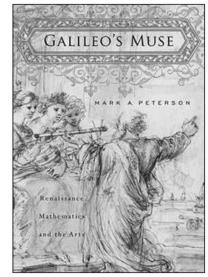
Galileo’s Muse: Renaissance Mathematics and the Arts



Reviewed by Anthony Phillips

Galileo’s Muse: Renaissance Mathematics and the Arts
Mark A. Peterson
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Galileo Galilei (1564–1642) is a pivotal figure in the development of Western science. Albert Einstein called him “the father of modern physics—indeed, of modern science altogether” [5]. More recently, Stephen Hawking wrote: “Galileo, perhaps more than any other single person, was responsible for the birth of modern science” [13].

Galileo’s trial and condemnation by the Inquisition in 1633 have become an international symbol of authority triumphing over knowledge. Since the issue was the structure of the solar system, the scandal has put in relief Galileo’s contributions to astronomy and cast into relative shadow the fundamental changes that he wrought in our intellectual approach to the natural sciences. These are surely what Einstein and Hawking refer to, and these are the subject of *Galileo’s Muse*.

Galileo’s stature as the father of modern science derives from his insistence on experiment as the way to verify hypotheses about nature and his recognition of mathematics as the medium in which hypotheses and experiments could be compared. As he says [7, Vol. 8, 212]:

> [reference to an experiment] is habitual and appropriate in those sciences which apply mathematical demonstrations to statements about nature, as we see with perspectivists, astronomers, mechanicians, musicians and others; they confirm their principles with experiments that may be perceived by the senses; these principles are the foundation for the whole ensuing structure.” [1]

We now know that advances in the sciences grow not only out of previous scientific work but also from the culture in which scientists live. Seeking out those roots in culture is especially interesting when the advances in question seem to break with scientific tradition. Gerald Holton, for example, set out “to explore how the cultural milieu Einstein found himself in resonated with and conditioned his science” [14]. He found a possible source of Einstein’s striving for uniform explanations of disparate phenomena in the influence of Goethe, a poet who had “an especially strong grip on the German imagination” of the day and who argued for “the primacy of unity in scientific thinking.”

Mark Peterson has carried out a similar program for Galileo. Rather than looking to literature, he maintains “that Galileo drew upon mathematical traditions in the arts in his scientific work.” (There is a powerful hint in the Galilean quotation above, which sets music and perspective on a par with astronomy and mechanics as experimental and mathematically deductive sciences.)

This review is organized to give an account of the main lines of Peterson’s argument, focusing more closely and more critically on the points most relevant to the development of mathematics. Full disclosure: I am a mathematician, which gives me a professional concern with scientific detail that may be out of place in judging a book written for a very wide audience, and I am also part Florentine,

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[1] *Translations from Italian are mine, except as noted. In this instance “experiments that may be perceived by the senses” for* sensate esperienze *differs from the “well-chosen experiments” that Peterson uses, following* [8]. *This conforms to the usage of the time* [6].

which gives me a personal interest in anything that concerns one of that city's most illustrious sons. I am not a historian of science. Some of the book's content, most notably the section on the *Oratio*, is possibly controversial in that context; I am not in a position to evaluate this material except to assess plausibility or nonplausibility for nonspecialists.

Peterson's strategy in *Galileo's Muse* is to sketch a selective panorama of the intellectual atmosphere in which Galileo developed, the late sixteenth century in Florence. Into this sketch is interpolated material about Galileo's life and scientific work, organized so as to highlight the effect of the artistic/literary milieu on the development of Galileo's scientific ideas.

Galileo's Muse begins with a section on Galileo as a humanist and on the classical legacy, those scientific works from (mainly Greek) antiquity that were available to Galileo and to his fellow scholars. Peterson reminds us that Galileo was the son of a musician and music theorist and that his early training was in the humanities. Galileo wrote elegant Latin, played the lute as well as any professional, developed an artistic taste so exquisite that the best painters of the period came to him for advice; his scientific career began only when he came upon Euclid, at about age twenty. Peterson might have added that Galileo was a connoisseur of literature and is still considered to be one of the best writers of Italian prose, if not the best of all.

There follow four two-chapter sections on the arts: Poetry, Painting, Music, Architecture. The book ends with three additional chapters and an epilogue. Two of those chapters cover the mathematics and science of the period and summarize how Galileo's humanist formation helped him transform them. The last concerns a document, the *Oration in Praise of Mathematics*, by Galileo's student Niccolò Aggiunti, which seems to reflect Galileo's thought and may even contain his own words. The epilogue is a brief examination of the Copernican controversy as seen in the context of Galileo's entire scientific life.

Peterson's "Poetry" section is mostly a search for mathematical elements in Dante's *Paradiso*. This is of interest in itself but does not seem to have any direct bearing on the subject of the book. I'll only say that I find his interpretation of the transcendental end of the poem in terms of the irrationality/unconstructability of π intriguing but too far-fetched to be useful.

The Visual Arts: Perspective, Error and *Chiaroscuro*

The great changes in art that occurred during the Renaissance included a new, quantitative approach to the visual world. As Peterson tells us, Filippo Brunelleschi was not satisfied with reading about classical proportions in Vitruvius: he went to Rome in 1405 and measured the remaining monuments. Leon Battista Alberti's *Della Pittura* (1435) gave a detailed explanation of how to draw a pavement correctly in perspective. It involved measuring the distance from the "eye" to the picture-plane and using that distance in a plane-geometric construction. Galileo trained as an artist and knew the perspective construction. In fact, the distinguished painter Cigoli (Lodovico Cardi, 1559–1613), a close lifelong friend, maintained that Galileo had taught him all the perspective he knew.

So art had become to some extent imbued with mathematics: mostly geometry, but with some actual measurement. Peterson suggests we look at it the other way: Renaissance art contributed to pulling geometry out of its abstract, platonic existence and into close contact with the real world of working artists; Galileo was part of this process. At the same time, as Peterson points out, artists like Piero della Francesca were explicitly aware of the accommodation necessary when applying a perfect theory to an imperfect medium; Peterson reasons that this sensibility was incorporated by Galileo into his understanding of the unavoidable error in any measurement (see below).

Peterson also remarks that Galileo's proficiency as an artist would have entailed a knowledge of *chiaroscuro*, the technique of using shading to convey relief. The training would have helped him correctly interpret the patterns of light and dark he observed on the surface of the moon as indications that the moon, far from being smooth, had mountain ranges and deep valleys. This line of thought appears, considerably elaborated, in a 1984 article by Samuel Edgerton [4], who writes, "I shall argue that we have here a clear case of cause and effect between the practice of Italian Renaissance Art and the development of modern experimental science."

Music: Tension on Strings

Galileo was closely tied to the contemporary community of artists, but his links to the world of music and music theory were even tighter. His father, Vincenzo Galilei (c. 1520–1591) was a renowned performer on the lute and a prominent music theorist; Marin Mersenne defers to him [18] in his 1625 *La Vérité des Sciences*. Most of Vincenzo's theoretical publications were devoted to the vexing and age-old problem of tuning.

The diatonic scale (our *do-re-mi-fa-sol-la-ti-do* or one of its cyclic permutations) was known to the Babylonians [16]; our names for the tones are more recent. The diatonic scale has five whole steps, *do-re-mi* and *fa-sol-la-ti*, along with two half steps, *mi-fa, ti-do*. Babylonians also had identified the most consonant musical intervals: the octave (eighth), the fifth, and the fourth, named for their span (8, 5, or 4 notes in the diatonic scale, starting from *do*). The Pythagorean discovery that these three intervals correspond to notes sounded by strings (of equal composition and tension) with length ratios 2:1, 3:2, and 4:3 brought music and mathematics together. But the alliance was always uneasy. Mathematicians wanted to make every interval correspond to a ratio of whole numbers. For example, the difference between the fourth and the fifth, i.e., the interval *fa-sol*, had to correspond to the ratio 9:8 (since $\frac{9}{8} \cdot \frac{4}{3} = \frac{3}{2}$). Since $\frac{9}{8}$ is not a perfect square, there was no rational way to make *mi-fa* half of *fa-sol*. And if some ratio of magnitudes could be found for *mi-fa*, there was still a problem, since $(\frac{9}{8})^6 \neq 2$. Many, many solutions were tried; all had to fall short somewhere. Artistic and technological progress in Europe (polyphony, placement of frets on lutes, tuning of harpsichords) meant that the theoretical tuning problems of antiquity had serious practical consequences. Peterson surveys this situation in some detail.

Vincenzo Galilei was an enthusiastic participant in the theoretical musical ferment of the day, but with a difference. In his writings on tuning, he constantly refers to *esperienza*, which sometimes means "experience" and sometimes definitely means "experiment". For example, in his 1589 *Discorso* … [10, p. 128] he writes, concerning the acoustical properties of metal strings versus gut: "Everyone can verify this as he pleases by experiment."

Vincenzo's experiments went beyond tinkering with instruments into a full-scale test of a physical law. Here the scientific innovation was profound enough (it made Vincenzo perhaps the first experimentalist in the history of European science [2]) that people have wanted to see Galileo involved in the process. One must keep in mind that Vincenzo and Galileo never mentioned each other in their published writings and that in Viviani's and Niccolò Gherardini's biographies [7, Vol. 19], which present themselves as based on conversations with Galileo, there is no mention of musical experiments at all; we only know that Galileo was living at home in the years directly before Vincenzo published the *Discorso* and that Galileo inherited Vincenzo's papers. Whether or not father and son performed these experiments together, as Peterson suggests, the story is worth repeating.

It had been thought throughout the Middle Ages that to raise the note produced by a plucked string to a note one octave higher, one could shorten the length by one-half *or double the tension*. The tension part was (probably incorrectly) attributed to Pythagoras by authorities of the late Roman Empire: Boethius, Macrobius, and Nichomachus, who wrote [17, p. 84]: "The weight on one string was twelve pounds, while on the other was six pounds. Being therefore in double ratio, it produced the octave, the ratio being evidenced by the weights themselves." Vincenzo challenged this statement in the *Discorso* [10, p. 104]: "Experiment…shows us that he who from two strings of equal length, thickness and quality would want to hear the Diapason [the octave], would need to hang from them weights that were not in the double…but in the quadruple proportion. The fifth will be heard whenever from the same strings one hangs weights in proportion 9:4 (*dupla sesquiquarta*), the fourth from those which would be [in proportion] 9:16 and the whole tone 9:8 from the [proportion] 64:81." On the next page, criticizing another traditional belief, he wrote: "This doctrine was published as the truth by Pythagoras, a man of very great authority; it was so well believed that even today it is accepted by some, who seek no further, satisfied just by Pythagoras having said it."

Vincenzo reported an additional experiment in one of the three essays, unpublished until recently [11], written after the *Discorso* and thus in the period 1588–1591. He reported that "the same thing [hearing the octave] will happen if equal weights are suspended from strings the thicknesses of which are in quadruple proportion, provided the length and goodness are the same."

Galileo's *Two New Sciences* was published some forty years after the experiments his father described. The late music historian Claude Palisca, after speculating on whether Vincenzo influenced Galileo or vice-versa, wrote, "While the possibility of such an influence is only conjectural, it is a striking fact that Galileo, in the section on consonances in the *Dialogues Concerning Two New Sciences*, repeats in the conversation between the two interlocutors, Sagredo and Salviati, the thought process that is documented in the discourses of Vincenzo Galilei" [18]. Galileo has Sagredo run through the relations between pitch and length, tension or thickness and justify them by reference to "true (*verissime*) experiments", but with no reference to where, by whom, or exactly how those experiments were carried out. In particular, Sagredo asserts that substituting a string of one fourth the thickness will give a note one octave higher. (Note that "thickness" must be interpreted as cross-sectional area for this to be correct). Salviati later suggests an improvement: the higher string should have one fourth the *weight*; this refinement obviates the thickness/cross-section

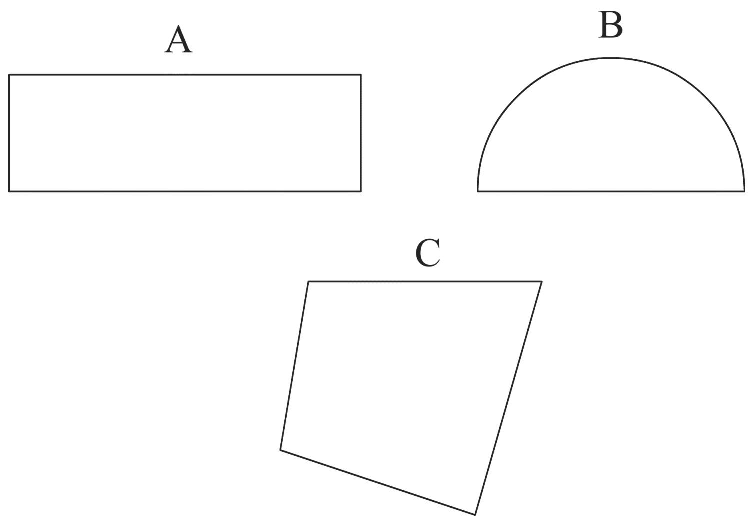

Figure 1. Galileo's illustration of three "magnitudes of the same type".

ambiguity and allows the rule to be applied to strings of different material. Galileo also goes beyond his father's experiments in relating pitch to frequency and in explaining the perception of consonance in terms of coherent vibrations of the eardrum.

From the evidence, to say that Galileo drew upon mathematical traditions in music theory would be a substantial understatement. He grew up exposed to reliance on physical experiment and also to the willingness to challenge traditional authority. This was in the domain of music theory, but the principles have wide application. Galileo spoke of his father's experiments as if they were his own and integrated them into his thought about periodic motion in general. On the other hand, the importance of the legacy of his father's scientific attitude towards tradition cannot be precisely gauged but should not be underestimated.

The Legacy of Antiquity

There is a substantial difference between the mathematics of 1500 and that of 1650. While algebra maintained a steady course through those times, the mathematics of magnitude changed radically. From a strictly mathematical viewpoint, the account of Galileo's participation in this change is the central part of *Galileo's Muse*, because it is part of the process by which we came to grips with real numbers.

The problem of how to extend arithmetic to quantities which might not be rational had already preoccupied the Greeks. Euclid's Book V, Definition 5, traditionally attributed to Eudoxus, states what it means for two *ratios of magnitudes* to be equal:

> Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.

In modern symbols, the definition of $\alpha : \beta = \gamma : \delta$ becomes

$$\forall\ m, n \in \mathbf{N}\ m\alpha \overset{>}{\underset{<}{=}} n\beta \Leftrightarrow m\gamma \overset{>}{\underset{<}{=}} n\delta.$$

Anachronistic replacement of β and δ by the unit gives

$$\forall\ \frac{n}{m} \in \mathbf{Q}^+\ \alpha \overset{>}{\underset{<}{=}} \frac{n}{m} \Leftrightarrow \gamma \overset{>}{\underset{<}{=}} \frac{n}{m},$$

suggestive of the modern definition of reals in terms of rationals *via* Dedekind cuts.

This definition did not have a smooth journey through the following centuries. As Peterson tells us [20, p. 43], it was badly garbled in the early translations into Arabic, and the ensuing nonsense assumed mystical significance during the Middle Ages. A correct rendition did not appear until around 1500 in Bartolomeo Zamberti's translation from the Greek, in time to be available to Galileo. Galileo considered it important enough to merit a long gloss in the projected Day Five of *Two New Sciences*, where he explains the composition of proportions using this example (Figure 1):

> Imagine two magnitudes A, B of the same type; the magnitude A will have a certain proportion to B; and now imagine another magnitude C placed amongst them, also of the same type: whatever proportion the magnitude A has to B is said to be composed of the two intermediate proportions, i.e., of that one which A has to C and of that one which C has to B" [7, Vol. 8, p. 360].

Galileo's choice of figures forces his readers to think of magnitudes, and their ratios, as having their own existence beyond the arithmetic of the counting numbers.

When he comes to physical laws, Galileo states them in terms of equality of ratios of magnitudes. The late Galileo scholar Stillman Drake considered this a drawback: "The price Galileo paid for rigor in the avoidance of algebra and the use of Eudoxian proportion theory was that his mathematical physics was restricted to comparisons of ratios" [9, Introduction, p. xxiii]. I believe this is a misunderstanding. Galileo measured time intervals using a water clock. There was no useful unit of time available to him, but working with a constant flow allowed him to correctly reckon the relative lengths of time intervals, exactly what he needed for a concise formulation of his law of falling bodies. The advantage of equations and functions in the development of calculus and in the full elaboration of laws of motion lay in the future.

The relations between ratios that Galileo published were based on ratios of measurements he had carried out. Folio 117v[2] in the collection of his manuscript notes in the Biblioteca Nazionale Centrale in Florence—first published in 1973 by Stillman Drake (see [3]; he dates it to 1608)—gives a nice example. In this experiment the inputs were the heights (300, 600, 800, 828, 1000 *punti*; a *punto* is slightly less than a millimeter) at which a ball was released to roll down a track. At the end of the track the ball was deflected to the horizontal and allowed to fall to the floor, 828 *punti* below. The outputs were the horizontal distances (800, 1172, 1238, 1340, 1500 *punti*) the ball had traveled after leaving the track and before hitting the ground. Galileo's theory of uniform acceleration due to gravity predicted that if H_1, H_2 were two release heights, D_1, D_2 the corresponding horizontal traces, then the ratio $D_1 : D_2$ should be the same as the ratio $\sqrt{H_1} : \sqrt{H_2}$. He used the first output measurement (800) to predict what the other four should be; he recorded in his notes the predicted values, the measurements, and the differences: 41, 22, 11, 40. Galileo famously wrote in *Il Saggiatore* that the book of nature is "written in mathematical language, and the characters are triangles, circles and other geometric figures…" [7, Vol. 6, p. 232]. What he did not write (the words did not then exist), and what this experiment beautifully exemplifies, was that the link between the geometric figures of theory and the book of nature as manifested in the laboratory was the exacting measurement of magnitudes in terms of real numbers.

There are in fact very few numbers in *Two New Sciences*, and those are all integers. Most of them are in Day Four, which contains three tables giving properties of trajectories with initial angles ranging from 1 to 89 degrees. Two of them give the height and the width of a semiparabola, assuming a constant initial velocity that is chosen so that initial angle 45° gives range 10000 (the units are not mentioned). The third gives the energy required to achieve the range 10000, assuming that initial angle 45° will require the kinetic energy of a fall from 5000 of the same units. Although the entries in the table are whole numbers, their large size guarantees several significant digits most of the time. In the first table, for example, the values go from 349 (for 1° and 89°) and 698 (for 2° and 88°) to 10000, with all the other angles giving four digits. This practice is congruent with the format of the mathematical tables of the time. In the table of tangents that Galileo used for the first table, the tangent of 45° was recorded as 10000, as he tells us. James Napier's use of the decimal point in his 1614 table of logarithms is said to have contributed to its rapid diffusion throughout Europe [15], but he used it sparingly: he records the sine of 90° as 1000000.0.

There is one work of Galileo's that he may have planned to publish but never did and that shows him at work with what are essentially finite decimals. Along with his 1596 *La Bilancetta*, he prepared two tables listing samples of various substances with their weights in air and in water. These tables were not published during his lifetime but are appended to *La Bilancetta* in Volume 1 of [7]. Let's call the two weights w_1 (in air) and w_2 (in water); the weights are recorded in *grani* (1 *grano* = 0.049g [1]) as integers plus a fraction, with denominators mostly low powers of 2. To be comparable, the pairs of weights are all normalized to have weight in air equal to 576, the weight of Galileo's Gold sample. Normalized relative weights $576 \times w_2/w_1$ are computed for seventeen of the samples. Of those, thirteen have denominator 100, two have denominator 60, one has denominator 2, and one is whole. For me, the predominance of denominator 100 strongly suggests that Galileo was carrying long division past the units and recording the next two digits with rounding. For example, one sample of copper has $w_1 = 179\frac{9}{16}$ and $w_2 = 159$. Division yields $576 \times w_2/w_1 = 510.039\ldots$; Galileo records $510\frac{4}{100}$.[3]

These three examples show Galileo at work with numbers. In the first the *punto*, his unit of length, is small enough with respect to the scale of his experiment for three or four significant figures to be achieved by whole numbers of *punti*. In the second his use of a trigonometric table with $\tan 45° = 10000$ similarly allows four significant figures from whole numbers. In the third we see decimal fractions with denominator 100 and again four or five significant figures. Why did Galileo, writing in 1638, never use the decimal point? It had been over twenty years since Simon Stevin's logarithmic tables; Galileo's friend Clavius

[2]*The page is reproduced in* [3]. *An interactive image is now available through the Biblioteca Nazionale Centrale via the Museo Galileo in Florence and the Max Plank Institute for the History of Science in Berlin,* `http://www.imss.fi.it/ms72/INDEX.HTM`.

[3]*Galileo records* $510\frac{15}{100}$ *and* $510\frac{75}{100}$ *for two other measurements from this sample and another. Peterson asserts that "Galileo only scaled the first sample in each pair [of samples]. Not scaling the second one, as if to avoid the embarrassment of seeing how different it was, is a bit odd, but psychologically not hard to understand." In this one case, his account is incorrect.*

had published a table with decimal points as far back as 1593 [12]. The answer may well be that he did not need it. One can speculate that he was leery of a notation that implied a possible infinity of digits trailing off to the right, but as far as I know there is no evidence one way or the other. Galileo's published works stay on traditional, solid ground with whole numbers and proportions of magnitudes, while his notebooks and his table of densities show the essential role that measurements, reported in numbers, played in his scientific work.

This is a different view of Galileo's mathematical position from that proposed in *Galileo's Muse*. Peterson emphasizes Galileo's use of proportions in lieu of numbers. "The notion of proportion, central in all the arts, took on a new significance in Galileo's work" [20, (p. 289)]. This statement has an additional problem: "proportion" in the arts means linear proportion; the proportions that Vincenzo and Galileo used, of one number with the square root or the $\frac{3}{2}$ power of another, do not make artistic sense. It is true that "is proportional to" is an essential connector in physics, but such proportionalities as $\ddot{x} \sim -\sin(x)$, the equation for the pendulum, are not those that artists consider; adding a friction term to produce $m\ddot{x} + k\dot{x} + a\sin(x) = 0$ generalizes proportions to linear combinations, which are still only special cases of equations of motion. So calling physics "the science of proportionalities in nature", as Peterson does (p. 291), is not completely appropriate.

The Architecture of Dante's *Inferno*: Scaling Problems

One of Galileo's most penetrating observations concerns scaling. This is in fact the first of the "Two New Sciences". As Peterson presents it, Galileo argues that the weight of a beam scales as its volume, but its strength as its cross-section, and concludes that any beam, scaled up geometrically, will eventually collapse under its own weight.[4] He extends the lesson to biology: "Nature could not make a horse as large as twenty, or a giant ten times as tall as a man, except either by miracle or by substantially changing the proportions of the limbs, and in particular of the bones, making them much, much thicker than ordinary ones" [7, Vol. 8, pp. 52–53]. Peterson relates this insight to Galileo's work on the architecture of Hell, as organized and depicted by Dante in the *Inferno* section of the *Divine Comedy*.

[4]*More specifically, Galileo argues with moments, and concludes that the strength is proportional to the diameter cubed and inversely proportional to the length. This is what is taught today, except that for rectangular beams, instead of diameter cubed we have width times the square of the height.*

Here are the relevant facts that are known today.

- Galileo gave two lectures in 1588 to the Florentine Academy, where he defended the model of Dante's Hell due to Antonio Manetti (1423–1497). Manetti had given a mathematically precise description of its location, extent, and structure. The overall shape was a cone with vertex at the center of the Earth, axis passing through Jerusalem and radius one twelfth of the earth's circumference, some 2,000 of our miles. The cone was essentially hollow (the Circles of Hell were terraces around the interior), with a roof some 500 miles thick. The lectures were very well received. In one of the lectures he argued from a scale model that the roof of Hell would be perfectly stable.
- In 1589 Galileo began a professorship at Pisa (controlled by Florence since 1509). He was appointed by Grand Duke Ferdinand I after being highly recommended by Guidobaldo dal Monte, a marquis and a well-respected mathematician.
- In 1592 Galileo was named professor of mathematics in Padua in the Venetian Republic. "In his first years there he…consulted for the Venetian Arsenal concerning the placement of oars on large ships" [20, p. 229].
- In 1594 Luigi Alamanni tried without success to have Galileo send him a copy of the *Inferno* lectures.
- In 1609 Galileo wrote to Antonio de' Medici: "And just lately I have succeeded in finding all the conclusions, with their proofs, pertaining to forces and resistances of pieces of wood of various lengths, sizes, and shapes, and by how much they would be weaker in the middle than at the ends, and how much more weight they can sustain if the weight were distributed over the whole rather than concentrated at one place, and what shape wood should have in order to be equally strong everywhere: which science is very necessary in making machines and all kinds of buildings, and which has never been treated before by anyone." (Translation from [19]; this item is not mentioned in *Galileo's Muse*.)
- Galileo's first biographer, Vincenzo Viviani, who lived "in Galileo's house during his last years, collecting Galileo's stories" [20, (p. 230)] does not mention the *Inferno* lectures.

- In 1638 Galileo published *Two New Sciences*, where the scaling of the strength with respect to linear dimensions is discussed in detail. He introduces the topic at the start of Day 1: his interlocutors are, or just were, at the Venice Arsenal and ponder what they had heard from one of the workmen in charge ("that good old fellow"): a large military galley under construction was in need of extra support to prevent "its breaking its back under the huge weight of its own vast bulk, a trouble to which smaller craft are not subject" [7, Vol. 8, pp. 49–50].
- When Galileo's stand-in, Salviati, explains that "there is a limit [in size] beyond which neither nature nor art can exceed: exceed, I say, while always keeping the same proportions and the same material," his interlocutor Sagredo reacts emotionally: "I feel my brain turning over and, like a cloud suddenly opened by lightning, my mind being flooded by a momentary and unusual light." [7, Vol. 8, pp. 49–50].

Peterson interprets the circumstantial evidence here quite reasonably. Galileo's defense of Manetti had been terribly flawed: the posited roof of Hell would have collapsed instantly under its own weight; he would have realized the error after moving to Padua, perhaps literally at the Venetian Arsenal. Since the *Inferno* lectures came just before he assumed his professorship at Pisa, knowledge of the error would have been extremely embarrassing to him and to his sponsors. That prospect would have concentrated his mind to work out the theory correctly; at the same time he would have done his best to avoid any further attention being paid to the lectures. One incongruity is Peterson's suggestion that the analysis may have been done "very soon after the Inferno Lectures;" this contrasts with the "recently" in Galileo's 1609 letter to Antonio de' Medici. It is also unfortunate that no reference is given for Peterson's assertion about Galileo's consulting for the Arsenal during his first years in the Venetian Republic. But this does not detract from the main point: it is very plausible that the detailed study of the architecture of Hell led Galileo to his trail-blazing study of strength of materials and of scaling laws.

The *Oration in Praise of Mathematics*

The *Oration* was written in Latin under the name of Niccolò Aggiunti and published in 1627. Peterson advances and defends the hypothesis that this work not only reflects the thoughts of Galileo (Aggiunti, twenty-seven years old at that time, had been "a devoted follower of Galileo for four years") but in many places reproduces the words of the master himself. Peterson's arguments are manifold and quite convincing (at least to me). Nevertheless, the *Oration* was not chosen by Antonio Favaro, the editor of Galileo's twenty-volume complete works, for inclusion in that series even though Favaro did include many ancillary documents besides Galileo's own writings. Some of Peterson's most compelling evidence comes from the *Oration* itself. Here is one of several quotations from the work:

> All earthly objects show forth the divine mathematics to those who observe them with close attention. They proclaim with utmost clarity that God is the Archgeometer: the movements of the stars; the balance of the earth; the absorption by plants of moisture from the ground through fibrous pipes; the penetration of the moisture to the leaves by means of veins running through the whole trunk and branches; the swimming, flight and crawling of fishes, birds and reptiles—obviously a subtle hidden mathematics underlies all these phenomena.

As Peterson remarks, "The Orator flatly contradicts Ptolemy's sharp distinction between philosophical mathematics and earthly physics," an opinion that "would be someday unremarkable but was close to unthinkable in 1627" and not what one might expect from "a young professor of mathematics".

Comparison of this passage with, say, the "Fable of the Sounds" in *Il Saggiatore*, keeping in mind the several translations involved,[5] makes it quite plausible that we are listening to Galileo's own twice-filtered voice.

Whether or not we accept the *Oration* as completely Galilean, it is hard not to enjoy the Orator's enthusiastic account of how "charming, thrilling and useful" it is to use a microscope, quoted at length by Peterson on p. 282.

Exposition, Divulgation, and the Appropriateness of Accuracy

Galileo's Muse is a good read. The style is comfortably conversational,[6] veering only occasionally into the colloquial. The book bubbles with ideas, insights, and delicious bits of historical detail. Most mathematicians will be amazed to learn that

[5] *Hypothetically, Aggiunti translated Galileo's Italian into Latin; this translation into English is due to Philippa Gould.*
[6] *The text is abundantly but discreetly footnoted; the notes are gathered at the back of the book and grouped by page number, an excellent practice.*

Dante's model for the entire universe was topologically a 3-sphere, with God and Satan at the poles; it's all spelled out in *Paradiso*, Canto XXVIII.

The easy reading and the immediacy of the story come at a price. In *Galileo's Muse* there is a hypothetical sentence on almost every page, usually correctly characterized as such. There are also hundreds of real facts, references, quotations from the period, and historically documented occurrences. The reader must constantly remember whether we are in "did" or "must have done" territory. Most of the time this is not a problem, but in places the boundary becomes fuzzy. The student reading "Pythagoras used his knowledge benevolently,..." (p. 168) may forget the "according to stories" seven sentences before and may not realize that almost nothing is known about the historical figure Pythagoras, certainly not whether he used his knowledge benevolently. This sentence, as well as "Pythagoras discovered the small integer ratios in music" on the next page, has to be interpreted as a statement about a semimythical figure. Similarly (I referred to this point earlier), when reading about Vincenzo Galilei's musical experiments with strings and weights, we find "His son Galileo...must have participated in these experiments." The "must have" is a red flag, of course. But then we read, "they repeated the experiments ...," "they found...," "they had to quadruple the weight...," "[Galileo's] first encounter with the issue of measurement uncertainty must have been this experiment in music," "the Pythagorean experiments on tension and musical pitch that Galileo conducted with his father..." (p. 257), as if it were a known fact that Galileo took part in his father's experiments. But it is not.

Scientists writing for a general audience, especially about the history of science, are in dangerous territory. Data are often lacking, the existing data are often unreliable, but the methods and logic have to be airtight; when they cannot be so, this circumstance should be fully understood by the reader. How to achieve this understanding without weighing down the text with a noxious drone of caveats is a conundrum. But creating false impressions must be avoided.

Readers of *Galileo's Muse* may expect, from the subtitle, a survey of the arts and mathematics in the Renaissance. I spoke above of a selective panorama; in fact, the choice of topics is somewhat more haphazard. Besides material that is germane to the thesis of the work, some is included just because it makes such a good story (the topology of Paradise mentioned above or the Cardano-Tartaglia *imbroglio* or speculation about what Luca Pacioli and Leonardo da Vinci actually thought of each other). But it is a good story and gives the reader a lively introduction to a period of ferment in art, music, and mathematics, and to Galileo, whose wide-encompassing intellect brought them all together.

References

[1] Roberto Caffarelli, *Il Laboratorio di Galileo Galilei*, Physics Department, Università di Pisa, 2005.

[2] Stillman Drake, Renaissance music and experimental science, *Journal of the History of Ideas* **31**, 483–500.

[3] Stillman Drake and James MacLachlan, Galileo's discovery of the parabolic trajectory, *Sci. Amer.* (March 1975), 102–110.

[4] Samuel Y. Edgerton, Galileo, Florentine "Disegno", and the "Strange Spottednesse" of the moon, *Art Journal* **44** (1984), 225–232.

[5] Albert Einstein, *Ideas and Opinions*, Crown, New York, 1954.

[6] John Florio, *Queen Anna's New World of Words, or Dictionarie of the Italian and English tongues, etc.*, London, 1611.

[7] Galileo Galilei, *Opere*, Firenze, 1898.

[8] ———, *Two New Sciences*, translated by Henry Crew and Alfonso de Savio, Dover, New York, 1954.

[9] ———, *Two New Sciences*, translated by Stillman Drake, University of Wisconsin Press, 1974.

[10] Vincenzo Galilei, *Discorso di Vincentio Galilei Nobile Fiorentino, intorno all'opere de messer Gioseffo Zarlino da Chioffia, et altri importanti particolari attenenti alla musica*, Marescotti, Florence, 1589. A useful translation is appended to Randall Goldberg's 2011 Indiana University thesis, *Where nature and art adjoin: Investigations into the Zarlino-Galilei dispute, including an annotated translation of Vincenzo Galilei's "Discorso intorno all'opere di Messer Gioseffo Zarlino"*.

[11] ———, *Discorso particolare intorno alla diuersita delle forme del diapason*, (with translation) in Claude Palisca, *The Florentine Camerata*, Yale Univ. Press, New Haven, 1989.

[12] Jekuthiel Ginsburg, On the early history of the decimal point, *Amer. Math. Monthly* **35** (1928), 347–349.

[13] Stephen Hawking, *A Brief History of Time: From the Big Bang to Black Holes*, Bantam Books, New York, 1988.

[14] Gerard Holton, Einstein and the cultural roots of modern science, *Daedalus* **127**, no. 1 (1998), 1–44.

[15] Victor J. Katz, *A History of Mathematics*, 2nd ed., Addison-Wesley, New York, 1998.

[16] Anne D. Kilmer, The discovery of an ancient Mesopotamian theory of music, *Proc. Amer. Philos. Soc.* **115** (1971), 131–149.

[17] Nichomachus, *The Manual of Harmonics of Nichomachus the Pythagorean*, translated by Flora R. Levin, Phanes Press, Grand Rapids, 1994.

[18] Claude V. Palisca, *Studies in the History of Italian Music and Music Theory*, Clarendon Press, Oxford, 1994.

[19] M. A. Peterson, Galileo's discovery of scaling laws, *Amer. J. Phys.* **70** (2002), 575–580.

[20] ———, *Galileo's Muse: Renaissance Mathematics and the Arts*, Harvard University Press, Cambridge, London, 2011.