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Notices

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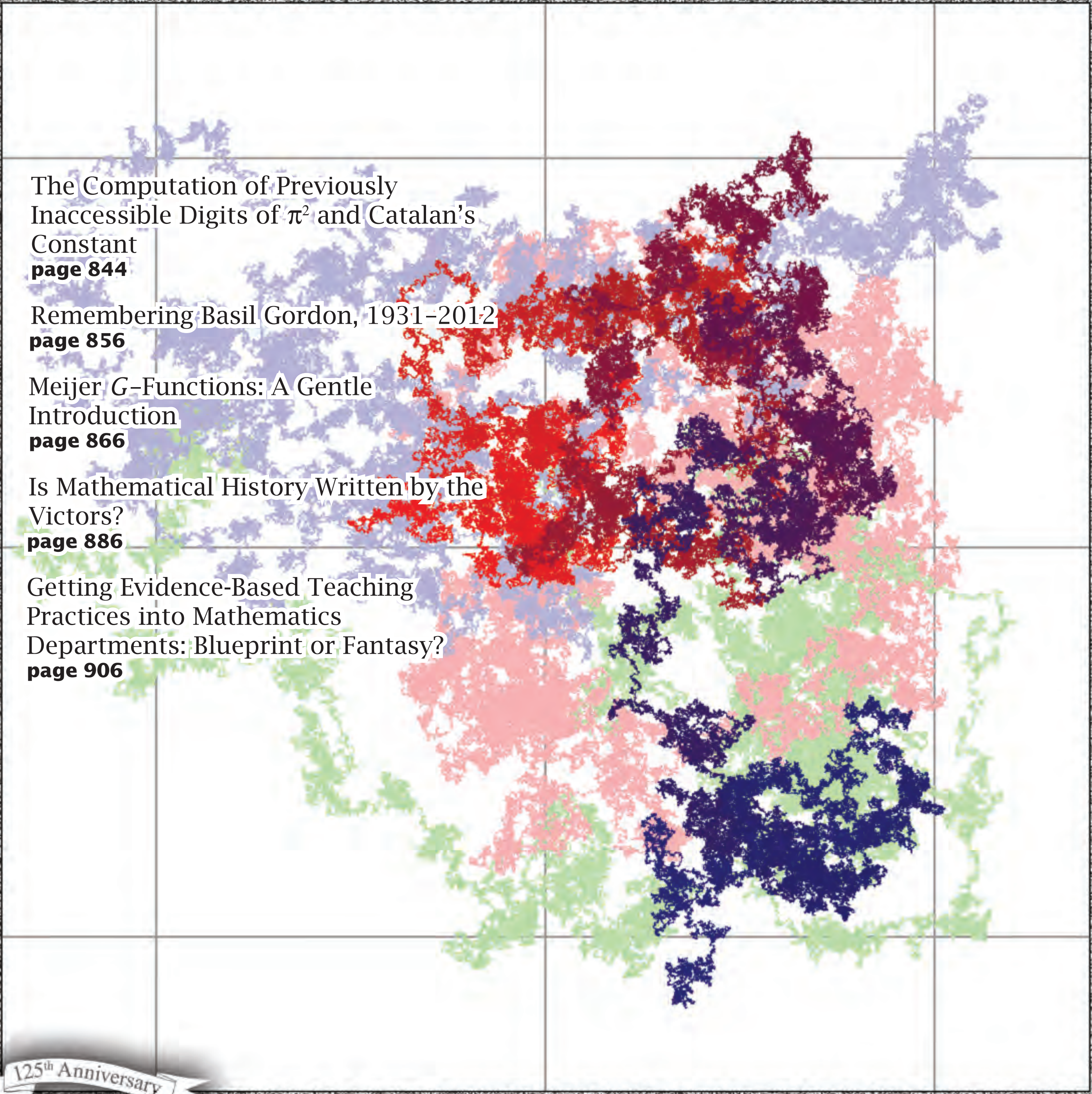

125th Anniversary
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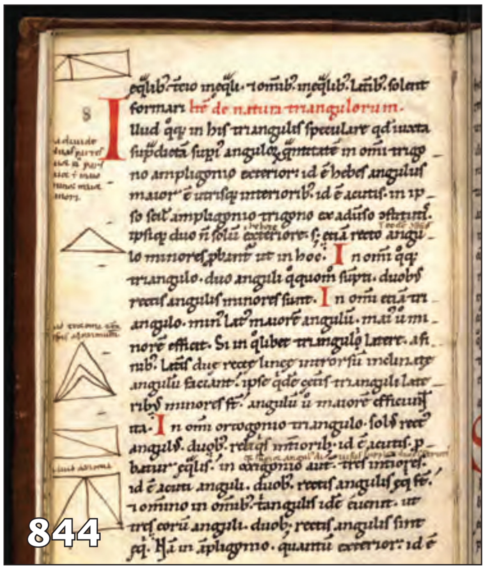
No surprises from Catalan's constant *(see page 948)*

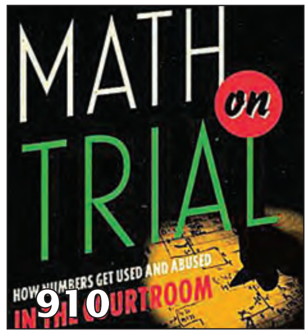
Notices
of the American Mathematical Society

August 2013

Communications

Commentary





This month features an article about Meijer G-functions and another about computing Catalan's constant. We feature a piece about evidence-based teaching and one that considers whether mathematical history is written by the victors. Finally, there is a memorial article for the algebraist Basil Gordon.

—*Steven G. Krantz, Editor*

Features

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of the American Mathematical Society

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Departments

From the AMS Secretary

Opinion

Two Views: How Much Math Do Scientists Need?

On April 5, 2013, The Wall Street Journal *published an essay by the Harvard biologist E. O. Wilson, "Great Scientist ≠ Good at Math". Berkeley mathematician Edward Frenkel responded to it in* Slate *on April 9, 2013. We reprint the two essays below, with permission from* The Wall Street Journal *and* Slate.

Great Scientist ≠ Good at Math

E. O. Wilson Shares a Secret: Discoveries Emerge from Ideas, Not Number-Crunching

For many young people who aspire to be scientists, the great bugbear is mathematics. Without advanced math, how can you do serious work in the sciences? Well, I have a professional secret to share: Many of the most successful scientists in the world today are mathematically no more than semiliterate.

During my decades of teaching biology at Harvard, I watched sadly as bright undergraduates turned away from the possibility of a scientific career, fearing that, without strong math skills, they would fail. This mistaken assumption has deprived science of an immeasurable amount of sorely needed talent. It has created a hemorrhage of brain power we need to stanch.

I speak as an authority on this subject because I myself am an extreme case. Having spent my precollege years in relatively poor Southern schools, I didn't take algebra until my freshman year at the University of Alabama. I finally got around to calculus as a thirty-two-year-old tenured professor at Harvard, where I sat uncomfortably in classes with undergraduate students only a bit more than half my age. A couple of them were students in a course on evolutionary biology I was teaching. I swallowed my pride and learned calculus.

I was never more than a C student while catching up, but I was reassured by the discovery that superior mathematical ability is similar to fluency in foreign languages. I might have become fluent with more effort and sessions talking with the natives, but being swept up with field and laboratory research, I advanced only by a small amount.

Fortunately, exceptional mathematical fluency is required in only a few disciplines, such as particle physics, astrophysics and information theory. Far more important throughout the rest of science is the ability to form concepts, during which the researcher conjures images and processes by intuition.

Everyone sometimes daydreams like a scientist. Ramped up and disciplined, fantasies are the fountainhead of all creative thinking. Newton dreamed, Darwin dreamed, you dream. The images evoked are at first vague. They may shift in form and fade in and out. They grow a bit firmer when sketched as diagrams on pads of paper, and they take on life as real examples are sought and found.

Pioneers in science only rarely make discoveries by extracting ideas from pure mathematics. Most of the stereotypical photographs of scientists studying rows of equations on a blackboard are instructors explaining discoveries already made. Real progress comes in the field writing notes, at the office amid a litter of doodled paper, in the hallway struggling to explain something to a friend, or eating lunch alone. Eureka moments require hard work. And focus.

Ideas in science emerge most readily when some part of the world is studied for its own sake. They follow from thorough, well-organized knowledge of all that is known or can be imagined of real entities and processes within that fragment of existence. When something new is encountered, the follow-up steps usually require mathematical and statistical methods to move the analysis forward. If that step proves too technically difficult for the person who made the discovery, a mathematician or statistician can be added as a collaborator.

In the late 1970s, I sat down with the mathematical theorist George Oster to work out the principles of caste and the division of labor in the social insects. I supplied the details of what had been discovered in nature and the lab, and he used theorems and hypotheses from his tool kit to capture these phenomena. Without such information, Mr. Oster might have developed a general theory, but he would not have had any way to deduce which of the possible permutations actually exist on earth.

Over the years, I have co-written many papers with mathematicians and statisticians, so I can offer the following principle with confidence. Call it Wilson's Principle No. 1: It is far easier for scientists to acquire needed collaboration from mathematicians and statisticians than it is for mathematicians and statisticians to find scientists able to make use of their equations.

This imbalance is especially the case in biology, where factors in a real-life phenomenon are often misunderstood or never noticed in the first place. The annals of theoretical biology are clogged with mathematical models that either can be safely ignored or, when tested, fail. Possibly no more than 10 percent have any lasting value. Only those linked solidly to knowledge of real living systems have much chance of being used.

If your level of mathematical competence is low, plan to raise it, but meanwhile, know that you can do outstanding scientific work with what you have. Think twice, though, about specializing in fields that require a close alternation of experiment and quantitative analysis. These include most of physics and chemistry, as well as a few specialties in molecular biology.

DOI: http://dx.doi.org/10.1090/noti1032

Newton invented calculus in order to give substance to his imagination. Darwin had little or no mathematical ability, but with the masses of information he had accumulated, he was able to conceive a process to which mathematics was later applied.

For aspiring scientists, a key first step is to find a subject that interests them deeply and focus on it. In doing so, they should keep in mind Wilson's Principle No. 2: For every scientist, there exists a discipline for which his or her level of mathematical competence is enough to achieve excellence.

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(Reprinted with permission from The Wall Street Journal*)*

Don't Listen to E. O. Wilson

Math Can Help You in Almost Any Career. There's No Reason to Fear It

E. O. Wilson is an eminent Harvard biologist and best-selling author. I salute him for his accomplishments. But he couldn't be more wrong in his recent piece in *The Wall Street Journal* (adapted from his new book *Letters to a Young Scientist*), in which he tells aspiring scientists that they don't need mathematics to thrive. He starts out by saying: "Many of the most successful scientists in the world today are mathematically no more than semi-literate … I speak as an authority on this subject because I myself am an extreme case." This would have been fine if he had followed with: "But you, young scientists, don't have to be like me, so let's see if I can help you overcome your fear of math." Alas, the octogenarian authority on social insects takes the opposite tack. Turns out he actually believes not only that the fear is justified, but that most scientists don't need math. "I got by, and so can you" is his attitude. Sadly, it's clear from the article that the reason Wilson makes these errors is that, based on his own limited experience, he does not understand what mathematics is and how it is used in science.

If mathematics were fine art, then Wilson's view of it would be that it's all about painting a fence in your backyard. Why learn how to do it yourself when you can hire someone to do it for you? But fine art isn't a painted fence, it's the paintings of the great masters. And likewise, mathematics is not about "number-crunching", as Wilson's article suggests. It's about concepts and ideas that empower us to describe reality and figure out how the world really works. Galileo famously said, "The laws of Nature are written in the language of mathematics." Mathematics represents objective knowledge, which allows us to break free of dogmas and prejudices. It is through math that we learned Earth isn't flat and that it revolves around the sun, that our universe is curved, expanding, full of dark energy, and quite possibly has more than three spatial dimensions. But since we can't really imagine curved spaces of dimension greater than two, how can we even begin a conversation about the universe without using the language of math?

Charles Darwin rightfully spoke of math endowing us "with something like a new sense." History teaches that mathematical ideas that looked abstract and esoteric yesterday led to spectacular scientific advances of today. Scientific progress would be diminished if young scientists were to heed Wilson's advice.

It is interesting to note that Wilson's recent article in *Nature* and his book claiming to show support for so-called group selection have been sharply criticized, by Richard Dawkins and many others. Some of the critics pointed out that one source of error was in Wilson's math. Since I'm not an expert in evolutionary theory, I can't offer an opinion, but I find this controversy interesting given Wilson's thesis that "great scientists don't need math."

One thing should be clear: While our perception of the physical world can always be distorted, our perception of the mathematical truths can't be. They are objective, persistent, necessary truths. A mathematical formula means the same thing to anyone anywhere—no matter what gender, religion, or skin color; it will mean the same thing to anyone a thousand years from now. And that's why mathematics is going to play an increasingly important role in science and technology.

One of the key functions of mathematics is the ordering of information. With the advent of the 3-D printing and other new technology, the reality we are used to is undergoing a radical transformation: Everything will migrate from the layer of physical reality to the layer of information and data. We will soon be able to convert information into matter on demand by using 3-D printers just as easily as we now convert a PDF file into a book or an MP3 file into a piece of music. In this brave new world, math will be king: It will be used to organize and order information and facilitate the conversion of information into matter.

It might still be possible to be "bad in math" (though I believe that anyone can be good at math if it is explained in the right way) and be a good scientist—in some areas and probably not for too long. But this is a handicap and nothing to be proud of. Granted, some areas of science currently use less math than others. But then practitioners in those fields stand to benefit even more from learning mathematics.

It would be fine if Wilson restricted the article to his personal experience, a career path that is obsolete for a modern student of biology. We could then discuss the real question, which is how to improve our math education and to eradicate the fear of mathematics that he is talking about. Instead, trading on that fear, Wilson gives a misinformed advice to the next generation, and in particular to future scientists, to eschew mathematics. This is not just misguided and counterproductive; coming from a leading scientist like him, it is a disgrace. Don't follow this advice—it's a self-extinguishing strategy.

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Letters to the Editor

On "A Revolutionary Material"

The history of the discovery of quasicrystals is not stated correctly in Radin's article ["A revolutionary material", by Charles Radin, *Notices*, March 2013]. Actually the mathematical theory came before the experimental discovery of quasicrystals.

The mathematical model for a 3-dimensional quasicrystal with icosahedral symmetry was first published by my father, P. Kramer, and his student R. Neri in the article "On periodic and nonperiodic space fillings of E^m obtained by projection", *Acta Cryst. Sect. A* **40** (1984), no. 5, 580–587. This paper was submitted on November 5, 1983, before Shechtman's experimental result, which earned him the Nobel Prize, was published.

In contrast, the paper by D. Levine and P. Steinhardt mentioned in the article was written and submitted afterwards. The review by M. Senechal (MR0768042) of the Kramer-Neri paper states this very clearly.

I want to make two points here. Firstly, the mathematical theory of quasicrystals predated the experiment. Secondly, the paper by Levine and Steinhardt is not at all the "initial report" on the theory of quasicrystals. It is unfortunate that the official press release of the Nobel Prize Committee contains the same historical and scientific inaccuracies.

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(Received March 14, 2013)

Errors in Papers: Math vs. Computer Science

In "Errors and corrections in mathematics literature" (*Notices*, April 2013), Joseph Grcar accepts as an axiom that "There is no reason to think…mathematicians make mistakes less often" in their published work than researchers in other branches of science. My experience in my own field, cryptography—which, although it involves a lot of math, inherits the disciplinary culture of computer science, not of mathematics—causes me to doubt the validity of this premise.

There are two reasons why the computer science literature is more prone to errors, including serious errors in important papers, than is mathematics. First, the tradition is to publish mainly in conference proceedings, not in journals. Authors write under deadline pressure and often submit their papers within hours (literally) of the deadline. Reviewers are also hurried—they each have to read and evaluate a couple dozen papers in the course of a few weeks.

In the second place, in computer science and related fields it is expected that successful researchers write papers at a frenetic pace, authoring or (more often) coauthoring a large number of papers each year. As I wrote in my article "The uneasy relationship between mathematics and cryptography" (*Notices*, September 2007), "Top researchers expect that practically every conference should include one or more quickie papers by them or their students." The heightened publish-or-perish pressures, which are much worse than in mathematics, contribute to quality control problems. Some examples of these problems can be found at `http://anotherlook.ca`.

Cryptography and computer science are not the only fields that seem to have more problems than mathematics with major errors in important papers. Grcar states that "biomedical and multidisciplinary journals are recognized for exemplary corrective policies." This is questionable. The reader will find a much less sanguine viewpoint in the article "Lies, damned lies, and medical science", by David H. Freedman (*The Atlantic*, November 2010).

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(Received March 29, 2013)

The Common Intellectual Property of Humankind

With respect to David A. Edwards's article "Platonism is the law of the land" (*Notices,* April 2013), in which he advocates that mathematical results should be patentable: Can one really imagine a world in which someone must obtain a license and pay royalties every time (s)he uses the Fundamental Theorem of Calculus or, for that matter, negative numbers?

—Steven H. Weintraub
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(Received March 19, 2013)

Sustainability or Collapse?

It appears to many observers that humanity is already moving into "ecological overshoot and collapse". If they are right, then Simon Levin's nice overview in "The mathematics of sustainability" (*Notices,* April 2013) needs a much more ambitious, interdisciplinary agenda.

The most outstanding application of mathematics to the ecological trajectory of modern civilization may still be the famous Limits-to-Growth study of the 1970s. This study (see *Limits to Growth—The 30-Year Update*, 2004) uses nonlinear dynamical systems that encode critical feedback and feedforward loops among a handful of global variables (population, resources, food, industrial output, pollution). It uses the methodology of scenarios that has been applied to great effect in climate modeling.

The Limits-to-Growth business-as-usual scenario, which has held up remarkably well, suggests that not only are we well into ecological overshoot but that some form of collapse could be imminent—starting within a decade or two. If so, mathematicians could make a major contribution by studying the chaotic aspects of these nonlinear systems, especially how they could be controlled to achieve a soft landing (= sustainability).

Mathematicians, especially those with expertise in complexity and scientific computing, could be part of interdisciplinary teams, similar to those assembled to model climate change. Unfortunately, though one might expect economists to be taking the lead when it comes to growth, they seem to be missing in action. However, Europe is starting to see conferences on "de-growth" as it deals with high fuel costs and one debt crisis after another.

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(Received April 2, 2013)

"New Chronology" of Nosovskii and Fomenko in the *Notices*

The April 2013 issue of the *Notices* contains an article by Florin Diacu, "Mathematical methods in the study of historical chronology". Most of this article is an exposition of the "research" of A. Fomenko and G. Nosovskii on mathematical methods in chronology. According to F. Diacu, this research is "published in a mathematical journal that has reasonably good ranking." F. Diacu laments that "So far, historians have ignored these studies…" and "So far, biblical scholars seem to have ignored Fomenko's conclusions …."

The reason scholars ignore this research is simple: it is the same reason physicists would ignore research concluding that the earth is flat.

We assume that most Western readers are not familiar with the "research" of Fomenko and Nosovskii on chronology, since most of the enormous output of these authors is published in Russian. So let us explain. Since the 1970s Fomenko has applied "mathematical methods" to revise the established historical chronology and historical events themselves. He comes to the remarkable conclusion that almost all history that we learn in school has been intentionally falsified, forged by some international conspiracy.

The following is our literal translation from Russian of a passage from a book by Fomenko and Nosovskii, preserving punctuation and capitalization of the original:

Section 2.7. Formation in 1776 of the United States of America on the American territories of the dissolved Moscow Tartary.

And now we ask the question: when and how did the United States come into existence? Let us pay special attention to the time of formation of the USA. The Encyclopedic Dictionary tells us that "in the process of the wars for independence in North America in 1775–1783…an independent state was created, the USA (1776)." And now we realize unexpectedly that formation of the USA surprisingly EXACTLY COINCIDES WITH THE END OF THE WAR WITH "PUGACHEV" IN RUSSIA. Let us recall that "Pugachev" was defeated in 1775. Now everything is in its place. Apparently, the "Independence war" in North America was a war with the weakening American Russian Horde. The Romanovs attacked the Horde from the East, and Americans "struggling for independence" attacked it from the West. Now they teach us that Americans were fighting for "independence from England". In reality this was a war for the partition of enormous American lands of the Moscow Tartary, which found themselves without the central Russian-Horde governance…. It is clear that the very fact of the war with the "Mongolian" Horde in America was carefully erased from the pages of textbooks of American history. As well as the very fact of the existence of the huge Moscow Tartary."

(Translated from G. V. Nosovskii and A. T. Fomenko, "Reconstruction of world history (New chronology)", *Business Express*, Moscow, 2001, p. 451 of 726 pp.)

In our opinion, the above quotation gives an adequate impression of the methods and conclusions of Fomenko and Nosovskii. We leave it to the readers to decide whether an account of this research deserves publication in the *Notices*.

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(Received April 2, 2013)

Flim-Flam in the Name of Science

Concerning the *Notices* article "Mathematical methods in the study of historical chronology," by Florin Diacu, April 2013 issue:

The *Notices* has disgraced itself by allowing its good name to be used in connection with the crackpot historical theories of Anatoly Fomenko. The fact that Fomenko is a mathematician does not in any way lend credibility to his pseudoscientific publications, which should interest the scientific community only insofar as they provide a cautionary illustration of the manner in which membership in a national scientific academy can be misused to promulgate pure nonsense. Suffice it to point out that Fomenko asserts that the entirety of Chinese history is a fabrication of eighteenth-century Jesuits, that all ancient Roman and Greek artifacts are actually forgeries produced during the Renaissance, and that the New Testament was written before the Old. Needless to say, such assertions are so thoroughly incompatible with vast troves of historical and archaeological evidence that they are not taken seriously by any competent experts in the relevant fields.

Of course, Fomenko has every right to pursue his bizarre hobbies and is free to make a public fool of himself to his heart's content. But the *Notices of the AMS* is not an appropriate forum for such pseudoscientific tomfoolery. The danger is that publication here could lend an air of legitimacy to work which would never be published in a scholarly journal refereed by competent experts in the fields in question.

Could the *Notices* please confine itself to articles with actual mathematical content? That way, we might reasonably hope to see more articles that have been refereed by scholars whose knowledge and training qualify them to accurately judge whether or not an article is substantially correct and interesting.

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(Received March 20, 2013)

Photo courtesy of the University of Hawaii Mathematics Department.

The Rudins in Hawaii

On behalf of the math faculty here at the University of Hawaii, I would like to express our sadness at the passing first of Walter Rudin and now too of Mary Ellen Rudin. The math department was quite fortunate to have the Rudins as very special and regular visitors to our department over more than twenty years. It all started with their sabbatical visit in the spring of 1982 and continued afterwards with biennial month-long stays typically during the months of January or February when the weather in Madison was particularly cold. Their retirements from the University of Wisconsin math department gave them the flexibility to escape those cold winters and enjoy our tropical weather and beautiful beaches. They spent most of their visits staying in beach cottages on Kailua Beach, which is located on the windward side of the island of Oahu. Their favorite activity was simply sitting on their beach chairs and enjoying the sunshine.

The Rudins became part of our departmental "family". Every visit included colloquiums from each of them along with "math talk" with faculty members: Walter mostly with the analysis group and Mary Ellen mostly with the logic, topology, and universal algebra groups. Of course there was always plenty of social interaction too. Every visit included a "banquet" at the New Chinese Restaurant in Kailua, when the math department would basically take over the small restaurant with a large crowd. This was not a fancy place, but it was Walter's and Mary Ellen's favorite spot to eat in Kailua.

Walter and Mary Ellen also made some extra trips so that their two daughters, Catherine and Eleanor, and their son, Charlie, would have a place to stay during their visits to Hawaii. Some of our faculty members provided windsurfing lessons for Charlie and his friends.

Their last visit to Hawaii was in 2005. By that time Walter's health problems had reached a point that travel was quite difficult for him. Mary Ellen on the other hand seemed like she would live forever at that time, but sadly all things must end. The passing of the Rudins marks the end of an era for us here in Hawaii as I'm sure it does also for many others around the world.

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(Received April 15, 2013)

Correction to Conway Interview

I thoroughly enjoyed the "Interview with John Horton Conway" in the May *Notices* (vol. 60, no. 5, p. 567). Conway's deep mathematical insights over a long lifetime are most impressive. But on page 573 Conway appears to mix up two famous integer sequences, namely the Hofstadter-Conway \$10000 sequence and the Hofstadter Q-sequence. These are, respectively, numbers A004001 and A005185 in the On-Line Encyclopedia of Integer Sequences. Conway's anecdote concerns the former sequence, but the formula corresponds to the latter. The recursion given is $f(0)=0$, $f(1)=1$, $f(n)=f(n-f(n-1))+f(n-f(n-2))$, where the initial values are also mistaken. Ironically this adds further terms to the sequence of mistakes recounted in the interview.

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(Received May 10, 2013)

Realizing Mathematical Reality

In *A Mathematician's Lament*, Paul Lockhart has written a provocative and important critique of contemporary mathematics education in America. In his review in the April 2013 *Notices*, William Schmidt calls Lockhart's book "realistic", saying, "I share in much of the author's lament" and proclaiming "the author provides an accurate characterization of mathematics instruction in the United States." Having accepted Lockhart's diagnosis—the dominant paradigm driving our system of mathematics education is irrevocably broken—the balance of Schmidt's review is, unfortunately, an exercise in avoidance.

To call Lockhart's "Mathematical Reality" too "abstract" and "unrealistic" is disingenuous. Schmidt repeatedly uses psychological projection, defensively ascribing the shortcomings of our current system onto Lockhart's vision for something new. Diversions such as these keep us from breaking out of the confines of an admittedly broken paradigm. As many examples in the history of science illustrate, if we continue blithely down this dysfunctional road, as Schmidt would seem to have us do, then we are Alice and have gone down the rabbit-hole, not Lockhart.

Lockhart is not simply an idealist who has thrown his hands up. Throughout his lament he carefully identifies some of the important assumptions that underlie the current paradigm which we should reject, and he identifies assumptions—which he collects under the moniker "Mathematical Reality"—that could serve as part of the foundation for a more successful paradigm.

Schmidt expects too much of one thoughtful messenger. It is the responsibility of our entire community to honestly respond to these challenges, to determine how educational practice may provide alternatives, and to help understand what new approaches could look like on a "day-to-day instructional level."

Existing programs developed under assumptions similar to Lockhart's "Mathematical Reality" can inform our efforts. One such initiative is the National Science Foundation-supported project Discovering the Art of Mathematics. Its inquiry-based approach requires students to "actually do some mathematics, and come up with their own ideas, opinions and reactions" in contexts that celebrate mathematics' "history, philosophy, thematic development, aesthetic criteria and current status" (Lockhart, p. 40). Curriculum materials sufficient to teach ten semester-long courses on entirely different mathematical subject areas are freely available. Professional development workshops and other supporting resources are also available. (See http://artofmathematics.westfield.ma.edu.)

Projects like this vividly illustrate the potential of alternatives, like Lockhart's "Mathematical Reality", to transform what has been a long "nightmare" for students into an intellectual experience in which they feel "exultation".

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(Received April 11, 2013)

Dyson's Imagined History

Some portions of Freeman Dyson's article about John von Neumann (*Notices*, February 2013), as physicists say, are not even wrong. Dyson imagines that von Neumann began an arms race by writing a report about the Atlas rocket.

The actual history is rather different. "The missile race was not the action-reaction situation so often imagined. Both the Soviet Union and the United States realized about the same time that they wanted intercontinental missiles, and each pursued its goal knowing very little about what the other was doing" (Norman Friedman, *The Fifty Years War*, p. 232).

Von Neumann's technical report written for the Air Force in 1954 helped design the Atlas, but it was James Killian's strategic report written for President Eisenhower in 1955 that recommended building the missile. Eisenhower doubted the strategic value of liquid fuel missiles, and only 129 Atlas missiles were ever deployed. Instead, Atlas rockets were used for space exploration, which may be another of von Neumann's legacies.

—Joseph Grcar
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(Received May 17, 2013)

Photo credit: NASA.

John Glenn atop a version of von Neumann's Atlas rocket.

Response to Grcar

I am grateful to Joseph Grcar for clarifying the history of the intercontinental missile programs of the United States and the Soviet Union. I do not find any substantial inconsistency between his account of the history and mine.

—Freeman Dyson
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(Received May 18, 2013)

Submitting Letters to the Editor

The *Notices* invites readers to submit letters and opinion pieces on topics related to mathematics. Electronic submissions are preferred (notices-letters@ams.org); see the masthead for postal mail addresses. Opinion pieces are usually one printed page in length (about 800 words). Letters are normally less than one page long, and shorter letters are preferred.

Identifications

Affiliations of authors of "Letters to the Editor" are provided for identification purposes only. Opinions expressed in letters are those of the authors and do not necessarily reflect those of their employers or, in the case of American Mathematical Society officers or committee members, policies of the Society. Committee reports to the Council of the Society and official communications of officers of the Society, when published in the *Notices*, appear in the section of the *Notices* "From the AMS Secretary".

The Computation of Previously Inaccessible Digits of π^2 and Catalan's Constant

David H. Bailey, Jonathan M. Borwein, Andrew Mattingly, and Glenn Wightwick

Introduction

We recently concluded a very large mathematical calculation, uncovering objects that until recently were widely considered to be forever inaccessible to computation. Our computations stem from the "BBP" formula for π, which was discovered in 1997 using a computer program implementing the "PSLQ" integer relation algorithm. This formula has the remarkable property that it permits one to directly calculate binary digits of π, beginning at an arbitrary position d, without needing to calculate any of the first $d-1$ digits. Since 1997 numerous other BBP-type formulas have been discovered for various mathematical constants, including formulas for π^2 (both in binary and ternary bases) and for Catalan's constant.

In this article we describe the computation of base-64 digits of π^2, base-729 digits of π^2, and base-4096 digits of Catalan's constant, in each case beginning at the ten trillionth place, computations that involved a total of approximately 1.549×10^{19} floating-point operations. We also discuss connections between BBP-type formulas and the age-old unsolved questions of whether and why constants such as $\pi, \pi^2, \log 2$, and Catalan's constant have "random" digits.

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Historical Background

Since the dawn of civilization, mathematicians have been intrigued by the digits of π [6], more so than any other mathematical constant. In the third century BCE, Archimedes employed a brilliant scheme of inscribed and circumscribed $3 \cdot 2^n$-gons to compute π to two decimal digit accuracy. However, this and other numerical calculations of antiquity were severely hobbled by their reliance on primitive arithmetic systems.

One of the most significant scientific developments of history was the discovery of full positional decimal arithmetic with zero by an unknown mathematician or mathematicians in India at least by 500 CE and probably earlier. Some of the earliest documentation includes the *Aryabhatiya*, the writings of the Indian mathematician Aryabhata dated to 499 CE; the *Lokavibhaga*, a cosmological work with astronomical observations that permit modern scholars to conclude that it was written on 25 August 458 CE [9]; and the Bakhshali manuscript, an ancient mathematical treatise that some scholars believe may be older still, but in any event is no later than the seventh century [7], [8], [2]. The Bakhshali manuscript includes, among other things, the following intriguing algorithm for computing the square root of q, starting with an approximation x_0:

$$a_n = \frac{q - x_n^2}{2x_n},$$

$$x_{n+1} = x_n + a_n - \frac{a_n^2}{2(x_n + a_n)}. \tag{1}$$

This scheme is quartically convergent in that it approximately *quadruples* the number of correct digits with each iteration (although it was never iterated more than once in the examples given in the manuscript) [2].

In the tenth century, Gerbert of Aurillac, who later reigned as Pope Sylvester II, attempted to introduce decimal arithmetic in Europe, but little headway was made until the publication of Fibonacci's *Liber Abaci* in 1202. Several hundred more years would pass before the system finally gained universal, if belated, adoption in the West. The time of Sylvester's reign was a very turbulent one, and he died in 1003, shortly after the death of his protector, Emperor Otto III. It is interesting to speculate how history would have changed had he lived longer. A page from his mathematical treatise *De Geometria* is shown in Figure 1.

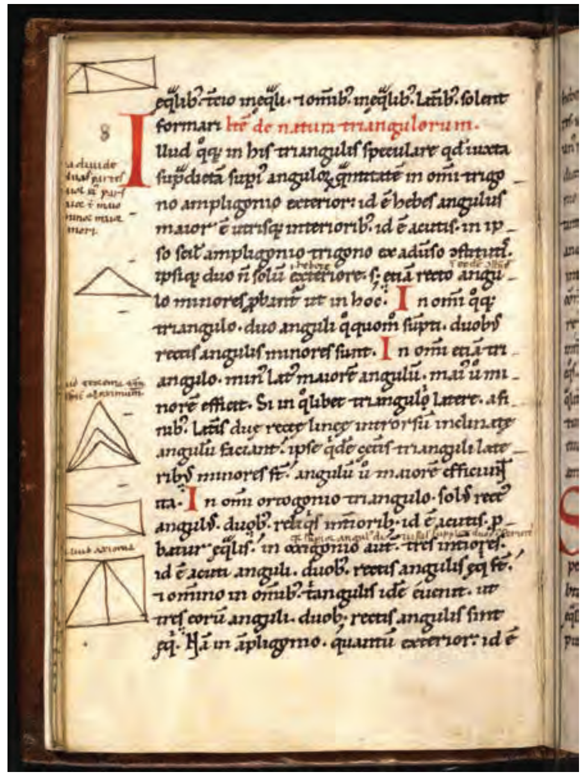

Figure 1. Excerpt from *De Geometria* by Pope Sylvester II (reigned 999–1003 CE).

The Age of Newton

Armed with decimal arithmetic and spurred by the newly discovered methods of calculus, mathematicians computed with aplomb. Again, the numerical value of π was a favorite target. Isaac Newton devised an arcsine-like scheme to compute digits of π and recorded 15 digits, although he sheepishly acknowledged, "I am ashamed to tell you to how many figures I carried these computations, having no other business at the time." Newton wrote these words during the plague year 1666, when, ensconced in a country estate, he devised the fundamentals of calculus and the laws of motion and gravitation.

All large computations of π until 1980 relied on variations of Machin's formula:

$$\frac{\pi}{4} = 4\arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{239}\right). \tag{2}$$

The culmination of these feats was a computation of π using (2) to 527 digits in 1853 by William Shanks, later (erroneously) extended to 707 digits. In the preface to the publication of this computation, Shanks wrote that his work "would add little or nothing to his fame as a Mathematician, though it might as a Computer" (until 1950 the word "computer" was used for a person, and the word "calculator" was used for a machine).

One motivation for such computations was to see whether the digits of π repeat, thus disclosing the fact that π is a ratio of two integers. This was settled in 1761, when Lambert proved that π is irrational, thus establishing that the digits of π do not repeat in any number base. In 1882 Lindemann established that π is transcendental, thus establishing that the digits of π^2 or any integer polynomial of π cannot repeat, and also settling once and for all the ancient Greek question of whether the circle could be squared—it cannot, because all numbers that can be formed by finite straightedge-and-compass constructions are necessarily algebraic.

The Computer Age

At the dawn of the computer age, John von Neumann suggested computing digits of prominent mathematical constants, including π and e, for statistical analysis. At his instigation, π was computed to 2,037 digits in 1949 on the Electronic

Division of Medicine & Science, National Museum of American History, Smithsonian Institution.

Figure 2. The ENIAC in the Smithsonian's National Museum of American History.

Table 1. Modern Computer-Era π Calculations.

Name	Year	Correct Digits
Miyoshi and Kanada	1981	2,000,036
Kanada-Yoshino-Tamura	1982	16,777,206
Gosper	1985	17,526,200
Bailey	Jan. 1986	29,360,111
Kanada and Tamura	Sep. 1986	33,554,414
Kanada and Tamura	Oct. 1986	67,108,839
Kanada et. al	Jan. 1987	134,217,700
Kanada and Tamura	Jan. 1988	201,326,551
Chudnovskys	May 1989	480,000,000
Kanada and Tamura	Jul. 1989	536,870,898
Kanada and Tamura	Nov. 1989	1,073,741,799
Chudnovskys	Aug. 1991	2,260,000,000
Chudnovskys	May 1994	4,044,000,000
Kanada and Takahashi	Oct. 1995	6,442,450,938
Kanada and Takahashi	Jul. 1997	51,539,600,000
Kanada and Takahashi	Sep. 1999	206,158,430,000
Kanada-Ushiro-Kuroda	Dec. 2002	1,241,100,000,000
Takahashi	Jan. 2009	1,649,000,000,000
Takahashi	Apr. 2009	2,576,980,377,524
Bellard	Dec. 2009	2,699,999,990,000
Kondo and Yee	Aug. 2010	5,000,000,000,000

Numerical Integrator and Calculator (ENIAC); see Figure 2. In 1965 mathematicians realized that the newly discovered fast Fourier transform could be used to dramatically accelerate high-precision multiplication, thus facilitating not only large calculations of π and other mathematical constants but research in computational number theory as well.

In 1976 Eugene Salamin and Richard Brent independently discovered new algorithms for computing the elementary exponential and trigonometric functions (and thus constants such as π and e) much more rapidly than by using classical series expansions. Their schemes, based on elliptic integrals and the Gauss arithmetic-geometric mean iteration, approximately *double* the number of correct digits in the result with each iteration. Armed with such techniques, π was computed to over one million digits in 1973, to over one billion digits in 1989, to over one trillion digits in 2002, and to over five trillion digits at the present time; see Table 1.

Similarly, the constants $e, \phi, \sqrt{2}, \log 2, \log 10, \zeta(3)$, Catalan's constant $G = \sum_{n=0}^{\infty}(-1)^n/(2n+1)^2$, and Euler's γ constant have now been computed to impressive numbers of digits; see Table 2 [10].

Table 2. Computations of Other Mathematical Constants.

Constant	Decimal digits	Researcher	Date
$\sqrt{2}$	1,000,000,000,000	S. Kondo	2010
ϕ	1,000,000,000,000	A. Yee	2010
e	500,000,000,000	S. Kondo	2010
$\log 2$	100,000,000,000	S. Kondo	2011
$\log 10$	100,000,000,000	S. Kondo	2011
$\zeta(3)$	100,000,001,000	A. Yee	2011
G	31,026,000,000	A. Yee and R. Chan	2009
γ	29,844,489,545	A. Yee	2010

One of the most intriguing aspects of this historical chronicle is the repeated assurances, often voiced by highly knowledgeable people, that future progress would be limited. As recently as 1963, Daniel Shanks, who himself calculated π to over 100,000 digits, told Philip Davis that computing one billion digits would be "forever impossible." Yet this feat was achieved less than thirty years later in 1989 by Yasumasa Kanada of Japan. Also in 1989, famous British physicist Roger Penrose, in the first edition of his best-selling book *The Emperor's New Mind*, declared that humankind likely will never know if a string of ten consecutive 7s occurs in the decimal expansion of π. This string was found just eight years later, in 1997, also by Kanada, beginning at position 22,869,046,249. After being advised of this fact by one of the present authors, Penrose revised his second edition to specify twenty consecutive 7s.

Along this line, Brouwer and Heyting, exponents of the "intuitionist" school of mathematical logic, proposed, as a premier example of a hypothesis that could never be formally settled, the question of whether the string "0123456789" appears in the decimal expansion of π. Kanada found this at the 17,387,594,880-th position after the decimal point. Even astronomer Carl Sagan, whose lead character in his 1985 novel *Contact* (played by Jodi Foster in the movie version) sought confirmation in base-11 digits of π, expressed surprise to learn, shortly after the book's publication, that π had already been computed to many millions of digits.

The BBP Formula for pi

A 1997 paper [3], [5, Ch. 3] by one of the present authors (Bailey), Peter Borwein and Simon Plouffe presented the following unknown formula for π, now known as the "BBP" formula for π:

(3)
$$\pi = \sum_{k=0}^{\infty} \frac{1}{16^k}\left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6}\right).$$

This formula has the remarkable property that it permits one to directly calculate binary or hexadecimal digits of π beginning at an arbitrary starting position without needing to calculate any of the preceding digits. The resulting simple algorithm requires only minimal memory, does not require multiple-precision arithmetic, and is very well suited to highly parallel computation. The cost of this scheme increases only slightly faster than the index of the starting position.

The proof of this formula is surprisingly elementary. First note that for any $k < 8$,

(4)
$$\int_0^{1/\sqrt{2}} \frac{x^{k-1}}{1-x^8}\,dx = \int_0^{1/\sqrt{2}} \sum_{i=0}^{\infty} x^{k-1+8i}\,dx = \frac{1}{2^{k/2}} \sum_{i=0}^{\infty} \frac{1}{16^i(8i+k)}.$$

Thus one can write

(5)
$$\sum_{i=0}^{\infty} \frac{1}{16^i}\left(\frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6}\right) = \int_0^{1/\sqrt{2}} \frac{4\sqrt{2} - 8x^3 - 4\sqrt{2}x^4 - 8x^5}{1-x^8}\,dx,$$

which on substituting $y := \sqrt{2}x$ becomes

(6)
$$\int_0^1 \frac{16y - 16}{y^4 - 2y^3 + 4y - 4}\,dy = \int_0^1 \frac{4y}{y^2-2}\,dy - \int_0^1 \frac{4y-8}{y^2-2y+2}\,dy = \pi,$$

reflecting a partial fraction decomposition of the integral on the left-hand side. In 1997 neither Maple nor Mathematica could evaluate (3) symbolically to produce the result π. Today both systems can do this easily.

Binary Digits of log 2

It is worth noting that the BBP formula (3) was not discovered by a conventional analytic derivation. Instead, it was discovered via a computer-based search using the PSLQ *integer relation detection algorithm* (see the section "Hunt for a pi Formula") of mathematician-sculptor Helaman Ferguson [4] in a process that some have described as an exercise in "reverse mathematical engineering". The motivation for this search was the earlier observation by the authors of [3] that log 2 also has this arbitrary position digit calculating property. This can be seen by analyzing the classic formula

(7)
$$\log 2 = \sum_{k=1}^{\infty} \frac{1}{k2^k},$$

which has been known at least since the time of Euler and which is closely related to the functional equation for the dilogarithm.

Let r mod 1 denote the fractional part of a nonnegative real number r, and let d be a nonnegative integer. Then the binary fraction of log 2 after the "decimal" point has been shifted to the right d places can be written as

(8)
$$\begin{aligned}(2^d \log 2) \bmod 1 &= \left(\sum_{k=1}^{d} \frac{2^{d-k}}{k} \bmod 1 + \sum_{k=d+1}^{\infty} \frac{2^{d-k}}{k} \bmod 1\right) \bmod 1 \\ &= \left(\sum_{k=1}^{d} \frac{2^{d-k} \bmod k}{k} \bmod 1 \right.\\ &\qquad \left. + \sum_{k=d+1}^{\infty} \frac{2^{d-k}}{k} \bmod 1\right) \bmod 1,\end{aligned}$$

where "mod k" has been inserted in the numerator of the first term since we are only interested in the fractional part of the result after division.

The operation $2^{d-k} \bmod k$ can be performed very rapidly by means of the *binary algorithm for exponentiation*. This scheme is the simple observation that an exponentiation operation such as 3^{17} can be performed in only five multiplications instead of 16 by writing it as $3^{17} = ((((3^2)^2)^2)^2) \cdot 3$. Additional savings can be realized by reducing all of the intermediate multiplication results modulo k at each step. This algorithm, together with the division and summation operations indicated in the first term, can be performed in ordinary double-precision floating-point arithmetic or for very large calculations by using quad- or oct-precision arithmetic.

Expressing the final fractional value in binary notation yields a string of digits corresponding to the binary digits of log 2 beginning immediately after the first d digits of log 2. Computed results can be easily checked by performing this operation for two slightly different positions, say $d-1$ and d, then checking to see that resulting digit strings properly overlap.

Hunt for a pi Formula

In the wake of finding the above scheme for the binary digits of log 2, the authors of [3] immediately wondered if there was a similar formula for π (none was known at the time). Their approach was to collect a list of mathematical constants (α_i) for which formulas similar in structure to the formula for log 2 were known in the literature and then to determine by means of the PSLQ integer relation algorithm if there exists a nontrivial linear relation of the form

(9)
$$a_0\pi + a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n = 0,$$

where a_i are integers (because such a relation could then be solved for π to yield the desired formula). After several months of false starts, the following relation was discovered:

(10)

$$\pi = 4 \cdot {}_2F_1\left(\begin{matrix}1, \frac{1}{4} \\ \frac{5}{4}\end{matrix} \middle| -\frac{1}{4}\right) + 2\arctan\left(\frac{1}{2}\right) - \log 5,$$

where the first term is a Gauss hypergeometric function evaluation. After writing this formula explicitly in terms of summations, the BBP formula for π was uncovered:

(11)

$$\pi = \sum_{k=0}^{\infty} \frac{1}{16^k}\left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6}\right).$$

One question that immediately arose in the wake of the discovery of the BBP formula for π was whether there are formulas of this type for π in other number bases—in other words, formulas where the 16 in the BBP formula is replaced by some other integer, such as 3 or 10. These computer searches were largely laid to rest in 2004, when one of the present authors (Jonathan Borwein), together with Will Galway and David Borwein, showed that there are no degree-1 BBP-type formulas of *Machin-type* for π, except those whose base is a power of two [5, pp. 131–133].

The BBP Formula in Action

Variants of the BBP formula have been used in numerous computations of high-index digits of π. In 1998 Colin Percival, then a 17-year-old undergraduate at Simon Fraser University in Canada, computed binary digits beginning at position one quadrillion (10^{15}). At the time, this was one of the largest, if not the largest, distributed computations ever done. More recently, in July 2010, Tsz-Wo Sze of *Yahoo! Cloud Computing*, in roughly 500 CPU-years of computing on *Apache Hadoop* clusters, found that the base-16 digits of π beginning at position 5×10^{14} (corresponding to binary position two quadrillion) are

```
0 E6C1294A ED40403F 56D2D764 026265BC
A98511D0 FCFFAA10 F4D28B1B B5392B8
```

In an even more recent 2013 computation along this line, Ed Karrels of Santa Clara University used a system with NVIDIA graphics cards to compute 26 base-16 digits beginning at position one quadrillion. His result: `8353CB3F7F0C9ACCFA9AA215F2`.

The BBP formulas have also been used to confirm other computations of π. For example, in August 2010, Shigeru Kondo (a hardware engineer) and Alexander Yee (an undergraduate software engineer) computed five trillion decimal digits of π on a home-built \$18,000 machine. They found that the last thirty digits leading up to position five trillion are

Figure 3. (T) Shigeru Kondo and his π-computer. (B) Alex Yee and his elephant.

```
7497120374 4023826421 9484283852
```

Kondo and Yee (see Figure 3) used the following Chudnovsky-Ramanujan series:

(12)

$$\frac{1}{\pi} = 12 \sum_{k=0}^{\infty} \frac{(-1)^k\,(6k)!(13591409 + 545140134k)}{(3k)!\,(k!)^3\,640320^{3k+3/2}}.$$

They did not merely evaluate this formula as written but instead employed a clever quasi-symbolic scheme that mostly avoids the need for full-precision arithmetic.

Kondo and Yee first computed their result in hexadecimal (base-16) digits. Then, in a crucial verification step, they checked hex digits near the end against the same string of digits computed using the BBP formula for π. When this test passed, they converted their entire result to decimal. The entire computation took ninety days, including sixty-four hours for the BBP confirmation and eight days for base conversion to decimal. Note that the much lower time for the BBP confirmation, relative to the other two parts, greatly reduced the overall computational cost. A description of their work is available at [11].

BBP-Type Formulas for Other Constants

In the years since 1997, computer searches using the PSLQ algorithm, as well as conventional analytic investigations, have uncovered BBP-type formulas for numerous other mathematical constants, including $\pi^2, \log^2 2, \pi \log 2, \zeta(3), \pi^3, \log^3 2, \pi^2 \log 2, \pi^4, \zeta(5)$ and Catalan's constant. BBP formulas are also known for many arctangents, as well as for $\log k$, $2 \le k \le 22$, although none is known for $\log 23$. These formulas and many others, together with references, are given in an online compendium [1].

One particularly intriguing fact is that, whereas only binary formulas exist for π, there are both binary and ternary (base-3) formulas for π^2:

(13)
$$\pi^2 = \frac{9}{8}\sum_{k=0}^{\infty}\frac{1}{64^k}\left(\frac{16}{(6k+1)^2} - \frac{24}{(6k+2)^2} - \frac{8}{(6k+3)^2} - \frac{6}{(6k+4)^2} + \frac{1}{(6k+5)^2}\right),$$

(14)
$$\pi^2 = \frac{2}{27}\sum_{k=0}^{\infty}\frac{1}{729^k}\left(\frac{243}{(12k+1)^2} - \frac{405}{(12k+2)^2} - \frac{81}{(12k+4)^2} - \frac{27}{(12k+5)^2} - \frac{72}{(12k+6)^2} - \frac{9}{(12k+7)^2} - \frac{9}{(12k+8)^2} - \frac{5}{(12k+10)^2} + \frac{1}{(12k+11)^2}\right).$$

Formula (13) appeared in [3], while formula (14) is due to Broadhurst. There are known binary BBP formulas for both $\zeta(3)$ and π^3, but no one has found a ternary formula for either.

Catalan's Constant

One other mathematical constant of central interest is Eugéne Charles Catalan's (1814–1894) constant,

(15)
$$G = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2} = 0.91596559417722\ldots,$$

which is arguably the most basic constant whose irrationality and transcendence (though strongly suspected) remain unproven. Note the close connection to this formula for π^2:

(16)
$$\frac{\pi^2}{8} = \sum_{n=0}^{\infty}\frac{1}{(2n+1)^2} = 1.2337005501362\ldots.$$

Formulas (15) and (16) can be viewed as the simplest Dirichlet L-series values at 2. Such considerations were behind our decision to focus the computation described in this paper on these two constants.

Catalan's constant has already been the subject of some large computations. As mentioned above, in 2009 Alexander Yee and Raymond Chan calculated G to 31.026 billion digits [10]. This computation employed two formulas, including this formula due to Ramanujan:

(17)
$$G = \frac{3}{8}\sum_{n=0}^{\infty}\frac{1}{\binom{2n}{n}(2n+1)^2} + \frac{\pi}{8}\log(2+\sqrt{3}),$$

which can be derived from the fact that

$$G = -T(\pi/4) = -3/2 \cdot T(\pi/12),$$

where $T(\theta) := \int_0^\theta \log\tan\sigma \, d\sigma$.

The BBP compendium lists two BBP-type formulas for G. The first was discovered numerically by Bailey, but both it and the second formula were subsequently proven by Kunle Adegoke, based in part on some results of Broadhurst.

For the present study, we sought a formula for G with as few terms as possible, because the run time for computing with a BBP-type formula increases roughly linearly with the number of nonzero coefficients. The two formulas in the compendium have twenty-two and eighteen nonzero coefficients, respectively. So we explored, by means of a computation involving the PSLQ algorithm, the linear space of formulas for G spanned by these two sets of coefficients, together with two known "zero relations" (BBP-type formulas whose sum is zero). These analyses and computations led to the following formula, which has only sixteen nonzero coefficients and which we believe to be the most economical BBP-type formula for computing Catalan's constant:

(18)
$$G = \frac{1}{4096}\sum_{k=0}^{\infty}\frac{1}{4096^k}\left(\frac{36864}{(24k+2)^2} - \frac{30720}{(24k+3)^2} - \frac{30720}{(24k+4)^2} - \frac{6144}{(24k+6)^2} - \frac{1536}{(24k+7)^2} + \frac{2304}{(24k+9)^2} + \frac{2304}{(24k+10)^2} + \frac{768}{(24k+14)^2} + \frac{480}{(24k+15)^2} + \frac{384}{(24k+11)^2} + \frac{1536}{(24k+12)^2} + \frac{24}{(24k+19)^2} - \frac{120}{(24k+20)^2} - \frac{36}{(24k+21)^2} + \frac{48}{(24k+22)^2} - \frac{6}{(24k+23)^2}\right).$$

BBP Formulas and Normality

One prime motivation in computing and analyzing digits of π and other well-known mathematical constants through the ages is to explore the age-old question of whether and why these digits appear "random". Numerous computer-based statistical checks of the digits of π—unlike those of e—so far have failed to disclose any deviation from reasonable statistical norms. See, for instance, Table 3, which presents the counts of individual hexadecimal digits among the first trillion hex digits, as obtained by Yasumasa Kanada.

Given some positive integer b, a real number α is said to be b-normal if every m-long string of base-b digits appears in the base-b expansion of α with precisely the expected limiting frequency $1/b^m$. It follows from basic probability theory that almost all real numbers are b-normal for any specific base b and even for all bases simultaneously. But proving normality for specific constants of interest in mathematics has proven remarkably difficult.

Interest in BBP-type formulas was heightened by the 2001 observation, by one of the present authors (Bailey) and Richard Crandall, that the normality of BBP-type constants such as $\pi, \pi^2, \log 2$ and G can be reduced to a certain hypothesis regarding the behavior of a class of chaotic iterations [5, pp. 141–173]. No proof is known for this general hypothesis, but even specific instances of this result would be quite interesting. For example, if it could be established that the iteration given by $w_0 = 0$ and

$$w_n = \left(2w_{n-1} + \frac{1}{n}\right) \bmod 1 \tag{19}$$

is equidistributed in $[0, 1)$ (i.e., is a "good" pseudorandom number generator), then, according to the Bailey-Crandall result, it would follow that $\log 2$ is 2-normal. In a similar vein, if it could be established that the iteration given by $x_0 = 0$ and

$$x_n = \left(16x_{n-1} + \frac{120n^2 - 89n + 16}{512n^4 - 1024n^3 + 712n^2 - 206n + 21}\right) \bmod 1 \tag{20}$$

is equidistributed in $[0, 1)$, then it would follow that π is 2-normal.

Giving further hope to these studies is the recent extension of these methods to a rigorous proof of normality for an uncountably infinite class of real numbers. Given a real number r in $[0, 1)$, let r_k denote the k-th binary digit of r. Then the real number

$$\alpha_{2,3}(r) = \sum_{k=0}^{\infty} \frac{1}{3^k 2^{3^k + r_k}} \tag{21}$$

is 2-normal. For example, the constant $\alpha_{2,3}(0) = \sum_{k \geq 0} 1/(3^k 2^{3^k}) = 0.541883680831502985\ldots$ is provably 2-normal. A similar result applies if 2 and 3 in this formula are replaced by any pair of coprime integers (b, c) greater than one [5, pp. 141–173].

A Curious Hexadecimal Conjecture

It is tantalizing that if, using (20), one calculates the hexadecimal digit sequence

$$y_n = \lfloor 16x_n \rfloor \tag{22}$$

Table 3. Digit counts in the first trillion hexadecimal (base-16) digits of π. Note that deviations from the average value 62,500,000,000 occur only after the first six digits, as expected.

Hex Digit	Occurrences
0	62499881108
1	62500212206
2	62499924780
3	62500188844
4	62499807368
5	62500007205
6	62499925426
7	62499878794
8	62500216752
9	62500120671
A	62500266095
B	62499955595
C	62500188610
D	62499613666
E	62499875079
F	62499937801
Total	1000000000000

(where $\lfloor \cdot \rfloor$ denotes greatest integer), then the sequence (y_n) appears to perfectly (not just approximately) produce the hexadecimal expansion of π. In explicit computations, we checked that the first 10,000,000 hexadecimal digits generated by this sequence are *identical* with the first 10,000,000 hexadecimal digits of $\pi - 3$. This is a fairly difficult computation, as it requires roughly n^2 bit-operations and is not easily performed on a parallel computer system. In our implementation, computing $2,000,000$ hex digits with (22) using Maple, required 17.3 hours on a laptop. Computing 4,100,000 using Mathematica, with a more refined implementation, required 46.5 hours. The full confirmation using a C++ program took 433,192 seconds (120.3 hours) on an IBM Power 780 (model: 9179-MHB, clock speed: 3.864 GHz). All these outputs were confirmed against stored hex digits of π in the software section of `http://www.experimentalmath.info`.

Conjecture 1. *The sequence* $\lfloor 16x_n \rfloor$, *where* (x_n) *is the sequence of iterates defined in equation* (20), *generates precisely the hexadecimal expansion of* $\pi - 3$.

We can learn more. Let $||x - y|| = \min(|x - y|, |1 - (x - y)|)$ denote the "wrapped" distance between reals x and y in $[0, 1)$. The base-16 expansion of π, which we denote π_n, satisfies

$$(23) \quad ||\pi_n - x_n|| \le \sum_{k=n+1}^{\infty} \cdot \frac{120k^2 - 89k + 16}{16^{k-n}(512k^4 - 1024k^3 + 712k^2 - 206k + 21)} \approx \frac{1}{64(n+1)^2},$$

so that ,upon summing from some N to infinity, we obtain the finite value

$$(24) \qquad \sum_{n=N}^{\infty} ||\pi_n - x_n|| \le \frac{1}{64(N+1)}.$$

Heuristically, let us assume that the π_n are independent, uniformly distributed random variables in $(0,1)$, and let $\delta_n = ||\alpha_n - x_n||$. Note that an error (i.e., an instance where x_n lies in a subinterval of the unit interval different from π_n so that the corresponding hex digits don't match) can only occur when π_n is within δ_n of one of the points $(0, 1/16, 2/16, \ldots, 15/16)$. Since $x_n < \pi_n$ for all n (where $<$ is interpreted in the wrapped sense when x_n is slightly less than one), this event has probability $16\delta_n$. Then the fact that the sum (24) has a finite value implies that, by the first *Borel-Cantelli* lemma, there can only be finitely many errors. Further, the small value of the sum (24), even when $N = 1$, suggests that it is unlikely that any errors will be observed. If we set $N = 10,000,001$ in (24), since we know there are no errors in the first 10,000,000 elements, we obtain an upper bound of 1.563×10^{-9}, which suggests it is truly unlikely that errors will ever occur.

A similar correspondence can be seen between iterates of (19) and the binary digits of $\log 2$. In particular, let $z_n = \lfloor 2w_n \rfloor$, where w_n is given in (19). Then since the sum of the error terms for $\log 2$, corresponding to (24), is infinite, it follows by the second Borel-Cantelli lemma that discrepancies between (z_n) and the binary digits of $\log 2$ can be expected to appear indefinitely but with decreasing frequency. Indeed, in computations that we have done, we have found that the sequence (z_n) disagrees with ten of the first twenty binary digits of $\log 2$, but in only one position over the range 5,000 to 8,000.

Computing Digits of π^2 and Catalan's Constant

In illustration of this theory, we now present the results of computations of high-index binary digits of π^2, ternary digits of π^2, and binary digits of Catalan's constant, based on formulas (13), (14), and (18), respectively. These calculations were performed on a 4-rack *BlueGene/P* system at IBM's Benchmarking Center in Rochester, Minnesota (see Figure 4). This is a shared facility, so calculations were conducted over a several-month period during which time, none, some, or all of the system was available. It was programmed remotely from Australia, which permitted the system to be used off-hours. Sometimes it helps to be in a different time zone!

Figure 4. Andrew Mattingly, Blue Gene/P, and Glenn Wightwick.

(1) *Base-64 digits of π^2 beginning at position 10 trillion.* The first run, which produced base-64 digits starting from position $10^{12} - 1$, required an average of 253,529 seconds per thread and was subdivided into seven partitions of 2048 threads each, so the total cost was $7 \cdot 2048 \cdot 253529 = 3.6 \times 10^9$ CPU-seconds. Each rack of the IBM system features 4096 cores, so the total cost is 10.3 "rack-days".

The second run, which produced base-64 digits starting from position 10^{12}, completed in nearly the same run time (within a few minutes). The two resulting base-8 digit strings appear in row A of Table X. (Each pair of base-8 digits corresponds to a base-64 digit.) Here the digits in agreement are delimited by |. Note that 53 consecutive base-8 digits (or, equivalently, 159 consecutive binary digits) are in perfect agreement.

(2) *Base-729 digits of π^2 beginning at position 10 trillion*. In this case the two runs each required an average of 795,773 seconds per thread, similarly subdivided as above, so that the total cost was 6.5×10^9 CPU-seconds, or 18.4 "rack-days". The two resulting base-9 digit strings are found in row B of Table X. (Each triplet of base-9 digits corresponds to one base-729 digit.) Note here that 47 consecutive base-9 digits (94 consecutive base-3 digits) are in perfect agreement.

(3) *Base-4096 digits of Catalan's constant beginning at position 10 trillion*. These two runs each required 707,857 seconds per thread, but in this case they were subdivided into eight partitions of 2048 threads each, so that the total cost was 1.2×10^{10} CPU-seconds, or 32.8 "rack-days". The two resulting base-8 digit strings are found in row C of Table X. (Each quadruplet of base-8 digits corresponds to one base-4096 digit.) Note that 47 consecutive base-8 digits (141 consecutive binary digits) are in perfect agreement.

These long strings of consecutively agreeing digits, beginning with the target digit, provide a compelling level of statistical confidence in the results. In the first case, for instance, note that the probability that thirty-two pairs of randomly chosen base-8 digits are in perfect agreement is roughly 1.2×10^{-29}. Even if one discards, say, the final six base-8 digits as a 1-in-262,144 statistical safeguard against numerical round-off error, one would still have twenty-four consecutive base-8 digits in perfect agreement, with a corresponding probability of 2.1×10^{-22}. Now strictly speaking, one cannot define a valid probability measure on digits of π^2, but nonetheless, from a practical point of view, such analysis provides a very high level of statistical confidence that the results have been correctly computed.

For this reason, computations of π and the like are a favorite tool for the integrity testing for computer system hardware and software. If either run of a paired computation of π succumbs to even a single fault in the course of the computation, then typically the final results will disagree almost completely. For example, in 1986 a similar pair of computations of π disclosed some subtle but substantial hardware errors in an early model of the Cray-2 supercomputer. Indeed, the calculations we have done arguably constitute the most strenuous integrity test ever performed on the BlueGene/P system. Table 4 gives some sense of the scale of the three record computations, which used more than 135 "rack-days", 1378 serial CPU-years, and more than 1.549×10^{19} floating point operations. This is comparable to the cost of the most sophisticated animated movies as of the present time (2011).

For the sake of completeness, in Table 5 we also record the one-, two-, and three-bit frequency counts from our Catalan computation.

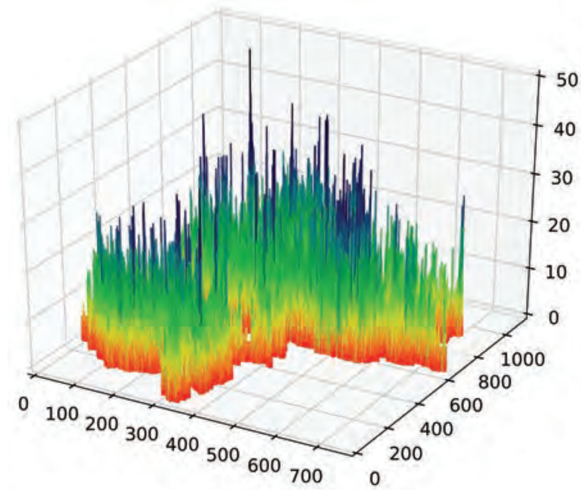

Figure 5. A "random" walk on a million digits of Catalan's constant.

Future Directions

It is ironic that, in an age when even pillars such as Fermat's Last Theorem and the Poincaré conjecture have succumbed to the brilliance of modern mathematics, one of the most elementary mathematical hypotheses, namely whether (and why) the digits of π or other constants such as $\log 2, \pi^2$, or G (see Figure 5) are "random", remains unanswered. In particular, proving that π (or $\log 2, \pi^2$, or G) is b-normal in some integer base b remains frustratingly elusive. Even much weaker results, for instance the simple assertion that a one appears in the binary expansion of π (or $\log 2, \pi^2$, or G) with limiting frequency $1/2$ (which assertion has been amply affirmed in numerous computations over the years), remain unproven and largely inaccessible at the present time.

Almost as much ignorance extends to simple algebraic irrationals such as $\sqrt{2}$. In this case it is now known that the number of ones in the

Table 4. (A) base-4 digits of π^2, (B) base-729 digits of π^2, and (C) base-4096 digits of Catalan's constant, in each case beginning at position 10 trillion.

A	75|601145053032364757245000057432627545303630524163506344|573227604
	|601145053032364757245000057432627545303630524163506344|220210566
B	001|122644850645485831771111352101628560483234534688|10565567635862
	|122644850645485831771111352101628560483234534688|04744867134524
C	0176|347050537747770511226133716201252573272173245222|6000177545727
	|347050537747770511226133716201252573272173245222|5703510516602

Table 5. The scale of our computations. We estimate 4.5 quad-double operations per iteration and that each costs 266 single-precision operations. The total cost in single-precision operations is given in the last column. This total includes overhead which is largely due to a rounding operation that we implemented using bit-masking.

CONSTANT	n'	d	#ITERS ($\times 10^{15}$)	TIME/ITER (microsec)	TIME (yr)	WITH VERIFY	TOTAL (yr)	O'HEAD (%)	FLOPS ($\times 10^{18}$)
π^2 base-2^6	5	10^{13}	2.16	1.424	97.43	194.87	230.35	18.2	2.58
π^2 base-3^6	9	10^{13}	3.89	1.424	175.38	350.76	413.16	17.8	4.65
G base-4^6	16	10^{13}	6.91	1.424	311.79	623.58	735.02	17.9	8.26

Table 6. Base-4096 digits of G beginning at position 10 trillion: digit proportions.

Digit	0	1	2	3	4	5	6	7
base-2 (141)	0.454	0.546	-	-	-	-	-	-
base-4 (70)	0.171	0.329	0.229	0.271	-	-	-	-
base-8 (47)	0.085	0.128	0.213	0.128	0.064	0.128	0.043	0.213

first n binary digits of $\sqrt{2}$ must be at least of the order of $\sqrt{n}$, with similar results for other algebraic irrationals [5, pp. 141–173]. But this is a very weak result, given that this limiting ratio is almost certainly $1/2$, not only for $\sqrt{2}$ but more generally for all algebraic irrationals.

Nor can we prove much about continued fractions for various constants, except for a few well-known results for special cases such as quadratic irrationals, ratios of Bessel functions, and certain expressions involving exponential functions.

For these reasons there is continuing interest in the theory of BBP-type constants, since, as mentioned, there is an intriguing connection between BBP-type formulas and certain chaotic iterations that are akin to pseudorandom number generators. If these connections can be strengthened, then perhaps normality proofs could be obtained for a wide range of polylogarithmic constants, possibly including $\pi, \log 2, \pi^2$, and G.

As settings change, so do questions. Until the question of efficient single-digit extraction was asked, our ignorance about such issues was not exposed. The case of the exponential series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{25}$$

is illustrative. For present purposes, the convergence rate in (25) is too good.

Conjecture 2. *There is no BBP formula for e. Moreover, there is no way to extract individual digits of e significantly more rapidly than by computing the first n digits.*

The same could be conjectured about other numbers, including Euler's constant $\gamma = 0.57721566490153\ldots$. In short, until vastly stronger mathematical results are obtained in this area, there will doubtless be continuing interest in computing digits of these constants. In the present vacuum, that is perhaps all that we can do.

Acknowledgments

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Remembering Basil Gordon, 1931–2012

Krishnaswami Alladi, Coordinating Editor

Basil Gordon, who made major research contributions to number theory, combinatorics, and algebra and who enhanced our understanding of Ramanujan's remarkable identities, passed away on January 12, 2012, at the age of eighty. Gordon, who was a professor of mathematics at UCLA, was an outstanding teacher at all levels. His legacy will continue, owing to the impact of his fundamental work and to the many students he groomed, as well as several other mathematicians he influenced. In this article, five noted mathematicians describe various contributions of Gordon and include personal reflections as well. Krishnaswami Alladi discusses Gordon's research on partitions and extensions of Ramanujan's identities. George Andrews describes Gordon's work on plane partitions. Ken Ono's article is on Gordon's work on modular forms, whereas Robert Guralnick talks about Gordon's contributions to algebra. Finally, Bruce Rothschild recalls how he and Basil Gordon ran the *Journal of Combinatorial Theory, Ser. A*, as managing editors.

Keith Kendig, a Ph.D. student of Professor Gordon in the 1960s, conducted a detailed interview of Gordon in 2011 dealing with his life and mathematical career. The text of this interview with pictures of Gordon from his childhood will appear in *Fascinating Mathematical People*, to be published by Princeton University Press in 2014. Some pictures of Gordon, courtesy of Keith Kendig, are included here.

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Photo courtesy of Keith Kendig.

Basil Gordon as a young faculty member at UCLA. Photo taken by Paul Halmos.

Krishnaswami Alladi

The Great Guru

Professor Basil Gordon was a towering figure in combinatorics and number theory. He made fundamental contributions to several areas, such as the theory of partitions, modular forms, mock theta functions, and coding theory. He was one of the very few who was at home with both combinatorial and modular form techniques. He was one of the

Photo courtesy of Krishnaswami Alladi.

The five mathematicians associated with the Capparelli Conjecture, its resolution, and generalizations all got together in Gainesville, Florida, in 2004. Clockwise from bottom right: Basil Gordon, Jim Lepowsky, Stefano Capparelli, George Andrews, and Krishnaswami Alladi.

leaders in the world of Ramanujan's mathematics. As managing editors for over two decades, he and his UCLA colleague Bruce Rothschild developed the *Journal of Combinatorial Theory-A* into a premier journal. A few years ago, the JCT-A came up with a special issue in honor of Gordon and Rothschild on their retirement from that editorial board. George Andrews, Ken Ono, Richard McIntosh, and I wrote a paper [3] about Gordon's work for that volume. The current article will be very different. I will give samples of some of Gordon's most appealing theorems and provide some personal reflections.

One of the finest examples of Gordon's fundamental research is his generalization of the Rogers-Ramanujan identities to odd moduli [11]. In analytic form, the Rogers-Ramanujan (R-R) identities provide product representations to two q-hypergeometric series (see [7]). The combinatorial interpretation of these identities is:

Theorem R-R. *For $i = 1, 2$, the number of partitions of an integer n into parts that differ by at least 2, with least part $\geq i$, equals the number of partitions of n into parts $\equiv \pm i (mod\, 5)$.*

In the 1960s Gordon [11] obtained the following beautiful generalization of the Rogers-Ramanujan partition theorem to all odd moduli ≥ 5:

Theorem 1. *For any pair of integers i and k satisfying $1 \leq i \leq k$ and $k \geq 2$, the number of partitions of an integer n of the form $b_1 + b_2 + \cdots + b_\nu$, where $b_j - b_{j+k-1} \geq 2$, and with at most $i - 1$ ones, is equal to the number of partitions of n into parts $\not\equiv 0, \pm i\ (mod\, 2k+1)$.*

In a partition we always write the parts b_i in descending order.

This result opened up a new direction of research (see [7]) on R-R type identities, namely identities which connect partitions with parts satisfying difference conditions to partitions with parts satisfying congruence conditions. George Andrews has been the leader in this field, and he was inspired by Gordon's generalization of the R-R identities.

One reason that the classical Rogers-Ramanujan identities are so important is because the ratio of the series in the two identities admits a continued fraction expansion, and this continued fraction has a lovely product representation:

$$R(q) = 1 + \cfrac{q}{1 + \cfrac{q^2}{1 + \cfrac{q^3}{\cdots}}} = \prod_{m=1}^{\infty} \frac{(1-q^{5m-2})(1-q^{5m-3})}{(1-q^{5m-1})(1-q^{5m-4})}. \tag{1}$$

In view of the product, the continued fraction plays an important role in the theory of modular forms in relation to the congruence subgroup $\Gamma_0(5)$ of the modular group.

Another gem that Gordon found was an analogue of (1) to the modulus 8. More precisely, by working with two q-hypergeometric identities that were analogous to the Rogers-Ramanujan identities but for the modulus 8 instead of the modulus 5 and by considering their ratio, Gordon [12] showed that

$$G(q) = 1 + q + \cfrac{q^2}{1 + q^3 + \cfrac{q^4}{1 + q^5 + \cfrac{q^6}{\cdots}}} = \prod_{m=1}^{\infty} \frac{(1-q^{8m-3})(1-q^{8m-5})}{(1-q^{8m-1})(1-q^{8m-7})}. \tag{2}$$

Like $R(q)$, the fraction $G(q)$ also has an important role in the theory of modular forms but to the congruence subgroup $\Gamma_0(8)$. This continued fraction identity was independently discovered by H. Göllnitz [10], and so this is now called the Göllnitz-Gordon continued fraction. The partition theorem that underlies the continued fraction in (2) is:

Theorem 2 (Göllnitz-Gordon)**.** *For $i = 1, 2$, the number of partitions of an integer n into parts that differ by at least 2, with least part $\geq 2i - 1$, and with no consecutive even numbers as parts, equals the number of partitions of n into parts $\equiv 4$ or $\pm (2i-1) (mod\, 8)$.*

Gordon told me that he was led to this fraction and Theorem 2 by his *meta theorem*: "What works for 5 works also for 8."

I always called Gordon "the great guru". His knowledge of mathematics was vast and deep and he shared his ideas generously. His contributions can not only be seen from his seminal papers but also in the work of his students whose careers he molded. During the Rademacher Centenary Conference at Penn State University in 1992 there were four of Gordon's Ph.D. students—Doug Bowman, Ken Ono, Richard McIntosh, and Sinai Robins—each presenting significant work on very different topics in number theory. That showed the breadth of Gordon's expertise.

Besides his Ph.D. students, there were several others like me who were influenced by his mathematical ideas and philosophy. In particular, it was due to his guidance that I was able to make the transition in the early nineties from classical analytic number theory to the theory of partitions and q-hypergeometric series, an area in which I continue to work today.

In December 1987 the Ramanujan Centennial was being celebrated, and several conferences were being conducted in India. I was asked to organize one such conference in Madras, and I invited Gordon to give a plenary talk. At that time I was working on classical analytic number theory, but I was charmed by the lectures of Gordon, Andrews, and others on partitions and q-hypergeometric series which I heard during the centennial. But I was in awe of the tantalizing q-hypergeometric identities and transformations being presented and a bit scared to enter this domain.

In 1989 I received a message from Gordon saying that he had a fully paid sabbatical and that he would like to spend a good part of it at the University of Florida. This was like a gift from heaven for me, because I realized that I could benefit from his visit by getting introduced into the world of partitions and q-hypergeometric series. And that is exactly what happened, and for this I am most grateful.

During that visit Gordon and I investigated a general continued fraction of Ramanujan, and through its study we obtained several results related to a number of classical identities in the theory of partitions and q-series from a unified perspective. This was my first paper [4] on partitions and q-series, and it appeared in the JCT-A. It was also during this visit that we started considering a very general approach to the celebrated 1926 partition theorem of Schur, which is:

Theorem S. *The number of partitions A(n) of an integer n into parts* $\equiv \pm 1 (mod\, 6)$ *is equal to the number of partitions B(n) of n into distinct parts* $\equiv \pm 1\ (mod\, 3)$*, and this is equal to the number of partitions C(n) of n into parts that differ by at least* 3 *but without consecutive multiples of* 3 *as parts.*

The condition "no consecutive multiples of 3 as parts" in Theorem S is analogous to the condition "no consecutive even numbers as parts" in Theorem 2. The equality $A(n) = C(n)$ in Schur's theorem can be considered as the next level result beyond the Rogers-Ramanujan partition theorem, because the gap 2 in Theorem R-R is replaced by 3 in Theorem S, and the modulus 5 in Theorem R-R is replaced by 6 in Theorem S. But Gordon told me that it is the equality $B(n) = C(n)$ that is more fundamental and capable of a significant generalization. He thus initiated me into his philosophy of "the method of weighted words," which is described in [3]. Guided by this philosophy of his, we found a generalization of Theorem S that involved words formed by colored integers satisfying certain gap conditions, and we were able to encapsulate it in the form of an elegant analytic *key identity* in two free parameters, a, b (see [5]).

Gordon's visit to the University of Florida in 1989 started our substantial collaboration. I visited him in Los Angeles over the next few years and worked on extensions of the method of weighted words to the deep partition theorem of Göllnitz [10]. We found a remarkable "key identity" in three free parameters, a, b, c, which contained our two-parameter identity for Schur's theorem as a special case. But we could not prove this three-parameter identity. In 1990, when George Andrews visited the University of Florida, I showed him the three-parameter identity Gordon and I had found. During that stay Andrews proved our identity, and that resulted in our triple joint paper [1]. Andrews told me that, in some sense as a pure partition result, Schur's theorem was more fundamental than the Rogers-Ramanujan partition theorem. Gordon and I worked out several ramifications of Schur's theorem [6] using the method of weighted words, and this confirmed Andrews's view of its importance.

I have already mentioned the Rademacher Centenary Conference of 1992 at Penn State and Gordon's presence at the conference with his students. I now state an interesting development that took place during that conference.

At the start of the Rademacher conference, Jim Lepowsky gave a talk on the connections between Lie algebras and partitions and stated a partition theorem as a conjecture made by his student Capparelli from a study of vertex operators in Lie algebras. Andrews went into hiding for the remainder of the conference to work on this conjecture, emerging from his hideout just to attend the talks. On the last day of the conference, Andrews outlined a generating function proof of Capparelli's conjecture and published it in the *Proceedings of the Rademacher Centenary Conference* [8]. Basil Gordon, who heard

Lepowsky's lecture, had noticed right away that the Capparelli conjecture could be generalized in the framework of the method of weighted words. He informed me about this a few weeks later as I was about to visit Penn State for my sabbatical in 1992–93 to work with Andrews. Gordon and I found a key identity for a generalized Capparelli theorem and provided a combinatorial bijective proof as well. This resulted in another triple paper [2], which we submitted to the *Journal of Algebra*, since Capparelli's work appeared there [9]. Neither Capparelli nor I were at the Rademacher Centenary Conference, but in fall 2004, Gordon, Andrews, Lepowsky, Capparelli, and I were all at a conference at the University of Florida. At that Florida conference, Gordon gave a beautiful lecture entitled "The return of the mock theta functions", in which he described among other things the work with his former student Richard McIntosh [14] on some new mock theta functions of order 8. Ramanujan, as is well known, had communicated his discovery of the mock theta functions in his last letter to Hardy in January 1920 just weeks before he died and in that letter gave examples of mock theta functions of orders 3, 5, and 7. Gordon and McIntosh investigated mock theta functions and their asymptotics in great detail. Many of their important results can be found in the survey paper [15]. There is also a good description of the Gordon-McIntosh work in [3], and all of this relates to the classical theory of mock theta functions. In the last few years dramatic advances have been made in a modern approach to mock theta functions which connects them to harmonic Maass forms, but I will not discuss that here.

Professor Gordon was very supportive of my effort to launch the *Ramanujan Journal* devoted to all areas of mathematics influenced by Ramanujan, and served on the editorial board since its inception in 1997. He contributed a fine paper to the very first issue, and I will describe this briefly.

The celebrated Euler's Pentagonal Numbers theorem has the combinatorial interpretation that if the set of partitions on an integer n into distinct parts is split into two subsets based on the parity of the number of parts, then the two subsets are equal in size except at the pentagonal numbers, in which case the difference is 1 between the sizes of the two subsets. Thus $Q(n)$, the number of partitions of n into distinct parts, is odd precisely at the pentagonal numbers. In 1995 I proved that

$$Q(n) = \sum_{k\geq 0} p_3(n;k)2^k, \tag{3}$$

where $p_3(n;k)$ is the number of partitions $b_1 + b_2 + \cdots + b_\nu$ of n into parts that differ by at least 3 and have precisely k gaps $b_i - b_{i+1}$ which are > 3, with $b_{\nu+1} = -1$. I told Gordon that this identity seems to indicate that, for each integer k ,

$$Q(n) \equiv 0 (mod\, 2^k) \quad \text{almost always}, \tag{4}$$

and one could prove (4) for small k using (3). But I did not know how to establish (4) for all $k \geq 1$. Gordon told me that such a congruence result is best approached through the theory of modular forms. Indeed he and Ken Ono established this conjecture using modular forms and contributed an important paper to the first issue of *The Ramanujan Journal* [13].

I had the pleasure of hosting Gordon at the University of Florida as well as in India. Whenever I visited UCLA to work with him, he would have me stay at his home or at least have me spend a substantial part of my stay working with him at his stately home in Santa Monica. In our ancient Hindu culture, we have the practice of *gurukula*. That is, the student lives with the guru and observes the guru in close quarters as he is practicing his art. By being so close to the guru, the student learns the nuances of whatever art form is being taught. My stay in Gordon's home was in some sense like a gurukula. I am sure his Ph.D. students must have had similar gurukula experiences.

Owing to his visits to Florida and to India, Gordon became close to my family. More than once when my parents passed through Los Angeles, he hosted them magnificently, and we were all touched by his gracious hospitality.

Gordon had great knowledge of Western classical music and knew much about art and world history. During his visits to India, when we took him around for sightseeing, he knew more about the history than most of us and would educate us by making comparisons with similar things in Europe. In 2004, before my visit to Italy with my wife and daughters, he instructed me that in Florence, in addition to the well-known sights such as Michelangelo's *David*, we should see the four statues at the Capelle Medicee sculpted by Michelangelo representing Dawn, Dusk, Day, and Night. Gordon said that, when G. N. Watson spoke about the mock theta functions of Ramanujan, he compared the grandeur of Ramanujan's identities to the beauty of these statues, and thus our visit to Florence would be incomplete if we did not see them. I am glad we followed the instructions of the guru!

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George E. Andrews

Basil Gordon was a reclusive, brilliant mathematician who proved some wonderful theorems on partitions which greatly inspired many, including me.

In the 1960s he published a number of innovative papers on the theory of partitions. Krishna Alladi has devoted much of his article to some of Basil's most prescient and spectacular achievements, including the generalization of the Rogers-Ramanujan identities, the Göllnitz-Gordon identities, and the method of weighted words. Gordon's later work on mock theta functions is one of the topics in Ken Ono's contribution.

Basil was a delightful and kind man but not a great correspondent. He directly answered very few of my letters to him. At first I feared that I might have offended him in some way. However, I was assured by others that his treatment of me was not unique. The late Marco Schützenberger told me that he had offered Basil a visiting position at Paris VII and received no response.

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However, lack of letters was made up for in personal interactions. In the late 1960s I attended an AMS meeting at Cornell University for the sole purpose of meeting Gordon and asking for clarification of some of his papers. He was open, gracious, and immensely helpful.

My most extended contact with Basil came in 1987 in India during the Ramanujan Centenary. We had several long car rides together connecting to the several conferences. During one such venture Basil entertained me and the other passengers with one of his hobbies. The object was to take a word or phrase, use its letters as an anagram, and produce a new word or phrase directly related to the original. The only one I remember was "Mosquitoes" morphing into "O, Moses quit!" This was long before computers dominated such games.

Basil had a wonderful sense of humor. Small absurdities delighted him. I recall one example (at my expense). This is from a rare letter to me, dated October 21, 1981:

> The following filler appeared in the *L.A. Times* of Oct. 18.
>
> **"Ramanujan, Mathematics Giant, Created Formulas**
> State College, Pa. (AP) Srinivasa Ramanujan who died about 60 years ago, when he was 32, is considered to be one of the giants of 20th century mathematics, said George E. Andrews of Penn State University. Ramanujan, a poor Indian, created his own math formulas."

As to Basil's mathematics, I shall restrict my comments to his work on plane partitions, much of it done with his students (Lorne Houten and others). The majority of this work is primarily contained in a sequence of papers titled "Notes on plane partitions" [8], [9], [10], [5], [6], [11]. There are four other papers [3], [4], [7], [12]. Of these, [4] is an early but interesting contribution on two-rowed partitions. The proof of the Bender-Knuth conjecture is given in [7].

The story of plane partitions dates back to P. A. MacMahon [13, p. 673]. There we find his first inkling that there might be appealing generating functions for plane partitions. More than twenty years later, MacMahon [14] effectively proved that

$$\prod_{n=1}^{\infty} \frac{1}{(1-q^n)^n} = 1 + q + 3q^2 + 6q^3 + \cdots,$$

where the coefficient of q^n is the number of plane partitions of n. For example, the six plane partitions of 3 are

$$3 \qquad 2\;\;1 \qquad \begin{matrix}2\\1\end{matrix} \qquad 1\;\;1\;\;1 \qquad \begin{matrix}1&1\\1&\end{matrix} \qquad \begin{matrix}1\\1\\1\end{matrix}$$

Although T. W. Chaundy wrote some papers on plane partitions in the 1930s, it fell to Basil (jointly with his student Lorne Houten) to develop serious methods that really opened up the subject. The first hint that something new was afoot came from their proof that

$$\prod_{n=1}^{\infty} \frac{1}{(1-q^n)^{[\frac{n+1}{2}]}} = 1 + q + 2q^2 + 4q^3 + \cdots$$

is the generating function for plane partitions with strictly decreasing parts along rows. Thus the four plane partitions of 3 subject to this description are

$$3 \qquad 2\;\;1 \qquad \begin{matrix}2\\1\end{matrix} \qquad \begin{matrix}1\\1\\1\end{matrix}$$

These papers illustrate the tremendous insights that Gordon developed in advancing from [4] to [7]. Indeed, it should be emphasized that there is much food for thought and many questions still to be answered arising from these papers. For example, in "Notes on plane partitions, IV" Gordon shows that $C_k(q)$, the generating function for k-rowed plane partitions with strictly decreasing parts along columns, can be evaluated in terms of certain classical infinite products and the false theta series

$$\sum_{n=0}^{\infty} (-1)^n q^{n(n+1)/2}.$$

Although Gordon promises [5, p. 98] that a more thorough investigation "will be undertaken elsewhere," unfortunately neither he nor anyone else has followed up on this unique appearance of false theta series in the world of plane partitions.

This wonderful, brilliant man illuminated many aspects of the theory of partitions. It was a joy to collaborate with him on two papers [1], [2] (also joint with Alladi). He gave the subject I love many new ideas and path-breaking insights. I and many others owe him a great debt.

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Robert Guralnick

Basil Gordon's Work in Algebra

Basil Gordon was a major figure in combinatorial number theory and especially in the theory of partition identities. This work is discussed by Alladi, Andrews, and Ono. I will focus on his work in algebra and group theory.

Gordon grew up in Baltimore and was a student at Johns Hopkins, spending a year as an undergraduate in Germany, where he studied with Ernst Witt and Emil Artin. He was advised to go to Caltech and work with Tom Apostol. Apostol has said that, as a graduate student, Gordon already knew as much as Apostol did. He graduated from Caltech in 1956 and spent one year there as a postdoc. He then accepted a job at UCLA but spent some time in the army before arriving there.

It is also important to note that Gordon was an outstanding teacher at all levels and was one of the most successful Ph.D. advisors at UCLA. He had twenty-six Ph.D. students, starting with David Cantor in 1960 and ending with Ken Ono in 1993. At the moment he has a total of seventy-four descendants, including forty-two grandstudents and six great-grandstudents. I was right in the middle. He was an outstanding advisor, giving precisely the right amount of guidance for each student. In particular, Ono (and Alladi, who was

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Photo courtesy of Keith Kendig.

Gordon as an undergraduate student at Johns Hopkins University.

not a Gordon student but quite influenced by him) says how influential he was in their lives and careers. Although most of Gordon's students wrote theses in number theory, he did produce about ten students whose theses were in algebra and group theory.

I had a different sort of relationship with him. I met another of Gordon's students, Michael Miller, while I was an undergraduate at UCLA (he was my TA in an undergraduate abstract algebra course). Mike suggested some problems to me, and he and I and Basil started working on them (with Mike communicating between us). We wrote a joint paper before I ever met Basil. I wrote a second paper on my own, took it to him, and asked him to be my advisor. He encouraged my independence.

He was also a wonderful teacher. He gave crystal-clear, beautiful lectures with no notes. Al Hales tells the story of being absolutely mesmerized watching him during office hours at Caltech. He ran the UCLA Putnam team for decades. The team usually did extremely well (most especially in 1968 with a third place finish). Gordon usually worked the solutions out within an hour or two. Mike Miller recalls that in 1969 Gordon finished all the problems in what seemed to be a matter of minutes.

While Gordon's work in algebra and group theory was not as significant as his other work, there were some gems. Below we discuss a few of these.

One of Gordon's earliest papers was a joint paper with Sol Golomb and Lloyd Welch on comma-free codes [2]. The motivation for this was the genetic code. At one point it was thought that nature gave an optimal solution to a coding theory problem. A set D of k-letter words from an n-letter alphabet is called a comma-free dictionary if when two such words are adjoined, no set of k consecutive letters, other than those in the chosen words, form a word in D. In the paper, various results about the number of words in a comma-free dictionary are obtained. Further results were obtained in the famous paper [7]. (Note that Jiggs is not the author; the authors of that paper were Baumert, Golomb, Gordon, Hales, Jewett, and Selfridge. As far as I know this is the first public acknowledgment of the true authors of that paper.) Gordon had several other interesting papers in coding theory.

In [6] Gordon and Straus consider the lattice of finite Galois extensions in an infinite Galois extension L/K. They elegantly describe the set of all possible degrees of finite Galois extensions of K'/K with $K < K' < L$.

In [3] Gordon and Motzkin generalize a result of Herstein about zeroes of polynomials of division algebras D. They prove that any degree n polynomial $f(x)$ in $D[x]$ either has at most n roots in D or has infinitely many roots (Herstein assumed the polynomials had central coefficients). They also consider polynomials where the variable is not assumed to commute with coefficients (and so evaluation maps are homomorphisms from the ring of such polynomials to D).

In [1] Fein and Gordon study a global Schur index for finite groups. Let G be a finite group and let $K(G)$ denote the field generated by all the entries in the character table of G. It is shown that $K(G)$ is an abelian extension of $\mathbb{Q}$ such that all residue fields are splitting fields. A splitting field in characteristic 0 contains a copy of $K(G)$, but $K(G)$ need not be a splitting field for G. They define $m(G)$ to be the minimum of $[K : K(G)]$ as K ranges over all possible splitting fields for G. They raise the question as to whether any abelian extension of $\mathbb{Q}$ is of the form $K(G)$ for some G.

In [4] Gordon and Schacher answer a question from Schacher's thesis. Let K be a number field and L/K be a cubic extension. The authors show that there exists a degree 4 polynomial over K that is irreducible for at least two completions of K, has Galois group A_4, and whose resolvent cubic polynomial defines L. A corollary shows the existence of a division algebra over K of dimension 144 containing a maximal subfield whose Galois group is A_4. In [5] the existence of a division algebra over $\mathbb{Q}$ with a maximal subfield Galois with Galois group A_5 was constructed. (Schacher in his thesis observed that $A_n, n > 7$, cannot occur in this way.)

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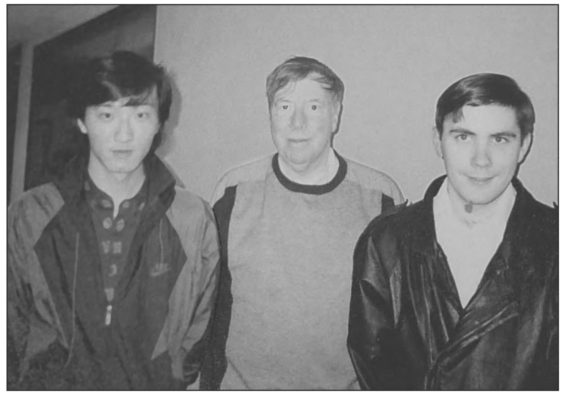

Photo courtesy of Keith Kendig.

Basil Gordon with two of his Ph.D. students—Ken Ono (on the left) and Doug Bowman (on the right)—at UCLA in spring 1993.

Ken Ono

Personal Reflections, and Gordon's Work on Modular Forms and Mock Theta Functions

Basil Gordon changed my life. After completing my undergraduate degree at the University of Chicago in 1989, I moved to Westwood to begin the Ph.D. program in mathematics at UCLA. I started the program with no vision. My passion up to that point had been bicycle racing. I certainly was not committed to the idea of pursuing a career in mathematics. Indeed, I almost dropped out of the program several times during my first year.

Basil Gordon's 1990 graduate course in number theory changed my life. His passionate lectures were beautiful and inspiring. Basil, my image of a great nineteenth-century scholar, saw the world through special lenses. I was mesmerized by his ability to make mathematics beautiful by making analogies with classical art, literature, and music. His encyclopedic knowledge of everything, combined with his obvious love of mathematics and his role in the subject, drew me into mathematics. Basil taught me how to find beauty in mathematical research, and he helped me find self-confidence and a genuine passion for mathematical research. He taught me these lessons in the idyllic setting of Santa Monica, in the reading room of his home (two blocks from the beach), in the Bagel Nosh, and in a cute Italian bistro that still serves delicious gnocchi.

Basil steered me in the direction of modular forms, a prescient choice, considering that Andrew Wiles would go on to use the subject in his proof of Fermat's Last Theorem, which he announced a few weeks after I defended my thesis in 1993. Basil encouraged me to think about congruences for the coefficients of modular forms, and he suggested deep works of Deligne, Serre, and Swinnerton-Dyer. Basil understood that these deep works would shed light on classical questions on partitions that date back to seminal works of Euler, Jacobi, and Ramanujan.

Basil was enamored with Ramanujan's work on the partition function $p(n)$, and he liked to say that he wanted "to do for $Q(n)$, the number of partitions of an integer n into distinct parts, everything that Ramanujan had done for $p(n)$." Basil succeeded. I am particularly fond of his work with Kim Hughes [1], which established analogs of Ramanujan's celebrated partition congruences modulo powers of 5. If $k \geq 0$ is an integer, then for every integer n with $24n \equiv -1 \pmod{5^{2k+1}}$ they proved that

$$Q(n) \equiv 0 \pmod{5^k}.$$

The partition functions $p(n)$ and $Q(n)$ are examples of coefficients of modular forms that can be represented as infinite products. Basil understood the importance of developing general theorems about such infinite products, which he referred to as eta-quotients and generalized eta-quotients[1] because of their connection to Dedekind's eta-function (note: $q := e^{2\pi i z}$)

$$\eta(z) := q^{1/24} \prod_{n=1}^{\infty} (1 - q^n).$$

In addition to proving general theorems about the modularity properties of such products, Basil was interested in the problem of classifying those

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[1] *Sinai Robins studied generalized eta-products in his Ph.D. thesis.*

products which resemble Jacobi's classical identity

$$\prod_{n=1}^{\infty}(1-q^n)^3 = \sum_{k=0}^{\infty}(-1)^k(2k+1)q^{(k^2+k)/2}.$$

This series is *lacunary*: it has the property that almost all of its coefficients are zero. Carrying out a generalization of an elegant paper [6] of Serre, Basil and his student Sinai Robins [4] classified all of the lacunary eta-products in certain families of modular forms.

Basil was also very interested in Ramanujan's mock theta functions, an enigmatic collection of q-series which Ramanujan described in his "death bed" letter to G. H. Hardy. Until the recent work of Zwegers [8], where these functions are described as holomorphic parts of weight 1/2 harmonic Maass forms, very little was known about the analytic properties of these series, which included such functions as

$$f(q) := 1 + \sum_{n=1}^{\infty}\frac{q^{n^2}}{(1+q)^2(1+q^2)^2\cdots(1+q^n)^2}.$$

In important work with his student Richard McIntosh, Basil defined a "universal" mock theta function:

$$F(\lambda, r, q) := \sum_{n=-\infty}^{\infty}\frac{(-1)^n q^{\lambda n(n+1)}}{1-q^{n+r}}.$$

They observed that most of Ramanujan's mock theta functions are related to specializations of this series, and they proceeded to determine the modular transformation laws for certain specializations [2], [3]. These results fit nicely into the comprehensive framework later discovered by Zwegers in his transformational work [8] in the subject (also see [5], [7]).

Basil Gordon was a great man. He taught me how to love mathematics. He taught me how to find the confidence to do mathematics. He is my image of the perfect advisor. I owe him so many debts, and I miss him terribly.

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Bruce Rothschild

Working with Basil Gordon

When I was just about to move to UCLA in 1969, Ted Motzkin was briefly visiting MIT from UCLA (I was there as a postdoc). Gian-Carlo Rota was involved at that time in reorganizing the *Journal of Combinatorial Theory* (JCT) into two series, JCT-A and JCT-B. He asked Motzkin if he would be interested in becoming the editor-in-chief of Series A. As Motzkin explained it, he knew that Basil Gordon would be available to support the effort, so he agreed to take the job. The decision took him about thirty seconds according to Rota. He also enlisted me, so Basil and I became managing editors for JCT-A.

I had met Basil briefly, but didn't know much about him except for his fearsome reputation among some of my friends in the graduate student body at UCLA. He was known as one who could solve almost any problem, especially the Putnam problems, in real time. As I got to know him, it became immediately obvious to me why Motzkin had been so confident.

Although Motzkin died unexpectedly shortly after I got to UCLA, Basil and I continued to manage JCT-A, first for several years with Marshall Hall at Caltech as editor-in-chief, and then without a "chief". We worked together on the journal for more than thirty years. This was surely, in a unique way, my most satisfying and rewarding collaboration. Although we never actually wrote a joint paper, the amount of mathematics we discussed was enormous. Mostly this meant I would learn about all kinds of things from him. Basil was an incredible scholar (in many things, not just mathematics), and when we had to figure out what to do with a paper submitted to JCT-A, I could always count on learning a great deal.

JCT-A operated in the usual way at the time, receiving papers, finding referees willing and able to review them, corresponding with all concerned about revisions, and ultimately making a decision whether to publish. Atypically for me, I was the one who kept things organized and moving along (with the essential support of our long-time secretary and assistant, Elaine Barth).

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Both of us had our dilatory episodes, not infrequently—but not always—due to a large number of papers recently submitted. Like many of our reviewers, Basil needed reminders about papers I doled out to him for review or for steering to reviewers, and more often than I'd admit under quantitative scrutiny, I failed to remind him in a timely manner. What was striking—as was so much about Basil—was that when I'd call, he'd immediately know the paper I was referring to, what it was about, and what its editorial problems and strengths were. In the next day or so those previously unrecorded comments would be in my hands. His comments would be written in his amazingly neat and legible handwriting between the lines and in the margins and when necessary in a manuscript on separate pages. His handwriting was so unique that it was essentially a signature. Finding an appropriate referee could also be quite difficult on occasion. At these times an appeal to Basil's familiarity with the area in question and, even more, with related areas in algebra or number theory made it possible to find the right reviewer.

Although Basil had extremely high standards, broad knowledge, and impeccable taste, his comments about papers that he thought were not strong enough would never be anything but constructive. He would never simply dismiss a paper, no matter how weak. His comments were kind and encouraging in such cases. He took all the mathematics seriously and responded accordingly. It was just a pleasure to work with him. Sometimes I would first see a paper that seemed perhaps too elementary or even trivial, but when I showed it to Basil he would see in it an example of a number theory issue and a connection to deep problems. Even though he might agree that the paper was not appropriate for JCT-A, he would himself be quite interested.

Basil and I stepped down from managing JCT-A in 2002, and the management moved, first to Arizona and then to Australia, and the publisher from Academic Press to Elsevier. Basil was retired by then, but I saw him fairly regularly right up to his last days. He was actively engaged in mathematics until the very end, and it continued to be enlightening, entertaining, and rewarding in general to talk to him.

Meijer G–Functions: A Gentle Introduction

Richard Beals and Jacek Szmigielski

The Meijer G–functions are a remarkable family $\mathcal{G}$ of functions of one variable, each of them determined by finitely many indices. Although each such function is a linear combination of certain special functions of standard type, they seem not to be well known in the mathematical community generally. Indeed they are not even mentioned in most books on special functions, e.g., [1], [18]. Even the new comprehensive treatise [15] devotes a scant 2 of its 900+ pages to them. (The situation is different in some of the literature oriented more toward applications, e.g., the extensive coverage in [6] and [9].)

The present authors were ignorant of all but the name of the G–functions until the second author found them relevant to his research [3]. As we became acquainted with them, we became convinced that they deserved a wider audience. Some reasons for this conviction are the following:

• The G–functions play a crucial role in a certain useful mathematical enterprise.

• When looked at conceptually, they are both natural and attractive.

• Most special functions, and many products of special functions, are G–functions or are expressible as products of G–functions with elementary functions. There are seventy-five such formulas in [4, sec. 5.6]; see also [6, sec. 6.2], [9, chap. 2], and [20]. Examples are the exponential function, Bessel functions, and products of Bessel functions (the notation will be explained below):

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$$e^x = G^{10}_{01}\left(\begin{matrix} \\ 1 \end{matrix} \middle| -x \right),$$

$$J_\nu(2x) = x^{-\nu} G^{10}_{02}\left(\begin{matrix} \\ \nu \quad 0 \end{matrix} \middle| x^2 \right),$$

$$J_{2\mu}(x)\,J_{2\nu}(x) = \frac{1}{\sqrt{\pi}} G^{12}_{24}\left(\begin{matrix} \frac{1}{2} \quad 0 \\ \mu+\nu \quad \nu-\mu \quad \mu-\nu \quad -\mu-\nu \end{matrix} \middle| x^2 \right).$$

• The family $\mathcal{G}$ of G–functions has remarkable closure properties: it is closed under the reflections $x \to -x$ and $x \to 1/x$, multiplication by powers, differentiation, integration, the Laplace transform, the Euler transform, and the multiplicative convolution. Thus, if G, G_1, and G_2 belong to $\mathcal{G}$ and the various transforms and the multiplicative convolution $G_1 * G_2$ exist, then the following also belong to $\mathcal{G}$:

$$(1) \qquad G(-x), \quad G(1/x), \quad x^a\,G(x), \quad G'(x);$$

$$(2) \qquad \int_c^x G(y)\,dy \quad \text{(for some choice of } c\text{)};$$

$$(3) \qquad \mathcal{L}G(x) \equiv \int_0^\infty e^{-xy}\,G(y)\,dy;$$

$$(4) \qquad \mathcal{E}_{a,b}G(x) \equiv \int_0^1 t^{a-1}(1-t)^{b-1}G(ty)\,dt;$$

$$(5) \qquad [G_1 * G_2](x) \equiv \int_0^\infty G_1\left(\frac{x}{y}\right) G_2(y)\,\frac{dy}{y}.$$

• The family $\mathcal{G}$ is minimal with respect to these properties. For example, the only nonzero multiple of e^x that belongs to $\mathcal{G}$ is e^x itself.

Closure under convolution, (5), is of particular importance for the mathematical enterprise alluded to above. It lies at the heart of the most comprehensive tables of integrals in print [16] and online, as well as the Mathematica integrator; see [19], [6], and [8].

In our view, the key to a conceptual understanding of a G–function is the differential equation that it satisfies: the *generalized hypergeometric equation*. We begin by noting what special property singles out precisely these equations among general linear homogeneous ODEs.

The Generalized Hypergeometric Equation

It is convenient here to replace the operator d/dx with the scale–invariant operator $D = x\,d/dx$, which is diagonalized by powers of x:

$$D[x^s] = s\,x^s. \tag{6}$$

The general homogeneous linear ODE

$$a_N(x)\,\frac{d^n(u)}{dx^n}(x) + a_{N-1}(x)\,\frac{d^{N-1}(u)}{dx^{N-1}}(x) + \cdots + a_0(x)\,u(x) = 0$$

can, after multiplication by x^N, be rewritten in the form

$$b_N(x)\,D^N u(x) + b_{N-1}(x)\,D^{N-1}u(x) + \cdots + b_0(x)\,u(x) = 0. \tag{7}$$

Suppose that the coefficients are analytic near $x = 0$ and consider the standard power series method: determine the coefficients of a formal power series solution $\sum_{n=0}^{\infty} u_n\,x^n$ by expanding (7) and collecting the coefficients of like powers of x. Carrying this out by hand can be quite tedious. For example, what are the coefficients u_5 and u_{10} in the series expansion of the solution of

$$u''(x) + e^x\,u(x) = 0,$$

given that $u_0 = 1$, $u_1 = 0$? This might lead one to ask the following question:

When do the linear equations for the coefficients $\{u_n\}$ in the series expansion reduce to a two-term recursion of the form $c_n\,u_n = d_n\,u_{n-1}$?

It is not difficult to show that the necessary and sufficient condition on the coefficients b_n is that each has the form $(\alpha_n\,x + \beta_n)\,x^k$ for some fixed k. It follows that (7) can be reduced to the form

$$Q(D)\,u(x) - \alpha\,x\,P(D)\,u(x) = 0,$$

where Q and P are monic polynomials. In view of (6), the recursion for the coefficients of a formal series solution is

$$Q(n)\,u_n = \alpha\,P(n-1)\,u_{n-1}. \tag{8}$$

If $\alpha = 0$, this trivializes: e.g., if the b_j are distinct, any solution is a linear combination of powers x^{1-b_j}. If $\alpha \neq 0$, we may take advantage of the scale invariance of the operator D to take αx as the independent variable and reduce to the case $\alpha = 1$.

Finally, the solution is trivial unless $Q(0) = 0$. Thus, excluding trivial cases and up to normalization, the answer to the question above is that (7) must be a generalized hypergeometric equation (GHGE), first version:

$$\left[D\prod_{j=1}^{q-1}(D + b_j - 1) - x\prod_{j=1}^{p}(D + a_j)\right]u(x) = 0. \tag{9}$$

Generalized Hypergeometric Functions

For equation (9), the recursion (8) is

$$n\prod_{j=1}^{q-1}(b_j + n - 1)\,u_n = \prod_{j=1}^{p}(a_j + n - 1)\,u_{n-1}. \tag{10}$$

(Here and subsequently we shall assume, usually without explicit statement, various conditions, such as the condition that no $b_j - 1$ be a negative integer.) The formal power series solution with constant term 1 is

$$\sum_{n=0}^{\infty}\frac{(a_1)_n(a_2)_n\cdots(a_p)_n}{(b_1)_n(b_2)_n\cdots(b_{q-1})_n\,n!}\,x^n, \tag{11}$$

where the extended factorial $(a)_n$ is defined by

$$a_0 = 1,$$

$$a_n = a(a+1)\cdots(a+n-1) = \frac{\Gamma(n+a)}{\Gamma(a)},\quad n \geq 1.$$

The series (11) diverges for all $x \neq 0$ if $p > q$, has radius of convergence 1 if $p = q$, and converges everywhere if $p < q$.

For $p \leq q$ the function defined by the series is the generalized hypergeometric function usually denoted

$${}_pF_{q-1}\left(\begin{matrix} a_1,\ldots,a_p \\ b_1,\ldots,b_{q-1}\end{matrix}\,\middle|\,x\right).$$

Since the equation here has order q, there should be an additional $q - 1$ linearly independent solutions. We shall come back to this point later.

A More Conceptual Route to a Solution

We start from a second, slightly generalized, version of (9). The factor D is replaced by $D + b_q - 1$:

$$\left[\prod_{j=1}^{q}(D + b_j - 1) - x\prod_{j=1}^{p}(D + a_j)\right]u(x) = 0. \tag{12}$$

Let us note two features of equation (12). First, D is invariant under $x \to -x$, so this sign change converts (12) to

$$\left[\prod_{j=1}^{q}(D + b_j - 1) + x\prod_{j=1}^{p}(D + a_j)\right]u(x) = 0. \tag{13}$$

Second, under the change of variables $y = 1/x$, D goes to $-D$. Therefore equation (12) is transformed into one having the same form or else the form (13) (depending on the sign of $(-1)^{q-p}$), but with the q parameters b_j replaced by the p parameters $1 - a_j$, and the p parameters a_j replaced by the

q parameters $1 - b_j$. This allows one to assume always that $p \le q$.

Since D diagonalizes over powers of x, while multiplication by x simply raises the power, we might try to find solutions in the form of (continuous) sums of powers:

$$u(x) = \frac{1}{2\pi i}\int_L \Phi(s)\, x^s\, ds, \tag{14}$$

where L is a suitable closed contour in the complex plane (or the Riemann sphere). We note in passing that integral representations of solutions have a clear advantage over series representations if one wants to determine behavior for large values of x or of the parameters.

Plugging the expression (14) into (12) and assuming that we may differentiate under the integral sign, we want

$$\begin{aligned} 0 &= \frac{1}{2\pi i}\int_L \Phi(s) \\ &\quad \times \left[\prod_{j=1}^{q}(b_j - 1 + s)\, x^s - \prod_{j=1}^{p}(a_j + s)\, x^{s+1}\right] ds \\ &= \frac{1}{2\pi i}\int_L \Phi(s)\prod_{j=1}^{q}(b_j - 1 + s)\, x^s\, ds \\ &\quad - \frac{1}{2\pi i}\int_{L+1} \Phi(s-1)\prod_{j=1}^{p}(a_j - 1 + s)\, x^s\, ds. \end{aligned}$$

If the contour L has the property that $L + 1$ can be deformed to L without crossing any singularities of the integrand, then we are led to the continuous version of the recursion (10):

$$\Phi(s)\prod_{j=1}^{q}(b_j - 1 + s) = \Phi(s-1)\prod_{j=1}^{p}(a_j - 1 + s). \tag{15}$$

We shall refer to this condition on L as the *translation condition*.

The various issues that arise in carrying out the construction of solutions of (12), by solving (15) and choosing a contour, arise already in the simplest examples.

Example: $q = 1,\ p = 0$**.** We follow the usual convention that the empty product equals 1. Therefore, with $q = 1$ and $p = 0$, equation (12) reduces to

$$(D + b - 1)u(x) - x\, u(x) = 0, \tag{16}$$

and the recursion equation (15) can be written

$$\frac{\Phi(s)}{\Phi(s-1)} = \frac{1}{b+s-1} = \frac{\Gamma(b+s-1)}{\Gamma(b+s)}.$$

Therefore we may take $\Phi(s)$ to be the entire function $1/\Gamma(b+s)$ and set

$$u(x) = \frac{1}{2\pi i}\int_L \frac{x^s\, ds}{\Gamma(b+s)}. \tag{17}$$

Now if, as we shall assume, the contour L is closed and does not cross a branch cut of x^s, then, by Cauchy's theorem, $u(x) = 0$. This leads us to look for another solution of (15).

The product $\varphi(s)\,\Phi(s)$ is a second solution of (15) if and only if $\varphi(s-1) = \varphi(s)$. In order to remain in the context of gamma functions, we may use Euler's reflection formula,

$$\Gamma(z)\,\Gamma(1-z) = \frac{\pi}{\sin \pi z}, \tag{18}$$

and multiply the kernel $1/\Gamma(b+s)$ in the integral (17) by

$$\varphi(s) = \frac{\pi}{\sin \pi(b+s)} = \Gamma(b+s)\,\Gamma(1-b-s), \tag{19}$$

leading to the function

$$u(x) = \frac{1}{2\pi i}\int_L \Gamma(1-b-s)\, x^s\, ds. \tag{20}$$

Note that the factor (19) is antiperiodic rather than periodic with period 1, i.e., $\varphi(s-1) = -\varphi(s)$. This corresponds to changing the sign of x in equation (16). We take as L the loop shown in Figure 1. In fact, the translation condition implies that the poles must lie on one side of L, and for a nontrivial result we want L to enclose the poles of the integrand, say in the negative direction. The residue of $\Gamma(1-b-s)$ at $s = 1 - b + n$ is $(-1)^n/n!$, so (20) is easily calculated:

$$u(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{1-b+n}}{n!} = x^{1-b}\, e^{-x}.$$

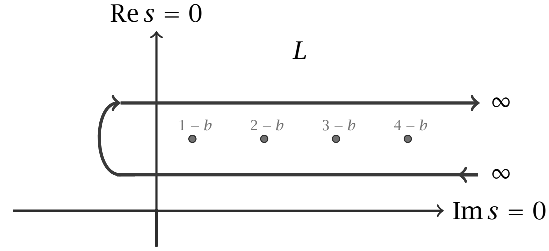

Figure 1. The contour L for (20).

Example: $q = p = 1$**.** Here equation (12) becomes

$$(D + b - 1)u(x) - x(D + a)u(x) = 0. \tag{21}$$

Equation (15) becomes

$$\frac{\Phi(s)}{\Phi(s-1)} = \frac{a+s-1}{b+s-1} = \frac{\Gamma(a+s)}{\Gamma(a+s-1)}\cdot\frac{\Gamma(b+s-1)}{\Gamma(b+s)},$$

so one solution is $\Phi(s) = \Gamma(a+s)/\Gamma(b+s)$:

$$u(x) = \frac{1}{2\pi i}\int_L \frac{\Gamma(a+s)}{\Gamma(b+s)}\, x^s\, ds. \tag{22}$$

This function is meromorphic with poles at $-a$, $-a-1$, $-a-2$, …. It has at most algebraic growth as $s \to \infty$ (see the discussion below). Therefore, for $0 < |x| < 1$ we may take L to be a loop to the right, as in Figure 1. Again, the translation condition requires that the poles lie on one side of this loop, and therefore outside it. Therefore, by Cauchy's theorem, $u(x) = 0$ for $0 < |x| < 1$.

If $|x| > 1$ we may take L to be the loop shown in Figure 2, enclosing the poles.

The residue of the integrand at the pole $s = -a - n$ is

$$\frac{(-1)^n}{n!} \cdot \frac{1}{\Gamma(b-a-n)} \cdot x^{-a-n}.$$

By (18),

(23)

$$\begin{aligned}\frac{1}{\Gamma(b-a-n)} &= \Gamma(1+a-b+n)\frac{(-1)^n \sin(b-a)}{\pi} \\ &= (-1)^n \frac{\Gamma(1+a-b+n)}{\Gamma(b-a)\Gamma(1+a-b)} \\ &= (-1)^n \frac{(1+a-b)_n}{\Gamma(b-a)}.\end{aligned}$$

Therefore, for $|x| > 1$ our solution is

$$\begin{aligned}(24)\qquad u(x) &= \frac{x^{-a}}{\Gamma(b-a)} \sum_{n=0}^{\infty} \frac{(1+a-b)_n}{n!} \left(\frac{1}{x}\right)^n \\ &= \frac{x^{-a}}{\Gamma(b-a)} \left(1 - \frac{1}{x}\right)^{b-a-1}.\end{aligned}$$

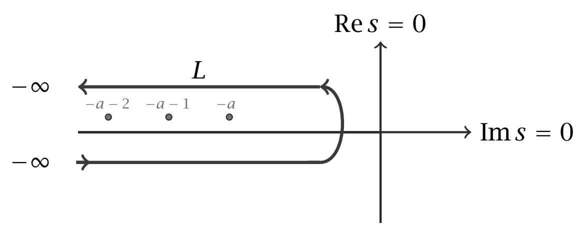

Figure 2. The contour L for (22) when $|x| > 1$.

As in the case $p = 0$, we may change the integrand, leading, for example, to

$$u(x) = \frac{1}{2\pi i} \int_L \frac{\Gamma(1-b-s)}{\Gamma(1-a-s)} x^s \, ds.$$

Again for $0 < |x| < 1$, we may integrate over a contour that encloses the poles going to the right, as in Figure 1. For $|x| > 1$ we choose a contour going to the left, as in Figure 2, but enclosing no poles. A calculation as in (23), (24), together with Cauchy's theorem, gives

$$u(x) = \begin{cases} \dfrac{x^{1-b}(1-x)^{b-a-1}}{\Gamma(b-a)}, & 0 < |x| < 1; \\ 0, & |x| > 1. \end{cases}$$

An interesting special case is $a = 0$, $b = 1$, which gives a step function:

$$u(x) = H(1 - |x|), \quad x \neq 0,$$

where H is the Heaviside function.

We leave it to the reader to consider two more possibilities, with respective kernels $\Gamma(a+s)\,\Gamma(1-b-s)$ and $1/[\Gamma(1-a-s)\,\Gamma(b+s)]$.

Representation by Generalized Hypergeometric Functions

Before proceeding to a discussion of the general case, we look briefly at the second order equation

$$\begin{aligned}[(D + b_1 - 1)(D + b_2 - 1) \\ - x(D+a)u(x)]u(x) = 0.\end{aligned}$$

One possibility for a solution is

(25)

$$u(x) = \frac{1}{2\pi i} \int_L \Gamma(a+s)\,\Gamma(1-b_1-s)\,\Gamma(1-b_2-s)\,x^s \, ds.$$

Another way to construct a solution is to look for $u_1(x) = x^{1-b_1} v_1(x)$. The corresponding equation for v_1 is

$$[D(D + b_2 - b_1) - x(D + a + 1 - b_1)]\, v_1(x) = 0,$$

which has a solution in standard form:

$$(26)\qquad v_1(x) = {}_1F_1\left(\begin{matrix} a+1-b_1 \\ b_2+1-b_1 \end{matrix} \,\middle|\, x\right),$$

and similarly with b_1 and b_2 interchanged. The solution (25) must be a linear combination of these two. The coefficients can be determined by looking at the residues of the integrand at the pole $s = 1 - b_1$ and at the pole $s = 1 - b_2$: $\Gamma(a+1-b_1)\,\Gamma(b_1-b_2)$ and $\Gamma(a+1-b_2)\Gamma(b_2-b_1)$ respectively. The result is

$$\begin{aligned}u(x) &= \Gamma(a+1-b_1)\,\Gamma(b_1-b_2) \\ (27)\qquad &\times x^{1-b_1}\,{}_1F_1\left(\begin{matrix} a+1-b_1 \\ b_2+1-b_1 \end{matrix} \,\middle|\, x\right) \\ &+ \Gamma(a+1-b_2)\,\Gamma(b_2-b_1) \\ &\times x^{1-b_2}\,{}_1F_1\left(\begin{matrix} a+1-b_2 \\ b_1+1-b_2 \end{matrix} \,\middle|\, x\right).\end{aligned}$$

There is one notable feature of this particular combination of the two standard solutions v_1, v_2. Each of the v_j has exponential growth as $x \to +\infty$, but (27) does not. In fact, the integrand decays exponentially in both directions on vertical lines, so we may deform the contour L into one that consists of a small circle around $s = a$, together with a vertical line $\operatorname{Re} s = c < \operatorname{Re} a$. The leading term can be computed from the residue at $s + a = 0$:

(28)

$$u(x) \sim \Gamma(a+1-b_1)\,\Gamma(a+1-b_2)\,x^{-a} \quad \text{as } x \to +\infty.$$

Thus this integral form singles out the unique (up to a multiplicative constant) linear combination of the two standard solutions that has algebraic behavior at $+\infty$.

General Considerations about Kernels and Contours

For each case of equation (12), the function Φ that satisfies the recursion equation (15) can and will be taken to be a quotient of products of gamma functions in the form $\Gamma(-c_j + s)$ or $\Gamma(d_j - s)$.

Therefore the poles of Φ, if any, will consist of finitely many sequences of points:

(**a**) $c_j, c_j-1, c_j-2, \ldots$ or (**b**) $d_j, d_j+1, d_j+2, \ldots$.

The translation condition requires that any such sequence must lie on one side of the contour L. The contour L is always chosen to separate the sequences **(a)** of poles that go to the left from the sequences **(b)** that go to the right. (A technical remark: We do not actually need the translation condition to deduce that these solutions of (15) give rise to solutions of the generalized hypergeometric equation; the condition that L separate the sequences of poles in this way is sufficient.)

We consider three types of contours L:

$L = L_I$: beginning at $-i\infty$, ending at $+i\infty$;

$L = L_\infty$: beginning and ending at $+\infty$, oriented clockwise;

$L = L_{-\infty}$: beginning and ending at $-\infty$, oriented counterclockwise.

In order to see which contours are available in a given case, we note here some basic facts about asymptotics.

First, Stirling's formula,

(29)
$$\Gamma(z) = \sqrt{\frac{2\pi}{z}}\left(\frac{z}{e}\right)^z\left[1 + O\left(\frac{1}{z}\right)\right] \quad \text{as } z \to \infty,$$

is valid uniformly in the right half-plane $\operatorname{Re} z \geq 0$. It implies that Γ grows faster than exponentially to the right along any horizontal line and decays exponentially in both directions along any vertical line in the right half-plane.

Second, the reflection formula (18), combined with (29), gives the asymptotic behavior in the left half-plane $\operatorname{Re} z \leq 0$: Γ decays faster than exponentially to the left along any horizontal ray other than the negative real axis and decays exponentially in either direction along any vertical line in the left half-plane as well. These facts imply that, for any fixed a, b,

(30)
$$\frac{\Gamma(a+s)}{\Gamma(b+s)} \sim s^{a-b} \quad \text{as } s \to \infty$$

along any ray that avoids the zeros and poles of the quotient.

Meijer G–functions

Here we consider the general hypergeometric equation (12) but with a difference from the standard notation: the indices $\{b_j\}$, $\{a_j\}$ are replaced by the reflections $\{1-b_j\}$, $\{1-a_j\}$:

(31)
$$\left[\prod_{j=1}^{q}(D-b_j) - x\prod_{j=1}^{p}(D+1-a_j)\right]u(x) = 0.$$

We assume again that $p \leq q$. The previous considerations lead us to a solution which, in Meijer's notation, is $G^{0,p}_{p,q}$:

(32)
$$G^{0,p}_{p,q}\left(\begin{matrix} a_1,\ldots,a_p \\ b_1,\ldots,b_q\end{matrix}\,\middle|\, x\right) = \frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{p}\Gamma(1-a_j+s)}{\prod_{j=1}^{q}\Gamma(1-b_j+s)}\, x^s\, ds.$$

Meijer [10] introduced a family of solutions of equation (31), denoted by

$$G^{m,n}_{p,q}\left(\begin{matrix} a'_1,\ldots,a'_p \\ b'_1,\ldots,b'_q\end{matrix}\,\middle|\, (-1)^{m+n+p}x\right), \quad 0 \leq m \leq q, \;\; 0 \leq n \leq p,$$

where the $\{a'_j\}$ and $\{b'_j\}$ are permutations of the original indices $\{a_j\}$ and $\{b_j\}$ respectively. The upper index m indicates that the *first m* factors in the *denominator* of the integrand have been changed to factors $\Gamma(b_j - s)$ in the numerator and the *last* $p-n$ factors in the *numerator* have been changed to factors $\Gamma(a_j - s)$ in the denominator. This results in a total of $m+n$ factors in the numerator.

Taking into account the invariance of the integrand under permutations of the indices a_j and b_j (separately) in the numerator and also in the denominator, one can obtain 2^{p+q} solutions of (31). However they are not necessarily distinct. If $p < q$, then the 2^p solutions with $m = 0$ vanish identically, as in our first solution (17) in the case $q = 1$, $p = 0$. If $p = q$, only the solution with $m = n = 0$ vanishes identically.

The choice of contours depends first on p and q. If $p < q$ (resp. $p > q$), the gamma function kernel Φ has faster than exponential decay to the right (resp. left) on horizontal lines. Thus, for $p < q$, one can take a contour $L = L_\infty$ as in Figure 1, and for $p > q$ a contour $L = L_{-\infty}$ as in Figure 2. In either case the contour is chosen to separate the sequences of poles going to $+\infty$ from those going to $-\infty$.

If there are more gamma factors in the numerator than in the denominator, i.e., $m+n > (p+q)/2$, then Φ decays exponentially along vertical lines and the contour can be deformed into L_I from $-i\infty$ to $+i\infty$. If $m+n = (p+q)/2$, Φ has algebraic behavior at ∞ by (30), and again the contour can be deformed, but this may require interpreting the integral in the sense of distributions.

When $p = q$, the kernel Φ itself has algebraic behavior as $s \to \infty$, just as in the case $p = q = 1$ above, so the choice of contour differs according to whether $0 < |x| < 1$ or $|x| > 1$.

A striking advantage of this method of producing solutions is the ease of finding solutions with prescribed behavior as $x \to 0$ or $x \to \infty$. Assuming again that $p \leq q$, the standard solutions of the form $x^{b_j}{}_pF_{q-1}$ that are $x^{b_j}[1 + (O(x)]$ as $x \to 0$ are multiples of the G-functions with $m = 1$. Solutions that are $x^{a_j-1}[1 + O(x^{-1}]$ as $x \to \infty$ are multiples of the G-functions with $n = 1$, $m + 1 > (p + q)/2$.

The Mellin Transform and the Convolution Theorem

The Mellin transform is the multiplicative version of the Laplace transform. We normalize it as

$$\mathcal{M}f(s) = \int_0^\infty f(x)\, x^{-s}\, \frac{dx}{x}.$$

The inverse transform is then

$$\mathcal{M}^{-1}g(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(s)\, x^s\, ds.$$

This suggests, correctly, that if a G-function

$$G(x) = \frac{1}{2\pi i} \int_L \Phi(s)\, x^s\, ds$$

has (in some suitable sense) a Mellin transform, then that transform is the kernel Φ. As an exercise, one can check this with (24): change the variable of integration to $y = 1/x$ to find that the Mellin transform is a beta function and therefore expressible in terms of gamma functions.

It is not difficult to verify that the Mellin transform takes multiplicative convolution to a product: in the notation of (5),

$$\mathcal{M}(G_1 * G_2) = \mathcal{M}G_1 \cdot \mathcal{M}G_2.$$

If G_1 and G_2 are G-functions with $\mathcal{M}G_j = \Phi_j$, then the product $\Phi_1\Phi_2$ is itself a quotient of products of gamma functions. If

$$G_j = G^{m_j,n_j}_{p_j,q_j}, \quad j = 1, 2,$$

then $G_1 * G_2$ is a G-function of type

$$G^{m_1+m_2,n_1+n_2}_{p_1+p_2,q_1+q_2}.$$

Some bookkeeping will identify the indices $\{a_j\}$, $\{b_j\}$ for G in terms of the indices $\{a_{j,1}\}$, $\{b_{j,1}\}$ for G_1 and $\{a_{j,2}\}$, $\{b_{j,2}\}$ for G_2.

This explains the convolution result (5). We leave verification of the closure under (1), (2), (3), and (4) to the reader.

We note in passing that the most commonly used integral transforms, such as the Laplace, Hankel, and fractional integral transforms, as well as the Mellin transform itself, can be viewed as special cases or variants of the general G-function transform

$$Tf(x) = \int_0^\infty G(xt)\, f(t)\, dt.$$

Conversely, the range of the G-function transform can be characterized in terms of the special cases just mentioned; see [17].

Discussion and History

The one bit of motivation that we have found in the literature is the remark in [4] that "the G-function provides an interpretation of the symbol ${}_pF_q$ when $p > q + 1$." Let us start from the equation

$$\left[\prod_{j=1}^{p}(D + a_j - 1) - x\prod_{j=1}^{q}(D + b_j)\right] v(x) = 0, \quad p \geq q. \tag{33}$$

As we noted earlier, under the changes of variables $x \to \pm 1/x$ the operator D goes to $-D$. Therefore if we set

$$u(x) = v\left(\frac{(-1)^{p-q}}{x}\right),$$

equation (33) is equivalent to equation (31). This explains the departure in (31) from the usual index notation (12).

We cannot help noting that the remark is of limited explanatory value (apart from the question of notation), since ${}_pF_{q-1}$, with $p \leq q$ and a given set of indices, denotes a single solution to (9), not 2^{p+q} solutions.

Meijer's original definition in 1936 [10] is the general version of (27): represent the function as a linear combination of the standard basis of solutions of (31). (This definition tacitly rules out the cases with index $m = 0$, which give the trivial solution.) Only in [11] did Meijer present the integral formulation above. The 1936 paper translates many formulas for special functions of the form

$$f(x) = h(x)\, k(x)$$

or

$$f(x) = h_1(x)\, k_1(x) + h_2(x)\, k_2(x)$$

into a form like

$$G(x) = G_1(x)\, G_2(x),$$

and many integral formulas into the convolution form

$$G(x) = G_1 * G_2(x).$$

It appears that Meijer may simply have done so many calculations that he could see exactly what linear combinations would make these formulas work out so neatly. (He may also have been guided by known examples that yield specified asymptotics, as in the example (27) above.) Only later did Meijer establish the integral formulas like (32) that are now taken as the definition [10]. In a lengthy series of papers he explored further relations with classical special functions, relations among the G-functions themselves, and questions of asymptotics [12], [13]; see also [5].

The idea to represent solutions of the GHGE as integrals of the kind we have been discussing—a type now known as Mellin–Barnes integrals—goes back to Barnes in 1908 for the Euler–Gauss case $q = p = 2$ [2] and to Mellin in 1910 for the general case [14]. Indeed, Mellin has a thorough discussion of the recursion equation (15) and its solutions.

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Beginning in 2014, the AMS is pleased to offer two new gold open access research journals. This advancement is motivated by encouragement from research libraries, prompting from the mathematical community, directives from research sponsors, and the need for a transition of scholarly publishing to open access. The two new journals, *Proceedings of the American Mathematical Society, Series B*, and *Transactions of the American Mathematical Society, Series B*, will serve as companions to the subscription journals *Proceedings* and *Transactions of the AMS*. Each of the new journals will be electronic-only and freely available online, supported by Article Processing Charges designed to cover the Society's publication costs (these charges are typically paid by a funding agency or by the author's institution, and are not expected to be paid by the author personally). In order to assure the high editorial standard of each new journal, *Proceedings B* and *Transactions B* will share common editorial boards, and thus submission procedures, with their companion journals. In addition, editorial decisions will be completely independent of whether a paper is intended for the open access journal or its subscription counterpart.

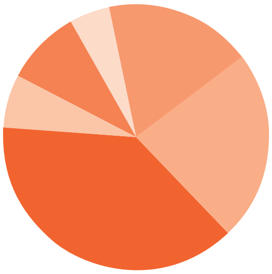
This entry into open access publishing is a significant first for the AMS, which prior to this point has not had a method for accommodating authors who wish to publish under a "gold" open access model. The two new journals will provide immediate unrestricted access to the publisher's final version of the paper and allows immediate deposit of the version of record in other repositories. Research and library communities in particular will benefit from this movement toward open access, as it provides the AMS with an opportunity to publish more high-quality literature in freely accessible research journals.

The open access publication option will be available starting September 1, 2013 for all articles accepted by the *Proceedings* and *Transactions of the AMS* editorial boards. New articles should be submitted via www.ams.org/procsubmit or www.ams.org/transubmit, and the open access publication decision may be determined by the author upon notification of acceptance.

The Article Processing Charges are:

- *Proceedings of the AMS, Series B*: List Price $1,500 (2014 discount price $750)
- *Transactions of the AMS, Series B*: List Price $2,750 (2014 discount price $1,400)

Further information will become available on **www.ams.org** as the journals approach their 2014 release date.



Report on the 2011-2012 New Doctoral Recipients

Richard Cleary, James W. Maxwell, and Colleen Rose

This report presents a statistical profile of recipients of doctoral degrees awarded by departments in the mathematical sciences at universities in the United States during the period July 1, 2011, through June 30, 2012. All information in the report was provided by the departments that awarded the degrees with additional information provided by the individual new doctoral recipients. The report includes an analysis of the fall 2012 employment plans of 2011–2012 doctoral recipients and a demographic profile summarizing characteristics of citizenship status, gender, and racial/ethnic group. This report is based on a complete census of the 2011–2012 new doctorates and includes information about 2011-2012 doctoral recipients that were not included in the preliminary report in the March 2013 issue of *Notices*.

Detailed information, including tables which traditionally appeared in this report, is available on the AMS website at `www.ams.org/annual-survey/survey-reports`.

Doctoral Degrees Awarded

1,798 Ph.D.'s were awarded by the 307 doctoral-granting departments. We are pleased to report that we had a 100% response rate for this survey and we thank the departments for their cooperation.

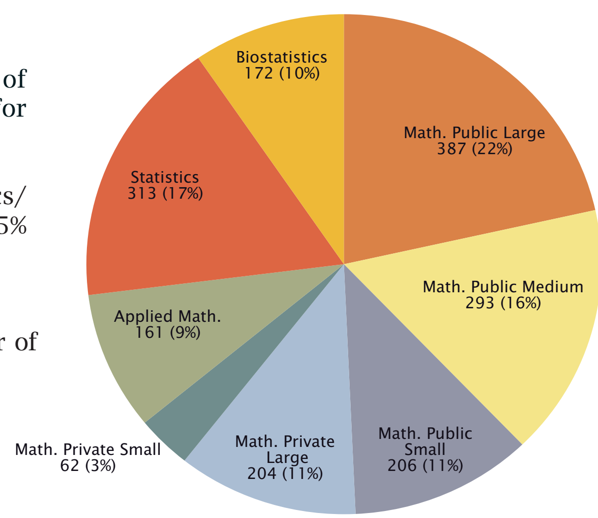
Biostatistics reported the largest increase in the number of doctoral recipients, up 57 over the total of 115 reported for 2010-2011.

32% (570) of the new Ph.D.'s had a dissertation in statistics/biostatistics, followed by applied mathematics (264) with 15% and algebra/number theory (227) with 13%.

Comparing Ph.D.'s awarded this year to last year, the number of Ph.D.s awarded:

- Increased about 9% from 1,653 to 1,798.
- Increased in all department groupings except for Math. Private Large, which awarded 7% fewer.
- Remained flat for degrees in statistics.

Figure A.1: Number and Percentage of Degrees Awarded by Department Grouping*



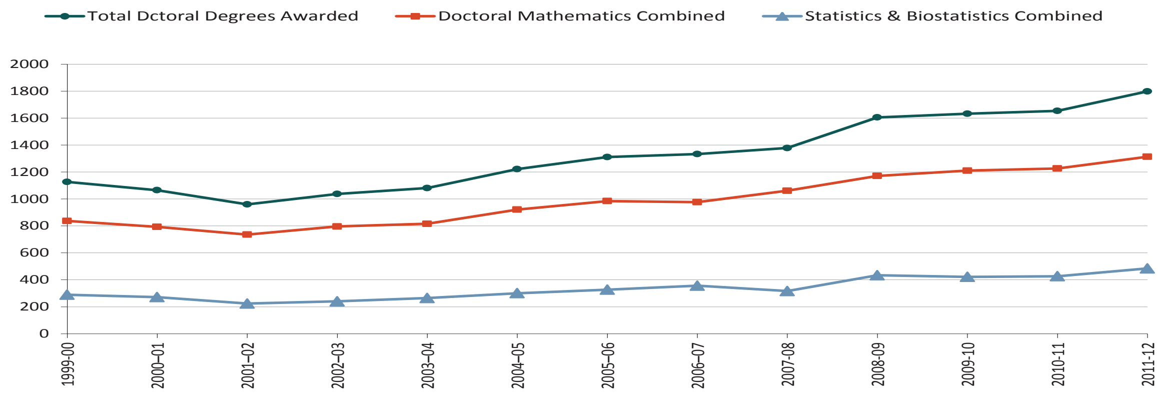
Total Degrees Awarded: 1,798

*See page 884 for a description of the department groupings.

Richard Cleary is a professor in the Division of Mathematics and Sciences at Babson College. James W. Maxwell is AMS associate executive director for special projects. Colleen A. Rose is AMS survey analyst.

Doctoral Degrees Awarded

Figure A.2: New Ph.D.'s Awarded by Group



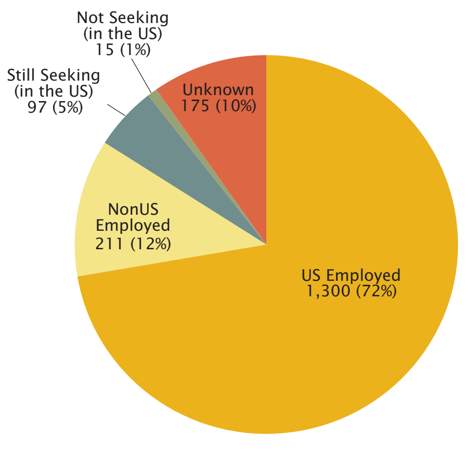
Comparing Ph.D.'s awarded this year with those awarded in 2001–2002:

- Ph.D.'s awarded have increased more than 87% over the last 10 years in all groups combined.
- Degrees awarded by Doctoral Mathematics combined and by Statistics & Biostatistics combined have increased 78% and 117%, respectively. Some of this latter increase may be attributed to the increase in response rate among the Statistics & Biostatistics departments.

Employment

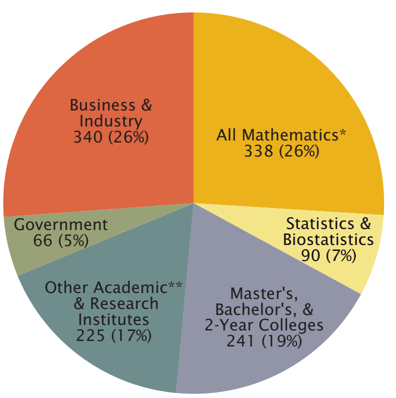
The overall U.S. unemployment rate for the new doctoral recipients is 6.9%, up from 4.3% last year. (Details on the calculations are on page 884.) The employment plans are known for 1,623 of the 1,798 new doctoral recipients. The number of new doctoral recipients employed in the U.S. is 1,300, up 9% from last year's number of 1,191. 68% of Ph.D.'s employed in Doctoral Math. departments are in postdoc positions, down from 71% last year. The number of new Ph.D.'s taking positions in Business & Industry has increased to 340 this year compared to 235 last year, and all groups showed an increase in Business & Industry.

Figure E.1: Employment Status



- 53% (685) of those who are employed in the U.S. are U.S. citizens, down slightly from 54% last year.
- 78% (615) of non-U.S. citizens whose employment status is known are employed in the U.S., the remaining 151 non-U.S. citizens are either employed outside of the U.S. or are unemployed.
- 8% (115) of all new Ph.D.'s are working at the institution which granted their degree, the same percent as last year. These individuals constitute 13% of total U.S. academic employed.

Figure E.2: U.S. Employed by Type of Employer

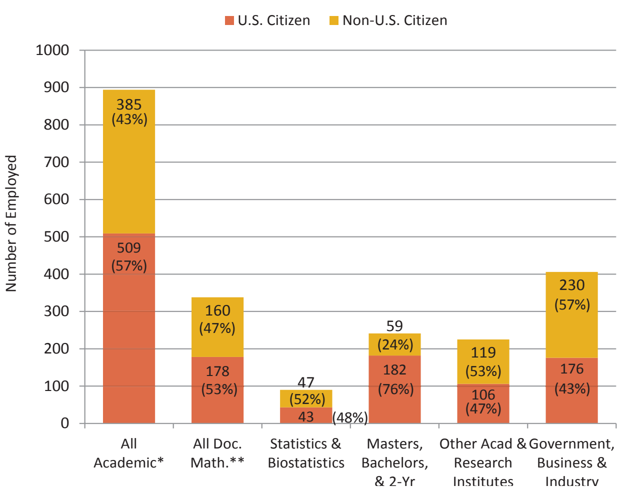


*Includes all Math. Public, Math. Private, and Applied Math. departments.

**Other Academic consists of departments outside the mathematical sciences including numerous medical related units.

- Total U.S. employed: 1,300
- U.S. academic hiring increased slightly to 894 and all hiring groups reported increases except Math. Public Small and Large, Math. Private Small, Master's, and 2-Year College.
- Goverment hiring decreased 19% (from 81 to 66); all doctoral granting groups except Math. Public Small, Applied Math. and Biostatistics showed a decrease in the number of Ph.D.'s taking positions in this sector.

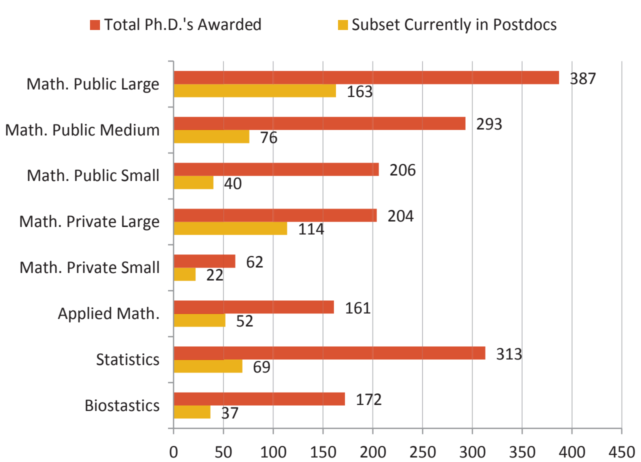
Employment

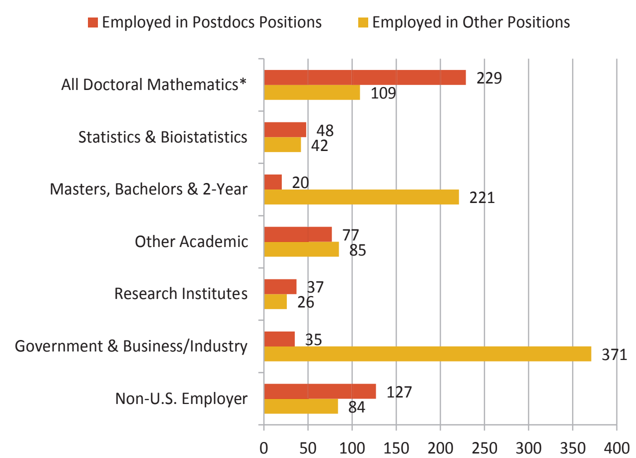


* Includes all Math. Public, Math. Private, and Applied Math., Master's, Bachelors, 2-Yr, Other academic and research institutes/nonprofit.
**Includes all Math. Public, Math. Private, and Applied Math. departments.

Looking at U.S. citizens whose employment status is known:

- 86% (685) are employed in the U.S., of these:
  - 32% are employed in Ph.D.-granting departments
  - 42% are employed in all other academic categories
  - 26% are employed in government, business and industry





*Includes all Math. Public, Math. Private, and Applied Math. departments.

- Total known to be employed: 1,511
- 38% (573) of the new Ph.D.'s that are employed are reported to be in postdoc positions, down from 41% last year.
- 56% of the new Ph.D.'s awarded by Math. Private Large departments are employed in postdocs, while only 22% of new Ph.D.'s awarded by Statistics departments are in postdocs.
- 46% of the new Ph.D.'s having U.S. academic employment are in postdocs; last year this percentage was 48%.
- 22% of the new Ph.D.'s in postdoc positions, are employed outside the U.S., last year this percentage was 24%.
- 68% of the new Ph.D.'s employed in Doctoral Math. departments are in postdoc positions, down from 71% last year. The analogous percent for Math. Private Large is 85%.

Employment

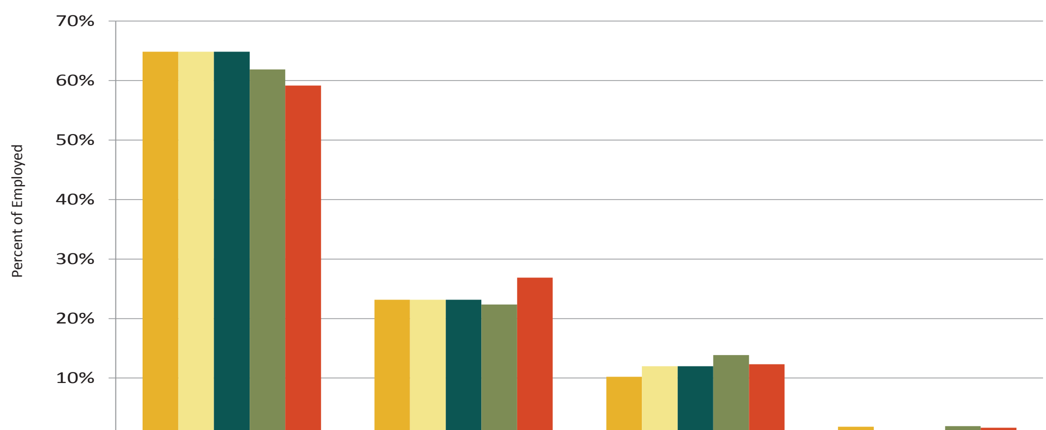
Figure E.6 displays the U.S. unemployment rate for new doctorates; details on the calculations are on page 884.

Figure E.6: Percentage of New Doctoral Recipients Unemployed 2003–2012*

*The difficult employment years of the 1990's are not shown here but are located on the AMS website at www.ams.org/annual-survey/2012Survey-NewDoctorates-Report.

- Unemployment among those whose employment status is known is 6.9%, up from 4.3% for Fall 2011.
- Math. Private Small reported the highest unemployment rate at 13%.
- Statistics reported the lowest unemployment at 3.5%.
- 7.8% of U.S. citizens are unemployed, compared to 4.6% in Fall 2011.
- 6.0% of non-U.S. citizens are unemployed, compared to 3.9% in Fall 2011.

Figure E.7: Percentage of Employed New Ph.D.'s by Type of Employer



	U.S. Academic*	U.S. Noncademic	Non-U.S. Academic	Non-U.S. Nonacademic
Fall 2008	65%	23%	10%	2%
Fall 2009	65%	23%	12%	1%
Fall 2010	65%	23%	12%	1%
Fall 2011	62%	22%	14%	2%
Fall 2012	59%	27%	12%	2%

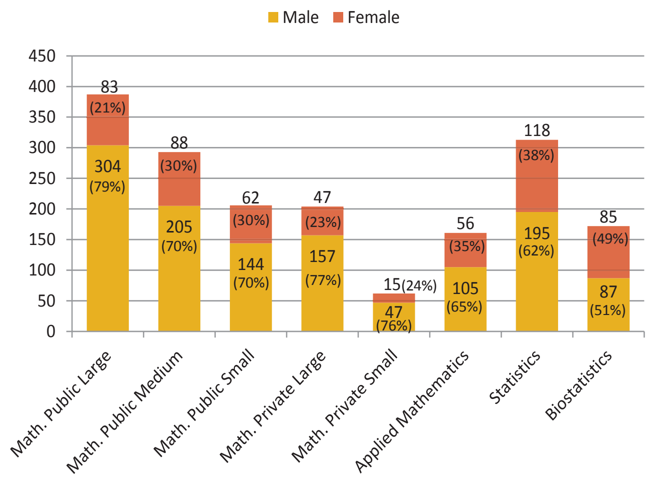
* Includes other academic departments and research institutes/other non-profits.

- U.S. academic hiring has dropped to 59% (a five-year low), while U.S. nonacademic hiring has jumped to 27% (a five-year high).
- Detailed information on new Ph.D.'s employed in the U.S. by degree-granting department group is available on the AMS website at www.ams.org/annual-survey/2012Survey-NewDoctorates-Report.

Demographics

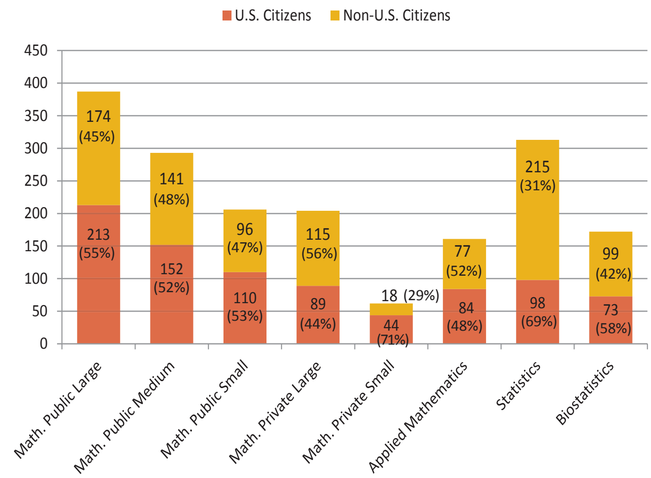
Gender and citizenship was known for all 1,798 new Ph.D.'s reported for 2011–2012. The number of U.S. citizens is 863 (48%) (down slightly from 49% last year). The number of females accounted for 28% of the U.S. citizen total (the same as last year). The number of non-U.S. citizens receiving a Ph.D. increased modestly to 52% from 51% last year; this is down 7 percentage points from the 10-year high of 59% reported in 2004–2005. 11% (65) of the non-U.S. citizens employed in the U.S. have permanent visa status (up from 9% last year).

Figure D.1: Gender of Doctoral Recipients by Degree-Granting Grouping



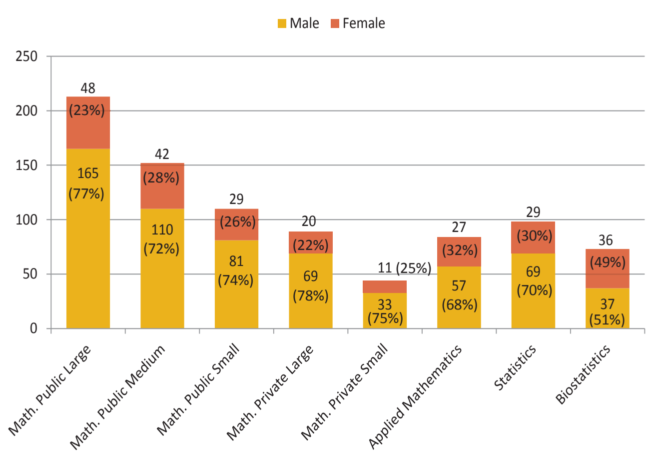
- Females account for 31% (554) of the 1,798 Ph.D.'s, down only marginally from last year's figure of 32%.

Figure D.2: Citizenship of Doctoral Recipients by Degree-Gtranting Grouping



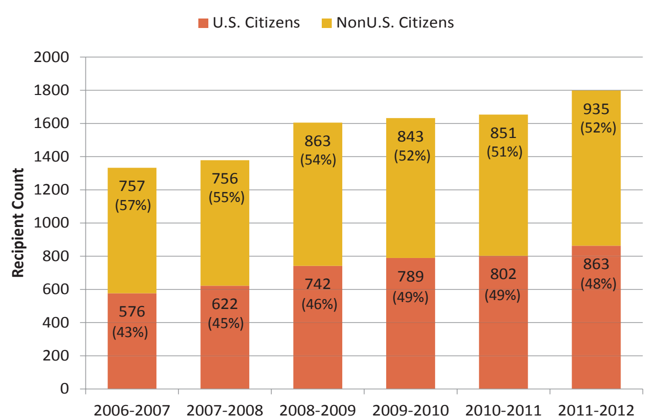
- All groups awarded more degrees to U.S. Citizens than Non-U.S. citizens except Math. Private Large, Statistics and Biostatitics which awarded 44%, 31% and 42% to U.S. citizens.

Figure D.3: Gender of U.S. Citizen Doctoral Recipients by Degree-Granting Grouping



- 50% of the males and 44% of the females are U.S. citizens.
- Females accounted for 28% of the U.S. citizens.
- Among the U.S. citizens: 7 are American Indian or Alaska Native, 62 are Asian, 28 are Black or African American, 33 are Hispanic or Latino, 5 are Native Hawaiian or Other Pacific Islander, 669 are White, and 59 are of unknown race/ ethnicity.

Figure D.4: Citizenship of New Ph.D.* Recipients, 2006–2012



*The increase shown from 2007–2008 to 2008–2009 is due in part to the increase in the response rate for statistics and biostatistics departments.

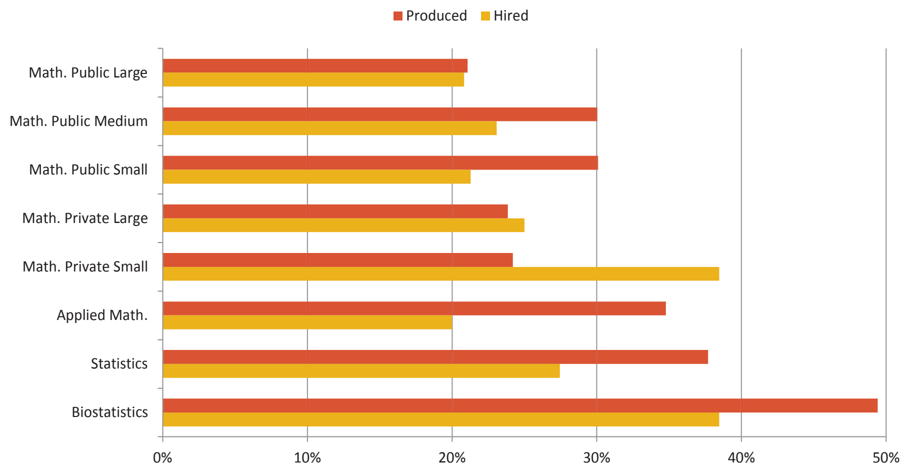
Looking at the last six years we see that:

- U.S. citizen counts have been increasing steadily, reaching a high of 863 this year. This is a 50% increase from Fall 2006–2007.
- Non-U.S. citizen counts which had been hovering around 850, are showing more variability, increasing to 935 this year. While this is a 24% increase from Fall 2006–2007, it represents a 10% increase from last year.

Female New Doctoral Recipients

After increasing to 32% last year, the number of female new doctoral recipients has decreased to 31% this year. Of the 894 new Ph.D.'s hired into academic positions 31% (279) were women, down from 33% last year. 25% of those hired into postdoc positions were women, with 42% of the women in postdocs being U.S. citizens, up from 41% last year. The U.S. unemployment rate for females is 7.4%, compared to 6.7% for males and 7% overall.

Figure F.1: Females as a Percentage of New Doctoral Recipients Produced by and Hired by Department Grouping



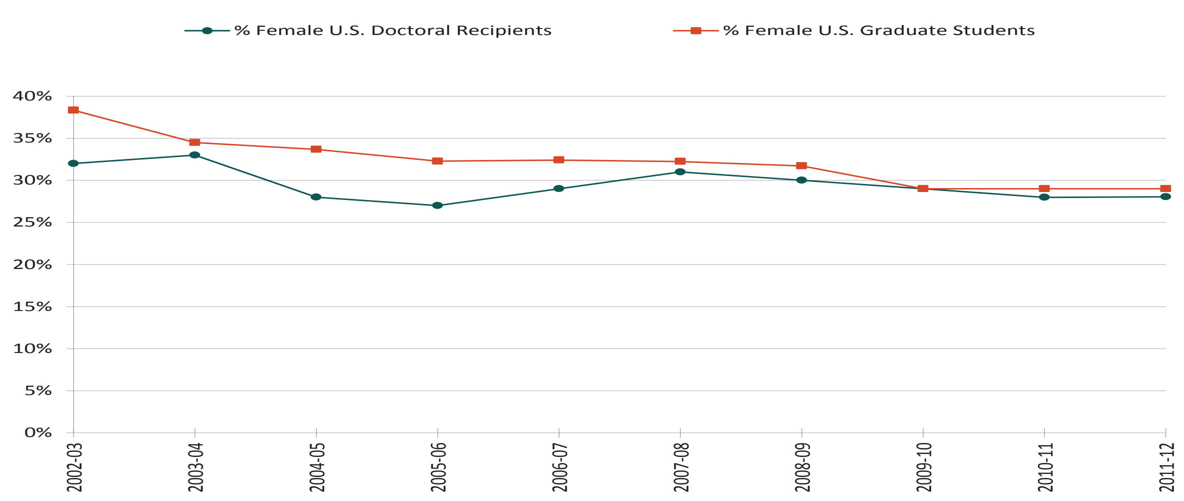
* For definitions of groups see page 884.

Table F.1: Number of Female New Doctoral Recipients Produced by and Hired by Department Groupings

Department Grouping	Females Produced	Females Hired
Math. Public Large	83	73
Math. Public Medium	88	75
Math. Public Small	62	49
Math. Private Large	47	44
Math. Private Small	15	13
Applied Math.	56	37
Statistics	118	102
Biostatistics	85	77

- 38% of those hired by Group B were women (up from 36% last year) and 29% of those hired by Group M were women (down from 33% last year).
- 37% of those hired into Research Institutes/Other non-profit positions were women (down from 62% last year but in line with 35% two years ago).
- 36% of those hired into Government positions were women (down slightly from 37% last year).
- 60% of the women employed in all doctoral groups are in postdoc positions, compared to 66% of males employed in postdocs in these groups.

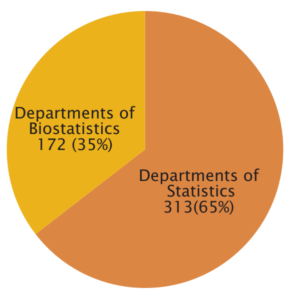
Figure F.2: Females as a Percentage of U.S. Citizen Doctoral Recipients



Ph.D.'s Awarded by Statistics and Biostatistics Departments

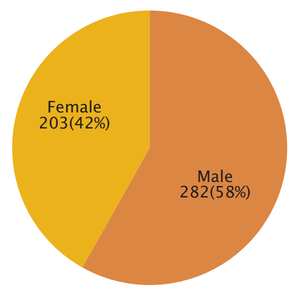
This section contains information about new doctoral recipients in these departments (59 statistics and 36 biostatistics departments). Statistics and Biostatistics departments produced 485 new doctorates, of which all but 6 had dissertations in statistics/biostatistics. This is a 28% increase in the number reported for fall 2011 of 375. In addition, Math Public, Math Private and Applied Math departments combined had 91 Ph.D. recipients with dissertations in statistics. 35% (171) of the new Ph.D.'s in Statistics and Biostatistics departments are U.S. citizens (while in the other groups combined 52% are U.S. citizens). The 90 departments responding last year and this year reported a total of 427 new doctoral recipients, an increase of 4% from last year. The unemployment among this group of new Ph.D.'s is 4.2% up from 3.8%.

Figure S.1: Ph.D.'s Awarded by Statistics/Biostatistics Departments



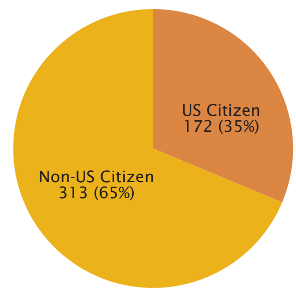
- 27% of all Ph.D.'s awarded were in Statistics/Biostatistics.
- Females account for 38% of statistics and 49% of biostatistics Ph.D.'s awarded.

Figure S.2: Gender of Ph.D. Recipients from Statistics/Biostatistics Departments



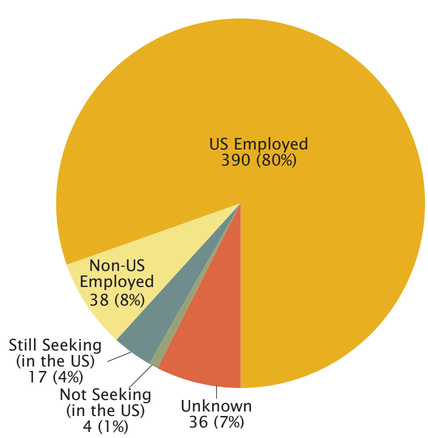
- Females accounted for 42% of the 485 Ph.D.'s in Statistics and Biostatistics, compared to all other groups combined, where 27% are female.

Figure S.3: Citizenship of Ph.D. Recipients from Statistics/Biostatistics Departments



- 38% of Statistics/Biostatistics U.S. citizen Ph.D. recipients are females, while in all other groups combined 26% of the U.S. citizens are females.

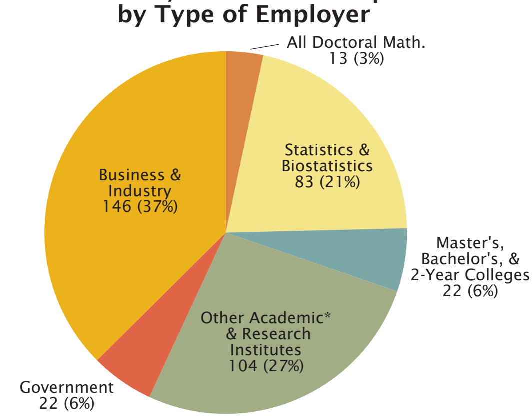
Figure S.4: Employment Status of Ph.D. Recipients from Statistics/Biostatistics Departments



Total Ph.D.'s Awarded: 485

- 4.2% of Statistics/Biostatistics Ph.D.'s are unemployed compared to 8.1% among all other groups. This is up from 3.8% last year.
- Unemployment among new Ph.D.'s with dissertations in statistics/probability is 4.0%, up from 3.6%. Among all other dissertation groupings 7.0% are unemployed.

Figure S.5: U.S.-Employed Ph.D. Recipients from Statistics/Biostatistics Departments by Type of Employer



*Other Academic consists of departments outside the mathematical sciences including numerous medical related units.

Total U.S. Employed: 390

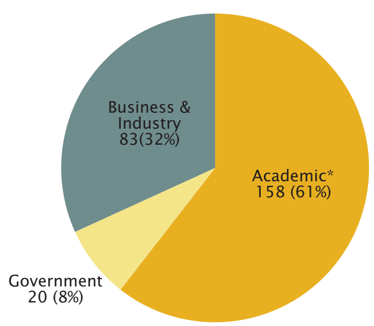
- 37% of Statistics/Biostatistics Ph.D.'s are employed in Business/Industry, compared to 22% in all other groups.
- 32% of those hired by statistics and biostatistics were females, compared to 24% in all other groups.

Information from the Employment Experiences of New Doctorates (EENDR) Survey

This section contains additional information on employment gathered from a subset of the 2011–2012 new Ph.D.'s on the EENDR Survey. It expands on the details of employment which are not available through the departments.

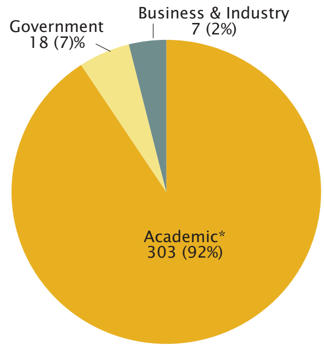
The 1,644 new Ph.D.'s reported in our Preliminary Report were sent this survey; of those individuals 709 (43%) responded. The employment status is known for 702 of these individuals, the U.S. unemployment among this group is 2.2%. Of the 681 who reported being employed, 31% indicated they were actively looking for new employment.

Figure EE.1: EENDR Respondents Reporting Permanent U.S. Employment by Sector



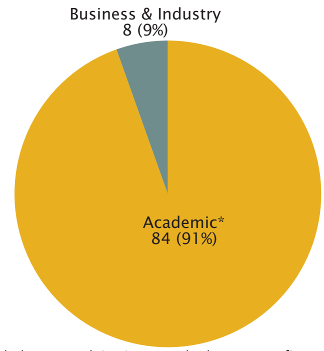
* Includes research institutes and other non-profits.

Figure EE.2: EENDR Respondents Reporting Temporary U.S. Employment by Sector



* Includes research institutes and other non-profits.

Figure EE.3. EENDR Respondents Employed Outside the U.S. by Sector



* Includes research institutes and other non-profits.

Of the 261 permanently employed:

- 35% are women.
- 67% of those reporting academic employment hold tenured/tenure-track positions.

Of the 328 temporarily employed:

- 28% are women.
- 33% were unable to find a suitable permanent position (up from 27% last year).
- 73% are employed in postdocs and 45% of these reported they could not find a suitable permanent position.

Of the 92 employed outside the U.S.:

- 27% are women.
- 28% are U.S. Citizens.
- 68% are employed in postdocs.

Table EE.1: Number and Percentage of EENDR Respondents Employed in the U.S. by Job Status

					Temporary		Temporary Postdocs				
Year	Perm Total	%	Temp Total	%	Perm Not Avail	% of Temp Total	Total	% of Temp Total	Perm Not Avail	% of Temp Postdocs	#(%) Unknown
Fall 2008	245	49%	222	45%	74	33%	172	77%	47	27%	29(6%)
Fall 2009	318	49%	326	51%	146	45%	234	72%	68	29%	0
Fall 2010	320	48%	341	52%	140	41%	246	72%	68	28%	0
Fall 2011	251	44%	319	56%	133	42%	225	71%	87	39%	0
Fall 2012	261	44%	328	56%	127	39%	242	74%	108	45%	0

Comparing the employment status of EENDR respondents employed in the U.S. over the last five years we see that:

- Permanent positions have leveled off at 44% this year. This is down 5 percentage points from the high reported in 2008 & 2009 and down in number by 59 (18%) from the high of 320 in 2010.
- Temporary positions remained unchanged at 56% this year, maintaining a five-year high.
- 39% of those holding temporary positions were unable to find suitable permanent positions, up 6 percentage points from 2008 and down 6 percentage points from the high in 2009.
- 45% of those holding postdoc positions were unable to find suitable permanent positions, a five-year high.

Information from the Employment Experiences of New Doctorates (EENDR) Survey

Table EE.2: Percentage of EENDR Respondents Employed in the U.S. by Employment Sector within Job Status

Year	Permanent			Temporary		
	Acad	Govn	B/I	Acad	Govn	B/I
Fall 2008	63%	6%	31%	95%	4%	1%
Fall 2009	64%	6%	29%	91%	5%	4%
Fall 2010	64%	8%	28%	93%	5%	2%
Fall 2011	61%	8%	31%	94%	5%	1%
Fall 2012	61%	8%	32%	92%	5%	2%

Looking at at Table EE.2 we see that

- Permanent academic employment has leveled off at 61%, while temporary employment in this sector has decreased to 92%.
- Permanent government employment has leveled off at 8%.
- Business/Industry permanent employment has increased to 32% (a five-year high), while temporary positions increased to 2%.

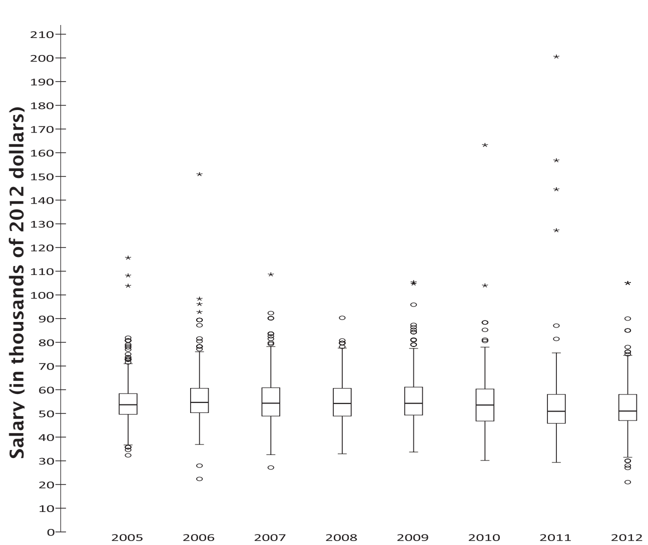
Starting Salaries of the 2011-2012 Doctoral Recipients

The starting salary figures were compiled from information gathered on the EENDR questionnaires sent to 1,644 individuals using addresses provided by the departments granting the degrees; 709 individuals responded between late October and April. Responses with insufficient data or from individuals who indicated they had part-time or non-U.S. employment were excluded. Numbers of usable responses for each salary category are reported in the following tables.

Readers should be warned that the data in this report are obtained from a self-selected sample, and inferences from them may not be representative of the full population.

Academic Teaching/Teaching and Research 9–10-Month Starting Salaries* (in thousands of dollars)

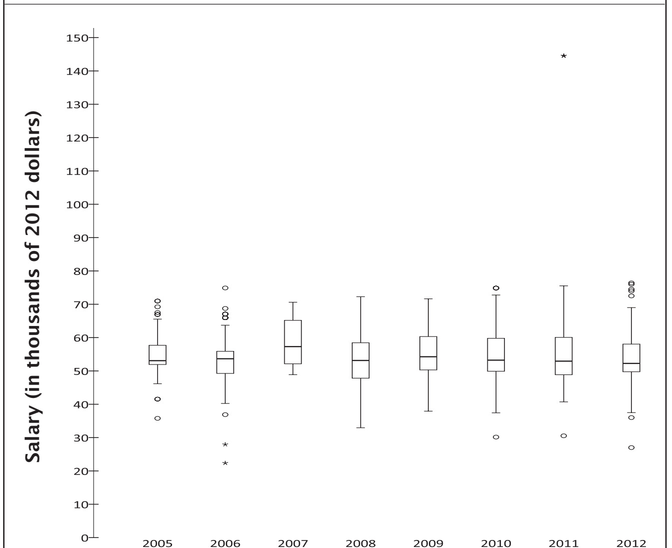
Ph.D. Year	Min	Q_1	Median	Q_3	Max
Total (203 male/78 female)					
2012 M	28.0	48.0	52.0	58.1	105.0
2012 F	21.0	47.0	51.0	56.0	78.0
One year or less experience (189 male/70 female)					
2012 M	28.0	47.0	50.8	57.0	78.0
2012 F	21.0	47.0	50.0	55.0	60.0



* Includes postdoctoral salaries.

Academic Postdoctorates Only* 9–10-Month Starting Salaries (in thousands of dollars)

Ph.D. Year	Min	Q_1	Median	Q_3	Max
Total (88 male/24 female)					
2012 M	49.5	50.0	55.1	60.0	76.4
2012 F	27.0	43.0	47.0	53.0	67.0
One year or less experience (86 male/23 female)					
2012 M	36.0	50.0	52.6	60.0	74.4
2012 F	40.0	47.0	50.0	54.5	67.0

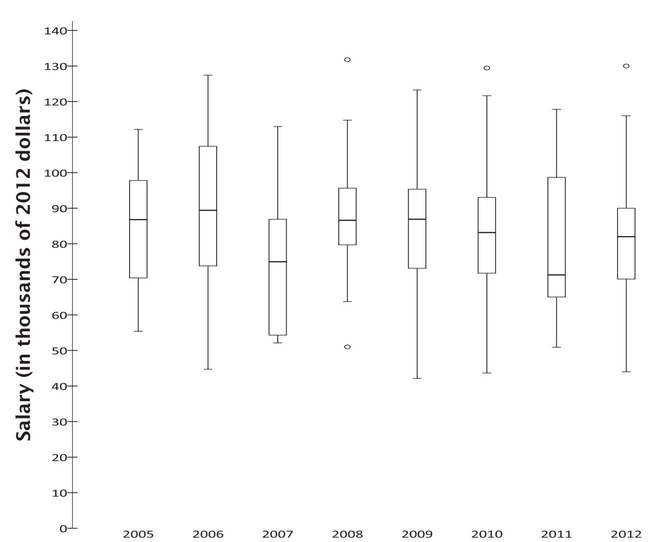


* A postdoctoral appointment is a temporary position primarily intended to provide an opportunity to extend graduate training or to further research experience.

Starting Salaries of the 2011-2012 Doctoral Recipients

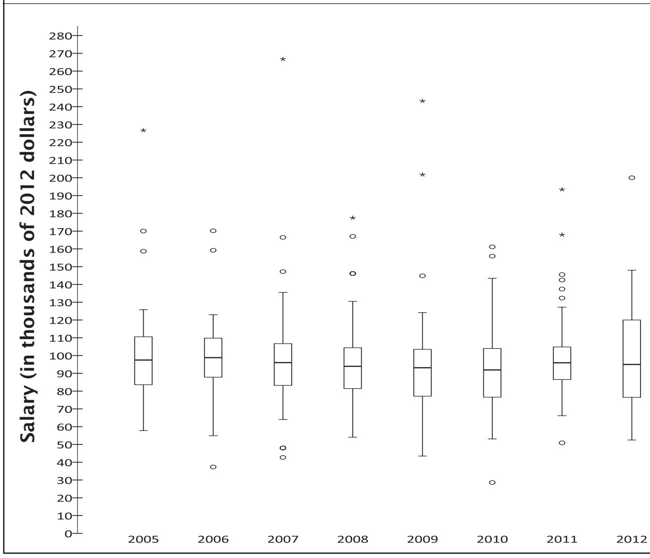
Government
11–12-Month Starting Salaries
(in thousands of dollars)

Ph.D. Year	Min	Q_1	Median	Q_3	Max
Total (23 male/14 female)					
2012 M	60.0	71.5	82.0	89.6	103.0
2012 F	44.0	68.3	81.2	91.5	116.0
One year or less experience (16 male/12 female)					
2012 M	65.6	70.1	85.5	84.3	97.3
2012 F	44.0	66.5	81.2	90.5	95.0



Business and Industry
11–12-Month Starting Salaries
(in thousands of dollars)

Ph.D. Year	Min	Q_1	Median	Q_3	Max
Total (55 male/25 female)					
2012 M	58.5	85.0	100.0	120.0	145.0
2012 F	52.5	68.5	83.1	94.8	105.0
One year or less experience (46 male/24 female)					
2012 M	63.0	85.0	99.5	120.0	200.0
2012 F	52.5	70.0	92.0	106.0	140.0



Remarks on Starting Salaries

Key to Tables and Graphs. Salaries are those reported for the fall immediately following the survey cycle. Years listed denote the survey cycle in which the doctorate was received—for example, survey cycle July 1, 2011–June 30, 2012, is designated as 2012. Salaries reported as 9–10 months exclude stipends for summer grants or summer teaching or the equivalent. M and F are male and female respectively. Male and female figures are not provided when the number of salaries available for analysis in a particular category was five or fewer. All categories of "Teaching/Teaching and Research" and "Research Only" contain those recipients employed at academic institutions only.

Graphs. The graphs show standard boxplots summarizing salary distribution information for the years 2005 through 2012. Values plotted for 2005 through 2012 are converted to 2012 dollars using the implicit price deflator prepared annually by the Bureau of Economic Analysis, U.S. Department of Commerce. These categories are based on work activities reported in EENDR. Salaries of postdoctorates are shown separately. They are also included in other academic categories with matching work activities.

For each boxplot the box shows the first quartile (Q1), the median (M), and the third quartile (Q3). The interquartile range (IQR) is defined as Q3–Q1. Think of constructing invisible fences 1.5 IQR below Q1 and 1.5 IQR above Q3. Whiskers are drawn from Q3 to the largest observation that falls below the upper invisible fence and from Q1 to the smallest observation that falls above the lower invisible fence. Think of constructing two more invisible fences, each falling 1.5 IQR above or below the existing invisible fences. Any observation that falls between the fences on each end of the boxplots is called an outlier and is plotted as ○ in the boxplots. Any observation that falls outside of both fences either above or below the box in the boxplot is called an extreme outlier and is marked as ✱ in the boxplot.

Remarks on U.S. Unemployment Rate Calculations

In the unemployment calculations provided in this report the individuals employed outside the U.S. have been removed from the denominator used in the calculation of the rate, in addition to the routine removal of all individuals whose employment status is unknown. This is a change from Annual Survey Reports prior to 2009. As a consequence, the unemployment rate now being reported more accurately reflects the U.S. labor market experienced by the new doctoral recipients. This change tends to increase the rate of unemployment over that reported in prior years.

In a further small change from prior years, those individuals reported as not seeking employment have also been removed from the denominator. The number of individuals so designated is small each year, and the impact of this change is to produce a slight increase in the rate over that reported in prior years.

The unemployment rates for years prior to 2009 shown in this report have been recalculated using this new method. One can view a comparison of the unemployment rates using the traditional method and the new method by visiting the AMS website at `www.ams.org/annual-survey/surveyreports.html`.

Departmental Groupings and Response Rates

Starting with reports on the 2012 AMS-ASA-IMS-MAA-SIAM Annual Survey of the Mathematical Sciences, the Joint Data Committee has implemented a new method for grouping the doctorate-granting mathematics departments. These departments are first grouped into those at public institutions and those at private institutions. These groups are further subdivided based on the size of their doctoral program as reflected in the average annual number of Ph.D.'s awarded between 2000 and 2010, based on their reports to the Annual Survey during this period. Furthermore, doctorate-granting departments which self-classify their Ph.D. program as being in applied mathematics will join with the other applied mathematics departments previously in Group Va to form their own group. The former Group IV will be divided into two groups, one for departments in statistics and one for departments in biostatistics.

For further details on the change in the doctoral department groupings see the article in the October 2012 issue of *Notices of the AMS* at `http://www.ams.org/notices/201209/rtx120901262p.pdf.`

Group Descriptions

Math. Public Large consists of departments with the highest annual rate of production of Ph.D.'s, ranging between 7.0 and 24.2 per year.

Math. Public Medium consists of departments with an annual rate of production of Ph.D.'s, ranging between 3.9 and 6.9 per year.

Math. Public Small consists of departments with an annual rate of production of Ph.D.'s of 3.8 or less per year.

Math. Private Large consists of departments with an annual rate of production of Ph.D.'s, ranging between 3.9 and 19.8 per year.

Math. Private Small consists of departments with an annual rate of production of Ph.D.'s of 3.8 or less per year.

Applied Mathematics consists of doctoral degree granting applied mathematics departments.

Statistics consists of doctoral degree granting statistics departments.

Biostatistics consists of doctoral granting biostatistics departments.

Group M contains U.S. departments granting a master's degree as the highest graduate degree.

Group B contains U.S. departments granting a baccalaureate degree only.

Listings of the actual departments which compose these groups are available on the AMS website at `www.ams.org/annual-survey/groups.`

Survey Response Rates by New Groupings

Doctorates Granted
Departmental Response Rates*

Math. Public Large	26 of 26 including	0 with no degrees
Math. Public Medium	40 of 40 including	0 with no degrees
Math. Public Small	64 of 64 including	10 with no degrees
Math. Private Large	24 of 24 including	0 with no degrees
Math. Private Small	28 of 28 including	5 with no degrees
Applied Math.	30 of 30 including	3 with no degrees
Statistics	59 of 59 including	5 with no degrees
Biostatistics	36 of 36 including	8 with no degrees
Total	307 of 307 including	31 with no degrees

NORTHWESTERN UNIVERSITY

Call for Nominations

Nemmers Prize in Mathematics

$200,000

Recipients of The Frederic Esser Nemmers Prize in Mathematics

John H. Conway
Ingrid Daubechies
Simon Donaldson
Mikhael Gromov
Joseph B. Keller
Robert P. Langlands
Yuri I. Manin
Yakov G. Sinai
Terence Tao
Edward Witten

Northwestern University invites nominations for the Frederic Esser Nemmers Prize in Mathematics to be awarded during the 2013-14 academic year. The award includes payment to the recipient of $200,000. Made possible by a generous gift to Northwestern by the late Erwin Esser Nemmers and the late Frederic Esser Nemmers, the award is given every other year.

Candidacy for the Nemmers Prize in Mathematics is open to those with careers of outstanding achievement in mathematics as demonstrated by major contributions to new knowledge or the development of significant new modes of analysis. Individuals of all nationalities and institutional affiliations are eligible except current or recent members of the Northwestern University faculty and recipients of the Nobel Prize.

The recipient of the 2014 Nemmers Prize in Mathematics will deliver a public lecture and participate in other scholarly activities at Northwestern University for 10 weeks during the 2014-15 academic year.

Nominations for the Frederic Esser Nemmers Prize in Mathematics will be accepted until December 1, 2013. Nominating letters of no more than three pages should describe the nominee's professional experience, accomplishments, and qualifications for the award. A brief *curriculum vitae* of the nominee is helpful but not required. Nominations from experts in the field are preferred to institutional nominations; direct applications will not be accepted.

Nominations may be sent to:

nemmers@northwestern.edu
or

Secretary
Selection Committee for the Nemmers Prizes
Office of the Provost
Northwestern University
633 Clark Street
Evanston, Illinois 60208-1119
U.S.A.

www.nemmers.northwestern.edu

Is Mathematical History Written by the Victors?

Jacques Bair, Piotr Błaszczyk, Robert Ely, Valérie Henry, Vladimir Kanovei, Karin U. Katz, Mikhail G. Katz, Semen S. Kutateladze, Thomas McGaffey, David M. Schaps, David Sherry, and Steven Shnider

The ABCs of the History of Infinitesimal Mathematics

The ABCs of the history of infinitesimal mathematics are in need of clarification. To what extent does the famous dictum "history is always written by the victors" apply to the history of mathematics as well? A convenient starting point is a remark made by Felix Klein in his book *Elementary Mathematics from an Advanced Standpoint* (Klein [72, p. 214]). Klein wrote that there are not one but two separate tracks for the development of analysis:

(A) the Weierstrassian approach (in the context of an *Archimedean* continuum) and

(B) the approach with indivisibles and/or infinitesimals (in the context of what we will refer to as a *Bernoullian* continuum).[1]

Klein's sentiment was echoed by the philosopher G. Granger, in the context of a discussion of Leibniz, in the following terms:

> Aux yeux des détracteurs de la nouvelle Analyse, l'insurmontable difficulté vient de ce que de telles pratiques font violence aux règles ordinaires de l'Algèbre, tout en conduisant à des résultats, exprimables en termes finis, dont on ne saurait contester l'exactitude. Nous savons aujourd'hui que deux voies devaient s'offrir pour la solution du problème:
>
> [A] Ou bien l'on élimine du langage mathématique le terme d'infiniment petit, et l'on établit, en termes finis, le sens à donner à la notion intuitive de 'valeur limite'. …
>
> [B] Ou bien l'on accepte de maintenir, tout au long du Calcul, la présence d'objets portant ouvertement la marque de l'infini, mais en leur conférant un statut propre qui

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[1] *Systems of quantities encompassing infinitesimal ones were used by Leibniz, Bernoulli, Euler, and others. Our choice of the term is explained in the subsection "Bernoulli, Johann". It* encompasses *modern non-Archimedean systems.*

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> les insère dans un système dont font aussi partie les grandeurs finies.…
>
> C'est dans cette seconde voie que les vues philosophiques de Leibniz l'ont orienté. (Granger 1981 [43, pp. 27–28])[2]

Thus we have two parallel tracks for conceptualizing infinitesimal calculus, as shown in Figure 1.

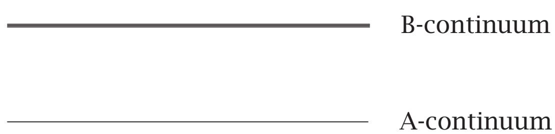

Figure 1. Parallel tracks: a thick continuum and a thin continuum.

At variance with Granger's appraisal, some of the literature on the history of mathematics tends to assume that the A-approach is the ineluctably "true" one, while the infinitesimal B-approach was, at best, a kind of evolutionary dead end or, at worst, altogether inconsistent. To say that infinitesimals provoked passions would be an understatement. Parkhurst and Kingsland, writing in *The Monist*, proposed applying a *saline solution* (if we may be allowed a pun) to the problem of the infinitesimal:

> [S]ince these two words [infinity and infinitesimal] have sown nearly as much faulty logic in the fields of mathematics and metaphysics as all other fields put together, they should be rooted out of both the fields which they have contaminated. And not only should they be rooted out, lest more errors be propagated by them: *a due amount of salt* should be ploughed under the infected territory, that the damage be mitigated as well as arrested. (Parkhurst and Kingsland 1925 [91, pp. 633–634]) [emphasis added—the authors]

Writes P. Vickers:

> So entrenched is the understanding that the early calculus was inconsistent that many authors don't provide a reference to support the claim, and don't present the set of inconsistent propositions they have in mind. (Vickers 2013 [108, section 6.1, p. 146])

Such an *assumption* of inconsistency can influence one's appreciation of historical mathematics, make a scholar myopic to certain significant developments due to their automatic placement in an "evolutionary dead-end" track, and inhibit potential fruitful applications in numerous fields ranging from physics to economics (see Herzberg 2013 [110]). One example is the visionary work of Enriques exploiting infinitesimals, recently analyzed in an article by David Mumford, who wrote:

> In my own education, I had assumed that Enriques [and the Italians] were irrevocably stuck.…As I see it now, Enriques must be credited with a nearly complete geometric proof using, as did Grothendieck, higher order infinitesimal deformations.…Let's be careful: he certainly had the correct ideas about infinitesimal geometry, though he had no idea at all how to make precise definitions. (Mumford 2011 [89])

Another example is important work by Cauchy (see the subsection "Cauchy, Augustin-Louis" below) on singular integrals and Fourier series using infinitesimals and infinitesimally defined "Dirac" delta functions (these precede Dirac by a century), which was forgotten for a number of decades because of shifting foundational biases. The presence of Dirac delta functions in Cauchy's oeuvre was noted in (Freudenthal 1971 [40]) and analyzed by Laugwitz (1989 [74]), (1992a [75]); see also (Katz and Tall 2012 [69]) and (Tall and Katz 2013 [107]).

Recent papers on Leibniz (Katz and Sherry [67], [68]; Sherry and Katz [100]) argue that, contrary to widespread perceptions, Leibniz's system for infinitesimal calculus was *not* inconsistent (see the subsection "Mathematical Rigor" for a discussion of the term). The significance and coherence of Berkeley's critique of infinitesimal calculus have been routinely exaggerated. Berkeley's sarcastic tirades against infinitesimals fit well with the ontological limitations imposed by the A-approach favored by many historians, even though Berkeley's opposition, on empiricist grounds, to an infinitely divisible continuum is profoundly at odds with the A-approach.

A recent study of Fermat (Katz, Schaps, and Shnider 2013 [66]) shows how the nature of his contribution to the calculus was distorted in recent Fermat scholarship, similarly due to an "evolutionary dead-end" bias (see the subsection "Fermat, Pierre de").

The Marburg school of Hermann Cohen, Cassirer, Natorp, and others explored the philosophical foundations of the infinitesimal method underpinning the mathematized natural sciences. Their versatile, and insufficiently known, contribution is analyzed in (Mormann and Katz 2013 [88]).

A number of recent articles have pioneered a reevaluation of the history and philosophy of mathematics, analyzing the shortcomings of received views, and shedding new light on the deleterious effect of the latter on the philosophy, the practice, and the applications of mathematics.

[2] *Similar views were expressed by M. Parmentier in (Leibniz 1989* [79, p. 36, note 92]*).*

Some of the conclusions of such a reevaluation are presented below.

Adequality to Chimeras

Some topics from the history of infinitesimals illustrating our approach appear below in alphabetical order.

Adequality

Adequality is a technique used by Fermat to solve problems of tangents, problems of maxima and minima, and other variational problems. The term *adequality* derives from the $\pi\alpha\rho\iota\sigma\acute{o}\tau\eta\varsigma$ of Diophantus (see the subsection "Diophantus"). The technique involves an element of approximation and "smallness", represented by a small variation E, as in the familiar difference $f(A+E)-f(A)$. Fermat used adequality in particular to find the tangents of transcendental curves such as the cycloid that were considered to be "mechanical" curves off-limits to geometry by Descartes. Fermat also used it to solve the variational problem of the refraction of light so as to obtain Snell's law. Adequality incorporated a procedure of discarding higher-order terms in E (*without* setting them equal to zero). Such a heuristic procedure was ultimately formalized mathematically in terms of the *standard part principle* (see the subsection "Standard Part Principle") in Robinson's theory of infinitesimals starting with (Robinson 1961 [94]). Fermat's adequality is comparable to Leibniz's *transcendental law of homogeneity* (see the subsection "Lex homogeneorum transcendentalis").

Archimedean Axiom

What is known today as the Archimedean axiom first appears in Euclid's *Elements*, Book V, as Definition 4 (Euclid [34, Definition V.4]). It is exploited in (Euclid [34, Proposition V.8]). We include bracketed symbolic notation so as to clarify the definition:

> Magnitudes [a, b] are said to have a ratio with respect to one another which, being multiplied [na] are capable of exceeding one another [$na > b$].

It can be formalized as follows:[3]

(1)
$$(\forall a, b)(\exists n \in \mathbb{N})\,[na > b], \text{ where } na = \underbrace{a + \cdots + a}_{n-times}.$$

Next, it appears in the papers of Archimedes as the following lemma (see Archimedes [2, I, Lamb. 5]):

> Of unequal lines, unequal surfaces, and unequal solids [a, b, c], the greater exceeds the lesser [$a < b$] by such a magnitude [$b - a$] as, when added to itself [$n(b - a)$], can be made to exceed any assigned magnitude [c] among those which are comparable with one another. (Heath 1897 [47, p. 4])

This can be formalized as follows:

(2) $$(\forall a, b, c)(\exists n \in \mathbb{N})\,[a < b \to n(b - a) > c].$$

Note that Euclid's definition V.4 and the lemma of Archimedes are *not* logically equivalent (see the subsection "Euclid's Definition V.4", footnote 11).

The Archimedean axiom plays no role in the plane geometry as developed in Books I–IV of *The Elements*.[4] Interpreting geometry in ordered fields, or in geometry over fields in short, one knows that $\mathbb{F}^2$ is a model of Euclid's plane, where $(\mathbb{F}, +, \cdot, 0, 1, <)$ is a Euclidean field, i.e., an ordered field closed under the square root operation. Consequently, $\mathbb{R}^* \times \mathbb{R}^*$ (where $\mathbb{R}^*$ is a hyperreal field) is a model of Euclid's plane as well (see the subsection below on *modern implementations*). Euclid's definition V.4 is discussed in more detail in the subsection "Euclid's Definition V.4".

Otto Stolz rediscovered the Archimedean axiom for mathematicians, making it one of his axioms for magnitudes and giving it the following form: if $a > b$, then there is a multiple of b such that $nb > a$ (Stolz 1885 [106, p. 69]).[5] At the same time, in his development of the integers, Stolz implicitly used the Archimedean axiom. Stolz's visionary realization of the importance of the Archimedean axiom and his work on non-Archimedean systems stand in sharp contrast to Cantor's remarks on infinitesimals (see the subsection below on *mathematical rigor*).

In modern mathematics, the theory of ordered fields employs the following form of the Archimedean axiom (see, e.g., Hilbert 1899 [51, p. 27]):

$$(\forall x > 0)\ (\forall \epsilon > 0)\ (\exists n \in \mathbb{N})\ [n\epsilon > x]$$

or equivalently

(3) $$(\forall \epsilon > 0)\ (\exists n \in \mathbb{N})\ \ [n\epsilon > 1].$$

A number system satisfying (3) will be referred to as an *Archimedean continuum*. In the contrary case, there is an element $\epsilon > 0$ called an infinitesimal

---

[3] *See, e.g., the version of the Archimedean axiom in (Hilbert 1899* [51, p. 19]*). Note that we have avoided using* "0" *in formula* (1)*, as in* "$\forall a > 0$"*, since* 0 *was not part of the conceptual framework of the Greeks. The term "multiplied" in the English translation of Euclid's definition V.4 corresponds to the Greek term* $\pi o\lambda\lambda\alpha\pi\lambda\alpha\sigma\iota\alpha\zeta\acute{o}\mu\epsilon\nu\alpha$*. A common formalization of the noun "multiple",* $\pi o\lambda\lambda\alpha\pi\lambda\acute{\alpha}\sigma\iota o\nu$*, is* $na = a + \cdots + a$.

[4] *With the exception of Proposition III.16, where so-called* horn angles *appear that could be considered as non-Archimedean magnitudes relative to rectilinear angles.*

[5] *See (Ehrlich* [32]*) for additional historical details concerning Stolz's account of the Archimedean axiom.*

such that no finite sum $\epsilon + \epsilon + \cdots + \epsilon$ will ever reach 1; in other words,

$$(\exists \epsilon > 0)\ (\forall n \in \mathbb{N})\ \left[\epsilon \le \tfrac{1}{n}\right]. \tag{4}$$

A number system satisfying (4) is referred to as a *Bernoullian continuum* (i.e., a non-Archimedean continuum); see the subsection "Bernoulli, Johann".

Berkeley, George

George Berkeley (1685–1753) was a cleric whose *empiricist* (i.e., based on sensations, or *sensationalist*) metaphysics tolerated no conceptual innovations, such as infinitesimals, without an *empirical* counterpart or referent. Berkeley was similarly opposed, on metaphysical grounds, to infinite divisibility of the continuum (which he referred to as *extension*), an idea widely taken for granted today. In addition to his outdated *metaphysical criticism* of the infinitesimal calculus of Newton and Leibniz, Berkeley also formulated a *logical criticism*.[6] Berkeley claimed to have detected a logical fallacy at the basis of the method. In terms of Fermat's E occurring in his *adequality* (see the subsection "Adequality"), Berkeley's objection can be formulated as follows:

> The increment E is assumed to be nonzero at the beginning of the calculation, but zero at its conclusion, an apparent logical fallacy.

However, E is not *assumed to be zero* at the end of the calculation, but rather is *discarded* at the end of the calculation (see the subsection "Berkeley's Logical Criticism" for more details). Such a technique was the content of Fermat's *adequality* (see the subsection "Adequality") and Leibniz's *transcendental law of homogeneity* (see the subsection "Lex homogeneorum transcendentalis"), where the relation of equality has to be suitably interpreted (see the subsection "Relation $\sqcap$"). The technique is equivalent to taking the *limit* (of a typical expression such as $\frac{f(A+E)-f(A)}{E}$ for example) in Weierstrass's approach and to taking the *standard part* (see the subsection "Standard Part Principle") in Robinson's approach.

Meanwhile, Berkeley's own attempt to explain the calculation of the derivative of x^2 in *The Analyst* contains a logical circularity: namely, Berkeley's argument relies on the determination of the tangents of a parabola by Apollonius (which is equivalent to the calculation of the derivative). This circularity in Berkeley's argument was analyzed in (Andersen 2011 [1]).

[6] *Berkeley's criticism was dissected into its logical and metaphysical components in (Sherry 1987* [98]*).*

Berkeley's Logical Criticism

Berkeley's *logical criticism* of the calculus amounts to the contention that the evanescent increment is first assumed to be nonzero to set up an algebraic expression and then is *treated as zero* in *discarding* the terms that contained that increment when the increment is said to vanish. In modern terms, Berkeley was claiming that the calculus was based on an inconsistency of type

$$(dx \neq 0) \wedge (dx = 0).$$

The criticism, however, involves a misunderstanding of Leibniz's method. The rebuttal of Berkeley's criticism is that the evanescent increment need *not* be "treated as zero" but, rather, is merely *discarded* through an application of the *transcendental law of homogeneity* by Leibniz, as illustrated in the subsection "Product Rule" in the case of the product rule.

While consistent (in the sense of the subsection "Mathematical Rigor", level (2)), Leibniz's system unquestionably relied on *heuristic* principles, such as the laws of continuity and homogeneity, and thus fell short of a standard of rigor *if* measured by today's criteria (see the subsection "Mathematical Rigor"). On the other hand, the consistency and resilience of Leibniz's system is confirmed through the development of *modern implementations* of Leibniz's heuristic principles (see the subsection "Modern Implementations").

Bernoulli, Johann

Johann Bernoulli (1667–1748) was a disciple of Leibniz's who, having learned an infinitesimal methodology for the calculus from the master, never wavered from it. This is in contrast to Leibniz himself, who throughout his career used both

(A) an Archimedean methodology (proof by exhaustion) and
(B) an infinitesimal methodology

in a symbiotic fashion. Thus Leibniz relied on the A-methodology to underwrite and justify the B-methodology, and he exploited the B-methodology to shorten the path to discovery (*Ars Inveniendi*). Historians often name Bernoulli as the first mathematician to have adhered systematically to the infinitesimal approach as the basis for the calculus. We refer to an infinitesimal-enriched number system as a *B-continuum*, as opposed to an Archimedean *A-continuum*, i.e., a continuum satisfying the *Archimedean axiom* (see the subsection "Archimedean Axiom").

Bishop, Errett

Errett Bishop (1928–1983) was a mathematical constructivist who, unlike his fellow intuitionist[7] Arend Heyting (see the subsection "Heyting, Arend"), held a dim view of classical mathematics in general and Robinson's infinitesimals in particular. Discouraged by the apparent nonconstructivity of his early work in complex analysis, Bishop believed he had found the culprit in the law of the excluded middle (LEM), the key logical ingredient in every proof by contradiction. He spent the remaining eighteen years of his life in an effort to expunge the reliance on LEM (which he dubbed "the principle of omniscience" in [11]) from analysis and sought to define *meaning* itself in mathematics in terms of such LEM-extirpation.

Accordingly, he described classical mathematics as both a *debasement of meaning* (Bishop 1973 [13, p. 1]) and *sawdust* (Bishop 1973 [13, p. 14]), and he did not hesitate to speak of both *crisis* (Bishop 1975 [11]) and *schizophrenia* (Bishop 1973 [13]) in contemporary mathematics, predicting an imminent demise of classical mathematics in the following terms:

> Very possibly classical mathematics will cease to exist as an independent discipline. (Bishop 1968 [10, p. 54])

His attack in (Bishop 1977 [12]) on calculus pedagogy based on Robinson's infinitesimals was a natural outgrowth of his general opposition to the logical underpinnings of classical mathematics, as analyzed in (Katz and Katz 2011 [63]). Robinson formulated a brief but penetrating appraisal of Bishop's ventures into the history and philosophy of mathematics, when he noted that

> the sections of [Bishop's] book that attempt to describe the philosophical and historical background of [the] remarkable endeavor [of Intuitionism] are more vigorous than accurate and tend to belittle or ignore the efforts of others who have worked in the same general direction. (Robinson 1968 [95, p. 921])

See the subsection "Chimeras" for a related criticism by Alain Connes.

[7] *Bishop was not an intuitionist in the narrow sense of the term, in that he never worked with Brouwer's continuum or "free choice sequences". We are using the term "intuitionism" in a broader sense (i.e., mathematics based on intuitionistic logic) that incorporates constructivism, as used for example by Abraham Robinson in the comment quoted at the end of this subsection.*

Cantor, Georg

Georg Cantor (1845–1918) is familiar to the modern reader as the *underappreciated* creator of the "Cantorian paradise", out of which David Hilbert would not be expelled, as well as the tragic hero, allegedly *persecuted* by Kronecker, who ended his days in a lunatic asylum. Cantor historian J. Dauben notes, however, an underappreciated aspect of Cantor's scientific activity, namely, his principled *persecution* of infinitesimalists:

> Cantor devoted some of his most vituperative correspondence, as well as a portion of the *Beiträge*, to attacking what he described at one point as the 'infinitesimal Cholera bacillus of mathematics', which had spread from Germany through the work of Thomae, du Bois-Reymond, and Stolz, to infect Italian mathematics.…Any acceptance of infinitesimals necessarily meant that his own theory of number was incomplete. Thus to accept the work of Thomae, du Bois-Reymond, Stolz, and Veronese was to deny the perfection of Cantor's own creation. Understandably, Cantor launched a thorough campaign to discredit Veronese's work in every way possible. (Dauben 1980 [27, pp. 216–217])

A discussion of Cantor's flawed investigation of the *Archimedean axiom* (see the subsection "Archimedean Axiom") may be found in the subsection "Mathematical Rigor".[8]

Cauchy, Augustin-Louis

Augustin-Louis Cauchy (1789–1857) is often viewed in the history of mathematics literature as a precursor of Weierstrass. Note, however, that contrary to a common misconception, Cauchy never gave an ϵ, δ definition of either limit or continuity (see the subsection "Variable Quantity" for Cauchy's definition of limit). Rather, his approach to continuity was via what is known today as *microcontinuity* (see the subsection "Continuity"). Several recent articles, (Błaszczyk et al. [14]; Borovik and Katz [16]; Bråting [20]; Katz and Katz [62], [64]; Katz and Tall [69]), have argued that a proto-Weierstrassian view of Cauchy is one-sided and obscures Cauchy's important contributions, including not only his infinitesimal definition of continuity but also such innovations as his infinitesimally defined ("Dirac") delta function, with applications in Fourier analysis and evaluation of singular integrals, and his study of orders of growth of infinitesimals that anticipated the work of Paul du Bois-Reymond,

[8] *Cantor's dubious claim that the infinitesimal leads to contradictions was endorsed by no less an authority than B. Russell; see footnote 15 in the subsection "Mathematical Rigor".*

Borel, Hardy, and ultimately (Skolem [102], [103], [104]) Robinson.

To elaborate on Cauchy's "Dirac" delta function, note the following formula from (Cauchy 1827 [23, p. 188]) in terms of an infinitesimal α:

$$\frac{1}{2}\int_{a-\epsilon}^{a+\epsilon} F(\mu)\,\frac{\alpha\, d\mu}{\alpha^2+(\mu-a)^2} = \frac{\pi}{2}F(a). \tag{5}$$

Replacing Cauchy's expression $\frac{\alpha}{\alpha^2+(\mu-a)^2}$ by $\delta_a(\mu)$, one obtains Dirac's formula up to trivial modifications (see Dirac [30, p. 59]):

$$\int_{-\infty}^{\infty} f(x)\delta(x) = f(0).$$

Cauchy's 1853 paper on a notion closely related to uniform convergence was recently examined in (Katz and Katz 2011 [62]) and (Błaszczyk et al. 2012 [14]). Cauchy handles the said notion using infinitesimals, including one generated by the null sequence $(\frac{1}{n})$.

Chimeras

Alain Connes (1947–) formulated criticisms of Robinson's infinitesimals between the years 1995 and 2007 on at least seven separate occasions (see Kanovei et al. 2012 [57, Section 3.1, Table 1]). These range from pejorative epithets such as "inadequate," "disappointing," "chimera," and "irremediable defect," to "the end of the rope for being 'explicit'."

Connes sought to exploit the Solovay model S (Solovay 1970 [105]) as ammunition against nonstandard analysis, but the model tends to boomerang, undercutting Connes's own earlier work in functional analysis. Connes described the hyperreals as both a "virtual theory" and a "chimera", yet acknowledged that his argument relies on the transfer principle (see the subsection "Modern Implementations"). In S, all definable sets of reals are Lebesgue measurable, suggesting that Connes views a theory as being "virtual" if it is not *definable* in a suitable model of ZFC. If so, Connes's claim that a theory of the hyperreals is "virtual" is refuted by the existence of a definable model of the hyperreal field (Kanovei and Shelah [59]). Free ultrafilters aren't definable, yet Connes exploited such ultrafilters both in his own earlier work on the classification of factors in the 1970s and 80s and in his magnum opus *Noncommutative Geometry* (Connes 1994 [26, Ch. V, Sect. 6.δ, Def. 11]), raising the question whether the latter may not be vulnerable to Connes's criticism of virtuality. The article [57] analyzed the philosophical underpinnings of Connes's argument based on Gödel's incompleteness theorem and detected an apparent circularity in Connes's logic. The article [57] also documented the reliance on nonconstructive foundational material, and specifically on the Dixmier trace $-\!\!\!\!\!\!\int$ (featured on the front cover of Connes's *magnum opus*) and the Hahn–Banach theorem, in Connes's own framework; see also [65].

See the subsection "Bishop, Errett" for a related criticism by Errett Bishop.

Continuity to Indivisibles

Continuity

Of the two main definitions of continuity of a function, Definition A (see below) is operative in either a B-continuum or an A-continuum (satisfying the Archimedean axiom; see the subsection "Archimedean Axiom"), while Definition B works only in a B-continuum (i.e., an infinitesimal-enriched or Bernoullian continuum; see the subsection "Bernoulli, Johann").

- *Definition A* (ϵ, δ approach): A real function f is continuous at a real point x if and only if

$$(\forall \epsilon > 0)\ (\exists \delta > 0)\ (\forall x')\quad [\ |x - x'| < \delta \to |f(x) - f(x')| < \epsilon\ ].$$

- *Definition B* (microcontinuity): A real function f is continuous at a real point x if and only if

$$(\forall x')\ [\ x' \sqcap x\ \to\ f(x') \sqcap f(x)\ ]. \tag{6}$$

In formula (6) the natural extension of f is still denoted f, and the symbol "$\sqcap$" stands for the relation of being infinitely close; thus, $x' \sqcap x$ if and only if $x' - x$ is infinitesimal (see the subsection "Relation $\sqcap$").

Diophantus

Diophantus of Alexandria (who lived about 1,800 years ago) contributed indirectly to the development of infinitesimal calculus through the technique called $\pi\alpha\rho\iota\sigma\acute{o}\tau\eta\varsigma$, developed in his work *Arithmetica*, Book Five, problems 12, 14, and 17. The term $\pi\alpha\rho\iota\sigma\acute{o}\tau\eta\varsigma$ can be literally translated as "approximate equality". This was rendered as *adaequalitas* in Bachet's Latin translation [4] and *adégalité* in French (see the subsection "Adequality"). The term was used by Fermat to describe the comparison of values of an algebraic expression, or what would today be called a function f, at nearby points A and $A+E$ and to seek extrema by a technique closely related to the vanishing of $\frac{f(A+E)-f(A)}{E}$ after discarding the remaining E-terms; see (Katz, Schaps, and Shnider 2013 [66]).

Euclid's Definition V.4

Euclid's Definition V.4 has already been discussed in the subsection "Archimedean Axiom". In addition to Book V, it appears in Books X and XII and is used in the method of exhaustion (see Euclid [34, Propositions X.1, XII.2]). The method of exhaustion was exploited intensively by both Archimedes and Leibniz (see the subsection "Leibniz's *De Quadratura*" on Leibniz's work *De Quadratura*). It was revived in the nineteenth century in the theory of the Riemann integral.

Euclid's Book V sets the basis for the theory of similar figures developed in Book VI. Great mathematicians of the seventeenth century, such as Descartes, Leibniz, and Newton, exploited Euclid's theory of similar figures of Book VI while paying no attention to its axiomatic background.[9] Over time Euclid's Book V became a subject of interest for historians and editors alone.

To formalize Definition V.4, one needed a formula for Euclid's notion of "multiple" and an idea of total order. Some progress in this direction was made by Robert Simson in 1762.[10] In 1876 Hermann Hankel provided a modern reconstruction of Book V. Combining his own historical studies with an idea of order compatible with addition developed by Hermann Grassmann (1861 [44]), he gave a formula that to this day is accepted as a formalization of Euclid's definition of proportion in V.5 (Hankel 1876 [46, pp. 389–398]). Euclid's proportion is a relation among four "magnitudes", such as

$$A : B :: C : D.$$

It was interpreted by Hankel as the relation

$$\begin{aligned}(\forall m, n)\,\big[&(nA >_1 mB \to nC >_2 mD)\\ &\wedge (nA = mB \to nC = mD)\\ &\wedge (nA <_1 mB \to nC <_2 mD)\big],\end{aligned}$$

where n, m are natural numbers. The indices on the inequalities emphasize the fact that the "magnitudes" A, B have to be of "the same kind", e.g., line segments, whereas C, D could be of another kind, e.g., triangles.

In 1880 J. L. Heiberg, in his edition of Archimedes' *Opera omnia* in a comment on a lemma of Archimedes, cites Euclid's Definition V.4, noting that these two are the same axioms (Heiberg 1880 [49, p. 11]).[11] This is the reason why Euclid's Definition V.4 is commonly known as the Archimedean axiom. Today we formalize Euclid's Definition V.4 as in (1), while the Archimedean lemma is rendered by formula (2).

[9] *Leibniz and Newton apparently applied Euclid's conclusions in a context where the said conclusions did not, technically speaking, apply: namely, to infinitesimal figures such as the characteristic triangle, i.e., triangle with sides dx, dy, and ds.*

[10] *See Simson's axioms that supplement the definitions of Book V as elaborated in (Simson 1762* [101, p. 114–115]*).*

[11] *In point of fact, Euclid's axiom V.4 and Archimedes' lemma are not equivalent from the logical viewpoint. Thus, the additive semigroup of positive appreciable limited hyperreals satisfies V.4 but not Archimedes' lemma.*

Euler, Leonhard

Euler's *Introductio in Analysin Infinitorum* (1748 [35]) contains remarkable calculations carried out in an extended number system in which the basic algebraic operations are applied to infinitely small and infinitely large quantities. Thus, in Chapter 7, "Exponentials and Logarithms Expressed through Series", we find a derivation of the power series for a^z starting from the formula $a^\omega = 1 + k\omega$, for ω infinitely small, and then raising the equation to the infinitely great power[12] $j = \frac{z}{\omega}$ for a finite (appreciable) z to give

$$a^z = a^{j\omega} = (1 + k\omega)^j$$

and finally expanding the right-hand side as a power series by means of the binomial formula. In the chapters following, Euler finds infinite product expansions factoring the power series expansion for transcendental functions (see the subsection "Euler's Infinite Product Formula for Sine"). By Chapter 10 he has the tools to sum the series for $\zeta(2)$ where $\zeta(s) = \sum_n n^{-s}$. He explicitly calculates $\zeta(2k)$ for $k = 1, \ldots, 13$ as well as many other related infinite series.

In Chapter 3 of his *Institutiones Calculi Differentialis* (1755 [37]), Euler deals with the methodology of the calculus, such as the nature of infinitesimal and infinitely large quantities. We will cite the English translation [38] of the Latin original [37]. Here Euler writes that

> even if someone *denies* that infinite numbers really exist in this world, still in mathematical speculations there arise questions to which answers cannot be given unless we admit an infinite number. (ibid., §82) [emphasis added—the authors]

Euler's approach, countenancing the possibility of *denying* that "infinite numbers really exist," is consonant with a Leibnizian view of infinitesimal and infinite quantities as "useful fictions" (see Katz and Sherry [67]; Sherry and Katz [100]). Euler then notes that "an infinitely small quantity is nothing but a vanishing quantity, and so it is really equal to 0." (ibid., §83)

Similarly, Leibniz combined a view of infinitesimals as "useful fictions" and *inassignable quantities*, with a generalized notion of "equality" that was an equality up to an incomparably negligible term. Leibniz sought to codify this idea in terms of his *transcendental law of homogeneity* (TLH); see the subsection "Lex homogeneorum transcendentalis". Thus, Euler's formulas such as

$$a + dx = a \tag{7}$$

[12] *Euler used the symbol i for the infinite power. Blanton replaced this by j in the English translation so as to avoid a notational clash with the standard symbol for $\sqrt{-1}$.*

(where a "is any finite quantity" ibid., §§86, 87) are consonant with a Leibnizian tradition (cf. formula (17) in the subsection "Lex homogeneorum transcendentalis"). To explain formulas like (7), Euler elaborated two distinct ways (arithmetic and geometric) of comparing quantities in the following terms:

> Since we are going to show that an infinitely small quantity is really zero, we must meet the objection of why we do not always use the same symbol 0 for infinitely small quantities, rather than some special ones…[S]ince we have two ways to compare them, either *arithmetic* or *geometric*, let us look at the quotients of quantities to be compared in order to see the difference.
>
> If we accept the notation used in the analysis of the infinite, then dx indicates a quantity that is infinitely small, so that both $dx = 0$ and $a\,dx = 0$, where a is any finite quantity. Despite this, the *geometric* ratio $a\,dx : dx$ is finite, namely $a : 1$. For this reason, these two infinitely small quantities, dx and $a\,dx$, both being equal to 0, cannot be confused when we consider their ratio. In a similar way, we will deal with infinitely small quantities dx and dy. (ibid., §86, pp. 51–52) [emphasis added—the authors]

Euler proceeds to clarify the difference between the arithmetic and geometric comparisons as follows:

> Let a be a finite quantity and let dx be infinitely small. The arithmetic ratio of equals is clear: Since $ndx = 0$, we have
>
> $$a \pm ndx - a = 0.$$
>
> On the other hand, the geometric ratio is clearly of equals, since
>
> $$\frac{a \pm ndx}{a} = 1. \tag{8}$$
>
> From this we obtain the well-known rule that *the infinitely small vanishes in comparison with the finite and hence can be neglected.* (Euler 1755 [38, §87]) [emphasis in the original—the authors]

Like Leibniz, Euler considers more than one way of comparing quantities. Euler's formula (8) indicates that his geometric comparison is identical with the Leibnizian TLH; namely, Euler's geometric comparison of a pair of quantities amounts to their ratio being infinitely close to 1. The same is true for TLH. Thus one has $a + dx = a$ in this sense for an appreciable $a \neq 0$, but *not* $dx = 0$ (which is true only *arithmetically* in Euler's sense). Euler's "geometric" comparison was dubbed "the principle of cancellation" in (Ferraro 2004 [39, p. 47]).

Euler proceeds to present the usual rules of infinitesimal calculus, which go back to Leibniz, L'Hôpital, and the Bernoullis, such as

$$a\,dx^m + b\,dx^n = a\,dx^m \tag{9}$$

provided $m < n$ "since dx^n vanishes compared with dx^m" (ibid., §89), relying on his "geometric" equality. Euler introduces a distinction between infinitesimals of different order and directly *computes*[13] a ratio of the form

$$\frac{dx \pm dx^2}{dx} = 1 \pm dx = 1$$

of two particular infinitesimals, assigning the value 1 to it (ibid., §88). Euler concludes:

> Although all of them [infinitely small quantities] are equal to 0, still they must be carefully distinguished one from the other if we are to pay attention to their mutual relationships, which has been explained through a geometric ratio. (ibid., §89).

The Eulerian hierarchy of orders of infinitesimals harks back to Leibniz's work (see the subsection "Nieuwentijt, Bernard" for a historical dissenting view).

Euler's Infinite Product Formula for Sine

The fruitfulness of Euler's infinitesimal approach can be illustrated by some of the remarkable applications he obtained. Thus, Euler derived an infinite product decomposition for the sine and sinh functions of the following form:

$$\sinh x = x\left(1+\frac{x^2}{\pi^2}\right)\left(1+\frac{x^2}{4\pi^2}\right) \cdot \left(1+\frac{x^2}{9\pi^2}\right)\left(1+\frac{x^2}{16\pi^2}\right), \dots \tag{10}$$

$$\sin x = x\left(1-\frac{x^2}{\pi^2}\right)\left(1-\frac{x^2}{4\pi^2}\right) \cdot \left(1-\frac{x^2}{9\pi^2}\right)\left(1-\frac{x^2}{16\pi^2}\right)\dots \tag{11}$$

Decomposition (11) generalizes an infinite product formula for $\frac{\pi}{2}$ due to Wallis [109]. Euler also summed the inverse square series: $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \frac{\pi^2}{6}$ (see [86]) and obtained additional identities. A common feature of these formulas is that Euler's computations involve not only infinitesimals but also infinitely large natural numbers, which Euler sometimes treats as if they

[13] *Note that Euler does not "prove that the expression is equal to 1"; such* indirect *proofs are a trademark of the ϵ, δ approach. Rather, Euler* directly *computes (what would today be formalized as the standard part of) the expression, illustrating one of the advantages of the B-methodology over the A-methodology.*

were ordinary natural numbers.[14] Similarly, Euler treats infinite series as polynomials of a specific infinite degree.

The derivation of (10) and (11) in (Euler 1748 [35, §156]) can be broken up into seven steps as follows.

Step 1. Euler observes that

$$2\sinh x = e^x - e^{-x} = \left(1+\frac{x}{j}\right)^j - \left(1-\frac{x}{j}\right)^j, \tag{12}$$

where j (or "i" in Euler [35]) is an infinitely large natural number. To motivate the next step, note that the expression $x^j - 1 = (x-1)(1+x+x^2+\cdots+x^{j-1})$ can be factored further as $\prod_{k=0}^{j-1}(x-\zeta^k)$, where $\zeta = e^{2\pi i/j}$; conjugate factors can then be combined to yield a decomposition into real quadratic terms.

Step 2. Euler uses the fact that $a^j - b^j$ is the product of the factors

$$a^2 + b^2 - 2ab\cos\frac{2k\pi}{j}, \quad \text{where} \quad k \geq 1, \tag{13}$$

together with the factor $a-b$ and, if j is an even number, the factor $a+b$ as well.

Step 3. Setting $a = 1+\frac{x}{j}$ and $b = 1-\frac{x}{j}$ in (12), Euler transforms expression (13) into the form

$$2 + 2\frac{x^2}{j^2} - 2\left(1-\frac{x^2}{j^2}\right)\cos\frac{2k\pi}{j}. \tag{14}$$

Step 4. Euler then replaces (14) by the expression

$$\frac{4k^2\pi^2}{j^2}\left(1+\frac{x^2}{k^2\pi^2}-\frac{x^2}{j^2}\right), \tag{15}$$

justifying this step by means of the formula

$$\cos\frac{2k\pi}{j} = 1 - \frac{2k^2\pi^2}{j^2}. \tag{16}$$

Step 5. Next, Euler argues that the difference $e^x - e^{-x}$ is divisible by the expression

$$1 + \frac{x^2}{k^2\pi^2} - \frac{x^2}{j^2}$$

from (15), where "we omit the term $\frac{x^2}{j^2}$ since even when multiplied by j, it remains infinitely small." (English translation from [36].)

Step 6. As there is still a factor of $a - b = 2x/j$, Euler obtains the final equality (10), arguing that then "the resulting first term will be x" (in order to conform to the Maclaurin series for $\sinh x$).

Step 7. Finally, formula (11) is obtained from (10) by means of the substitution $x \mapsto ix$. □

We will discuss modern formalizations of Euler's argument in the next subsection.

[14] *Euler's procedure is therefore consonant with the Leibnizian* law of continuity *(see the subsection "*Lex continuitatis*"), though apparently Euler does not refer explicitly to the latter.*

Euler's Sine Factorization Formalized

Euler's argument in favor of (10) and (11) was formalized in terms of a "nonstandard" proof in (Luxemburg 1973 [82]). However, the formalization in [82] deviates from Euler's argument, beginning with Steps 3 and 4, and thus circumvents the more problematic Steps 5 and 6.

A proof in the framework of modern nonstandard analysis formalizing Euler's argument step-by-step throughout appeared in (Kanovei 1988 [56]); see also (McKinzie and Tuckey 1997 [86]) and (Kanovei and Reeken 2004 [58, §2.4a]). This formalization interprets problematic details of Euler's argument on the basis of general principles of modern nonstandard analysis, as well as general analytic facts that were known in Euler's time. Such principles and facts behind some early proofs in infinitesimal calculus are sometimes referred to as "hidden lemmas" in this context; see (Laugwitz [73], [74]), (McKinzie and Tuckey 1997 [86]). For instance, the "hidden lemma" behind Step 4 above is the fact that, for a fixed x, the terms of the Maclaurin expansion of $\cos x$ tend to 0 faster than a convergent geometric series, allowing one to infer that the effect of the transformation of Step 4 on the product of the factors (14) is infinitesimal. Some "hidden lemmas" of a different kind, related to basic principles of nonstandard analysis, are discussed in [86, pp. 43ff.].

What clearly stands out from Euler's argument is his explicit use of infinitesimal expressions such as (14) and (15), as well as the approximate formula (16), which holds "up to" an infinitesimal of higher order. Thus, Euler used infinitesimals *par excellence*, rather than merely ratios thereof, in a routine fashion in some of his best work.

Euler's use of infinite integers and their associated infinite products (such as the decomposition of the sine function) were interpreted in Robinson's framework in terms of hyperfinite sets. Thus Euler's product of j-infinitely many factors in (11) is interpreted as a hyperfinite product in [58, formula (9), p. 74]. A hyperfinite formalization of Euler's argument involving infinite integers and their associated products illustrates the successful remodeling of the *arguments* (and not merely the *results*) of classical infinitesimal mathematics, as discussed in the subsection "Mathematical Rigor".

Fermat, Pierre de

Pierre de Fermat (1601–1665) developed a pioneering technique known as *adequality* (see the subsection "Adequality") for finding tangents to curves and for solving problems of maxima and minima. Katz, Schaps, and Shnider (2013 [66]) analyze some of the main approaches in the literature to the method of adequality, as well as its source in the $\pi\alpha\rho\iota\sigma\acute{o}\tau\eta\varsigma$ of Diophantus (see the subsection "Diophantus"). At least some of the manifestations of adequality, such as Fermat's treatment of transcendental curves and Snell's law, amount to variational techniques exploiting a small (alternatively, infinitesimal) variation E. Fermat's treatment of geometric and physical applications suggests that an aspect of approximation is inherent in adequality, as well as an aspect of smallness on the part of E.

Fermat's use of the term *adequality* relied on Bachet's rendering of Diophantus. Diophantus coined the term *parisotes* for mathematical purposes. Bachet performed a semantic calque in passing from *par-isoō* to *ad-aequo*. A historically significant parallel is found in the similar role of, respectively, adequality and the *transcendental law of homogeneity* (see the subsection "Lex homogeneorum transcendentalis") in the work of, respectively, Fermat and Leibniz on the problems of maxima and minima.

Breger (1994 [21]) denies that the idea of "smallness" was relied upon by Fermat. However, a detailed analysis (see [66]) of Fermat's treatment of the cycloid reveals that Fermat did rely on issues of "smallness" in his treatment of the cycloid and reveals that Breger's interpretation thereof contains both mathematical errors and errors of textual analysis. Similarly, Fermat's proof of Snell's law, a variational principle, unmistakably relies on ideas of "smallness".

Cifoletti (1990 [25]) finds similarities between Fermat's adequality and some procedures used in smooth infinitesimal analysis of Lawvere and others. Meanwhile, J. Bell (2009 [9]) seeks the historical sources of Lawvere's infinitesimals mainly in Nieuwentijt (see the subsection "Nieuwentijt, Bernard").

Heyting, Arend

Arend Heyting (1898–1980) was a mathematical intuitionist whose lasting contribution was the formalization of the intuitionistic logic underpinning the Intuitionism of his teacher Brouwer. While Heyting never worked on any theory of infinitesimals, he had several opportunities to present an expert opinion on Robinson's theory. Thus, in 1961, Robinson made public his new idea of nonstandard models for analysis and "communicated this almost immediately to…Heyting" (see Dauben [28, p. 259]). Robinson's first paper on the subject was subsequently published in *Proceedings of the Netherlands Royal Academy of Sciences* [94]. Heyting praised nonstandard analysis as "a standard model of important mathematical research" (Heyting 1973 [50, p. 136]). Addressing Robinson, he declared:

> [Y]ou connected this extremely abstract part of model theory with a theory apparently so far apart as the elementary calculus. In doing so you threw new light on the history of the calculus by giving a clear sense to Leibniz's notion of infinitesimals. (ibid)

Intuitionist Heyting's admiration for the application of Robinson's infinitesimals to calculus pedagogy is in stark contrast with the views of his fellow constructivist E. Bishop (subsection "Bishop, Errett").

Indivisibles versus Infinitesimals

Commentators use the term *infinitesimal* to refer to a variety of conceptions of the infinitely small, but the variety is not always acknowledged. It is important to distinguish the infinitesimal methods of Archimedes and Cavalieri from those employed by Leibniz and his successors. To emphasize this distinction, we will say that tradition prior to Leibniz employed *indivisibles*. For example, in his heuristic proof that the area of a parabolic segment is 4/3 the area of the inscribed triangle with the same base and vertex, Archimedes imagines both figures to consist of perpendiculars of various heights erected on the base. The perpendiculars are indivisibles in the sense that they are limits of division and so one dimension less than the area. In the same sense, the indivisibles of which a line consists are points, and the indivisibles of which a solid consists are planes.

Leibniz's infinitesimals are not indivisibles, for they have the same dimension as the figures that compose them. Thus, he treats curves as comprising infinitesimal line intervals rather than indivisible points. The strategy of treating infinitesimals as dimensionally homogeneous with the objects they compose seems to have originated with Roberval or Torricelli, Cavalieri's student, and to have been explicitly arithmetized in (Wallis 1656 [109]).

Zeno's *paradox of extension* admits resolution in the framework of Leibnizian infinitesimals (see the subsection "Zeno's Paradox of Extension"). Furthermore, only with the dimensionality retained is it possible to make sense of the fundamental theorem of calculus, where one must think about the rate of change of the *area* under a curve, another reason why indivisibles had to be abandoned in favor of infinitesimals so as to enable the development of the calculus (see Ely 2012 [33]).

Leibniz to Nieuwentijt

Leibniz, Gottfried

Gottfried Wilhelm Leibniz (1646–1716), the co-inventor of infinitesimal calculus, is a key player in the parallel infinitesimal track referred to by Felix

Klein [72, p. 214] (see the section "The ABCs of the History of Infinitesimal Mathematics").

Leibniz's *law of continuity* (see the subsection "Lex continuitatis") and his *transcendental law of homogeneity* (which he had already discussed in his response to Nieuwentijt in 1695, as noted by M. Parmentier [79, p. 38], and later in greater detail in a 1710 article [78] cited in the seminal study of Leibnizian methodology by H. Bos [17]) form a basis for implementing the calculus in the context of a B-continuum.

Many historians of the calculus deny significant continuity between infinitesimal calculus of the seventeenth century and twentieth-century developments such as Robinson's theory (see further discussion in Katz and Sherry [67]). Robinson's hyperreals require the resources of modern logic; thus many commentators are comfortable denying a historical continuity. A notable exception is Robinson himself, whose identification with the Leibnizian tradition inspired Lakatos, Laugwitz, and others to consider the history of the infinitesimal in a more favorable light. Many historians have overestimated the force of Berkeley's criticisms (see the subsection "Berkeley, George") by underestimating the mathematical and philosophical resources available to Leibniz.

Leibniz's infinitesimals are fictions—not logical fictions, as (Ishiguro 1990 [54]) proposed, but rather pure fictions, like imaginaries, which are not eliminable by some syncategorematic paraphrase; see (Sherry and Katz 2012 [100]) and the subsection "Leibniz's *De Quadratura*" below.

In fact, Leibniz's defense of infinitesimals is more firmly grounded than Berkeley's criticism thereof. Moreover, Leibniz's system for differential calculus was free of logical fallacies (see the subsection "Berkeley's Logical Criticism"). This strengthens the conception of modern infinitesimals as a formalization of Leibniz's strategy of relating inassignable to assignable quantities by means of his *transcendental law of homogeneity* (see the subsection "Lex homogeneorum transcendentalis").

Leibniz's *De Quadratura*

In 1675 Leibniz wrote a treatise on his infinitesimal methods, *On the Arithmetical Quadrature of the Circle, the Ellipse, and the Hyperbola*, or *De Quadratura*, as it is widely known. However, the treatise appeared in print only in 1993 in a text edited by Knobloch (Leibniz [80]).

De Quadratura was interpreted by R. Arthur [3] and others as supporting the thesis that Leibniz's infinitesimals are mere fictions, eliminable by long-winded paraphrase. This so-called syncategorematic interpretation of Leibniz's calculus has gained a number of adherents. We believe this interpretation to be a mistake. In the first place, Leibniz wrote the treatise at a time when infinitesimals were despised by the French Academy, a society whose approval and acceptance he eagerly sought. More importantly, as (Jesseph 2013 [55]) has shown, *De Quadratura* depends on infinitesimal resources in order to construct an approximation to a given curvilinear area less than any previously specified error. This problem is reminiscent of the difficulty that led to infinitesimal methods in the first place. Archimedes' method of exhaustion required one to determine a value for the quadrature *in advance* of showing, by *reductio* argument, that any departure from that value entails a contradiction. Archimedes possessed a heuristic, indivisible method for finding such values, and the results were justified by exhaustion, but only after the fact. By the same token, the use of infinitesimals is "just" a shortcut only if it is entirely eliminable from quadratures, tangent constructions, etc. Jesseph's insight is that this is not the case.

Finally, the syncategorematic interpretation misrepresents a crucial aspect of Leibniz's mathematical philosophy. His conception of mathematical fiction includes imaginary numbers, and he often sought approbation for his infinitesimals by comparing them to imaginaries, which were largely uncontroversial. There is no suggestion by Leibniz that imaginaries are eliminable by long-winded paraphrase. Rather, he praises imaginaries for their capacity to achieve universal harmony by the greatest possible systematization, and this characteristic is more central to Leibniz's conception of infinitesimals than the idea that they are mere shorthand. Just as imaginary roots both unified and extended the method for solving cubics, likewise infinitesimals unified and extended the method for quadrature so that, e.g., quadratures of general parabolas and hyperbolas could be found by the same method used for quadratures of less difficult curves.

Lex continuitatis

A heuristic principle called *The law of continuity* (LC) was formulated by Leibniz and is a key to appreciating Leibniz's vision of infinitesimal calculus. The LC asserts that whatever succeeds in the finite succeeds also in the infinite. This form of the principle appeared in a letter to Varignon (Leibniz 1702 [77]). A more detailed form of LC in terms of the concept of *terminus* appeared in his text *Cum Prodiisset*:

> In any supposed continuous transition, ending in any terminus, it is permissible to institute a general reasoning, in which the final terminus may also be included. (Leibniz 1701 [76, p. 40])

$$\begin{Bmatrix} \textit{assignable} \\ \textit{quantities} \end{Bmatrix} \overset{\text{LC}}{\leadsto} \begin{Bmatrix} \textit{inassignable} \\ \textit{quantities} \end{Bmatrix} \overset{\text{TLH}}{\leadsto} \begin{Bmatrix} \textit{assignable} \\ \textit{quantities} \end{Bmatrix}$$

Figure 2. Leibniz's law of continuity (LC) takes one from assignable to inassignable quantities, while his transcendental law of homogeneity (TLH; the subsection "Lex homogeneorum transcendentalis") returns one to assignable quantities.

To elaborate, the LC postulates that whatever properties are satisfied by ordinary or assignable quantities should also be satisfied by inassignable quantities (see the subsection "Variable Quantity") such as infinitesimals (see Figure 2). Thus the trigonometric formula $\sin^2 x + \cos^2 x = 1$ should be satisfied for an inassignable (e.g., infinitesimal) input x as well. In the twentieth century this heuristic principle was formalized as the *transfer principle* (see the subsection "Modern Implementations") of Łoś–Robinson.

The significance of LC can be illustrated by the fact that a failure to take note of the law of continuity often led scholars astray. Thus, Nieuwentijt (see the subsection "Nieuwentijt, Bernard") was led into something of a dead end with his nilpotent infinitesimals (ruled out by LC) of the form $\frac{1}{\infty}$. J. Bell's view of Nieuwentijt's approach as a precursor of nilsquare infinitesimals of Lawvere (see Bell 2009 [9]) is plausible, though it could be noted that Lawvere's nilsquare infinitesimals cannot be of the form $\frac{1}{\infty}$.

Lex homogeneorum transcendentalis

Leibniz's *transcendental law of homogeneity*, or *lex homogeneorum transcendentalis* in the original Latin (Leibniz 1710 [78]), governs equations involving differentials. Leibniz historian H. Bos interprets it as follows:

> A quantity which is infinitely small with respect to another quantity can be neglected if compared with that quantity. Thus all terms in an equation except those of the highest order of infinity, or the lowest order of infinite smallness, can be discarded. For instance,
>
> (17) $$a + dx = a$$ $$dx + ddy = dx$$
>
> etc. The resulting equations satisfy this...requirement of homogeneity. (Bos 1974 [17, p. 33])

For an interpretation of the equality sign in the formulas above, see the subsection "Relation $\sqcap$".

The TLH associates to an inassignable quantity (such as $a + dx$) an assignable one (such as a); see Figure 2 for a relation between LC and TLH.

Mathematical Rigor

There is a certain lack of clarity in the historical literature with regard to issues of fruitfulness, consistency, and rigorousness of mathematical writing. As a rough guide and to be able to formulate useful distinctions when it comes to evaluating mathematical writing from centuries past, we would like to consider three levels of judging mathematical writing:

(1) potentially fruitful but (logically) inconsistent,
(2) (potentially) consistent but informal,
(3) formally consistent and fully rigorous according to currently prevailing standards.

As an example of level (1) we would cite the work of Nieuwentijt (see the subsection "Nieuwentijt, Bernard" for a discussion of the inconsistency). Our prime example of level (2) is provided by the Leibnizian *laws of continuity and homogeneity* (see subsections "Lex continuitatis" and "Lex homogeneorum transcendentalis"), which found rigorous implementation at level (3) only centuries later (see the subsection "Modern Implementations").

A foundational rock of the received history of mathematical analysis is the belief that mathematical rigor emerged starting in the 1870s through the efforts of Cantor, Dedekind, Weierstrass, and others, thereby replacing formerly unrigorous work of infinitesimalists from Leibniz onward. The philosophical underpinnings of such a belief were analyzed in (Katz and Katz 2012a [64]), where it was pointed out that in mathematics, as in other sciences, former errors are eliminated through a process of improved conceptual understanding, evolving over time, of the key issues involved in that science.

Thus no scientific development can be claimed to have attained perfect clarity or rigor merely on the grounds of having eliminated earlier errors. Moreover, no such claim for a single scientific development is made either by the practitioners or by the historians of the natural sciences. It was further pointed out in [64] that the term *mathematical rigor* itself is ambiguous, its meaning varying according to context. Four possible meanings for the term were proposed in [64]:

(1) it is a shibboleth that identifies the speaker as belonging to a clan of professional mathematicians;
(2) it represents the idea that, as a scientific field develops, its practitioners attain greater and more conceptual understanding of key issues and are less prone to error;

(3) it represents the idea that a search for greater correctness in analysis *inevitably* led Weierstrass specifically to epsilontics (i.e., the A-approach) in the 1870s;

(4) it refers to the establishment of what are perceived to be the ultimate foundations for mathematics by Cantor, eventually explicitly expressed in axiomatic form by Zermelo and Fraenkel.

Item (1) may be pursued by a fashionable academic in the social sciences but does not get to the bottom of the issue. Meanwhile, item (2) would be agreed upon by historians of the other sciences.

In this context it is interesting to compare the investigation of the Archimedean property as performed by the would-be rigorist Cantor, on the one hand, and the infinitesimalist Stolz on the other. Cantor sought to derive the Archimedean property as a consequence of those of a linear continuum. Cantor's work in this area was not only unrigorous but actually erroneous, whereas Stolz's work was fully rigorous and even visionary. Namely, Cantor's arguments "proving" the inconsistency of infinitesimals were based on an implicit assumption of what is known today as the Kerry-Cantor axiom (see Proietti 2008 [92]). Meanwhile, Stolz was the first modern mathematician to realize the importance of the *Archimedean axiom* (see the subsection "Archimedean Axiom") as a separate axiom in its own right (see Ehrlich 2006 [32]) and, moreover, to develop some non-Archimedean systems (Stolz 1885 [106]).

In his *Grundlagen der Geometrie* (Hilbert 1899 [51]), Hilbert did not develop a new geometry, but rather remodeled Euclid's geometry. More specifically, Hilbert brought rigor into Euclid's geometry in the sense of formalizing both Euclid's *propositions* and Euclid's style of *procedures and style of reasoning*.

Note that Hilbert's system works for geometries built over a non-Archimedean field, as Hilbert was fully aware. Hilbert (1900 [52, p. 207]) cites Dehn's counterexamples to Legendre's theorem in the absence of the Archimedean axiom. Dehn planes built over a non-Archimedean field were used to prove certain cases of the independence of Hilbert's axioms (see Cerroni 2007 [24]).[15]

Robinson's theory similarly formalized seventeenth- and eighteenth-century analysis by remodeling both its *propositions* and its *procedures and reasoning*. Using Weierstrassian ϵ, δ techniques, one can recover only the *propositions* but not the proof procedures. Thus, Euler's *result* giving an infinite product formula for sine (see the subsection "Euler's Infinite Product Formula for Sine") admits numerous proofs in a Weierstrassian context, but Robinson's framework provides a suitable context in which Euler's *proof*, relying on infinite integers, can also be recovered. This is the crux of the historical debate concerning ϵ, δ versus infinitesimals. In short, Robinson did for Leibniz what Hilbert did for Euclid. Meanwhile, epsilontists failed to do for Leibniz what Robinson did for Leibniz, namely, formalizing the procedures and reasoning of the historical infinitesimal calculus. This theme is pursued further in terms of the internal/external distinction in the subsection "Variable Quantity".

Modern Implementations

In the 1940s Hewitt [48] developed a modern implementation of an infinitesimal-enriched continuum extending $\mathbb{R}$ by means of a technique referred to today as the ultrapower construction. We will denote such an infinitesimal-enriched continuum by the new symbol $\mathbb{I\!R}$ ("thick-R").[16] In the next decade, Łoś (Łoś 1955 [81]) proved his celebrated theorem on ultraproducts, implying in particular that elementary (more generally, first-order) statements over $\mathbb{R}$ are true if and only if they are true over $\mathbb{I\!R}$, yielding a modern implementation of the Leibnizian *law of continuity* (see the subsection "Lex continuitatis"). Such a result is equivalent to what is known in the literature as the *transfer principle*; see Keisler [70]. Every finite element of $\mathbb{I\!R}$ is infinitely close to a unique real number; see the subsection "Standard Part Principle". Such a principle is a mathematical implementation of Fermat's *adequality* (see the subsection "Adequality"), of Leibniz's *transcendental law of homogeneity* (see the subsection "Lex homogeneorum transcendentalis"), and of Euler's *principle of cancellation* (see the discussion between formulas (7) and (9) in the subsection "Euler, Leonhard").

Nieuwentijt, Bernard

In Nieuwentijt's *Analysis Infinitorum* (1695), the Dutch philosopher (1654–1718)[17] proposed a system containing an infinite number, as well as infinitesimal quantities formed by dividing finite numbers by this infinite one. Nieuwentijt postulated that the product of two infinitesimals should be exactly equal to zero. In particular, an infinitesimal quantity is nilpotent. In an exchange of

[15] *It is a melancholy comment to note that fully three years later the philosopher-mathematician Bertrand Russell was still claiming, on Cantor's authority, that the infinitesimal "leads to contradictions" (Russell 2003* [97, p. 345]*). This set the stage for several decades of anti-infinitesimal vitriol, including the* saline solution *of Parkhurst and Kingsland (see the section "The ABCs of the History of Infinitesimal Mathematics").*

[16] *A more traditional symbol is* $^*\mathbb{R}$ *or* $\mathbb{R}^*$.

[17] *Alternative spellings are Nieuwentijdt or Nieuwentyt.*

publications with Nieuwentijt on infinitesimals (see Mancosu 1996 [84, p. 161]), Leibniz and Hermann claimed that this system is consistent only if all infinitesimals are equal, rendering differential calculus useless. Leibniz instead advocated a system in which the product of two infinitesimals is incomparably smaller than either infinitesimal. Nieuwentijt's objections compelled Leibniz in 1696 to elaborate on the hierarchy of infinite and infinitesimal numbers entailed in a robust infinitesimal system.

Nieuwentijt's nilpotent infinitesimals of the form $\frac{1}{\infty}$ are ruled out by Leibniz's *law of continuity* (see the subsection "Lex continuitatis"). J. Bell's view of Nieuwentijt's approach as a precursor of nilsquare infinitesimals of Lawvere (see Bell 2009 [9]) is plausible, though it could be noted that Lawvere's nilsquare infinitesimals cannot be of the form $\frac{1}{\infty}$.

Product Rule to Zeno

Product Rule

In the area of Leibniz scholarship, the received view is that Leibniz's infinitesimal system was logically faulty and contained internal contradictions allegedly exposed by the cleric George Berkeley (see the subsection "Berkeley, George"). Such a view is fully compatible with the A-track-dominated outlook, bestowing supremacy upon the reconstruction of analysis accomplished through the efforts of Cantor, Dedekind, Weierstrass, and their rigorous followers (see the subsection "Mathematical Rigor"). Does such a view represent an accurate appraisal of Leibniz's system?

The articles (Katz and Sherry 2012 [67], [68]; Sherry and Katz [100]), building on the earlier work (Sherry 1987 [98]), argued that Leibniz's system was in fact consistent (in the sense of level (2) of the subsection "Mathematical Rigor")[18] and featured resilient heuristic principles such as the *law of continuity* (see the subsection "Lex continuitatis") and the *transcendental law of homogeneity* (TLH) (see the subsection "Lex homogeneorum transcendentalis"), which were implemented in the fullness of time as precise mathematical principles guiding the behavior of modern infinitesimals.

How did Leibniz exploit the TLH in developing the calculus? We will now illustrate an application of the TLH in the particular example of the derivation

Figure 3. Thick-to-thin: applying the law of homogeneity or taking standard part (the thickness of the top line is merely conventional).

of the product rule. The issue is the justification of the last step in the following calculation:

$$\begin{aligned} d(uv) &= (u+du)(v+dv) - uv \\ &= udv + vdu + du\,dv \qquad (18) \\ &= udv + vdu. \end{aligned}$$

The last step in the calculation (18), namely,

$$udv + vdu + du\,dv = udv + vdu,$$

is an application of the TLH.[19]

In his 1701 text *Cum Prodiisset* [76, pp. 46–47], Leibniz presents an alternative justification of the product rule (see Bos [17, p. 58]). Here he divides by dx and argues with differential quotients rather than differentials. Adjusting Leibniz's notation to fit with (18), we obtain an equivalent calculation:[20]

$$\begin{aligned} \frac{d(uv)}{dx} &= \frac{(u+du)(v+dv) - uv}{dx} \\ &= \frac{udv + vdu + du\,dv}{dx} \\ &= \frac{udv + vdu}{dx} + \frac{du\,dv}{dx} \\ &= \frac{udv + vdu}{dx}. \end{aligned}$$

Under suitable conditions the term $\left(\frac{du\,dv}{dx}\right)$ is infinitesimal, and therefore the last step

$$\frac{udv + vdu}{dx} + \frac{du\,dv}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} \qquad (19)$$

is legitimized as a special case of the TLH. The TLH interprets the equality sign in (19) and (17) as the relation of being infinitely close, i.e., an equality up to infinitesimal error.

Relation $\sqcap$

Leibniz did not use our equality symbol but rather the symbol "$\sqcap$" (see McClenon 1923 [85, p. 371]). Using such a symbol to denote the relation of being infinitely close, one could write the calculation

[18]*Concerning the status of Leibniz's system for differential calculus, it may be more accurate to assert that it was* not inconsistent*, in the sense that the contradictions alleged by Berkeley and others turn out not to have been there in the first place once one takes into account Leibniz's generalized notion of equality and his transcendental law of homogeneity.*

[19]*Leibniz had two laws of homogeneity: one for dimension and the other for the order of infinitesimalness. Bos notes that they "disappeared from later developments"* [17, p. 35]*, referring to Euler and Lagrange. Note, however, the similarity to Euler's* principle of cancellation *(see Bair et al.* [5]*).*

[20]*The special case treated by Leibniz is* $u(x) = x$*. This limitation does not affect the conceptual structure of the argument.*

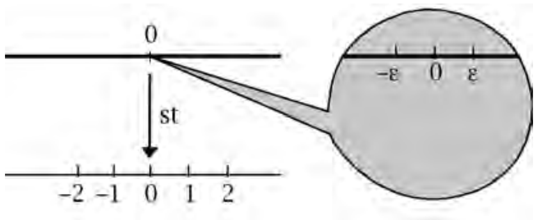

Figure 4. Zooming in on infinitesimal ε (here $\mathrm{st}(\pm\varepsilon) = 0$). The standard part function associates to every finite hyperreal the unique real number infinitely close to it. The bottom line represents the "thin" real continuum. The line at top represents the "thick" hyperreal continuum. The "infinitesimal microscope" is used to view an infinitesimal neighborhood of 0. The derivative $f'(x)$ of $f(x)$ is then defined by the relation $f'(x) \sqcap \frac{f(x+\varepsilon)-f(x)}{\varepsilon}$.

of the derivative of $y = f(x)$ where $f(x) = x^2$ as follows:

$$\begin{aligned} f'(x) &\sqcap \frac{dy}{dx} \\ &= \frac{(x+dx)^2 - x^2}{dx} \\ &= \frac{(x+dx+x)(x+dx-x)}{dx} \\ &= 2x + dx \\ &\sqcap 2x. \end{aligned}$$

Such a relation is formalized by the *standard part function*; see the subsection "Standard Part Principle" and Figure 3.

Standard Part Principle

In any totally ordered field extension E of $\mathbb{R}$, every finite element $x \in E$ is infinitely close to a suitable unique element $x_0 \in \mathbb{R}$. Indeed, via the total order, the element x defines a Dedekind cut on $\mathbb{R}$, and the cut specifies a real number $x_0 \in \mathbb{R} \subset E$. The number x_0 is infinitely close to $x \in E$. The subring $E_f \subset E$ consisting of the finite elements of E therefore admits a map

$$\mathrm{st}: E_f \to \mathbb{R},\ x \mapsto x_0,$$

called the *standard part function*.

The standard part function is illustrated in Figure 3. A more detailed graphic representation may be found in Figure 4.[21]

The key remark, due to Robinson, is that the limit in the A-approach and the standard part function in the B-approach are essentially equivalent tools. More specifically, the limit of a sequence (u_n) can be expressed, in the context of a hyperreal enlargement of the number system, as the standard part of the value u_H of the natural extension of the sequence at an infinite hypernatural index $n = H$. Thus,

$$\lim_{n\to\infty} u_n = \mathrm{st}(u_H). \tag{20}$$

Here the standard part function "st" associates to each finite hyperreal the unique finite real infinitely close to it (i.e., the difference between them is infinitesimal). This formalizes the natural intuition that for "very large" values of the index, the terms in the sequence are "very close" to the limit value of the sequence. Conversely, the standard part of a hyperreal $u = [u_n]$ represented in the ultrapower construction by a Cauchy sequence (u_n) is simply the limit of that sequence:

$$\mathrm{st}(u) = \lim_{n\to\infty} u_n. \tag{21}$$

Formulas (20) and (21) express limit and standard part in terms of each other. In this sense, the procedures of taking the limit and taking the standard part are logically equivalent.

Variable Quantity

The mathematical term *μέγεθος* in ancient Greek has been translated into Latin as *quantitas*. In modern languages it has two competing counterparts: in English, *quantity*, *magnitude*;[22] in French, *quantité*, *grandeur*; in German, *Quantität*, *Grösse*. The term *grandeur* with the meaning *real number* is still in use in (Bourbaki 1947 [19]). *Variable quantity* was a primitive notion in analysis as presented by Leibniz, l'Hôpital, and later Carnot and Cauchy. Other key notions of analysis were defined in terms of variable quantities. Thus, in Cauchy's terminology a variable quantity *becomes* an *infinitesimal* if it eventually drops below any *given* (i.e., constant) quantity (see Borovik and Katz [16] for a fuller discussion). Cauchy notes that the *limit* of such a quantity is zero. The notion of *limit* itself is defined as follows:

> Lorsque les valeurs successivement attribuées à une même variable s'approchent indéfiniment d'une valeur fixe, de manière à finir par en différer aussi peu que l'on voudra, cette dernière est appelée la limite de toutes les autres. (Cauchy, *Cours d'Analyse* [22])

Thus, Cauchy defined both infinitesimals and limits in terms of the primitive notion of a variable quantity. In Cauchy, any variable quantity q that does not tend to infinity is expected to

[21] *For a recent study of optical diagrams in nonstandard analysis, see (Dossena and Magnani* [31], [83]*) and (Bair and Henry* [8]*).*

[22] *The term "magnitude" is etymologically related to μέγεθος. Thus, μέγεθος in Greek and* magnitudo *in Latin both mean "bigness", "big" being* mega *(μέγα) in Greek and* magnum *in Latin.*

decompose as the sum of a given quantity c and an infinitesimal α:

$$q = c + \alpha. \tag{22}$$

In his 1821 text [22], Cauchy worked with a hierarchy of infinitesimals defined by polynomials in a base infinitesimal α. Each such infinitesimal decomposes as

$$\alpha^n (c + \varepsilon) \tag{23}$$

for a suitable integer n and infinitesimal ε. Cauchy's expression (23) can be viewed as a generalization of (22).

In Leibniz's terminology, c is an *assignable* quantity, while α and ε are *inassignable*. Leibniz's *transcendental law of homogeneity* (see the subsection "Lex homogeneorum transcendentalis") authorized the replacement of the inassignable $q = c + \alpha$ by the assignable c, since α is negligible compared to c:

$$q \sqcap c \tag{24}$$

(see the subsection "Relation $\sqcap$"). Leibniz emphasized that he worked with a generalized notion of equality where expressions were declared "equal" if they differed by a negligible term. Leibniz's procedure was formalized in Robinson's B-approach by the *standard part function* (see the subsection "Standard Part Principle"), which assigns to each finite hyperreal number the unique real number to which it is infinitely close. As such, the standard part allows one to work "internally" (not in the technical NSA sense but) in the sense of exploiting concepts already available in the toolkit of the historical infinitesimal calculus, such as Fermat's *adequality* (see the subsection "Adequality"), Leibniz's *transcendental law of homogeneity* (see the subsection "Lex homogeneorum transcendentalis"), and Euler's *principle of cancellation* (see Bair et al. [5]). Meanwhile, in the A-approach as formalized by Weierstrass, one is forced to work with "external" concepts such as the multiple-quantifier ϵ, δ *definitions* (see the subsection "Continuity") which have no counterpart in the historical infinitesimal calculus of Leibniz and Cauchy.

Thus, the notions of *standard part* and *epsilontic limit*, while logically equivalent (see the subsection "Standard Part Principle"), have the following difference between them: the standard part principle corresponds to an "internal" development of the historical infinitesimal calculus, whereas the epsilontic limit is "external" to it.

Zeno's Paradox of Extension

Zeno of Elea (who lived about 2,500 years ago) raised a puzzle (the *paradox of extension*, which is distinct from his better-known paradoxes of motion) in connection with treating any continuous magnitude as though it consists of infinitely many indivisibles; see (Sherry 1988 [99]), (Kirk et al. 1983 [71]). If the indivisibles have no magnitude, then an *extension* (such as space or time) composed of them has no magnitude; but if the indivisibles have some (finite) magnitude, then an extension composed of them will be infinite. There is a further puzzle: If a magnitude is composed of indivisibles, then we ought to be able to add or concatenate them in order to produce or increase a magnitude. But indivisibles are not next to one another: as limits or boundaries, any pair of indivisibles is separated by what they limit. Thus, the concept of addition or concatenation seems not to apply to indivisibles.

The paradox need not apply to infinitesimals in Leibniz's sense however (see the subsection "Indivisibles versus Infinitesimals") for, having neither zero nor finite magnitude, infinitely many of them may be just what is needed to produce a finite magnitude. And in any case, the addition or concatenation of infinitesimals (of the same dimension) is no more difficult to conceive of than adding or concatenating finite magnitudes. This is especially important, because it allows one to apply arithmetic operations to infinitesimals (see the subsection "Lex continuitatis" on the *law of continuity*). See also (Reeder 2012 [93]).

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Getting Evidence-Based Teaching Practices into Mathematics Departments: Blueprint or Fantasy?

Robert Reys

How do you teach mathematics in your college classes? Are undergraduate classes taught the same as graduate classes? Do you teach mathematics as you were taught? Are your instructional practices aligned with what research says about how students learn mathematics? Do you use evidence-based instructional practices? That is, does evidence exist that your instructional practices help students learn the mathematics you are teaching? How often do you exchange ideas about teaching mathematics with colleagues? The purpose of this paper is to encourage discussion among college faculty about effective instructional practices with the intent of improving the teaching and student learning of mathematics.

The Need to Improve Mathematics Teaching

Since the days of E. H. Moore and his 1902 presidential address to the AMS there have been many calls for improved teaching of mathematics [1]. More recently David Bressoud in the *MAA Launchings* [2] shares some ways that physicists have worked to utilize evidence-based teaching methods in their classes. Bressoud also provides resources that represent multiple perspectives from mathematicians engaged in the scholarship of teaching. Yet, despite the desire to promote better mathematics learning in STEM courses and the growing body of evidence-based research that has implications for teaching practices, changing collegiate instructional practices has been slow and does not happen easily [3].

In September 2011 the Association of American Universities announced a five-year initiative to improve the quality of undergraduate teaching and learning in science, technology, engineering, and mathematics (STEM) at its member institutions. In February 2012 the President's Council of Advisors on Science and Technology (PCAST) issued a report [4] that forecast the need for producing "1 million more college graduates in STEM fields" during the next decade. Among other things, the report noted the important role that college mathematics courses play in either opening or closing the doors to different STEM fields. To address this issue, the PCAST report called for more research into the best ways to teach and learn mathematics and urged "widespread adoption of empirically validated teaching practices" by STEM faculty in higher education. It also recommended the launch of "a national experiment in postsecondary mathematics education to address the mathematics-preparation gap."

In June 2012 the National Science Foundation issued a Dear Colleague Letter for Widening Implementation and Demonstration of Evidence-based Reforms (WIDER) calling for proposals. The program is intended to promote improvement in undergraduate STEM instructional practices and bring to scale successful instructional practices within and across departments. This is just one of the initiatives designed to stimulate greater knowledge of and widespread use of evidence-based instructional practices. This initiative provides mathematics departments an opportunity to focus on teaching practices. The visual image of most collegiate-level mathematics courses (regardless

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of its accuracy) is a professor writing on a black/whiteboard with his/her back to the class while students are copying feverishly and others have their hand raised but their questions going unanswered [5]. There are many factors that influence the instructional methods faculty members employ in their teaching practices. Probably the single most influential factor impacting how teachers teach is their prior experience as a student [6]. Stigler and Hiebert claim that breaking away from the cultural model of mathematics teaching that is so prevalent in the United States, typically lecture and note taking, is a major challenge. In the research related to evidence-based teaching in STEM undergraduate courses, it is worth noting that the focus has been almost exclusively on science-related courses [7], [8]. Despite the work of R. L. Moore and the legacy of the Moore method, it can be argued that inquiry-based learning is much more prevalent in science courses than in mathematics courses at the undergraduate level [9], [10], [11]. Perhaps opportunities to engage in laboratory environments provides experiences that engage learners and promotes more inquiry-based practices in science and engineering, whereas mathematics is too often viewed as a spectator sport; i.e., show me what to do or tell me how to do it, and I will memorize and practice the procedures. Even though there have been many calls by professional organizations to make the learning of mathematics a sense-making activity (National Council of Teachers of Mathematics, Mathematical Association of America, American Mathematical Society), for many college students mathematics learning is viewed as assuming a passive rather than an active role in learning.

Challenges Related to Changing Instructional Approaches

What are some of the hurdles to be cleared in promoting evidence-based change in teaching practices in collegiate mathematics classes? Here are a few broad challenges:

- *We tend to teach as we have been taught.* If lecture is the primary model experienced for learning mathematics, as Stigler and Hiebert suggest, it is difficult to break this cycle. Since professors teaching mathematics courses were successful in learning mathematics via a lecture method, it is difficult for them to relate to learning difficulties of their students. A frequently voiced sentiment is, "if it ain't broke, don't fix it." However, other people, particularly those outside mathematics, who cite the high rate of attrition in college mathematics courses argue that "actually it is broke" and there are some fundamental problems regarding the learning and teaching of mathematics that need to be addressed.

- *Doctoral programs in mathematics are heavily weighted toward research, with limited systematic attention to teaching.* In order to do original research in mathematics, one needs to delve deeply into a topic within a discipline. Completion of a Ph.D. in mathematics requires extensive research that focuses on mathematics, and thus few doctoral programs in mathematics have specific requirements regarding competence in teaching.[1] The goal of covering many chapters each semester still exists, and fixed pacing requirements often make it difficult for instructors to break away from lectures and engage their students in open discussions about the mathematical concepts being examined. Furthermore, heavy emphasis on mathematics research leaves little time for TAs to focus on evidenced-based instructional practices. In fact, Project NExT[2] was established in part to help provide some support for further development of teaching skills by new faculty members in mathematics departments.

- *The climate and environment of the department/college/institution provides a powerful context to either support or discourage emphasis on good teaching.* Some research suggests this is the single most important factor inhibiting change in teaching practices [12], [13]. For example, does scheduling of large classes with several hundred students limit the use of evidence-based instructional options? Are specific service courses, such as business calculus or linear algebra, viewed as service courses? If so, it may discourage a focus on inquiry when a course is supposed to help students learn specific skills. Does the chair/dean/chancellor encourage and value good teaching? Are their values made clear? Are there support systems for helping faculty learn about evidence-based research related to effective teaching practices? Is there motivation/incentive to engage in learning about different instructional models?

- *The reward system in comprehensive research-oriented institutions favors faculty members who gain national visibility via their research and scholarship rather than based on the quality of their teaching.* An average or below-average teacher may be promoted because of a sterling record of scholarship. However, it is rare (virtually

[1]*Nearly fifty years ago while I was a doctoral student, I taught college algebra and calculus for two years as a teaching assistant in a mathematics department at a research university. During that time the extent of instructional support was limited to giving the TAs a book and specific expectations about what chapters to cover. We did not know what classes we were assigned to teach until the night before classes started! Thank goodness progress has been made and support for TAs has improved in mathematics departments since I was a TA.*

[2]*Project NExT (New Experiences in Teaching) is a program founded over twenty years ago and is sponsored by the Mathematical Association of America. For more information see* `http://archives.math.utk.edu/projnext/`.

impossible) for a faculty member who is a great teacher but has done little research in mathematics to be promoted in a research-oriented institution. Thus, the argument offered by Earnest Boyer [14] more than twenty years ago for rewarding the scholarship of teaching has not made much headway.

• *Faculty members face pressures throughout their careers but may react differently depending on the stage of their career.* While publication pressure is usually the heaviest early in their careers, once that hurdle is cleared faculty members may focus attention elsewhere, including teaching [15]. So while posttenured faculty members may be interested in teaching, their instructional approaches may have become so entrenched that significant change is very difficult [16]. On the other hand, some senior faculty members choose to give more attention to improving teaching practices. This has been sparked in some cases by calls for collaboration between higher-education faculty and K–12 school systems (e.g., NSF Math Science Partnerships).

• *Institutions of higher education have created new types of faculty appointments to address teaching.* This may be viewed as a blessing or a curse. New faculty tracks have been created for specialized roles (e.g., adjunct, clinical, visiting, fixed-term, nonregular, and postdocs). Many of these appointments are designed to address the teaching needs of institutions, so it seems reasonable that these people would be highly motivated to learn about and use evidence-based teaching practices [17], [7]. However, hiring "others" to focus on teaching, may in fact decrease interest by faculty in tenure-track positions to explore ways to improve their teaching.

In an effort to gain information about what some institutions know about the teaching practices employed in their mathematics classes, I contacted a department chair and a provost at two research-oriented institutions. I know both of these people well. I felt I could be honest with them and that they would be honest in their responses.

Questions Asked and What Was Learned from Two Institutions

Here is how I situated the discussion: I am trying to learn about the instructional practices being used in undergraduate mathematics classes at your universities. I want to identify issues and challenges that would most likely be encountered in gathering information about instructional practices used by your mathematics faculty members teaching undergraduate courses. I then asked:

• Do you regularly collect information about instructional practices used in teaching your undergraduate mathematics courses at your institution? (This would include teachers who are graduate students as well as adjunct and regular faculty members.)

Neither institution collected any data on teaching practices. The provost said their institution collects information to support a state mandate to provide an "Institution Effectiveness Plan", but this amounted to collecting course syllabi that highlight content. While some of these syllabi provide clues about instructional practices, there is no specific requirement or expectation that instructional practices be addressed. While each institution gathers course evaluations from students, they have no current structure that systematically collects information about teaching practices nor do they have any mechanism in place that communicates information about best or effective teaching practices to their faculty members.

More specifically, the mathematics department chair said: "No, we currently do not have such information. The department is initiating ways to collect such information in a more formal manner than 'coffee room' chatter about what somebody did in class. We are planning to have a department 'retreat' (a couple of hours) this year where interested faculty will share their encouraging teaching practices, and this is actually a pretty big step for us."

• What do you think would be the most significant barriers in getting faculty members to participate in efforts to learn about evidence-based instructional practices?

Each of the administrators mentioned lack of time and uncertainty of the perceived value/importance of the activity. The mathematics department chair went on to say:

"Many mathematicians are so 'involved in their research' that their teaching is something they do; there is a belief that teaching is inherently good (they teach the way they were taught and that IS GOOD, by definition)—the students are ill-prepared and that causes the problem. The department culture about teaching is the most significant barrier as I see it. Promotion requires publications/grants, etc.; teaching is regarded as good unless there is some major issue that surfaces…good teaching practices rarely have an important role in the process. So, the culture has to change; it must respect more than traditional thinking about research contributions.

"I have also found that many research mathematicians have really good ideas about teaching and learning—and they practice these regularly, but they really don't share them with colleagues in part because they don't have an opportunity to do so. As the culture changes and instruction is understood to become part of the equation for advancement, faculty will want to be better teachers and share what they do."

• Are there incentives to systematically collect information about teaching practices used by mathematics faculty members?

The provost suggested an appeal to professional responsibility. She also addressed the importance of selling this need to top-level administrators and convincing them how this information would be useful to them in making policy decisions. She said the data collection would need to include all STEM disciplines and not be limited to a single department. Furthermore, any such survey would need to be free of implied or stated value judgments regarding particular instructional practices. The need to avoid any bias with regard to teaching practices was reiterated by the mathematics department chair. In addition, the mathematics chair said:

"In reality, there are probably not any incentives. The culture has to change. It is rare that a person will get a good raise because they were a good teacher. We give a lot of lip service to good teaching, but there is no concerted effort to encourage or reward it in the department."

This quote prompted a reviewer of this paper to say, "We need a culture that encourages faculty to think about the effectiveness of their teaching, to share their personal insights, and to provide support that enables faculty to adopt easily implementable ideas and helps faculty monitor the effectiveness of what they are doing." I say, Amen! Although, I don't think we should limit ourselves to "easily implementable ideas", as some of the evidence-based practices may not be easy to implement.

Where to from Here?

This discussion provides a limited glimpse about mathematics teaching, including perspectives from administrators from a Carnegie doctoral/research university and the other from a research university (high research activity). Yet these perspectives represent only two of more than 3,000 four-year institutions of higher education and none of the nearly 2,000 two-year institutions. This discussion suggests that structuring a framework that will provide accurate profiles of the current teaching practices in mathematics courses in various institutions is a big challenge. Using that information to inform faculty members in institutions of higher education about evidence-based teaching practices and helping to stimulate systematic change in teaching practices are likely to pose a far more demanding challenge.

The stakes of not doing everything possible to improve mathematics teaching and learning are high. Millions of students are taking courses in mathematics at institutions of higher education this year. Whether the courses are remedial, satisfying a general education requirement, calculus or beyond, there is increasing interest and pressure to make learning of mathematics meaningful and to help students make sense of whatever mathematics is being learned. While students have a responsibility to seek understanding of the mathematics, faculty members have a responsibility to utilize evidence-based teaching practices to help facilitate mathematics learning.

So what might a mathematics department do? One valuable first step would be to have an open and frank discussion about teaching among members of the department. This might include: addressing some of the opening questions, sharing teaching approaches and techniques that professors use, visiting and observing other professors as they teach, learning more about how people learn, and examining evidence-based instructional practices that have been shown to be successful. Any or all of the above might be eye opening and intellectually stimulating and perhaps most importantly be recognition that the department is serious about improving the teaching and learning of mathematics at its institution.

As the department chair interviewed said, "The culture has to change." The big question is how and whether his statement reflects a blueprint or fantasy for his institution and thousands of other mathematics departments throughout the U.S. My hope is that this article will stimulate some discussion about your departmental culture and help bring teaching and learning of mathematics to the forefront.

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Book Review

Burden of Proof: A Review of *Math on Trial*

Reviewed by Paul H. Edelman

Math on Trial
Leila Schneps and Coralie Colmez
Basic Books, 2013
US$26.99, 272 pages
ISBN-13: 978-0465032921

In *Math on Trial*, Leila Schneps and Coralie Colmez write about the abuse of mathematical arguments in criminal trials and how these flawed arguments "have sent innocent people to prison" (p. ix). Indeed, people "saw their lives ripped apart by simple mathematical errors." The purpose of focusing on these errors, despite mathematics' "relatively rare use in trials" (p. x), is "that many of the common mathematical fallacies that pervade the public sphere are perfectly represented by these trials. Thus they serve as ideal illustrations of these errors and of the drastic consequences that faulty reasoning has on real lives" (p. x). The authors' strategy is to identify common mathematical errors and then illustrate how those errors arose in trials. They seek to accomplish two goals: first, to impress upon the general public the importance of being able to "distinguish whether the numbers brandished in our faces are legitimately providing information or being misused for dangerous ends"; second, "to identify the most important errors that have actually occurred" so that such mistakes can be eliminated in the future.

These are worthy if anodyne goals, and I would not dare argue against them. But the claims that Schneps and Colmez make are strong ones and prompt many questions. Do they adequately support their contention that mathematics has a "disastrous record of causing judicial error?" How influential are mathematical arguments, anyway? Are mathematical arguments more problematic

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than other expert testimony? What role should mathematics play in the judicial system?

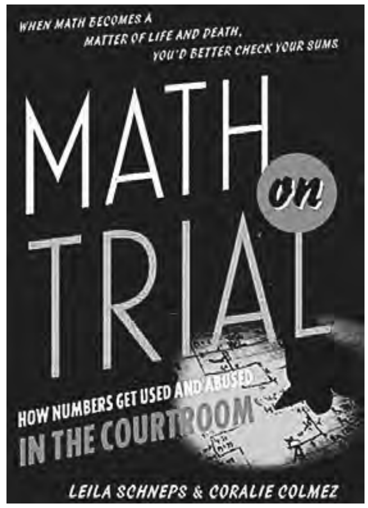
I will get to these questions shortly but, first, a brief description of the book. It consists of ten chapters. Each begins with a short introduction to a particular faulty mathematical argument and then illustrates the error with a discussion of a criminal case in which that argument was advanced. For instance, the first chapter, titled "Math Error Number 1, Multiplying Nonindependent Probabilities", discusses The Case of Sally Clark: Motherhood under Attack. In this case, Ms. Clark, whose first child died in the crib, was charged with the murder of her second child, who also died in her care. The error appears in the guise of testimony by an expert witness that the likelihood of two children dying innocently in her care could be computed by taking the square of the likelihood that a random baby dies innocently while in the care of the family. But that computation relies on the independence of the probabilities, which, if there is some underlying medical issue that caused the death, may well not be the case.

The range of cases presented, both geographically and historically, is in many ways the best feature of the book. Six of the ten cases arise in the United States, while the remainder are from Europe. There are three quite old cases, pre-World War I, and three from the twenty-first century, including one still in litigation. This breadth makes for a very good read, but it also leads to some questions. Do we really think that mathematical errors from the 1860s are as salient as ones from last year? Might advances in knowledge in the intervening one hundred fifty years mediate our concern about such errors? Moreover, continental Europe's legal regime is rather different than that of Britain and the United States. Will those differences have any effect on how mathematics is used? None of these questions is addressed by the book.

There is a lot to like about *Math on Trial*. It is an easy and fun read. The cases, like so many criminal cases, are fascinating in their details. The older cases, in particular, are entertaining, and the mathematical hooks bring a different perspective to the Dreyfus affair (Ch. 10) and the story of Charles Ponzi (Ch. 8). The writing tends toward the breathless, as is common in the true crime genre, but rarely goes over the top. The mathematics is well presented and well integrated into the narrative. Some of the explications are excellent: the discussion of the probabilistic issues in searching DNA databases (Ch. 5) and how Simpson's paradox manifests itself in sex discrimination cases (Ch. 6) are especially noteworthy in this regard.

As entertaining and informative as *Math on Trial* is, have Schneps and Colmez mustered sufficient evidence to justify their claim that mathematics has a "disastrous record of causing judicial error," let alone the claim that "the misuse of mathematics can be deadly?" I think not. To see why, we have to read the presented cases with a more critical eye.

Some of the cases actually do not exhibit any mathematical errors at all. The supposed mathematical issue arising in the case of Charles Ponzi, whose eponymous scheme bilked thousands of people, is that people "got fooled because they did not realize the implications of the incredible rapidity of exponential growth" (p. 149). While that may be a shortcoming of individuals, it is hardly a misuse of mathematics by the legal system. It is also difficult to understand exactly how this led to people being defrauded by Ponzi. After all, investments are all about exponential growth—often in the 5%–10% range but exponential nevertheless.



The chapter about sex discrimination, which describes Simpson's paradox in a very accessible way, actually illustrates a triumph for mathematics in the legal context. The University of California was able to show that what first appeared to be discrimination against women in graduate admissions was, in fact, anything but. Why this chapter is included is a bit of a mystery to me actually, since much of it describes the well-known sex discrimination allegations made by Jenny Harrison against the UC Berkeley mathematics department, a case which never went to trial and in which statistics played little role, if any.

But what of the cases that do, in fact, exhibit material mathematical mistakes? It is one thing for there to be evidence in the record exhibiting a faulty mathematical argument; it is quite another to assume that such an argument was decisive in the outcome of the case. Consider the Dreyfus Affair, which gets quite a nice discussion in the book. Alphonse Bertillon, a handwriting expert, was called to testify on whether the critical memo was written by Dreyfus. He "built up an extraordinary, well-argued theory that Dreyfus had purposely forged an imitation of his own handwriting so that if he were caught, he could attempt to explain away any evidence against him by claiming he had been framed" (p. 196). The authors focus on this theory, which is quite elaborate and quite clearly daft, and conclude, "Bertillon's testimony contributed to Dreyfus' conviction." But they give no reason to believe this, and given the machinations of the French military, it is difficult to imagine that the outcome would have been different if no such "evidence" had been admitted.

Schneps and Colmez make similar jumps throughout. Just because evidence is admitted to the record does not mean that it played a role in the outcome. In the 1865 case of Hetty Green (another fabulously interesting case), who was trying to enforce a contested will, the Harvard mathematician Benjamin Peirce was called in to decide if a certain signature was a forgery. He presented an elaborate, but seriously flawed, model purporting to demonstrate that fact. Perhaps this would be disconcerting but for the fact that the court decided against Ms. Green on purely legal grounds having nothing to do with the signature itself.[1] (I can't help but also note that Ms. Green hardly falls into the category of people whose lives have been ripped apart by a mathematical mistake. She was already worth several million dollars or so in the 1860s.)

Even the most important case in the book, People v. Collins, is subject to this criticism. People v. Collins is the first case in the United States to explicitly consider the role of probability in evidence and is a staple in every evidence course in every law school in the country. The basic facts are the following: On June 18, 1964, Juanita Brooks had her purse snatched while walking home from the grocery store in Los Angeles. She and another witness reported that the assailant was a woman with a blond ponytail who was subsequently picked up by a bearded African American man in a yellow car. A couple meeting that description, Janet and Malcolm Collins, were soon located in the vicinity and were eventually charged and tried.

One thing the district attorney, Ray Sinetar, had going for him was his intuition that there was unlikely to be more than one couple who fit this very uncommon description. In order to push this insight he managed to adduce through the testimony of Daniel Martinez, a professor of mathematics at California State Long Beach, the following table expressing the likelihood of various observations:[2]

	Characteristic	Individual Probability
A.	Partly yellow automobile	1/10
B.	Man with mustache	1/4
C.	Girl with ponytail	1/10
D.	Girl with blond hair	1/3
E.	Negro man with beard	1/10

Sinetar then concluded that the likelihood of such a couple existing is the product of these probabilities and hence about 1 in 12 million. Ultimately the Collinses were convicted, but an appeal was soon filed to challenge, among other things, this probabilistic approach to evidence.

There are, of course, so many things wrong with this argument it is hard to keep count. First of all, the numbers themselves were produced by surveying Sinetar's secretaries (I guess we would call this crowd-sourcing now) and had no factual basis. The probabilities themselves are clearly not independent, so multiplication is obviously inappropriate. And even if both of these problems can be overcome, what exactly it all proves is quite problematic. The argument exhibited what is known as the prosecutor's fallacy: at best he computed the likelihood that a random couple matched the characteristics of the Collinses, not the likelihood that the Collinses were guilty of the crime. Schneps and Colmez do a fine job of explaining the plethora of confusions in the argument.

All that is well and good, and all of it became recognized after the California Supreme Court overturned the verdict. This was just bad mathematics, and it certainly deserves to be excoriated. But did it make a difference in the outcome of the case? One juror is quoted as saying, "I don't remember our discussing the professor much when we deliberated. Maybe we were overwhelmed by the numbers." And a reporter who covered the trial wrote, "Jurors said they disregarded Martinez's testimony,…, and found the couple guilty on evidence given by other witnesses."[3]

Obviously, I cannot go through chapter by chapter, but I think the point is made that many things happen at trial and that to focus on only one aspect of the evidence as the "but-for" cause of the outcome is a mistake. Nevertheless, having bogus mathematical arguments entered into the record is disturbing and the authors ask a legitimate question as to how such arguments can be effectively prevented or countered. They argue that "it is probably going to be necessary to educate the public, from which juries are drawn, to recognize some of the most common mathematical principles that forensic analysis cannot do without" (p. 224). While promoting education is always good, raising the numeracy of the general public is not easy. Fortunately, I do not think it is really necessary in order for the legal system to work adequately.

Mathematical arguments appear in a wide range of legal disputes. They appear in the analysis of race and sex discrimination, anti-trust, stock fraud, and torts, to name a just a few. The vast majority of these applications are not terribly controversial, although any particular model will be subject to criticism and interpretation. *Math on Trial* focuses on the introduction of probabilistic evidence in criminal trials, a very narrow, although important,

[1] *What eventually undid Hetty Green was an interpretation of the parol evidence rule which would not permit her testimony to confirm a contract with her deceased aunt. See* The Howland Will Case, *4 Amer. L. Rev. 625 (1869).*

[2] *Details of how this table was produced are somewhat murky. The reader should read Chapter 2 of the book being reviewed and George Fisher, "The Green Felt Jungle: The Story of People v. Collins", in* Evidence Stories *(Richard Lempert, ed.), Foundation Press, 7 (2006).*

[3] *These quotes are reported in Fisher on page 16.*

area.[4] The argument put forward in People v. Collins is one such example; testimony about the likelihood of a DNA sample coming from a particular individual is another.

Even within this narrow area, the significance of an error in mathematics can easily be overstated. A recent study looked at eighty-six cases in which people were convicted of serious crimes but were later exonerated on the basis of DNA evidence. In 71% of those cases, there were erroneous eyewitness identifications, 63% had forensic science testing errors, 44% had police misconduct, and 28% had prosecutorial misconduct.[5] So there would seem to be many more important problems in the criminal justice system than bad mathematics. Indeed, these more mundane problems arise in most of the cases discussed in *Math on Trial*.

Before we worry about remedying the problem of bad mathematics in criminal trials, we should probably consider what mechanisms are already in place to prevent bad evidence from being introduced. Judges and juries routinely have to cope with evidence of a very technical nature. One cannot hope for them to be adequately educated in all of the areas of knowledge that will be put before them. That is why the legal system provides various procedural safeguards to control what information is put into the legal record. Rules of evidence, standards for the admittance of expert testimony, and other procedural devices all provide means for blocking or refuting bad evidence. It is not a coincidence that one basis for the California Supreme Court overturning the Collinses' conviction, not mentioned in *Math on Trial*, was the inadmissibility of the mathematical testimony, since no empirical support was presented for any of the claims. On reading the cases in *Math on Trial,* one is struck by how ineffectual these devices were, either because of inadequate counsel (another common thread in faulty convictions, as mentioned above) or the failure of the judge to enforce the appropriate rules.

Lawyers have another way to deal with bad testimony, particularly expert testimony—they can provide their own experts to dispute the bad information.[6] In the adversarial system in the United States and Britain we would expect each side to challenge the other by providing their own experts on questions such as the likelihood of some event. This is not a perfect system, since poor defendants may not have the resources to hire such an expert, and that might be the reason there is little evidence of this practice in the cases discussed in *Math on Trial*. Another problem with this approach is that the trial can then become a battle of the experts, the result being that juries (and judges) throw up their hands and ignore the expert testimony altogether. This very reaction was noted in the Hetty Green case, where a contemporaneous account noted that "[T]he result of so much labor of experts, their skill, their ingenuity, their patience, their anxiety, simply demonstrates to the profession their inutility as witnesses in a court of justice."[7]

Should we think that mathematical error is any more prevalent than any other kind of error? One might argue that because they are innumerate, lawyers are worse at coping with mathematical issues than with others. This possibility has been raised before,[8] but I am not sure that I am persuaded by this argument. I know of no studies that indicate that lawyers are less competent at dealing with elementary probability theory than with, say, sophisticated economic modeling questions in anti-trust. And if they are no more prone to error in mathematics than in any other technical area, would it not make more sense to address the issue at the broader level than at the discipline-specific level?

Despite these criticisms, I would agree with the authors that education can certainly play an important role. Indeed, in the case of DNA evidence, it already has. "Immediately after DNA's first courtroom appearance in the 1980s, scientists from disciplines as varied as statistics, psychology, and evolutionary biology debated the strengths and limitations of forensic DNA evidence. Blue-ribbon panels were convened, conferences were held, unscientific practices were identified, data were collected, critical papers were written, and standards were developed and implemented.…Most exaggerated claims and counterclaims about DNA evidence have been replaced by scientifically defensible propositions. Although some disagreement remains, the scientific process worked."[9]

[4] *Three of the ten cases presented are not of this type, but they are also the least persuasive of the chapters: the aforementioned chapters on Ponzi, Hetty Green, and Jenny Harrison.*

[5] *Data reported in Michael J. Saks & Jonathan J. Koehler,* The Coming Paradigm Shift in Forensic Identification Science*, 309 Science 892 (2005). The remainder of issues on the list are false/misleading testimony by forensic scientists (27%), dishonest informants (19%), incompetent defense representation (19%), false testimony by lay witnesses (17%), and false confessions (17%).*

[6] *At least this is true in the United States and Britain. The situation is somewhat different in civil law regimes of continental Europe.*

[7] The Howland Will Case *on page 643.*

[8] *Most recently in Lisa Milot*, Illuminating Innumeracy, *63 Case Western L. Rev. 1 (2013)*

[9] *Saks & Koehler on page 893. I would note here that Colmez and Schneps base two chapters on what they claim to be faulty use of DNA evidence. One is a case involving database trawling and the other is on the methods of DNA testing. While the first case plausibly represents a mathematical error, the latter discussing the Meredith Kercher case (more commonly referred to as the Amanda Knox case in the U.S.) seems to me to be better described as a dispute over testing protocol rather than over mathematics.*

The moral of this tale is that education can be successful, but it is a result of experts working among themselves and coming to a consensus on these highly technical issues. The results are then promulgated through the legal system via these experts. The authors of *Math on Trial* themselves are part of such a project, the "Bayes and the Law" Research Consortium, to develop "a set of criteria and a set of analytic tools that should ensure that probability will henceforth be used correctly" (p. 224). I wish them luck. It takes time (thirty years in the case of DNA) for best practices to be adopted, both because the scientific process is slow and because the legal system is a distributed one and so information disperses slowly through it. Mistakes are going to happen—it is unfortunate but inevitable.

Finally, it is worth thinking about what the role of mathematics in the law should be in a perfect world of sophisticated jurors, judges, and lawyers. There is considerable debate within the legal academy as to whether it is possible to put formal probabilistic foundations under the theory of evidence.[10] And for those who think such a theory can be laid, there are a number of different candidates for how it should be developed.[11] The fact of the matter is that rigorous mathematical thinking is sometimes not in accord with the workings of the judicial system.

The Conjunction Paradox is an example of the kind of problem that arises when trying to establish a probabilistic theory of the burden of proof. In civil actions, such as tort, the plaintiff typically has to establish his case by a preponderance of the evidence, which is usually interpreted to mean that the probability of the offense exceeds 0.5. But sometimes the offense consists of two or more elements. For example, in a common negligence claim, the plaintiff might have to show both that the defendant was negligent and that the plaintiff's injuries resulted from the defendant's actions. Suppose that the plaintiff can establish both claims, the first with probability 0.7 and the second with a probability 0.6. By doing so he has met the burden required to demonstrate his claim, and he should recover his damages. This is the way most courts would analyze the case.

On the other hand, traditional probability would argue that the likelihood of both elements being true is closer to $0.6 \times 0.7 = 0.42$ (assuming the independence of these two events, which seems plausible in this situation), which would not meet the threshold required. Since the conjunction of the events does not exceed the 0.5 probability threshold, the plaintiff should lose. Mathematically this argument seems unexceptionable, but it is not recognized by the legal system.

So where does this leave us with *Math on Trial*? I think it is unconvincing in its claim that the misuse of mathematics in evidence is either particularly significant or novel. It seems much like the other technical expert testimony and is subject to similar costs and benefits. Their proposed solution of educating jurors sounds unpromising to me, but educating lawyers and judges, not so much in the mathematics itself but rather in how to be an educated consumer of the information, is a very reasonable approach. Part of that education is the development of techniques and analyses that gain acceptance within the scientific community. An even more productive response would be to better train the lawyers and judges in law and provide greater access to legal counsel. This would address errors in using evidence across the whole spectrum of disciplines.

Its analysis and prescriptions notwithstanding, *Math on Trial* is an entertaining and informative read for those interested in true crime with a mathematical hook. Perhaps it will impress upon the general public the importance of numeracy and inspire them to look beyond and behind the numbers that are trumpeted around us. If so, that would be all to the good.

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[10] *See, e.g., Ronald J. Allen,* A Reconceptualization of Civil Trials*, 66 B. U. L. Rev. 401 (1986) and Richard Lempert,* The New Evidence Scholarship: Analyzing the Process of Proof*, 66 B. U. L. Rev. 439 (1986).*

[11] *Recent work includes Edward K. Cheng,* Reconceptualizing the Burden of Proof*, 122 Yale L. J. 1254 (2013) and Kevin Clermont,* Death of Paradox: The Killer Logic Beneath the Standards of Proof*, 88 Notre Dame L. Rev. (2012).*

Book Review

The Evolution of an Idea

Reviewed by Robyn Arianrhod

The Noether Theorems: Invariance and Conservation Laws in the Twentieth Century
Yvette Kosmann-Schwarzbach, translated from the French by Bertram E. Schwarzbach
Springer, 2011, US$119.00, 218 pages
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Emmy Noether, the most famous female mathematician of the twentieth century, was the daughter of mathematician Max Noether. She received her doctorate at Erlangen in 1907 (when she was twenty-five) for a thesis on the topic of algebraic invariants. She soon became part of the brilliant circle led by Felix Klein and David Hilbert at Göttingen, and popular legend says she influenced even Einstein. Of course, legends tend to abound about pioneering female mathematicians, but in this case there is some substance to the claim, as *The Noether Theorems* shows. In the interest of clarifying the intended readership, let me hasten to add that this book is not a series of portraits of colorful personalities; rather, it is a deeply scholarly work, tersely written but generously footnoted, that celebrates Noether's most famous achievements. It begins with an English translation of Noether's paper "Invariant variational problems", published originally in German ("Invariante Variationsprobleme") in 1918, which will delight mathematicians and mathematical physicists who use these theorems without having read the original version.

But *The Noether Theorems* does more than celebrate Emmy Noether and her theorems: as the subtitle suggests, it traces what Yvette Kosmann-Schwarzbach calls the "evolution of ideas" about the relationship between mathematical invariance and conservation laws, and it will be of great interest to historians of mathematics. For those who simply wish to get a feel for the work and influence of this famous female mathematician but who are not expert in the mathematics behind these theorems (namely, Lagrangian dynamics, calculus of variations, and Lie groups), let me provide a short summary. A physical law has symmetries when it is unchanged or invariant under certain coordinate or gauge transformations. Noether showed that these symmetries indicate which quantities are conserved under the law. While Noether's theorems apply to laws of physics formulated in terms of a Lagrangian, the principle can be illustrated by a well-known example from undergraduate mathematics. A particle with momentum $\mathbf{p}$ moves in a conservative force field $\mathbf{F}(\mathbf{x}) = -\nabla V(\mathbf{x})$ according to Newton's law $d\mathbf{p}/dt = \mathbf{F}$. If V is invariant under translations in the x_1 direction, then the x_1-component p_1 of the momentum is constant, since $dp_1/dt = -\partial V/\partial x_1 = 0$, so p_1 is "conserved". Another relevant undergraduate result is that mechanical energy too is conserved under a "conservative" force, because the "work" integral of such a force is path independent. The mathematical role of the boundary in Noether's theorems is illustrated by line integrals that depend only on the end-points of the relevant path.

These elementary examples are more than simplifying analogies. They illustrate what Kosmann-Schwarzbach calls the "strange story" of the Noether theorems, which were generally overlooked in the early years precisely because their significance lies in their generalizing a host of already-known results like these. These known results had been built up slowly and separately over the centuries, and it took physicists some time to appreciate the importance of Noether's discovery that a single fundamental principle united mathematical symmetries and physical conservation laws.

Even the concept of energy itself had been long contested: when Leibniz first proposed the existence and conservation of the mysterious *vis viva*, which he represented by the formula mv^2, there was a heated and sometimes partisan debate about why we would need any consequence of force other than the Cartesian/Newtonian concept of momentum, mv! In 1740 another pioneering female mathematician, Émilie du Châtelet, played an

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important role [1] in defending Leibniz's concept. Following Jean Bernouilli's lead, mathematicians eventually resolved the *vis viva* debate by showing that integrating $F = dp/dt$ with respect to t gives the expression for momentum, while integration with respect to x yields the formula for (what we now call) kinetic energy. However, identifying the latter as a "work" integral came much later (as did definitive experimental confirmation of the conservation of mechanical energy). Indeed, the quest to understand the concept of energy is another example of the fascinating evolution of scientific ideas. But I digress. Kosmann-Schwarzbach's historical survey begins with Lagrange, who first connected conservation laws with symmetries in the laws of dynamics.

He took the first step towards this connection in his *Mécanique Analytique* (1788) when he outlined what we now call Lagrangian dynamics, and showed that the equations of motion derived in this way lead naturally to the known conservation laws. Kosmann-Schwarzbach writes (p. 33),

> While, before Lagrange, the various conservation results had been taken to be first principles belonging to the foundations of dynamics, Lagrange viewed them as consequences of the equations of dynamics, an important shift of point of view.

I will return to this shift towards the mathematization of physics later in this review. Kosmann-Schwarzbach adds (p. 34),

> It is only in the second edition of *Mécanique Analytique* (1811) that Lagrange observed a correlation between symmetries and the principles of conservation of certain quantities, in particular, energy.

Others made incremental generalizations of these ideas, but it was Emmy Noether who took the final step, proving a general, purely mathematical connection between symmetry (or invariance) and formal "conservation laws". However, she realized immediately that her second theorem could be applied to the conservation of energy in the then-new theory of general relativity, her first theorem being applicable to special relativity and classical dynamics. The difference in the two theorems concerns the nature and dimension of the symmetry groups of transformations and the nature of the associated conservation laws. (The first theorem applies to theories whose conservation laws are "proper" because they can be derived from a flux integral over the boundary of a surface, which leads, via Gauss's law, to a "continuity equation"; the field equations give a divergence-free quantity that expresses the relevant local conservation law. Noether's first theorem showed, for example, that "proper" conservation of energy is a consequence of the invariance of the Lagrangian under time-translations. Her second theorem applies to gauge theories whose conservation laws are "improper" because the vanishing of ordinary divergences does not yield meaningful local conservation laws, but the second theorem identifies invariant identities that can be used to define a concept of conservation. See, for example, Bergmann [2, p. 194], Byers [3].) As Kosmann-Schwarzbach puts it (p. 26), "Noether thus emphasized an essential difference between special relativity and general relativity...."

The problem of energy in general relativity had perplexed both Einstein and Hilbert, and finding a global law remains problematic because of the difficulty of including gravitational energy as well as "local" conservation of the stress-energy tensor **T**. (See Kosmann-Schwarzbach, p. 127, and for more detail, see, e.g., Wald [4, p. 286].) Local conservation can be expressed in terms of the vanishing divergence of **T**, although use of the term "divergence" refers to the symbolic similarity with vector divergences, because in general relativity one is really talking about contracted covariant derivatives of the tensor **T**, as Kosmann-Schwarzbach points out (pp. 127–128). She also notes (p. 43) that, in 1924, Schouten and Struik showed that when Noether's second theorem is applied to the Lagrangian of general relativity, "The identities obtained were also consequences of the Bianchi identities, which were well known in Riemannian geometry."

The Bianchi identities also lead to the vanishing of the contracted covariant derivatives of **T** via Einstein's field equations, and herein lies an interesting tale, which Kosmann-Schwarzbach sketches very briefly: the story of Einstein's field equations versus Hilbert's. It is important in Emmy Noether's story, because she arrived in Göttingen, at Klein and Hilbert's invitation, in the spring of 1915, when Einstein and Hilbert were each trying to derive the gravitational field equations. They would both succeed in late November of that year, although Kosmann-Schwarzbach makes the point (pp. 40–41) that not only did Einstein lay the foundation for Hilbert's work, but he also (just!) has priority in the discovery of the field equations.

The fascinating thing about the Einstein-Hilbert field equations is the very different approaches of the two men, the physicist and the mathematician. I confess that I have always been drawn to Einstein's use of physical principles: the relativity principle, together with the constancy of the speed of light in special relativity, and the "equivalence principle" in general relativity. He illustrated the latter with the example of a freely falling lift: if you drop a ball, it will appear stationary in the falling lift, because it is falling under the influence of gravity at the same rate as the lift itself. So, by changing reference frames (from the ground to the falling lift), the "force" of gravity on the ball is transformed away. Similarly, according to Einstein [5, p. 114], "we are able to 'produce' a gravitational field merely by changing the system of coordinates." In this way he showed that gravitation could be treated in a relativistic theory, in which the laws of physics retain their form (that is, remain invariant) under coordinate transformations between reference frames. The mathematical language he used to make this idea precise was that of tensor calculus (which Ricci and Levi-Civita had pioneered in the late nineteenth century). By contrast, Hilbert's approach was to use a Lagrangian-style action principle.

Kosmann-Schwarzbach shows (p. 70) that, as early as 1910, Hilbert's colleague Klein, who had been working on the geometry of Lorentz groups which are fundamental in Einstein's special theory, claimed that one could, "*if one really wanted to*, replace the term 'theory of invariants with respect to a group of transformations' with the term 'relativity with respect to a group'." The italics are mine: what a perfect illustration of the priorities of Klein the mathematician, for whom the theory of relativity is a purely mathematical theory of invariants and groups, in contrast to Einstein's use of physical principles! In fact, in 1916 Einstein implicitly acknowledged the superior elegance of the Lagrangian/Hamiltonian formulation used by Hilbert but concluded [5, p. 118]:

> It is not my purpose in this discussion to represent the general theory of relativity as a system that is as simple and logical as possible, and with the minimum number of axioms; but my main object is to develop this theory in such a way that the reader will feel that the path we have entered upon is psychologically the natural one, and that the underlying assumptions will seem to have the highest possible degree of security.

In July 1915 Hilbert had invited Einstein to spend a week at Göttingen. Einstein likely met Emmy Noether there, and even if he did not, he soon became aware of her work. Although his and Hilbert's field equations were published at the end of 1915, in the following year they were still trying to clarify the relativistic concept of conservation of energy, and both appealed to Noether for clarification, which she provided. (Like Hilbert, Einstein used a Lagrangian (or rather, a Hamiltonian) mathematical approach in examining conservation of energy [5, pp. 145–149].) By July 1918 she had completed the "Noether theorems", which impressed Einstein so much that he wrote to Klein in December:

> I once again feel that refusing her the right to teach [because of her gender] is a great injustice. I would be very favorable to taking energetic steps [on her behalf] before the ministry. If you do not think that this is possible, then I will go to the trouble of doing it alone.

Kosmann-Schwarzbach gives this quote on page 72, although she doesn't say whether or not Einstein carried out his offer, but she notes (pp. 48–49) that the following year the Weimar Republic's new Ministry of Science, Arts and Education allowed Noether to take up an appointment as a (poorly paid) *Privatdozentin*, or assistant professor of mathematics, at Göttingen. She seems to have been popular there, but within just a few years she had moved on from the "Noether theorems" to other topics in algebra, in which she was "one of the most important mathematicians of her time" (Kosmann-Schwarzbach, p. 53). In 1933 the Nazis removed her from her teaching position—like Einstein, she was Jewish, and like him, she migrated to the U.S. in 1933. Sadly, she died in 1935.

Most of *The Noether Theorems*—five of its seven chapters—is an overview of the response to these two theorems from Noether's contemporaries (I have mentioned only Einstein's reaction here) and from later physicists and mathematicians. The book traces the influence of the theorems—which first waned, and then waxed after 1980—through a detailed, highly technical survey of various attempts to modernize and extend their applicability, as well as tracing recent progress in the mathematical understanding of symmetries and conservation laws in general. A notable example is the rise of gauge theories, in which "Noether's theorem is an essential tool" (p. 130). It is significant that Kosmann-Schwarzbach concludes that resistance to Noether's theorems had little to do with sexism or racism, because her later work in algebra was "immediately recognized and admired" (p. 146); rather, it was a product of the scientific styles and interests of those who were driving progress at that time. *The Noether Theorems* thus highlights the role of historical and personal contingencies in the evolution of scientific ideas in general.

For instance, a mathematical result such as Noether's may play a relatively small role in the process of creating physical theories but can become important in later refinements of the language of the theory. Kosmann-Schwarzbach quotes (p. 89) Vizgin's 1985 assessment that Hilbert's conservation law in general relativity is "a special case of Noether's second theorem, proved two and a half years later by Emmy Noether." Similarly, we learn (pp. 80–83) that in 1927, when Eugene Wigner used parity symmetry to derive the quantum mechanical law associated with parity conservation, apparently he had not heard of Noether's theorems. (In any case, these theorems would not in themselves have applied to the discrete symmetry he used, although discrete analogues were developed in the 1970s (Kosmann-Schwarzbach, pp. 80–81, 148). In 1972, however, Wigner claimed, "We physicists pay lip service to the great accomplishments of Emmy Noether, but we do not really use her work…." This quote is given on page 82, and, as I mentioned, *The Noether Theorems* then shows how this perception slowly changed, especially after 1980. The book concludes with a list of the many recent fields of application of these theorems, not only in quantum and classical mechanics and in relativity and quantum field theory but also in elasticity theory, fluid mechanics, geometric optics, the mechanics of nonholonomic systems, locomotion, and numerical analysis.

I think Kosmann-Schwarzbach's perspective is that physicists were unduly slow in recognizing the importance of mathematical concepts like Noether's. She mentions (p. 146) that, in a 1980 address, the physicist Chen Ning Yang quoted "an amusing letter from Faraday to Maxwell as 'a good example of [our] resistance to the mathematization of physics'," as he put it with good-humored self-deprecation.

Clearly Lagrange's mathematical approach—and therefore Hilbert's and Noether's—has been marvelously successful in physics. On the other hand, Einstein preferred to ground his theory in a "psychologically natural" physical framework, and it's interesting that his point of view is similar to Maxwell's. In his 1873 *Treatise on Electricity and Magnetism*, Maxwell gave a summary of Lagrangian dynamics (before using it to examine electromagnetic energy and its conservation), but he concluded his summary as follows [6, p. 210]:

> Lagrange and most of his followers, to whom we are indebted for these methods, have endeavoured to banish all ideas except those of pure quantity, so as not only to dispense with diagrams, but even to get rid of the ideas of velocity, momentum and energy.…

He went on to say that mathematics has given science many ideas that would not have been possible otherwise, but that in exploring dynamics,

> we must have our minds imbued with dynamical truths as well as mathematical methods…[This is in order] to avoid inconsistency with what is already established, and also that when our views become clearer, the language we have adopted may be a help to us and not a hindrance.

Maxwell had a genius for choosing "helpful" mathematical language, notably his innovative use of differential vector calculus to represent the intuitive field concept created by the "mathematically illiterate" Faraday. Kosmann-Schwarzbach doesn't quote Yang's "amusing letter from Faraday to Maxwell," but it's worth noting that, despite his lack of formal education, Faraday instinctively knew that in trying to describe electromagnetic effects, mathematicians like Ampère placed too much faith in the action-at-a-distance mathematical methods of Newtonian gravity theory.

Of course, no matter which approach is more aesthetically or psychologically pleasing to us, a theory stands or falls on its ability to make suitably accurate predictions. But one of the fascinating things about scientific history is the often serendipitous nature of discovery, and to a mathematician surely the most thrilling of all such contingencies is the "unreasonable effectiveness" of mathematical language in physics, to use Wigner's well known but still evocative expression [7]. This effectiveness is evident in the "mathematized" approach of Lagrange, Hilbert, and Noether, as well as in Einstein's and Maxwell's (and many others') combination of experiment, physical intuition, and appropriate mathematical language.

But, to sum up *The Noether Theorems* in its own terms, it is an important study of the work of Emmy Noether, the evolution of ideas about conservation and symmetry, and the extraordinary fertility of mathematical language.

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Simplicity, in Mathematics and in Art

Allyn Jackson

Simplicity is as hard to pinpoint in mathematics as it is in art. Certainly both subjects have their great exemplars of the quality. But is there a definition of simplicity? A criterion? A measure? Or a sure path to it?

These kinds of questions were in the air at a conference called Simplicity: Ideals of Practice in Mathematics and the Arts, which took place at the Graduate Center of the City University of New York in early April 2013. Instead of trying to definitively answer such questions—surely a doomed prospect anyway—the participants gave in to the sheer joy of discussion in the stimulating atmosphere of each other's company. The conference featured lectures and panel discussions by an eclectic group of twenty-five artists, architects, art historians, mathematicians, and mathematically inclined philosophers, as well as a film program. The audience included academics from nearby institutions and local artists; as the conference offered easy and free online registration, a random smattering of folks wandered in out of curiosity.

Not an Absolute Notion

Simplicity often seems to be a timeless, absolute quality, and for good reason. Peter Sarnak, Institute for Advanced Study and Princeton University, offered Euclid's proof of the infinitude of primes as simplicity par excellence. The stark elegance of this ancient proof is as striking today as it must have been to people encountering it through the millennia. Of course, the proof is an exemplar of simplicity, not a definition. Indeed, Curtis Franks, University of Notre Dame, argued against the possibility of ever establishing for all time an absolute notion of simplicity. What we think of as simple emerges from conventions that are deeply embedded in how we live and how we see the world, and they have a long genetic history. "Our thinking occurs within those conventions," he said. "There is not really a way out of them."

As conventions evolve, so do notions of simplicity. Franks mentioned Gauss's 1831 paper that established the respectability of complex numbers. The problem Gauss was working on—concerning quadratic and biquadratic residues—had only unsatisfyingly complicated and piecemeal solutions over $\mathbb{Z}$. Over $\mathbb{C}$, a far simpler solution emerged. The complex numbers revealed simplicity where previously there had seemed to be none.

Mathematics is not engaged in a straightforward march toward absolute simplicity. Rather, by discovering simplicity anew, Franks said, "We will be more awake to the changing landscape of mathematical thought." He noted a parallel in art, where something new—like the work of Andy Warhol or Marcel Duchamp—acts as a sort of "shock treatment" that compels a new perspective.

Several conference speakers mentioned the art of Fred Sandback, who used taut lengths of yarn to represent outlines of three-dimensional shapes. In photos, the works look unimpressive; as philosopher Juliet Floyd, Boston University, noted, they are "unphotographable". But walking around and through the constructions, she found them to be "extremely moving objects". Finnish architect Juhani Pallasmaa described how a Sandback construction, merely "a few lines stretched in space", sets off a chain reaction in the viewer's mind, causing the viewer to see figures of specific material shapes, to feel their weight and texture. "The air inside the imaginary figure seems to get

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denser and to have a slightly different consistency from the air outside," he said. Simple constructions that hold much complexity and meaning: That's just what mathematicians seek in their work.

Pallasmaa's erudite lecture contained many striking quotations, including this one of Balthus: "The more anonymous painting is, the more real it is." The same can be said for architecture, Pallasmaa stated. Could a similar statement be made for mathematics? Are there mathematical results that are so natural, so pristine that one cannot perceive the fingerprints of the mathematicians who discovered them? Perhaps one example would be the previously mentioned proof of the infinitude of primes, its attribution to Euclid notwithstanding. Perhaps others are found in what Paul Erdős famously called "proofs from the Book".

Pallasmaa also quoted the philosopher Gaston Bachelard, who in his book *The Philosophy of No: A Philosophy of the New Scientific Mind*, stated that scientific thought "develops along a predestined path, from animism through realism, rationalism, and complex rationalism, to dialectical rationalism." Pallasmaa did not say that mathematics develops in this way; his point rather was that art aspires to develop in the opposite direction, from the rational back towards "a unifying, mythical, and animistic experience". Perhaps mathematics shuttles back and forth between the two endpoints.

Visceral Encounters

Bachelard's "predestined path" at times echoed through the conference in comments that seemed to derive from the misconception, common outside of mathematics, that the subject consists entirely of proofs, progressing inexorably from one logical step to the next. This misconception was vividly countered at various points during the conference. In an open microphone session, Blaise Heltai pointed out that mathematics and art are actually very similar in process: When you are thinking about a mathematical object, you are right inside the thing, trying to puzzle out its structure and secrets. You're not thinking about how to prove anything—that comes later. The puzzling-out resembles the conceptual part of doing art. Heltai has a special perspective, as he is a painter with a Ph.D. in mathematics; he makes a living as a management consultant.

The kind of visceral encounter with mathematics that Heltai referred to emerged at various times, such as in the lecture of Dennis Sullivan, CUNY Graduate Center and Stony Brook University. When as a graduate student he was preparing for the preliminary examination, Sullivan studied John Milnor's book *Topology from the Differential Viewpoint*. Sullivan knew the book inside and out, every definition, every proof. The day before the exam, as he took a final glance through the book, it suddenly occurred to him that he could compress the contents into a single, simple picture. Moving back and forth across the stage, he used gesticulations to indicate a 2-sphere on one side, a 3-sphere on the other, and a "slinky" curve between them. This curve, representing the preimage of a regular value of a map from the 3-sphere to the 2-sphere, provided a mental image summarizing the Pontryagin–Thom construction. If one knows the language of manifolds and transversality, Sullivan claimed, one can reconstruct the whole theory of cobordism in differential topology just from the intuition conveyed by his slinky picture. This experience made him realize, "*That's* what it means to understand a piece of mathematics."

The visceral component of mathematical work surely evokes strong feelings, but mathematicians usually do not discuss their feelings about their work, at least not in public lectures. In an earlier panel discussion, Riikka Stewen, Finnish Academy of Fine Arts, asked whether mathematicians have strong love/hate feelings about their work. "Yes, very strong feelings," came the immediate reply from a mathematician on the panel, Andrés Villaveces, National University of Colombia. There is a loneliness in the work of an artist, and much mathematical work shares this quality. Just as a painter faces an empty canvas, he said, "Mathematicians are up against the empty page every day."

The longing, even desperation, that is implicit in the remarks of Villaveces also emerged in Sarnak's lecture, titled "Is there a place for 'ugly' mathematics?". Sarnak considered the situation where the only known route to a proof is ugly, in the sense of being strewn with long and complicated calculations and verifications. The question then becomes, How desperate are we for a proof? When Sarnak gave an example of an ugly calculation connected with a beautiful result in the theory of automorphic forms, Mikhail Gromov, Institut des Hautes Études Scientifiques and New York University, piped up to say: "Maybe the mathematics is fine, it's your mind that's ugly."

Then there was Gromov's lecture. A fish says: "You want to understand what water is? Jump in and find out." Instead of plunging in, you could study the chemical and physical properties of water. But without the experience of plunging into water, you have no frame in which to talk about what water really is. Similarly, when the experience of plunging into mathematics is absent, there is no frame in which to talk about what mathematics is—much less what simplicity in mathematics is.

That's a verbose description of one moment that flashed by in an instant in Gromov's stream-of-consciousness lecture. He jumped into Descartes's timeless statement, "Cogito ergo sum [I think therefore I am]". The important thing here, Gromov said, is the *ergo*, the *therefore*. In a sense, dogs *think*: Much of what goes on in a human brain is very similar to what goes on in the brain of a dog. Surely dogs *are*. But dogs do not understand *ergo*. This *ergo* is a major source of the kind of thinking that is characteristic of humans, Gromov said. And yet, "it is completely hidden from us. And there is a good reason why it is hidden. If it surfaces, you die. You will not survive. It's against survival, it's against evolution, it's against [natural] selection."

So it went. Gromov passed so quickly over so many topics, diving to the depths, all the while leavening the presentation with flashes of subversive humor. The effect was dizzying. Afterward, during the open microphone session, an audience member demanded a one-sentence summary—with an example. An impossible request to fulfill. Nevertheless it can be said that one of Gromov's main messages was: Guard against the delusion of false simplicity. Many things that we assume at first glance to be simple are in fact highly complex.

After seeing Gromov's effervescent mind bubble over for thirty minutes, audience member Al Thaler, known to many for his long service at the National Science Foundation and now an adjunct faculty member at CUNY's Hunter College, commented, "I could never live like that."

Contrasting Groups

The Simplicity conference was the brainchild of mathematician Juliette Kennedy, University of Helsinki, and two CUNY mathematicians, Roman Kossak of the Graduate Center and Philip Ording of Medgar Evers College. The conference was something of a follow-up to a 2007 symposium called Aesthetics and Mathematics, which took place in Utrecht and was organized by Kennedy and two University of Utrecht mathematicians, Rosalie Iemhoff and Albert Visser (Iemhoff was one of the lecturers at Simplicity). Participants in the 2007 symposium could drop in at an art exhibition at the Mondriaanhuis, *Logic Unfettered—European and American Abstraction Now*, which was curated by Kennedy.

In addition to the film program at the Simplicity conference, there was an installation of a few works by artist Kate Shepherd in the lobby outside the hall where the lectures were given (Shepherd also participated in one of the panel discussions). But space constraints there, as well as the difficulty of securing exhibit space in New York City, meant that Simplicity offered few opportunities to experience art. As a result, art was represented mainly through the presence and words of the artists themselves. By contrast, the mathematicians could actually present pieces of mathematics by using a computer and a beamer, or even just a blackboard, in the case of Sullivan. They tried mightily to avoid technical details, with imperfect success.

Another contrast was socio-economic. As Kennedy pointed out in a panel discussion, the mathematicians and philosophers at the conference all work in academia, which provides economic security and social acceptability, while artists often lead far more precarious lives on the fringes of society. She noted the "heroic" efforts that many artists must put forth in order to carry out their work.

What did each group absorb from the other? It's difficult to say. One participant observed that mathematicians tend to have a high opinion of themselves and their own knowledge and are therefore not so open to new ideas, while artists are pretty much the opposite: Receptiveness to impressions and influences from a wide variety of sources is the artist's lifeblood. One artist who attended Simplicity, Miyuki Tsushima, said she didn't follow all the details of the math lectures. She could simply sit and let the impressions wash over her as she made some sketches for her latest work.

An inspiration for the conference was the so-called twenty-fourth problem of David Hilbert. This problem, which Hilbert considered adding to his famous list of twenty-three problems that he presented at the International Congress of Mathematicians in Paris in 1900, was unearthed by Rüdiger Thiele, University of Leipzig, from papers at the library of the University of Göttingen. Part of Hilbert's description of the problem reads: "Criteria of simplicity, or proof of the greatest simplicity of certain proofs. Develop a theory of the method of proof in mathematics in general. Under a given set of conditions there can be but one simplest proof" (translation by Thiele from his article "Hilbert's 24th Problem", *American Mathematical Monthly*, January 2003).

Etienne Ghys, École Normale Supérieure de Lyon, pointed out the naiveté of imagining that such ultimate simplicity is possible. Yet, as the conference highlighted, simplicity as a dream, as an ideal, remains a powerful guiding light in mathematics and the arts. As Franks said, there are no absolute notions of simplicity. But do not relinquish the quest. "On the contrary, I want to say yes, find criteria for simplicity, continue to do so," said Franks. Don't imagine that the matter will ever be settled definitively; rather, "return to the task often."

Materials from some of the lectures are on the Simplicity conference website, `http://s-i-m-p-l-i-c-i-t-y.org`, *and videos of some of the lectures will be posted soon.*

Revisiting an Outreach Mathematician

Jerry Dwyer and Lawrence Schovanec

Two articles describing early experiences of an outreach mathematician and the chairperson who advocated for such a role appeared in the *Notices of the American Mathematical Society* in 2001 [1], [2]. Several years later, it is timely to reflect again on the evolving nature of this endeavor. The first author (Dwyer) of this essay is the outreach mathematician involved in the earlier articles. The second author (Schovanec) was the chair of the Department of Mathematics and Statistics (M&S) at Texas Tech University (TTU) at the time that Dwyer was hired in 2003. Schovanec now serves as the interim president of TTU and previously was the dean of the College of Arts & Sciences. In each of his administrative roles he has promoted outreach, engagement, and the associated reward structures within the university.

Background

Herein we define outreach for a mathematics department as any activity that enhances the teaching and learning of mathematics outside the department, in particular in K–12 education and community colleges. An outreach mathematician is not a mathematics education researcher but is likely to spend considerable time working on matters that are sometimes within the domain of mathematics education. A significant difference between outreach and traditional mathematics scholarship is dependence on the cooperation of local school districts and state educational agencies and of teachers at all levels.

Reaction to the 2001 articles was positive. Many institutions have faculty dedicated to outreach, but in many cases these faculty were hired in nontenure-track roles or pursued outreach activities only when the traditional requirements for tenure and promotion were met. This is because the typical reward system may not recognize that an outreach mathematician should be tenured and promoted in a manner cognizant of his or her outreach role as is the case in a traditional track of pure mathematical or educational research. Addressing this issue is critical to any effort to create an outreach position in a mathematics department.

At the time of the earlier articles, Dwyer was a tenure-track faculty member in the Department of Mathematics at the University of Tennessee. In spite of satisfaction with the job and a high level of support from the Tennessee mathematics faculty, for personal reasons, he moved to the M&S department at TTU in 2003. When this position became vacant in 2002, there was mixed support to add faculty expertise in mathematics education, but Schovanec, then the department chair, recognized the potential of adding an outreach faculty member. Moving to a new position had its challenges, including building confidence with the local K–12 community, obtaining support of new colleagues, and a delay in the tenure and promotion process for the outreach mathematician.

Outreach Roles and Recognition

At TTU, there are basically three components in the outreach work: (1) on-campus activity teaching courses, supervising students, and interacting

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with faculty in other departments who have interests that are aligned with outreach activities; (2) organizing locally and regionally funded outreach activities such as K–12 school visits and summer programs; and (3) serving on national committees and participating on panels at various meetings. There has been a progression in how these roles have been developed. Master's theses have been completed and a mathematics education concentration has been added to the mathematics Ph.D. program that provides opportunities for training mathematicians in the newer outreach roles. Personal school visits have been reduced, while teacher workshops and collaboration with school districts have increased. Grant writing has evolved. Though proposals are still submitted to local foundations with a history of supporting educational programs unique to TTU, a large commitment is now tied to substantial federal funding requests, often of a collaborative nature.

The major issues facing an outreach mathematician are similar to those of traditional faculty. There is a need to seek funding and to publish scholarly articles. A progression from small funded proposals to larger ones has resulted in significant support for outreach projects. Some of the activities for which funding has been obtained include girls' math clubs, pre-engineering and mathematics academy summer programs, and several National Science Foundation programs primarily related to teacher training and scholarship programs. Common objectives of these programs include: increasing the number and diversity of students in STEM programs; enhancing undergraduate research opportunities in the sciences; and growing the number of teachers in STEM areas while enhancing opportunities for teacher preparation.

One award, which is reflective of the maturation of outreach activities at TTU, is the Integrated STEM Initiative on the South Plains [3, `http://www.depts.ttu.edu/stem/isisp.php`]. This NSF program provides funding for the integration of outreach programs and stimulates changes in the campus culture related to STEM education and outreach. It may be argued that the hiring of an outreach mathematician provided the catalyst and leadership for a more cohesive campus-wide approach to outreach grant writing. As a result, over the last several years more than $12 million in funding has significantly affected STEM education and outreach projects at TTU. Somewhat ironically, in light of increased funding, a new challenge has been to maintain a focus on the original departmental expectations of the outreach position. Dwyer has had to curtail his role in grass roots activities in order to administer existing projects and coordinate new collaborative programs.

The topic of publication can be a challenge for one in a nontraditional academic role such as an outreach mathematician. The demands of administering outreach must be reconciled with the logistics of producing outreach scholarship. For example, one must address the use of human subjects, Institutional Review Board (IRB) approval, and access to K–12 populations, issues often foreign to traditional mathematics research. There are also both a shortage of appropriate publication outlets and reservations from colleagues concerning the quality of scholarship that results from outreach activities.

A significant development in support of outreach at TTU occurred in 2012 when the university adopted a revised tenure and promotion policy that recognized outreach and community engagement as part of a faculty member's contributions to teaching, research, or service. Even if a discussion of outreach and community engagement does not rise to the university level, a mathematics department should consider amending the reward and promotion structure to take into account the nontraditional role of an outreach mathematician. The mathematics department at TTU has recently adopted such a clause in its promotion and tenure documents. For the record, Dwyer obtained tenure and eventually promotion to full professor, the first such case in his department.

Institutional Perspective

From the viewpoint of Schovanec as chair and then dean, there are new opportunities, rewards, and frustrations related to outreach mathematics. As chair of a traditional research department at a large state university, he found enthusiasm and appreciation for mathematics outreach at the higher levels of university administration not always commensurate with departmental support.

The goodwill and publicity derived from outreach activities translated into enhanced financial support of the department and a perception of departmental vitality. The Texas Senate recognized the M&S department when it was awarded the Texas Association of Partners in Education Award, in large part due to activities initiated by Dwyer and his colleagues in the department. Publicity events for major grants garnered significant attention and increased the visibility of the M&S department at local and regional levels. Furthermore, collaborations with local school districts and regional colleges and alliances with teachers enhanced student recruitment and presented greater opportunities for research and funding.

Since Dwyer arrived at TTU, he has played a critical role in the growing culture of outreach and engagement that now receives greater institutional recognition. Most recently Dwyer has been featured as an Integrated Scholar [4], a distinction that TTU has enlisted to recognize contributions to teaching, research, and service, where outreach is recognized as a component of all three areas. In 2006, TTU was the first Texas university to be included in the Community Engagement

Classification of the Carnegie Foundation for the Advancement of Teaching; it is regularly recognized in the president's Higher Education Community Service Honor Roll. This distinction is partially based on data reflective of TTU's strategic priority to expand community engagement and evidence of extensive faculty-led community collaborations. Since 2009, TTU has annually assessed the institution's community engagement activity utilizing Michigan State's Outreach and Engagement Measurement Instrument (OEMI). Texas Tech has been represented in national and international conversations on institutional mission and assessment of community engagement.

Another promising development is the creation of a TTU university-wide multidisciplinary STEM Center for Outreach, Research, and Education. The center, for which Dwyer serves as director, is supported by six colleges at TTU. This university-wide participation in the center in some ways reflects both authors' vision for the recognition and institutionalization of outreach at TTU.

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What We Are Doing about the High Cost of Textbooks

The AIM Editorial Board

Let's begin with the obvious: The price of textbooks has risen much faster than the cost of living over the last thirty years, but there has not been a significant increase in their quality. We don't propose to analyze the economic and educational factors that underlie this phenomenon. Instead, we will describe our efforts to help lower the cost of textbooks for standard undergraduate mathematics courses in North American colleges and universities.

The AIM Editorial Board consists of David Austin, George Jennings, Kent E. Morrison, Frank Sottile, and Katherine Yoshiwara.

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We make up the AIM (American Institute of Mathematics) Editorial Board, which is part of a larger NSF project[1] to develop open source software and curriculum materials for undergraduate mathematics education. Open textbooks can be "open access", meaning they are freely available in digital format, or "open source", meaning their source files are freely available. Our project hopes to overcome two obstacles faced by textbooks: It's hard to find them, and it's hard to know which ones are good.

Let's say you are scheduled to teach abstract algebra next term. How do you choose the text? You may use the book you used last time, or the book you had as a student (yes, Herstein is still available in paperback for $111), or you may ask your colleague next door for a recommendation. Or you may decide to look for an open source textbook.

Search for free abstract algebra texts and you will get two million results or more. You will find that the top 150 entries or so actually look promising, but then you notice that some are duplicates and some are links to pirated versions. Many of them point to supplemental notes, exercise solutions, and other material related to the books. Very few are actually links to entire textbooks that

[1] *Information about Project UTMOST (Undergraduate Teaching in Mathematics with Open Software and Texts) can be found at* http://utmost.aimath.org.

can be obtained without cost. Then it takes time to follow the links and read enough to decide whether or not a book is a viable candidate for your course.

With our project we are working to find, evaluate, and promote open textbooks in mathematics. We have created a short "approved list" of open textbooks organized by course name, with a link to each book's website. We encourage authors to maintain an active website with more detailed information about their books, as well as errata sheets, links to reviews, course adoption lists, and contact information for comments and corrections. Such a website can help create an ecosystem around a book so that others can contribute to its improvement. An open textbook can be a shared community resource that remains current and won't go out of print.

The criteria we use to approve books are described in detail on our website.[2] In brief, we are looking for books that can serve well as the required text in a course. We contact faculty who have actually used the books to find out what they think. Do they recommend the book in question? Would they use it again? Although we do not require that the books be available in printed format, we recommend that authors arrange for printing and distribution by companies such as Lightning Source, Lulu, Amazon, or Barnes & Noble.

There are currently twenty-one books in thirteen courses on the approved list. Some of them began as commercially published textbooks whose copyrights were returned to the author. Others began as lecture notes and evolved into polished texts while remaining open access.

Tom Judson's *Abstract Algebra: Theory and Applications*, first published by PWS in 1994, is an example of how a textbook can become open source. After getting the copyright back, Judson released the text as open source with a GNU Free Documentation License. The mathematics department of Virginia Commonwealth University took the LaTeX source, invested some time in formatting and editing, and arranged for Lightning Source to print and bind the book and for Amazon to sell it. The price is only $20 (hard cover) and the entire PDF version is available for no cost at the book's website. You and your students can print your own copies or just the pages you want. You can arrange for a photocopy store or your campus bookstore to print copies for all of you, at a cost as low as $10. Of course, you don't have to print it at all; you can read it on your computer or notebook.

There is also a digital version in which each chapter is a Sage notebook that allows immediate computation with Sage, and there are Sage notebooks that contain just the exercises. (Sage is free open source software.)

[2] http://www.aimath.org/textbooks/

Some of the books are released with the GNU Free Documentation License mentioned above, while many have a Creative Commons license that is more restrictive in that it does not allow others to sell the book for profit but does allow them to charge the nominal cost of printing. Finally, a few of the books are commercially published, so that the publishers retain all the rights to print and distribute hard copies, but with open access PDF versions that can be printed for individual use.

What motivates authors to forego the opportunity to earn money from their hard work? For many it is the experience gained from writing a commercially published book that did not yield rich returns, so the profit motive is weak for the next book. Authors would like their work to be used and appreciated, so they would like to make it widely available. For many a primary motivation is the desire to do something beneficial for the world.

If you'd like to contribute to the open source movement, here are some things you can do. Give serious consideration to open textbooks, and let your colleagues know about them, too. Let the authors know when you use their texts and become part of the community by giving feedback, tracking errors and typos, and contributing supplementary material. Contribute a book review to the Open Academic Catalog at the University of Minnesota (`http://open.umn.edu`), where reviews and ratings of open texts in all subjects—not just mathematics—are being collected.

Now that we have evaluated a number of books, we see the need for practical advice and guidance for authors who want to produce open source materials. We are creating a collection of recommended practices for all aspects of writing and publishing open mathematics textbooks, both in the current environment, where most books are read in a static version as PDF files or as printed copy, and in the new environment of constant Internet connectivity, mobile devices, and cloud computing. In a future column we would like to discuss the challenges and opportunities that an author faces under these rapidly changing conditions.

Mathematics People

Smith Awarded Adams Prize

IVAN SMITH of the University of Cambridge has been awarded the 2013 Adams Prize. This year's topic was topology.

According to Tim Gowers, chairman of the Adams Prize Adjudicators, Smith "has proved several beautiful and important results in symplectic topology. With Simon Donaldson, he found new proofs of some major results of Taubes that were simpler and that avoided delicate use of machinery from outside symplectic topology. With Paul Seidel, he attacked the problem of understanding the nature of Khovanov cohomology, a mysterious but very useful invariant. They developed a geometric definition that was later shown, by Smith and Abouzaid, to be an alternative definition of Khovanov cohomology. Also with Abouzaid, he showed that the famous homological mirror symmetry conjecture of Kontsevich is true for any product when it is true for the factors: this yielded new examples of manifolds for which the conjecture holds. With Seidel he proved a conjecture of Eliashberg and Gromov, showing that there are well-behaved exotic symplectic structures on Euclidean space. These are just a few of the achievements that caused Smith to stand out from a very strong field."

The Adams Prize is awarded each year jointly by the Faculty of Mathematics at the University of Cambridge and St. John's College to a young researcher or researchers based in the United Kingdom doing first-class international research in the mathematical sciences. The prize is named after the mathematician John Couch Adams and was endowed by members of St. John's College. It carries a cash prize of approximately £14,000 (about US$21,000), of which one-third is awarded to the prizewinner on announcement of the prize, one-third is provided to the prizewinner's institution (for research expenses of the prizewinner), and one-third is awarded to the prizewinner on acceptance for publication in an internationally recognized journal of a substantial (normally at least twenty-five printed pages) original survey article of which the prizewinner is an author.

—From a University of Cambridge announcement

Goldblatt Awarded Jones Medal

ROBERT GOLDBLATT of the Victoria University of Wellington has been awarded the 2012 Jones Medal by the Royal Society of New Zealand. According to the prize citation, Goldblatt was honored "for his profound and world-leading research in modal logic and category theory, and his lifetime of dedicated service to mathematics." He "has become one of the world's leading authorities in modal logic. In this system, statements can be much more than simply true or false: they can be, say, necessarily true, possibly true, or eventually true. This flexible logic is at the heart of basic software engineering and the commercial program and chip verification industry. Modal logic is interdisciplinary, overlapping mathematics, philosophy, linguistics, and computer science."

—From a Royal Society of New Zealand announcement

Bondarenko Awarded Popov Prize 2013

ANDRIY BONDARENKO of Kiev University has been awarded the 2013 Vasil A. Popov Prize. According to the prize citation, he was honored "for his outstanding contributions to approximation theory. He along with Radchenko and

Viazovska solved the spherical t-design conjecture by Korevaar and Meyers concerning optimal approximation of integrals over the sphere by arithmetic means of values of the integrand. This result beautifully illustrates the power of the fixed-point method to approximation problems. Andriy Bondarenko has also advanced powerful new ideas in other areas of approximation theory, in particular, in monotone rational approximation, one of Vasil A. Popov's favorite research areas." The prize consists of a marble pyramid trophy and a cash award of US$2,000.

—From a University of South Carolina press release

AWM Awards Inaugural Research Prizes

The Association for Women in Mathematics (AWM) has announced the awarding of two new major research prizes. The AWM-Microsoft Research Prize in Algebra and Number Theory has been awarded to Sophie Morel of Princeton University "in recognition of her exceptional research in number theory." The prize recognizes exceptional research in algebra and number theory by a woman early in her career.

The prize citation reads in part: "Morel is a powerful arithmetic algebraic geometer who has made fundamental contributions to the Langlands program. Her research has been called 'spectacularly original, and technically very demanding'. Her research program has been favorably compared to that of several Fields medalists. She accomplished one of the main goals of the Langlands program by calculating the zeta functions of unitary and symplectic Shimura varieties in terms of the L-functions of the appropriate automorphic forms. To achieve this, she introduced an innovative t-structure on derived categories which had been missed by many experts. Her book *On the Cohomology of Certain Noncompact Shimura Varieties*, published in the Annals of Mathematics Studies series, is described as a tour de force.

"Morel found another remarkable application of her results on weighted cohomology. She gave a new geometric interpretation and conceptual proof of Brenti's celebrated but mysterious combinatorial formula for Kazhdan-Lusztig polynomials, which are of central importance in representation theory."

Morel received her Ph.D. from the Université Paris-Sud. She has also held positions at the Institute for Advanced Study, the Clay Mathematics Institute, Harvard University, and the Radcliffe Institute for Advanced Studies.

Svitlana Mayboroda of the University of Minnesota has been awarded the 2012 AWM Sadosky Research Prize in Analysis in recognition of her fundamental contributions to harmonic analysis and PDEs. The award, named for Cora Sadosky, a former president of AWM, recognizes exceptional research in analysis by a woman early in her career.

The prize citation reads in part: "Mayboroda's research has centered on boundary value problems for second and higher order elliptic equations in nonsmooth media. Elliptic equations in nonsmooth media model a variety of physical systems and thus play a central role in science and engineering. Her research addresses fundamental problems aimed at understanding how irregular geometries or internal inhomogeneities of media affect the behavior of the physical system in question. Her talent and imagination, which have been praised by world leaders in the field, is also evident in her recent work with Vladimir Maz'ya on regularity in all dimensions for the polyharmonic Green's function in general domains and of the Wiener test for higher order elliptic equations, which in turn relies on a new notion of capacity in this case. This is the first result of its kind for higher order equations, showing remarkable creativity and deep insight. For higher order elliptic operators the situation on nonsmooth domains is quite different than in the second-order case, and much less is known. Mayboroda's contributions have opened up fundamental new paths in this uncharted territory and she has been a major driving force behind it."

Mayboroda received her Ph.D. from the University of Missouri at Columbia. She has been the recipient of a Sloan Research Fellowship and an NSF CAREER grant, with which she ran a Workshop for Women in Analysis and PDE in 2012 designed to support early-career women in their passage from graduate school to postdoctoral or faculty positions.

–From AWM announcements

Lubetzky and Sly Awarded Rollo Davidson Prize

Eyal Lubetzky of Microsoft Research and the University of Washington and Allan Sly of the University of California Berkeley have been jointly awarded the 2013 Rollo Davidson Prize "for their work on the dynamics of the Ising model, and especially their remarkable proof of the cut-off phenomenon." The Rollo Davidson Trust was founded in 1975 and awards the annual prize to young mathematicians working in the field of probability.

—From a Rollo Davidson Trust announcement

Shoham and Tennenholtz Receive ACM/AAAI Newell Award

Yoav Shoham of Stanford University and Moshe Tennenholtz of Technion-Israel Institute of Technology and Microsoft Research have been named the recipients of the 2012 Allen Newell Award of the Association for Computing Machinery (ACM) and the Association for the Advancement of Artificial Intelligence (AAAI). They were recognized for contributions to multiagent systems spanning computer science, game theory, and economics.

According to the prize citation, "Shoham's pioneering work provided a methodology for specifying distributed multiagent systems. His research on game theory includes advances in combinatorial auctions, where participants place bids on combinations of discrete items. He also advanced mechanism design, sometimes known as reverse game theory, which sets formal rules that design a game as well as predicting how a game will be played.

"Tennenholtz pioneered several approaches to the design and analysis of interactions between decision-makers in computational settings. He also created RMax, a general efficient algorithm applicable to learning by interacting with an environment. In addition, he introduced the concept of program equilibrium, an ingenious application of computer science to the analysis of Internet economies. He is acknowledged as a central contributor to many of Microsoft's pricing algorithms for online advertising."

—From an ACM announcement

Ibragimov Awarded Anassilaos Prize

Zair Ibragimov of California State University, Fullerton, has been awarded the 2012 International Anassilaos Prize in Mathematics from the Associazione Culturale Anassilaos. He was honored for his "distinguished contributions to analysis and geometry, including geometric function theory, metric geometry and hyperbolization of metric spaces." The prize was instituted in honor of the twentieth-century Italian geometer Renato Calapso.

—From a Cal State Fullerton announcement

2013 Clay Research Awards Announced

The Clay Mathematics Institute (CMI) has awarded the 2013 Clay Research Award to Rahul Pandharipande of ETH Zurich. He was honored "for his recent outstanding work in enumerative geometry, specifically for his proof in a large class of cases of the MNOP conjecture that he formulated with Maulik, Okounkov, and Nekrasov. The conjecture relates two methods of counting curves in an algebraic variety, one given by Gromov-Witten theory and the other by Donaldson-Thomas invariants. By building in particular on joint work with Thomas on stable pairs, Pandharipande and his student Aaron Pixton proved the conjecture for many (possibly most) Calabi-Yau threefolds." Aaron Pixton was awarded a 2013 Clay Research Fellowship.

—From a CMI announcement

Dick and Pillichshammer Awarded 2013 Information-Based Complexity Prize

Josef Dick of the University of New South Wales and Friedrich Pillichshammer of Johannes Kepler University have been named recipients of the 2013 Prize for Achievement in Information-Based Complexity (IBC). The prize consists of US$3,000 and a plaque. This annual prize is given for outstanding contributions to information-based complexity.

—Joseph Traub, Columbia University

Prizes of the Mathematical Society of Japan

The Mathematical Society of Japan (MSJ) has awarded several prizes for 2013.

The Algebra Prize has been awarded to Tomoyuki Arakawa of Kyoto University for work on representation theory of infinite dimensional Lie algebras and *W*-algebras and to Atsushi Ichino of Kyoto University for work in the theory of automorphic representations and periods. The Algebra Prize is awarded to researchers who have made significant contributions to the development of algebra in a broad sense by obtaining outstanding results.

The Spring Prize was awarded to Masayuki Asaoka of Kyoto University for his outstanding contributions to the study of hyperbolic dynamical systems and related geometry. The Spring Prize is awarded to researchers under the age of forty who have obtained outstanding mathematical results.

The Prize for Science and Technology in Research was awarded to Masafumi Akahira of the University of Tsukuba for his outstanding contributions to statistical higher order asymptotic theory. The Prizes in Research recognize researchers for highly original research or developments that contribute to the development of science and technology in Japan.

—From MSJ announcements

USA Math Olympiad

The 2013 USA Mathematical Olympiad (USAMO) was held April 30–May 1, 2013. The students who participated in the Olympiad were selected on the basis of their performances on the American High School and American Invitational Mathematics Examinations. The twelve highest scorers in this year's USAMO, listed in alphabetical order, were: Calvin Deng, North Carolina School of Science and Mathematics, Durham; Andrew He, Monta Vista High School, Cupertino, California; Ravi Jagadeesan, Phillips Exeter Academy, Exeter, New Hampshire; Pakawut Jiradilok, Phillips Exeter Academy, Exeter, New Hampshire; Kevin Li, West Windsor-Plainsboro High School South, West Windsor, New Jersey; Ray Li, Phillips Exeter Academy, Exeter, New Hampshire; Mark Sellke, William Henry Harrison High School, Evansville, Indiana; Bobby Shen, Dulles High School, Sugar Land, Texas; Zhuoqun Song, Phillips Exeter Academy, Exeter, New Hampshire; David Stoner, South Aiken High School, Aiken, South Carolina; Thomas Swayze, Canyon Crest Academy, San Diego, California; and Victor Wang, Ladue Horton Watkins High School, Ladue, Missouri.

The twelve USAMO winners will attend the Mathematical Olympiad Summer Program (MOSP) at the University of Nebraska, Lincoln. Ten of the twelve will take the team selection test to qualify for the U. S. team. The six students with the highest combined scores from the test and the USAMO will become members of the U. S. team and will compete in the International Mathematical Olympiad (IMO) to be held in Santa Marta, Colombia, July 18–28, 2013.

—From Mathematical Association of America announcements

Moody's Mega Math Challenge

The winners of the 2013 Mega Math Challenge for high school students have been announced. The topic for this year's competition dealt with recycling. Each group had to quantify the plastic waste filling U.S. landfills, come up with the best recycling methods for U.S. cities to implement based on their demographics, and recommend guidelines for nationwide recycling standards.

A team from Wayzata High School in Plymouth, Minnesota, was awarded the Summa Cum Laude Team Prize of US$20,000 in scholarship money. The members of the team were Jenny Lai, Abram Sanderson, Amy Xiong, Lynn Zhang, and Roy Zhao. Their coach was Thomas Kilkelly.

The Magna Cum Laude Team Prize of US$15,000 in scholarship money was awarded to a team from North Carolina School of Science and Mathematics, Durham, North Carolina. The team members were Jeffrey An, Dayton Ellwanger, Christie Jiang, and Anne Kelley. They were coached by Daniel Teague.

The Cum Laude Team Prize of US$10,000 was awarded to a team from North Penn High School, Lansdale, Pennsylvania. The team members were Priya Kikani, Scott Landes, Patrick Nicodemus, Julianna Supplee, and Francis Walsh. Their coach was Dianne Wakefield.

The Meritorious Team Prize of US$7,500 was awarded to a team from T. R. Robinson High School, Tampa, Florida. The team members were Lauren Lopez, Ravi Patel, Chris Sipes, Dylan Wang, and Anna Yannakopoulos. They were coached by Judi Charley-Sale.

The Exemplary Team Prize of US$5,000 was awarded to a team from Evanston Township High School, Evanston, Illinois. The team members were Maggie Davies, Caroline Duke, Laura Goetz, Katie Latimer, and Dina Sinclair. Their coach was Mark Vondracek.

The First Honorable Mention Team Prize of US$2,500 was awarded to a team from Montgomery Blair High School, Silver Spring, Maryland. The team members were Alexander Bourzutschky, Alan Du, Tatyana Gubin, Lisha Ruan, and Audrey Shi. They were coached by David Stein.

The Mega Math Challenge invites teams of high school juniors and seniors to solve an open-ended, realistic, challenging modeling problem focused on real-world issues. The top five teams receive awards ranging from US$5,000 to US$20,000 in scholarship money. The competition is sponsored by the Moody's Foundation, a charitable foundation established by Moody's Corporation, and organized by the Society for Industrial and Applied Mathematics (SIAM).

—From a Moody's Foundation/SIAM announcement

Malloy and Rubillo Receive NCTM Lifetime Achievement Awards

The National Council of Teachers of Mathematics (NCTM) has presented Mathematics Education Trust (MET) Lifetime Achievement Awards for Distinguished Service to Mathematics Education to Carol E. Malloy, Wilmington, North Carolina, and James M. Rubillo, Bucks County Community College, Newtown, Pennsylvania. Malloy has been a voice and a leader in mathematics education. Throughout her career she has worked to address the inequities that African American, Latino, and Native American students face in learning mathematics. She has served on the NCTM Board of Directors, edited NCTM yearbooks, reviewed journal manuscripts, written journal articles, served on committees, given presentations, and been elected president of the Benjamin Banneker Association, an NCTM affiliate. She served on the writing team for the publication *Principles and Standards for School Mathematics*. In 2010 she was awarded the first annual UNC–Chapel Hill School of Education Black Alumni Impact Award. She is currently serving as a lead author for Glencoe/McGraw-Hill K–12 school mathematics programs.

Rubillo has been an inspirational leader, communicator, and advocate for mathematics education for more than forty-five years. He has made numerous contributions to the mathematics education community, with a special emphasis on technology and teaching mathematics at the community college and high school levels. He participated in developing NCTM's *An Agenda for Action*, released in 1980, the first publication to focus on problem solving as a basic skill, which changed the direction of mathematics education in the United States. His vision for improving instruction extended to the use of technology to reach more educators through such initiatives as Math in the Media and Math Matters. He served as executive director of NCTM from 2001 to 2009. He received an honorary doctor of science degree from West Chester University in 2004, and in 2008 the National Council of Supervisors of Mathematics presented him with NCSM's Ross Taylor/Glenn Gilbert National Leadership Award.

—From NCTM announcements

National Academy of Sciences Elections

The National Academy of Sciences (NAS) has elected eighty-four new members and twenty-one foreign associates for 2013. Following are the new members whose work involves the mathematical sciences: Manjul Bhargava, Princeton University; S. James Gates Jr., University of Maryland, College Park; Juris Hartmanis, Cornell University; Victor Kac, Massachusetts Institute of Technology; Gregory F. Lawler, University of Chicago; Juan Maldacena, Institute for Advanced Study; James A. Sethian, University of California Berkeley; Éva Tardos, Cornell University; David A. Vogan Jr., Massachusetts Institute of Technology; Avi Wigderson, Institute for Advanced Study; and Horng-Tzer Yau, Harvard University. Peter G. Hall, University of Melbourne, Australia, was elected as a foreign associate.

—From an NAS announcement

American Academy of Arts and Sciences Elections

The American Academy of Arts and Sciences (AAAS) has elected 186 new fellows and 12 foreign honorary members for 2013. Following are the new fellows whose work involves the mathematical sciences.

Lawrence D. Brown, University of Pennsylvania, Wharton School; Philip J. Hanlon, University of Michigan/Dartmouth College; Herve Jacquet, Columbia University; H. Blaine Lawson Jr., Stony Brook University, State University of New York; Duong H. Phong, Columbia University; Sorin Popa, University of California, Los Angeles; Walter A. Strauss, Brown University; Richard A. Tapia, Rice University; and Bin Yu, University of California, Berkeley. Elected as a foreign honorary member was Henri Berestycki, École des Hautes Études en Sciences Sociales, Paris.

—From an AAAS announcement

AWM Essay Contest Winners Announced

The Association for Women in Mathematics (AWM) has announced the winners of its 2013 essay contest, "Biographies of Contemporary Women in Mathematics".

The grand prize was awarded to Rebecca Myers, High Tech High International, San Diego, California, for her essay "Sara Billey: The Most Famous 'Sara in Math'". The essay won first place in the high school category and will be published in the *AWM Newsletter*.

First place in the undergraduate-level category went to Joy Otobo, Benue State University, Kaduna, Nigeria, for her essay "Destined to Count". First place in the middle school-level category went to Emmanuel Martinez, Lyford Middle School, Lyford, Texas, for his essay "A Teacher of Miracles".

—From an AWM announcement

Royal Society Elections

The Royal Society of London has elected its new fellows for 2013. The new fellows whose work involves the mathematical sciences are: Keith Ball, University of Warwick; Raymond Goldstein, University of Cambridge; Gareth Roberts, University of Warwick; Alan Turnbull, National Physical Laboratory; and Julia Yeomans, University of Oxford. Elected as a foreign member was Elliott Lieb, Princeton University.

—From a Royal Society announcement

Mathematics Opportunities

AMS Travel Grants for ICM 2014

The American Mathematical Society has applied to the National Science Foundation (NSF) for funds to permit partial travel support for U.S. mathematicians attending the 2014 International Congress of Mathematicians (ICM 2014), August 13–21, 2014, in Seoul, Korea. Subject to the award decision by the NSF, the Society is preparing to administer the selection process, which would be similar to previous programs funded in 1990, 1994, 1998, 2002, 2006, and 2010.

Instructions on how to apply for support will be available on the AMS website at http://www.ams.org/programs/travel-grants/icm. The application period will be **September 1–November 15, 2013**. This travel grants program, if funded, will be administered by the Membership and Programs Department, AMS, 201 Charles Street, Providence, RI 02904-2294. You can contact us at ICM2014@ams.org; 800-321-4267, ext. 4113; or 401-455-4113.

This program is open to U.S. mathematicians (those who are currently affiliated with a U.S. institution). Early-career mathematicians (those within six years of their doctorate), women, and members of U.S. groups underrepresented in mathematics are especially encouraged to apply. ICM 2014 Invited Speakers from U.S. institutions should submit applications if funding is desired.

Applications will be evaluated by a panel of mathematical scientists under the terms of a proposal submitted to the National Science Foundation by the Society.

Should the proposal to the NSF be funded, the following conditions will apply: mathematicians accepting grants for partial support of the travel to ICM 2014 may not supplement them with any other NSF funds. Currently, it is the intention of the NSF's Division of Mathematical Sciences to provide no additional funds on its other regular research grants for travel to ICM in 2014. However, an individual mathematician who does not receive a travel grant may use regular NSF grant funds, subject to the usual restrictions and prior approval requirements.

Visit http://www.ams.org/programs/travel-grants/icm for more details. All information currently available about the ICM 2014 program, organization, and registration procedure is located on the ICM 2014 website, http://www.icm2014.org/.

—AMS Membership and Programs Department

Committee on Education Launches New Award

At its January 2013 meeting, the AMS Council gave final approval to a new award proposed by the Committee on Education. The award recognizes outstanding contributions by mathematicians to education in mathematics at the pre-college level and during the first two years of college.

> **Award for Impact on the Teaching and Learning of Mathematics**
>
> The Award for Impact on the Teaching and Learning of Mathematics was established by the AMS Committee on Education in 2013. The award is given annually to a mathematician (or group of mathematicians) who has made significant contributions of lasting value to mathematics education. Priorities of the award include recognition of (a) accomplished mathematicians who have worked directly with pre-college teachers to enhance teachers' impact on mathematics achievement for all students, or (b) sustainable and replicable contributions by mathematicians to improving the mathematics education of students in the first two years of college.

The endowment fund that supports the award was established by a contribution from Kenneth I. and Mary Lou Gross in honor of their daughters Laura and Karen.

The US$1,000 award is given annually. The recipient is selected by the Committee on Education.

Nominations for the first award are now open online at ams.org/profession/prizes-awards/prizes. The nomination deadline is **September 15, 2013**.

The Society gratefully acknowledges Professor and Mrs. Gross for their generosity in creating the endowed fund. Their gift demonstrates their steadfast commitment to mathematics and to preparing our nation's educators to teach it. The Kenneth I. and Mary Lou Gross Fund will provide a perpetual source of support for the new award.

—AMS Committee on Education

AMS Scholarships for "Math in Moscow"

The Math in Moscow program at the Independent University of Moscow (IUM) was created in 2001 to provide foreign students (primarily from the United States, Canada, and Europe) with a semester-long, mathematically intensive program of study in the Russian tradition of teaching mathematics, the main feature of which has always been the development of a creative approach to studying mathematics from the outset—the emphasis being on problem solving rather than memorizing theorems.

Indeed, discovering mathematics under the guidance of an experienced teacher is the central principle of the IUM, and the Math in Moscow program emphasizes in-depth understanding of carefully selected material rather than broad surveys of large quantities of material. Even in the treatment of the most traditional subjects, students are helped to explore significant connections with contemporary research topics. The IUM is a small, elite institution of higher learning focusing primarily on mathematics and was founded in 1991 at the initiative of a group of well-known Russian research mathematicians, who now make up the Academic Council of the university. Today, the IUM is one of the leading mathematical centers in Russia. Most of the Math in Moscow program's teachers are internationally recognized research mathematicians, and all of them have considerable teaching experience in English, typically in the United States or Canada. All instruction is in English.

With funding from the National Science Foundation (NSF), the AMS awards five US$9,000 scholarships each semester to U.S. students to attend the Math in Moscow program. To be eligible for the scholarships, students must be either U.S. citizens or enrolled at a U.S. institution at the time they attend the Math in Moscow program. Students must apply separately to the IUM's Math in Moscow program and to the AMS Math in Moscow Scholarship program. Undergraduate or graduate mathematics or computer science majors may apply. The deadlines for applications for the scholarship program are **September 15, 2013**, for the spring 2014 semester and **April 15, 2014**, for the fall 2014 semester.

Information and application forms for Math in Moscow are available on the Web at `http://www.mccme.ru/mathinmoscow` or by writing to: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: `mim@mccme.ru`. Information and application forms for the AMS scholarships are available on the AMS website at `http://www.ams.org/programs/travel-grants/mimoscow` or by writing to: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2294; email: `student-serv@ams.org`.

—AMS Membership and Programs Department

Call for Nominations for AWM-Birman Research Prize in Topology and Geometry

The Executive Committee of the Association for Women in Mathematics (AWM) has established the AWM–Joan & Joseph Birman Research Prize in Topology and Geometry to highlight exceptional research in some area of topology/geometry by a woman early in her career. The prize will be awarded every other year, with the first prize presented at the AWM reception at the Joint Mathematics Meetings in San Antonio, Texas, in January 2015.

The prize is made possible by a generous contribution from Joan Birman, whose work has been in low-dimensional topology, and her husband, Joseph, who is a theoretical physicist whose specialty is applications of group theory to solid state physics. Dr. Birman says, "Mathematical research has played a central role in my own life and has been a source of deep personal satisfaction. In addition, some of my closest friendships have come about through joint work. Finally, as a teacher I felt privileged to be there when my students had their own 'aha' moments. From my own life I know that creative research in mathematics can present special difficulties when women have young children. I felt the conflict personally when my young children were pulling at my clothing to get my attention, but I was in 'math mode'. Everything I know suggests that women have greater difficulty handling this particular conflict than men. I also know that children grow up and develop interests of their own, and when that happens the conflict slowly diminishes. Also, if you have experienced the rich satisfaction of creative research at an early career time, you never forget it. Moreover, the math community will almost certainly be welcoming if you have taken a break but then start to have good research ideas again. Those are the reasons why it was an easy decision for us to use money that we'd set aside to encourage research by talented young women through this AWM early career prize. What better use could we find for our money?"

When reviewing nominations for this award, the field will be broadly interpreted to include topology, geometry, geometric group theory, and related areas. Candidates should be women within ten years of receiving their Ph.D.'s or having not yet received tenure. Nominations should be submitted by **February 15, 2014**.

For further information on the AWM–Joan & Joseph Birman Research Prize and nomination materials, please visit `http://www.awm-math.org`.

—AWM announcement

Call for Nominations for Sloan Fellowships

Nominations for candidates for Sloan Research Fellowships, sponsored by the Alfred P. Sloan Foundation, are due by **September 16, 2013**. A candidate must be a member of the regular faculty at a college or university in the United States or Canada and must have received the Ph.D. or equivalent within the six years prior to the nomination. For information write to: Sloan Research Fellowships, Alfred P. Sloan Foundation, 630 Fifth Avenue, Suite 2550, New York, New York 10111-0242; or consult the foundation's website: `http://www.sloan.org/fellowships`.

—From a Sloan Foundation announcement

NSF Focused Research Groups

The Focused Research Groups (FRG) activity of the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) supports small groups of researchers in the mathematical sciences.

The deadline date for full proposals is **September 20, 2013**. The FRG solicitation may be found on the Web at `http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5671`.

—From an NSF announcement

NSF Mathematical Sciences Postdoctoral Research Fellowships

The Mathematical Sciences Postdoctoral Research Fellowship program of the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) awards fellowships each year that are designed to permit awardees to choose research environments that will have maximal impact on their future scientific development. Awards of these fellowships are made for appropriate research in areas of the mathematical sciences, including applications to other disciplines. Fellows may opt to choose either a research fellowship or a research instructorship. The deadline for this year's applications is **October 16, 2013**. Applications must be submitted via FastLane on the World Wide Web. For more information see the website `http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5301`.

—From an NSF announcement

NSA Mathematical Sciences Grants and Sabbaticals Program

As the nation's largest employer of mathematicians, the National Security Agency (NSA) is a strong supporter of the academic mathematics community in the United States. Through the Mathematical Sciences Program, the NSA provides research funding and sabbatical opportunities for eligible faculty members in the mathematical sciences.

Grants for Research in Mathematics. The Mathematical Sciences Program (MSP) supports self-directed, unclassified research in the following areas of mathematics: algebra, number theory, discrete mathematics, probability, and statistics. The Research Grants program offers three types of grants: the Young Investigators Grant, the Standard Grant, and the Senior Investigators Grant. The program also supports conferences and workshops (typically in the range of US$15,000–$20,000) in these five mathematical areas. The program does not entertain research or conference proposals that involve cryptology. A Special Situation Proposal category is for research experience for undergraduates or events that do not fall within the typical "research" conference format. In particular, MSP is interested in supporting efforts that increase broader participation in the mathematical sciences, promote wide dissemination of mathematics, and promote the education and training of undergraduates and graduate students. Principal investigators, graduate students, and all other personnel supported by NSA grants must be U.S. citizens or permanent residents of the United States at the time of proposal submission. Proposals should be submitted **electronically** by **October 15, 2013**, via the program website: `http://www.nsa.gov/research/math_research/index.shtml`.

Sabbatical Program. NSA's Mathematics Sabbatical Program offers mathematicians, statisticians, and computer scientists the unique opportunity to develop skills in directions that would be nearly impossible anywhere else. Sabbatical employees work side by side with other NSA scientists on projects that involve cryptanalysis, coding theory, number theory, discrete mathematics, statistics and probability, and many other subjects. Visitors spend 9–24 months at NSA, and most find that within a very short period of time they are able to make significant contributions.

NSA pays 50 percent of salary and benefits during academic months and 100 percent of salary and benefits during summer months of the sabbatical detail. A housing supplement is available to help offset the cost of local lodging.

Applicants must be U. S. citizens and must be able to obtain a security clearance. A complete application includes a cover letter and curriculum vitae with list of significant publications. The cover letter should describe the applicant's research interests, programming experience and level of fluency, and how an NSA sabbatical would affect teaching and research upon return to academia. Additional information about the Sabbatical Program is available at `http://www.nsa.gov/research/math_research/sabbaticals/index.shtml`.

For more information about the Grants or Sabbaticals Program, please contact the program office at 301-688-0400. You may also send email to `mspgrants@nsa.gov`.

—Mathematical Sciences Program announcement

Research Experiences for Undergraduates

The Research Experiences for Undergraduates (REU) program supports active research participation by undergraduate students in any of the areas of research funded by the National Science Foundation (NSF). Student research may be supported in two forms: REU supplements and REU sites.

REU supplements may be requested for ongoing NSF-funded research projects or may be included in proposals for new or renewal NSF grants or cooperative agreements.

REU sites are based on independent proposals to initiate and conduct undergraduate research participation projects for a number of students. REU site projects may be based in a single discipline or academic department or on interdisciplinary or multidepartment research opportunities with a strong intellectual focus. Proposals with an international dimension are welcomed. A partnership with the Department of Defense supports REU sites in research areas relevant to defense. Undergraduate student participants supported with NSF funds in either supplements or sites must be citizens or permanent residents of the United States or its possessions.

Students may not apply to NSF to participate in REU activities. Students apply directly to REU sites and should consult the directory of active REU sites on the Web at http://www.nsf.gov/crssprgm/reu/reu_search.cfm. The deadline for full proposals for REU sites is **August 28, 2013**. Deadline dates for REU supplements vary with the research program; contact the program director for more information. The full program announcement can be found at the website http://www.nsf.gov/pubs/2009/nsf09598/nsf09598.htm.

—From an NSF announcement

Call for Nominations for 2012 Sacks Prize

The Association for Symbolic Logic (ASL) invites nominations for the 2012 Sacks Prize for the most outstanding doctoral dissertation in mathematical logic. The Sacks Prize consists of a cash award and five years' free membership in the ASL. Dissertations must have been defended by September 30, 2013.

General information about the prize is available at http://www.aslonline.org/info-prizes.html. For details about nomination procedures, see http://www.aslonline.org/Sacks_nominations.html.

—From an ASL announcement

Call for Nominations for Otto Neugebauer Prize

The European Mathematical Society (EMS) is seeking nominations for the Otto Neugebauer Prize for the History of Mathematics. The prize will be awarded "for highly original and influential work in the field of history of mathematics that enhances our understanding of either the development of mathematics or a particular mathematical subject in any period and in any geographical region." The award comprises a certificate including the citation and a cash prize of 5,000 euros (approximately US$6,500). The deadline for nominations is **December 31, 2013**. For further information see the website http://www.euro-math-soc.eu/otto_neugebauer_prize.html.

—From an EMS announcement

News from PIMS

The Pacific Institute for the Mathematical Sciences (PIMS) invites nominations of outstanding young researchers in the mathematical sciences for postdoctoral fellowships for the year 2014–2015.

Nominees must have a Ph.D. or equivalent (or expect to receive a Ph.D. by December 31, 2014) and must be within three years of receipt of the Ph.D. at the time of the nomination (i.e., Ph.D. received on or after January 1, 2011). The fellowship may be taken up at any time between September 1, 2014, and January 1, 2015. The fellowship is for one year and is renewable, contingent on satisfactory progress, for at most one additional year. PIMS postdoctoral fellows are expected to participate in all PIMS activities related to their areas of expertise and will be encouraged to spend time at more than one site.

Candidates must be nominated by at least one scientist or by a department (or departments) affiliated with PIMS. The fellowships are intended to supplement support provided by the sponsor and are tenable at any of the PIMS Canadian member universities: the University of Alberta, the University of British Columbia, the University of Calgary, the University of Lethbridge, the University of Regina, the University of Saskatchewan, Simon Fraser University, and the University of Victoria, as well as at the PIMS affiliate, the University of Northern British Columbia.

Complete applications must be uploaded to MathJobs by **December 1, 2013**. For further information, see the website http://www.pims.math.ca/scientific/postdoctoral or contact assistant.director@pims.math.ca.

—From a PIMS announcement

Inside the AMS

Math in Moscow Scholarships Awarded

The AMS has made awards to six mathematics students to attend the Math in Moscow program in the fall of 2013. Following are the names of the undergraduate students and their institutions: Alexander Dunlap, University of Chicago; Vishesh Jain, Stanford University; Jonathan Lai, University of Texas at Austin; Quan Nguyen, Hendrix College; David Richman, Massachusetts Institute of Technology; and Forrest Thurman, University of Central Florida.

Math in Moscow is a program of the Independent University of Moscow that offers foreign students (undergraduate or graduate students specializing in mathematics and/or computer science) the opportunity to spend a semester in Moscow studying mathematics. All instruction is given in English. The fifteen-week program is similar to the Research Experiences for Undergraduates programs that are held each summer across the United States.

The AMS awards several scholarships for U.S. students to attend the Math in Moscow program. The scholarships are made possible through a grant from the National Science Foundation. For more information about Math in Moscow, consult `http://www.mccme.ru/mathinmoscow` and the article "Bringing Eastern European mathematical traditions to North American students," *Notices*, November 2003, pages 1250–4.

—Elaine Kehoe

AMS Congressional Fellow Chosen

The American Mathematical Society is pleased to announce that Karen Saxe has been selected as its 2013–14 Congressional Fellow.

The fellowship provides a unique public policy learning experience, demonstrates the value of science-government interaction, and brings a technical background and external perspective to the decision-making process in Congress.

Saxe is currently chair of the mathematics, statistics, and computer science department at Macalester College. She received her Ph.D. in mathematics from the University of Oregon. The AMS will sponsor her fellowship through the Congressional Fellowship program administered by the American Association for the Advancement of Science (AAAS).

Fellows spend a year on the staff of a member of Congress or a congressional committee working as a special legislative assistant in legislative and policy areas requiring scientific and technical input. The fellowship program includes an orientation on congressional and executive branch operations and a year-long seminar series on issues involving science, technology, and public policy.

—AMS Washington Office

AMS Sponsors Exhibit on Capitol Hill

The AMS sponsored an exhibit at the nineteenth annual Coalition for National Science Funding (CNSF) exhibition and reception on Capitol Hill on May 7, 2013. Philip T. Gressman, University of Pennsylvania, presented work on "The Boltzmann Equation: Where Mathematics and Science Collide". The exhibition drew more than 285 people, including 10 members of Congress, to view thirty-five exhibits on research funded by the National Science Foundation.

Gressman and his colleague Robert M. Strain have found solutions to a 140-year-old, seven-dimensional equation that were not known to exist for more than a century despite its widespread use in modeling the behavior of gases.

Professor Philip T. Gressman, University of Pennsylvania, with Representative Eddie Bernice Johnson (D-TX-30), Ranking Member of the House Science, Space & Technology Committee.

The Boltzmann equation was developed to predict how gaseous material distributes itself in space and how it responds to changes in things like temperature, pressure, or velocity. Using modern mathematical techniques from the fields of partial differential equations and harmonic analysis, Gressman and Strain proved the global existence of classical solutions and rapid time decay to equilibrium for the Boltzmann equation with long-range interactions. Global existence and rapid decay imply that the equation correctly predicts that the solutions will continue to fit the system's behavior and not undergo any mathematical catastrophes such as a breakdown of the equation's integrity caused by a minor change within the equation. Rapid decay to equilibrium means that the effect of an initial small disturbance in the gas is short lived and quickly becomes unnoticeable.

The study also provides a new understanding of the effects due to grazing collisions, when neighboring molecules just glance off one another rather than collide head on. These glancing collisions turn out to be a dominant type of collision for the full Boltzmann equation with long-range interactions.

The Coalition for National Science Funding (CNSF) is an alliance of more than one hundred twenty-five scientific and professional societies and universities united by a concern for the future vitality of the national science, mathematics, and engineering enterprise. The coalition is chaired by Samuel M. Rankin III, associate executive director of the AMS and the director of its Washington office.

—AMS Washington Office

From the Public Awareness Office

Blog on Math Blogs—*Two mathematicians tour the mathematical blogosphere*. Editors Brie Finegold (University of Arizona) and Evelyn Lamb (freelance math and science writer) blog on blogs related to math in the news, mathematics research, applied mathematics, mathematicians, mathematics education, math and the arts, and more. Finegold and Lamb, both past AAAS-AMS Mass Media Fellows and Ph.D. mathematicians, select and write their thoughts on interesting blogs from around the world, as well as invite reactions from readers. Among the topics: "This Week in Number Theory" and "Building the World Digital Mathematical Library".
`blogs.ams.org/blogonmathblogs`

Mathematical Imagery. Recently added: Selected works from the 2013 Mathematical Art Exhibition held at the Joint Mathematics Meetings in San Diego, additional views of Carlo Séquin's "125 tetrahedra in 25 projected 5-cells", commissioned work to celebrate the AMS 125th anniversary, and more origami works by Robert J. Lang.
`ams.org/mathimagery`

—Annette Emerson and Mike Breen
AMS Public Awareness Officers
`paoffice@ams.org`

Deaths of AMS Members

Yousef Alavi, of Western Michigan University, died on May 21, 2013. Born on March 19, 1928, he was a member of the Society for 57 years.

Anne H. Allen, of Bennington, VT, died on May 23, 2012. Born on December 21, 1932, she was a member of the Society for 50 years.

Kenneth I. Appel of Dover, NH, died on April 19, 2013. Born on October 8, 1932, he was a member of the Society for 54 years.

Fariborz Asadian, of Warner Robins, Georgia, died on January 9, 2013. Born in August 1960, he was a member of the Society for 23 years.

S. Elwood Bohn, of Green Valley, Arizona, died on April 16, 2013. Born on March 11, 1927, he was a member of the Society for 54 years.

Don L. Burkholder, professor, University of Illinois, died on April 14, 2013. Born on January 19, 1927, he was a member of the Society for 56 years.

Maxon Buscher, of Cambridge, Massachusetts, died on February 8, 2013. Born on July 16, 1943, he was a member of the Society for 4 days.

Amos Joel Carpenter, of Indianapolis, Indiana, died on October 30, 2012. Born on May 12, 1939, he was a member of the Society for 20 years.

Robert W. Carroll, professor, University of Illinois, died on December 8, 2012. Born on May 10, 1930, he was a member of the Society for 54 years.

William R. Fuller, of Lafayette, Indiana, died on January 7, 2013. Born on October 27, 1920, he was a member of the Society for 62 years.

Wilfred Martin Greenlee, of Tucson, Arizona, died on March 11, 2013. Born on December 31, 1936, he was a member of the Society for 51 years.

Joan T. Hallett, of Reno, Nevada, died on November 27, 2012. Born on April 21, 1936, she was a member of the Society for 35 years.

Seymour Kass, of Brookline, Massachusetts, died on April 12, 2013. Born on April 13, 1926, he was a member of the Society for 52 years.

John F. Kellaher, of Sherborn, Massachusetts, died on September 19, 2012. Born on March 11, 1930, he was a member of the Society for 52 years.

Shoshichi Kobayashi, professor, University of California Berkeley, died on August 29, 2012. Born on January 4, 1932, he was a member of the Society for 56 years.

Horace Komm, of Bethesda, Maryland, died on February 19, 2013. Born in December 1916, he was a member of the Society for 74 years.

Edward H. Larson, of Eastham, Massachusetts, died on April 2, 2013. Born on October 14, 1927, he was a member of the Society for 53 years.

Jean-Pierre G. Meyer, professor, Johns Hopkins University, died on April 24, 2013. Born on August 5, 1929, he was a member of the Society for 62 years.

Ernest A. Michael, professor, University of Washington, died on April 29, 2013. Born on August 26, 1925, he was a member of the Society for 63 years.

Edward W. Odell, professor, University of Texas at Austin, died on January 9, 2013. Born on March 15, 1947, he was a member of the Society for 35 years.

Tanjiro Okubo, of Victoria, British Columbia, died on March 17, 2012. Born on December 23, 1915, he was a member of the Society for 51 years.

Frank W. J. Olver, professor, University of Maryland, died on April 23, 2013. Born on December 15, 1924, he was a member of the Society for 51 years.

Willard Parker, of Rock Hill, South Carolina, died on April 16, 2013. Born on August 10, 1938, he was a member of the Society for 47 years.

William M. Perel, of Pawnee, Oklahoma, died on January 20, 2013. Born on October 17, 1927, he was a member of the Society for 57 years.

James F. Reineck, professor, State University of New York at Buffalo, died on April 13, 2013. Born on May 31, 1957, he was a member of the Society for 32 years.

Ira S. Rubin, of Edison, New Jersey, died on February 6, 2013. Born on March 3, 1930, he was a member of the Society for 13 years.

Francis D. Ryan, of Toledo, Ohio, died on January 20, 2011. Born on December 9, 1926, he was a member of the Society for 48 years.

Paul T. Schaefer, of Honeoye, New York, died on December 25, 2012. Born on March 7, 1930, he was a member of the Society for 60 years.

Stephen T. Schibell, of Elkridge, Maryland, died on March 24, 2013. Born on July 7, 1953, he was a member of the Society for 31 years.

James B. Serrin, professor, University of Minnesota, died on August 23, 2012. Born on November 1, 1926, he was a member of the Society for 63 years.

John P. van Alstyne, of Ashville, North, Carolina, died on April 16, 2010. Born on September 12, 1921, he was a member of the Society for 59 years.

William P. Wardlaw, professor, U. S. Naval Academy, died on January 2, 2013. Born on March 3, 1936, he was a member of the Society for 48 years.

Mario Wschebor, of Uruguay, died on September 16, 2011. Born on December 3, 1939, he was a member of the Society for 25 years.

Andrei V. Zelevinsky, professor, Northeastern University, died on April 10, 2013. Born on January 30, 1953, he was a member of the Society for 21 years.

For Your Information

Museum of Mathematics Awards Rosenthal Prize

Scott Goldthorp and Patrick Honner, respectively, have been chosen the winner of and runner-up for the 2012 Rosenthal Prize for Innovation in Math Teaching, awarded by the Museum of Mathematics (MoMath). Goldthorp, a teacher at Rosa International Middle School in Cherry Hill, New Jersey, was awarded a US$25,000 cash prize; and Honner, a teacher at Brooklyn Technical High School in Brooklyn, New York, was awarded a US$10,000 cash prize. The annual Rosenthal Prize for Innovation in Math Teaching is designed to recognize and promote hands-on math teaching in upper elementary and middle school classrooms. Each year the winning teacher is awarded a cash prize of US$25,000, and the winning activity is shared with interested teachers across the country.

—MoMath announcement

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—MoMath announcement

The State of the *Proceedings of the American Mathematical Society*

Ken Ono

The American Mathematical Society launched the *Proceedings of the American Mathematical Society* (PAMS) in 1950. Its very first issue contained gems such as the classical notes: "On a theorem of Erdős and Turán" on the distribution of prime numbers by Alfréd Rényi, and "On the radical of a Lie algebra" by Harish-Chandra. Such articles established the tradition of excellence which PAMS now enjoys. PAMS has now published approximately 26,000 excellent short research articles in all areas of pure and applied mathematics. Digitized versions of PAMS legacy articles, which include papers published from 1950 to 1995, as well as all articles published prior to 2007, are freely available on the AMS website.

The first volume of PAMS was edited by Gustav A. Hedlund, Nathan Jacobson, and A. C. Schaeffer, with the assistance of ten auxiliary editors. The editorial structure of PAMS has evolved over time, and it is now a large enterprise. Today the board consists of almost forty mathematicians whose expertise covers all areas of pure and applied mathematics. The editors are divided into the following five broad areas of mathematics:

- ODE, PDE, Global Analysis, and Dynamical Systems
- Topology and Geometry
- Analysis
- Algebra, Number Theory, Combinatorics, and Logic
- Applied Mathematics, Probability, and Statistics.

Each of these five categories has a coordinating editor, and I serve as the managing editor who oversees the entire enterprise. I was fortunate to follow in the footsteps of Ron Fintushel, my predecessor as managing editor, who did an exemplary job steering PAMS. I have been on the job since February 2010.

PAMS has undergone a number of fairly recent changes which I wish to explain. All submissions are now handled by *Editflow*, a Web-based editorial tool which keeps track of all documents and information related to submitted articles. Editors have at their fingertips complete email histories, a database of referees, etc. Authors also have easy access to information pertaining to their submissions. Of course, no system is perfect, and we are continually revising and improving *Editflow*. The AMS staff has been doing a great job of helping the editors with the daily operations of PAMS. Thanks to their efforts and *Editflow*, we can realistically aim to secure referee reports for submitted articles within three months of submission. Of course, this is subject to the case-by-case goodwill of referees. Manuscripts are in the hands of a cognizant editor within 1–2 days of submission, and referees are approached shortly thereafter.

In 2008 PAMS began accepting papers up to fifteen pages in length, a significant increase compared to the previous limit of ten pages. This change was approved for a variety of reasons. The change in policy had several noticeable implications, the most important being the opportunity to receive many more high quality submissions, some of which would otherwise have been submitted to the *Transactions of the American Mathematical Society* (TAMS), or to a nonAMS journal. And indeed, the number of submissions to PAMS has skyrocketed, and PAMS currently receives many more quality submissions than it can publish. To address this problem the Editorial Board has tightened acceptance standards. To be considered for publication in PAMS, a paper must be of high quality, original, well written, and be of fundamental interest to a wide range of mathematicians. Articles which fall short are not sent out for review. After a few months of this new policy, we have seen

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DOI: http://dx.doi.org/10.1090/noti1035

our backlog drop, and I have high hopes that PAMS will reach a healthy and sustainable equilibrium within 1–2 years.

There are further changes afoot with respect to PAMS. In April the Council of the AMS discussed a proposal to launch open access companions to PAMS and TAMS. This proposal is in response to various factors, including mandates worldwide concerning the free access of government funded research (most notably in the UK), and advocacy for open access by the communities of researchers and librarians. I look forward to these projects with optimism, and I firmly believe that the future is bright for PAMS. The traditional PAMS, combined with the new open access companion journal, will together continue to provide an outstanding forum for short high quality research articles. Let's continue the great tradition established by Hedlund, Jacobson, and Schaeffer in 1950 by writing and publishing the next generation's analogs of the wonderful articles by Rényi and Harish-Chandra.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.wustl.edu in the case of the editor and smf@ams.org in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

July 24, 2013: Proposals for NSF CAREER Awards. See the website http://www.nsf.gov/pubs/2011/nsf11690/nsf11690.htm.

July 31, 2013: Nominations for SASTRA Ramanujan Prize. See the website http://www.math.ufl.edu/sastra-prize/nominations-2013.html.

August 1, 2013: Applications for August review for National Academies Research Associateship Programs. See the website http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

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Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

August 13, 2013: Full proposals for NSF Scholarships in Science, Technology, Engineering, and Mathematics (S-STEM) program. See http://www.nsf.gov/pubs/2012/nsf12529/nsf12529.htm.

August 28, 2013: Full proposals for NSF REU sites. See "Mathematics Opportunities" in this issue.

September 1–November 15, 2013: Applications for travel grants for ICM 2014. See "Mathematics Opportunities" in this issue.

September 15, 2013: Applications for spring 2014 semester of Math in Moscow. See http://www.mccme.ru/mathinmoscow or write to: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; e-mail: mim@mccme.ru. Information and application forms for the AMS scholarships are available on the AMS website at http://www.ams.org/programs/travel-grants/mimoscow or by writing to: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence RI 02904-2294; email student-serv@ams.org.

September 15, 2013: Deadline for nominations for AMS Award for Impact on the Teaching and Learning of Mathematics. See "Mathematics Opportunities" in this issue.

September 16, 2013: Nominations for Sloan Fellowships. Contact Sloan Research Fellowships, Alfred P. Sloan Foundation, 630 Fifth Avenue, Suite 2550, New York, NY 10111-0242, or consult the foundation's website: http://www.sloan.org/fellowships.

September 20, 2013: Full proposals for NSF Focused Research Groups. See "Mathematics Opportunities" in this issue.

September 30, 2013: Nominations for W. K. Clifford Prize. See http://www.wkcliffordprize.org.

October 1, 2013: Applications for AWM Travel Grants and Mathematics Education Research Travel Grants. See https://sites.google.com/site/awmmath/programs/travel-grants or telephone: 703-934-0163; email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

October 1, 2013: Nominations for Parzen Prize. Send to Thomas Wehrly, Department of Statistics, 3143 TAMU, Texas A&M University, College Station, Texas 77843-3143. For more information see the website http://www.stat.tamu.edu/awards-and-prize-details.php?prizeid=7.

October 4, 2013: Letters of intent for NSF Program ADVANCE Institutional Transformation and Institutional Transformation Catalyst awards. See http://www.nsf.gov/pubs/2012/nsf12584/nsf12584.htm?WT.mc_id=USNSF_36&WT.mc_ev=click.

October 15, 2013: Proposals for NSA Grants for Research in Mathematics. See "Mathematics Opportunities" in this issue.

October 16, 2013: Proposals for National Science Foundation (NSF) Postdoctoral Research Fellowships. See http://www.nsf.gov/pubs/2012/nsf12496/nsf12496.htm.

November 1, 2013: Applications for November review for National Academies Research Associateship Programs. See the website http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

November 12, 2013: Full proposals for NSF Program ADVANCE Institutional Transformation and Institutional Transformation Catalyst awards. See http://www.nsf.gov/pubs/2012/nsf12584/nsf12584.htm?WT.mc_id=USNSF_36&WT.mc_ev=click.

December 1, 2013: Applications for PIMS postdoctoral fellowships. See "Mathematics Opportunities" in this issue.

December 2, 2013: Nominations for Ferran Sunyer i Balaguer Prize. See http://ffsb.iec.cat.

December 31, 2013: Nominations for Otto Neugebauer Prize. See "Mathematics Opportunities" in this issue.

February 15, 2014: Nominations for AWM–Joan & Joseph Birman Prize in Topology and Geometry. See "Mathematics Opportunities" in this issue.

April 15, 2014: Applications for fall 2014 semester of Math in Moscow. See "Mathematics Opportunities" in this issue.

Contact Information for Mathematics Institutes

American Institute of Mathematics
360 Portage Avenue
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Fax: 650-845-2074
email: conrey@aimath.org
http://www.aimath.org

Stefan Banach International Mathematical Center
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P.O. Box 21
00-956 Warszawa, Poland
Telephone: 48 22 522-82-32
Fax: 48 22 622-57-50
email: Banach.Center.Office@impan.pl
http://www.impan.gov.pl/BC

Banff International Research Station
University of British Columbia
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Vancouver, BC V6T 1Z4, Canada
Telephone: 604-822-1649
Fax: 604-822-0883
email: birs-director@birs.ca
http://www.birs.ca

Banff International Research Station
c/o The Banff Centre
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Banff, AB T1L 1H5, Canada
Telephone: 403-763-6999
Fax: 403-763-6990
email: birsmgr@birs.ca
http://www.birs.ca

Basque Center for Applied Mathematics (BCAM)
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48009 Bilbao, Basque Country, Spain
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Fax: +34 946 567 843
email: info@bcamath.org
http://www.bcamath.org

Center for Discrete Mathematics and Theoretical Computer Science (DIMACS)
CoRE Building, 4th Floor
Rutgers University
96 Frelinghuysen Road
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Telephone: 732-445-5930
Fax: 732-445-5932
email: center-admin@dimacs.rutgers.edu
http://dimacs.rutgers.edu

Center for Scientific Computation and Mathematical Modeling (CSCAMM)
University of Maryland
4146 CSIC Building #406
Paint Branch Drive
College Park, MD 20742-3289
Telephone: 301-405-0652
Fax: 301-314-6674
email: info@cscamm.umd.edu
http://www.cscamm.umd.edu/

Centre International de Rencontres Mathématiques (CIRM)
163, avenue de Luminy, Case 916
F-13288 Marseille Cedex 09, France
Telephone: 33 04 91 83 30 00
Fax: 33 04 91 83 30 05
http://www.cirm.univ-mrs.fr

Centre for Mathematics and Its Applications (CMA)
Building 27
Australian National University
ACT 0200, Australia
Telephone: 61 2 6125 2897
Fax: 61 2 6125 5549
email: admin.research.msi@anu.edu.au
http://maths.anu.edu.au/about-us/structure/centre-mathematics-its-applications

Centre for Quantum Geometry of Moduli Spaces
Aarhus University
Ny Munkegade 118, Bldg. 1530
DK-8000 Aarhus C, Denmark
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Fax: 45 8613 1769
email: qgm@au.dk
http://qgm.au.dk/

Centre de Recerca Matemática (CRM)
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Barcelona, Spain
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email: crm@crm.cat
http://www.crm.cat

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Fax: 514-343-2254
email: CRM@CRM.UMontreal.ca
http://www.crm.umontreal.ca

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Fax: 050 509177
email: crm@crm.sns.it
http://www.crm.sns.it/

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Fax: 91-44-2747-0225
email: office@cmi.ac.in
http://www.cmi.ac.in

Chern Institute of Mathematics
Nankai University
Tianjin 300071, China
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Fax: 86-22-2350-1532
email: cim@nankai.edu.cn
http://www.nim.nankai.edu.cn/nim_e/index.htm

Euler International Mathematical Institute
nab. Fontanki, 27
St. Petersburg 191023, Russia
Telephone: 7 960 279 10 37
Fax: 7 812 234 05 74
email: admin@euler.pdmi.ras.ru
http://www.pdmi.ras.ru/EIMI/index.html

Fields Institute for Research in Mathematical Sciences
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Fax: 416 348 9714
email: inquiries@fields.utoronto.ca
http://www.fields.utoronto.ca/

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8092 Zürich, Switzerland
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Fax: 41 44 632 1614
email: admin@fim.math.ethz.ch
http://www.fim.math.ethz.ch/

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Le Bois Marie 35, route de Chartres
F-91440 Bures sur Yvette, France
Telephone: 33 1 60 92 66 00
Fax: 33 1 60 92 66 69
http://www.ihes.fr

Institut Henri Poincaré
11, rue Pierre et Marie Curie
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Telephone: 33 01 44 27 67 89
Fax: 33 01 44 07 09 37
http://ihp.fr/en

Institut Mittag-Leffler
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http://www.math.ias.edu/

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Fax: 82 2 958 3770
http://www.kias.re.kr/en/index.jsp

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http://www.mbi.osu.edu

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`http://www.kurims.kyoto-u.ac.jp/`

Erwin Schrödinger International Institute for Mathematical Physics
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Fax: 91 22 22804610/4611
`http://www.tifr.res.in/index.php/en/`

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Fax: +44 (0)24 7652 4182
email: `Miles.Reid@maths.warwick.ac.uk`
`http://www.maths.warwick.ac.uk/mrc/index.html`

Weierstrass Institute for Applied Analysis and Stochastics
Mohrenstrasse 39
10117 Berlin, Germany
Telephone: 49 30 20372 0
Fax: 49 30 20372 303
email: `contact@wias-berlin.de`
`http://www.wias-berlin.de/`

Book List

The Book List highlights recent books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. Suggestions for books to include on the list may be sent to `notices-booklist@ams.org`.

*Added to "Book List" since the list's last appearance.

**An Accidental Statistician: The Life and Memories of George E. P. Box,* by George E. P. Box. Wiley, April 2013. ISBN-13: 978-1-118-40088-3.

American Mathematicians as Educators, 1893–1923: Historical Roots of the "Math Wars", by David Lindsay Roberts. Docent Press, July 2012, ISBN-13: 978-09837-004-49.

Assessing the Reliability of Complex Models: Mathematical and Statistical Foundations of Verification, Validation, and Uncertainty Quantification, by the National Research Council. National Academies Press, 2012. ISBN-13: 978-0-309-25634-6.

The Best Writing on Mathematics 2012, edited by Mircea Pitici. Princeton University Press, November 2012. ISBN-13: 978-06911-565-52.

The Big Questions: Mathematics, by Tony Crilly. Quercus, April 2011. ISBN-13:978-18491-624-01. (Reviewed October 2012.)

Charles S. Peirce on the Logic of Number, by Paul Shields. Docent Press, October 2012. ISBN-13: 978-0-9837004-7-0.

The Continuity Debate: Dedekind, Cantor, du Bois-Reymond, and Peirce on Continuity and Infinitesimals, by Benjamin Lee Buckley. Docent Press, December 2012. ISBN-13: 978-0-9837004-8-7.

Decoding the Heavens: A 2,000-Year-Old Computer—and the Century-Long Search to Discover Its Secrets, by Jo Marchant. Da Capo Press, February 2009. ISBN-13: 978-03068-174-27. (Reviewed June/July 2013).

The Foundations of Geometry and Religion from an Abstract Standpoint, by Salilesh Mukhopadhyay. Outskirts Press, July 2012. ISBN-13: 978-1-4327-9424-8.

The Fractalist: Memoir of a Scientific Maverick, by Benoit Mandelbrot.

Pantheon, October 2012. ISBN-13: 978-03073-773-57.

Fueling Innovation and Discovery: The Mathematical Sciences in the 21st Century, by the National Research Council. National Academies Press, 2012. ISBN-13: 978-0-309-25473-1.

Galileo's Muse: Renaissance Mathematics and the Arts, by Mark Austin-Peterson. Harvard University Press, October 2011. ISBN-13: 978-06740-597-26. (Reviewed November 2012.)

Game Theory and the Humanities: Bridging Two Worlds, by Steven J. Brams. MIT Press, September 2012. ISBN-13: 978-02625-182-53.

Games and Mathematics: Subtle Connections, by David Wells. Cambridge University Press, November 2012. ISBN-13: 978-11076-909-12.

Girls Get Curves: Geometry Takes Shape, by Danica McKellar. Plume, July 2013. ISBN-13: 978-04522-987-43.

The Golden Ticket: P, NP, and the Search for the Impossible, by Lance Fortnow. Princeton University Press, March 2013. ISBN-13: 978-06911-564-91.

Google's PageRank and Beyond: The Science of Search Engine Rankings, by Amy Langville and Carl Meyer. Princeton University Press, February 2012. ISBN-13: 978-06911-526-60.

Guesstimation 2.0: Solving Today's Problems on the Back of a Napkin, by Lawrence Weinstein. Princeton University Press, September 2012. ISBN-13: 978-06911-508-02.

Heavenly Mathematics: The Forgotten Art of Spherical Trigonometry, by Glen Van Brummelen. Princeton University Press, December 2012. ISBN-13: 978-06911-489-22.

Henri Poincaré: Impatient Genius, by Ferdinand Verhulst. Springer, August 2012. ISBN-13: 978-14614-240-62.

Henri Poincaré: A Scientific Biography, by Jeremy Gray. Princeton University Press, November 2012. ISBN-13: 978-06911-527-14.

How to Study as a Mathematics Major, by Lara Alcock. Oxford University Press, March 2013. ISBN-13: 978-0199661312.

I Died for Beauty: Dorothy Wrinch and the Cultures of Science, by Marjorie Senechal. Oxford University Press, December 2012. ISBN-13:978-01997-325-93.

Ibn al-Haytham's Theory of Conics, Geometrical Constructions and Practical Geometry, by Roshdi Rashed. Routledge, February 2013. ISBN-13: 978-0-415-58215-5.

In Pursuit of the Unknown: 17 Equations That Changed the World, by Ian Stewart. Basic Books, March 2012. ISBN-13: 978-04650-297-30. (Reviewed December 2012.)

Infinity: New Research Frontiers, edited by Michael Heller and W. Hugh Woodin. Cambridge University Press, February 2011. ISBN-13: 978-11070-038-73.

The Infinity Puzzle: Quantum Field Theory and the Hunt for an Orderly Universe, by Frank Close. Basic Books, November 2011. ISBN-13: 978-04650-214-44. (Reviewed September 2012.)

Introduction to Mathematical Thinking, by Keith Devlin. Keith Devlin, July 2012. ISBN-13: 978-06156-536-31.

Invisible in the Storm: The Role of Mathematics in Understanding Weather, by Ian Roulstone and John Norbury. Princeton University Press, February 2013. ISBN-13: 978-06911-527-21.

The Joy of x: A Guided Tour of Math, from One to Infinity, by Steven Strogatz. Eamon Dolan/Houghton Mifflin Harcourt, October 2012. ISBN-13: 978-05475-176-50.

Late Style: Yuri I. Manin Looking Back on a Life in Mathematics. A DVD documentary by Agnes Handwerk and Harrie Willems. Springer, March 2012. ISBN NTSC: 978-3-642-24482-7; ISBN PAL: 978-3-642-24522-0. (Reviewed January 2013.)

Lemmata: A Short Mathematical Thriller, by Sam Peng. CreateSpace, December 2011. ISBN-13: 978-14681-442-39.

Levels of Infinity: Selected Writings on Mathematics and Philosophy, by Hermann Weyl. Edited by Peter Pesic. Dover Publications, February 2013. ISBN-13: 978-0486489032.

The Logician and the Engineer: How George Boole and Claude Shannon Created the Information Age, by Paul J. Nahin, Princeton University Press, October 2012. ISBN-13: 978-06911-510-07.

Manifold Mirrors: The Crossing Paths of the Arts and Mathematics, by Felipe Cucker. Cambridge University Press, April 2013. ISBN-13: 978-0521728768.

Math Goes to the Movies, by Burkard Polster and Marty Ross. Johns Hopkins University Press, July 2012. ISBN-13: 978-14214-048-44.

Math Is Murder, by Robert C. Brigham and James B. Reed. Universe, March 2012. ISBN-13: 978-14697-972-81.

**Math on Trial: How Numbers Get Used and Abused in the Courtroom,* by Leila Schneps and Coralie Colmez. Basic Books, March 2013. ISBN-13: 978-04650-329-21. (Reviewed in this issue.)

Mathematical Excursions to the World's Great Buildings, by Alexander J. Hahn. Princeton University Press, July 2012. ISBN-13: 978-06911-452-04.

The Mathematical Writings of Évariste Galois, edited by Peter M. Neumann. European Mathematical Society, October 2011. ISBN-13: 978-3-03719-104-0. (Reviewed December 2012.)

A Mathematician Comes of Age, by Steven G. Krantz. Mathematical Association of America, December 2011. ISBN-13: 978-08838-557-82.

A Mathematician's Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form, by Paul Lockhart. Bellevue Literary Press, April 2009. ISBN-13: 978-1-934137-17-8. (Reviewed April 2013.)

Mathematics in Popular Culture: Essays on Appearances in Film, Fiction, Games, Television and Other Media, edited by Jessica K. Sklar and Elizabeth S. Sklar. McFarland, February 2012. ISBN-13: 978-07864-497-81.

**Mathematics under the Microscope: Notes on Cognitive Aspects of Mathematical Practice,* by Alexandre V. Borovik. AMS, January 2010. ISBN-13: 978-0-8218-4761-9.

Maverick Genius: The Pioneering Odyssey of Freeman Dyson, by Phillip F. Schewe. Thomas Dunne Books, February 2013. ISBN-13:978-03126-423-58.

Meaning in Mathematics, edited by John Polkinghorne. Oxford University Press, July 2011. ISBN-13: 978-01996-050-57. (Reviewed May 2013.)

Measurement, by Paul Lockhart. Belknap Press of Harvard University

About the cover

No surprises from Catalan's constant

This month's cover was suggested by the article in this issue written by David H. Bailey *et alia*. It exhibits two ways to portray 1,000,000 "digits" taken from somewhere in the base 4 expansion of the Catalan constant. In the foreground on the cover is the track of a random walk based on the expansion, with 0 equal to a step right, 1 a step up, etc. It is colored red at the start and dark blue at the end. In the background these digits are laid out, top to bottom and left to right, as shaded pixels.

The track of the random walk overlays three pseudo-random walks in pastel colors, and the behavior of the digits' track is not visibly inconsistent with a random distribution. At first this might not seem like an interesting way to look at such an expansion, but the article "Walking on real numbers", written by Jon Borwein et al. and published recently in the *Mathematical Intelligencer*, demonstrates that it can often reveal interesting information. It sometimes leads to curious patterns—as for example, the walk illustrated below (redrawn from an image in that article), based on the binary expansion

$$0.10111111011\ldots$$

of the concatenation of successive primes 2, 3, 5, 7, 11…. This is known to be a normal number, but the expansion does not appear otherwise to be very random.

We thank Jon Borwein and Fran Aragon for supplying us with the expansion of the Catalan constant, as well as helpful comments.

—*Bill Casselman*
Graphics Editor
(notices-covers@ams.org)

Press, September 2012. ISBN-13: 978-06740-575-55.

Nine Algorithms That Changed the Future: The Ingenious Ideas That Drive Today's Computers, by John MacCormick.Princeton University Press, December 2011. ISBN-13: 978-06911-471-47.

The Noether Theorems: Invariance and Conservation Laws in the Twentieth Century, by Yvette Kosmann-Schwarzbach. Springer, December 2010. ISBN-13: 978-03878-786-76. (Reviewed in this issue.)

On the Formal Elements of the Absolute Algebra, by Ernst Schröder (translated and with additional material by Davide Bondoni; with German parallel text). LED Edizioni Universitarie, 2012. ISBN-13: 978-88-7916-516-7.

Paradoxes in Probability Theory, by William Eckhardt. Springer, September 2012. ISBN-13: 978-94007-513-92. (Reviewed March 2013.)

Peirce's Logic of Continuity: A Conceptual and Mathematical Approach, by Fernando Zalamea. Docent Press, December 2012. ISBN-13: 978-0-9837004-9-4.

Proving Darwin: Making Biology Mathematical, by Gregory Chaitin. Pantheon, May 2012. ISBN-13: 978-03754-231-47.

Relations between Logic and Mathematics in the Work of Benjamin and Charles S. Peirce, by Allison Walsh. Docent Press, October 2012. ISBN-13: 978-0-9837004-6-3.

The Search for Certainty: A Journey through the History of Mathematics, 1800-2000, edited by Frank J. Swetz. Dover Publications, September 2012. ISBN-13: 978-04864-744-27.

Secrets of Triangles: A Mathematical Journey, by Alfred S. Posamentier and Ingmar Lehman. Prometheus Books, August 2012. ISBN-13: 978-16161-458-73.

Seduced by Logic: Emilie Du Châtelet, Mary Somerville and the Newtonian Revolution, by Robyn Arianrhod. Oxford University Press, September 2012. ISBN-13: 978-01999-316-13. (Reviewed June/July 2013).

Selected Papers: Volume II: On Algebraic Geometry, including correspondence with Grothendieck, by David Mumford. Edited by Amnon Neeman, Ching-Li Chai, and Takahiro Shiota. Springer, July 2010. ISBN-13: 978-03877-249-11. (Reviewed February 2013.)

The Signal and the Noise: Why So Many Predictions Fail—But Some Don't, by Nate Silver. Penguin Press, September 2012. ISBN-13:*978-15942-041-11.*

Simon: The Genius in My Basement, by Alexander Masters. Delacorte Press, February 2012. ISBN-13: 978-03853-410-80.

Thinking in Numbers: On Life, Love, Meaning, and Math, by Daniel Tammet. Little, Brown and Company, July 2013. ISBN-13: 978-03161-873-74.

Thinking Statistically, by Uri Bram. CreateSpace Independent Publishing Platform, January 2012. ISBN-13: 978-14699-123-32.

Transcending Tradition: Jewish Mathematicians in German Speaking Academic Culture, edited by Birgit Bergmann, Moritz Epple, and Ruti Ungar. Springer, January 2012. ISBN-13: 978-36422-246-38. (Reviewed February 2013.)

Turbulent Times in Mathematics: The Life of J. C. Fields and the History of the Fields Medal, by Elaine McKinnon Riehm and Frances Hoffman. AMS, November 2011. ISBN-13: 978-0-8218-6914-7.

Uneducated Guesses: Using Evidence to Uncover Misguided Education Policies, by Howard Wainer. Princeton University Press, August 2011. ISBN-13: 978-06911-492-88. (Reviewed June/July 2012.)

The Universe in Zero Words: The Story of Mathematics as Told through Equations, by Dana Mackenzie. Princeton University Press, April 2012. ISBN-13: 978-06911-528-20. (Reviewed December 2012.)

Visions of Infinity: The Great Mathematical Problems, by Ian Stewart. Basic Books, March 2013. ISBN-13: 978-04650-224-03.

A Wealth of Numbers: An Anthology of 500 Years of Popular Mathematics Writing, edited by Benjamin Wardhaugh. Princeton University Press, April 2012. ISBN-13: 978-06911-477-58. (Reviewed March 2013.)

Who's #1?: The Science of Rating and Ranking, by Amy N. Langville and Carl D. Meyer. Princeton University Press, February 2012. ISBN-13: 978-06911-542-20. (Reviewed January 2013.)

Doctoral Degrees Conferred

2011–2012

ALABAMA

Auburn University (12)

Department of Mathematics and Statistics

Duncan, Bryce, Bell numbers of graphs

Gong, Yankun, Rank based methods for repeated measurement data

Kwessi Nyandjou, Eddy, Efficient rank regression with wavelet estimated scores

McClanahan, Stacy, The metamorphosis of 2-fold triple systems into maximum packings of $2K_n$ with 4-cycles

Noble, Matthew, Colorful results on Euclidean distance graphs and their chromatic numbers

Rogers, Julie, Generalizing clatworthy group divisible designs

Scible, Greggory, Finitely generated modules over noncommutative chain rings

Sehgal, Nidhi, Cycle systems

Varagona, Scott, Simple techniques for detecting decomposability or indecomposability of generalized inverse limits

Yin, Shuxin, Nonparametric methods for classification and related feature selection procedures

Yuceturk, Guven, Gregarious path decomposition of some graphs

Zhang, Aijun, Spatial spread and front propagation dynamics of nonlocal monostable equations in periodic habitats

University of Alabama (5)

Department of Mathematics

Bishop-Ross, Rachel, The road trip property: An aid in classifying groups with quadratic isoperimetric inequalities

Ginting, Maydison, A strategy to control the running risk in hedging a long-term supply commitment with short-dated futures contracts

Green, Michael, Graphs of groups

Winkles, Nathan, Performance evaluation of inexact GMRES

Yao, Pengfei, Matched interface boundary (MIB) enhanced multiresolution time-domain (MRTD) algorithm for electromagnetic simulations

University of Alabama at Birmingham (5)

Department of Biostatistics

Bentley, John, An examination of statistical methods for longitudinal mediation modeling

Birkner, Thomas, Hierarchical and Bayesian approaches to estimating prevalence based on pool screening

Li, Jun, Bayesian hierarchical generalized linear models for detecting (rare) haplotype-haplotype and haplotype-environment interactions in genetic association analysis

Phadnis, Milind, Robust non-parametric regression approach for competing cause censored survival mortality data with covariates

Wineinger, Nathan, Statistical methods in the analysis of copy number variation data

University of Alabama-Huntsville (3)

Department of Mathematical Sciences

Jackson, Tobin, Convergence analysis of fully discrete finite element approximations for an unsteady doubly diffusive convection model

O'Neal, Frank Allen, Neighborhood sum parameters on graphs

Wu, Yinshu, Traveling wave fronts for a diffusive competition model with time delay

University of Alabama-Tuscaloosa (2)

Information Systems, Statistics, and Management Science Department

Dovoedo, Yinaze, Contributions to outlier detection methods: Some theory and applications

Yuan, Fang, Construction of estimation-equivalent second-order split-split-plot designs

ARIZONA

Arizona State University (13)

School of Human Evolution and Social Change

Kareva, Irina, Niche construction, sustainability and evolutionary ecology of cancer

Lopez, Raquel, Integrability of quadratic non-autonomous quantum linear systems

Morin, Benjamin, Computational and analytical mathematical techniques for modeling heterogeneity

Soho, Edme, Immune response in the study of infectious diseases (co-infection) in an endemic region

Torres-Garcia, Griselle, Size structured epidemic models

School of Mathematical and Statistical Sciences

Bland, Adam, Reachability in K-colored tournaments

Chang, Shaojie, Computational study of the cone-horizontal cell feedback mechanism in the outer-plexiform layer of cat retina

DeBiasio, Louis, Optimal degree conditions for spanning subgraphs

Han, Zhun, A chemostat model of bacteriophage-bacteria interaction with infinite distributed delays

Karl, Andrew, A correlated random effects model for nonignorable missing data in value-added assessment of teacher effects

Milovanovic, Jelena, Chi-square orthogonal components for assessing goodness-of-fit of multidimensional multinomial data

Sanborn, Barbara, Symplectic topology and geometric quantum mechanics

The above list contains the names and thesis titles of recipients of doctoral degrees in the mathematical sciences (July 1, 2011, to June 30, 2012) reported in the 2012 Annual Survey of the Mathematical Sciences by 197 departments in 143 universities in the United States. Each entry contains the name of the recipient and the thesis title. The number in parentheses following the name of the university is the number of degrees listed for that university.

Weber, Eric, Students' ways of thinking about two-variable functions and rate of change in space

University of Arizona (11)

Department of Mathematics

Gorlina, Yuliya, Weighted Delaunay triangulations of piecewise-flat surfaces

Huang, Chuan, Novel methods for T_2 estimation using highly undersampled radial MRI data

Johnson, Matthew, A classification of all Hecke eigenform product identities

Lin, Lizhen, Nonparametric inference for bioassay

Petrov, Aleksandar, On A-expansions of Drinfeld modular forms

Piercey, Victor, Resolutions of collinearity among four points in the complex projective plane

Schettler, Jordan, The change in lambda invariants for cyclic p-extensions of $\mathbb{Z}_p$-fields

Program in Applied Mathematics

Chiquete, Carlos, Stability and receptivity of idealized detonation

Durickovic, Bojan, Waves on elastic rods and helical spring problems

Gemmer, John, Swelling thin elastic sheets and the hyperbolic plane

Xiong, Hui, Nonparametric statistical approaches for benchmark dose estimation in quantitative risk assessment

ARKANSAS

University of Arkansas at Fayetteville (2)

Mathematical Sciences Department

Griffin, Heather, Pointwise Schauder estimates for parabolic equations in Carnot groups

Rosell, Pablo, Limiting behavior of nondeterministic fillings of the torus by colored squares

CALIFORNIA

California Institute of Technology (7)

Department of Applied and Computational Mathematics

Hu, Xin, Multiscale modeling and computation of 3D incompressible turbulent flows

Huang, Jinghao, Discrete differential form subdivision and vector field generation over volumetric domain

Lintner, Stephane, High-order integral equation methods for diffraction problems involving screens and apertures

Department of Mathematics

Simanek, Brian, Asymptotic properties of orthogonal and extremal polynomials

Venka Teswaran, Vidya, Vanishing results for Hall-Littlewood polynomials

Walker, Alden, Surface maps into free groups

Wu, Yitao, On the p-adic local invariant cycle theorem

Claremont Graduate University (7)

School of Mathematical Sciences

Ambrose, Martin, Adaptive Monte Carlo algorithms for continuous and discrete transport problems

Caballero, David, Discrete variable representation of the angular variables in quantum three-body scattering

Chambers, Dwayne, Topological symmetry groups of complete graphs

Jalali, Sammuel, Wireless channel equalization in digital communication systems

Paladugu, Sri, Functional inference from molecular networks systems biology

Torres Barba, David, Assessment of functional activity in isolated cardiomyocytes using computational methods

Vochanel, Michael, Problems in GPS accuracy

Stanford University (22)

Department of Mathematics

Bormashenko, Olena, Permutations with interval restrictions

Cheng, Man Chuen, A duality theorem for Deligne-Mumford stacks with respect to Morava K-theory

Chowdhury, Atoshi, Compactifying Picard stacks over degenerations of surfaces

Fadnavis, Sukhada, Graph colorings and graph limits

Fong, Tsz Ho, New results on the singularity analysis of the Kaehler-Ricci flow

Georgieva, Penka, Orientability of moduli spaces and open Gromov-Witten invariants

Hough, Robert, Distribution problems in number theory

Krummel, Brian, Existence and regularity of branched minimal submanifolds

Lee, Jonathan, Stratifications and equivalent cohomology of spaces of upper-triangular square-zero matrices

Li, Man Chun, On a free boundary problem for embedded minimal surfaces and instability theorems for manifolds with positive isotropic curvature

Li, Xiannan, The behavior of L-functions at the edge of the critical strip and applications

Nguyen, Dung, Characteristic numbers of genus one space curves

Sher, David, Conic degeneration and the determinant of the Laplacian

Shkolnikov, Mykhaylo, Competing particle systems and their applications

Smith, Aaron, Some analyses of Markov chains by the coupling method

Department of Statistics

Chen, Su, Consistence and convergence rate of Markov chain quasi Monte Carlo with examples

Li, Jun, Differential expression identification and false discovery rate estimation in RNA-Seq data

Rivera, Camilo, Detection of bumps on the intensity function of an inhomogeneous Poisson process

Shen, Jeremy, Change-point models on point processes and applications in genomics

Sun, Yunting, On latent systemic effects in multiple hypotheses

Tibshirani, Ryan, The solution path of the generalized lasso

Zhang, Feng, Cross-validation and regression analysis in high-dimensional sparse linear models

University of California, Berkeley (31)

Department of Mathematics

Antunovic, Tonci, Two probabilistic models of competition

Bayer, Robertson, Low for speed

Brown, Morgan, Cox rings and partial amplitude

Chan, Melody, Tropical curves and metric graphs

Chen, Yann-shin, Impulse control and optimal stopping

Choi, Ka-Lun, Constructing a broken Lefschetz fibration of S^4 with a spun or twist-spun torus knot fiber

Doker, Jeffrey, Geometry of generalized permutohedra

Goerner, Matthias, Visualizing regular tesselations: Principal congruence links and equivariant morphisms from surfaces to 3-manifolds

Isaacson, Erica, Some periodic solutions of the two-dimensional Stokes-Oldroyd-B system with stress diffusion

Jubin, Benoit, The tangent functor monad and folations

Kleinman, Aaron, Combinatorial phylogenetics for reconstruction algorithms

Lee, Chul-hee, Algebraic structures in modular q-hypergeometric series

Liu, Yi, Nonzero degree maps between three dimensional manifolds

Marks, Andrew, Recursion theory and countable Borel equivalence relations

Penneys, David, Planar structure for inclusions of finite von Neumann algebras

Pomerleano, Daniel, Curved string theory

Qvilodran, Rene, On extremizers for adjoint Fourier restriction inequalities and a result in incidence geometry

Rincon, Edgard, Tropical linear spaces and applications

Rizzolo, Douglas, Scaling limits of random trees

Schaeffer, George, The Hecke stability method and ethereal forms

Slofstra, William, Strong Macdonald theory and the Brylinski filtration for affine Lie algebras

Ventura, Ivan, Applications of semiclassical analysis to partial differential equations

Wilder, Alan, Smooth field theories and homotopy field theories

Zhou, Junjie, Essays on microeconomics

Department of Statistics

Lim, Chinghway, Modeling high dimensional data: Prediction, sparsity, and robustness

Miratrix, Luke, Three statistical methods for the social sciences

Uhler, Caroline, Geometry of maximum likelihood estimation in Gaussian graphical models

Xu, Ying, Regularization methods for canonical correlation analysis, rank correlation matrices and Renyi correlation matrices

Group in Biostatistics

Brooks, Jordan, Super learner and targeted maximum likelihood estimation for longitudinal data structures with applications to atrial fibrillation

Jamshidian, Farid, Applications to semiparametric estimation methods in causal inference and prediction

McKeown, Karen, Topics in current status data

University of California, Davis (20)

Department of Mathematics

Berg, Sonya, A quantum algorithm for the quantum Schur transform

Dueck, Jean-Pierre, Spectral properties of Wigner matrices

Ferreira, Jeffrey, Row-strict quasisymmetric Schur functions, characterizations of Demazure atoms, and permuted basement nonsymmetric MacDonald polynomials

Hunt, Thomas, A proof of the higher order accuracy of the patchy method for solving the Hamilton-Jacobi-Bellman equation

Ifrim, Mihaela, Normal form transformations for quasilinear wave equations

Lee, Eunghyun, Bethe ansatz solvable interacting particle system on Z

Liou, Jia-Ming, Topology of the Krichever map

Nakatsukasa, Yuji, Algorithms and perturbation theories of matrix eigenvalue problems and the singular value decomposition

Oyoung, Jen Keng, Totally asymmetric simple exclusion process for particles of different hopping rates

Renfrew, David, Outliers of finite rank deformations to random matrices and related functionals

Shinault, Gregory, Inhomogeneous tilings of the Aztec diamond and the Airy process

Stamps, Matthew, Topological methods in matroid theory

Tseng, Hsiao-Chieh, Compressive sensing and its applications in radar imaging and rough surface scattering

Vershynina, Anna, Existence of the thermodynamic limit and asymptotic behavior of some irreversible quantum dynamical systems

Woei, Ernest, Characterization and clustering of dendrite trees using morphological features extracted by graph spectra

Department of Statistics

Hyun, Jung Won, Local polynomial estimation for a smooth spatial random process with a stochastic trend and a stationary noise

Jiang, Yun Kai, Topics on Bayesian analysis of missing data

Lee, Lawrence, Iterative estimation equation approach for nonlinear mixed effects models

Loux, Travis, Causal inference and estimation of the odds ratio

Mou, Jiani, Two-stage fence methods for longitudinal data

University of California, Irvine (19)

Department of Mathematics

Campbell, Robert, Realizing cubic hypersurfaces

Chen, Wei-Kuo, Chaos problem in mean field spin glasses

Chen, Ying, Modeling solid tumor growth in complex dynamic geometries

Farhat, Aseel, Analytical study of the Hasegawa-Mima model, a multi-layer and a continuously stratified geostrophic model of ocean dynamics

Kong, Li, Long range stochastic volatility with slow and fast scales in option pricing

Le, Anh, Minami estimates for a finite rank Anderson model

Liang, Jian, Solving partial differential equations on point clouds and geometric understanding of point clouds

Liu, Yu-Yu, Turbulent flame speeds of G-equations

Lv, Hua, Modeling, calibration, and simulation of spot price paths

Marx, Christoph, Quasi-periodic Jacobi cocycles, dynamics, continuity, and applications to extended Harper's model

Mavi, Rajinder, Quantum mechanized models with strictly ergodic disorder

Pecharich, Jeremy, A deformation complex for modules over deformation quantization

Peng, Yuyu, Multiscale modeling of cell populations and intracellular gene regulatory network

Pham, Kara, Predictions of the morphological stability of growing tumors: A theoretical analysis and experimental validation

Pipan, John, Periodic non-autonomous second-order Hamiltonian systems

Sorace, Ronald, Accumulation of mutation in stochastically growing colonies: Theory and applications

Wu, Zhiwei, Solution equations and geometric curve flows

Xue, Xun (Sean), An anti-classification theorem for ergodic homeomorphisms of the torus

Yu, Meng, Multi-channel enhancement by regularized optimization

University of California, Los Angeles (46)

Department of Biostatistics, Fielding School of Public Health

Kim, Soeun, Multiple imputations for missing covariates in regression models in the presence of interactions

Lee, Jihey, Bayesian analyses of longitudinal self-reported counts of sexual behavior

Lin, Sherry T., Joint Bayesian modeling of irregularly measured multivariate longitudinal nutrient consumption and longitudinal outcome data

Patel, Trina R., Bayesian methods in the quantitative risk assessment and toxicity profiling of engineered nanomaterials

Tom, Jennifer A., Bayesian hierarchical modeling for massive sequence datasets

Zhu, Yuda, Hierarchical and semi-parametric Bayesian models for the study of longitudinal HIV behavior and tuberculosis incidence data

Department of Mathematics

Allen, Patrick, Modularity of nearly ordinary 2-adic residually dihedral Galois representations

Blinstein, Semyon, Cohomological invariants of algebraic tori

Buttcane, Jack, Sums of $SL(3,Z)$ Kloosterman sums

Cantarero, Alejandro, Numerical methods and inverse problems in elliptic PDEs

Hegemann, Rachel, Spatially embedded social networks: Dynamic models and data reconstruction

Hellrung, Jeffrey, On embedded methods for crack propagation, virtual surgery, shattered objects in computer animation and elliptic partial differential equations

Huang, Hao, Various problems in extremal combinatorics

Jacobson, Judah, L^1 minimization for sparse audio processing

Keegan, Matthew, Models and methods for multiphase segmentation

Kovac, Vjekoslav, Applications of the Bellman function technique in multilinear and nonlinear harmonic analysis

Lederman, Carl, Finite element and meshfree applications to image processing

Lee, Choongbum, Problems in extremal and probabilistic combinatorics

Lewis, Erik, Estimation techniques for self-exciting point processes with applications to criminal behavior

Lidman, Tye, Triple cup products in Heegaard Floer homology

Mahboubi, Pejman, Intermittency of the Malliavin derivatives and regularity of the densities for a stochastic heat equation

Massey, Adam, The KH-theory of complete simplicial toric varieties and the algebraic K-theory of weighted projective spaces

Merton, Gabriel, Codazzi tensors with two eigenvalue functions

Palamourdas, Konstantinos, $1, 2, 3, \ldots, 2n+1, \infty$

Reduzzi, Davide, Shifting Hecke eigensystems in positive characteristic

Ruozzi, Anthony, Algebraic tori and essential dimension

Seyalioglu, Hakan, Reducing trust when trust is essential

Shih, Justin, On the negative *K*-theory of singular varieties

Sizemore, James, Orbit equivalence and von Neumann rigidity for actions of wreath product groups

Smith, Laura, Incorporating spatial information into density estimates and street gang models

Tiruviluamala, Neelesh, On the passage of Gaussian beams through cusps in ray paths

Von Brecht, James, Particle formation in particle interactions

Waters, Alden, A parametrix construction for low regularity wave equations and spectral rigidity for two dimensional periodic Schrödinger operators

Wilson, Stedman, Embeddings of polytopes and polyhedral complexes

Winchester, Adam, Gap rigidity and unique prime decomposition

Wong, Wanshun, Essential dimension of finite groups

Yan, Ming, Image and signal processing with non-Gaussian noise: EM-type algorithms and adaptive outlier pursuit

Yang, Xiaokui, Positivity and vanishing theorems in complex and algebraic geometery

Yao, Yao, Aggregation equation with degenerate diffusion

Department of Statistics

Balderama, Earvin, Spatial-temporal branching point process models in the study of invasive species

Rundel, Colin Witter, Bayesian methods for spatial assignment of migratory birds

Shen, Jie, Additive mixed modelling of HIV patient outcomes across multiple studies

Tsai, Wei Tan, Multilinear approximation with Kronecker weights

Wu, Tianfu, Integration and goal-guided scheduling of bottom-up and top-down computing processes in hierarchical models

Yao, Zhenyu, Learning spatial-temporal models for understanding actions and events in video

Zhao, Mingtian, A statistical and computational theory for the art of painting

University of California, Riverside (25)

Department of Mathematics

Bennett, Matthew, On tilting modules for the current algebra associated to a simple Lie algebra

Carlson, Christopher, Foliations, contact structures and finite group actions

Herzog, Barbara, Sub-index for the critial points of the Riemannian distance function

Katz, Adam, PBW bases for dioperads

Lal, Nishu, Spectral zeta functions of Laplacians on self-similar fractals

Lee, Kwangwoo, Transfer theorems on tautological modules of Hilbert schemes of nodal curves and de Jonquieres' formulas

Manning, Nathaniael, Global Wehl modules for twisted and untwisted loop algebras

Niemeyer, Robert, On the properties of sequences of compatible orbits of Koch snowflake prefractal billiard tables and particular periodic orbits of the Koch snowflake fractal billiard

Oeser IV, Paul, Monoidal extensions of a locally quasi-unmixed domain

Pro, Curtis, Topics on Riemannian submersions and diffeomorphism stability

Sill, Michael, Average distance functions and their applications

Thomas, Bradley, Boundary characterization of a smooth domain with noncompact automorphism

Zaragoza, Juan, Orthogonal partial conformal change

Department of Statistics

Benecke, Scott, Bayes neutral zone classification in unsupervised and semisupervised settings

Che, Xiaohong, Bayesian statistics and its application to quantitative trait loci mapping

De Palma, Elijah, Sequential hypothesis testing with spatially correlated presence-absence data and the corridor problem

Dutta, Santanu, Optimum designs for identification and discrimination within a class of computing linear regression models

Flores, Analisa, Characterization of special variance structures for designs in model identification and discrimination

Gan, Lu (Rebecca), Optimal longitudinal cohort designs and variance parameter estimation

Ghosh, Indranil, Inference for the bivariate and multivariate hidden truncated Pareto (type II) and Pareto (type IV) distribution and some measures of divergence related to incompatibility

Le, Rebecca, Proposed: Alternative approaches in multi-label neutral zone classification problems

Shi, Nigie, Estimation and clustering on longitudinal data using penalized spline models

Song, Huiming, Proposed: Bayesian analysis of MTD/BMTD models

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Zhang, Zhanpan, Clustering: Algorithm, optimization and inference

University of California, San Diego (16)

Department of Mathematics

Berglund, James, Z-graded maximal orders of GK 3

Brik, Alex, Extensions of answer set programming

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Duong, Son, Transversality of CR mappings between CR submanifolds of complex spaces

Foley, John, Comparing Kac-Moody groups over the complex numbers and fields of positive characteristic via homotopy theory

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Johnson, Alan, Reductions and propositional proofs for total NP search problems

Laetsch, Thomas, An approximation of Wiener measure on manifolds with non-positive sectional curvature

Lee, Seung, Determinants of intertwining operators between genuine principal series representations of nonlinear real split groups

Pollock, Sara, Convergence of goal-oriented adaptive finite element methods

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Reed, Joseph, Methods for PDE-constrained optimization

Schultheis, Daniel, Virtual invariants on Quot schemes over Fano surfaces

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Tiefenbruck, Mark, Patterns and statistics on words

Young, Alexander, Examples of algebras of small Gelfand-Kirillov dimension

University of California, Santa Barbara (5)

Department of Mathematics

Fisher, Jordan, Efficiently removing stiffness in the immersed boundary methods

Wu, Peng, Studies on Einstein manifolds and Ricci solitons

Department of Statistics and Applied Probability

Ghofrani, Hamid, Latent degree graph models for social networks

Sau, Raj, Rebalancing portfolios under transaction costs

Xu, Yan, Some contributions to multidimensional scaling and unfolding

University of California, Santa Cruz (11)

Applied Mathematics and Statistics Department

Acevedo-Arreguin, Luis Antonio, The magnetohydrodynamics of the solar tachocline

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Farah, Marian, Bayesian nonparametric methods for emulation, sensitivity analysis, and calibration of computer simulators

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Liang, Waley, Bayesian nonstationary Gaussian process models via treed process convolutions

Department of Mathematics

Dai, Mimi, The nematic liquid crystal systems and magneto-hydrodynamics system: The properties of their solutions

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Hein, Doris, Variations on the theme of the Conley conjecture

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University of Southern California (12)

Department of Mathematics

Chebotarov, Dmytro, Classification of transitive vertex algebroids

DeSalvo, Stephen, Probabilistic divide-and-conquer—a new method for exact simulation—and lower bound expansions for random Bernoulli matrices via novel integer partitions

Ghosh, Subhankar, Stein couplings for Berry-Essen bounds and concentration inequalities

Ignatova, Mihaela, Quantitative unique continuation and complexity of solutions to partial differential equations

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Lin, Wei, Survival analysis with missing data and high dimensionality

Pavelescu, Andrei, On the proportion of derangements in cosets of primitive permutation groups

Reis, Ednei, Asymptotic expansion for solutions of the Navier-Stokes equations with potential forces

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Zhong, Changlong, Comparison of dualizing complexes

COLORADO

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Lauriski-Karriker, Tonya, Optional risk set sampling designs for case-crossover studies with applications to studies involving environmental exposures

Colorado State University (5)

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Burch, Nathanial, Probabilistic foundation of nonlocal diffusion and formulation and analysis for elliptic problems on uncertain domains

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University of Colorado, Denver (6)

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CONNECTICUT

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Wesleyan University (1)

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Henshaw, Glenn, Search bounds for points in linear and quadratic spaces

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Biostatistics Division

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DELAWARE

Delaware State University (2)

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DISTRICT OF COLUMBIA

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Florida Institute of Technology (6)

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Florida State University (24)

Department of Mathematics

Armstrong, Johnathon, Principal elements of mixed-sign Coxeter systems

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University of Central Florida (3)

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Ahmadi Abhari, Sayed Hamed, Quantum algorithms for: Quantum phase estimation, approximation of the Tutte polynomial and black-box structures

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University of Florida (20)

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Ozrazgat, Tezcan, The stereotype model and the existence of maximum likelihood estimates in categorical regression models

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Song, Jihyun, Some contributions to the equivalence of prospective and retrospective inferences in case-control studies

Steorts, Rebecca, Bayes and empirical Bayes benchmarking for small area estimation

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University of Miami (7)

Department of Mathematics

Gao, Daozhou, Transmission dynamics of some epidemiological patch models

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Ryan, Daniel, Fitness dependent dispersal in intraguild predation communities

Scheib, Douglas, The effects of harvesting on ecological communities

Zohar, Dahlia, Embeddings of curves into Euclidean spaces

University of South Florida (4)

Department of Mathematics and Statistics

Assad, Magdy, Pfaffian and Wronskian solutions to integrable generalized nonlinear partial differential equations

Hackett, Keith, Statistical analysis and mechanistic modeling of water quality: Hillsborough Bay, Florida

Molinares, Carlos, Parametric and Bayesian modeling of reliability and survival analysis

Xu, Yong, Statistical models for environmental and health sciences

GEORGIA

Emory University (21)

Department of Biostatistics and Bioinformatics

Blackstock, Anna, Techniques for pattern recognition in high-throughput metabolomic data

Hart, Kari, Robust latent class analysis for longitudinal data

Li, Li, Semiparametric efficient and robust estimation of treatment effects from observational data

Li, Ruosha, Varying-coefficient regression analyses for semi-competing risks data

Lin, Ji, Likelihood based methods in logistic regression with missing data

McClintock, Shannon, Model-based statistical methods for public health surveillance subject to imperfect observations

Pan, Yi, A new permutation-based method for assessing observer agreement with replicated and repeated observations

Peng, Hesen, High-dimensional universal dependence discovery

Qin, Tielin, Bayesian analysis for repeated compositional data and approaches for correcting measurement errors in general multivariate linear model

Switchenko, Jeffrey, Estimation of epidemic model parameters: A spatial analysis using Bayesian techniques

Xu, Zhiheng, Bayesian space-time analysis in carcinogenesis

Zhu, Ming, Survival analysis with covariate measurement error

Mathematics and Computer Science Department

D'Elia, Marta, Assimilation of velocity data into fluid dynamic simulations, an applicaton to computational hemodynamics

Dellamonica Jr., Domingos, Topics in Ramsey theory

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Janiszewski, Susan, 3-connected, claw-free generalized net-free graphs are Hamiltonian

Lee, Sang June, Results on Sidon and B_h sequences

Palta, Hasan, Some mathematical problems in design of free-form mirrors and lenses

Thomas, Sean, Complex iso-length spectriality in arithmetic hyperbolic 3-manifolds

Wang, Zhen, Preconditioning techniques for the incompressible Navier-Stokes equations

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Georgia Institute of Technology (11)

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Chen, Kenneth, Topics in group methods for integer programming

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Lu, Nan, Normally elliptic singular perturbation problems: Local invariant manifolds and applications

Ma, Jie, Judicious partitions of graphs and hypergraphs

Pascoe-Streib, Amanda, Markov chains at the interface of combinatorics, computing and statistical physics

Restrepo Lopez, Ricardo, Topics in spatial and dynamical mixing of interacting particle systems

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Streib, Noah, Planar and Hamiltonian cover graphs

Wu, Jialiang, Hybrid modeling and analysis of multiscale biochemical reaction networks

Georgia State University (4)

Mathematics and Statistical Department

Jalil, Sajiya, Stability analysis of phase-locked bursting in inhibitory neuron networks

Wang, Binhuan, Evaluation of continuous-scale diagnostic tests with missing data

Zhao, Kun, Mathematical models for network analysis, proteomics, and disease prevention

Zhou, Haochuan, Statistical inferences for the Youden index

University of Georgia (7)

Department of Mathematics

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Manning, Justin, Axiomatic quantum fields on the de Sitter surface with a local spectral condition

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Samples, Brandon, Components of Springer fibers for the exceptional groups G_2 and F_4

Stankewicz, James, Twists of Shimura curves

Walters, Nathan, Some capacity-theoretic results extended to algebraic curves

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HAWAII

University of Hawaii at Manoa (2)

Department of Mathematics

Chasse, Matthew, Linear preservers and entire functions with restricted zero loci

DeMeo, William, Congruence lattices of finite algebras

IDAHO

Idaho State University (1)

Department of Mathematics

Clements, Nathan, Hypercyclic operators on Banach spaces

University of Idaho (1)

Department of Mathematics

Peng, Yan-Ling, The chromaticity of K_4-homeomorphs

ILLINOIS

Illinois Institute of Technology (3)

Applied Mathematics Department

Iyigunler, Ismail, Topics in counterparty risk and dynamic conic finance

Rodriguez, Rodrigo, Dynamic conic finance: No-arbitrage pricing and no-good deal pricing for dividend-paying securities in discrete-time markets with transaction costs

Ye, Qi, Analyzing reproducing kernel approximation methods via a Green function approach

Illinois State University (3)

Department of Mathematics

Rumsey, Chepina, Advancing fourth-grade students' understanding of arithmetic properties with instruction that promotes mathematical argumentation

Staebler-Wiseman, Heidi, Problem posing as a pedagogical strategy: A teacher's perspective

Wucherer, Robin, The mathematical beliefs and instructional practices of four community college remedial mathematics instructors

Northern Illinois University (6)

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Das, Ujjwal, MDL procedure, Lasso penalty and their statistical risk for type-1 censored data

Kettlestrings, David, Some results on heights and extensions of function fields over a finite field

Kettner, Laura, Problems with application to well-posedness

Pearson, Abigail, Noncommutative piecewise Noetherian rings

Sarkar, Suchitrita, Unified competing risks

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Northwestern University (17)

Department of Mathematics

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Nendorf, Robert, Power operations for Moreva *E*-theory of height 2 and characteristic 3

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Skirvin, Michael, A global analogue of the Springer resolution for SL_2

Stadnik, Maria, Ray class fields of conductor p

Stadnik, Ted, Constructions for differential operators in positive characteristic

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Mendes, Eduardo, Convergence rates and model selection for mixture of experts

Engeering Science and Applied Mathematics Department

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Miklius, Matthew, From foams to fruit flies—A mathematical analysis of cellular structures

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Southern Illinois University Carbondale (5)

Department of Mathematics

Haynes, Elizabeth, Smale flows on three dimensional manifolds

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Katugampola, Udita, On generalized fractional integrals and derivatives

Wilathgamuwa, Gayan, Stochastic fractional differential equations driven by fractional Brownian motion and its generalizations

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University of Chicago (14)

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Chung, Francis, A partial data result for the magnetic Schrödinger inverse problem

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INDIANA

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Indiana University-Purdue University Indianapolis (2)

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Sega, Laurentiu, Integrating immunological and epidemiological models

Vaidyanathan, Prahlad, Invariants of continuous fields of C^* algebras

Wegman, Nicholas, Plurisubharmonic domination in Fréchet spaces

You, Young Hwan, The necessary condition for ∂-equation in Hölder space C^3

Zadeh, Joseph, Iterated fractional Brownian motion

Department of Statistics

He, Tianhong, Lasso and general L_1 regularized regression under linear equality and inequality

Leaf, Duncan, Inferential models and restricted spaces

Lim, Eunjung, Bayesian linkage analysis for autotetraploids

Lin, Yanzhu, Supervised dimension reduction for high-dimensional generalized linear models

Mehta, Shraddha, Bayesian nonparametric density estimation and sufficient dimension reduction in survival analysis

Pungpapong, Vitara, Empirical Bayes variable selection in regression models

Wang, Yong, A multivariate spatial modeling approach using semi-parametric covariance functions

Xia, Jin, Divide and recombine (D&R) for the analysis of large and complex data sets with application to VoIP

Zhang, Zuoyi, The detection of clusters with spatial heterogeneity

University of Notre Dame (11)

Applied and Computational Mathematics and Statistics

Salyers, Tanya, Network based modeling in social sciences and biology

Department of Mathematics

Ambler, Stuart M., A bundle gerbe construction of a spinor bundle from the smooth free loop of a vector bundle

Boyle, Bernadette M., On the unimodality of pure *O*-sequences

Brower, Donald A., Aspects of stability in simple theories

Carson, Jacob M., Computational complexity of automatic structures

Ge, Jian, Low dimension soul theorems

Grady, Ryan E., On geometric aspects of topological quantum mechanics

Huang, Yen-Chang, Singular solutions of the sigma-k-Ricci equation

Karapetyan, David A., On the well-posedness of the hyperelastic rod equation

Porter, Christopher P., Mathematical and philosophical perspectives on algorithmic randomness

Rowekamp, Brandon M., Planar pixelations and shape reconstruction

IOWA

Iowa State University (23)

Department of Mathematics

Bickner, Devin, On normal networks

Hong, Jyy-I (Joy), Coalescence in Bellman-Harris and multi-type branching processes

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Kramer, Dominic, Basis identification through convex optimization

Lastrina, Michelle, List-coloring and sum-list-coloring problems on graphs

Liu, Sijia, Novel data clustering methods and applications

McKay, Tracy, The edit distance function for graphs: An exploration of the case of forbidden induced $K_{2,t}$ and other questions

Ozer, Ahmet Ozkan, Exact controllability and feedback stabilization for a multilayer Rao-Nakra beam

Peters, Travis, Positive semidefinite maximum nullity and zero forcing number

Rastegar, Reza, Topics in self-interacting random walks

Smith, Jason, Induced saturation number

Stines, Elijah, Abelian qo-groups and atomic pseudo-valuation domains

Vidden, Chad, The direct discontinuous Galerkin method with symmetric structure for diffusion problems

Ylvisaker, Andrew, A formalization of logic in diagonal-free cylindric algebras

Department of Statistics

Adrian, Daniel, Some contributions to Ricean and complex-valued modeling of functional MRI time series

Beverlin, Lucas, Exploitation of advanced statistical methods and analysis to improve nonlinear dynamic glucose modelling

Chen, Wei-Chen, Overlapping codon model, phylogenetic clustering and alternative partial expectation conditional maximization algorithm

Guo, Jiqiang, Bayesian methods for system reliabililty and community detection

He, Wei, A combined linkage disequilibrium and cosegregation method for fine mapping of QTL approaches to study long-term accuracy of genetic selection

Kim, Young Min, On nonparametric likelihood methods for weakly and strongly dependent time series

Larson, Nicholas, Investigation and development of statistical methods for gene expression data

Lund, Steven, Statistical methods for identifying differentially expressed genes using hierarchical models

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University of Iowa (22)

Applied Mathematical and Computational Sciences

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Yong, Kamuela, A mathematical model of the interactions between pollinators and their effects on pollination of almonds

Department of Biostatistics

Acion, C. Laura, Criteria for generalized linear model selection based on Kullback's symmetric divergence

Cheng, Gang, The nonparametric least-squares method for estimating monotone functions with interval-censored observations

Jang, MiJin, Working correlation selection in GEE

Jiang, Dingfeng, Concave selection in generalized linear models

Porter, Aaron T., A path-specific approach to SEIR modeling

Xie, Diqiong, Bias and variance estimators of average treatment effect using propensity-score matching

Yang, Ming, Statistical models for count time series with excess zeros

Zhang, Jingyang, Making diagnosis with multiple tests under no gold standard

Department of Mathematics

Flournoy, Cecil, N-parameter Fibonacci AF C^*-algebras

Gillespie, Timothy, Superposition of zeros of automorphic L-functions and functoriality

Williams, Kristopher, The Milnor fiber associated to an arrangement of hyperplanes

Department of Statistics and Actuarial Science

Ahn, Jae Youn, Non-parametric inference of risk measures

Chen, Kun, Regularized multivariate stochastic regression

Liu, Jin, Penalized methods in genome-wide association studies

Su, Fei, Statistical analysis of nonlinear diffusion processes

Tang, Xiongwen, Two-level lognormal frailty model and competing risks model with missing cause of failure

Xie, Yunlong, Likelihood-based inference for antedependence (Markov) models for categorical longitudinal data

KANSAS

Kansas State University (9)

Department of Mathematics

Ahmad, Muhammad Naeem, Cobordism theory of semifree circle actions on complex N-Spin manifolds

Ghimire, Santosh, Classification of image pixels based on minimum distance and hypothesis testing

Hoang, Nguyen, Numerical solutions to some ill-posed problems

Ostapyuk, Olena, Backward iteration in the unit ball

Department of Statistics

Dey, Rajarshi, Inference for the K-sample problem based on precedence probabilities

Magalla, Champa, Model adequacy tests for exponential family regression models

Paranagama, Dilan, Correlation and variance stabilization in the two group comparison case in high dimensional data under dependencies

Samarakoon, Nishantha, Conditional variance function checking in heteroscedastic regression models

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University of Kansas (3)

Department of Mathematics

Dorn, Timothy, Shear flows in liquid crystal models

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Wichita State University (3)

Department of Mathematics, Statistics, and Physics

Banerjee, Arijit, Testing homogeneity of a parameter matrix some of the rows of which are under synchronized order restrictions

Ingle, William, Stability estimates for inverse problems of some elliptic equations

Kropf, Everett, Numerical conformal mapping of Schwarz-Christofel transformations and slit maps for multiply connected domains

KENTUCKY

University of Kentucky (11)

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Aleams, Barra, Equivalence theorems and the local-global property

Cook II, David, Lefschetz properties and enumerations

Geng, Jun, Elliptic boundary value problems on non-smooth domains

Jung, JiYoon, Analytic and topological combinations of partition posets and permutations

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Mattingly, Christopher, Rational approximation on compact nowhere dense sets

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Weaver, Elizabeth, Minimality and duality of tail-biting trellises for linear codes

Zhi, Weifeng, Spectral analysis of nonlinear dimensionality reduction

Department of Statistics

Lin, Yushun, Regression analysis of longitudinal failure time data

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Department of Bioinformatics and Biostatistics

Ray, Herman E., Design and inference in phase II/III clinical trials incorporating monitoring of multiple endpoints

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Zapf, Matthew, Cantor set approximations and dimension computations in hyperspaces

LOUISIANA

LSU Health Science Center, New Orleans (1)

Department of Biostatistics

Lee, Myungok, Analysis of longitudinal categorical data with non-ignorable dropout using shared random effects models

Louisiana State University, Baton Rouge (11)

Department of Mathematics

Bakhova, Maiia, A numerical investigation of Apéry-like equations and related Picard-Fuchs equations

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Dann, Susanna, Paley-Wiener theorems with respect to the spectral parameter

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Warshauer, Lisa, Some classes of graphs that are nearly cycle-free

Louisiana Tech University (9)

Program of Mathematics and Statistics

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Department of Biostatistics and Bioinformatics

Dornelles, Adrianna, Built environment and obesity risk factors: Do where you live, work and commute influence your weight status? A spatial analysis of elementary school personnel in New Orleans

Hagan, Joseph, Comparison of supervised learning methods for classification of microarray data

He, Hao, Modeling data from cluster randomized trials with a small number of big clusters and a random-split method

Department of Mathematics

Calmes, Jason, Estimating the probability of accurate phylogeny reconstruction by quartet aggregation

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Maggio, Charles, Interactions of calcium dynamics, muscle forces, and tissue properties in a model of uterine fluid flow

Miller, Jason, Central-upwind schemes for shallow water models and tsunami wave dynamics

O'Keefe, Augustine, Cohen-Macaulay toric rings arising from finite graphs

Pollack, Michael, Semi-discrete central-upwind schemes for elasticity in a heterogeneous media

Pond, Cody, Effective boundary conditions on coated bodies

Pullen, Jeffrey, Complete coverage of polyhedra

Straub, Armin, Arithmetic aspects of random walks and methods in definite integration

Todorov, Todor, Desuspension of involutions on $S^1 \times S^n$ and concordance classes of homeomorphisms of RP^n

Yang, Xiangfeng, Large deviation expansions

University of Louisiana at Lafayette (4)

Department of Mathematics

Denton, Zachary, Qualititative theory of Riemann-Liouville fractional differential equations

Huang, Qihua, Deterministic and stochastic structured population models with applications to green tree frogs

Ramirez-Valdez, Juan Diego, Generalized monotone iterative techniques for fractional differential equations with boundary conditions

Zhang, Pei, Discrete stage-structured models with application to plants and amphibians

MARYLAND

Johns Hopkins University (5)

Department of Biostatistics

Bobb, Jennifer, Statistical analysis of multisite time series data for estimating health effects of environmental exposures

Goldsmith, A. Jeffrey, Cross-sectional and longitudinal penalized functional regression

Goldsmith, Russell, Robust statistical methods for the study of disease through complex structural outcomes

Kiihl, Samara, Statistical methods applied to synthetic lethality analysis by microarrays

Ning, Yang, Nonstandard likelihood-based inference

Johns Hopkins University, Baltimore (6)

Department of Applied Mathematics and Statistics

Bindjeme, Patrick, A repertoire of cost functions for the QuickSort algorithm

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Yoruk, Erdem, Learning graphical models with limited observations of high-dimensional data

Department of Mathematics

Ariturk, Sinan, Concentration and vanishing of eigenfunctions on a manifold with boundary

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Department of Mathematics and Statistics

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Yang, Yushu, Tau leaping methods for stiff stochastic chemical systems

University of Maryland, College Park (27)

Department of Mathematics

Aulicino, David, Progress toward classifying Teichmüller disks with completely degenerate Kontsevich-Zorich spectrum

Barnes, Sean, An agent-based approach to controlling pathogenic transmission in medical facilities

Cao, Lingyan, Optimal variance swaps portfolios and estimating Greeks for variance

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Frazier, Jeffrey, Length spectral rigidity of non-positively curved surfaces

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Kirn, Malina, Using machine learning to measure the cross section of top quark pairs in the muon+jets channel at the compact muon solenoid

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Trevino, Rodrigo, Beyond orientability and compactness: New results on the dynamics of flat surfaces

Vaczlavik, Lucaci, Computational methods for game options

Voulgaraki, Anastasia, Semiparametric regression and mortality rate prediction

Wilson, Shelby, Mathematical models of immune regulation and cancer vaccines

Yeager, Joseph, Geometric and topological ellipticity in cohomogeneity two

Zhou, Yun, Trading advantages of structured products and option implied liquidity

MASSACHUSETTS

Boston University (7)

Department of Mathematics and Statistics

Barry, Anna, Vortex crystals in fluids

Ding, Qi, Statistical topics relating to computer network anomaly detection

Hayden, Douglas, Causality uncertainty, and falsification in clinical research

Ho, Man Ho, On the differential Grothendieck-Riemann-Roch theorems

Larraín-Hubach, Andrés, Pseudodifferential bundles

Pencina, Karol, Quantification of improvement in risk prediction models

Zan, Yue, Machine learning methods in construction of transcriptional regulatory networks

Boston University School of Public Health (8)

Department of Biostatistics

Demler, Olga, Impact of new variable on discrimination of risk prediction models

Hartley, Stephen, Bayesian methods for multivariate modeling of pleiotropic single-nucleotide polymorphisms and genetic risk prediction

Hendricks, Audrey, Exploration of gene region simulation, correction for multiple testing, and summary methods

Li, Rong "Renny", Synchrony of time series, with applications to surveillance data

Miao, Xiaopeng, Statistical evaluation of surrogate markers in randomized clinical trials

Reichmann, William, Methods in subgroup analysis: Estimation of risk and implications for randomized controlled trial design

Xanthakis, Vanessa, Assessing methods for inclusion of new disease markers into risk prediction models

Zhuang, Wei Vivian, Using phenotyped but ungenotyped relatives in genetics association tests

Brandeis University (7)

Department of Mathematics

Broderick, Ryan, Incompressibility and fractals

Gainer-Dewar, Andrew, Gamma-species, quotients and graph enumeration

Luo, Zhenbin, Contou-Carrere symbol and its reciprocity laws via iterated integrals

Nelson, Dawn, Variations of Leopoldt's conjecture

Senguupta, Tathagata, Supersingular $K3$ surfaces in characteristic 3

Stambaugh, Nathanial, Toward an outer space for right-angled Artin groups

Wang, Chao-Jen, Application of the Goulden-Jackson method to counting Dyck paths by occurrences of subwords

Harvard University (17)

Department of Mathematics

Barlev, Jonathan David, D-modules on spaces of rational maps and on other generic data

Bozic, Ivana, Mathematical models of cancer

Caraiani, Ana, Local-global compatibility and the action of monodromy on nearby cycles

Deopurkar, Anand R., Alternate compactifications of Hurwitz spaces

Huizenga, Jack W., Restrictions of Steiner bundles and divisors on the Hilbert scheme of points in the plane

Khodorovskiy, Tatyana, Symplectic rational blow-up and embeddings of rational homology balls

Koberda, Thomas Michael, Mapping class groups, homology and finite covers of surfaces

Silberstein, Aaron Michael, Anabelian intersection theory

Street, Ethan J., Towards an instanton Floer homology for tangles

Thorne, Jack, The arithmetic of simple singularities

Department of Statistics

Du, Chao, Stochastic modeling and Bayesian inference with application in biophysics

Kwon, Tae Yeon, Three essays on credit risk models and their Bayesian estimation

Loong, Bronwyn, Topics and applications in synthetic data

Lysy, Martin, The method of batch inference for multivariate diffusions

Nesterko, Sergiy, Respondent-driven sampling and homophily in network data

Pattanayak, Cassandra, The critical role of covariate balance in causal inference with randomized experiments and observational studies

Zhu, Li, Some optimal and sequential experimental designs with potential applications to nanostructure synthesis and beyond

Harvard University, School of Public Health (18)

Biostatistics Department

Adhikari, Kaustubh, Statistical methodology for sequence analysis

Claggett, Brian, Statistical methods for clinical trials with multiple outcomes, HIV surveillance, and nonparametric meta-analysis

Cui, Rain, Variable selection methods for longitudinal data

De, Gourab, Contributions to analysis of family-based designs for gene-environment interaction and rare variants

Hsu, Jessie, Outcome-driven clustering of microarray data

Huang, Yen-Tsung, Statistical analysis of genomic data in lung cancer and other complex diseases

Hund, Lauren, Survey designs and spatio-temporal methods for disease surveillance

Larson, Jessica, Hidden Markov models predict epigenetic chromatin domains

Lei, Quanhong, Improved double-robust estimation of missing data and causal inference models and efficient estimation of the average treatment effect on the treated

Marino, Miguel, Statistical issues in large-scale community-based studies

Matsouaka, Roland, Contributions to imputation methods based on ranks and to treatment selection methods in personalized medicine

Minnier, Jessica, Inference and prediction for high dimensional data via penalized regression and kernel machine methods

Parast Bartroff, Layla, Landmark prediction of survival

Snavely, Anna, Multivariate data analysis with applications to cancer

Sofer, Tamar, Statistical methods for high dimensional data in environmental genomics

Stephens, Alisa, Enhancing statistician power: Flexible covariate-adjusted semiparametric inference for randomized studies with multivariate outcomes

Yansane, Alfa, Statistical methods for panel studies with applications in environment epidemiology

Zhao, Sihai Dave, Survival analysis with high-dimensional covariates with applications to cancer genomics

Massachusetts Institute of Technology (14)

Department of Mathematics

Chen, Tsao-Hsien, Geometric Langlands in prime characteristic

Ganatra, Sheel, Symplectic cohomology and duality for the wrapped Fukaya category

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Kamburov, Nikola, A free boundary problem inspired by a conjecture of De Giorgi

Lewis, Joel, Pattern avoidance for alternating permutations and reading words of tableaux

Moeller, Niels Martin, Mean curvature flow self-shrinkers with genus and asymptotically conical ends

Morales, Alejandro, Combinatorics of colored factorizations, flow polytopes and of matrices over finite fields

Preygel, Anatoly, Thom-Sebastiani and duality for matrix factorizations, and results on the higher structures of the Hochschild invariants

Sam, Steven, Free resolutions, combinatorics, and geometry

Sheridan, Nicholas, Homological mirror symmetry for a Calabi-Yau hypersurface in projective space

Speh, Peter, A classification of real and complex nilpotent orbits of reductive groups in terms of complex even nilpotent orbits

Travkin, Roman, Quantum geometric Langlands correspondence in positive characteristic: The GL_N case

van Ekeren, Jethro, Modular invariance for vertex operator superalgebras

Zakharevich, Inna, Scissors congruence and *K*-theory

Northeastern University (2)

Department of Mathematics

Carroll, Andrew, Semi-invariants for gentle string algebras

Sutar-Deshpande, Kavita, Orbit closures of quiver representations

Tufts University (1)

Department of Mathematics

Plante, Donald, On the interior of “fat” Sierpinski triangles

University of Massachusetts, Amherst (6)

Department of Mathematics and Statistics

Gamzon, Adam, Local torsion on abelian surfaces

Gerding, Aaron, Spectral methods for higher genus constant mean curvature surfaces

Giorgi, Elena, A mathematical growth model of the viral population in early HIV-1 infections

Hwang, Sungha, Spatial evolutionary game theory: Deterministic approximations, decompositions, and hierarchical multiscale models

McGibbon, Jason, Monodromy in knot contact homology

Resendes, David, Statistical methods for nonlinear dynamic models with measurement error using the Ricker model

Worcester Polytechnic Institute (1)

Mathematical Sciences Department

Maris, Razvan Florian, Random homogenization for the Stokes flow through a leaky membrane

MICHIGAN

Central Michigan University (5)

Department of Mathematics

Alshawarbeh, Etaf, Beta-Cauchy distribution and its applications

Blodgett, Aaron, The stable rationality of fixed fields determined by lattices over cyclic groups

Oyapero, Mojisola, Wavelet analysis of DNA sequences using integer representation

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Sharawi, Yousra, Minimum semidefinite rank of a graph

Michigan State University (15)

Department of Mathematics

Bellsky, Thomas, Nonlinear asymptotic stability for a generalized reaction-diffusion system

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Cornwell, Christopher, Invariants of topological and Legendrian links in lens spaces with a universally tight contact structure

Dresch, Jacqueline, Multi-scale, multi-dimension, DNA-based mathematical modeling of gene expression

Hayrapetyan, Gurgen, Geometric evolution of single-layer interfaces in the functionalized Cahn-Hilliard equation

Jaeger, Thomas, Topics in link homology

Lin, Junshan, Near-field imaging via inverse scattering

Wei, Yan, Direct analysis of implied volatility for European options

Willenbring, Robert, The Möbius function of generalized factor order

Zou, Yuting, On the estimation of Poincaré maps of three-dimensional vector fields near a hyperbolic critical point

Department of Statistics and Probability

Chang, Hsiu-Ching, Latent class profile analysis: Inference, estimation and its application

Li, Shaoyu, Statistical genetics

Ma, Shujie, Theory of spline regression with applications

Sinha, Sumit, The continuous-time principal-agent problem with moral hazard recursive preferences

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Michigan Technological University (4)

Department of Mathematics and Science

Clark, David, Applications of finite geometries to designs and codes

Gong, Yejun, Large eddy simulation of dispersed multiphase flow

Jiang, Beini, Employing graphics processing unit (GPU) technology to speed up multi-scale and multi-resolution agent-based brain cancer model

Liu, Chang, Problems in the classical calculus of variations

Oakland University (2)

Department of Mathematics and Statistics

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Purcell, John, Modeling, numerical analysis, and simulations of nonlinear dynamic beams

University of Michigan (51)

Department of Biostatistics

Ahn, Jaeil, Bayesian modeling of epidemiologic data under complex sampling schemes

Bai, Yun, Joint composite estimating functions in spatial and spatio-temporal models

Chen, Wei, Statistical methods and analysis in genome wide association studies and next-generation sequencing

Gopalakrishnan, Shyam, Methods for statistics and population genetics analyses

Ha, Jinkyung, Effects of a misattributed cause of death on cancer mortality

Hu, Tianle, Time-dependent cross-ratio estimation for bivariate failure times

Kang, Jian, Some novel spatial stochastic models for functional neuroimaging analysis

Liu, Xiaohong, Imputation and dynamic models in semiparametric survival analysis

Park, Yong Seok, On estimation and inference under order restrictions

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Wu, Jincao, Joint Bayesian image and prediction modeling

Zawistowski, Matthew Stephen, Statistical methods and models for modern genetic analysis

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Department of Mathematics

Eisenstein, Eugene, Inversion of adjunction in high codimension

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Izbicki Jennings, Geri, Efficient numerical methods for water wave propagation in unbounded domains

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Kang, Hyosang, Cofinite classifying spaces for lattices in $\mathbb{R}$-rank one semisimple Lie groups

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Masarik, Matthew, Decay of solutions to the wave equation in static spherically symmetric spacetimes

Parra, Manuel Rodrigo, Currents and equidistribution in holomorphic dynamics

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Marchenko, Olga V., Joint models and tests for time to tumor recurrence and disease stage in oncology clinical trials

Tang, Runlong, Adaptive and multistage procedures for inference on monotone regression functions in designed studies and observed data settings

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Wayne State University (8)

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He, Qi, Large deviation of stochastic system and its applications

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Phinezy, Bryan, Variations on a graph coloring theme

Department of Statistics

Bilgic, Yusuf, Rank-based estimation and prediction for mixed effects models in nested designs

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Division of Biostatistics, School of Public Health

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Johnson, Leif, Geometric ergodicity of a random-walk metropolis algorithm via variable transformation and computer aided reasoning in statistics

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Yang, Fan, Likelihood ratio tests for high-dimensional normal distributions

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Department of Mathematics and Statistics

Goddard II, Jerome, Classes of reaction diffusion equations with nonlinear boundary conditions

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Estes, John, On k-trees and special classes of k-trees

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Alferink, Steven, Lifetime prediction and confidence bounds in accelerated degradation testing for lognormal distributions with an Arrhenius rate relationship

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Anderson, Robert, Analysis of gradiometer tensor fields of anomalous mass distributions

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Shan, Guosen, Statistical methods and designs for use in clinical studies based on small sample sizes

State University of New York at Buffalo (12)

DEPARTMENT OF MATHEMATICS

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Zhao, Guowei, Dipole-dipole correlations for the sine-Gordon model

Stony Brook University (35)

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Ahn, Hyeong Jun, Hardy-Weinberg deviation and EM-based haplotype frequency estimation

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Su, Yi, A stochastic segmentation model for joint DNA-RNA microarray data analysis

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Tu, Xiawei, The power of detecting a mixture in the treatment groups through analysis of dose response data

Wang, Ruixue, Performance of model selection statistics in growth mixture modeling

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Zhang, Guangxiang, Adaptive fitting of mixed-effects models with correlated random-effects

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Green, Christopher, The Ahlfors iteration for conformal mapping

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Koca, Caner, On conformal geometry of Kähler surfaces

Malkoun, Joseph, Hyperkähler $4n$-manifolds with n commuting quaternionic killing fields

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Song, Ki, New complex coordinates on the Teichmüller space of a punctured disk

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Diveris, Kosmas, On the eventual vanishing of self-extensions

Molenje, Levi, High school teachers' use of graphing calculators when teaching linear and quadratic functions: Professed beliefs and observed practice

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Department of Mathematics and Statistics

Adamczak, William, The alcove path model and tableaux

Bianco, Scott, $X_{n+1} = A_n X_n + B_n (\mathrm{mod} p)$ in two dimensions

Cade, Patrick, Projective spectrum and cohomology

Kaufmann, Nissim, Entropic priors: Corrections, new observations, and applications

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Lu, Tao, ODE based statistical models for dynamic gene regulator network identification

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Cho, Hyunjoo, Existence of almost contact structures on manifolds with G_2-structures and generalizations

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Taylor, Krystal, Continuous incidence theory and its applications to number theory and geometry

Towsley, Adam, Reduction of orbits

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Brynjarsdottir, Jenny, Dimension reduced modeling of spatio-temporal processes with applications to statistical downscaling

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Zhuang, Lili, Bayesian dynamical modeling of count data

University of Cincinnati (4)

Department of Mathematical Sciences

Chen, Chen, Bayesian analyses of mediational models for survival outcome

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Ren, Yan, A non-parametric Bayesian method for hierarchical clustering of longitudinal data

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University of Toledo (1)

Department of Mathematics and Statistics

Stephens, Robert, The Steenrod algebra is a prime ring and the Krull dimensions of the Steenrod algebra

OKLAHOMA

Oklahoma State University (4)

Department of Mathematics

Adhikari, Dhanapati, Global regularity problem for the 2D Boussinesq equations

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Department of Statistics

Hatfield, Gary, An analysis of measures of spatial autocorrelation

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Biostatistics and Epidemiology Department

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PENNSYLVANIA

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McCready, Karen, Sharpness of exponent bounds for $SU(n)$

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Wang, Xingbin, Genomic meta-analysis combining microarray studies with confounded clinical variables: Application to depression analysis

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Department of Mathematics

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Department of Mathematics

Palladino, Frank, On rational differential equations

SOUTH CAROLINA

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Division of Biostatistics and Epidemiology

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South Dakota State University (4)

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University of Tennessee, Knoxville (11)

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Lewis, Paul, Lagrangian representations of (p, p, p)-triangle groups

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Li, Zhiqiang, Stability of nonlinear filters and branching particle approximations to the filtering problem

Ogle, Jacob, Counting reducible composites of polynomials

Sass, Chris, Circle packings on affine tori

Tsvietkova, Anastasiia, Hyperbolic structures from link diagrams

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Department of Mathematics

Chaynikov, Vladimir V., Properties of hyperbolic groups: Free normal subgroups, quasiconvex subgroups and actions of maximal growth

Schroeder, Justin Z., Hamilton cycle embeddings of complete tripartite graphs and their applications

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TEXAS

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Department of Mathematics

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Department of Statistical Sciences

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Renfro, Lindsay, Bayesian evaluation and adaptive trial design for surrogate time-to-event endpoints in clinical trials

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Computational and Applied Mathematics Department

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Department of Mathematics

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Zhang, Zhi, Optimal control theory on space of functions of bounded variation

Department of Statistics

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DeHoff, Kevin, Live cell compartment tracking: Object tracking in oscillating intensity images

Kahle, David, Minimum distance estimation in categorical conditional independence models

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Liu, Dajiang, Statistical methods for analyzing rare variant complex trait association via sequence data

Southern Methodist University (4)

Department of Mathematics

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Statistical Science Department

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Texas A&M University (27)

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Allmaras, Moritz, Modeling aspects and computational methods for some recent problems of tomographic imaging

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Barney, Bradley, Bayesian joint modeling of binomial and rank response data

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Department of Mathematics

Goodson, Joshua, Effects of new mathematics research on mathematics majors' beliefs

McBroom, Ewelina Suchacka, Teachers' technological pedagogical content knowledge development

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Texas Tech University (8)

Department of Mathematics and Statistics

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Biostatistics Department

Guennel, Tobias, Statistical methods for normalization and analysis of high-throughput genomic data

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Arnold, Rachel, The discrete Hodge star operator and Poincaré duality

Ben Romdhane, Mohamed, Higher-degree immersed finite elements for second-order elliptic interface problems

Conrad, Katarina, Analysis of the buckling states of an infinite plate conducting current

Garret, Flagg, Interpolation methods for the model reduction of bilinear systems

Hinkelmann, Franziska, Algebraic framework for discrete models in systems biology

Kapil, Ahuja, Recycling Krylov subspaces and preconditioners

McBee, Brian, Computational approaches to improving room heating and cooling for energy efficient buildings

Mechai, Idir, A posteriori error analysis for a discontinuous Galerkin method applied to hyperbolic problems on tetrahedral meshes

Saenz-Maldonado, Edgar, On nearly Euclidean Thurston maps

Van Wyk, Hans-Werner, A variational approach for identifying uncertain, distributed parameters in elliptic systems

Wang, Xiaojun, Well posedness for complex fluids problems in the high Weissenberg number limit

Wang, Zhu, Reduced-order modeling of complex engineering and geophysical flows: Analysis and computations

Wilkerson, Mary, Finite subdivision rules from mating of quadratic functions: Existence and constructions

Wyatt, Sarah, Issues in interpolatory model reduction: Inexact solves, second-order systems, and DAE's

Yu, Haofeng, A numerical investigation of the canonical duality method for non-convex variational problems

Department of Statistics

Huang, Wandi, GLR control charts for monitoring a proportion

Velasco-Cruz, Ciro, Spatially correlated model selection

Wang, Sai, GLR control charts for monitoring the mean vector or the dispersion of a multivariate normal process

Zhang, Huaiye, Bayesian approach dealing with mixture model problems

Zheng, Hongzhang, On development and performance evaluation of some bio-surveillance methods

WASHINGTON

University of Washington (27)

Applied Mathematics Department

Goldwyn, Joshua, Mathematical modeling of cochlear implants: From single neurons to psychoacoustics

Kim, Woo Hyun, A thermodynamic characterization of a diffusion process in nonequilibrium steady state

Kirkinis, Eleftherios, Renormalization group methods in applied mathematics

Lind Cole, Christine, Mathematical models for facilitated diffusion and the Brownian ratchet

Mandli, Kyle, Finite volume methods for the multilayer shallow water equations with applications to storm surges

Matrajt Arbetman, Laura, Optimal vaccine allocation for pandemic influenza

Vasan, Vishal, Some boundary-value problems for water waves

Williams, Matthew, Exploiting low dimensionality in nonlinear optics and other physical systems

Biostatistics Department

Bansal, Aasthaa, Combining biomarkers to improve performance in diagnostic medicine

Devlin, Sean, Nonparametric regression of the ROC

Gard, Charlotte, Bayesian methods for modeling distributions of data

Juraska, Michal, Sieve analysis

Sachs, Michael, Statistical methods to assess the prospective predictive accuracy of a medical test

Sitlani, Colleen, Evaluation and selection of treatment in longitudinal surgical trials

Wu, Kenneth, Pathophysiology and management of thromboembolic disorders

Department of Mathematics

Crites, Andrew, Pattern avoidance and affine permutations

Duarte Espinoza, Mauricio, Stationary distribution for spinning reflected diffusions

Patakfalvi, Zsolt, Moduli spaces of higher dimensional varieties

Suryanarayan, Sweta, Local cohomology at generic singularities of Schubert varieties in cominuscule flag varieties

Tokle, Joshua, Global heat kernel estimates related to the fractional Laplacian

Willge, Travis, Parallel tractor extension and split G_2 holonomy

Wolcott, Frank Lucas, A tensor-triangulated approach to derived categories of non-Noetherian rings

Department of Statistics

Evans, Robin, Parametrizations of discrete graphical models

Gruhl, Jonathan, Bayesian moduling for multivariate mixed outcomes with applications to cognitive testing data

Kim, Albert, A Bayesian surveillance system for detecting clusters of non-infectious diseases

Klimova, Anna, Coordinate-free exponential families on contingency tables

Ryu, Soyoung, Statistical approaches to analyze mass spectrometry data

Washington State University (6)

Department of Mathematics

Alzalg, Baha Mahmoud, Optimization over symmetric cones under uncertainty

Dichone, Bonnie Jean Kealy, A nonlinear stability analysis of vegetative turning pattern formation for an interaction-diffusion plant-surface water model system in an arid flat environment

Hamlin, Nathan, Recurrence representations

Haydock, Thomas, Super fair divisions: A minimum number of cuts

Labrum, Matthew, Mathematical models of species responses to habitat destruction

Zhu, Li, Asymptotic analysis and bounds of multi-coherence risk

WEST VIRGINIA

West Virginia University (4)

Department of Mathematics

Beck-Roth, Stephanie, Reduced order model of a spouted fluidized bed utilizing proper orthogonal decomposition

Musser, Jordan, Modeling of heat transfer and reactive chemistry for particles in gas-solid flow utilizing continuum-discrete methodology

Ngonghala, Calistus, Mathematical modeling and analysis of epidemiological and chemical systems

Wu, Qin, B-splines in EMD and graph theory in pattern recognition

WISCONSIN

Marquette University (1)

Department of Mathematics, Statistics, and Computer Science

Pandiyan, Balamurugan, Mathematical modeling and dynamical analysis of the hypothalamus-pituitary-thyroid (HPT)

University of Wisconsin, Madison (27)

Department of Mathematics

Ache, Antonio, Obstruction-flat asymptotically locally Euclidean metrics and asymptotics of the self-dual complex

Blackhurst, Jonathan, Random Frobenius elements and p-extensions of the rationals

Dabkowski, Michael, Eventual regularity of solutions to the supercritical dissipative surface quasi-geostrophic equation

Georgiou, Nicos, Finite and zero temperature polymer models

Godjali, Ali, Hessenberg pair of linear transformations

Guo, Jingwei, Two bounds in the lattice point problem

Hu, Jingwei, Numerical methods for quantum kinetic equations and high frequency waves

Johnson, Evan Alec, Simulation of magnetic reconnection with a ten-moment two-fluid plasma model

Kiernan, Kathleen, Exponents of matrices

Lin, Hao, Properties of the limit shape for some last passage growth models in random environments

Meyer, Seth, Zero forcing sets and bipartite circulants

Schroeder, Michael, Symmetric Hamilton cycle decompositions of graphs

Tu, Junwu, Koszul duality and its applications in algebraic geometry

Wang, Yuanqi, On Ricci solitons and Ricci flows

Woodbury, Michael, Explicit trilinear forms and subconvexity of the triple product L-function

Worawannotai, Chalermpong (Boyd), Dual polar graphs, the quantum algebra $U_q(\mathfrak{sl}_2)$, and Leonard systems

Yan, Bokai, Asymptotic-preserving schemes for kinetic-fluid coupling model

Yuan, Fang, The weak compactness of Ricci flow with Ricci curvature bounded from below

Department of Statistics

Dawson, John, Statistical methods for differential co-expression and the genomic analysis of time-to-event outcomes

Eng, Kevin, Statistical methods for neutral inference in comparative experiments

He, Xu, Identification of subgroups with large differential treatment effects in genome-wide association studies

Niu, Min, Spatially correlated unbalanced longitudinal energy billing data analysis

Ozeki, Akichika, Efficient inference in semiparametric models

Shang, Zuofeng, Bayesian variable selection: Theory and application

Wu, Zhuang, Regression tree algorithms for longitudinal and censored survival data

Zhao, Jiwei, Identifiability and estimation in statistical models with nonignorable missing data

Zhao, Yang, Hierarchical Bayesian modeling of species richness in biological communities

University of Wisconsin, Milwaukee (3)

DEPARTMENT OF MATHEMATICAL SCIENCES

Du, Hong, Nonlinear functional discriminant analysis

Harris, Pamela, Combinatorial problems related to Kostant's weight multiplicity formula

Wood, John, A random switch model for heavy-tailed distributions

WYOMING

University of Wyoming (5)

DEPARTMENT OF MATHEMATICS

Fernando, Bandhisattambige, Stochastic Navier-Stokes equation with Ito-Levy noise

Mukherjee, Saikat, Composition operators on Paley-Wiener type spaces

Nair, Reshmi, Acyclic matrices with an extreme number of distinct eigenvalues

Wijeratne, Chandana, On stochastic shell models of turbulence

Xu, Meng, Stochastic analysis and nonlinear filtering of point vortex dynamics subjected to jump noise

FROM THE
AMS SECRETARY

ATTENTION ALL AMS MEMBERS

Voting Information for 2013 AMS Election

AMS members who have chosen to vote online will receive an email message on or shortly after August 26, 2013, from the AMS Election Coordinator, Survey & Ballot Systems.

The From Line will be “AMS Election Coordinator,” the Sender email address will be noreply@directvote.net, and the Subject Line will be “AMS 2013 Election - login information below.” If you use a spam filter you may want to use the above address or subject information to configure your spam filter to ensure this email will be delivered to you.

The body of the message will provide your unique voting login information and the address (URL) of the voting website. New in 2013, members will also receive a unique embedded login link as an alternative means to securely access the ballot with one simple click. This new feature was added to make it an easier and more convenient voting experience for our members.

AMS members who have chosen to vote by paper should expect to receive their ballot by the middle of September. Unique voting login information will be printed on the ballot, should you wish to vote online.

At midnight (U.S. Eastern Daylight Time) on November 8, 2013, the website will stop accepting votes. Paper ballots received after this date will not be counted.

Additional information regarding the 2013 AMS Election is available on the AMS website:
www.ams.org/about-us/governance/elections/election-info;
or by contacting the AMS: election@ams.org, 800-321-4267 (US & Canada), 401-455-4000 (worldwide).

Thank you and . . . please remember to vote.

Carla D. Savage

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at http://www.ams.org/cgi-bin/mathcal-submit.pl. The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

August 2013

* 12–16 **Masterclass Pressure and Weil-Petersson metrics by Martin Bridgeman (Boston), Dick Canary (Michigan), and Andrés Sambarino (Paris-Sud 11)**, QGM, Aarhus University, Aarhus, Denmark.
Speakers: Martin Bridgeman (Boston), Dick Canary (Michigan), Andrés Sambarino (Paris-Sud 11). On their recent work with Francois Labourie proving that the pressure metric from statistical mechanics agrees with the complex Weil-Petersson metric on the SL(n)-representation variety. As usual with the QGM masters classes, the class will begin from first principles and then accelerate once or twice during the week.
Preliminary schedule: 10:00–10:45, Lecture; 11:15–12:00, Lecture; 12:00–2:00, Lunch; 2:00–2:45 Lecture; 3:15–4:00, Lecture. Social programme Monday August 12th at 6pm. Social networking dinner: Free of charge, Thursday August 15th at 6pm. Special dinner at a restaurant in town, free of charge but you must be signed up for this.
Information: http://qgm.au.dk/events/show/artikel/masterclass-august-2013/.

* 19–22 **Conference: Pressure metric and Higgs bundles**, QGM, Aarhus University, Aarhus, Denmark.
Description: Preliminary schedule 10:00–10:45, Lecture; 11:15–12:00, Lecture; 12:00–2:00, Lunch; 2:00–2:45, Lecture; 3:15–4:00, Lecture. Social programme Monday August 19th at 6pm: Social networking dinner. Free of charge. Thursday August 21 at 6pm: Special dinner at a restaurant in town. Free of charge, but you must be signed up for this.
Information: http://qgm.au.dk/events/show/artikel/conference-august-2013/.

September 2013

* 6–8 **National Seminar To Commemorate the Sesquicentennial Birth Anniversary of "Sir Asutosh Mookerjee" (1864-1924) (NSAM-2013)**, Calcutta Mathematical Society, Asutosh Bhavan AE-374, Sector-I, Salt Lake City, Kolkata-700064, West Bengal, India.
Description: Sir Asutosh Mookerjee was the founder president of this society. The society started its journey in the year 1908 under the able leadership of Sir Asutosh Mookerjee, popularly known as "the tiger of Bengal". He was a versatile educationist, mathematical genius and a jurist. It's our proud privilege to get the opportunity to celebrate his 150th birthday. His genius as a mathematician has not been highlighted so much so far. This seminar is planned to plumb into his mathematical works in detail along with the activities in other fields too. Those who want to present a paper in this seminar must send the title of the paper and abstract by July 15, 2013, to cmsconf@gmail.com.
Registration fee: INR 400 (for members), INR 600 (for non-members).
Information: Contact No.: +91-33-2337 8882; email: cmsconf@gmail.com; http://www.calmathsoc.org/index.php.

* 9–13 **Workshop & Summer School on Finite Semifields**, Università degli Studi di Padova, Padova, Italy.
Description: The theory of finite Semifields has received a great amount of attention and many interesting results and new connections to other mathematical structures have been established during the last decade. This workshop is aimed at students and researchers who take an interest in this rich theory as well as at established researchers active in this area.
Information: http://www.combinatorics.it.

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

* 16–20 **5th International Conference on Analytic Number Theory and Spatial Tessellations**, National Pedagogical Dragomanov University, Kyiv, Ukraine.
Description: The conference is devoted to the 145th anniversary of the eminent Ukrainian mathematician George Voronoi (1868–1908). As at all previous Kyiv conferences, the topics of this conference are related to all research areas to which Voronoi contributed: number theory, geometry of numbers, Voronoi method of summability, spatial tessellations, applications of Voronoi diagrams in natural sciences.
Information: `http://www.fmi.npu.edu.ua/voronoi2013`.

* 16–20 **LAP 2013: Logic and Applications**, Inter University Center Dubrovnik, Dubrovnik, Croatia.
Description: The conference brings together researchers from various fields of logic with applications in computer science. Topics of interest include, but are not restricted to: formal systems of classical and non-classical logic, category theory, proof theory, Lambda calculus, Pi calculus, behavioural types, systems of reasoning in the presence of incomplete, imprecise and/or contradictory information, computational complexity, interactive theorem provers. The first conference proof systems was held in Dubrovnik on June 28, 2012, co-located with the conference LICS 2012.
Information: `http://imft.ftn.uns.ac.rs/math/cms/LAP2013`.

* 16–20 **Summer School of Mathematics for Economics and Social Sciences organized by the Mathematics Research Centre "Ennio De Giorgi", partially supported by the International Doctoral Program in Economics of the Scuola Superiore Sant'Anna**, Conservatorio Di Santa Chiara, San Miniato, Italy.
Description: The School aims to improve the knowledge of mathematical methods among graduate students in economics and social sciences, with a focus on those techniques which albeit widespread in use are not properly covered in typical graduate programs. The School is an interdisciplinary venue intended to foster the interaction of people coming from the too often separated communities of mathematical and social scientists.
Topics: Information theory, chaos and ergodicity with application to data analysis.
Lecturer: Stefano Marmi, Fabrizio Lillo (Scuola Normale Superiore). Participation is subject to selection, 20–25 positions available.
Support: Financial support for board and accommodation. Online applications should be made at `http://crm.sns.it/event/276/financial.html`. All applications must include a CV that shall be sent to: `crm@crm.sns.it`. Applications without a CV will not be considered.
Deadlines: For application: August 2, 2013. Decision on the application will be communicated: August 9, 2013.
Information: `http://crm.sns.it/event/276/documents.html#title`.

* 16–20 **The 34th International Conference on Quantum Probability and Related Topics**, Steklov Mathematical Institute, Moscow, Russia.
Description: The conference continues a traditional series of yearly conferences on quantum probability and related topics. Conference topics include recent developments in quantum probability and quantum dynamics such as quantum Markov processes and semigroups, white noise calculus, quantum stochastic calculus, stochastic limit and applications to physics, quantum information theory, quantum control, Levy Laplacians and associated processes, free probability, independences, Hilbert modules, non-commutative geometry, interacting Fock spaces, infinite dimensional Lie algebras, infinite dimensional analysis, quantum field theory, quantum optics, and other related subjects.
Information: `http://qp34.mi.ras.ru`.

* 19–20 **DIMACS Workshop on Algorithmic Information Fusion and Data Mining (WAIFDM)**, DIMACS Center, CoRE Building, Rutgers University, Piscataway, New Jersey.
Description: Presented under the auspices of the Special Focus on Algorithmic Decision Theory and in partnership with the European Consortium ALGODEC. Information fusion and data mining are fundamental in the scientific discovery process of data acquisition, information integration, and knowledge discovery. Although methods for information fusion and data mining have been used for hundreds of years, it remains a challenging problem to understand when, what, and how to optimally mine data, fuse information and discover knowledge. Among others, the DIMACS Workshop on Algorithmic Information Fusion and Data Mining (WAIFDM) will address the following two types of problems: Given a complex problem in a data-rich environment, how to extract variables and how to perform variable selection and combination? Here "variable" includes feature, attribute, cue, indicator, and parameter. Given two machine learning or data mining systems A and B, when and how to best combine A and B? Given many possible decisions systems for a solution, how to best select and combine a subset of these systems?
Organizers: Frank Hsu, Fordham University, `hsu@cis.fordham.edu`; Fred Roberts, DIMACS, `froberts@dimacs.rutgers.edu`; Alexis Tsoukias, University of Paris and LAMSADE (CNRS), `tsoukias@lamsade.dauphine.fr`.
Local Arrangements: Workshop Coordinator, DIMACS Center, `workshop@dimacs.rutgers.edu`, 732-445-5928.
Information: `http://dimacs.rutgers.edu/Workshops/InformationFusion/index.html`.

* 21–22 **The 33nd Annual Southeastern-Atlantic Regional Conference on Differential Equations**, University of Tennessee, Knoxville, Tennessee.
Plenary Speakers: Lawrence Craig Evans, University of California, Berkeley; Wilfrid Gangbo, Georgia Institute of Technology; Yuriko Renardy, Virginia Polytechnic Institute and State University; and Glenn Webb, Vanderbilt University. In addition to the plenary speakers, there will be sessions of twenty minute contributed talks.
Funding: From the National Science Foundation has been requested to provide travel support for advanced graduate students and recent Ph.D. recipients. The conference also welcomes and encourages the contributions of undergraduate students. Women and minorities are especially encouraged to participate in this conference and apply for support.
Deadline: For abstracts and early registration is August 30, 2013.
Information: For more information, please contact: `swise@math.utk.edu`; `lenhart@math.utk.edu` or `phan@math.utk.edu`; `http://www.math.utk.edu/SEARCDE2013/Files/Main.html`.

* 24 **ICERM Public Lecture: On Growth and Form: Mathematics, Physics and Biology**, Salomon Hall, Brown University, Providence, Rhode Island.
Description: The diversity of living forms led Darwin to state that it is "enough to drive the sanest man mad". How can we describe this variety? How can we understand the origin and evolution of these "endless forms most beautiful?" And how do these forms link to function and physiology at the organismic level and beyond? Mathematics, and geometry in particular, provides a natural language to express these questions and answer them. Motivated by biological observations on different scales from molecules to organisms to swarms, I will show how a combination of quantitative experiments, physical analogies, mathematical theories and computational models allow us to begin to unravel the mechanistic basis for aspects of morphogenesis and thence towards physiology, pathophysiology and biomimetics.
Speaker: L. Mahadevan, Harvard University.

Sponsor: Simons Foundation.
Information: http://icerm.brown.edu/simonslecture.

October 2013

* 1–5 **II International Seminar: Nonlinear Phenomenology Advances**, St. Petersburg State Polytechnical University, Saint Petersburg, Russia.
Description: There will be a discussion on some modern advances, approaches and tools for studying nonlinear problems in different fields of science (mathematics, physics, chemistry, biology, economics and others).
Scope: Nonlinear dynamical systems; spatio-temporal structures patterning in complex media, fractals; number theory and cryptography; nonlinear statistics; wavelet analysis; self-organization and cooperative phenomena; classical and quantum chaos, controlling chaos, turbulence; wave turbulence and complexity; mesoscopic and low-dimensional systems; nonlinear dynamics in chemistry, biology, economics and social sciences; soils nonlinear dynamics and evolution; neural network modeling of nonlinear systems and phenomena.
Information: http://www.hmath.spbstu.ru/index.php/seminary.

* 11–12 **Workshop on Mathematics of Electoral Systems: Voting, Apportioning and Districting**, Budapest, Hungary.
Description: Researchers and advanced Ph.D. students are invited to submit papers to the Workshop. Both theoretical and applied contributions are welcome.
Keynote speakers: Felix Brandt (TU München), Friedrich Pukelsheim (Universität Augsburg).
Deadline: Papers should be submitted to email: mesvad@uni-corvinus.hu by August 20, 2013. Authors of accepted papers will be notified by September 1, 2013.
Program committee: Clemens Puppe (Karlsruhe Institute of Technology), Attila Tasnádi (Corvinus University).
Information: http://mes-vad.uni-corvinus.hu.

* 23–25 **International Conference on Advanced Computing and Applications (ACOMP 2013)**, Ho Chi Minh City University of Technology (HCMUT), Ho Chi Minh City, Vietnam.
Description: ACOMP is an annual international forum for the exchange of ideas, techniques, and state-of-the-art applications in the field of advanced computing among scientists, engineers, and practitioners. For ACOMP2013, we will provide a friendly environment where researchers can discuss current and future trends in research areas such as Security and Information Systems, Software Engineering, Embedded Systems and VLSI Design, High Performance Computing, Image Processing and Visualization, Scientific Computing and other interesting topics.
Information: http://www.cse.hcmut.edu.vn/acomp2013/.

* 23–27 **Conference on integrable systems, random matrix theory, and combinatorics**, University of Arizona, Tucson, Arizona.
Description: The main goal of this conference is to foster interaction between researchers in integrable nonlinear partial differential equations and researchers working in random matrix theory and combinatorics. This will be accomplished by lectures in the morning, and then dividing into working groups in the afternoons. We are planning a conference with the main participants being younger researchers, with a few more senior researchers scattered in for good measure. A second aspect, and one that will be celebrated in the evenings, is the 60th birthday of Nicholas Ercolani, who is always a strong advocate for research at the boundary between diverse areas of mathematics such as these.
Information: http://math.arizona.edu/~mcl/CombConf.htm.

* 24–25 **International Conference on Mathematical Techniques in Engineering Applications (ICMTEA 2013)**, Graphic Era University, Dehradun, Uttarakhand, India.
Description: International Conference on Mathematical Techniques in Engineering Applications's aims at publishing refereed, original research studies and articles that bring up the latest development and research in the mathematical sciences and engineering area. The goal of the conference is to bring together leading engineers, researchers and academics from the world to discuss novel theories, technologies and applications of mathematics in major engineering fields. The conference will feature prominent plenary speakers as well as technical sessions. The conference covers a broad spectrum of topics related to mathematics applications in engineering. ICMTEA 2013 is a peer-reviewed conference, with experts from different parts of world.
Information: http://www.geu.ac.in/graphicneprd.aspx?pgid=104&nid=489.

* 28–31 **International Conference on Complex Analysis and Geometry In the Honor of Hassine Elmir AGC 2013**, Monastir University, Rue Salem Bchir, Al Munastir 5000, Tunisia.
Description: This conference, to be held at Monastir University, will focus on recent developments in complex analysis and geometry.
Information: http://www.fsg.rnu.tn/AGC_2013.htm.

* 30–November 1 **56th Annual Congress of the South African Mathematical Society**, University of KwaZulu-Natal, Pietermaritzburg, South Africa.
Description: The annual congress of the South African Mathematical Society provides a unique platform for South African mathematicians and international partners to share ideas, showcase cutting edge research in the mathematical sciences and to start new collaborations and friendships. As well as the core areas of applied and pure mathematics, we encourage significant interdisciplinary collaborations that showcase contributions that mathematics can make in all areas of science.
Information: http://www.sams2013.ukzn.ac.za/.

November 2013

* 4–8 **Waves in Science and Engineering**, Huatulco, Mexico.
Description: Conference Waves in Science and Engineering 2013 will be held November 4-8. It is intended to bring together experts from different fields of the general area of classical wave theory and applications including acoustic, electromagnetic, and elastic wave propagation. The mathematical and numerical modeling procedures in these fields contribute to a considerable number of applied physical and engineering problems, over a large range of length scales. Among these are problems in sonar, radar, medical imaging, detection, materials, and wave interactions with surfaces and obstacles. The conference will cover many of the current mathematical and numerical techniques that are applied across disciplines. Mathematicians, physicists, and engineers of varying backgrounds and occupations will present recent developments in wave phenomena in science and engineering.
Information: http://www.wise.esimez.ipn.mx.

December 2013

* 16–19 **deLeonfest 2013**, Institute for the Mathematical Sciences, ICMAT (CSIC-UAM-UC3M-UCM), Madrid, Spain.
Description: We would like to draw your attention to the workshop deLeónfest 2013. This event is to commemorate the 60th birthday of Professor Manuel de León. Manuel de León has enormously contributed in many aspects of mathematics and research, and still does. Among many of his facets we could highlight his research on symplectic geometry, Poisson manifolds, nonholonomic mechanics, geometric integrators, optimal control theory, etc, his active role in the diffusion of mathematics, in mathematical organizations at the international and national levels, in mathematics popularization, edition and direction of scientific journals.
Registration: The registration is now open. The registration fee is 100 euros for seniors and 50 euros for students. More

information about this event is available at http://www.icmat.es/deLeonfest. If you need any further information, please contact us at deleonfest@icmat.es. We hope to see you in Madrid for this tribute to Manuel de León.
Information: http://www.icmat.es/deLeonfest/.

January 2014

* 27–31 **AIM Workshop: Arithmetic statistics over finite fields and function fields**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the study of statistical questions about objects of arithmetic geometry, especially algebraic varieties over function fields and finite fields.
Information: http://www.aimath.org/ARCC/workshops/arithstatffield.html.

April 2014

* 10–12 **University of Arkansas Spring Lecture Series in Mathematics – "Multi-parameter Geometry and Analysis"**, University of Arkansas, Fayetteville, Arkansas.
Description: Each year, the Department of Mathematical Sciences at the University of Arkansas hosts a small conference called the Arkansas Spring Lecture Series in Mathematics. The conference focuses on a special topic selected by a principal lecturer who delivers five talks on the subject. The main speaker in 2014 will be Professor Alexander Nagel of the University of Wisconsin-Madison, and the conference topic is going to be: "Multi-parameter Geometry and Analysis". There are ten additional invited talks, and early career researchers and finishing graduate students may give contributed talks.
Information: http://math.uark.edu/3742.php.

May 2014

* 26–30 **8th European Conference on Elliptic and Parabolic Problems**, Hotel Serapo, Gaeta, Italy.
Description: Besides elliptic and parabolic issues, the topics of the conference include geometry, free boundary problems, fluid mechanics, evolution problems in general, calculus of variations, homogenization, control, modeling and numerical analysis. In addition to the plenary talks parallel sessions and minisymposia will be organized.
Information: http://www.math.uzh.ch/index.php?konferenzdetails0&key1=32&L=1.

June 2014

* 5–7 **Number Theory at Illinois: A Conference in Honor of the Batemans**, University of Illinois, Urbana, Illinois.
Description: A Number Theory Conference in memory of Paul and Felice Bateman will be held at the University of Illinois. The Batemans were long-time members of the faculty and Paul was department head for 14 years. Paul was a member of the American Mathematical Society for 71 years and among his other services, was a Trustee of the AMS. This meeting continues a long tradition of number theory conferences at Illinois.
Invited talks: There will be twenty invited talks as well as opportunities for contributed talks. These will cover a broad spectrum of number theory, representing Paul's many interests. A banquet will be held on June 6. There will be a refereed proceedings volume of conference talks. The conference will be preceded by the Midwest Number Theory Conference for Graduate Students, June 3–4, 2014 (which is being announced separately).
Information: http://www.math.illinois.edu/nt2014.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

August 2014

* 28–29 **Connections for Women: Geometric Representation Theory**, Mathematical Sciences Research Institute, Berkeley, California.
Description: Within the broad range of geometric representation theory, the Connections Workshop will focus on three research topics in which we expect particularly striking new developments within the next few years: Categorical and geometric structures in representation theory and Lie superalgebras; Geometric construction of representations via Shimura varieties and related moduli spaces; Hall algebras and representations. The workshop will bring together researchers from these different topics within geometric representation theory and will thus facilitate a successful start of the semester program. It will give junior researchers from each of these parts of geometric representation theory a broader picture of possible applications and of new developments, and will establish a closer contact between junior and senior researchers. This workshop is aimed at encouraging and increasing the active participation of women and members of under-represented groups in the MSRI program.
Information: http://www.msri.org/workshops/706.

November 2014

* 17–21 **Categorical Structures in Harmonic Analysis Workshop**, Mathematical Sciences Research Institute, Berkeley, California.
Description: The workshop will focus on the role of categorical structures in number theory and harmonic analysis, with an emphasis on the setting of the Langlands program. Celebrated examples of this theme range from Lusztig's character sheaves to Ngo's proof of the Fundamental Lemma. The workshop will be a forum for researchers from a diverse collection of fields to compare problems and strategies for solutions.
Information: http://www.msri.org/web/msri/scientific/workshops/all-workshops/show/-/event/Wm9805.

December 2014

* 1–5 **Automorphic forms, Shimura varieties, Galois representations and L-functions**, Mathematical Sciences Research Institute, Berkeley, California.
Description: L-functions attached to Galois representations coming from algebraic geometry contain subtle arithmetic information (conjectures of Birch and Swinnerton-Dyer, Deligne, Beilinson, Bloch and Kato, Fontaine and Perrin-Riou). Langlands has predicted the existence of a correspondence relating these L-functions to L-functions of automorphic forms which are much better understood. The workshop will focus on recent developments related to Langlands correspondence (construction of Galois representations attached to automorphic forms via the cohomology of Shimura varieties, modularity of Galois representations…) and arithmetic of special values of L-functions. It will be dedicated to Michael Harris as a tribute to his enormous influence on the themes of the workshop.
Information: http://www.msri.org/workshops/719.

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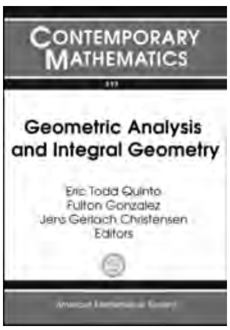
Analysis



Geometric Analysis and Integral Geometry

Eric Todd Quinto, **Fulton Gonzalez**, and **Jens Gerlach Christensen**, *Tufts University, Medford, MA*, Editors

This volume contains the proceedings of the AMS Special Session on Radon Transforms and Geometric Analysis, in honor of Sigurdur Helgason's 85th Birthday, held from January 4–7, 2012, in Boston, MA, and the Tufts University Workshop on Geometric Analysis on Euclidean and Homogeneous Spaces, held from January 8–9, 2012, in Medford, MA.

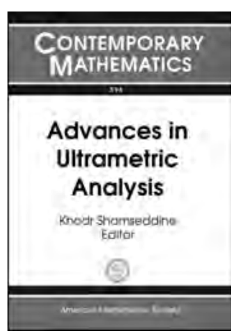
This volume provides an historical overview of several decades in integral geometry and geometric analysis as well as recent advances in these fields and closely related areas. It contains several articles focusing on the mathematical work of Sigurdur Helgason, including an overview of his research by Gestur Ólafsson and Robert Stanton. The first article in the volume contains Helgason's own reminiscences about the development of the group-theoretical aspects of the Radon transform and its relation to geometric analysis. Other contributions cover Radon transforms, harmonic analysis, Penrose transforms, representation theory, wavelets, partial differential operators on groups, and inverse problems in tomography and cloaking that are related to integral geometry.

Many articles contain both an overview of their respective fields as well as new research results. The volume will therefore appeal to experienced researchers as well as a younger generation of mathematicians. With a good blend of pure and applied topics the volume will be a valuable source for interdisciplinary research.

Contents: *Historical articles:* **S. Helgason**, Some personal remarks on the Radon transform; **G. Ólafsson** and **R. J. Stanton**, On the life and work of S. Helgason; *Research and expository articles:* **G. Ambartsoumian**, **J. Boman**, **V. P. Krishnan**, and **E. T. Quinto**, Microlocal analysis of an ultrasound transform with circular source and receiver trajectories; **N. B. Andersen** and **M. Flensted–Jensen**, Cuspidal discrete series for projective hyperbolic spaces; **S. Bernstein** and **I. Z. Pesenson**, The Radon transform on $SO(3)$: Motivations, generalizations, discretization; **J. G. Christensen**, Atomic decompositions of Besov spaces related to symmetric cones; **M. Eastwood**, A double fibration transform for complex projective space; **T. Kakehi**, Magnetic Schrödinger equation on compact symmetric spaces and the geodesic Radon transform of one forms; **T. Kobayashi**, F-method for constructing equivariant differential operators; **H. Liu**, Schiffer's conjecture, interior transmission eigenvalues and invisibility cloaking: Singular problem vs. nonsingular problem; **W. R. Madych**, Approximate reconstruction from circular and spherical mean Radon transform data; **G. Ólafsson**, **A. Pasquale**, and **B. Rubin**, Analytic and group-theoretic aspects of the cosine transform; **H. Oda** and **T. Oshima**, Quantization of linear algebra and its application to integral geometry; **F. Rouvière**, Mean value theorems on symmetric spaces; **B. Rubin**, Semyanistyi fractional integrals and Radon transforms; **H. Sekiguchi**, Radon-Penrose transform between symmetric spaces; **J. A. Wolf**, Principal series representations of infinite dimensional Lie groups, II: construction of induced representations.

Contemporary Mathematics, Volume 598

August 2013, 280 pages, Softcover, ISBN: 978-0-8218-8738-7, LC 2013013624, 2010 *Mathematics Subject Classification:* 22E30, 43A85, 44A12, 45Q05, 92C55; 22E46, 32L25, 35S30, 65R32, **AMS members US$80**, List US$100, Order code CONM/598



Advances in Ultrametric Analysis

Khodr Shamseddine, *University of Manitoba, Winnipeg, Manitoba, Canada*, Editor

This volume contains papers based on lectures given at the 12th International Conference on p-adic Functional Analysis, which was held at the University of Manitoba on July 2–6, 2012.

The articles included in this book feature recent developments in various areas of non-archimedean analysis: branched values and zeros of the derivative of a p-adic meromorphic function, p-adic meromorphic functions $f'P'(f), g'P'(g)$ sharing a small function, properties of composition of analytic functions, partial fractional differentiability, morphisms between ultrametric Banach algebras of continuous functions and maximal ideals of finite dimension, the p-adic q-distributions, Banach spaces over fields with an infinite rank valuation, Grobman-Hartman theorems for diffeomorphisms of Banach spaces over valued fields, integral representations of continuous linear maps on p-adic spaces of continuous functions, non-Archimedean operator algebras, generalized Keller spaces over valued fields, proper multiplications on the completion of a totally ordered abelian group, the Grothendieck approximation theory in non-Archimedean functional analysis, generalized power series spaces, measure theory and the study of power series and analytic functions on the Levi-Civita fields.

Through a combination of new research articles and survey papers, this book provides the reader with an overview of current developments and techniques in non-archimedean analysis as well as a broad knowledge of some of the sub-areas of this exciting and fast-developing research area.

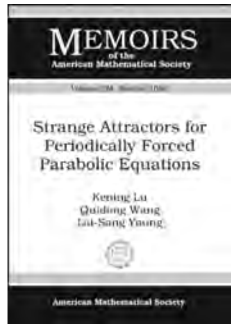
This item will also be of interest to those working in algebra and algebraic geometry.

Contents: **M. Berz** and **S. Troncoso**, Affine invariant measures in Levi-Civita vector spaces and Erdös obtuse angle theorem; **J.-P. Bézivin**, **K. Boussaf**, and **A. Escassut**, Some old and new results on zeros of the derivative of a p-adic meromorphic function; **K. Boussaf**, **A. Escassut**, and **J. Ojeda**, Survey on p-adic meromorphic functions $f'P'(f), g'P'(g)$ sharing a small function and additional properties; **B. Diarra**, The p-adic q-distributions; **A. Escassut** and **N. Maïnetti**, Morphisms between ultrametric Banach algebras and maximal ideals of finite codimension; **A. Escassut** and **J. Ojeda**, Survey on branched values and exceptional values for p-adic meromorphic functions; **H. Glöckner**, Grobman-Hartman theorems for diffeomorphisms of Banach spaces over valued fields; **A. K. Katsaras**, Integral representations of continuous linear maps on p-adic spaces of continuous functions; **H. A. Keller**, Subfields of valued, complete fields; **A. N. Kochubei**, On some classes of non-Archimedean operator algebras; **H. Maïga** and **F. Tangara**, Some identities and congruences for Stirling numbers of the second kind; **H. M. Moreno**, Non-measurable sets in the Levi-Civita field; **E. Nagel**, Partial fractional differentiability; **H. Ochsenius** and **E. Olivos**, A generalized space over a field with a valuation of rank $\alpha > \omega$; **H. Ochsenius** and **E. Olivos**, A comprehensive survey of non-archimedean analysis in Banach spaces over fields with an infinite rank valuation; **E. Olivos** and **W. H. Schikhof**, All proper multiplications on the completion of a totally ordered albelian group; **C. Perez-Garcia**, The Grothendieck approximation theory in non-archimedean functional analysis; **K. Shamseddine**, A brief survey of the study of power series and analytic functions on the Levi-Civita fields; **W. Śliwa**, On non-archimedean generalized power series spaces.

Contemporary Mathematics, Volume 596

September 2013, approximately 289 pages, Softcover, ISBN: 978-0-8218-9142-1, 2010 *Mathematics Subject Classification:* 46S10, 30G06, 12J25, 32P05, 26E30, 11S80, 30D35, 47L10, 46G10, 06F05, **AMS members US$80**, List US$100, Order code CONM/596

Differential Equations



Strange Attractors for Periodically Forced Parabolic Equations

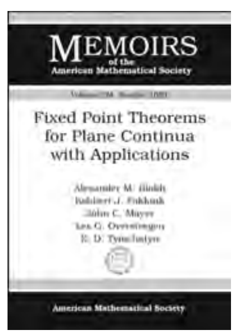
Kening Lu, *Brigham Young University, Provo, UT*, **Qiudong Wang**, *University of Arizona, Tucson, AZ*, and **Lai-Sang Young**, *Courant Institute of Mathematical Sciences, New York University, NY*

Contents: Introduction; Basic definitions and facts; Statement of theorems; Invariant manifolds; Canonical form of equations around the limit cycle; Preliminary estimates on solutions of the unforced equation; Time-T map of forced equation and derived 2-D system; Strange attractors with SRB measures; Application: The Brusselator; Appendix A. Proofs of Propositions 3.1–3.3; Appendix B. Proof of Proposition 7.5; Appendix C. Proofs of Proposition 8.1 and Lemma 8.2; Bibliography.

Memoirs of the American Mathematical Society, Volume 224, Number 1054

June 2013, 85 pages, Softcover, ISBN: 978-0-8218-8484-3, LC 2013006850, 2010 *Mathematics Subject Classification:* 37L30; 37D45, **AMS members US$55.20**, List US$69, Order code MEMO/224/1054

Geometry and Topology



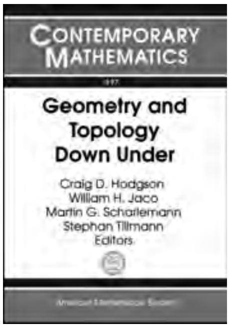
Fixed Point Theorems for Plane Continua with Applications

Alexander M. Blokh, *University of Alabama, Birmingham, AL*, **Robbert J. Fokkink**, *Delft Institute of Applied Mathematics, Netherlands*, **John C. Mayer** and **Lex G. Oversteegen**, *University of Alabama, Birmingham, AL*, and **E. D. Tymchatyn**, *University of Saskatchewan, Saskatoon, SK, Canada*

Contents: Introduction; *Part 1. Basic Theory:* Preliminaries and outline of Part 1; Tools; Partitions of domains in the sphere; *Part 2. Applications of Basic Theory:* Description of main results of Part 2; Outchannels and their properties; Fixed points; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 224, Number 1053

June 2013, 97 pages, Softcover, ISBN: 978-0-8218-8488-1, LC 2013006837, 2010 *Mathematics Subject Classification:* 37C25, 54H25; 37F10, 37F50, 37B45, 54C10, **AMS members US$55.20**, List US$69, Order code MEMO/224/1053



Geometry and Topology Down Under

Craig D. Hodgson, *University of Melbourne, Parkville, Victoria, Australia*, **William H. Jaco**, *Oklahoma State University, Stillwater, OK*, **Martin G. Scharlemann**, *University of California, Santa Barbara, CA*, and **Stephan Tillmann**, *University of Sydney, NSW, Australia*, Editors

This book contains the proceedings of the conference Geometry & Topology Down Under, held July 11–22, 2011, at the University of Melbourne, Melbourne, Australia, in honour of Hyam Rubinstein.

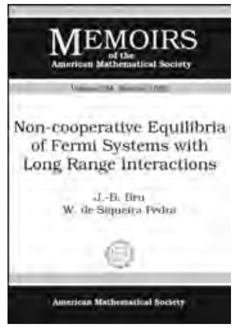
The main topic of the book is low-dimensional geometry and topology. It includes both survey articles based on courses presented at the conferences and research articles devoted to important questions in low-dimensional geometry. Together, these contributions show how methods from different fields of mathematics contribute to the study of 3-manifolds and Gromov hyperbolic groups. It also contains a list of favorite problems by Hyam Rubinstein.

Contents: *Survey and expository papers:* **J. Hass**, What is an almost normal surface?; **D. Calegari**, The ergodic theory of hyperbolic groups; **S. Hong** and **D. McCullough**, Mapping class groups of 3-manifolds, then and now; **B. H. Bowditch**, Stacks of hyperbolic spaces and ends of 3-manifolds; **E. Carberry**, Harmonic maps and integrable systems; **H. Rubinstein**, Some of Hyam's favourite problems; *Research papers:* **D. Bachman**, **R. Derby-Talbot**, and **E. Sedgwick**, Almost normal surfaces with boundary; **B. A. Burton**, Computational topology with Regina: Algorithms, heuristics and implementations; **A. Clay** and **M. Teragaito**, Left-orderability and exceptional Dehn surgery on two-bridge knots; **A. Deruelle**, **M. Eudave-Muñoz**, **K. Miyazaki**, and **K. Motegi**, Networking Seifert surgeries on knots IV: Seiferters and branched coverings; **S. Friedl**, Commensurability of knots and L^2-invariants; **J. A. Hillman**, The groups of fibred 2-knots; **C. Hodgson** and **H. Masai**, On the number of hyperbolic 3-manifolds of a given volume; **K. Ichihara** and **I. D. Jong**, Seifert fibered surgery and Rasmussen invariant; **F. Luo**, Existence of spherical angle structures on 3-manifolds; **J. H. Rubinstein** and **A. Thompson**, 3-manifolds with Heegaard splittings of distance two; **M. Scharlemann**, Generating the genus $g+1$ Goeritz group of a genus g handlebody.

Contemporary Mathematics, Volume 597

August 2013, approximately 383 pages, Softcover, ISBN: 978-0-8218-8480-5, LC 2013012326, 2010 *Mathematics Subject Classification:* 57M25, 57M27, 57M50, 57N10, 57Q15, 57Q45, 20F65, 20F67, 53A10, 53C43, **AMS members US$98.40**, List US$123, Order code CONM/597

Mathematical Physics



Non-cooperative Equilibria of Fermi Systems with Long Range Interactions

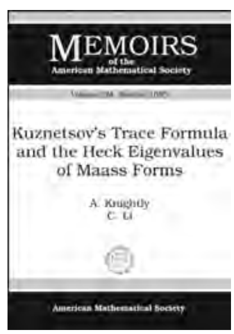
J.-B. Bru, *Universidad del Pais Vasco, Bilbao, Spain*, and **W. de Siqueira Pedra**, *Universität Mainz, Germany*

Contents: *Part 1. Main Results and Discussions:* Fermi systems on lattices; Fermi systems with long-range interactions; *Part 2. Complementary Results:* Periodic boundary conditions and Gibbs equilibrium states; The set $E_{\vec{\ell}}$ of $\vec{\ell}.\mathbb{Z}^d$–invariant states; Permutation invariant Fermi systems; Analysis of the pressure via t.i. states; Purely attractive long–range Fermi systems; The max–min and min–max variational problems; Bogoliubov approximation and effective theories; Appendix; Bibliography; Index of notation; Index.

Memoirs of the American Mathematical Society, Volume 224, Number 1052

June 2013, 155 pages, Softcover, ISBN: 978-0-8218-8976-3, LC 2013009060, 2010 *Mathematics Subject Classification:* 82B10, 91A40; 46A55, 58E30, **AMS members US$64**, List US$80, Order code MEMO/224/1052

Number Theory



Kuznetsov's Trace Formula and the Hecke Eigenvalues of Maass Forms

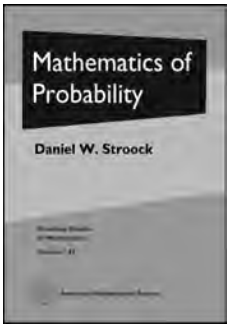
A. Knightly, *University of Maine, Orono, ME*, and **C. Li**, *The Chinese University of Hong Kong, China*

Contents: Introduction; Preliminaries; Bi-K_∞-invariant functions on $\mathrm{GL}_2(\mathbf{R})$; Maass cusp forms; Eisenstein series; The kernel of $R(f)$; A Fourier trace formula for GL(2); Validity of the KTF for a broader class of h; Kloosterman sums; Equidistribution of Hecke eigenvalues; Bibliography; Notation index; Subject index.

Memoirs of the American Mathematical Society, Volume 224, Number 1055

June 2013, 132 pages, Softcover, ISBN: 978-0-8218-8744-8, LC 2013006851, 2010 *Mathematics Subject Classification:* 11F72, 11F70, 11F41, 11F37, 11F30, 11L05, 11F25, 22E55, **AMS members US$58.40**, List US$73, Order code MEMO/224/1055

Probability and Statistics



Mathematics of Probability

Daniel W. Stroock, *Massachusetts Institute of Technology, Cambridge, MA*

This book covers the basics of modern probability theory. It begins with probability theory on finite and countable sample spaces and then passes from there to a concise course on measure theory, which is followed by some initial applications to probability theory, including independence and conditional expectations. The second half of the book deals with Gaussian random variables, with Markov chains, with a few continuous parameter processes, including Brownian motion, and, finally, with martingales, both discrete and continuous parameter ones.

The book is a self-contained introduction to probability theory and the measure theory required to study it.

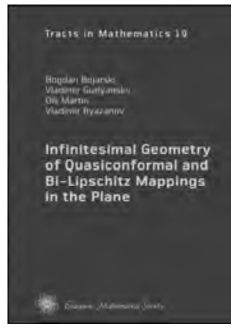
Contents: Some background and preliminaries; Probability theory on uncountable sample spaces; Some applications to probability theory; The central limit theorem and Gaussian distributions; Discrete parameter stochastic processes; Some continuous-time processes; Martingales; Notation; Bibliography; Index.

Graduate Studies in Mathematics, Volume 149

August 2013, 284 pages, Hardcover, ISBN: 978-1-4704-0907-4, LC 2013011622, 2010 *Mathematics Subject Classification:* 60A99, 60J10, 60J99, 60G42, 60G44, **AMS members US$60**, List US$75, Order code GSM/149

New AMS-Distributed Publications

Analysis



Infinitesimal Geometry of Quasiconformal and Bi-Lipschitz Mappings in the Plane

Bogdan Bojarski, *Institute of Mathematics, PAN, Warsaw, Poland*, **Vladimir Gutlyanskii**, *National Academy of Sciences of Ukraine, Donetsk, Ukraine*, **Olli Martio**, *Finnish Academy of Science and Letters, Helsinki, Finland*, and **Vladimir Ryazanov**, *National Academy of Sciences of Ukraine, Donetsk, Ukraine*

This book is intended for researchers interested in new aspects of local behavior of plane mappings and their applications. The presentation is self-contained, but the reader is assumed to know basic complex and real analysis.

The study of the local and boundary behavior of quasiconformal and bi-Lipschitz mappings in the plane forms the core of the book. The concept of the infinitesimal space is used to investigate the behavior of a mapping at points without differentiability. This concept, based on compactness properties, is applied to regularity problems of quasiconformal mappings and quasiconformal curves, boundary behavior, weak and asymptotic conformality, local winding properties, variation of quasiconformal mappings, and criteria of univalence. Quasiconformal and bi-Lipschitz mappings are instrumental for understanding elasticity, control theory and tomography, and the book also offers a new look at the classical areas such as the boundary regularity of a conformal map. Complicated local behavior is illustrated by many examples.

The text offers a detailed development of the background for graduate students and researchers. Starting with the classical methods to study quasiconformal mappings, this treatment advances to the concept of the infinitesimal space and then relates it to other regularity properties of mappings in Part II. The new unexpected connections between quasiconformal and bi-Lipschitz mappings are treated in Part III. There is an extensive bibliography.

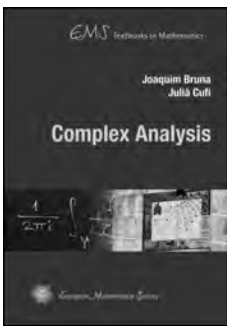
This item will also be of interest to those working in differential equations.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: *I. Quasiconformal Mappings in the Plane:* Background of the theory; Conformal invariants; Definitions of quasiconformal maps; Compactness and convergence theory; Beltrami differential equation; *II. Infinitesimal Geometry of Quasiconformal Maps:* Infinitesimal space; Asymptotically conformal curves; Conformal differentiability; Points of maximal stretching; Lipschitz continuity of quasiconformal maps; Regularity of quasiconformal curves; Regularity of conformal maps at the boundary; *III. Applications of Quasiconformal Maps:* John's rotation problem; Variation of quasiconformal maps; Criteria of univalence; Bibliography; Index.

EMS Tracts in Mathematics, Volume 19

May 2013, 214 pages, Hardcover, ISBN: 978-3-03719-122-4, 2010 *Mathematics Subject Classification:* 30C65, 30C75, 35J46, 35J50, 35J56, 35J70, 35Q35, 35Q60, 37F30, 37F40, 37F45, 57R99, **AMS members US$62.40**, List US$78, Order code EMSTM/19



Complex Analysis

Joaquim Bruna and **Julià Cufí**, *Universitat Autonoma de Barcelona, Spain*

The theory of functions of a complex variable is a central theme in mathematical analysis that has links to several branches of mathematics. Understanding the basics of the theory is necessary for anyone interested in general mathematical training or for anyone who wants to use mathematics in applied sciences or technology.

The book presents the basic theory of analytic functions of a complex variable and their points of contact with other parts of mathematical analysis. This results in some new approaches to a number of topics when compared to the current literature on the subject.

Some issues covered are: a real version of the Cauchy–Goursat theorem, theorems of vector analysis with weak regularity assumptions, an approach to the concept of holomorphic functions of real variables, Green's formula with multiplicities, Cauchy's theorem for locally exact forms, a study in parallel of Poisson's equation and the inhomogeneous Cauchy–Riemann equations, the relationship between Green's function and conformal mapping, the connection between the solution of Poisson's equation and zeros of holomorphic functions, and the Whittaker–Shannon theorem of information theory.

The text can be used as a manual for complex variable courses of various levels and as a reference book. The only prerequisite is a working knowledge of the topology of the plane and the differential calculus for functions of several real variables. A detailed treatment of harmonic functions also makes the book useful as an introduction to potential theory.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Arithmetic and topology in the complex plane; Functions of a complex variable; Holomorphic functions and differential forms; Local properties of holomorphic functions; Isolated singularities of holomorphic functions; Homology and holomorphic functions; Harmonic functions; Conformal mapping; The Riemann mapping theorem and Dirichlet's problem; Runge's theorem and the Cauchy–Riemann equations; Zeros of holomorphic functions; The complex Fourier transform; References; Symbols; Index.

EMS Textbooks in Mathematics, Volume 14

May 2013, 576 pages, Hardcover, ISBN: 978-3-03719-111-8, 2010 *Mathematics Subject Classification:* 30-01, 31-01, **AMS members US$62.40**, List US$78, Order code EMSTEXT/14

Erwin Schrödinger—50 Years After

Wolfgang L. Reiter and **Jakob Yngvason**, *University of Vienna, Austria*, Editors

Erwin Schrödinger (1887–1961) was an Austrian physicist famous for the equation named after him and which earned him the Nobel Prize in 1933. This book contains lectures presented at the international symposium "Erwin Schrödinger —50 Years After", held at the Erwin Schrödinger International Institute for Mathematical Physics in January 2011 to commemorate the 50th anniversary of Schrödinger's death.

The text covers a broad spectrum of topics ranging from personal reminiscences to foundational questions about quantum mechanics and historical accounts of Schrödinger's work. Besides the lectures presented at the symposium the volume also contains articles specially written for this occasion.

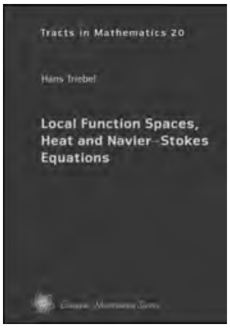
The contributions give an overview of Schrödinger's legacy to the sciences from the standpoint of some contemporary leading scholars in the field.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: W. Thirring, Erwin Schrödinger: Personal reminiscences; **J. Renn**, Schrödinger and the genesis of wave mechanics; **J. Fröhlich** and **B. Schubnel**, Do we understand quantum mechanics—finally?; **A. J. Leggett**, Schrödinger's cat and her laboratory cousins; **M. Müller** and **P. Zoller**, Digital and open system quantum simulation with trapped ions; **R. Kaltenbaek** and **M. Aspelmeyer**, Optomechanical Schrödinger cats—a case for space; **H. Kragh**, A quantum discontinuity: the Bohr–Schrödinger dialogue; **A. J. Knox**, The debate between Hendrik A. Lorentz and Schrödinger on wave mechanics; **O. Darrigol**, A few reasons why Louis de Broglie discovered Broglie's waves and yet did not discover Schrödinger's equation; Chronology; List of contributors; Name index; Subject index.

ESI Lectures in Mathematics and Physics, Volume 9

April 2013, 195 pages, Hardcover, ISBN: 978-3-03719-121-7, 2010 *Mathematics Subject Classification:* 01-02, 81-02, 81-03, 81P05, 81P15, **AMS members US$62.40**, List US$78, Order code EMSESILEC/9



Local Function Spaces, Heat and Navier–Stokes Equations

Hans Triebel, *University of Jena, Germany*

In this book a new approach is presented to exhibit relations between Sobolev spaces, Besov spaces, and Hölder–Zygmund spaces on the one hand and Morrey–Campanato spaces on the other. Morrey–Campanato spaces extend the notion of functions of bounded mean oscillation. These spaces play an important role in the theory of linear and nonlinear PDEs.

Chapters 1–3 deal with local smoothness spaces in Euclidean n-space based on the Morrey–Campanato refinement of the Lebesgue spaces. The presented approach relies on wavelet decompositions. This is applied in Chapter 4 to Gagliardo–Nirenberg inequalities. Chapter 5 deals with linear and nonlinear heat equations in global and local function spaces. The obtained assertions about function spaces and nonlinear heat equations are used in Chapter 6 to study Navier–Stokes equations.

The book is addressed to graduate students and mathematicians with a working knowledge of basic elements of (global) function spaces and an interest in applications to nonlinear PDEs with heat and Navier–Stokes equations as prototypes.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Global and local spaces; Local spaces: Properties; Morrey–Campanato spaces; Gagliardo–Nirenberg inequalities; Heat equations; Navier–Stokes equations; Bibliography; Symbols; Index.

EMS Tracts in Mathematics, Volume 20

May 2013, 241 pages, Hardcover, ISBN: 978-3-03719-123-1, 2010 *Mathematics Subject Classification:* 46-02, 46E35, 42B35, 42C40, 35K05, 35Q30, 76D03, 76D05, **AMS members US$67.20**, List US$84, Order code EMSTM/20

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ALL SOULS COLLEGE
University of Oxford
Senior Research Fellowship in Mathematics

Salary: £76,850 to £84,446 (according to whether and how much University lecturing the Fellow undertakes) plus £5,997 housing allowance for eligible Fellows and other benefits. All Souls College, Oxford, proposes to elect four Senior Research Fellows with effect from October 2014 (or an agreed later date) in four subjects, one of which is mathematics, and invites applications from suitably qualified candidates. A Senior Research Fellowship is of comparable academic standing to a statutory Professorship in the University of Oxford. Applicants are expected to have a correspondingly distinguished record of achievement in research and those elected may apply within the university for the title of Professor.

For further particulars and to complete the online application, see the Appointments section of the college's website: `http://www.all-souls.ox.ac.uk`.

Closing dates: Applications: September 20, 2013. References: September 27, 2013.

Meetings with Fellows: February 14–15, 2014 and February 21–22, 2014.

All Souls College is an Equal Opportunities Employer and particularly encourages applications from women and those with a legally protected characteristic.

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Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Louisville, Kentucky

University of Louisville

October 5–6, 2013

Saturday – Sunday

Meeting #1092

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: June 2013
Program first available on AMS website: August 22, 2013
Program issue of electronic *Notices*: October 2013
Issue of *Abstracts*: Volume 34, Issue 3

Deadlines

For organizers: Expired
For abstracts: August 13, 2013

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael Hill, University of Virginia, *Framed manifolds and equivariant homotopy: A solution to the Kervaire Invariant One problem.*

Suzanne Lenhart, University of Tennessee and NIMBioS, *Using optimal control of PDEs to investigate population questions.*

Ralph McKenzie, Vanderbilt University, *A perspective on fifty years of work, delight and discovery in general algebra.*

Victor Moll, Tulane University, *2-adic valuations of classical sequences: A collection of examples.*

Special Sessions

Algebraic Coding Theory (Code: SS 5A), **Steve Szabo**, Eastern Kentucky University, and **Heide Gluesing-Luerssen**, University of Kentucky.

Algebraic Cryptography (Code: SS 12A), **Daniel Smith**, University of Louisville.

Applied Analysis and Inverse Problems (Code: SS 9A), **Peijun Li**, Purdue University, **Jiguang Sun**, Michigan Technological University, and **Yongzhi Steve Xu**, University of Louisville.

Combinatorial Commutative Algebra (Code: SS 4A), **Juan Migliore**, University of Notre Dame, and **Uwe Nagel**, University of Kentucky.

Commutative Rings, Ideals, and Modules (Code: SS 3A), **Ela Celikbas** and **Olgur Celikbas**, University of Missouri-Columbia.

Extremal Graph Theory (Code: SS 2A), **Jozsef Balogh**, University of Illinois at Urbana-Champaign, and **Louis DeBiasio** and **Tao Jiang**, Miami University, Oxford, OH.

Finite Universal Algebra (Code: SS 6A), **Ralph McKenzie**, Vanderbilt University, and **Matthew Valeriote**, McMaster University.

Fixed Point Theorems and Applications to Integral, Difference, and Differential Equations (Code: SS 8A), **Jeffrey W. Lyons**, Nova Southeastern University, and **Jeffrey T. Neugebauer**, Eastern Kentucky University.

Harmonic Analysis and Partial Differential Equations (Code: SS 10A), **Russell Brown** and **Katharine Ott**, University of Kentucky.

History of Mathematics and Its Use in Teaching (Code: SS 20A), **Daniel J. Curtin**, Northern Kentucky University, and **Daniel E. Otero**, Xavier University.

Homogenization of Partial Differential Equations (Code: SS 14A), **Zhongwei Shen**, University of Kentucky, and **Yifeng Yu**, University of California, Irvine.

Mathematical Analysis of Complex Fluids and Flows (Code: SS 15A), **Xiang Xu**, Carnegie Mellon University, and **Changyou Wang**, University of Kentucky.

Mathematical Issues in Ecological and Epidemiological Modeling (Code: SS 19A), **K. Renee Fister**, Murray State University, and **Suzanne Lenhart**, University of Tennessee.

Mathematical Models in Biology and Physiology (Code: SS 21A), **Yun Kang**, Arizona State University, and **Jiaxu Li**, University of Louisville.

Partial Differential Equations from Fluid Mechanics (Code: SS 16A), **Changbing Hu**, University of Louisville, and **Florentina Tone**, University of West Florida.

Partially Ordered Sets (Code: SS 18A), **Csaba Biro** and **Stephen J. Young**, University of Louisville.

Recent Advances on Commutative Algebra and Its Applications (Code: SS 11A), **Hamid Kulosman** and **Jinjia Li**, University of Louisville, and **Hamid Rahmati**, Miami University.

Set Theory and Its Applications (Code: SS 1A), **Paul Larson**, Miami University, **Justin Moore**, Cornell University, and **Grigor Sargsyan**, Rutgers University.

Spreading Speeds and Traveling Waves in Spatial-Temporal Evolution Systems (Code: SS 17A), **Bingtuan Li**, University of Louisville, and **Roger Lui**, Worcester Polytechnic Institute.

The Work of Mathematicians and Mathematics Departments in Mathematics Education (Code: SS 22A), **Benjamin Braun**, **Carl Lee**, and **David Royster**, University of Kentucky.

Topological Dynamics and Ergodic Theory (Code: SS 13A), **Alica Miller**, University of Louisville, and **Joe Rosenblatt**, University of Illinois at Urbana-Champaign.

Weak Convergence in Probability and Statistics (Code: SS 7A), **Cristina Tone**, **Ryan Gill**, and **Kiseop Lee**, University of Louisville.

Philadelphia, Pennsylvania

Temple University

October 12–13, 2013

Saturday – Sunday

Meeting #1093

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June 2013
Program first available on AMS website: August 29, 2013
Program issue of electronic *Notices*: October 2013
Issue of *Abstracts*: Volume 34, Issue 3

Deadlines

For organizers: Expired
For abstracts: August 20, 2013

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Patrick Brosnan, University of Maryland, *Normal functions.*

Xiaojun Huang, Rutgers University at New Brunswick, *Equivalence problems in several complex variables.*

Barry Mazur, Harvard University, *Arithmetic statistics: Elliptic curves and other mathematical objects* (Erdős Memorial Lecture).

Robert Strain, University of Pennsylvania, *On the Boltzmann equation without angular cut-off.*

Special Sessions

Analysis and Computing for Electromagnetic Waves (Code: SS 10A), **David Ambrose** and **Shari Moskow**, Drexel University.

Combinatorial Commutative Algebra (Code: SS 12A), **Tái Huy Há**, Tulane University, and **Fabrizio Zanello**, Massachusetts Institute of Technology and Michigan Technological University.

Contact and Symplectic Topology (Code: SS 5A), **Joshua M. Sabloff**, Haverford College, and **Lisa Traynor**, Bryn Mawr College.

Difference Equations and Applications (Code: SS 9A), **Michael Radin**, Rochester Polytechnic Institute, and **Faina Berezovskaya**, Howard University.

Geometric Aspects of Topology and Group Theory (Code: SS 17A), **David Futer**, Temple University, and **Ben McReynolds**, Purdue University.

Geometric Topology of Knots and 3-manifolds (Code: SS 16A), **Abhijit Champanerkar**, **Ilya Kofman**, and **Joseph Maher**, College of Staten Island and The Graduate Center, City University of New York.

Geometric and Spectral Analysis (Code: SS 3A), **Thomas Krainer**, Pennsylvania State Altoona, and **Gerardo A. Mendoza**, Temple University.

Higher Structures in Algebra, Geometry and Physics (Code: SS 2A), **Jonathan Block**, University of Pennsylvania, **Vasily Dolgushev**, Temple University, and **Tony Pantev**, University of Pennsylvania.

History of Mathematics in America (Code: SS 4A), **Thomas L. Bartlow**, Villanova University, **Paul R. Wolfson**, West Chester University, and **David E. Zitarelli**, Temple University.

Mathematical Biology (Code: SS 8A), **Isaac Klapper**, Temple University, and **Kathleen Hoffman**, University of Maryland, Baltimore County.

Meshfree, Particle, and Characteristic Methods for Partial Differential Equations (Code: SS 21A), **Toby Driscoll** and **Louis Rossi**, University of Delaware, and **Benjamin Seibold**, Temple University.

Modular Forms and Modular Integrals in Memory of Marvin Knopp (Code: SS 20A), **Helen Grundman**, Bryn Mawr College, and **Wladimir Pribitkin**, College of Staten Island and the Graduate Center, City University of New York.

Multiple Analogues of Combinatorial Special Numbers and Associated Identities (Code: SS 11A), **Hasan Coskun**, Texas A&M University Commerce.

Nonlinear Elliptic and Wave Equations and Applications (Code: SS 15A), **Nsoki Mavinga**, Swarthmore College, and **Doug Wright**, Drexel University.

Parabolic Evolution Equations of Geometric Type (Code: SS 18A), **Xiaodong Cao**, Cornell University, **Longzhi Lin**, Rutgers University, and **Peng Wu**, Cornell University.

Partial Differential Equations, Stochastic Analysis, and Applications to Mathematical Finance (Code: SS 14A), **Paul Feehan** and **Ruoting Gong**, Rutgers University, and **Camelia Pop**, University of Pennsylvania.

Recent Advances in Harmonic Analysis and Partial Differential Equations (Code: SS 1A), **Cristian Gutiérrez** and **Irina Mitrea**, Temple University.

Recent Developments in Noncommutative Algebra (Code: SS 6A), **Edward Letzter** and **Martin Lorenz**, Temple University.

Representation Theory, Combinatorics and Categorification (Code: SS 19A), **Corina Calinescu**, New York City College of Technology, City University of New York, **Andrew Douglas**, New York City College of Technology and Graduate Center, City University of New York, and **Joshua Sussan** and **Bart Van Steirteghem**, Medgar Evers College, City University of New York.

Several Complex Variables and CR Geometry (Code: SS 7A), **Andrew Raich**, University of Arkansas, and **Yuan Zhang**, Indiana University-Purdue University Fort Wayne.

The Geometry of Algebraic Varieties (Code: SS 13A), **Karl Schwede**, Pennsylvania State University, and **Zsolt Patakfalvi**, Princeton University.

St. Louis, Missouri

Washington University

October 18–20, 2013

Friday – Sunday

Meeting #1094

Central Section
Associate secretary: Georgia M. Benkart
Announcement issue of *Notices*: August 2013
Program first available on AMS website: September 5, 2013
Program issue of electronic *Notices*: October 2013
Issue of *Abstracts*: Volume 34, Issue 4

Deadlines

For organizers: Expired
For abstracts: August 27, 2013

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Ronny Hadani, University of Texas at Austin, *Title to be announced*.

Effie Kalfagianni, Michigan State University, *Title to be announced*.

Jon Kleinberg, Cornell University, (Einstein Lecture) *Title to be announced*.

Vladimir Sverak, University of Minnesota, *Title to be announced*.

Special Sessions

Advances in Difference, Differential, and Dynamic Equations with Applications (Code: SS 12A), **Elvan Akin**, Missouri S&T University, **Youssef Raffoul**, University of Dayton, and **Agacik Zafer**, American University of the Middle East.

Advances in Mathematical Methods for Disease Modeling (Code: SS 21A), **Jimin Ding**, Washington University in St. Louis, **Necibe Tuncer**, University of Tulsa, and **Naveen K. Vaidya**, University of Missouri-Kansas City.

Algebraic Cycles and Coherent Sheaves (Code: SS 19A), **Roya Beheshti**, **Matt Kerr**, and **N. Mohan Kumar**, Washington University, St. Louis.

Algebraic and Combinatorial Invariants of Knots (Code: SS 1A), **Heather Dye**, McKendree University, **Allison Henrich**, Seattle University, **Aaron Kaestner**, North Park University, and **Louis Kauffman**, University of Illinois.

Automorphic Forms and Representation Theory (Code: SS 7A), **Dubravka Ban** and **Joe Hundley**, Southern Illinois University, and **Shuichiro Takeda**, University of Missouri, Columbia.

Commutative Algebra (Code: SS 11A), **Lianna Sega**, University of Missouri, Kansas City, and **Hema Srinivasan**, University of Missouri, Columbia.

Computability Across Mathematics (Code: SS 2A), **Wesley Calvert**, Southern Illinois University, and **Johanna Franklin**, University of Connecticut.

Convex Geometry and its Applications (Code: SS 16A), **Susanna Dann**, **Alexander Koldobsky**, and **Peter Pivovarov**, University of Missouri.

Geometric Aspects of 3-Manifold Invariants (Code: SS 10A), **Oliver Dasbach**, Louisiana State University, and **Effie Kalfagianni**, Michigan State University.

Geometric Topology in Low Dimensions (Code: SS 4A), **William H. Kazez**, University of Georgia, and **Rachel Roberts**, Washington University in St. Louis.

Groupoids in Analysis and Geometry (Code: SS 6A), **Alex Kumjian**, University of Nevada at Reno, **Markus Pflaum**, University of Colorado, and **Xiang Tang**, Washington University in St. Louis.

Interactions between Geometric and Harmonic Analysis (Code: SS 3A), **Leonid Kovalev**, Syracuse University, and **Jeremy Tyson**, University of Illinois, Urbana-Champaign.

Linear and Non-linear Geometry of Banach Spaces (Code: SS 13A), **Daniel Freeman** and **Nirina Lovasoa Randrianarivony**, St. Louis University.

Noncommutative Rings and Modules (Code: SS 5A), **Greg Marks** and **Ashish Srivastava**, St. Louis University.

Operator Theory (Code: SS 9A), **John McCarthy**, Washington University in St. Louis.

PDEs of Fluid Mechanics (Code: SS 17A), **Roman Shvydkoy**, University of Illinois Chicago, and **Vladimir Sverak**, University of Minnesota.

Spectral, Index, and Symplectic Geometry (Code: SS 15A), **Alvaro Pelayo** and **Xiang Tang**, Washington University, St. Louis.

Statistical Properties of Dynamical Systems (Code: SS 14A), **Timothy Chumley** and **Renato Feres**, Washington University in St. Louis, and **Hongkun Zhang**, University of Massachusetts, Amherst.

Topological Combinatorics (Code: SS 20A), **John Shareshian**, Washington University, St. Louis, and **Russ Woodroofe**, Mississippi State University.

Wavelets, Frames, and Related Expansions (Code: SS 8A), **Marcin Bownik**, University of Oregon, **Darrin Speegle**, Saint Louis University, and **Guido Weiss**, Washington University, St. Louis.

p-local Group Theory, Fusion Systems, and Representation Theory (Code: SS 18A), **Justin Lynd**, Rutgers University, and **Julianne Rainbolt**, Saint Louis University.

Session for Contributed Talks

There also will be a session for 10-minute contributed talks. Please see the abstracts submission form at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`. The deadline for all submissions is **August 27, 2013**.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include hotel tax. Participants must state that they are with the **American Mathematical Society (AMS) Meeting at Washington University or "AMS Meet Me in St. Louis"** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

Knight Center, 1 Brookings Drive, St. Louis, MO 63130**,** 314-933-9400; `www.ahl-knightcenter.com/`. Rates are US$139 per night for single/double occupancy. Please note that hotel tax in St. Louis includes a 7.25% convention tax as well as a 6.925% city tax. Amenities include free wireless Internet in rooms, continental breakfast, complimentary guest snack pantry, garage parking, in-room dining, fitness center, guest laundry, and 24-hour business center. The hotel is located on the campus of Washington University. Cancellation and early check out policies vary; be sure to check when you make your reservation. Check in time is 3:00 p.m. and check out time is 12:00 p.m. **The deadline for reservations at this rate is September 17, 2013.**

Crowne Plaza/St. Louis–Clayton, 7750 Carondelet Avenue, Clayton, MO 63105, 314-726-5400; `www.cpclayton.com`. Rates are US$109 per night for singles/doubles. Please note that hotel tax is 15.43%. Amenities include complimentary wireless access, a hot breakfast buffet, as well as a heated indoor/outdoor swimming pool, 24-hour fitness and business centers. The hotel offers complimentary shuttle service to and from Lambert-St. Louis International Airport and the immediate Clayton business district, daily from 6:00 a.m.-10:00 p.m. This property is located within 2 miles of campus. Cancellation and early check out policies vary; be sure to check when you make your reservation. Check in time is 4:00 p.m. and check out time is 12:00 p.m. **The deadline for reservations at this rate is September 17, 2013.**

Sheraton Clayton Plaza Hotel St. Louis, 7730 Bonhomme Avenue, Clayton, MO 63105, 314-863-0400; `www.sheratonclaytonhotel.com/`. Rates are US$104 per night for single/double occupancy. Please note that hotel tax is 7.25% occupancy tax and 7.33% sales tax. Amenities include complimentary fitness facility, indoor heated pool, business center, and complimentary shuttle service to and from the airport (call for details.) Self-parking is US$14/day and valet parking is US$22/day. In-room fee for Internet is US$9.95/24 hours; complimentary Internet access in the lobby. This property is located within 2 miles of campus. Cancellation and early check out policies vary; be sure to check when you make your reservation. Check in time is 3:00 p.m. and check out time is 12:00 p.m. **The deadline for reservations at this rate is September 17, 2013.**

The Comfort Inn, 4630 Lindell Blvd., St. Louis, MO 63108, 314-361-4900; `www.comfortinn.com/hotel-saint_louis-missouri-MO310`. Rates are US$89 per night for single and double occupancy. Please note that hotel tax is 15.2%. Amenities include free wireless high-speed Internet access, business center, free hot breakfast. This property is located approximately 3 miles from the campus. Cancellation and early check out policies vary; be sure to check when you make your reservation. Check in time is 3:00 p.m. and check out time is 11:00 a.m. **The deadline for reservations at this rate is September 17, 2013.**

Drury Inn & Suites, St. Louis Forest Park, 2111 Sulphur Avenue, St. Louis, MO, 63139, 314-646-0770; `www.druryhotels.com/PropertyHotelServices.aspx?property=0136`. Rates are US$102 per night for single and double occupancy. Please note that there is an 8.491% hotel tax and 7.25% city tax. **Reference LTI Group Number: 2162009 for rate.** Amenities include indoor/outdoor pool, free breakfast, free wireless Internet, on-site parking, 24 hour fitness center. This property is located approximately 4.6 miles from the campus. Cancellation and early check out policies vary; be sure to check when you make your reservation. Check in time is 3:00 p.m. and check out time is 11:00 a.m. **The deadline for reservations at this rate is September 17, 2013.**

Pear Tree Inn–St. Louis Union Station, 2211 Market Street, St. Louis, MO 63103, 314-241-3200; `www.druryhotels.com/propertyoverview.aspx?property=0064`. Rates are US$63 per night plus 15.741% tax. **Reference LTI Group Number: 2162024 for this rate.** Amenities include free hot breakfast, free Wi-Fi Internet access, and indoor pool. This property is located approximately 7.7 miles from the campus. Cancellation and early check out policies vary; be sure to check when you make your reservation. Check in time is 3:00 p.m. and check out time is at 11:00 a.m. **The deadline for reservations at this rate is September 17, 2013.**

Drury Inn – St. Louis Union Station, 201 South 20th Street, St. Louis, MO 63103, 314-231-3900; `www.druryhotels.com/propertyoverview.aspx?property=0057`. Rates are US$79 per night plus 15.96% tax. **Reference LTI Group Number: 2162025 for this rate.** Amenities include free hot breakfast, free Wi-Fi Internet access, and indoor pool. This property is located approximately 8.2 miles from the campus. Cancellation and early check out policies vary; be sure to check when you make your reservation. Check in time is 3:00 p.m. and check out time is 11:00 a.m. **The deadline for reservations at this rate is September 17, 2013.**

The Parkway Hotel, 4450 Forest Park Parkway, St. Louis, MO 63108, 314-256-7777; `www.theparkwayhotel.com/about-us`. Rates are US$119 per night plus approximatly US$21 in tax. Amenities include free hot breakfast buffet, free Wi-Fi Internet access in the lobby, free shuttle service within a five-mile radius (ask front desk for hours). This property is located approximately 3.4 miles from the campus. Cancellation and early check out policies vary; be sure to check when you make your reservation. Check in time is at 4:00 p.m.; and check out time is at 12:00 p.m. **The deadline for reservations at this rate is September 17, 2013.**

Food Services

On Campus: The **Danforth University Center (DUC) offers a food court** featuring Asian, Mediterranean, Latin, vegan, vegetarian, and specialty grill selections. It also houses **Bergson Café**, which is a convenient stop for coffee, espresso, cappuccino, and cold drinks as well as pastries, salads, sandwiches, and smoothies. Another dining option in the DUC is **Ibby's**, a bistro offering a unique selection of menu items. Most options are open Friday and Sunday from 11:00 a.m.-7:30 p.m. and Saturday 11:00 a.m.-5:00 p.m.

Whispers Café in Olin Library provides an eclectic selection of sandwiches, salads, and pastries. Enjoy gourmet coffee while checking your email or surfing the Web in the café's wireless environment. Café hours are Friday and Saturday 9:00 a.m.-3:00 p.m. and Sunday 10:00 a.m.-12:00 p.m.

Off Campus: There are many choices for dining convenient to campus and hotels.

Blueberry Hill, 6504 Delmar, 314-727-4444. St. Louis landmark filled with pop culture memorabilia. Voted #1 hamburger and #1 décor. Open Monday through Sunday 11:00 a.m.-1:30 a.m.

Eclipse Restaurant, 5177 Delmar, 314-726-2222. Offering creative, cosmic twist of casual fine dining cuisine. Open daily 6:30 a.m.- 3:00 a.m.

Nico, 6525 Delmar, 314-222-2258. Mediterranean cuisine in the heart of The Loop. Open Friday-Saturday, 11:00 a.m.-11:00 p.m. and Sunday-Thursday, 11:00 a.m.-10:00 p.m.

Pi, 6144 Delmar, 314-727-6633. Offering San Francisco's original deep-dish, cornmeal crust pizza; appetizers & salads. The restaurant is open 11:00 a.m.-12:00 a.m. Monday-Saturday and 11:00 a.m.-11:00 p.m. on Sunday.

Tavolo V, 6118 Delmar, 314-721-4333. Offering Italian with a vegetarian twist. Hours are daily 11:30 a.m.-2:30 p.m. and 5:00 p.m.- 9:30 p.m.

Please visit `http://visittheloop.com/wp/wp-content/uploads/2008/11/2012-Loop-business-directory.pdf`, for more casual and fine dining options in St. Louis.

Registration and Meeting Information

Registration and the AMS book exhibit will be located in Holmes Lounge. Special Sessions and all Invited Addresses will be held in Carver Hall and the Einstein Lecture will be held in Graham Memorial Chapel. Please refer to the campus map `www.wustl.edu/community/visitors/maps/danforthmap.pdf` for specific locations. The registration desk will be open on Friday, October 18, 3:30 p.m.-4:00 p.m. and Saturday, October 19, 7:30 a.m.-4:00 p.m. Fees are US$54 for AMS members, US$76 for nonmembers; and US$5 for students, unemployed mathematicians, and emeritus members. Fees are payable on-site via cash, check, or credit card; advance registration is not available.

Other Activities

Book Sales: Stop by the on site AMS bookstore and review the newest titles from the AMS, enjoy up to 25% off all AMS publications, or take home an AMS t-shirt! Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

AMS Einstein Public Lecture in Mathematics: The Einstein Public Lecture will be given by Jon Kleinberg, Cornell University on Saturday at 5:05 p.m. at Graham Memorial Chapel. The title of his talk is to be determined.

A reception hosted by the Department of Mathematics and the AMS will take place between 6:15 p.m. - 7:15 p.m. in Holmes Lounge. The AMS thanks our hosts for their gracious hospitality.

Local Information and Maps

This meeting will take place on the Danforth Campus of Washington University. A campus map for all of Washington University can be found at `www.wustl.edu/community/visitors/maps/danforthmap.pdf`. Information about the Department of Mathematics may be found at `http://wumath.wustl.edu/`. Please visit the Washington University website at `http://wustl.edu/` for additional information on the campus.

Parking

Visitors are encouraged to use the visitor parking meters located on campus or park in designated visitor parking lots. All visitor parking is indicated in purple on the Danforth Campus parking map found here: `parking.wustl.edu/parkingmap_2010.pdf`.

Visitor parking meters are are located in a variety of locations on campus. The cost is US25 cents for 15 minutes (with a maximum of two hours). The enforcement schedule and time limits are indicated on each meter. Many of the parking meters on campus are double-sided, which means that one parking meter accepts coins and marks time for two separate parking spaces. If you are parking at one of these two-sided meters, please make certain that you select the appropriate space after inserting your coins. Visitor parking meters are designated as purple on the Campus Parking Map.

Hourly parking for visitors is conveniently located in the Danforth University Center Garage. The charge is US$1.00 per hour, with a maximum charge of US$8.00 for the day. Payment will be required upon exiting the garage; cash and credit cards are accepted. The Danforth University Center Garage is designated as number 82 on the Campus Parking Map.

Day passes: Visitors to campus may purchase daily parking permits, which allow for parking in most yellow zones. Daily parking permits are US$5.00, and are available at the office of Parking Services or at the Campus Bookstore. Please note that Daily Permits are not valid in the Danforth University Center Garage. You can contact Parking Services at (314) 935-5601 for further information.

Weekend Parking: Visitors who will be parking on campus during weekends (from 6:00 p.m. Friday to 7:00 a.m. Monday) are permitted to park in yellow zones without displaying a parking permit (with the exception of the Danforth University Center Garage).

Travel

Washington Unviersity is located in St. Louis, Missouri. The Lambert St. Louis International Airport is located 11 miles north of Washington University.

By Car: From the Lambert St. Louis International Airport: Take I-70 east toward St. Louis. Merge onto I-170 south via exit 238B, Clayton. Merge onto Forest Park Parkway via exit 1E. Turn right onto Big Bend Blvd. Turn left onto Forsyth Blvd.

By Commuter Train: MetroLink, St. Louis' light rail commuter train system, has two stops on the Danforth Campus: Skinker station, located at the northeast corner of Skinker and Forest Park Parkway near One Brookings Drive, and University City-Big Bend station, located at the northwest corner of Big Bend and Forest Park Parkway near the Village. To reach these from Lambert St. Louis Airport: Board the light rail train headed toward the Forest Park-DeBaliviere station. Exit at Forest Park-DeBaliviere and board the next train headed for Shrewsbury. Take this train either one or two stops to Skinker station or University City-Big Bend station.

By Bus: MetroBuses also stop outside Malinckrodt Center on Forsyth. Please visit the Metro St. Louis website: `www.metrostlouis.org/Default.aspx` for the latest schedules and routes.

Car Rental: Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at `www.hertz.com`, click on the box "I have a discount", and type in our convention number (CV): 04N30003. You can also call Hertz directly at 800-654-2240 (U.S. and Canada) or 405-749-4434 (other countries). At the time this announcement was prepared, rates were US$24.00 to US$74.00 per day on the weekend. At the time of your reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

Weather

The average high temperature for October is approximately 70 degrees Fahrenheit and the average low is approximately 50 degrees Fahrenheit. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at `sites.nationalacademies.org/pga/biso/visas/` and `travel.state.gov/visa/visa_1750.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `mkl@ams.org`.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Riverside, California

University of California Riverside

November 2–3, 2013

Saturday – Sunday

Meeting #1095

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2013
Program first available on AMS website: September 19, 2013
Program issue of electronic *Notices*: November 2013
Issue of *Abstracts*: Volume 34, Issue 4

Deadlines

For organizers: Expired
For abstracts: September 10, 2013

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael Christ, University of California, Berkeley, *Title to be announced*.

Mark Gross, University of California, San Diego, *Title to be announced*.

Matilde Marcolli, California Institute of Technology, *Title to be announced*.

Paul Vojta, University of California, Berkeley, *Title to be announced*.

Special Sessions

Algebraic Structures in Knot Theory (Code: SS 19A), **Allison Henrich**, Seattle University, and **Sam Nelson**, Claremont McKenna College.

Analysis and Geometry of Metric Spaces (Code: SS 12A), **Asuman G. Aksoy**, Claremont McKenna College, and **Zair Ibragimov**, California State University, Fullerton.

Categorification in Representation Theory (Code: SS 15A), **Aaron Lauda** and **David Rose**, University of Southern California.

Commutative Algebra and its Interaction with Algebraic Geometry and Combinatorics (Code: SS 10A), **Kuei-Nuan Lin** and **Paolo Mantero**, University of California, Riverside.

Computational Problems on Large Graphs and Applications (Code: SS 16A), **Kevin Costello** and **Laurent Thomas**, University of California, Riverside.

Computer, Mathematics, Imaging, Technology, Network, Health, Big Data, and Statistics (Code: SS 3A), **Subir Ghosh**, University of California, Riverside.

Developments in Markov Chain Theory and Methodology (Code: SS 2A), **Jason Fulman**, University of California, Riverside, and **Mark Huber**, Claremont McKenna College.

Diophantine Geometry and Nevanlinna Theory (Code: SS 14A), **Aaron Levin**, Michigan State University, **David McKinnon**, University of Waterloo, and **Paul Vojta**, University of California, Berkeley.

Dynamical Systems (Code: SS 13A), **Nicolai Haydn**, University of Southern California, and **Huyi Hu**, Michigan State University.

Fluids and Boundaries (Code: SS 5A), **James P. Kelliher**, **Juhi Jang**, and **Gung-Min Gie**, University of California, Riverside.

Fractal Geometry, Dynamical Systems, and Mathematical Physics (Code: SS 9A), **Michel L. Lapidus**, University of California, Riverside, **Erin P. J. Pearse**, California State Polytechnic University, San Luis Obispo, and **John A. Rock**, California State Polytechnic University, Pomona.

From Harmonic Analysis to Partial Differential Equations: In Memory of Victor Shapiro (Code: SS 11A), **Alfonso Castro**, Harvey Mudd College, **Michel L. Lapidus**, University of California, Riverside, and **Adolfo J. Rumbos**, Pomona College.

Geometric Analysis (Code: SS 4A), **Zhiqin Lu**, University of California, Irvine, **Bogdan D. Suceava**, California State University, Fullerton, and **Fred Wilhelm**, University of California, Riverside.

Geometric and Combinatorial Aspects of Representation Theory (Code: SS 8A), **Wee Liang Gan** and **Jacob Greenstein**, University of California, Riverside.

Geometry of Algebraic Varieties (Code: SS 6A), **Karl Fredrickson**, University of California, Riverside, **Mark Gross**, University of California, San Diego, and **Ziv Ran**, University of California, Riverside.

Heights, Diophantine Problems, and Lattices (Code: SS 17A), **Lenny Fukshansky**, Claremont McKenna College, and **David Krumm**, University of Georgia and Claremont McKenna College.

Homotopy Theory and K-Theory (Code: SS 7A), **Julie Bergner**, University of California, Riverside, and **Christian Haesemeyer**, University of California, Los Angeles.

Teaching ODEs: Best Practices from CODEE (Community of Ordinary Differential Equations Educators) (Code: SS 18A), **Nishu Lal**, Pomona College and Pitzer College, and **Ami Radunskaya**, Pomona College.

The Mathematics of Planet Earth (Code: SS 1A), **John Baez**, University of California, Riverside.

Session for Contributed Talks

There also will be a session for 10-minute contributed talks. Please see the abstracts submission form at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`. The deadline for all submissions is September 10, 2013.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include hotel tax. Participants must state that they are with the **American Mathematical Society (AMS) Meeting at the University of California, Riverside** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

The Mission Inn Hotel and Spa, 3649 Mission Inn Avenue, Riverside, CA 92501, 951-784-0300; www.missioninn.com/. Rates are US$135 per night for single/double occupancy. Please note the occupancy tax is 11%. Amenities include heated pool area featuring a hot tub spa and poolside menu, modern fitness center, complimentary Wi-Fi in lobby, and 6 award-winning dining options. The hotel is located approximately 2.6 miles from the campus. Check in time is 3:00 p.m. and check out time is 12:00 p.m. **The deadline for reservations at this rate is October 2, 2013.**

Hyatt Place Riverside Downtown, 3500 Market Street, Riverside, California, 92501, 951-321- 3500; riversidedowntown.place.hyatt.com/en/hotel/home.html. Rates are US$109 per night for singles/doubles. Please note that hotel tax is 13%. Amenities include complimentary wireless access, a hot breakfast buffet, 24-hour fitness, and complimentary computers and printers. This property is located within 2 miles of campus. Check in time is 3:00 p.m. and check out time is 12:00 p.m. **The deadline for reservations at this rate is October 11, 2013.**

Courtyard by Marriott, 1510 University Avenue Riverside, California 92507, 314-863-0400; www.marriott.com/hotels/travel/ralcy-courtyard-riverside-downtown-ucr-area/. Rates are US$102 per night for single/double occupancy. Please note that hotel tax is 7.25% occupancy tax and 7.33% sales tax. Amenities include complimentary breakfast buffet, lobby and in-room high-speed Internet, and overnight self parking. This property is located less than a mile from campus. Check in time is 3:00 p.m. and check out time is 12:00 p.m. **The deadline for reservations at this rate is October 27, 2013.**

Riverside Marriott, 3400 Market Street, Riverside, California 92501, 951-784-8000; www.marriott.com/hotels/travel/ralmc-riverside-marriott/. Rates are US$105 per night for single and double occupancy. Please note that hotel tax is 15.2% Amenities include breakfast for one, in-room Internet access, free self-parking, shuttle service to/from campus with 24-hour reservation. This property is located approximately 2.3 miles from the campus. Check in time is 3:00 p.m. and check out time is 12:00 p.m. **The deadline for reservations at this rate is October 2, 2013.**

Food Services

On Campus: The University Village is across the street from campus and offers a wide variety of dining options including **At Thai**, **Boba Cafe**, **Denny's**, **Fatburger**, **Frice Seafood**, **Oven 450**, **Ray's Pizza**. Hours are generally 11:00 a.m.–10:00 p.m., but vary by establishment.

Coco's Bakery Restaurant is located at 1303 University Ave., and is open Friday–Saturday 6:30 a.m.–12:00 p.m. and Sunday 6:30 a.m.–11:00 p.m.

Getaway Cafe is located at 3615 Canyon Crest Drive and is open 10:00 a.m.–12:00 a.m. Monday–Sunday.

The Sub Station is located at 3663 Canyon Crest Drive and is open 8:00 a.m.–7:45 p.m. on Saturday and 11:00 a.m.–6:00 p.m. on Sunday.

Off Campus: There are many choices for dining convenient to campus and hotels.

Anchos Southwest Grill, 10773 Hole Avenue, Riverside, 951-352-0240. Decorated as a pueblo-styled village, Anchos serves some of the best Southwestern influenced food. Open Friday-Saturday 11:30 a.m.-10:00 p.m. and Sunday-Thursday 11:30 a.m.-9:00 p.m.

Angel's Thai Restaurant, 6739 Brockton Avenue, Riverside; 951-788-1995. This small family-owned restaurant features delicious Thai cuisine with a menu which reflects both traditional and and original Thai dishes. Open daily 11:30 a.m.-9:00 p.m.

Cask'n Cleaver Steakhouse, 1333 University Avenue, Riverside; 951-682-4580. For 45 years, Cask'n Cleaver steakhouses have provided us with great classic steak and seafood entrees in a relaxing "California Casual" ambiance. Open for lunch Monday-Friday 11:30 a.m.-2:00 p.m. and dinner Friday-Saturday 5:00 p.m.-10:00 p.m. and Sunday 4:00 p.m.-9:00 p.m.

Market Broiler, 3525 Merrill Ave, Riverside; 951-276-9007. Market Broiler continues to be Riverside's #1 restaurant for fresh fish and seafood since their opening 22 years ago. The restaurant is open daily 11:00 a.m.-10:00 p.m.

Salted Pig, 3700 12th Street, Riverside; 951-848-4020. A hip and trendy casual dining restaurant located in dowtown Riverside is a cross between a pub, fine brasserie, and fine dining restaurant utlizing locally-grown and sustainable ingredients. Hours are Monday-Wednesday 11:00 a.m-12:00 a.m., Thursday-Friday 11:00 a.m.-2:00 a.m., and Saturday, 5:00 p.m.-2:00 a.m. Closed Sunday.

Please visit www.exploreriverside.com/wheretodine.asp, for more casual and fine dining options in Riverside.

Registration and Meeting Information

Registration will be located in Room 277 and the AMS book exhibit will be located in Room 284 of the Surge Building. Special Sessions and Invited Address locations to be determined. Please refer to the campus map campusmap.ucr.edu/imap/index.html for specific locations. The registration desk will be open on Saturday, November 2, 7:30 a.m.-4:00 p.m. and Sunday, November 3, 8:00 a.m.-12:00 p.m. Fees are US$54 for AMS members, US$76 for nonmembers; and US$5 for students, unemployed mathematicians, and emeritus members. Fees are payable on-site via cash, check, or credit card; advance registration is not available.

Other Activities

Book Sales: Stop by the on-site AMS bookstore and review the newest titles from the AMS, enjoy up to 25% off all

AMS publications, or take home an AMS t-shirt! Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

Local Information and Maps

This meeting will take place on the Riverside campus of the University of California, Riverside. A campus map for all of the university can be found at `campusmap.ucr.edu/imap/index.html`. Information about the Department of Mathematics may be found at `mathdept.ucr.edu/`. Please visit the University of California, Riverside website at `www.ucr.edu/` for additional information on the campus.

Parking

Visitors are encouraged to use Lot 24 for daily parking. All parking on campus requires a permit. Time limits are strictly enforced 24/7.

For short-term parking options you will find both Pay-By-Space and Pay & Display permit dispensers located in designated parking lots throughout the campus. Time limits vary depending on the lot and are clearly posted at each pay station.

Attendants at the main Campus Kiosk, located at the University Avenue entrance on W. Campus Drive, are available to assist and direct you to a parking lot best suited for your campus visit.

Pay & Display dispenser permit options vary by location and are only valid in non-reserved spaces of the parking lot where they are purchased. Options are clearly posted at the Pay & Display dispensers. Read through your options prior to purchasing your permit. Permits must be displayed in your vehicle and clearly visible to lot enforcement officials.

Parking Lot 24–Blue Permit Lot offers parking all day on weekends for US$1.00 per hour or US$5.00 for a daily permit.

Unless otherwise noted, Pay-By-Space and Pay & Display Dispensers only accept Visa or MasterCard as payment.

You can purchase your permit ahead of time here: `ebusiness.parking.ucr.edu/ebusiness/cmn/index.aspx`.

A map of the parking lot options is located here: `parking.ucr.edu/docs/parking_lot_map_web_version.pdf`

Travel

University of California, Riverside's main campus is in Riverside in the geographical center of Inland Southern California. Located near the junction of the 91, 215, and 60 freeways, University of California, Riverside is approximately 50 miles east of Los Angeles and 100 miles north of San Diego. The address for the campus is 900 University Ave., Riverside, CA 92521, phone: 951-827-1012.

There are two major airports near Riverside: Los Angeles International Airport (LAX) and Ontario International Airport (ONT). Below you'll find approximate travel times to UC Riverside. Please note that these are estimates. To ensure proper arrival for check-in, please factor in time for traffic and construction.

Los Angeles International Airport to UCR: approx. 1 hour and 15 minutes, 70.70 miles.

Ontario International Airport to UCR: approx. 30 minutes, 19.65 miles.

By Car:

From Ontario Airport:

From the 10 Freeway: Take the I-10 east, to the I-15 south and then to the CA-60 east. Exit at University Avenue and turn left. At the second light, take a right onto West Campus Drive.

From the 60 Freeway: Take the CA-60 east. Exit at University Avenue and turn left. At the second light, take a right onto West Campus Drive.

From Los Angeles County:

From the 91 Freeway: Take CA-91 east to the CA-60 east. Exit at University Avenue and turn left. At the second light, take a right onto West Campus Drive.

From the 10 Freeway: Take the I-10 east to the I-15 south and then to CA-60 east. Exit at University Avenue and turn left. At the second light, take a right onto West Campus Drive.

From the 60 Freeway: Take the CA-60 east. Exit at University Avenue and turn left. At the second light, take a right onto West Campus Drive.

By Shuttle: If you plan on taking a shuttle or taxi to UC Riverside, be sure to make arrangements prior to your arrival at the airport.

Prime Time Shuttle (800-RED-VANS) offers a rate of US$31/person one-way and US$10/each additional person from Ontario International Airport.

Super Shuttle (800-BLUE-VAN) offers a rate of US$35/person one-way and US$9/each additional person from Ontario International Airport. All reservations must be made 24 hours in advance and pre-paid with a credit card.

By Taxi: Happy Taxi (951-781-8294) offers rides in the Riverside area. Call for more information regarding your departure point and fees.

AAA Yellow Cab (951-684-2100) offers rides in the Riverside area. Call for more information regarding your departure point and fees.

San Gabriel Transit (1800-340-TAXI). All passengers ride for the price of one. Reservations over the Internet require 2-hour notice. This service also includes Bell Cab Co. and Yellow Cab.

By Bus: Riverside Transit Agency (800-800-7821) offers rides to the UCR campus from various locations for US$1.25 per route.

Car Rental: Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at `www.hertz.com`, click on the box "I have a discount", and type in our convention number (CV): 04N30003. You can also call Hertz directly at 800-654-2240 (U.S. and Canada) or 405-749-4434 (other countries). At the time this announcement was prepared, rates were US$24.00 to US$74.00 per day on the weekend. At the time of your reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

Weather

The average high temperature for November is approximately 76 degrees Fahrenheit and the average low is approximately 48 degrees Fahrenheit. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at `sites.nationalacademies.org/pga/biso/visas/` and `travel.state.gov/visa/visa_1750.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `mkl@ams.org`.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Baltimore, Maryland

Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 15–18, 2014

Wednesday – Saturday

Meeting #1096

Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Georgia M. Benkart
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: Expired
For abstracts: September 17, 2013

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/national.html`.

Joint Invited Addresses

Benson Farb, University of Chicago, *Title to be announced* (AMS-MAA).

Eitan Grinspun, Columbia University, *Title to be announced* (MAA-AMS-SIAM Gerald and Judith Porter Public Lecture).

Carl Pomerance, Dartmouth College, *Title to be announced* (AMS-MAA).

AMS Invited Addresses

Andrew Blake, Microsoft Research Cambridge, *Title to be announced* (AMS Josiah Willard Gibbs Lecture).

Emmanuel Candès, Stanford University, *Title to be announced.*

Christopher Hacon, University of Utah, *Title to be announced.*

Dusa McDuff, Columbia University, *Title to be announced* (AMS Colloquium Lectures).

Paul Seidel, Massachusetts Institute of Technology, *Title to be announced.*

H.-T. Yau, Harvard University, *Title to be announced.*

AMS Special Sessions

Some sessions are cosponsored with other organizations. These are noted within the parenthesis at the end of each listing, where applicable.

Accelerated Advances in Higher Order Invexities/Univexities with Applications to Optimization and Mathematical Programming (Code: SS 8A), **Ram U. Verma**, International Publications USA, and **Alexander J. Zaslavski**, Technion-Israel Institute of Technology.

Advances in Analysis and PDEs (Code: SS 49A), **Tepper L . Gill** and **Daniel A. Williams**, Howard University.

Algebraic Geometry (Code: SS 50A), **Christopher Hacon**, University of Utah, and **Zsolt Patakfalvi**, Princeton University.

Algebraic Structures Motivated by Knot Theory (Code: SS 17A), **Mieczyslaw K. Dabkowski**, University of Texas at Dallas, **Jozef Przytycki**, George Washington University, and **Radmila Sazdanovic**, University of Pennsylvania.

Algebraic and Analytic Aspects of Integrable Systems and Painleve Equations (Code: SS 32A), **Anton Dzhamay**, University of Northern Colorado, **Kenichi Maruno**, University of Texas-Pan America, and **Christopher Ormerod**, California Institute of Technology.

Analytic Number Theory (Code: SS 23A), **Angel Kumchev**, Towson University, **Scott Parsell**, West Chester University, and **Gang Yu**, Kent State University.

Applied Harmonic Analysis: Large Data Sets, Signal Processing, and Inverse Problems (Code: SS 12A), **Mauro Maggioni**, Duke University, and **Naoki Saito** and **Thomas Strohmer**, University of California, Davis.

Banach Spaces, Metric Embeddings, and Applications (Code: SS 16A), **Mikhail Ostrovskii**, St. John's University, and **Beata Randrianantoanina**, Miami University.

Big Data: Mathematical and Statistical Modeling, Tools, Services, and Training (Code: SS 18A), **Ivo Dinov**, University of California Los Angeles.

Categorical Topology (Code: SS 42A), **Frédéric Mynard**, Georgia Southern University, and **Gavin Seal**, École Polytechnique Fédérale de Lausanne.

Classification Problems in Operator Algebras (Code: SS 38A), **Ionut Chifan**, University of Iowa, and **David Penneys**, University of Toronto.

Communication of Mathematics via Interactive Activities (Code: SS 47A), **Benjamin Levitt** and **Glen Whitney**, National Museum of Mathematics.

Computability in Geometry and Topology (Code: SS 39A), **Mieczyslaw Dabkowski**, University of Texas at Dallas, and **Rumen D. Dimitrov**, Western Illinois University.

De Bruijn Sequences and Their Generalizations (Code: SS 53A), **Abbas Alhakim**, American University of Beirut, and **Steven Butler**, Iowa State University.

Deformations Spaces of Geometric Structures on Low-dimensional Manifolds (Code: SS 40A), **Caleb Ashley**, Howard University, **Michelle Lee** and **Melissa Macasieb**, University of Maryland, and **Andy Sanders**, University of Illinois at Chicago.

Difference Equations and Applications (Code: SS 9A), **Michael A. Radin**, Rochester Institute of Technology.

Dispersive and Geometric Partial Differential Equations (Code: SS 1A), **Shuanglin Shao**, University of Kansas, **Chongchun Zeng**, Georgia Institute of Technology, and **Shijun Zheng**, Georgia Southern University.

Ergodic Theory and Symbolic Dynamics (Code: SS 31A), **Aimee Johnson**, Swarthmore College, and **Cesar Silva**, Williams College.

Fractal Geometry: Mathematics of Fractals and Related Topics (Code: SS 11A), **Michel Lapidus**, University of California Riverside, **Erin Pearse**, California State Polytechnic University, San Luis Obispo, **Robert Strichartz**, Cornell University, and **Machiel Van Frankenhuijsen**, Utah Valley University.

Fractional, Stochastic, and Hybrid Dynamic Systems with Applications (Code: SS 7A), **John Graef**, University of Tennessee at Chattanooga, **Gangaram S. Ladde**, University of South Florida, and **Aghalaya S. Vatsala**, University of Louisiana at Lafayette.

Geometric Applications of Algebraic Combinatorics (Code: SS 48A), **Elizabeth Beazley**, Haverford College, and **Kristina Garrett**, St. Olaf College (AMS-AWM).

Global Dynamics and Bifurcations of Difference Equations (Code: SS 37A), **Mustafa Kulenovic** and **Orlando Merino**, University of Rhode Island.

Heavy Tailed Probability Distributions and Their Applications (Code: SS 22A), **Tuncay Alparslan** and **John P. Nolan**, American University.

Highlighting Achievements and Contributions of Mathematicians of the African Diaspora (Code: SS 34A), **Asamoah Nkwanta**, Morgan State University, and **Talitha M. Washington**, Howard University.

History of Mathematics (Code: SS 29A), **Sloan Despeaux**, Western Carolina University, **Della Dumbaugh**, University of Richmond, and **Glen van Brummelen**, Quest University.

Homological and Characteristic p Methods in Commutative Algebra (Code: SS 4A), **Neil Epstein**, George Mason University, **Sean Sather-Wagstaff**, North Dakota State University, and **Karl Schwede**, Penn State University.

Homotopy Theory (Code: SS 20A), **Niles Johnson**, Ohio State University at Newark, **Mark W. Johnson**, Penn State University, Altoona, **Nitu Kitchloo**, Johns Hopkins University, **James Turner**, Calvin College, and **Donald Yau**, Ohio State University at Newark.

Hyperplane Arrangements and Applications (Code: SS 41A), **Takuro Abe**, Kyoto University, **Max Wakefield**, United States Naval Academy, and **Masahiko Yoshinaga**, Hokkiado University.

Logic and Probability (Code: SS 2A), **Wesley Calvert**, Southern Illinois University, **Doug Cenzer**, University of Florida, **Johanna Franklin**, University of Connecticut, and **Valentina Harizanov**, George Washington University (AMS-ASL).

Mathematics and Mathematics Education in Fiber Arts (Code: SS 14A), **Sarah-Marie Belcastro**, Smith College, and **Carolyn Yackel**, Mercer University.

Mathematics in Natural Resource Modeling (Code: SS 43A), **Shandelle Henson**, Andrews University, and **Catherine Roberts**, College of the Holy Cross.

Mathematics of Computation: Differential Equations, Linear Algebra, and Applications (Code: SS 30A), **Susanne C. Brenner**, Louisiana State University, and **Chi-Wang Shu**, Brown University (AMS-SIAM).

My Favorite Graph Theory Conjectures (Code: SS 35A), **Craig Larson**, Virginia Commonwealth University, and **Ralucca Gera**, Naval Postgraduate School.

Nineteenth Century Algebra and Analysis (Code: SS 10A), **Frank D. Grosshans**, West Chester University, **Karen H. Parshall**, University of Virginia, and **Paul R. Wolfson**, West Chester University.

Nonlinear Systems: Polynomial Equations, Nonlinear PDEs, and Applications (Code: SS 27A), **Wenrui Hao**, University of Notre Dame.

Outreach for Mathematically Talented Youth (Code: SS 45A), **Christina Eubanks-Turner**, University of Louisiana at Lafayette, **Virginia Watson**, Kennesaw State University, and **Daniel Zaharopol**, Art of Problem Solving Foundation.

Progress in Free Probability (Code: SS 26A), **Dmitry Kaliuzhnyi-Verbovetskyi**, Drexel University, and **Todd Kemp**, University of California San Diego.

Quantum Walks, Quantum Computation, and Related Topics (Code: SS 6A), **Chaobin Liu**, Bowie State University, **Takuya Machida**, University of Tokyo, **Nelson Petulante**, Bowie State University, and **Salvador E. Venegas-Andraca**, Tecnológico de Monterrey, Campus Estado de México.

Random Matrices: Theory and Applications (Code: SS 13A), **Paul Bourgade** and **Horng-Tzer Yau**, Harvard University.

Reaction Diffusion Equations and Applications (Code: SS 44A), **Jerome Goddard II**, Auburn University Montgomery, and **Ratnasingham Shivaji**, University of North Carolina Greensboro.

Recent Advances in Homogenization and Model Reduction Methods for Multiscale Phenomena (Code: SS 21A), **Silvia Jiménez Bolaños** and **Burt S. Tilley**, Worcester Polytechnic Institute.

Recent Progress in Geometric and Complex Analysis (Code: SS 3A), **Zheng Huang**, City University of New York, Graduate Center and College of Staten Island, **Longzhi Lin**, Rutgers University, and **Marcello Lucia**, City University of New York, Graduate Center & College of Staten Island.

Recent Progress in Multivariable Operator Theory (Code: SS 46A), **Ron Douglas**, Texas A&M University, and **Michael Jury**, University of Florida.

Recent Progress in the Langlands Program (Code: SS 15A), **Moshe Adrian**, University of Utah, and **Shuichiro Takeda**, University of Missouri.

Representation Theory of p-adic Groups and Automorphic Forms (Code: SS 28A), **Arsalan Chademan**, University of Kurdistan, and **Manouchehr Misaghian**, Prairie View A&M University.

Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs (Code: SS 25A), **Bernard Brooks** and **Jobby Jacobs**, Rochester Institute of Technology, **Jacqueline Jensen-Vallin**, Slippery Rock University, and **Carl Lutzer**, **Darran Narayan**, and **Tamas Wiandt**, Rochester Institute of Technology.

Set-Valued Optimization and Variational Problems with Applications (Code: SS 24A), **Akhtar Khan**, Rochester Institute of Technology, **Mau Nam Nguyen**, Portland State University, **Miguel Sama**, Universidad Nacional de Educacin e Distancia, and **Christiane Tammer**, Martin Luther University of Halle-Wittenberg.

Structural and Extremal Problems (Code: SS 19A), **Daniel Cranston**, Virginia Commonwealth University, and **Gexin Yu**, College of William & Mary.

Symplectic and Contact Structures on Manifolds with Special Holonomy (Code: SS 51A), **Sergey Grigorian**, University of Texas Pan American, **Sema Salur**, University of Rochester, and **Albert J. Todd**, University of California Riverside.

The Changing Education of Pre-service Teachers in Light of the Common Core (Code: SS 52A), **William McCallum**, University of Arizona, **Kristin Umland**, University of New Mexico, and **Ellen Whitesides**, University of Arizona.

The Ubiquity of Dynamical Systems (Code: SS 33A), **Edray H. Goins**, Purdue University, and **Talitha M. Washington**, Howard University.

Topological Graph Theory: Structure and Symmetry (Code: SS 5A), **Jonathan L. Gross**, Columbia University, and **Thomas W. Tucker**, Colgate University.

Trends in Graph Theory (Code: SS 36A), **Ralucca Gera**, Naval Postgraduate School.

Knoxville, Tennessee

University of Tennessee, Knoxville

March 21–23, 2014

Friday – Sunday

Meeting #1097

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: January 2014
Program first available on AMS website: February 6, 2014
Program issue of electronic *Notices*: March 2014
Issue of *Abstracts*: Volume 35, Issue 2

Deadlines

For organizers: August 21, 2013
For abstracts: January 28, 2014

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Maria Chudnovsky, Columbia University, *Title to be announced* (Erdős Memorial Lecture).

Ilse Ipsen, North Carolina State University, *Title to be announced.*

Daniel Krashen, University of Georgia, *Title to be announced.*

Suresh Venapally, Emory University, *Title to be announced.*

Special Sessions

Commutative Ring Theory (in honor of the retirement of David E. Dobbs) (Code: SS 1A), **David Anderson**, University of Tennessee, Knoxville, and **Jay Shapiro**, George Mason University.

Diversity of Modeling and Optimal Control: A Celebration of Suzanne Lenhart's 60th Birthday (Code: SS 3A), **Wandi Ding**, Middle Tennessee State University, and **Renee Fister**, Murrray State University.

Fractal Geometry and Ergodic Theory (Code: SS 2A), **Mrinal Kanti Roychowdhury**, University of Texas Pan American.

Randomized Numerical Linear Algebra (Code: SS 4A), **Ilse Ipsen**, North Carolina State University.

Baltimore, Maryland

University of Maryland, Baltimore County

March 29–30, 2014

Saturday – Sunday

Meeting #1098

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: January 2014
Program first available on AMS website: February 26, 2014
Program issue of electronic *Notices*: March 2014
Issue of *Abstracts*: Volume 35, Issue 2

Deadlines

For organizers: August 29, 2013
For abstracts: January 28, 2014

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Maria Gordina, University of Connecticut, *Title to be announced.*

L. Mahadevan, Harvard University, *Title to be announced.*

Nimish Shah, Ohio State University, *Title to be announced.*

Dani Wise, McGill University, *Title to be announced.*

Albuquerque, New Mexico

University of New Mexico

April 5–6, 2014

Saturday – Sunday

Meeting #1099

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: January 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: April 2014
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 5, 2013
For abstracts: February 11, 2014

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Anton Gorodetski, University of California, Irvine, *To be announced.*

Fan Chung Graham, University of California, San Diego, *To be announced.*

Adrian Ioana, University of California, San Diego, *To be announced.*

Karen Smith, University of Michigan, Ann Harbor, *To be announced.*

Special Sessions

Interactions in Commutative Algebra (Code: SS 4A), **Louiza Fouli**, New Mexico State University, **Bruce Olberding**, New Mexico State University, and **Janet Vassilev**, University of New Mexico.

Progress in Noncommutative Analysis (Code: SS 2A), **Anna Skripka**, University of New Mexico, and **Tao Mei**, Wayne State University.

Stochastics and PDEs (Code: SS 5A), **Juraj Foldes**, Institute for Mathematics and Its Applications, **Nathan Glatt-Holtz**, Institute for Mathematics and Its Applications and Virginia Tech, and **Geordie Richards**, Institute for Mathematics and Its Applications and University of Rochester.

The Inverse Problem and Other Mathematical Methods Applied in Physics and Related Sciences (Code: SS 1A), **Hanna Makaruk**, Los Alamos National Laboratory, and **Robert Owczarek**, University of New Mexico and Enfitek, Inc.

Topics in Spectral Geometry and Global Analysis (Code: SS 3A), **Ivan Avramidi**, New Mexico Institute of Mining and Technology, and **Klaus Kirsten**, Baylor University.

Lubbock, Texas

Texas Tech University

April 11–13, 2014

Friday – Sunday

Meeting #1100

Central Section
Associate secretary: Georgia M. Benkart
Announcement issue of *Notices*: February 2014
Program first available on AMS website: February 27, 2014
Program issue of electronic *Notices*: April 2014
Issue of *Abstracts*: Volume 35, Issue 2

Deadlines

For organizers: September 18, 2013

For abstracts: February 10, 2014

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Nir Avni, Northwestern University, *To be announced.*

Alessio Figalli, University of Texas at Austin, *To be announced.*

Jean-Luc Thiffeault, University of Wisconsin-Madison, *To be announced.*

Rachel Ward, University of Texas at Austin, *To be announced.*

Special Sessions

Analysis and Applications of Dynamic Equations on Time Scales (Code: SS 2A), **Heidi Berger**, Simpson College, and **Raegan Higgins**, Texas Tech University.

Complex Function Theory and Special Functions (Code: SS 7A), **Roger W. Barnard** and **Kent Pearce**, Texas Tech University, **Kendall Richards**, Southwestern University, and **Alex Solynin** and **Brock Williams**, Texas Tech University.

Fractal Geometry and Dynamical Systems (Code: SS 3A), **Mrinal Kanti Roychowdhury**, The University of Texas-Pan American.

Issues Regarding the Recruitment and Retention of Women and Minorities in Mathematics (Code: SS 5A), **James Valles Jr.** and **Doug Scheib**, Saint Mary-of-the-Woods College.

Qualitative Theory for Non-linear Parabolic and Elliptic Equations (Code: SS 6A), **Akif Ibragimov**, Texas Tech University, and **Peter Polacik**, University of Minnesota.

Recent Advancements in Differential Geometry and Integrable PDEs, and Their Applications to Cell Biology and Mechanical Systems (Code: SS 4A), **Giorgio Bornia**, **Akif Ibragimov**, and **Magdalena Toda**, Texas Tech University.

Topology and Physics (Code: SS 1A), **Razvan Gelca** and **Alastair Hamilton**, Texas Tech University.

Tel Aviv, Israel

Bar-Ilan University, Ramat-Gan and Tel-Aviv University, Ramat-Aviv

June 16–19, 2014

Monday – Thursday

Meeting #1101

The Second Joint International Meeting between the AMS and the Israel Mathematical Union.
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: January 2014
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Special Sessions

Mirror Symmetry and Representation Theory, **David Kazhdan**, Hebrew University, and **Roman Bezrukavnikov**, Massachusetts Institute of Technology.

Nonlinear Analysis and Optimization, **Boris Mordukhovich**, Wayne State University, and **Simeon Reich** and **Alexander Zaslavski**, The Technion-Israel Institute of Technology.

Qualitative and Analytic Theory of ODE's, **Yosef Yomdin**, Weizmann Institute.

Eau Claire, Wisconsin

University of Wisconsin-Eau Claire

September 20–21, 2014

Saturday – Sunday

Meeting #1102

Central Section
Associate secretary: Georgia M. Benkart
Announcement issue of *Notices*: June 2014
Program first available on AMS website: August 7, 2014
Program issue of electronic *Notices*: September 2014
Issue of *Abstracts*: Volume 35, Issue 3

Deadlines

For organizers: March 20, 2014
For abstracts: August 5, 2014

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Matthew Kahle, Ohio State University, *To be announced.*

Markus Keel, University of Minnesota, *To be announced.*

Svitlana Mayboroda, University of Minnesota, *To be announced.*

Halifax, Canada

Dalhousie University

October 18–19, 2014

Saturday – Sunday

Meeting #1103

Eastern Section

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: August 2014
Program first available on AMS website: September 5, 2014
Program issue of electronic *Notices*: October 2014
Issue of *Abstracts*: Volume 35, Issue 3

Deadlines

For organizers: March 18, 2014
For abstracts: August 19, 2014

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

François Bergeron, Université du Québec à Montréal, *Title to be announced.*

Sourav Chatterjee, New York University, *Title to be announced.*

William M. Goldman, University of Maryland, *Title to be announced.*

Sujatha Ramdorai, University of British Columbia, *Title to be announced.*

San Francisco, California

San Francisco State University

October 25–26, 2014

Saturday – Sunday

Meeting #1104

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2014
Program first available on AMS website: September 11, 2014
Program issue of electronic *Notices*: October 2014
Issue of *Abstracts*: Volume 35, Issue 4

Deadlines

For organizers: March 25, 2014
For abstracts: September 3, 2014

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Kai Behrend, University of British Columbia, Vancouver, Canada, *To be announced.*

Kiran S. Kedlaya, University of California, San Diego, *To be announced.*

Julia Pevtsova, University of Washington, Seattle, *To be announced.*

Burt Totaro, University of British Columbia, Vancouver, Canada, *To be announced.*

Special Sessions

Algebraic Geometry (Code: SS 1A), **Renzo Cavalieri**, Colorado State University, **Noah Giansiracusa**, University of California, Berkeley, and **Burt Totaro**, University of California, Los Angeles.

Geometry of Submanifolds (Code: SS 3A), **Yun Myung Oh**, Andrews University, **Bogdan D. Suceava**, California State University, Fullerton, and **Mihaela B. Vajiac**, Chapman University.

Polyhedral Number Theory (Code: SS 2A), **Matthias Beck**, San Francisco State University, and **Martin Henk**, Universität Magdeburg.

Greensboro, North Carolina

University of North Carolina, Greensboro

November 8–9, 2014

Saturday – Sunday

Meeting #1105

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: August 2014
Program first available on AMS website: September 25, 2014
Program issue of electronic *Notices*: November 2014
Issue of *Abstracts*: Volume 35, Issue 4

Deadlines

For organizers: April 8, 2014
For abstracts: September 16, 2014

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday

Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015

Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014
For abstracts: To be announced

Washington, District of Columbia

Georgetown University

March 7–8, 2015

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 7, 2014
For abstracts: To be announced

Huntsville, Alabama

University of Alabama in Huntsville

March 20–22, 2015

Friday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 20, 2014
For abstracts: To be announced

Las Vegas, Nevada

University of Nevada, Las Vegas

April 18–19, 2015

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 18, 2014
For abstracts: To be announced

Porto, Portugal

University of Porto

June 11–14, 2015

Thursday – Sunday
First Joint International Meeting involving the American Mathematical Societry (AMS), the European Mathematical Society (EMS), and the Sociedade de Portuguesa Matematica (SPM).
Associate secretary: Georgia M. Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For abstracts: To be announced

Chicago, Illinois

Loyola University Chicago

October 3–4, 2015

Saturday – Sunday
Central Section
Associate secretary: Georgia M. Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: October 2015
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 10, 2015
For abstracts: To be announced

Fullerton, California

California State University, Fullerton

October 24–25, 2015

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: October 2015
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 27, 2015
For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday

Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For abstracts: To be announced

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday

Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Brian D. Boe
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday

Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Georgia M. Benkart
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2017
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 16–19, 2019

Wednesday – Saturday

Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM)and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2018
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2018
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Central Section: **Georgia Benkart**, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: **Steven H. Weintraub**, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: **Brian D. Boe,** Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, e-mail: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/.

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 274 in the the February 2013 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences in Cooperation with the AMS: (see http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

July 22–26, 2013: Samuel Eilenberg Centenary Conference (E100), Warsaw, Poland.

November 1–3, 2013: Sixth International Conference on Science and Mathematics Education in Developing Countries, Mandalay, Myanmar.

A Mathematical Spectrum

From self-study texts for undergraduates to career guides to brilliant biographies (and much more!), the AMS Bookstore offers texts from all reaches of the mathematical sphere. Below, find a sampling of our classic and current works.

An Epsilon of Room, II

pages from year three of a mathematical blog

Terence Tao, *University of California, Los Angeles, CA*

This remarkable collection of articles drawn from the third-year postings of Tao's popular blog surveys important and challenging topics in current mathematics, with deep results presented in an accessible format.

2011; 248 pages; Softcover; ISBN: 978-0-8218-5280-4; List US$42; AMS members US$33.60; Order code MBK/77

Algebraic Geometry

A Problem Solving Approach

Thomas Garrity et al.

A problem-driven introduction to algebraic geometry that requires its readers to *think* through the mathematics, thereby truly grasping the concepts.

Student Mathematical Library, Volume 66; 2013; 335 pages; Softcover; ISBN: 978-0-8218-9396-8; List US$53; AMS members US$42.40; Order code STML/66

TEXTBOOK

Difference Sets

Connecting Algebra, Combinatorics, and Geometry

Emily H. Moore, *Grinnell College, IA*, and **Harriet S. Pollatsek**, *Mount Holyoke College, South Hadley, MA*

The goal of this book is to serve prospective undergraduate researchers of difference sets, as well as to provide a rich text for a senior seminar or capstone course in mathematics, with the hope that readers will acquire a solid foundation that will empower them to explore the literature on difference sets independently.

Student Mathematical Library, Volume 67; 2013; 298 pages; Softcover; ISBN: 978-0-8218-9176-6; List US$49; AMS members US$39.20; Order code STML/67

Euler Through Time

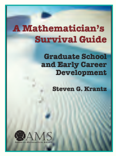
A New Look at Old Themes

V. S. Varadarajan, *University of California, Los Angeles, CA*

...[T]he author has admirably managed to organize the text in such a manner that an interested non-specialist will find the whole story comprehensible, absorbing, and enjoyable. This book has been written with the greatest insight, expertise, experience, and passion on the part of the author, and it should be seen as what it really is: a cultural jewel in the mathematical literature as a whole!

—Zentralblatt MATH

2006; 302 pages; Hardcover; ISBN: 978-0-8218-3580-7; List US$61; AMS members US$48.80; Order code EULER



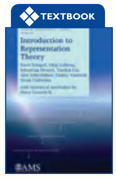
A Mathematician's Survival Guide

Graduate School and Early Career Development

Steven G. Krantz, *Washington University, St. Louis, MO*

In his inimitable and forthright style, Steven Krantz addresses the major issues of graduate school, including choosing a program, passing the qualifying exams, finding an advisor, writing a thesis, and getting that first job, all the while avoiding generalities and giving clear advice on how to handle real situations.

2003; 222 pages; Softcover; ISBN: 978-0-8218-3455-8; List US$30; AMS members US$24; Order code GSCM



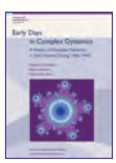
TEXTBOOK

Introduction to Representation Theory

Pavel Etingof et al. with historical interludes by **Slava Gerovitch**, Massachusetts Institute of Technology, Cambridge, MA

The goal of this book is to give a "holistic" introduction to representation theory, presenting it as a unified subject which studies representations of associative algebras and treating the representation theories of groups, Lie algebras, and quivers as special cases.

Student Mathematical Library, Volume 59; 2011; 228 pages; Softcover; ISBN: 978-0-8218-5351-1; List US$42; AMS members US$33.60; Order code STML/59



Early Days in Complex Dynamics

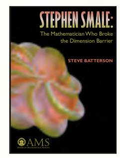
A history of complex dynamics in one variable during 1906–1942

Daniel S. Alexander, *Drake University, Des Moines, IA*, **Felice Iavernaro**, *Università di Bari, Italy*, and **Alessandro Rosa**

This book paints a robust picture of the field of complex dynamics between 1906 and 1942 through detailed discussions of the work of Fatou, Julia, Siegel, and several others.

A co-publication of the AMS and the London Mathematical Society.

History of Mathematics, Volume 38; 2011; 454 pages; Hardcover; ISBN: 978-0-8218-4464-9; List US$99; AMS members US$79.20; Order code HMATH/38



Stephen Smale: The Mathematician Who Broke the Dimension Barrier

Steve Batterson, *Emory University, Atlanta, GA*

To write this biography involved tremendous work, but the result is brilliant … accessible for nonmathematicians … On the other hand, the author's skillful presentation of Smale's mathematical discoveries will surely attract the interest of professional mathematicians … In many respects it is a masterpiece.

—European Mathematical Society Newsletter

2000; 306 pages; Softcover; ISBN: 978-0-8218-2696-6; List US$41; AMS members US$32.80; Order code MBDB.S

TEXTBOOK

Understanding Numbers in Elementary School Mathematics

Hung-Hsi Wu, *University of California, Berkeley, CA*

A textbook for pre-service and current elementary school teachers that develops the basic facts about numbers from the beginning and thoroughly covers the subject matter for grades K through 7.

2011; 551 pages; Hardcover; ISBN: 978-0-8218-5260-6; List US$79; AMS members US$63.20; Order code MBK/79

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