A Mathematician's Lament

Reviewed by William Schmidt

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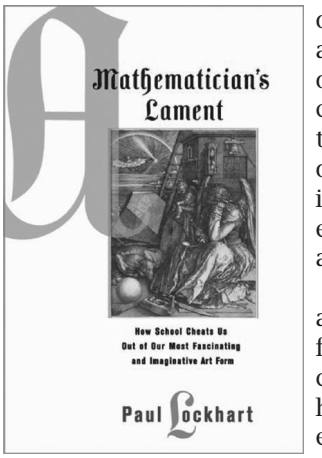
Paul Lockhart
Bellevue Library Press, 2009
US$12.95, 140 pages
ISBN-13: 978-1934137178

A Mathematician's Lament by Paul Lockhart provides a searing and pointed critique of mathematics education in the United States. The book is built on the assumption that mathematics is art. But, unlike what we traditionally consider the arts, where the patterns depend on properties of the physical world, mathematics is about making patterns of ideas. The book is a lament because the author believes that school mathematics essentially destroys the creative aspects of mathematics. Since mathematics is art, he argues, it is more than the memorization of definitions, formulas, and algorithms and should therefore be taught in a way that encourages exploration and discovery.

My reading of this book leaves me with two distinctly different reactions—that the lament is both realistic and unrealistic. First, the author describes the characteristics that led him to conclude that school mathematics is a "senseless tragedy". I find his characterizations to be realistic criticisms of schooling in America and consistent with my own research. On the other hand, he makes unrealistic assumptions about the typical student's appreciation of the beauty of mathematics as an art form, and this leads him to an unrealistic perspective on the implications for school mathematics.

Let me first focus on the author's criticisms about school mathematics in the United States. School mathematics, he claims, stunts the imaginations of children through a "sterile set of facts to be memorized and procedures to be followed" instead



of providing "a creative and rewarding process of invention and discovery." He suggests that, by concentrating on the what and leaving out the why, "mathematics is reduced to an empty shell."

The author further asserts that reform efforts are often ill-conceived and do more harm than good. For example, he suggests that attempts to make math interesting and relevant to kids' lives often end up appearing forced and contrived. He believes that the "glory" of mathematics is that it is irrelevant to our lives, and this is what makes mathematics fun. Algebra is not about daily life, he says, but rather about numbers and symmetry. He also criticizes attempts that resort to what he calls "cutesiness" in order to reduce mathematics anxiety.

Another point the author makes, which I think is particularly important, is that mathematics is one of the few school subjects that is typically taught without reference to its history and philosophical underpinnings. He also criticizes the way school mathematics is considered by many—including parents, teachers, and students—to be a race in which some students are ahead of others. He then asks the crucial question: Where does this race lead? This is an important criticism, because treating mathematics instruction as a race facilitates poor educational practices, such as tracking in the middle grades.

The author reserves some of his strongest criticism for high school geometry, which he considers an "instrument of the devil." In fact, his criticism is even more strongly stated: "A geometry class is by far the most mentally and emotionally destructive component of the entire K–12 mathematics

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DOI: http://dx.doi.org/10.1090/noti981

curriculum." Forcing students to follow a rigid and dogmatic format in laying out their proofs, he argues, destroys the very essence of what geometric proofs should be and undermines the students' intuition. His view is that a proof is like a work of fiction or a poem in that its goal is to satisfy. He actually goes on to say, "A well-written, well-crafted argument should feel like a splash of cool water, and be a beacon of light—it should refresh the spirit and illuminate the mind. And it should be charming."

I share in much of the author's lament. It correctly characterizes much of mathematics instruction as found, unfortunately, in both the international and national research studies in which I have been involved. On the other hand, I believe the same research suggests that the author's assumption about the typical student's comfort level with mathematics is unrealistic and therefore cannot serve as a sound basis for public schooling in math. In the last section he describes an imaginary place he calls "Mathematical Reality", which by implication is where school mathematics should take students. In the chapter titled "Exultation", the author takes the reader on several explorations of number and shape in "Mathematical Reality". One example refers to the seventeenth-century invention of projective geometry as a means for expanding Euclidean geometry by removing parallelism. In sharing this example, the author's words cited below make very clear his view of what "doing" mathematics should look like.

> The point is that there is no reality to any of this, so there are no rules or restrictions other than the ones we care to impose. And the aesthetic here is very clear, both historically and philosophically: if a pattern is interesting and attractive, then it's good. (And if it means having to work hard to bend your mind around a new idea, so much the better.) Make up anything you want, so long as it isn't boring. Of course this is a matter of taste, and tastes change and evolve. Welcome to art history! Being a mathematician is not so much about being clever (although lord knows that helps); it's about being aesthetically sensitive and having refined and exquisite taste.

I found this part of the book fun and an interesting read. It posits a very different vision, although one that is very abstract, of what mathematics education should look like, one I personally could enjoy, as I am sure many others would too. But I wonder whether the world of "Mathematical Reality" would be accessible and interesting to all students.

Given his view of school mathematics, I am not surprised by the author's rejection of what he calls the "ladder myth", which is the idea that mathematics can be arranged as a sequence of topics, with each being more advanced than the other. He believes the path through such a curriculum prevents teachers and students from seeing mathematics as an organic whole. He then goes on to advise mathematics teachers to play around in "Mathematical Reality" and "throw the stupid curriculum and textbook out the window!" Mathematics is, in general, hierarchical; I am therefore confused about what the implications would be for the organization of curricula. Is he suggesting mathematics teaching should be more episodic? The "ladder myth" is pretty universally present in school mathematics in most of the countries that I have studied. Without some structure, I fear many, if not most, students would find an episodic journey through "Mathematical Reality" to be similar to Alice's adventures in Wonderland. Such an approach also could exacerbate the inequalities in mathematics education that are currently rampant in U.S. schools (see *Inequality for All: The Challenge of Unequal Opportunity in American Schools*, Schmidt & McKnight, 2012).

All in all, I found this short book an easy, thoughtful, and entertaining read. The author clearly is passionate about mathematics as an art, as a creative process. In reading this book, one can easily get the impression that mathematics instruction should be more like an unfettered journey into a jungle where an individual can make his or her own way through that terrain. I do not disagree with his view of mathematics as a special place for one's imagination. Using this vision as a guiding principle for structuring mathematics education would enrich the study of mathematics for some. But it would desert many others.

This brings us back to where I started this review. The author provides an accurate characterization of mathematics instruction in the United States. His passion makes the critique compelling. He persuasively rails against textbooks. But, other than the suggestion to explore the land of "Mathematical Reality" and a clever and enlightening discussion of several examples, what school mathematics should look like in a more specific sense is left to the reader's imagination. I have a hunch this is what the author would like. I am left with two questions: (1) What would school mathematics look like—not at the vision level, but at the day-to-day instructional level—if explorations of "Mathematical Reality" formed the basis for teaching in classrooms around the world? (2) Who would end up being excluded from playing in the unique place known as "Mathematical Reality?" If truly accessible to all, it would be a wonderful place in which to learn mathematics.