The Universe in Zero Words and In Pursuit of the Unknown

Reviewed by Gerald B. Folland

The Universe in Zero Words: The Story of Mathematics as Told through Equations
Dana Mackenzie
Princeton University Press, April 2012
224 pages, US$27.95
ISBN-13: 978-0691152829

In Pursuit of the Unknown: 17 Equations That Changed the World
Ian Stewart
Basic Books, March 2012
352 pages, US$26.99
ISBN-13:978-0465029730

A decade ago, Graham Farmelo got a dozen scientists and science writers to contribute essays on the "great equations of modern science" and compiled them into an interesting and informative book titled *It Must Be Beautiful* [2]. That book has now, it seems, engendered twin children. This year, two well-known expositors of mathematics, Dana Mackenzie and Ian Stewart, have simultaneously and independently produced books that employ a list of "great equations" (24 of them for Mackenzie, 17 for Stewart) as a framing device for discussing mathematics and its impact on civilization. Do we have a case of overkill here? Perhaps. But let me address that question after examining the evidence.

The first point to be made is that, in spite of their common theme, the books are quite dissimilar. Farmelo's book has already been reviewed in the *Notices* by Bill Faris [1], so I will say little about it here except to note a few points of comparison. Most importantly, the fact that it is the work of multiple authors, only one of whom is a mathematician, gives it a decidedly different flavor from the books of Mackenzie and Stewart under review, and its restriction to twentieth-century science gives it a narrower scope. Stewart does not assume much mathematical background on the part of his readers, and he tries to be very careful about explaining the meaning of the ingredients in his equations. In one of the early chapters he explains the concept of derivative, and thereafter he is willing to use it frequently so that some differential equations can be put on his list, but that is about as high as the mathematics goes. (Even integrals are described only briefly and used in only one chapter.) Mackenzie operates at a somewhat higher level of mathematical sophistication. He starts out simply enough—his first equation is "$1 + 1 = 2$", which leads into a discussion of the development of arithmetic in ancient times—but in the later chapters he is not afraid to include some equations whose precise meaning will be over the heads of many of his readers, as long as he can say something interesting about their general significance on a nontechnical level.

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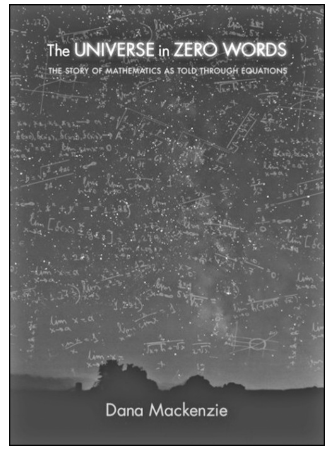
Now, what sorts of equations have the authors found worthy of inclusion in these "Top n" lists?

To begin with, the winner of the popularity contest—the one equation that is featured in all three books—is of little intrinsic interest in mathematics and only a small part of a bigger picture in physics, but it is so famous that it could not be omitted. Can you guess what it is? I'll give you a hint: one would be wrong, but not far wrong, to guess that the E on its left-hand side is in honor of Einstein.

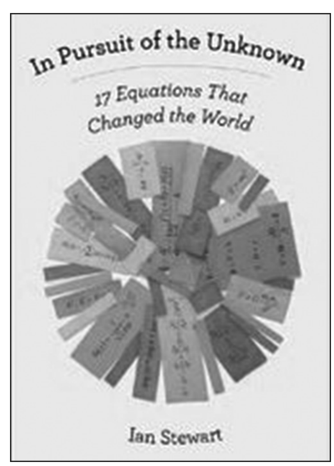
The other equations common to both Stewart and Mackenzie have a lot more mathematical meat attached to them: the Pythagorean theorem, Newton's law of gravity, Maxwell's equations, and the Black-Scholes equation. (The last of these, as the concluding topic of both books and one of the technical ideas that contributed to the recent global economic crisis, functions as a *memento mori*: an admonition that the graces of mathematics can turn into disgraces as well.) There are also other chapters with a parallel purpose in the two books, although the featured equations are different. For example, both books have a chapter on chaotic dynamics, but Stewart introduces it with the discrete logistic equation $x_{n+1} = kx_n(1 - x_n)$, whereas Mackenzie uses the Lorenz differential equations with their "strange attractor" solutions. And they each have a chapter on quantum mechanics: Stewart chooses Schrödinger's equation to frame it; Mackenzie chooses Dirac's. (Farmelo has both.)

As for the rest, one will find most of the usual suspects either in the chapter headings or elsewhere in the texts, but the authors have had to stretch things a bit to fulfill their purposes. Five of Stewart's seventeen "equations" are not actually equations, at least if an equation is an assertion that two apparently distinct things are equal. Three of them are just the definitions of the derivative, the normal distribution, and the Fourier transform, and one is just the defining relation $i^2 = -1$; the other ringer is an inequality, $dS \geq 0$. (S stands for entropy, in case you were wondering.) Similarly, one of Mackenzie's equations is $\pi = 3.1415926535\ldots$, and while π is a worthy topic of discussion, its decimal expansion is hardly its most interesting feature. In all these cases as well as others it quickly becomes clear that the equation that heads a chapter is merely what Alfred Hitchcock called "the McGuffin": the device that sets the plot in motion.

And most of those plots are quite engaging. For example, Stewart's chapter on the Pythagorean theorem begins with some remarks on its emergence in the geometry of ancient Greece and elsewhere, then leads the reader through a bit of trigonometry to Cartesian geometry, nonEuclidean geometry, and finally the apotheosis of the Pythagorean theorem in its infinitesimal form as the cornerstone of Riemannian geometry. Mackenzie's most arcane offering is the Chern-Weil-Allendoerfer-Gauss-Bonnet formula, which leads into an account of the eventful life of S. S. Chern and his role in the development of modern differential geometry and its connections to quantum physics.



Stewart is writing for the general literate reader. He tries hard to keep the technicalities at a minimum, and he spends a lot of time discussing historical developments and the phenomena of the "real world", whose study involves the mathematics in his equations. As a mathematician I sometimes wished that he would get to the point more quickly, but his discursive style is probably quite agreeable to his intended audience. As one would expect from such an experienced writer, he is generally successful in getting the essential points across in a nontechnical way without too much distortion. A few sentences here and there are obscure, and one equation (the Einstein gravitational equation including the cosmological constant, on page 236) is seriously garbled. The one chapter that I think quite unsatisfactory is the one on the second law of thermodynamics; I found Stewart's description of the relation between heat and temperature puzzling and off-kilter, and his explanation of situations where the second law apparently fails is inadequate.



Stewart's chapter on logarithms (equation: $\log xy = \log x + \log y$) focuses on their original employment as a (powerful!) labor-saving device for numerical calculations. But computers have now rendered that use obsolete, and Stewart is at a bit of a loss to come up with modern applications. The chapter ends rather lamely by pointing out that the decibel scale of sonic intensity is a logarithmic one and that to find the half-life of an isotope from its decay law $N = N_0e^{-kt}$, one needs to know what $\log 2$ is. I think Stewart has missed a trick by not picking up on the great structural feature of his logarithmic equation: it is, to use a professional term that he would probably not want to employ, one of the earliest examples of a group homomorphism. With a little work, he could have used it to introduce one of the great themes of modern mathematics—the exploitation of correspondences between apparently different structures and some of its many recent applications.

Mackenzie's ideal readers have a little more mathematical background than Stewart's; I envision them as bright undergraduates or high school seniors who are oriented toward mathematics and science or people to whom this description would have applied in the not-too-distant past. They should probably know some calculus, and they should be comfortable enough with symbolic expressions to be able to look at an unfamiliar one with more curiosity than distaste. Mackenzie's chapters are pithier and generally more mathematically adventurous than Stewart's (and therefore, for my taste, more fun to read). For the ideal reader just described, they should serve as inviting doorways into many intriguing areas of mathematics. Like Stewart, Mackenzie is skillful at sketching things with clarity and simplicity. His weak spot is the chapter on the Dirac equation, which contains several confusions and misstatements.

In summary, the books of Mackenzie and Stewart, as well as Farmelo, are all worthy additions to the popular scientific literature, and they are of sufficiently diverse character that their considerable overlap is not mere duplication. However, individually and collectively, they do demonstrate that the "great equations" conceit is not particularly natural or productive and that the attempt to shoehorn a wide range of mathematics into this format is a procrustean one. I don't think we now have a surfeit of "great equations" books, but we do have a sufficiency.

References

[1] W. G. Faris, Review of *It Must Be Beautiful: Great Equations of Modern Science*, by G. Farmelo, *Notices Amer. Math. Soc.* **50** (2003), 361–367.

[2] G. Farmelo (ed.), *It Must Be Beautiful: Great Equations of Modern Science*, Granta Books, London and New York, 2002.