



David Mumford— Selected Papers, Volume II

Reviewed by Frans Oort

David Mumford—Selected Papers, Volume II: On Algebraic Geometry, including Correspondence with Grothendieck

Edited by Ching-Li Chai, Amnon Neeman, and Takahiro Shiota

Springer, July 2010

Price: US\$99.00, 767 pages

ISBN: 978-0-387-72491-1

The volume under review reproduces all papers by David Mumford in algebraic geometry not already included in Volume I (see [1]) and mathematical correspondence between Grothendieck and Mumford, plus several letters from Grothendieck to other mathematicians. Hence all papers in algebraic geometry by David Mumford are now collected and available in these two volumes [1], [2]. Let me first say a few words about the mathematics of Mumford.

The Style of Mumford

From the *Autobiography of David Mumford*: “At Harvard a classmate said ‘Come with me to hear Professor Zariski’s first lecture, even though we won’t understand a word’ and Oscar Zariski bewitched me. When he spoke the words ‘algebraic variety’, there was a certain resonance in his voice that said distinctly that he was looking into a secret garden. I immediately wanted to be able to do this too. It led me to 25 years of struggling to make this world tangible and visible. Especially, I became obsessed with a kind of passion flower in this garden, the moduli space of Riemann. I was always trying to find new angles from which I could see them better.” (1996; see [5], p. 225.)

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DOI: <http://dx.doi.org/10.1090/noti951>

During the period starting around 1960 and ending in the 1980s, David Mumford amazed us with new approaches to old problems in algebraic geometry. He rekindled interest in classical geometric ideas using modern methods.

Meeting Mumford and attending lectures by him meant an encounter with sparkling new ideas. The same holds true for reading the twelve books and more than sixty papers Mumford wrote in the years 1959–1982, when he was active in algebraic geometry. These carry fundamentally new insights, such as:

- revival of old ideas and techniques, long forgotten and reactivated by Mumford in a new spirit;
- unexpected views, questions, and directions in mathematics; and
- a deep understanding of the material studied, developing a sure and precise grip on the essence of topics.

His publications give us back that beautiful geometric feeling that was getting more and more lost in the algebraization and the functorialization of geometry.

His style is unique and fascinating. In a period when writing mathematics was increasingly done in a way where every symbol had many indices, where trees of definitions and concepts were difficult to climb, Mumford found a mathematical language that is clear and leads you straight to the central idea without losing precision. In reading Mumford, you’d better have a piece of paper and pencil at hand, because many arguments have to be worked out in greater detail by the reader himself, only to discover at the end that the author is correct, that he must have thought through all details of the situation being considered. This alone makes reading Mumford’s papers fascinating and stimulating.

In many cases Mumford does not aim at the greatest generality. The basic idea, the immediate

intuitive approach to the problem studied is central. Often he brings new insight and a fresh approach to classical questions. His work opens windows and gives rise to new developments.

As John Tate describes this: “Mumford has carried forward, after Zariski, the project of making algebraic and rigorous the work of the Italian school ... Mumford’s main interest [is] the theory of varieties of moduli. This is a central topic in algebraic geometry having its origins in the theory of elliptic integrals. The development of the algebraic and global aspects of this subject in recent years is due mainly to Mumford, who attacked it with a brilliant combination of classical, almost computational, methods and Grothendieck’s new scheme-theoretic techniques.” (1974, see [4]).

It would be wonderful to document developments arising from and stimulated by his pioneering work. He is generous to many of us, in contact and in writing, producing beautiful ideas and results and leaving open roads to new thoughts. We can hardly underestimate the influence this has had on all of us.

From his rich source of ideas, he often awarded inspiration to other mathematicians, who then finished the details. I have myself experienced this twice, and I am still very grateful for those opportunities.

About Selected Papers, Volume I

For a review of the first volume, see [7] and [8]. In that book, just about half of the papers by Mumford in algebraic geometry were published. As those two reviews point out, there were several flaws. It was not clear why some papers were not reproduced, and the papers did not appear in chronological order. That volume contains five different lists of references. No two agree. There are painful mistakes, such as page numbers that are omitted or wrong; names that are misspelled; references that are unsystematically abbreviated, even within the same list; papers from the volume itself that are referred to by different titles. We missed important papers by Mumford, such as the paper with Deligne, “The irreducibility of the space of curves of given genus”, [69e], appearing now in Volume II (with 325 citations, one of the most influential papers in modern algebraic geometry). But we also missed papers that were hard to find.

About Selected Papers, Volume II: Papers and Notes

The present volume corrects these flaws. The book contains a precise bibliography of Mumford. This is very useful; we can really trust this list. Furthermore, this volume contains all papers not appearing in Volume I.

The editors have done a great job of writing notes about the papers. In addition to correcting misprints, the notes indicate new developments and comment

on information not available at the time a particular paper was written. This additional material makes reading these articles even more interesting.

Just sit down with this volume and be overwhelmed by ideas from, say, fifty years ago, still new and inspiring now. Read through the paper [61a] “The topology of normal singularities ...” or [65d] “Picard groups of moduli problems”, just to mention two of the papers featured here, and you see the fresh look, the powerful approach, and the inspiration communicated to the reader. But also read [78d], “Fields Medals (IV): An instinct for the key idea”, where Mumford and Tate describe work by the 1978 Fields Medalist Pierre Deligne; we see the unique quality of work by Deligne, its place in the history of algebraic geometry, and interaction with mathematics by Grothendieck in just two pages. Such papers collected in this volume give insight into this field in the years 1960–1980.

The Correspondence

Mumford wrote in 2008: “For me, personally, Grothendieck’s letters were priceless and enabled me to understand many of his ideas in their raw form before they were generalized too far and embedded in the daunting machinery of his ‘Élements’.” See [2], p. 5.

In this volume we see the fruit of a fascinating labor: reproducing a correspondence. It is a unique source of information and inspiration. We can compare a paper with the comments by Grothendieck on a preliminary version. Moreover, the editors have provided more than 160 footnotes explaining ideas in those letters and providing recent developments and references. Reviewing, typesetting, and editing this material was a big task, and we can be very grateful to the editors for this valuable work. Together with the Grothendieck-Serre correspondence [6], this book gives a beautiful picture and stimulating ideas around the development of algebraic geometry in the years 1960–1985. We know that Grothendieck destroyed many personal papers; as a consequence we mainly have the letters of Grothendieck to Mumford, while the greater part of the other half of the correspondence is lacking. This makes guessing even more interesting, though we would have appreciated seeing the other half.

Let me say some words about the interaction between Grothendieck and Mumford, the deep respect on both sides, and the difference in their style of research, thinking, and writing.

First Interaction between Mumford and Grothendieck

I remember meeting Grothendieck in 1961 in a Paris street; both of us were going to the same lecture. Grothendieck mentioned a construction made by a young American mathematician. In a 1961 letter to Grothendieck, Mumford described

his proof of “the key theorem in a construction of the arithmetic scheme of moduli of curves of any genus.” Grothendieck was excited about this idea, apparently completely new to him. Later Mumford pinned down the notion of a “coarse moduli scheme”, necessary in case the obvious moduli functor is not representable by a variety (or by a scheme). (See [2], pp. 635–638, where we see this excitement of Grothendieck reflected in several letters to Mumford.) Grothendieck explained that for “higher levels” he could represent moduli functors, but for all levels he could not perform the necessary construction. Mumford announced this method and explained it in his paper [61c]. Grothendieck, although positive about the new ideas of the young Mumford, wrote in 1961: “It seems to me that, because of your lack of some technical background on schemata, some proofs are rather awkward and unnatural” ([2], p. 636).

To French mathematicians of that period a construction in algebraic geometry should be done by solving a “universal problem”, in their terminology, by “representing a functor”. Igusa constructed (*Ann. Math.* 72 (1960), 621–649, in terminology developed by Mumford later) the *coarse moduli scheme* of curves of genus 2 over \mathbb{Z} . To the “functorial thinking” of Paris mathematicians in 1961 this was strange, and we taste this atmosphere in the description by Samuel of this construction by Igusa: “Signalons aussitôt que le travail d’Igusa ne résoud pas, pour les courbes de genre 2, le ‘problème des modules’ tel qu’il a été posé par Grothendieck à diverses reprises dans ce Séminaire. (We note that the work by Igusa for curves of genus 2 does not solve the problem of ‘moduli of curves’ as proposed by Grothendieck several times in this seminar.)” See the first lines of [3]. This is the background of the difference between Paris and Harvard mathematics at that time. Grothendieck classified work by Igusa as “most discouraging to read”; see [2], p. 636. (I like that paper of Igusa; it was on my desk on many occasions.)

Mumford completed this construction of moduli spaces over any base scheme (curves of any genus, polarized abelian varieties); it was a starting point of new developments. A clear line led from classical invariant theory to ideas by Igusa for $g = 2$ to Mumford’s geometric invariant theory, thereby creating an aspect of research that complemented Grothendieck’s work on these topics.

The stimulating difference between these two giants, Grothendieck and Mumford, their insight, and the respect they had for each other are a source of rich ideas. What a privilege for us to feel stimulating aspects contained in Mumford’s papers reproduced in these volumes and to be able to enjoy the exchange of their ideas, surviving in (part of) their correspondence, which is now available thanks to this beautiful volume.

Some Remarks about Grothendieck

Alexandre (Alexander) Grothendieck was active in algebraic geometry in the period 1958–1970. His style was fundamental. Anything he considered would be done in the most general situation. If there is a general solution, we can be sure Grothendieck puts us on the right track. The revolution he started (in algebraic geometry) has been fruitful, although not many of us can perform on the same heights and at the same level of abstraction.

We now have (partial) access to a fascinating biography. *Who Is Alexandre Grothendieck?* is a projected 3-volume biography of Alexandre Grothendieck.

Volume I.: Anarchy (by W. Scharlau) covers the story of Grothendieck’s parents and his life 1928–1948.

Volume II.: Mathematics (by L. Schneps) covers Grothendieck’s life and mathematics 1948–1970.

Volume III.: Spirituality (by W. Scharlau) covers the years 1970–1991.

Volume I is complete and available in German and English (2009). Volume III is complete and available in German (2010). Volume II is in preparation.

A few years ago Grothendieck sought to block publication of his unpublished writings:

Declaration of intent of non-publication. I do not intend to publish or republish any work or text of which I am the author, in any form whatsoever, printed or electronic, whether in full or in excerpts, texts of personal nature, of scientific character, or otherwise, or letters addressed to anybody, and any translation of texts of which I am the author. Any edition or dissemination of such texts which have been made in the past without my consent, or which will be made in the future and as long as I live, is against my will expressly specified here and is unlawful in my eyes. As I learn of these, I will ask the person responsible for such pirated editions, or of any other publication containing without permission texts from my hand (beyond possible citations of a few lines each), to *remove from commerce* these books; and librarians holding such books to *remove these books* from those libraries.

If my intentions, clearly expressed here, should go unheeded, then the shame of it falls on those responsible for the illegal editions, and those responsible for the libraries concerned (as soon as they have been informed of my intention).

Written at my home, January 3, 2010, Alexandre Grothendieck.

(Translated by Scott Morrison; see <http://sbseminar.wordpress.com/2010/02/09/grothendiecks-letter/>. For the original French

version of this letter, see <http://tqft.net/misc/Grothendieck%27s%20Declaration.pdf>.)

In 1985 Mumford wrote to Grothendieck: "...the letters that you wrote me are by far the most important things which explained your ideas and insights. The letters are vivid and clear and unencumbered by the customary style of formal French publications...My proposal would be to approach someone with a broad knowledge of your theories...and give...permission...I feel sure that such a collection would be extremely useful to the younger generation." See [2], p. 750. We can be glad that the editors of the present volume (clearly with "broad knowledge of your theories") got the permission of J. Malgoire (who had power-of-attorney for Grothendieck) to publish (part of) the correspondence of Grothendieck contained in this volume; see [2], pp. v and xii.

I put this beautiful volume back on my shelf, and I am sure I will consult it again many times.

References

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