Nominations for President

Nomination of Robert L. Bryant

David Eisenbud

Robert Bryant is an intensely engaged mathematician with broad interests both in and beyond core mathematics. He was my successor as Director of MSRI, where he has served from 2007 until now—his term will end in August 2013, so, if elected, he would be able to concentrate his attention on the AMS Presidency. Robert is justly recognized for his major research achievements: He is a Fellow of the American Academy of Arts and Sciences and a member of the National Academy of Sciences. Phillip Griffiths and Mike Eastwood describe Robert's research below, so I will focus on other aspects of his activity.

Robert has been very much oriented toward serving the mathematics community. Before becoming Director of MSRI, he had been Chair of MSRI's board, and had served as Director of the Park City Mathematics Institute. In 2002, he was appointed by then-President Bush to serve on the Board of Directors of the Vietnam Education Foundation, and he currently serves on the International Committee for the National Mathematics Center of Nigeria. In all these roles he has thought carefully and deeply about programs to support a broad range of mathematicians and prospective mathematicians: at Park City, for example, he was responsible for programs ranging from the teaching of high school teachers and undergraduates to the frontiers of research.

Again during his service at MSRI, Robert has demonstrated his powerful commitment to the fostering of underrepresented groups in mathematics. He has worked with MSRI's Deputy Director to support and enhance the activities of the Human Resources Advisory Committee, first set up by Bill Thurston and Lenore Blum in the 90's. Under his leadership, MSRI has greatly extended the National Math Circles movement, both on local and national levels. (Math Circles, borrowing and adapting an East European tradition, bring research mathematicians, scientists, and others with mathematical sophistication into contact with kids with a view to engaging them in mathematics well beyond that of the conventional classroom.)

Robert has also continued to foster the engagement of the mathematical community with other sciences, and with societal needs. For example his team at MSRI has collaborated with a large group of other Mathematics Institutes to engage the support of the Simons Foundation and sponsor an international series of high-profile lectures on the Mathematics of Planet Earth, which is now ongoing. The first of these, held in Australia, was aired on two national Australian television channels, and others will follow. (Video of all the lectures, and much else of similar relevance, is posted at `http://www.mpe2013.org/`.)

Robert has a very wide experience of life and of the mathematical community. A North Carolina native, Robert grew up on a farm. Remaining in North Carolina, he received his Ph.D. in Mathematics in 1979 at the University of North Carolina at Chapel Hill, working under Robert B. Gardner. He served on the faculty at Rice University for seven years, and then moved to Duke University in 1987, where he held the Juanita M. Kreps Chair in Mathematics until moving to the University of California at Berkeley in July 2007. He has held numerous visiting positions at universities and research institutes around the world.

From experience I know that one of the most important activities of the AMS President is to reach out to invite mathematicians to serve on the many AMS committees that knit the mathematical community together. Robert's experience with the enormous number of mathematicians who come to MSRI, from graduate students to senior leaders, and especially with members from underrepresented groups, will give him an excellent basis for doing this.

He has my strongest endorsement for the important role of President of the AMS.

David Eisenbud is professor of mathematics at the University of California at Berkeley. His email address is `de@math.berkeley.edu`.

Phillip Griffiths

Robert Bryant would make an excellent President of the AMS and I strongly support his candidacy. The following are some personal observations on his scientific work.

Robert Bryant is the leading geometer of our time in the grand tradition of Elie Cartan and S.-S. Chern. His research shows an unexcelled combination of geometric insight and taste, together with computational power. Bryant's work exhibits both remarkable depth and breadth: he has solved a wide range of deep and frequently longstanding geometric questions, and he has developed fundamental general theory and techniques. Bryant's work has been pioneering in multiple areas, a sample of which I shall mention here. One is his analysis of the Willmore conjecture, proving the amazing result that it is an integrable system and opening the door to significant progress on the problem. Another is the work of Bryant and collaborators on the local smooth isometric embedding of Riemannian manifolds of dimension n in Euclidean space in the best

Phillip Griffiths is professor of mathematics at the Institute for Advanced Study. His email address is `pg@math.ias.edu`.

possible embedding dimension $n(n+1)/2$, combining both differential and algebraic geometry and hard analysis. Algebraic geometry enters through the subtle analysis of the characteristic variety. This work solved a large part of the problem when $n = 3$ and clarified the general situation. For example, for n at least 3, it can never be an elliptic or hyperbolic system. On another front, Bryant's determination of the surfaces of constant Finsler curvature spawned major progress and was instrumental in the revitalization of the subject. He has made significant contributions in dynamical systems in which the global analysis of the solution dynamics to the classical problem of elastica was obtained. Bryant has also made a contribution to algebraic geometry in the analysis of the variations of the Hodge structure of Calabi-Yau varieties.

A theme that runs throughout much of Bryant's work is the "geometry of differential equations". This is a rich and deep subject, beginning with Lie and his contemporaries in the late nineteenth century and continuing into the twentieth century through the works of Cartan and others. In this area, Bryant is a leading current practitioner. The emphasis in the subject is more on the understanding of special equations, usually of geometric origin, than on developing a vast general theory. There is a general approach for addressing problems in the subject, namely Cartan's so-called method of equivalence. Its application is more of an art than a science; one must use considerable geometric insight to guide the calculations, which are frequently of great subtlety and intricacy. The differential equations of interest are frequently overdetermined, and the relevant calculations have a cohomological as well as a geometric aspect. Bryant is the undisputed master in the use of the equivalence method, as demonstrated in his work on exceptional holonomy that Mike Eastwood will describe. Another illustration of the range of problems into which Bryant has brought the equivalence method is his work on the geometry of Euler-Lagrange equations, in which a complete set of invariants is obtained and using which one may characterize those equations that have special properties, such as additional conservation laws and exceptional symmetries.

The above comments have been about Robert Bryant's contributions, which are at the highest level, to our field. As chair of the Board of Trustees of MSRI for the past four years, I have been able to observe at first hand Bryant's leadership, both scientific and administrative, of that institution which is so central to the mathematical community. His commitment to service to our community is exceptional and complete. Robert Bryant would be an outstanding President of the AMS.

Mike Eastwood

I am delighted to contribute to the nomination of Robert Bryant for President of the American Mathematical Society. I have been fortunate enough to have known Robert for more than 30 years and he is undoubtedly one of the most impressive and exceptional individuals that I have ever met. Although I shall mostly confine my remarks to his mathematics, I would also like to attest to his outstanding leadership abilities. His vision and depth of knowledge are surely evident to all who meet him, most recently in his capacity as Director of MSRI. In short, I cannot imagine a more suitable President.

Robert works in differential geometry, especially from the point of view of "exterior differential systems". Indeed, he is one of the authors of the classic text of the same title (but known affectionately as BCG^3 after Bryant, Chern, Gardner, Goldschmidt, and Griffiths). Exterior differential systems provide the modern framework for understanding the notoriously difficult but pioneering works of Élie Cartan from the early twentieth century. Bryant is one of the very few mathematicians to have absorbed and effectively utilized the methods of Cartan, starting with his 1979 Ph.D. thesis where he analyzed generic 3-plane distributions on 6-manifolds in the spirit of Cartan's famous 1910 "five variables" article (which analyses generic 2-plane distributions on 5-manifolds). These numbers are very special. The maximally symmetric instance of a 2-plane distribution in 5 dimensions has symmetries lying in the split form of the exceptional Lie algebra. For 3-planes in 6 dimensions, the corresponding local symmetries also constitute a simple Lie algebra, this time the split form Spin(4, 3).

It is no coincidence that the compact Lie groups G_2 and Spin(7) star in Bryant's seminal 1987 *Annals* article on exceptional holonomy. He shows the existence of Riemannian manifolds with holonomy G_2 in 7 dimensions and Spin(7) in 8 dimensions, thereby, after having been open for more than 30 years, settling Berger's list of potential holonomies. For me, this article also secures Robert's reputation as a superb expositor. The original comments sprinkled throughout ensure that it is highly readable. It is a style that he has maintained in all his work. Several other articles from this time are concerned with holonomy. Complete metrics with exceptional holonomy are constructed in joint work with Salamon. Bryant also constructs torsion-free connections with "exotic holonomies" (remarkable non-metric connections lying outside Berger's list), which have been the inspiration for many other authors.

Much of Robert's work is driven by his intimate knowledge of the simple Lie groups and their genesis in the geometry of ordinary differential equations. In recent years, my own interest has been in the development of "parabolic geometry". These are differential geometries modelled, in the sense of Cartan, on homogeneous spaces of the form G/P for G semisimple and P parabolic. There is now a well-developed theory of such geometries due to Čap, Slovák, Souček, et al. with origins in the classical works of Cartan, Tanaka, et al. Bryant's work provides significant impetus in this theory, going right back to his Ph.D. thesis. This impact continues and not only through his published work, as two of my current postdocs can confirm: both recently posted what I thought to be hard questions on "MathOverflow", a place for mathematicians to ask and answer questions. Both questions were answered within a day by Robert Bryant (current "reputation" 20,189)!

Mike Eastwood is professor of mathematics at the Institute for Advanced Study. His email address is pg@math.ias.edu.

Finally, I should mention Robert's extraordinary skill as a lecturer and public speaker. In particular, I recall a colloquium he gave in Auckland in 2008 entitled "The idea of holonomy". In addition to rolling cubes around, the main visual aid for this talk was a porter's trolley (which undergoes holonomy when manipulated)! It was a very fine demonstration and, of course, much more appropriate for a colloquium than a bunch of equations.

In conclusion, I wholeheartedly support Robert Bryant's nomination for President of the AMS both as an extraordinary mathematician and leader.

Nomination of Benedict Gross

Joe Buhler and Ken Ribet

It is a pleasure to write in support of the nomination of Benedict (Dick) Gross for the Presidency of the American Mathematical Society. Professor Gross is a superb mathematician who has risen to the top of our profession. In addition, he has the rare quality of being equally at ease with people as he is with mathematical ideas. His warm personality and empathy for others go hand in hand with his inspired teaching and clear exposition. These same qualities have made him a valued colleague and administrator.

It is natural to begin with Gross's mathematics. Gross wrote his Ph.D. dissertation on the arithmetic of complex multiplication on elliptic curves, working with John Tate at Harvard University. His thesis was published in the Lecture Notes in Mathematics series, and has been influential in many ways. In particular, Gross's thesis inaugurated the study of "**Q**-curves", which continue to be studied actively.

Gross's Ph.D. thesis represented only a fraction of the work that he carried out as a graduate student. Before writing his thesis, he gave the first conceptual proof via arithmetic geometry of a famous formula that Chowla and Selberg first proved in 1947, thereby answering a question of André Weil and sparking Deligne's theorem to the effect that Hodge cycles on abelian varieties are "absolutely Hodge". At roughly the same time, Gross studied the arithmetic of Jacobians of Fermat curves with David Rohrlich, and he proved an identity relating Gauss sums and special values of Morita's p-adic Γ-function in joint work with Neal Koblitz.

Gross's rather startling output as a graduate student launched a sustained and rich research career that has produced more than 100 mathematical articles touching on number theory, arithmetic geometry, representation theory and many other subjects. In this short article, we will focus on some highlights of his work.

In the early 1980s, Gross and Don Zagier proved a celebrated formula that relates the heights of special points on modular curves to the derivatives of suitable L-series attached to modular forms. The Gross–Zagier formula is an extremely deep result whose proof interweaves analytic and algebraic techniques. The formula, in addition to its intrinsic beauty, has several important applications. When combined with work from the same period by Kolyvagin and others, it implies a statement toward the conjecture of Birch and Swinnerton-Dyer for elliptic curves that has not been improved significantly in subsequent decades. Recall that the BSD Conjecture postulates an equality between the algebraic and analytic ranks of an elliptic curve. The results of Gross–Zagier and others establish this equality whenever the analytic rank is at most 1.

In addition, the Gross–Zagier formula implies the existence of an elliptic curve with analytic rank 3. A few years earlier, Dorian Goldfeld had shown that the existence of such a curve would imply an effective lower bound on the class number of imaginary quadratic fields—given any $a < 1$ there would be a c such that $h > c\,(\log(|D|))a$, where h is the class number and D is the discriminant of the quadratic field. Gauss had implicitly raised the question of effectively enumerating all imaginary quadratic fields with a given class number, and the Goldfeld–Gross–Zagier result gives an explicit solution. In 1987, the Frank Nelson Cole Prize in Number Theory was awarded to Goldfeld, Gross, and Zagier for their resolution of Gauss's problem.

The Gross–Zagier theorem has been the basis of much further work; the formula and its proof continue to be at the center of current research on the arithmetic of abelian varieties and automorphic representations.

Roughly during the same period, Gross formulated an analogue of conjectures of H. Stark that relate values of L-series and their derivatives to objects of interest in algebraic number theory. Gross's conjecture was for the p-adic analogue of L-functions that were constructed by Iwasawa. Gross's elegant formulation connects the values of certain p-adic L-functions over totally real number fields with what have become known as "Gross–Stark" units. Over the last thirty years, a number of mathematicians have proposed refinements and extensions of the Gross–Stark conjecture.

In the early 1990s, Gross and Dipendra Prasad studied the automorphic representations that arise in the Langlands program. They began with the idea that the restriction of automorphic representations of $\mathbf{SO}_n$ to $\mathbf{SO}_n$-1 should be describable in terms of arithmetic data, both over local and global fields. Their study uncovered an incredibly rich mathematical lode, with surprising applications and the potential for numerous generalizations.

Over the last several years, Gross and Manjul Bhargava have obtained new results in a subject that might be termed Arithmetic Invariant Theory. In particular, they have extended Bhargava's results on Selmer groups and average ranks of elliptic curves to the family of hyperelliptic curves of a fixed genus. Their extension has applications to counting points on hyperelliptic curves; for example, they prove that among hyperelliptic curves $y_2 = f(x)$, of fixed odd degree > 5, at least half have no more than twenty rational points. The Bhargava–Gross work on Selmer groups has recently been used by Poonen and Stoll to prove that a positive proportion of such curves have

Joe Buhler is professor of mathematics at Reed College. His email address is `jpb@reed.edu`.

Ken Ribet is professor of mathematics at the University of California at Berkeley. His email address is `ribet@math.berkeley.edu`.

exactly one rational point, and that the fraction of curves with this property approaches 1 exponentially fast as the genus goes to infinity. These striking results have sparked considerable interest, and are likely to be improved and extended in the near future.

We stress again that the above summary touches on only a small fraction of Gross's mathematical output. His research achievements have been recognized in a number of ways. In addition to the AMS Cole Prize, Gross received a Sloan fellowship and a MacArthur fellowship, and was elected a fellow of the American Academy of Arts and Sciences and a member of the U.S. National Academy of Sciences.

Gross has not only an extraordinarily penetrating intellect, but also a remarkable capacity for collaborating with others in a way that elevates both his work and theirs. Gross's collaborators since 2000 include Bryan Birch, Pierre Deligne, Noam Elkies, Edward Frenkel, Wee Teck Gan, Joe Harris, Eriko Hironaka, Mark Lucianovic, Curt McMullen, Gabriele Nebe, Ariel Pacetti, James Parson, David Pollack, Mark Reeder, Fernando Rodriguez Villegas, Gordan Savin and Nolan Wallach.

In both his oral and written exposition, Gross is deeply committed to clarity, elegance, and depth. It is hard to separate his mathematics from his pedagogical sensibilities and teaching, since exactly these qualities permeate his approach to exposition at all levels.

For instance, Gross enjoys teaching introductory mathematics courses to non-majors. His course with Joe Harris on number theory led to the charming book *The Magic of Numbers*. More recently, he developed Fat Chance, a course with Joe Harris and Nathan Kaplan on probabilistic and statistical reasoning. Two years ago Gross was named a Harvard College Professor for a five-year term; this title is bestowed on selected faculty to recognize them for exceptional undergraduate teaching.

On the other end of the spectrum, he has taught a wide array of graduate courses, and has advised thirty-four graduate students, many of whom are now well known mathematicians in their own right. The Mathematics Genealogy Project shows that quite a few of Gross's students have had doctoral students of their own; it lists seventy-two descendants for Benedict Gross.

We now turn to Professor Gross's service to the mathematics profession and to the academic community more broadly. His common sense, charm, good taste, and outgoing personality have made Gross a compelling choice as committee member and administrator. He has served Harvard University, the AMS, numerous editorial boards, the Sloan Foundation and the Mathematical Sciences Research Institute; he is now a trustee of the Institute for Advanced Study at Princeton. Fans of films with mathematical connections know that Gross coached Jill Clayburgh on her proof of the Snake Lemma in the 1980 film "It's My Turn".

Gross served as the Chair of the Mathematics Department at Harvard from 1999 to 2002. His skill in this position, and perhaps especially his rapport with members of the university community outside the department, led to his appointment as Dean of Harvard College. This appointment came at an especially critical time for Harvard because the college was going through the process of revising its undergraduate curriculum. Gross became the overall chair of this endeavor, and coordinated efforts that led to the most thoroughgoing curricular changes in the last thirty years. As Dean of Harvard College, Gross had to manage a large staff, cope with innumerable crises, represent the College to external audiences, and mediate among a diverse group of constituencies. His success as Dean stemmed no doubt from his knack for giving a fair hearing to all sides of an issue before setting a course of action.

On the personal side, we note that Dick is an all-around great guy who is never too busy to pitch in to help solve a problem. His broad experience, talent at social interaction, and wide range of interests are unusual among top-notch mathematicians. Gross's intelligence, wisdom, and interpersonal skills make him extraordinarily well qualified to be the next President of the AMS.