 1, France*

This book is concerned with the dynamics of rational transformations of projective varieties and meromorphic transformations of compact Kähler manifolds. Four main viewpoints are developed.

The first article describes the geometry of the varieties which admit a rational transformation with interesting dynamical properties; the geometry constrains the existence of such dynamical systems, but interesting examples with rich dynamics are described.

The second article explains how complex analysis, potential theory, and Hodge theory can be married with methods from dynamical systems to describe the stochastic properties of meromorphic transformations of Kähler manifolds. Then, arithmetic aspects of algebraic dynamical systems are described in a third article; in particular, equidistribution theorems in diophantine geometry and dynamical systems are analyzed and compared.

The fourth article describes the basics of p-adic dynamics in one variable.

This item will also be of interest to those working in number theory.

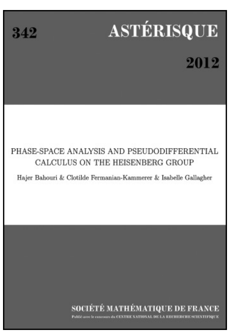
A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: S. Cantat, Introduction; **S. Cantat**, Quelques aspects des systèmes dynamiques polynomiaux (Existence, exemples, rigidité); **V. Guedj**, Propriétés ergodiques des applications rationnelles; **A. Chambert-Loir**, Théorèmes d'équidistribution pour les systèmes dynamiques d'origine arithmétique; **S. Cantat** and **A. Chambert-Loir**, Dynamique p-adique (d'après les exposés de Jean-Christophe Yoccoz).

Panoramas et Synthèses, Number 30

February 2012, 341 pages, Softcover, ISBN: 978-2-85629-338-6, 2010 *Mathematics Subject Classification:* 14E07, 14J50, 32H50, 11-02, 11G50, 14K15, 37P30, 37F10, 32U15, 32U40, **Individual member US$54**, List US$60, Order code PASY/30

Differential Equations



Phase-Space Analysis and Pseudodifferential Calculus on the Heisenberg Group

Hajer Bahouri and **Clotilde Fermanian-Kammerer**, *Université Paris-Est Créteil, France*, and **Isabelle Gallagher**, *Université Paris Diderot, France*

A class of pseudodifferential operators on the Heisenberg group is defined. As it should be, this class is an algebra containing the class of differential operators. Furthermore, those pseudodifferential operators act continuously on Sobolev spaces and the loss of derivatives may be controlled by the order of the operator. Although a large number of works have been devoted in the past to the construction and the study of algebras of variable-coefficient operators, including some very interesting works on the Heisenberg group, the authors' approach is different, and in particular puts into light microlocal directions and completes, with the Littlewood-Paley theory initiated in 2000 by Bahouri, Gérard, and Xu: a microlocal analysis of the Heisenberg group.

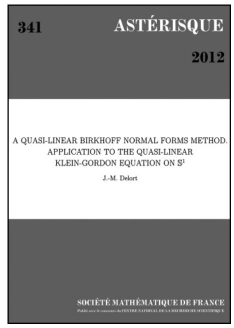
This item will also be of interest to those working in analysis.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction and main results; Fundamental properties of pseudodifferential operators; The algebra of pseudodifferential operators; Littlewood-Paley theory; The action of pseudodifferential operators on Sobolev spaces; Appendix A. Some useful results on the Heisenberg group; Appendix B. Weyl–Hörmander symbolic calculus on the Heisenberg group; Bibliography.

Astérisque, Number 342

February 2012, 127 pages, Softcover, ISBN: 978-2-85629-334-8, 2010 *Mathematics Subject Classification:* 35S05, 43A80, 35A27, **Individual member US$46.80**, List US$52, Order code AST/342



A Quasi-Linear Birkhoff Normal Forms Method. Application to the Quasi-Linear Klein-Gordon Equation on $\mathbb{S}^1$

J.-M. Delort, *Université Paris 13, Villetaneuse, France*

Consider a nonlinear Klein-Gordon equation on the unit circle, with smooth data of size $\epsilon \to 0$. A solution u which, for any $\kappa \in \mathbb{N}$, may be extended as a smooth solution on a time-interval $]-c_\kappa \epsilon^{-\kappa}, c_\kappa \epsilon^{-\kappa}[$ for some $c_\kappa > 0$ and for $0 < \epsilon < \epsilon_\kappa$, is called an almost global solution. It is known that when the nonlinearity is a polynomial depending only on u, and vanishing at order at least 2 at the origin, any smooth small Cauchy data generate, as soon as the mass parameter in the equation stays outside a subset of zero measure of $\mathbb{R}_+^*$, an almost global solution, whose Sobolev norms of higher order stay uniformly bounded. The goal of this book is to extend this result to general Hamiltonian *quasi-linear* nonlinearities. These are the only *Hamiltonian* nonlinearities that depend not only on u but also on its space derivative. To prove the main theorem, the author develops a Birkhoff normal form method for quasi-linear equations.

This item will also be of interest to those working in analysis.

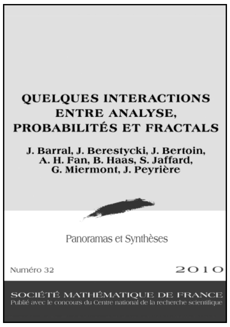
A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Almost global existence; Symbolic calculus; Composition and Poisson brackets; Symplectic reductions; Proof of almost global existence; Bibliography; Index.

Astérisque, Number 341

February 2012, 113 pages, Softcover, ISBN: 978-2-85629-335-5, 2010 *Mathematics Subject Classification:* 35L70, 35B45, 37K05, 35S50, **Individual member US$40.50**, List US$45, Order code AST/341

Number Theory



Quelques Interactions Entre Analyse, Probabilités et Fractals

J. Barral, *Université Paris 13, Villetaneuse, France*, **J. Berestycki** and **J. Bertoin**, *Université Pierre et Marie Curie, Paris, France*, **A. H. Fan**, *Université de Picardie, Amiens, France*, **B. Haas**, *Université Paris-Dauphine, France*, **S. Jaffard**, *Université Paris Est Créteil, France*, **G. Miermont**, *Université Paris-Sud Bâtiment, Orsay, France*, and **J. Peyrière**, *Université Paris-Sud 11, Orsay, France*

Following the seminal contributions of Benoît Mandelbrot in the 1970s, concepts derived from fractal geometry gave a new impulse to several areas of mathematics. The goal of this volume is to present syntheses on two subjects where important advances occurred in the last 15 years: multiplicative processes and fragmentation. One arose from harmonic analysis (Riesz products) and the other from a probabilistic model proposed by N. Kolmogorov to explain experimental observations on rock fragmentation. However, they share analogies and use common mathematical tools issued from the study of random fractals.

The first paper introduces basic concepts in fractal analysis. It starts with the description of the historical developments that led to their introduction and interactions. The definitions of fractional dimensions are introduced, and pertinent tools in geometric measure theory are recalled. Examples of multifractal functions and measures are studied. Finally, ubiquity systems, which play an increasing role in multifractal analysis, are introduced.

The second paper deals with fine geometric properties of measures obtained as limits of multiplicative processes. It starts by showing in which contexts they appear and by describing their key properties. The notions of dimension of a measure and of multifractal analysis are introduced in a general setting and illustrated on the aforementioned examples. Finally, the efficiency of these measures for the description of percolation on trees, and for dynamical or random coverings, is shown.

The third paper describes the time evolution of objects that disaggregate in a random way, and the fragments of which evolve independently. A statistical self-similarity assumption endows them with a structure of random fractal. The foundations of fragmentation theory are given, and the laws of these processes are shown to be characterized by a self-similarity index, a dislocation measure, and an erosion coefficient. Then, a random tree endowed with a distance is considered, which leads to a description of the genealogy of the process. Finally, the speed with which the fragment containing a given point decays is studied. This leads to the introduction of a multifractal spectrum of speeds of fragmentation.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: **S. Jaffard**, Introduction; **J. Barral**, **A. H. Fan**, and **J. Peyrière**, Mesures engendrées par multiplications; **J. Berestycki**, **J. Bertoin**, **B. Haas**, and **G. Miermont**, Quelques aspects fractals des fragmentations aléatoires.

Panoramas et Synthèses, Number 32

February 2012, 243 pages, Softcover, ISBN: 978-2-85629-313-3, 2010 *Mathematics Subject Classification:* 11J83, 11K06, 26A15, 26A30, 28A78, 28A80, 37B40, 43A25, 60G18, 60J80, **Individual member US$54**, List US$60, Order code PASY/32

Classified Advertisements

Positions available, items for sale, services available, and more

SOUTH CAROLINA

UNIVERSITY OF SOUTH CAROLINA
Call for Nominations
for Vasil Popov Prize—2013

The Vasil Popov Prize is awarded every three years for outstanding research in fields related to the work of Vasil A. Popov, who is best known for his contributions to Approximation Theory. Candidates must have received their Ph.D. within the previous six years. Nominations, to include a brief description of the relevant work and a vita of the nominee, should be sent to Pencho Petrushev, Chair, Popov Prize Selection Committee, Interdisciplinary Mathematics Institute, University of South Carolina, Columbia, SC 29208; email: `popov.prize@gmail.com`. The deadline for nominations is November 15, 2012. The prize will be awarded in April of 2013 at the Fourteenth International Conference in Approximation Theory in San Antonio, Texas. For further information, visit `http://imi.cas.sc.edu/popov-prize-call-nominations/`.

000030

CHINA

SHANGHAI CENTER FOR MATHEMATICAL SCIENCES (SCMS)
Chair-Professorships, Research Professorships, and Postdoctoral Research Fellowships

The Shanghai Center for Mathematical Sciences (SCMS) invites applicants for positions at the levels of chair-professorships, research professorships, and postdoctoral fellowships in pure and applied mathematics. SCMS is a recently founded mathematical research center, located on the campus of Fudan University, Shanghai, China. The center is committed to establishing itself as one of the first-class mathematical research centers in the world. SCMS has several openings in chair-professors, numerous openings in research professorships, and in postdoctoral fellowships. The annual salaries of these positions are comparable with most of the established mathematical research institutions in the world. For more information please visit `http://www.scms.fudan.edu.cn`. Applicants should send a cover letter specifying the research area, a curriculum vitae with a list of publications, and a summary of research plan. Applicants for research professorships and postdoctoral fellowships should also arrange three recommendation letters to be sent to: Mrs. Yingying Cai at `scms@fudan.edu.cn`. Email applications are strongly encouraged.

000032

NETHERLANDS

UTRECHT UNIVERSITY
Two Full Professors of Mathematics
(0.8–1.0 fte)

The Mathematical Institute of Utrecht University invites applications for two full professorships. It is anticipated that one appointment will be made in the section of Fundamental Mathematics, currently comprising algebra, analysis and geometry; and one in the section of Mathematical Modelling, currently comprising applied analysis, stochastics and mathematics of computation. The search however is not limited to the listed areas, and, furthermore, in case of exceptional candidates, both appointments may be made in the same section. We are looking for outstanding candidates who will invigorate and enrich the pool of expertise in the institute and the university at large. The institute has a long-standing tradition of crossing borders into other scientific fields. Interdisciplinary activity includes, but is not limited to, theoretical physics, theoretical biology, and life sciences. Appointees are expected to play an active role in all aspects of academic life. Candidates should demonstrate excellence in research, including grant-earning capacity, and be skilled in teaching and student supervision. Furthermore, we expect a willingness to take up administrative responsibilities. The appointments are, in principle, permanent at the level of full professor on a Core Chair. However, the institute may offer more junior candidates of exceptional promise a Profile Chair, which is subject to review after a 5-year period. Utrecht University specifically encourages female candidates to apply. Closing date for applications: August 1, 2012. See `http://www.math.uu.nl/jobs` for a complete job description and `http://www.math.uu.nl/facts.html` for a fact sheet concerning the institute.

000031

BOOK SALE

BOOK ANNOUNCEMENT

Introduction to Dynamical Systems and Geometric Mechanics, `http://www.solarcrestpublishing.com`. 30% off with Coupon Code AMS30.

000029

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2012 rate is \$3.50 per word with a minimum two-line headline. No discounts for multiple ads or the same ad in consecutive issues. For an additional \$10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: August 2012 issue–May 29, 2012; September 2012 issue–June 28, 2012; October 2012 issue–July 26, 2012; November 2012 issue–August 30, 2012; December 2012 issue–October 1, 2012.; January 2013 issue–October 29, 2012.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Rochester, New York

Rochester Institute of Technology

September 22–23, 2012

Saturday – Sunday

Meeting #1082

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June/July 2012
Program first available on AMS website: July 19, 2012
Program issue of electronic *Notices*: September 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: July 10, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Steve Gonek, University of Rochester, *Title to be announced*.

James Keener, University of Utah, *The mathematics of life—decisions, decisions*.

Dusa McDuff, Barnard College, Columbia University, *Embedding questions in symplectic geometry*.

Peter Winkler, Dartmouth College, *New directions for random walk on a graph*.

Special Sessions

Analytic Number Theory (Code: SS 5A), **Steve Gonek**, University of Rochester, and **Angel Kumchev**, Towson University.

Applied and Computational Mathematics (Code: SS 11A), **Ludwig Kohaupt**, Beuth University of Technology, and **Yan Wu**, Georgia Southern University.

Continuum Theory (Code: SS 3A), **Likin C. Simon Romero**, Rochester Institute of Technology.

Difference Equations and Applications (Code: SS 10A), **Michael Radin**, Rochester Institute of Technology.

Financial Mathematics (Code: SS 1A), **Tim Siu-Tang Leung**, Columbia University.

Frontiers in Applied and Industrial Mathematics (Code: SS 13A), **Kara L. Maki** and **David S. Ross**, Rochester Institute of Technology.

Geometric, Categorical and Combinatorial Methods in Representation Theory (Code: SS 12A), **David Hemmer** and **Yiqiang Li**, State University of New York at Buffalo.

Geometric Evolution Equations (Code: SS 21A), **Mihai Bailesteanu**, University of Rochester, and **Mao-Pei Tsui**, University of Toledo.

Inverse Problems and Nonsmooth Optimization: Celebrating Zuhair Nashed's 75th Birthday (Code: SS 7A), **Patricia Clark**, **Baasansuren Jadama**, and **Akhtar A. Khan**, Rochester Institute of Technology, and **Hulin Wu**, University of Rochester.

Mathematical Image Processing (Code: SS 20A), **Nathan Cahill**, Rochester Institute of Technology, and **Lixin Shen** and **Yuesheng Xu**, Syracuse University.

Microlocal Analysis and Nonlinear Evolution Equations (Code: SS 2A), **Raluca Felea**, Rochester Institute of Technology, and **Dan-Andrei Geba**, University of Rochester.

Modern Relativity (Code: SS 6A), **Manuela Campanelli** and **Yosef Zlochower**, Rochester Institute of Technology.

New Advances in Graph Theory (Code: SS 9A), **Jobby Jacob**, Rochester Institute of Technology, and **Paul Wenger**, University of Colorado Denver.

Nonlinear Dynamics of Excitable Media (Code: SS 18A), **Elizabeth Cherry**, Rochester Institute of Technology.

Nonlinear Partial Differential Equations in the Physical and Biological Sciences (Code: SS 14A), **Tony Harkin**, Rochester Institute of Technology, and **Doug Wright**, Drexel University.

Operator Theory and Function Spaces (Code: SS 4A), **Gabriel T. Prajitura** and **Ruhan Zhao**, State University of New York at Brockport.

Permutations Patterns, Algorithms, and Enumerative Combinatorics (Code: SS 19A), **Howard Skogman** and **Rebecca Smith**, State University of New York at Brockport.

Probability and Statistical Physics (Code: SS 16A), **Wenbo Li**, University of Delaware, and **Carl Mueller** and **Shannon Starr**, University of Rochester.

Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs (Code: SS 8A), **Bernard Brooks**, **Darren Narayan**, and **Tamas Wiandt**, Rochester Institute of Technology.

Symplectic and Contact Topology (Code: SS 15A), **Dusa McDuff**, Barnard College, and **Vera Vertesi**, Massachusetts Institute of Technology.

Wavelet and Frame Theoretic Methods in Harmonic Analysis and Partial Differential Equations in Memory of Daryl Geller (Code: SS 17A), **Alex Iosevich**, University of Rochester, and **Azita Mayeli**, City University of New York.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice as early as possible. Special discounted rates have been negotiated with the hotels listed below. Rates quoted do not include the combined New York state sales tax (8%) and occupancy tax (5%). Participants must state that they are with the **American Mathematical Society (AMS) Meeting at Rochester Institute of Technology** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details when making your reservation**.

Country Inn & Suites, Rochester-Henrietta, 4635 West Henrietta Road, Rochester, NY 14467; 585-486-9000; 800-596-2375 (toll free); `http://www.countryinns.com/rochester-hotel-ny-14467/nyrochso/locations.` Rates are US$99 for a standard room with two queen beds, double occupancy. This entire property is non-smoking. Amenities include complimentary continental breakfast; free coffee/cookies all day; free shuttle service to and from Greater Rochester International Airport Monday–Friday (with advanced registration); business center; in-room refrigerator, coffee maker, microwave oven; complimentary parking; free high-speed Internet in guest rooms; and fitness room. This property is located approximately four miles from the campus. **Cancellation and early check-out policies vary**; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 21, 2012**.

Best Western, 940 Jefferson Rd., Rochester, NY 14623; 585-427-2700; `www.bestwestern.com/rochestermarketplaceinn.` Rates are US$84.99 for single/double occupancy. Amenities include complimentary hot breakfast with omelet station; limited free shuttle service within five miles of the hotel (9 a.m.–5 p.m.); in-room coffee maker, microwave, and refrigerator; free in-room wireless Internet; 24-hour complimentary cookies, coffee, and tea in lobby; complimentary parking; and fitness center. This property is located approximately three miles from the campus. **Cancellation and early check-out policies vary**; be sure to check when you make your reservation. The deadline for reservations at this rate is **September 6, 2012**.

RIT Inn and Conference Center, 5257 West Henrietta Road, Henrietta, NY 14467; 585-359-1800; `www.RITINN.com`. Rates are US$94 per night for single or double/double occupancy for the dates of this meeting. Amenities include cable TV in guest rooms; in-room coffee maker; on-site business center; outdoor and indoor heated swimming pools, fitness center; two restaurants on property serving breakfast, lunch, and dinner; and free shuttle service to RIT and the airport. This property is located approximately one mile from the campus. **Cancellation and early check-out policies vary**; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 31, 2012**.

Courtyard by Marriott Brighton, 33 Corporate Woods, Rochester, NY 14623; 585-292-1000; `www.Marriott.com/ROCCH`. Rates are US$109 for single or double occupancy. Amenities include an in-room coffee maker; fitness center; indoor heated pool; business center; "The Market" 24-hour snack shop; on-property restaurant serving breakfast and dinner, featuring Starbucks coffee; and complimentary airport shuttle. This property is located approximately four miles from the campus. **Cancellation and early check-out policies vary**; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 31, 2012**.

Fairfield Inn by Marriott Airport, 1200 Brooks Avenue, Rochester, NY 14624; 585-529-5000; `www.Marriott.com/ROCFA.` Rates are US$114 per night for single/double occupancy. Amenities include an in-room coffee maker, complimentary deluxe continental breakfast, fitness center, indoor heated pool, complimentary wireless Internet, "The Market" 24-hour snack shop, and complimentary coffee service in the lobby. This property is located approximately four and one half miles from the campus. **Cancellation and early check-out policies vary**; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 31, 2012**.

RIT has arranged for **Monroe Transportation Services** to provide a limited shuttle service to and from these properties on September 22 and 23. Please inquire at the front desk of your hotel to learn of pick-up locations and times.

Dining on Campus

There are several options for dining available in the Student Alumni Union (Building 4), which is approximately a five-minute walk from the location of the meeting. Other options also very nearby are the Global Village Cantina and Crossroads located in the Global Village and the Ctrl Alt Deli located in Golisano Hall (Building 70). Hours of operation for the fall for these vendors will be posted on the RIT website sometime in late July or early August, `http://finweb.rit.edu/diningservices/hourofoperation.html`.

Local Information and Maps

A campus map may be found at `http://maps.rit.edu`. Information about Rochester Institute of Technology School of Mathematical Sciences may be found at `http://www.rit.edu/cos/math/welcome.php`. Please watch the website available at `http://www.ams.org/meetings/sectional/sectional.html` for additional information on this meeting. Please visit the RIT website at `http://www.rit.edu` for additional information on the campus.

Other Activities

Book Sales: Stop by the on-site AMS bookstore and review the newest titles from the AMS, enjoy up to 25% off all AMS publications, or take home an AMS t-shirt! Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

Parking

Hotels are all approximately between one and five miles driving distance from the RIT campus. Parking for the meeting is in Parking Lot U, located off Eleanor Gleason Circle and approximately 2 blocks from Thomas Gosnell Hall. This parking lot is open to anyone on weekends, and visitors will not require a parking permit.

Registration and Meeting Information

Registration will be held in the Atrium of Gosnell Hall. The AMS book exhibit will also be held in Gosnell Hall in Room 8-1300. All Invited Addresses will be held in the Van Peursem Auditorium in Gosnell Hall, room number 8-1250. Special Sessions will be held in Gosnell Hall, Gleason Hall, and Orange Hall. Please refer to the campus map at `http://maps.rit.edu` for specific locations. The registration desk will be open on Saturday, September 22, 7:30 a.m.-4:00 p.m.; and Sunday, September 23, 8:00 a.m.-12:00 p.m. Fees are US$53 for AMS members, US$74 for nonmembers; and US$5 for students, unemployed mathematicians, and emeritus members. Fees are payable on-site via cash, check, or credit card.

There will be a **reception** sponsored by the Rochester Institute of Technology, School of Mathematical Sciences. It will take place between 6:00 p.m. and 8:00 p.m. on Saturday, September 22, 2012, in the Alfred L. Davis room in the Student Alumni Union. The AMS thanks our hosts for their gracious hospitality.

Travel

RIT is approximately five miles from Greater Rochester International Airport (ROC).

By Air: The Greater Rochester International Airport (ROC) is located at 1200 Brooks Ave., Rochester, NY, approximately four miles southwest of the city of Rochester. Rochester is serviced by Air Canada, AirTran, American Eagle/American Airlines, Delta, Jet Blue, United and US Airways. Their website is `http://www.rochesterintlairport.com/`. Ground transportation options from the airport include shuttle service, public buses, taxi, and rental cars.

Some hotels offer complimentary shuttle service to the airport; please check with your hotel when you make your reservation. Shuttle service for hire to and from the airport is available through Around Town Shuttle service (585-227-9334).

Taxicabs are available through Advance Airport Taxi Service (585-235-3333). The taxi booth is located in front of the main terminal, center entranceway, lower level; rates are US$10.00 minimum to and from the airport, with US$2.50 for each additional passenger and US$3.00 per mile.

By Train: Amtrak provides service to the city of Rochester. The train station is located at 320 Central Avenue, Rochester, NY 14605 (800-872-7245). Information on schedules and fares can be found at `http://www.amtrak.com`.

By Bus: Bus service is offered by New York Trailways and Greyhound Bus Line. Both bus companies share a terminal at Midtown Plaza, which is located in downtown Rochester at 168 Cumberland St., Rochester, NY (585-232-5121). Information on schedules and fares can be found at `www.greyhound.com` and `www.trailwaysny.com`.

By Car:

From the Airport: Turn right onto Brooks Avenue, then right onto Interstate 390 South. From 390, take the Scottsville Road exit and turn right. Drive for approximately three miles, then turn left onto Jefferson Road. Travel east for approximately one-half mile to the campus.

From Interstate 90: Take Exit 46 and proceed north on Interstate 390 to Exit 13 (Hylan Drive). Turn left on Hylan and continue north to Jefferson Road. Turn left on Jefferson and proceed west for approximately two miles to the campus.

Car Rental: There are five car rental agencies with offices at the Greater Rochester International Airport. For reservations with AVIS please dial toll-free 800-831-2847 or visit their website at `www.avis.com`. For reservations with Hertz please dial toll-free 800-654-3131 or visit their website at `www.hertz.com`. For reservations with Budget please dial toll-free 800-527-0700 or visit their website at `www.budgetrentacar.com`. For reservations with National please call toll-free 877-222-9058 or visit their website at `www.nationalcar.com`. For reservations with Enterprise please call toll-free 800-325-8007 or visit their website at `www.enterprise.com`.

Local Transportation

Taxi Service:

Sentry Taxicab (585-235-7777; 800-taxicab (toll-free)) provides service throughout Monroe County 24 hours a day. This company provides taxicab service in both cars and vans which hold five to seven passengers.

Bus Service within Rochester: The airport and the city of Rochester are serviced by the Rochester Transit Service (RTS). Bus schedules are available at the Visitors Information Booth at the airport, can be obtained by calling RTS directly at 585-288-1700, or can be viewed by visiting the RTS website at `http://www.rgrta.com/`. The bus shelter at the airport is located on lower level roadway, east end of airport. RIT is also a stop on Route 24 of the RTS bus service. The current bus fare rate is US$1.

Weather

The average high temperature for September is approximately 72 degrees and the average low is approximately 52 degrees. Rain is common for this time of year. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at `http://sites.nationalacademies.org/pga/biso/visas/` and `http://travel.state.gov/visa/visa_1750.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `mac@ams.org`. If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns.

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country.

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application.

* Include a letter of invitation from the meeting organizer or the U.S. host specifying the subject, location and dates of the activity, and how travel and local expenses will be covered.

* If travel plans will depend on early approval of the visa application, specify this at the time of the application.

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

New Orleans, Louisiana

Tulane University

October 13–14, 2012

Saturday – Sunday

Meeting #1083

Southeastern Section
Associate secretary: Robert J. Daverman
Announcement issue of *Notices*: June/July 2012
Program first available on AMS website: September 6, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 3, 2012
For abstracts: August 28, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Anita Layton, Duke University, *Mathematical modeling of renal hemodynamics: Feedback dynamics and coupled oscillators*.

Lenhard Ng, Duke University, *From holomorphic curves to knot invariants via the cotangent bundle*.

Henry K. Schenck, University of Illinois at Urbana-Champaign, *From approximation theory to algebraic geometry: The ubiquity of splines*.

Milen Yakimov, Louisiana State University, *The Andruskiewitsch-Dumas Conjecture*.

Special Sessions

Algebraic Combinatorics: Rook Theory and Applications (Code: SS 4A), **Mahir Bilen Can** and **Michael Joyce**, Tulane University, and **Jeff Remmel**, University of California at San Diego.

Algebraic Structures over Commutative Rings (Code: SS 9A), **Lee Klingler**, Florida Atlantic University, **Aihua Li**, Montclair State University, and **Ralph Tucci**, Loyola University New Orleans.

Algebraic and Topological Combinatorics (Code: SS 10A), **Alexander Engstrom** and **Matthew Stamps**, Aalto University.

Analysis of Pattern Formation in Partial Differential Equations (Code: SS 8A), **Xuefeng Wang**, Tulane University.

Application of Functional Analytic Techniques to Nonlinear Boundary Value Problems (Code: SS 11A), **John R. Graef** and **Lingju Kong**, University of Tennessee at Chattanooga, and **Bo Yang**, Kennesaw State University.

Approximation Theory, Geometric Modelling, and Algebraic Geometry (Code: SS 7A), **Henry Schenck**, University of Illinois at Urbana-Champaign.

Biological Fluid Dynamics: Modeling, Computations, and Applications (Code: SS 5A), **Anita T. Layton**, Duke University, and **Sarah D. Olson**, Worcester Polytechnic Institute.

Combinatorial Commutative Algebra (Code: SS 1A), **Chris Francisco**, Oklahoma State University, **Tai Huy Ha**, Tulane University, and **Adam Van Tuyl**, Lakehead University.

Combinatorial Methods in Knot Theory (Code: SS 13A), **Heather Russell**, University of Southern California, and **Oliver Dasbach**, Louisiana State University.

Diffusion Processes in Biology (Code: SS 2A), **Gustavo Didier**, Tulane University, and **Greg Forest**, University of North Carolina, Charlotte.

Geometric and Algebraic Aspects of Representation Theory (Code: SS 12A), **Pramod N. Achar**, Louisiana State University, and **Dijana Jakeli'c**, University of North Carolina at Wilmington.

Interactions of Geometry and Topology in Low Dimensions (Code: SS 3A), **John Etnyre**, Georgia Tech, **Rafal Komendarczyk**, Tulane University, and **Lenhard Ng**, Duke University.

Quantum Groups and Noncommutative Algebraic Geometry (Code: SS 6A), **Kailash C. Misra**, North Carolina State University, and **Milen Yakimov**, Louisiana State University.

Stochastic Analysis: Current Directions and Applications (Code: SS 14A), **Hui-Hsiung Kuo**, **Ambar Sengupta**, and **P. Su**, Louisiana State University.

Session for Contributed Talks

There also will be a session for 10-minute contributed talks. Please see the abstracts submission form at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`. The deadline for all submissions is **August 28, 2012**.

Accommodations

Participants should make their own arrangements directly with the properties listed below. Special rates for the meeting have been negotiated and are available at the properties shown below for the period of October 12–14, 2012. When making reservations **participants should state that they are with the AMS/Tulane meeting**. Unfortunately, there are two citywide conventions in New Orleans at the same time as our meeting, so room blocks in the following hotels will sell out. **Please book early.** Hotels have varying cancellation or early checkout penalties; be sure to ask for details when making your reservation. **The room rates listed do not include applicable taxes; the current tax rate on hotel rooms is 13% + US$1 per night**.

All of these properties are within one to two blocks of the St. Charles Streetcar line, which goes through the beautiful Garden District and directly by the Tulane campus. The street cars run approximately every 20 minutes, 24 hours per day. The fare is US$1.25 each way (exact change), or you may purchase a Jazzy pass from the conductor, good for unlimited rides from the time of purchase until about 4:00 a.m. the next day for US$3 (exact change required).

Hampton Inn-Garden District Hotel, 3626 St. Charles Ave., New Orleans, LA 70115; phone: 504-899-9990; fax: 504-894-6414; `www.neworleanshamptoninns.com/garden-district`. US$139 for a king standard room; there is no charge for a second person. Rooms are equipped with a coffee maker and microfridge. Rates include complimentary hot breakfast buffet or on-the-run breakfast bag, free wireless Internet access, complimentary tea and cheese reception each evening, access to Touro Hospital Fitness Center, outdoor lap pool, and complimentary parking. Distance to the meeting site is 2.35 miles. **Hotels have varying cancellation or early checkout penalties**; be sure to ask for details when making your reservation. **The deadline for reservations is September 12, 2012**.

La Quinta Inn & Suites, 301 Camp St., New Orleans, LA 70130; phone: 504-598-9978; fax: 504-598-9978; `www.lq.com`. US$79 for king or double; coffeemaker in room. Rates include complimentary hot continental breakfast, free wireless access; fitness center and outdoor pool on premises. The distance to the meeting site is 4.75 miles. **Hotels have varying cancellation or early checkout penalties**; be sure to ask for details when making your reservation. **The deadline for reservations is September 21, 2012**.

Maison St. Charles, 1319 St. Charles Ave., New Orleans, LA 70130; phone: 504-522-0187; fax: 504-529-4379; `www.maisonstcharles.com`. US$145 for a double queen room (one or two persons). Rates include free wireless Internet access throughout the hotel, complimentary continental breakfast, fitness club access, and outdoor swimming pool with spa. On-site gated parking is US$17+tax/day. The distance to the meeting site is four miles. **Hotels have varying cancellation or early checkout penalties**; be sure to ask for details when making your reservation. **The deadline for reservations is August 28, 2012**.

Bed & Breakfasts in the Garden District on or near the St. Charles Streetcar line: While no contracts have been made with the following properties, and neither the AMS nor Tulane University can vouch for the suitability of them, participants may wish to check availability. Be sure to ask if rooms have private baths. Please note that the range of rates quoted may be seasonally adjusted and not available at the time of our meeting. Also be aware that **these properties usually have a 30-day cancellation policy** (one night's deposit will be forfeited), so please be sure to check. Some rates include taxes.

Avenue Inn Bed & Breakfast, 4125 St. Charles Ave; 504-269-2640; US$99-299.

Creole Gardens Guest House, 1415 Prytania St.; 866-569-8700; US$89-350.

Green House Inn, 1212 Magazine St. (downtown but two blocks from streetcar line), 504-525-1333; US$89-189.

Mandevilla Bed And Breakfast, 7716 St. Charles Ave., 800-288-0484 (toll-free) or 504-862-6396; US$99-179; about eight blocks from campus.

Magnolia Mansion, 2127 Prytania St., 504-412-9500; US$159-229.

Maison Perrier, 4117 Perrier St., 888-610-1807 or 504-897-1807; US$170-225 inclusive of all fees/taxes (three night minimum).

Park View Guest House, 7004 St. Charles Ave., 888-533-0746 (toll-free) or 504-861-7564; US$169-189; across from campus.

St. Charles Guest House, 1748 Prytania Ave., 504-523-6556; US$45-105.

Sully Mansion Bed and Breakfast, 2631 Prytania St., 800-364-2414 (toll-free) or 504-891-0457; US$119-230.

Terrell House, 1441 Magazine St., 866-261-9687 (toll-free) or 504-237-2076; US$150 and up.

Many other inns/bed and breakfast listings for the New Orleans area may be found at `www.bedandbreakfast.com/new-orleans-louisiana.html`. Be sure to check if your selection is on the St. Charles Streetcar line for the easiest and least expensive accessibility to the Tulane campus. For checking locations and the distance to campus using an Internet navigation tool (Google maps, Mapquest), please use 6823 St. Charles Ave., New Orleans, LA 70118, as Tulane's address.

Dining on/near Campus

The Food Court in the Lavin-Bernick Center will be open on Saturday and Sunday. There is also dining on the Loyola University campus next door to Tulane in the Danna Student Center (Orleans Room and Flambeaux's Grill) and La Divinia Gelateria in the bottom floor of Carrollton Residence Hall.

Local Information and Maps

The Tulane Department of Mathematics and Statistics website is found at `http://tulane.edu/sse/math/`. A campus map is found at `http://tulane.edu/about/visiting/uptown-campus-map.cfm`.

Other Activities

AMS Book Sale: Stop by the on-site AMS bookstore and review the newest titles from the AMS, enjoy up to 25% off all AMS publications, or take home an AMS t-shirt! Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

Parking

Parking on Saturday and Sunday is free in all campus lots and in metered spaces. You must avoid loading zones and other marked, restricted places.

Registration and Meeting Information

The meeting will take place on the uptown campus of Tulane University. Invited Addresses, meeting registration, and the AMS book exhibit will be held in the Lavin-Bernick Center. Special Sessions and Contributed Talks will be held in Jones, Boggs, and Newcomb Halls.

The registration desk will be open Saturday, 7:30 a.m.–4:00 p.m.; and Sunday, 8:00 a.m.–noon. Fees are US$53 for AMS members, US$74 for nonmembers; and US$5 for students, unemployed mathematicians, and emeritus members. Fees are payable on-site by cash, check, or credit card.

Travel Information

The nearest airport is Louis Armstrong New Orleans International Airport (**MSY**), 900 Airline Dr., Kenner, LA 70062; `http://www.flymsy.com`. The airport is about 12 miles from the campus, a 25-minute drive.

A cab ride costs US$35-40 from the airport to the Central Business District (CBD) or to campus for one or two persons and $14 (per passenger) for three or more passengers. Pickup is on the lower level, outside the baggage claim area. There may be an additional charge for extra baggage.

Airport Shuttle: Shuttle service is available from the airport to the hotels in the CBD or Garden District for US$20 (per person, one-way) or US$38 (per person, round-trip) with a limit of three bags per person. Call 1-866-596-2699 or (504) 522-3500 for more details or to make a reservation. Advance reservations are required 48 hours prior to travel for all ADA accessible transfers. Please call in advance of your travel date for the specially equipped shuttle to be reserved. Ticket booths are located on the lower level in the baggage claim area. `www.airport-shuttleneworleans.com`.

Driving Directions to Campus from I-10: As you enter the downtown area, follow the signs to Hwy 90 Business/West Bank. Exit at St. Charles Ave./Carondelet St. (do not cross the bridge). At the second traffic light turn right onto St. Charles Ave. and follow for four miles. Drive about four blocks past Gibson Hall and its half-circle driveway and turn right at the stoplight at the intersection of St. Charles and Broadway. Drive up Broadway about 3/4 mile (about eight blocks) and turn right onto Willow St. Drive two blocks and turn right onto Newcomb Drive. You may park anywhere along the roadside or in the lots; metered spots are free on weekends. The Lavin-Bernick Center is about a block away from the parking areas.

Car Rental

Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at `www.hertz.com`, click on the box "I have a discount", and type in our convention number (CV): **04N30002**. You can also call Hertz directly at 800-654-2240 (U.S. and Canada) or 405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

Weather

October in New Orleans is beautiful. The weather is cool and dry with bright blue skies overhead. Average high is 72°, average low is 63°, and the rainfall is low.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at `http://sites.nationalacademies.org/pga/biso/visas/` and `http://travel.state.gov/visa/visa_1750.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `dls@ams.org`.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns.

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country.

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application.

* Include a letter of invitation from the meeting organizer or the U.S. host specifying the subject, location and dates of the activity, and how travel and local expenses will be covered.

* If travel plans will depend on early approval of the visa application, specify this at the time of the application.

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Akron, Ohio

University of Akron

October 20–21, 2012

Saturday – Sunday

Meeting #1084

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: August 2012
Program first available on AMS website: September 27, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 10, 2012
For abstracts: September 4, 2012
The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Tanya Christiansen, University of Missouri, *Title to be announced.*

Tim Cochran, Rice University, *Title to be announced.*

Ronald Solomon, Ohio State University, *Title to be announced.*

Ben Weinkove, University of California San Diego, *Title to be announced.*

Special Sessions

Additive and Combinatorial Number Theory (Code: SS 14A), **Tsz Ho Chan**, University of Memphis, **Kevin O'Bryant**, City University of New York, and **Gang Yu**, Kent State University.

Applied Topology (Code: SS 9A), **Peter Bubenik**, Cleveland State University, and **Matthew Kahle**, Ohio State University.

Cayley Graph Computations and Challenges for Permutation Puzzle Groups (Code: SS 20A), **Morley Davidson**, Kent State University, and **Tomas Rokicki**, Radical Eye Software.

Commutative Algebra (Code: SS 8A), **Livia Hummel**, University of Indianapolis, and **Sean Sather-Wagstaff**, North Dakota State University.

Complex Analysis and Its Broader Impacts (Code: SS 5A), **Mehmet Celik**, University of North Texas at Dallas, **Alexander Izzo**, Bowling Green State University, and **Sonmez Sahutoglu**, University of Toledo.

Complex Geometry and Partial Differential Equations (Code: SS 4A), **Gabor Szekelyhidi**, University of Notre Dame, **Valentino Tosatti**, Columbia University, and **Ben Weinkove**, University of California San Diego.

Extremal Graph Theory (Code: SS 2A), **Arthur Busch**, University of Dayton, and **Michael Ferrara**, University of Colorado Denver.

Geometry of Algebraic Varieties (Code: SS 12A), **Ana-Marie Castravet**, **Emanuele Macrí**, and **Hsian-Hua Tseng**, The Ohio State University.

Graphs and Polytopes in Algebraic Combinatorics (Code: SS 10A), **Stefan Forcey**, University of Akron, and **Forest Fisher**, NOVA-Manassas.

Groups, Representations, and Characters (Code: SS 1A), **Mark Lewis**, Kent State University, **Adriana Nenciu**, Otterbein University, and **Ronald Solomon**, Ohio State University.

Harmonic Analysis and Convexity (Code: SS 7A), **Benjamin Jaye**, **Dmitry Ryabogin**, and **Artem Zvavitch**, Kent State University.

Interactions between Geometry and Topology (Code: SS 22A), **Dan Farley**, Miami University, **Jean-Francois Lafont**, Ohio State University, and **Ivonne J. Ortiz**, Miami University.

Issues in the Preparation of Secondary Teachers of Mathematics (Code: SS 21A), **Laurie A. Dunlap** and **Antonio R. Quesada**, University of Akron.

Knot Theory and 4-Manifolds (Code: SS 15A), **Tim Cochran** and **Christopher Davis**, Rice University, and **Kent Orr**, Indiana University.

Noncommutative Ring Theory (Code: SS 6A), **S. K. Jain**, Ohio University, and **Greg Marks** and **Ashish Srivastava**, St. Louis University.

Nonlinear Partial Differential Equations and Harmonic Analysis (Code: SS 19A), **Diego Maldonado**, Kansas State University, **Truyen Nguyen**, University of Akron, and **Nguyen Cong Phuc**, Louisiana State University.

Nonlinear Waves and Patterns (Code: SS 11A), **Anna Ghazaryan** and **Vahagn Manukian**, Miami University.

Separate versus Joint Continuity (Code: SS 23A), **Zbigniew Piotrowski** and **Eric J. Wingler**, Youngstown State University.

Spectral, Scattering, and Inverse Scattering Theory (Code: SS 3A), **Tanya Christiansen**, University of Missouri, and **Peter Hislop** and **Peter Perry**, University of Kentucky.

Statistical Genetics and Applications (Code: SS 17A), **Omar De La Cruz**, Case Western Reserve University.

Stochastic Processes and Applications (Code: SS 16A), **Oana Mocioalca**, Kent State University.

A Survey of Lattice-Valued Mathematics and Its Applications (Code: SS 18A), **Austin Melton**, Kent State University, and **Stephen E. Rodabaugh**, Youngstown State University.

Toric Algebraic Geometry and Beyond (Code: SS 13A), **Kiumars Kaveh**, University of Pittsburgh, **Benjamin Nill**, Case Western Reserve University, and **Ivan Soprunov**, Cleveland State University.

Tucson, Arizona

University of Arizona, Tucson

October 27–28, 2012

Saturday – Sunday

Meeting #1085

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2012
Program first available on AMS website: October 4, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 17, 2012
For abstracts: September 11, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael Hutchings, University of California Berkeley, *Quantitative invariants in four-dimensional symplectic geometry*.

Kenneth McLaughlin, University of Arizona, Tucson, *Random matrices, integrable systems, asymptotic analysis, combinatorics*.

Ken Ono, Emory University, *Adding and counting* (Erdős Memorial Lecture).

Jacob Sterbenz, University of California San Diego, *Regularity of hypberbolic gauge field equations*.

Goufang Wei, University of California Santa Barbara, *Comparison results for Ricci curvature*.

Special Sessions

Analytical and Numerical Approaches in Nonlinear Systems: Collapses, Turbulence, Nonlinear Waves in Mathematics, Physics, and Biology (Code: SS 9A), **Alexander Korotkevich** and **Pavel Lushnikov**, University of New Mexico.

Asymptotic Analysis of Random Matrices, Integrable Systems, and Applications (Code: SS 13A), **Ken McLaughlin** and **Nick Ercolani**, University of Arizona.

Biomathematics (Code: SS 17A), **Jim M. Cushing** and **Joseph Watkins**, University of Arizona.

The B.S. Degree in Mathematics in Industry (Code: SS 19A), **William Velez**, University of Arizona.

Differential Equations and Biological Systems (Code: SS 16A), **Patrick Shipman**, Colorado State University, and **Zoi Rapti**, University of Illinois at Urbana-Champaign.

Dispersion in Heterogeneous and/or Random Environments (Code: SS 2A), **Rabi Bhattacharya**, Oregon State University, Corvallis, and **Edward Waymire**, University of Arizona.

Geometric Analysis and Riemannian Geometry (Code: SS 4A), **David Glickenstein**, University of Arizona, **Guofang Wei**, University of California Santa Barbara, and **Andrea Young**, Ripon College.

Geometrical Methods in Mechanical and Dynamical Systems (Code: SS 3A), **Akif Ibragimov**, Texas Tech University, **Vakhtang Putkaradze**, Colorado State University, and **Magdalena Toda**, Texas Tech University.

Harmonic Maass Forms and q-Series (Code: SS 1A), **Ken Ono**, Emory University, **Amanda Folsom**, Yale University, and **Zachary Kent**, Emory University.

Hyperbolic Geometry (Code: SS 18A), **Julien Paupert**, Arizona State University, and **Domingo Toledo**, University of Utah.

Inverse Problems and Wave Propagation (Code: SS 7A), **Leonid Kunyansky**, University of Arizona.

Mathematical Fluid Dynamics and Its Application in Geosciences (Code: SS 20A), **Bin Cheng**, Arizona State University, and **Nathan Glatt-Holtz**, Indiana University.

Mathematical Physics: Spectral and Dynamical Properties of Quantum Systems (Code: SS 6A), **Bruno Nachtergaele**, University of California Davis, **Robert Sims**, University of Arizona, and **Günter Stolz**, University of Alabama, Birmingham.

Mathematics of Optical Pulse Propagation: Modeling, Analysis, and Simulations (Code: SS 8A), **Jason Fleischer**, Princeton University, and **Moysey Brio**, **Karl Glasner**, and **Shankar Venkataramani**, University of Arizona.

Motives, Algebraic Cycles, and K-Theory (Code: SS 11A), **Deepam Patel**, Indiana University, Bloomington, and **Ravindra Girivaru**, University of Missouri, St. Louis.

Representations of Groups and Algebras (Code: SS 5A), **Klaus Lux** and **Pham Huu Tiep**, University of Arizona.

Special Functions, Combinatorics, and Analysis (Code: SS 15A), **Diego Dominici**, SUNY New Paltz, **Tim Huber**, University of Texas - Pan American, and **Robert Maier**, University of Arizona.

Spectral Theory and Global Analysis (Code: SS 12A), **Lennie Friedlander**, University of Arizona, and **Klaus Kirsten**, Baylor University.

Topics in Commutative Algebra (Code: SS 10A), **Kristen Beck** and **Silvia Saccon**, The University of Arizona.

The Ubiquitous Laplacian: Theory, Applications, and Computations (Code: SS 14A), **Bin Dong** and **Lotfi Hermi**, University of Arizona.

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday

Meeting #1086

Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: August 7, 2012
For abstracts: September 25, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/national.html`.

Joint Invited Addresses

Robin Pemantle, David Rittenhouse Laboratories, *Zeros of polynomials and their importance in combinatorics and probability* (AMS-MAA Invited Address).

Emily Shuckburgh, Cambridge University, *Title to be announced* (AMS-MAA Invited Address).

AMS Invited Addresses

Gerard Ben Arous, Courant Institute- NYU, *Title to be announced.*

Jean Bourgain, Institute for Advanced Study, *Title to be announced.*

Laura DeMarco, University of Illinois at Chicago, *Title to be announced.*

Jordan Ellenberg, University of Wisconsin, *Title to be announced.*

Alice Guionnet, École Normale Supérieure de Lyon, *Title to be announced* (AMS Colloquium Lectures).

Robert Guralnick, University of Southern California, *Title to be announced.*

Cedric Villani, Institut Henri Poincaré, *Title to be announced* (AMS Josiah Willard Gibbs Lecture).

Call for MAA Contributed Papers

The MAA Committee on Contributed Paper Sessions solicits contributed papers pertinent to the sessions listed below. Contributed Paper Session presentations are limited to fifteen minutes, except in the general session, where they are limited to ten minutes. Please note that the dates and times scheduled for these sessions remain tentative.

Contributed Paper Sessions with Themes

Actuarial Education, organized by **Robert Buck**, Slippery Rock University, and **Thomas Wakefield**, Youngstown State University; Friday afternoon. Interest in actuarial science has increased tremendously over the past few years, with many institutions trying to start programs or upgrade existing programs. This session invites papers/

talks that focus on starting an actuarial science program, sharing ideas on various ways to structure actuarial science programs, describing how institutions can adjust to the constantly changing requirements of the actuarial organizations, and outlining specific and/or unique details of your actuarial science program.

Also appropriate would be information on helping students find jobs/internships and the expectations of actuarial employers as well as discussion of VEE credit, the SOA/CAS exam structure, meeting Associate or Fellowship requirements, and approaching the new CERA designation. In addition, information on available resources for actuarial education is always welcome as well as ideas for motivating student interest in actuarial science, such as actuarial science club activities or other outreach.

Many institutions are interested in offering actuarial science as an option for their students, and these types of papers/talks would help them get started. The session will focus primarily on what the SOA refers to as Introductory Undergraduate Actuarial Science Programs or Advanced Undergraduate Actuarial Science Programs, as opposed to Graduate or Research Programs. Sponsored by PRIMUS: Problems, Resources, and Issues in Undergraduate Mathematics Studies. Papers from the session may be considered for a special issue of PRIMUS on actuarial education.

Adding Modern Ideas to an Introductory Statistics Course, organized by **Brian T. Gill**, Seattle Pacific University; **Scott Alberts,** Truman State University; and **Andrew Zieffler**, University of Minnesota; Friday afternoon. Modern introductory statistics courses have evolved to place much greater emphasis on conceptual understanding, active learning in the classroom, use of real data, and use of technology. We invite submissions that provide details about learning activities, new technologies, resources, or new teaching methods that have proven successful in teaching introductory statistics courses. We particularly encourage submissions related to the use of (1) big datasets in introductory statistics, (2) randomization or bootstrap methods, (3) modeling, or (4) open source software. We encourage submissions related to a variety of types of intro courses, including face-to-face, online, or hybrid as well as courses for specialized audiences such as business, engineering, or biology. Submissions related to introductory courses for math and statistics majors are also welcome. Sponsored by the SIGMAA on Statistics Education. Presenters will be considered for the Dex Whittinghill Award for Best Contributed Paper.

Assessing the Effectiveness of Online Homework, organized by **Jason Aubrey**, University of Missouri; **John Travis**, Mississippi College; and **Joanne Peeples**, El Paso Community College; Saturday morning. Online homework systems such as open source systems WeBWorK and WAMAP, commercial systems such as WebAssign, MapleTA and others have matured over the past decade to the point where the use of such systems has become mainstream within the service curriculum in mathematics. Anecdotal evidence indicates that there are significant benefits.

This session provides an opportunity to report on efforts to assess the effectiveness of online homework. Instructors will have an opportunity to share innovative uses of online homework systems and to report on how successful these new approaches have been. Papers will focus on the use of metrics for assessing changes in student learning and behavior, including factors such as persistence, self-efficacy, and retention. Sponsored by the MAA Committee on Technologies in Mathematics Education (CTME), MAA Committee on Two-Year Colleges (CTYC), and the SIGMAA on Mathematics Instruction Using the Web.

Bridging the Gap: Designing an Introduction to Proofs Course, organized by **Sarah L. Mabrouk**, Framingham State University; Thursday afternoon. This session invites papers regarding the creation of "bridge" and introductory proofs courses and the effects of such courses on students' abilities to read, analyze, and write proofs in subsequent courses such as number theory, abstract algebra, and real/complex analysis courses in addition to numerical and applied mathematics courses. Information about textbooks, assignments/projects, and activities that help students to read and analyze statements as well as to understand when it is appropriate to use, for example, the contrapositive or proof by contradiction are of particular interest. Papers providing information about approaches that have not been successful are welcome as are those about how ineffective initial attempts were modified to help students to understand statement analysis, to recognize/write equivalent statements, to select appropriate rather than inappropriate methods of proof, to realize when proofs are complete or incomplete, and to use meaningful language and terminology in good proof writing while minimizing student frustration and the student's view that the instructor is being picky about sentence structure and diction. Papers providing evidence of course effectiveness in helping students to read, analyze, and write proofs are particularly encouraged.

Communicating Mathematics, organized by **Brian Katz**, Augustana College, and **Elizabeth Thoren**, University of California Santa Barbara; Saturday afternoon. Increasingly, college graduates are expected to have a suite of communication skills in addition to the technical skills specific to their majors. Simultaneously pedagogy at many high schools, colleges, and universities is shifting towards student-centered methods that require students to write and speak more in their math classes. As a result, students are being asked to develop and use significantly more communication skills in these classes. In this session, we will explore the ways that mathematics instructors support students as they speak and write for our classes, as well as the ways we prepare them to communicate after they leave the classroom.

Ideally the session will include scholarly presentations on (1) successful methods or assignments designed to improve written or oral communication skills and (2) the consequences of using writing and speaking in class for the students' skills, attitudes, and beliefs.

Computational Modeling in the Undergraduate Curriculum, organized by **Kurt Matthew Bryan**, **Joseph Eichholz**, and **Jeffery Leader**, Rose-Hulman Institute of Technology; Wednesday morning. The extraordinary growth of computing power is transforming how engineering, science, and mathematics are done. Math majors stepping into industry or applied graduate programs need to be proficient with the tools and modes of thought needed to exploit this power. This training often starts too late; however, inexpensive computing power is inspiring new undergraduate courses and programs in computational science, often within mathematics programs, and can and should change the way undergraduate mathematics courses like linear algebra, differential equations, and probability are taught.

We seek presenters to share examples illustrating the incorporation of high-performance computing into the undergraduate mathematics curriculum. Especially welcome are class activities and projects that illuminate how computing power is used to attack realistic problems previously inaccessible at the undergraduate level or lessons that use computing power to give a fresh take on traditional topics.

Developmental Mathematics Education, organized by **J. Winston Crawley**, and **Kimberly J. Presser**, Shippensburg University; Saturday morning. In recent years, the number of underprepared or math-anxious students coming to our colleges and universities has been growing. In order to help these students to be successful, we need to undertake new strategies for support services, courses offered, and perhaps even in our programs themselves. This session invites papers on all aspects of developmental mathematics education. In particular, what classroom practices are effective with such students and how does research in student learning inform these practices? For students interested in math-intensive majors such as the sciences, how can we best prepare these students for several subsequent mathematics courses? How can we best coordinate support services with the courses offered in our mathematics departments? We are interested in hearing presentations from across the spectrum of community colleges through four-year universities at this session.

Effective Strategies and Programs for Mentoring Women and Minorities in Mathematics, organized by **Jenna Price Carpenter**, Louisiana Tech University; **Jessica M. Deshler**, West Virginia University; and **Elizabeth A. Burroughs**, Montana State University; Thursday afternoon. Women (~45%) and minorities (ranging from ~6% for African American and Hispanic students to 0.4% for Native American students) have long been underrepresented in mathematics, from the B.S. to the Ph.D. level, as well as in the faculty ranks. There are, however, examples of initiatives which do successfully mentor women and minorities to success at all levels. This session focuses on strategies and programs (from one-on-one mentoring to funded programs) that effectively mentor these students or faculty in mathematics. Papers should refer to relevant research and include assessment where possible, share lessons learned, as well as focus on aspects that could be adopted by others. Sharing of example materials, brochures, websites, etc., are also encouraged.

Fostering Mathematical Habits of Mind, organized by **Kien H. Lim**, University of Texas at El Paso; **Ayse A. Sahin**, DePaul University; and **Holly Hirst**, Appalachian State University; Friday afternoon. The term "Mathematical Habits of Mind" (MHoM) refers to the need to help students think about mathematics the way mathematicians do. There has been considerable interest among mathematics educators and mathematicians in helping students develop MHoM. This session allows mathematicians and mathematics educators to share their scholarship, their teaching, and their perspectives related to fostering MHoM. We seek papers that focus on at least one of these areas: theoretical frameworks for analyzing MHoM, empirical studies on MHoM, pedagogical challenges and strategies for fostering specific MHoM, creating a classroom culture that is conducive to MHoM, or philosophical perspectives associated with MHoM. Sponsored by the MAA Committee on the Mathematical Education of Teachers (COMET).

The History of Geometry, Its Applications, and Their Uses in the Classroom, organized by **Amy Shell-Gellasch**, Hood College, and **Glen Van Brummelen**, Quest University; Saturday afternoon. The roots of geometry go back before recorded time, and almost all cultures have used it for some significant purpose (astrology, navigation, architecture, ritual, etc.). Although the art of geometry is currently waning in the high school curriculum, its relevance to practical applications continues to grow in the sciences and beyond. This session solicits papers that address topics relevant to the history of geometry and its applications. This might include (but is not limited to) physics, chemistry, biology, astronomy, navigation and its devices, architecture, cartography, networks, and trigonometry. Papers may be scholarly or pedagogical in nature. Sponsored by the SIGMAA on the History of Mathematics

How Assessment Results Changed Our Program, organized by **Miriam Harris-Botzum**, Lehigh Carbon Community College, and **Bonnie Gold**, Monmouth University; Wednesday morning. One of the purposes of assessment in higher education is to improve student learning and to improve our programs. Is there evidence that program assessment has made a positive difference in student learning in mathematics?

This session will provide faculty teaching mathematics, statistics, or quantitative literacy/reasoning courses the opportunity to disseminate how they have "closed the loop" in program assessment, making changes that have resulted in improvements in their programs, in their teaching, and ultimately in student learning. Presenters may talk about changes that have already been implemented and their impact or changes that are under way and their plans to assess the impact. Sponsored by the MAA Committee on Assessment.

Innovative and Effective Ways to Teach Linear Algebra, organized by **David M. Strong**, Pepperdine University; Friday morning. Linear algebra is one of the most interesting and useful areas of mathematics, due to its beautiful theory and the enormous importance it plays in understanding and solving many real-world problems. Many valuable and creative ways to teach its rich theory and applications are continually being developed and refined. This session will serve as a forum in which to share and discuss these ideas and approaches. Innovative and effective ways to teach linear algebra include, but are not limited to, (1) hands-on, in-class demos; (2) effective use of technology, such as Matlab, Maple, Mathematica, Java Applets or Flash; (3) interesting and enlightening connections between ideas that arise in linear algebra and ideas in other mathematical branches; (4) interesting and compelling examples and problems involving particular ideas being taught; (5) comparing and contrasting visual (geometric) and more abstract (algebraic) explanations of specific ideas; and (6) other novel and useful approaches or pedagogical tools.

Innovative Ideas for Courses in the First Two Years, organized by **Andrew Granville Bennett**, Kansas State University; Wednesday afternoon. With the increasing focus on retention and completion and calls for sharply increasing the number of students who pursue STEM majors, many programs are looking at revisions to their introductory mathematics program. This session looks to share ideas for content, instruction, and assessment for courses taken in the first two years.

Talks should not be purely aspirational but should include a discussion of how at least some segment of the proposal was implemented and the impact on students, either for better or for worse. We particularly encourage submissions about (1) innovative instructional techniques that increase student success, (2) new approaches to precalculus courses that better prepare students for calculus, (3) changes in pedagogy and/or curriculum that encourage more students to pursue additional coursework in mathematics, (4) methods to identify and remediate holes in students' knowledge, (5) better assessment techniques to identify conceptual understanding; and (6) other innovative ideas for teaching college mathematics. Sponsored by the MAA Committee on Calculus Reform and the First Two Years (CRAFTY).

Integrating the Mathematics of Planet Earth 2013 in the College Mathematics Curriculum, organized by **Ben Galluzzo**, Shippensburg University; Wednesday afternoon. Planet Earth is dynamic and complex; mathematics is a tool we can use to understand it. The NSF-funded North American Mathematical Sciences Institutes are sponsoring the theme of The Mathematics of Planet Earth in 2013 (MPE 2013) with the goal of showcasing the role that mathematics plays in recognizing, investigating, and solving planetary problems. In support of MPE 2013, this session seeks proposals that discuss methods for integrating Environmental Mathematics issues into the typical college curriculum. Accepted papers will be published on the SIGMAA EM website to increase awareness and encourage conversation about theme-related topics throughout the year. Sponsored by the SIGMAA on Environmental Mathematics and MPE 2013.

Learning Centers: Problems and Creative Solutions, organized by **James M. Sobota**, **Karoline Auby**, and **Maighread McHugh**, University of Wisconsin–La Crosse; Thursday morning. This session will deal with Learning Centers, primarily dealing with tutoring lower-level mathematics courses. We are looking to share creative solutions to Learning Center issues, such as the recruitment and training of quality tutors; how Learning Centers can help tutors develop their mathematical skills and understanding to better prepare them for careers in the teaching profession; how to deal with budget problems in these times of tight budgets; how to incorporate appropriate technology into the Learning Center; how to involve more faculty in the Learning Centers; and how to cooperate with various "special service" tutoring centers, possibly including those in other disciplines.

Learning Centers are becoming more and more important in the lower-level curriculum, especially with the increasing number of remedial students. This comes at a time when there are more and more demands on already tight budgets. Presentation proposals should focus on how your college or university has or is planning on dealing with some of these issues and how others can be helped by what you have learned.

Mathematics and the Arts: Practice, Pedagogy, and Discovery, organized by **Douglas Norton**, Villanova University; Thursday morning. This session provides the opportunity to share and learn from experiences at the intersection of mathematics and any of the visual, performing, musical, architectural, literary, fiber, sculptural, or other arts. Those who explore aesthetic consequences of mathematics, incorporate mathematical motivations or structures in their practice of the arts, teach modules or entire courses on math and one or more arts, or carry out investigations at the interface of the arts and mathematics are invited to share their experiences. Sponsored by SIGMAA on Mathematics and the Arts.

Mathematics Experiences in Business, Industry, and Government, organized by **Carla D. Martin**, James Madison University; **Phil Gustafson**, Mesa State College; and **Michael Monticino**, University of North Texas; Saturday morning. The MAA Business, Industry and Government Special Interest Group (BIG SIGMAA) provides resources and a forum for mathematicians working in business, industry, and government (BIG) to help advance the mathematics profession by making connections, building partnerships, and sharing ideas. BIG SIGMAA consists of mathematicians in BIG as well as faculty and students in academia who are working on BIG problems.

Mathematicians, including those in academia, with BIG experience are invited to present papers or discuss projects involving the application of mathematics to BIG problems. The goal of this contributed paper session sponsored by BIG SIGMAA is to provide a venue for mathematicians with experience in business, industry, and government to share projects and mathematical ideas in this regard. Anyone interested in learning more about BIG practitioners, projects, and issues will find this session of interest. Sponsored by the SIGMAA on Business, Industry, and Government.

Mathematics and Sports, organized by **R. Drew Pasteur**, College of Wooster; Thursday afternoon. Applications of mathematics are plentiful in sports, relating to probability, statistics, linear algebra, calculus, and numerical analysis, among other areas. This contributed paper session will feature various uses of mathematics to study phenomena arising from multiple sports. The expanding availability of play-by-play data for professional and some collegiate sports is leading to innovative kinds of analysis. This session will include both expository talks and presentations of original research; undergraduate students and their mentors are particularly encouraged to submit abstracts for consideration. With a broad audience in mind, all talks are requested to be accessible to undergraduate mathematics majors.

Mentoring Graduate Students: Pathways to Success, organized by **Jenna Price Carpenter**, Louisiana Tech University, and **Molly Fenn**, North Carolina State University; Friday afternoon. The goal of this contributed paper session is to share best practices, tips, resources, strategies, and answer questions about successfully mentoring graduate students. We will be looking for presenters who can share perspectives representing a variety of institution and degree sizes and types, as well as talks that focus on research-related issues and those that address larger professional development aspects of mentoring graduate students to become successful professionals. We hope to provide faculty with examples of multiple pathways that enable them to be great mentors. Sponsored by the MAA Professional Development Committee.

Philosophy, Mathematics, and Progress, organized by **Thomas Drucker**, University of Wisconsin Whitewater, and **Dan Sloughter**, Furman University; Friday afternoon. Mathematics as a discipline seems to make progress over time, while philosophy is often taken to task for not having made such progress over the millennia. When philosophy confronts issues related to mathematics, one natural topic is how mathematics succeeds in making progress while philosophy does not. One question to be addressed in this session is whether philosophy can help to explain the apparent progress displayed by mathematics. Another is whether the mismatch in progress between the disciplines is more apparent than real. As currents of mathematical change gather speed, perhaps a philosophical perspective is needed to make sure that contemporary practitioners do not lose their footing. Papers addressing issues of progress in mathematics and philosophical ways of understanding that progress will help to continue the conversation between mathematicians and philosophers. Sponsored by the SIGMAA on the Philosophy of Mathematics.

Preparing Elementary School Mathematics Specialists, organized by **Steve Morics**, University of Redlands, and **Klay T. Kruczek**, Southern Connecticut State University; Saturday afternoon. Over the last decade, there have been numerous calls for the use of mathematics specialists in elementary and middle schools. These specialists use their expertise to oversee the delivery of mathematics instruction in their schools by taking direct responsibility for classroom time and by mentoring their colleagues in their own mathematics instruction. Recently some institutions have begun degree or certificate programs to educate these mathematics specialists.

Papers will report on the preparation, placement, and support of mathematics specialists in the elementary grades. Papers may describe programs to prepare preservice or in-service teachers to become mathematics specialists, or may describe efforts with school districts to create positions and support for these specialists. Reports on the successful installation and implementation of mathematics specialists are also welcome. Papers should include evidence of success or the potential for application to other institutions or districts. Sponsored by the MAA Committee on the Mathematical Education of Teachers (COMET).

Projects, Demonstrations, and Activities That Engage Liberal Arts Mathematics Students, organized by **Sarah L. Mabrouk**, Framingham State University; Thursday morning. Many colleges and universities offer liberal arts mathematics courses (lower-level courses other than statistics, college algebra, precalculus, and calculus) designed for students whose majors are in disciplines other than mathematics, science, social science, or business. Students taking such courses have a variety of backgrounds, strengths, and levels of interest/comfort with mathematics.

This session invites papers regarding projects, demonstrations, and activities that can be used to enhance the learning experience for students taking liberal arts mathematics courses. Papers should include information about the topic(s) related to the project/demonstration/activity, preliminary information that must be presented, and the goal(s)/outcome(s) for the project/demonstration/activity. Presenters discussing demonstrations and activities are encouraged to give the demonstration or perform the activity if time and equipment allow, and to discuss the appropriateness of the demonstration/activity for the learning environment and the class size. Presenters discussing projects are encouraged to address how the project was conducted, presented, evaluated, as well as grading issues, if any, and the rubric used to appraise the students' work. Each presenter is encouraged to discuss how the project/demonstration/activity fits into the course, the use of technology, if any, the students' reactions, and the effect of the project/demonstration/activity on the students' attitudes towards and understanding of mathematics.

Research on the Teaching and Learning of Undergraduate Mathematics, organized by **Kyeong Hah Roh**, Arizona State University; **Stacy Brown**, Pitzer College; and **Mike Oehrtman**, University of Northern Colorado; Thursday morning.

This session presents papers that address issues concerning the teaching and learning of undergraduate mathematics, including theoretical and empirical investigations that employ quantitative and qualitative methodologies.

Proposals for reports of Research on Undergraduate Mathematics Education are invited. The research should build on the existing research literature and use established methodologies to investigate important issues in undergraduate mathematics teaching and learning. The goals of the session are to share high-quality research on undergraduate mathematics education with the broader mathematics community. The session will feature research in a number of mathematical areas, including linear algebra, advanced calculus, abstract algebra, and mathematical proof. Sponsored by the SIGMAA on Research in Undergraduate Mathematics Education.

The Scholarship of Teaching and Learning in Collegiate Mathematics, organized by **Jacqueline Dewar**, Loyola Marymount University; **Thomas Banchoff,** Brown

University; **Curtis Bennett**, Loyola Marymount University; **Pam Crawford**, Jacksonville University; and **Edwin Herman**, University of Wisconsin–Stevens Point; Friday morning. The scholarship of teaching and learning is a growing field of inquiry in which faculty bring disciplinary knowledge to bear on questions of teaching and learning that arise from classroom practice and use student-generated evidence to support their conclusions. Work in this area includes examination of the efficacy of pedagogical techniques, assignments, or technology, as well as probes of student understanding.

The goals of this session are to: (1) feature scholarly work focused on the teaching of postsecondary mathematics, (2) provide a venue for teaching mathematicians to make public their scholarly investigations into teaching/learning, and (3) highlight evidence-based arguments for the value of teaching innovations or in support of new insights into student learning.

Appropriate for this session are preliminary or final reports of postsecondary classroom-based investigations of teaching methods, student learning difficulties, curricular assessment, or insights into student (mis)understandings. Abstract submissions should have a clearly stated question that was or is under investigation and should give some indication of the type of evidence that has been gathered and will be presented. For example, papers might reference the following types of evidence: student work, participation or retention data, pre/post tests, interviews, surveys, think-alouds, etc.

Student Success in Quantitative Reasoning, organized by **Ray Collings**, Georgia Perimeter College; Thursday afternoon. Quantitative reasoning at the freshman/sophomore level is freshly emerging in new courses. Reports of developmental, management, teaching/learning outcomes, and success in serving other disciplines with these courses are encouraged. Sponsored by the MAA Committee on Two-Year Colleges (CTYC), and the SIGMAA on Quantitative Literacy.

Touch It, Feel It, Learn It: Tactile Learning Activities in the Undergraduate Mathematics Classroom, organized by **Jessica M. Libertini,** University of Rhode Island, and **Julie Barnes**, Western Carolina University; Friday morning. This session invites presentations describing activities that use tactile teaching methods in any mathematics classes. Some examples of tactile methods could include props that students can touch to understand concepts better, projects where students create physical models that represent a concept, or in-class activities where students work together to create a hands-on demonstration of their understanding of a particular concept. This session seeks presentations that focus on engaging students through interaction with props, use of manipulative materials, or even inviting students to physically become a part of a function or concept; this does not include technology demonstrations such as computer visualizations. We seek innovative and creative ways for physically involving students in mathematics. Presentations detailing how to integrate a particular activity into class, student reactions, educational benefits, difficulties to avoid, and possible modifications of the activity are desired.

Transition from High School to College: Alternative Pathways, organized by **Gail Burrill**, Michigan State University; Saturday afternoon. Should all students be prepared to take a traditional sequence of calculus courses? If not, what alternatives provide a mathematically rich, useful, and relevant experience for students? The session will highlight different mathematical pathways by sharing concrete examples of courses that provide options for mathematical experiences closely tied to a variety of student interests and career aspirations.

The issue of what high school mathematics prepares which students for which courses at colleges/universities has been of concern in the past. Recent evidence indicates the transition from high school to postsecondary mathematics is becoming even more problematic. In addition, the Common Core State Standards describe the mathematical expectations for all high school graduates and identify additional topics as necessary for the preparation of students intending to take advanced mathematics. Consequently, high school graduates will enter college with different backgrounds. The MAA/NCTM Committee on Mutual Concerns and the MAA Committee on Articulation and Placement are seeking papers that address, from either the high school or introductory college/university perspective, this transition challenge, ranging from rethinking the calculus sequence to the role of statistics courses to mathematically challenging quantitative literacy requirements. In all cases, the goals, prerequisites, and intended trajectory should be made explicit. Sponsored by the MAA/NCTM Committee on Mutual Concerns and the MAA Committee on Articulation and Placement.

Trends in Undergraduate Mathematical Biology Education, organized by **Timothy D. Comar**, Benedictine University; Saturday morning. This session highlights successful implementations of biomathematics courses and content in the undergraduate curriculum, entire biomathematics curricula, efforts to recruit students into biomathematics courses, undergraduate research projects, preparation for graduate work in biomathematics and computational biology or for medical careers, and assessment of how these courses and activities impact the students.

Several recent reports emphasize that aspects of biological research are becoming more quantitative and that life science students, including pre-med students, should be introduced to a greater array of mathematical, statistical, and computational techniques and to the integration of mathematics and biological content at the undergraduate level. Mathematics majors also benefit from coursework at the intersection of mathematics and biology because there are interesting, approachable research problems, and mathematics students need to be trained to collaborate with scientists in other disciplines, particularly biology.

Topics may include scholarly work addressing the issues related to the design of effective biomathematics courses and curricula, how to gear content toward pre-med students, integration of biology into mathematics courses, collaborations between mathematicians and biologists that have led to new courses, course modules, or undergraduate research projects, effective use of technology in biomathematics courses, and assessment issues. Sponsored by the SIGMAA on Mathematical and Computational Biology.

Using Inquiry-Based Learning in Mathematics for Liberal Arts Courses, organized by **Julian F. Fleron**, **Volker Ecke**, **Philip K. Hotchkiss**, and **Christine von Renesse**, Westfield State University; Friday morning. One of the biggest challenges in Mathematics for Liberal Arts (MLA) courses is engaging the students with the mathematics, since these courses are generally terminal courses that, usually are not connected to their major. Inquiry-Based Learning (IBL), a student-centered approach to teaching where the students are encouraged to learn mathematics without reliance on direct instruction from an authority, has shown to be a successful way to engage this audience.

In this session we are interested in seeing innovative ways of integrating IBL techniques into MLA courses. We are interested in presentations that model a successful activity that was used in an inquiry-based MLA course. In particular, presentations should illustrate the following: how the students were engaged in the mathematics; the specific questions/problems on which the students were working; how the class period was structured, and how the students responded to this type of instruction. Sponsored by PRIMUS: Problems, Resources, and Issues in Undergraduate Mathematics Studies. Papers from the session may be considered for a special issue of PRIMUS on Inquiry-Based Learning in Mathematics for Liberal Arts Courses.

Using Mobile Communication Devices for Mathematics Education, organized by **Lawrence Moore**, Duke University, and **Lila Roberts**, Clayton State University; Friday afternoon. The nature of communication has changed substantially in the last twenty years. In particular, the proliferation of mobile communication devices (cell phones, smart phones, tablets, laptops, etc.) has had a profound effect on the way people communicate. Many instructors view this proliferation as a challenge, for example, text messaging in class. This evolution of communication can also present new learning opportunities for our students. This session will give instructors who are using these communication systems in an innovative manner an opportunity to share their experiences using these new systems to enhance student learning and to report on their effectiveness.

Mobile communication devices can include cell phones, smart phone, tablets, networked calculators, or any other personal device having the ability to communicate wirelessly. The focus of the reports should be on how the use of these communication devices/tools improves student learning of mathematics inside or outside the classroom.

Depending on the number of papers submitted, all or some of the contributors will be asked to demonstrate their projects at an informal reception organized by the WEB SIGMAA. Sponsored by the Committee on Technologies in Mathematics Education (CTME) and the SIGMAA on Mathematics Instruction Using the Web.

Writing the History of the MAA, organized by **Victor J. Katz**, University of the District of Columbia; **Amy Shell-Gellasch**, Hood College; and **Janet Beery**, Redlands University; Wednesday morning. The session Writing the History of the MAA at the 2012 JMM provided opportunities for members to discuss their progress in writing histories of their sections or other aspects of the MAA. But as the MAA centennial approaches, it is important to complete these histories for publication. Thus, we invite section historians or individuals who have been researching this history to present more fully developed findings. We welcome section officers who presented in 2012 as well as members of sections not represented then. Furthermore, we invite those who have been working on other topics related to the MAA's history to present, especially those who can deal with the history of any MAA-sponsored projects or the accomplishments of a particular committee. This session is sponsored by the History Subcommittee of the Centennial Committee and is a follow up to the contributed paper session of the same name at the 2012 JMM. Sponsored by the History Subcommittee of the MAA Centennial Planning Committee.

General Contributed Paper Sessions

General Contributed Paper Sessions, organized by **Stephen Davis**, Davidson College; **Gizem Karaali,** Pomona College; and **Douglas Norton**, Villanova University; Wednesday, Thursday, Friday, and Saturday mornings and afternoons. This session accepts contributions in all areas of mathematics, curriculum, and pedagogy. When you submit your abstract you will be asked to classify it into one of the following areas: *Assessment and Outreach, Calculus, History and Philosophy of Mathematics, Interdisciplinary Topics, Mathematics Education, Mathematics and Technology, Modeling and Applications of Mathematics, Probability and Statistics, Research in Algebra and Topology, Research in Analysis, Research in Applied Mathematics, Research in Geometry and Linear Algebra, Research in Graph Theory and Combinatorics, Research in Number Theory, Teaching Introductory Mathematics, Teaching Mathematics beyond the Calculus Sequence, Assorted Other Topics* (does not fit into one of the stated topical general sessions).

Submission Procedures for MAA Contributed Paper Abstracts

Abstracts must be submitted electronically at `http://jointmathematicsmeetings.org/meetings/abstracts/abstract.pl?type=jmm`. Simply fill in the number of authors, click "New Abstract", and then follow the step-by-step instructions. **The final deadline for abstracts is Tuesday, September 25, 2012**; it is highly advised that you **submit your abstract well before the final deadline.**

You may give at most two talks in the "topical" sessions. If your paper cannot be accommodated in the session in which it is submitted, it will automatically be considered for the general session. You may give at most one talk in the general session, and the general session is open only to those who are not already speaking in one of the topical contributed paper sessions. Each session room is equipped with a computer projector, an overhead projector, and a screen.

N.B. Laptops are not provided; speakers should bring their own, or contact your organizer.

The organizer(s) of your session will automatically receive a copy of the abstract, so it is not necessary for you to send it directly to the organizer. All accepted abstracts are published in a book that is available to registered participants at the meeting. Questions concerning the submission of abstracts should be addressed to abs-coord@ams.org.

Oxford, Mississippi

University of Mississippi

March 1–3, 2013

Friday – Sunday

Meeting #1087

Southeastern Section
Associate secretary: Robert J. Daverman
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Patricia Hersh, North Carolina State University, *Title to be announced*.

Daniel Krashen, University of Georgia, *Title to be announced*.

Washington Mio, Florida State University, *Title to be announced*.

Slawomir Solecki, University of Illinois at Urbana-Champaign, *Title to be announced*.

Special Sessions

Algebraic Combinatorics (Code: SS 1A), **Patricia Hersh**, North Carolina State University, and **Dennis Stanton**, University of Minnesota.

Chestnut Hill, Massachusetts

Boston College

April 6–7, 2013

Saturday – Sunday

Meeting #1088

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 6, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Roman Berukavnikov, Massachusetts Institute of Technology, *Title to be announced*.

Marston Conder, University of Auckland, *Title to be announced*.

Alice Guionnet, École Normale Supérieure de Lyon, *Title to be announced*.

Yanir Rubinstein, Stanford University, *Title to be announced*.

Special Sessions

Algebraic and Geometric Structures of 3-Manifolds (Code: SS 3A), **Ian Biringer**, Yale University, and **Tao Li** and **Robert Meyerhoff**, Boston College.

Complex Geometry and Microlocal Analysis (Code: SS 2A), **Victor W. Guillemin** and **Richard B. Melrose**, Massachusetts Institute of Technology, and **Yanir A. Rubinstein**, Stanford University.

Homological Invariants in Low-Dimensional Topology. (Code: SS 1A), **John Baldwin** and **Joshua Greene**, Boston College.

Boulder, Colorado

University of Colorado Boulder

April 13–14, 2013

Saturday – Sunday

Meeting #1089

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 12, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: February 19, 2013

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Gunnar Carlsson, Stanford University, *Title to be announced*.

Joseph A. De Loera, University of California, Davis, *Title to be announced*.

Brendan Hassett, Rice University, *Title to be announced*.

Raphael Rouquier, University of California Los Angeles, *Title to be announced*.

Special Sessions

Associative Rings and Their Modules (Code: SS 1A), **Greg Oman** and **Zak Mesyan**, University of Colorado, Colorado Springs.

Dynamics and Arithmetic Geometry (Code: SS 2A), **Su-ion Ih**, University of Colorado at Boulder, and **Thomas J. Tucker**, University of Rochester.

Extremal Graph Theory (Code: SS 3A), **Michael Ferrara**, University of Colorado Denver, **Stephen Hartke**, University of Nebraska-Lincoln, and **Michael Jacobson**, University of Colorado Denver.

Themes in Applied Mathematics: From Data Analysis through Fluid Flows and Biology to Topology (Code: SS 4A), **Hanna Makaruk**, Los Alamos National Laboratory, and **Robert Owczarek**, University of New Mexico and Enfitek, Inc.

Ames, Iowa

Iowa State University

April 27–28, 2013

Saturday – Sunday

Meeting #1090

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: April 2013
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 27, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Kevin Costello, Northwestern University, *Title to be announced*.

Marianne Csornyei, University of Chicago, *Title to be announced*.

Vladimir Markovic, California Institute of Technology, *Title to be announced*.

Eitan Tadmor, University of Maryland, *Title to be announced*.

Special Sessions

Algebraic and Geometric Combinatorics (Code: SS 4A), **Sung Y. Song**, Iowa State University, and **Paul Terwilliger**, University of Wisconsin-Madison.

Cluster Algebras and Related Combinatorics (Code: SS 5A), **Gregg Musiker**, University of Minnesota, **Kyungyong Lee**, Wayne State University, and **Li Li**, Oakland University.

Generalizations of Nonnegative Matrices and Their Sign Patterns (Code: SS 3A), **Minerva Catral**, Xavier University, **Shaun Fallat**, University of Regina, and **Pauline van den Driessche**, University of Victoria.

Operator Algebras and Topological Dynamics (Code: SS 1A), **Benton L. Duncan**, North Dakota State University, and **Justin R. Peters**, Iowa State University.

Zero Forcing, Maximum Nullity/Minimum Rank, and Colin de Verdiere Graph Parameters (Code: SS 2A), **Leslie Hogben**, Iowa State University and American Institute of Mathematics, and **Bryan Shader**, University of Wyoming.

Alba Iulia, Romania

June 27–30, 2013
Thursday – Sunday

Meeting #1091

First Joint International Meeting of the AMS and the Romanian Mathematical Society, in partnership with the "Simion Stoilow" Institute of Mathematics of the Romanian Academy.
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses

Viorel Barbu, Universitatea Cuza, *Title to be announced.*

Sergiu Klainerman, Princeton University, *Title to be announced.*

George Lusztig, Massachusetts Institute of Technology, *Title to be announced.*

Stefan Papadima, Institute of Mathematics of the Romanian Academy of Sciences, *Title to be announced.*

Dan Timotin, Institute of Mathematics of the Romanian Academy of Sciences, *Title to be announced.*

Srinivasa Varadhan, New York University, *Title to be announced.*

Louisville, Kentucky

University of Louisville

October 5–6, 2013
Saturday – Sunday

Meeting #1092

Southeastern Section
Associate secretary: Robert J. Daverman
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 5, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Michael Hill, University of Virginia, *Title to be announced.*

Suzanne Lenhart, University of Tennessee, *Title to be announced.*

Ralph McKenzie, Vanderbilt University, *Title to be announced.*

Victor Moll, Tulane University, *Title to be announced.*

Philadelphia, Pennsylvania

Temple University

October 12–13, 2013
Saturday – Sunday

Meeting #1093

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 12, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Barry Mazur, Harvard University, *Title to be announced* (Erdős Memorial Lecture).

St. Louis, Missouri

Washington University

October 18–20, 2013
Friday – Sunday

Meeting #1094

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 20, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Ronny Hadani, University of Texas at Austin, *Title to be announced*.

Effie Kalfagianni, Michigan State University, *Title to be announced*.

Jon Kleinberg, Cornell University, *Title to be announced*.

Vladimir Sverak, University of Minnesota, *Title to be announced*.

Special Sessions

Algebraic and Combinatorial Invariants of Knots (Code: SS 1A), **Heather Dye**, McKendree University, **Allison Henrich**, Seattle University, and **Louis Kauffman**, University of Illinois.

Riverside, California

University of California Riverside

November 2–3, 2013

Saturday – Sunday

Meeting #1095

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: September 10, 2013

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael Christ, University of California Berkeley, *Title to be announced*.

Mark Gross, University of California San Diego, *Title to be announced*.

Matilde Marcolli, California Institute of Technology, *Title to be announced*.

Paul Vojta, California Institute of Technology, *Title to be announced*.

Baltimore, Maryland

Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15–18, 2014

Wednesday – Saturday

Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Albuquerque, New Mexico

University of New Mexico

April 5–6, 2014

Saturday – Sunday

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: April 2014
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 5, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: February 11, 2014

Lubbock, Texas

Texas Tech University

April 11–13, 2014
Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: September 18, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Tel Aviv, Israel

Bar-Ilan University, Ramat-Gan and Tel-Aviv University, Ramat-Aviv

June 16–19, 2014
Monday – Thursday
The 2nd Joint International Meeting between the AMS and the Israel Mathematical Union.
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

AMS Special Sessions

Nonlinear Analysis and Optimization, **Boris Mordukhovich**, Wayne State University, and **Simeon Reich** and **Alexande Zaslavski**, The Technion - Israel Institute of Technology.

Eau Claire, Wisconsin

University of Wisconsin-Eau Claire

September 20–21, 2014
Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: February 20, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: August 5, 2014

San Francisco, California

San Francisco State University

October 25–26, 2014
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: October 2014
Issue of *Abstracts*: To be announced

Deadlines
For organizers: March 25, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: September 3, 2014

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015
Saturday – Tuesday
Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015
Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Porto, Portugal

University of Porto

June 11–14, 2015

Thursday – Sunday
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday
Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday
Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday
Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2017
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Robert J. Daverman, Department of Mathematics, University of Tennessee, Knoxville, TN 37996-6900, e-mail: daverman@math.utk.edu; telephone: 865-974-6900.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/.

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 111 in the the January 2012 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

June 10–June 30, 2012: MRC Research Communities, Snowbird, Utah. (Please see http://www.ams.org/amsmtgs/mrc.html for more information.)

October 3–8, 2012: International Conference on Group Theory, Combinatorics, and Computing, at Florida Atlantic University, Boca Raton, FL (held in cooperation with the AMS). Please see http://www.math.fau.edu/ for more information.

AMERICAN MATHEMATICAL SOCIETY

For the Avid Reader

What's Happening in the Mathematical Sciences, Volume 8

Dana Mackenzie

Major mathematical developments made understandable to the general reader, bringing alive the presence of mathematics in the everyday world

What's Happening in the Mathematical Sciences, Volume 8; 2011; 129 pages; Softcover; ISBN: 978-0-8218-4999-6; List US$23; AMS members US$18.40; Order code HAPPENING/8

A Moscow Math Circle

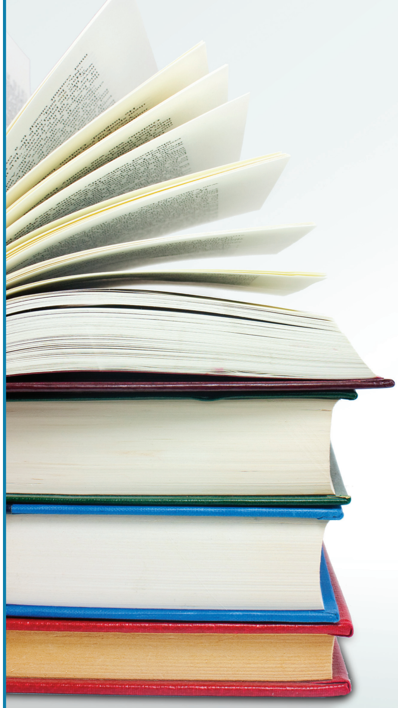
Week-by-week Problem Sets

Sergey Dorichenko, *Moscow Schools 57 and 179, Russia, and Kvant Magazine, Moscow, Russia*

Describes in detail how to run math circles, choose problems, and handle any difficulties

Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

MSRI Mathematical Circles Library, Volume 8; 2012; 240 pages; Softcover; ISBN: 978-0-8218-6874-4; List US$49; AMS members US$39.20; Order code MCL/8



The Mathematics of Voting and Elections: A Hands-On Approach

TEXTBOOK

Jonathan K. Hodge, *Grand Valley State University, Allendale, MI*, and **Richard E. Klima**, *Appalachian State University, Boone, NC*

This book answers perplexing questions about voting procedures, such as why a perfectly fair election can produce an outcome that nobody likes

Mathematical World, Volume 22; 2005; 226 pages; Softcover; ISBN: 978-0-8218-3798-6; List US$39; AMS members US$31.20; Order code MAWRLD/22

Stefan Banach

Remarkable Life, Brilliant Mathematics

Emilia Jakimowicz, *Gdańsk University, Gdańsk-Oliwa, Poland*, and **Adam Miranowicz**, *Adam Mickiewicz University, Poznań, Poland*, Editors

A meticulously researched and detailed account of the life of the Polish mathematician, revealing previously unknown facts

A publication of Gdańsk University Press. Distributed non-exclusively worldwide by the American Mathematical Society.

2010; 186 pages; Hardcover; ISBN: 978-83-7326-451-9; List US$55; AMS members US$44; Order code BANACH

Quanta of Maths

Etienne Blanchard, *University of Paris 7, France*, **David Ellwood**, *Clay Mathematics Institute, Cambridge, MA*, **Masoud Khalkhali**, *University of Western Ontario, London, ON, Canada*, **Matilde Marcolli**, *California Institute of Technology, Pasadena, CA*, **Henri Moscovici**, *Ohio State University, Columbus, OH*, and **Sorin Popa**, *University of California, Los Angeles, CA*, Editors

A marvelous tribute to the breadth and depth of Alain Connes' contributions to mathematics

Titles in this series are co-published with the Clay Mathematics Institute (Cambridge, MA).

Clay Mathematics Proceedings, Volume 11; 2010; 675 pages; Softcover; ISBN: 978-0-8218-5203-3; List US$129; AMS members US$103.20; Order code CMIP/11

Early Days in Complex Dynamics

A history of complex dynamics in one variable during 1906–1942

Daniel S. Alexander, *Drake University, Des Moines, IA*, **Felice Iavernaro**, *Università di Bari, Italy*, and **Alessandro Rosa**

This book paints a robust picture of the field of complex dynamics between 1906 and 1942

Co-published with the London Mathematical Society beginning with Volume 4. Members of the LMS may order directly from the AMS at the AMS member price. The LMS is registered with the Charity Commissioners.

History of Mathematics, Volume 38; 2011; 454 pages; Hardcover; ISBN: 978-0-8218-4464-9; List US$99; AMS members US$79.20; Order code HMATH/38

Understanding Numbers in Elementary School Mathematics

TEXTBOOK

Hung-Hsi Wu, *University of California, Berkeley, CA*

A thorough presentation of the basic facts about numbers for pre-service and current elementary school teachers

2011; 551 pages; Hardcover; ISBN: 978-0-8218-5260-6; List US$79; AMS members US$63.20; Order code MBK/79

History of Mathematics

Fifth Edition

Florian Cajori

A concise, interesting, and reliable account of mathematics history essential to every personal and departmental library

AMS Chelsea Publishing, Volume 303; 1991; 524 pages; Hardcover; ISBN: 978-0-8218-2102-2; List US$61; AMS members US$54.90; Order code CHEL/303.H

Mathematics Everywhere

Martin Aigner and **Ehrhard Behrends**, *Freie Universität Berlin, Germany*, Editors

Translated by Philip G. Spain

Everyday examples of how mathematics permeates the real world, in far-ranging areas from compact discs to climate change

2010; 330 pages; Softcover; ISBN: 978-0-8218-4349-9; List US$49; AMS members US$39.20; Order code MBK/72

AMS
AMERICAN MATHEMATICAL SOCIETY
BOOKSTORE
ams.org/bookstore

American Mathematical Society Distribution Center
35 Monticello Place, Pawtucket, RI 02861 USA

0002-9920(201206)59:6;1-J