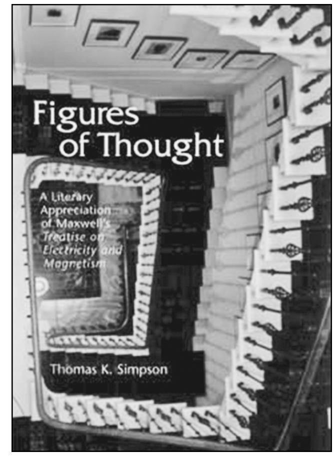
# Figures of Thought

*Reviewed by Brian Hayes*




**Figures of Thought: A Literary Appreciation of Maxwell's *Treatise on Electricity and Magnetism***
*Thomas K. Simpson*
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Mathematics has the most carefully curated literature in the entire world of learning. *Mathematical Reviews* and the *Zentralblatt für Mathematik* endeavor to catalogue, classify, and summarize every research publication in mathematics, even filling the occasional much-needed gap. But one thing that's *not* commonly done with the mathematical literature is to treat it as literature—to examine a piece of mathematical writing by the methods of literary scholarship and close textual analysis, the way a critic would explicate a poem.

Mathematical discourse is very different from poetry. The meaning of a poem is inextricably linked to the particular words that compose it. Mathematical truths, in contrast, are supposed to transcend the manner of their expression. Poems resist paraphrase and translation, but a theorem is a theorem, however you state it. As a measure of the distance between the two genres, consider the set of all badly written good poems. If this set is not empty, its members must be few and very peculiar. Badly written good mathematics, on the other hand, is all too common. (As for well-written bad mathematics: If your proof has a logical flaw, no amount of eloquent persiflage will redeem it.)

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In mathematics and the sciences, style and substance seem to be orthogonal variables—but on this point Thomas K. Simpson disagrees. Speaking of scientific works generally and with particular reference to James Clerk Maxwell's *Treatise on Electricity and Magnetism*, Simpson writes:

> It seems to be generally assumed that the literary and the scientific aspects of the work will factor, so to speak, and remain separable—thus the literary form will not bear significantly upon the scientific content. As it turns out, Maxwell in the *Treatise* is demonstrating precisely the opposite: so far from being divided, Maxwell's literary and scientific efforts are conjoined, in their aims as in their means.

Simpson's book *Figures of Thought* undertakes to support this claim through a literary and rhetorical examination of Maxwell's writing. The lit-crit approach to science is a novelty for most readers but not for Simpson. For many years he was a tutor at St. John's College, a school whose Great Books Program immerses students in original sources across the curriculum. He has been a student of Maxwell's work for fifty years.

### The Electrodynamic Wrestling Match

The *Treatise on Electricity and Magnetism* is a major landmark in physics, standing as Maxwell's last word on the subject; the first edition came out in 1873, and he was at work on a revision when he died (at age 48) in 1879. The book is also a minor milestone in mathematics, as it shows Maxwell and his contemporaries groping toward the system of ideas and notations that would soon become vector calculus. (Maxwell coined the terms *gradient* and *curl*; with a change of sign, his *convergence* operator became the *divergence*.)

The *Treatise* is one of those immense and encyclopedic Victorian testaments: two volumes, four parts, fifty-six chapters, 866 numbered articles, more than a thousand pages. I must confess that I had never read a word of it before taking up Simpson's commentary; what I knew of electrodynamics came from modern textbooks. Having now delved into the *Treatise*, I can report that it's not quite what I expected. It is not a summing up or a tidying up of Maxwell's earlier research papers on electromagnetic themes. It's a record of his ongoing wrestling match with problems he still finds challenging. It's science in progress and mathematics in the making.

Simpson's *Figures of Thought* covers only a small part of the *Treatise*, but it's the part of greatest interest to modern readers. We get a guided tour through the first nine chapters of Part IV, where electrical and magnetic phenomena are shown to be two aspects of a single concept: the electromagnetic field. Then we skip ahead to a later chapter that introduces the set of relations now known as Maxwell's equations.

Simpson's kind of literary analysis is not a critique of words and sentences or other aspects of small-scale prose style. As a matter of fact, there are few direct quotations of Maxwell's text. Instead the focus is on structure and rhetoric—on how Maxwell frames his arguments and tries to win the reader over to his views.

Simpson sees the *Treatise* as a drama in three acts, or as "a classic trilogy on the pattern of the *Oresteia*: opening with confidence, passing into darkness and confusion, but then emerging with a resolution that is new to the world and which could not have been foreseen at the outset." The drama has a hero: Michael Faraday, the unlettered, visionary genius of nineteenth-century British science, who intuited the relation between electricity and magnetism but resisted all urgings to put his discoveries in mathematical form. (Maxwell nonetheless eulogized Faraday as "a mathematician of a very high order.")

There's no real villain in the story, but there *is* a figure who serves as a dark shadow providing contrast for Faraday's brilliance. He is André-Marie Ampère, the French claimant to the title of founder of electrodynamics. "Embodied in the characters of Ampère and Faraday are not just two styles but two contrasting stances toward life itself: Ampère's imperious, dictating to nature; Faraday's modest, open, and sensitive to nature's voice."

The disagreements between these rivals were matters of substance as well as style. Ampère endeavored to explain electrical and magnetic phenomena with the kind of central force law that prevails in Newtonian gravitation. This action-at-a-distance scheme works well in the case of pointlike charged particles, where an electrostatic force proportional to $ee'/r^2$ is just like the Newtonian law $mm'/r^2$ (except that electric charge $e$ comes in negative and positive varieties whereas mass $m$ is always positive). The simplicity is lost, however, when the sources are no longer pointlike. When describing the force between two current-carrying loops of wire, Ampère becomes ensnared in a thicket of sines and cosines:

$$dF = \frac{ii'}{r^2}(\sin\theta\sin\theta'\cos\eta - \frac{1}{2}\cos\theta\cos\theta')\,ds\,ds'.$$

Here $i$ and $i'$ are the currents, $s$ and $s'$ are segments of the loops, and the angles $\theta$, $\theta'$ and $\eta$ define the relative orientation of the loops.[1]

Faraday had a different vision. Inspired by the patterns he observed when iron filings are sprinkled on a permanent magnet, he imagined "lines of force" extending throughout space. Two electric currents would interact not by means of direct forces transmitted through empty space but through the intermediary agency of electric and magnetic fields. Each current, magnet, or charge generates its own field and responds to the fields around it.

Act I of Simpson's three-part drama has Maxwell constructing a mathematical version of Faraday's lines of force—essentially a vector field, although modern notations and a few key concepts are lacking. The effort is a success, in the sense that the calculations based on the field concept correctly predict various experimental results. But Simpson calls it a "fragile victory" because the field is static; this version of the theory can accommodate only unvarying electric currents and magnetic fields.

With Act II, the drama turns to melodrama. This is "the darkest moment, the point of crisis." "We are in the dark wood of the *Treatise*," Simpson writes, evoking the *selva oscura* of Dante's *Inferno*. The source of all this consternation is Faraday's discovery of induction: A change in the current flowing through one circuit induces a momentary current in another nearby circuit. There is even self-induction, where the same circuit both generates and responds to the disturbance. Maxwell compares the effect to the momentum of water flowing in a garden hose, which resists changes in velocity. However, the analogy is imperfect because the effects of momentum in water "will be the same whether the hose is coiled or stretched in a straight line; but those of self-induction depend altogether on the configuration of the conductor."

Act III will eventually resolve this puzzle, but the ending is not one of those operatic climaxes where all the players suddenly drop their disguises,

[1] *I show the equation in the form given by Ampère. Simpson writes it* $(\cos\theta\cos\theta'\sin\eta - \frac{1}{2}\cos\theta\cos\theta')$. *I suspect this is a transcription error.*

lovers are reunited, and troublemakers promise to reform. Getting to a satisfactory theory takes seven dense chapters, including a long digression into the celestial mechanics of Joseph-Louis Lagrange. The key idea is to associate energy and momentum not with the current flowing through a wire but with the electric and magnetic fields that surround the wire. From this novelty we are led to an even more remarkable idea in the denouement: We can dispense with the hardware of wires and magnets altogether and watch as disembodied electric and magnetic fields act and react, then dance across the universe as light waves.

### Whodunit

Simpson introduces his literary appreciation of Maxwell with this declaration:

> A scientific work evidences literary character when it is imbued with a vision or a goal towards which its parts are organized throughout. Achieving this organization is the business of the art of poetics, which teaches us that there must be a story line with a beginning, middle, and end.

Thus we are asked to believe that the three-act drama outlined above is something that Maxwell plotted out in advance, in the way that the author of a murder mystery knows who "done" it long before the reader begins to pick up clues. The *Treatise*, says Simpson, is "not simply a linear argument that deposits a result but a poetic work that prizes its beginnings in order to appreciate its conclusions."

This is not a view of Maxwell's authorial method and intent that I would have come to on my own, if I had been reading the *Treatise* without Simpson's guidance. My impression, for what it's worth, is that Maxwell is not working from a carefully constructed outline but rather is exploring ideas as they arise, testing how various pieces of the puzzle might fit together, starting over when a strategy doesn't work out. This impression of a tentative and *ad hoc* narrative becomes all the stronger when I draw back from the 150 pages that Simpson analyzes in detail and consider the rest of the *Treatise*, in which Maxwell reviews, digests, and attempts to formulate mathematical explanations for two centuries' worth of experimental findings.

I am not entirely alone in this opinion of Maxwell's aims and technique. C. W. F. Everitt, writing on Maxwell for the *Dictionary of Scientific Biography*, noted that Maxwell "gave the *Treatise* a loose-knit structure, organized on historical and experimental rather than deductive lines. Ideas are exhibited at different phases of growth in different places; different sections are developed independently, with gaps, inconsistencies, or even flat contradictions in argument. It is a studio rather than a finished work of art."

The question of rhetorical premeditation becomes particularly troubling in Simpson's Act II. The passage quoted above about the "dark wood" continues as follows:

> We are in the dark wood of the *Treatise*; and it is a sure sign of Maxwell's plan that he insists on leading us into it before he offers a way out. He might, after all, have structured the *Treatise* otherwise, in the manner of a linear textbook, marshaling a repertory of equations to yield the required result with no detour into aporia. That would have been the "direct" style, and it would have made far easier reading for Maxwell's contemporaries and for generations of students since. Maxwell's purpose, however, lies in another direction. He intends us, as readers, to experience this impasse the way Faraday experienced it.

It's not altogether a happy revelation that Maxwell would be so cavalier about the needs of those generations of students—especially since Simpson tells us that the *Treatise* was in fact meant to serve as a textbook for Cambridge students taking a new tripos examination in electricity and magnetism.

In spite of these misgivings, I have been grateful to have Simpson whispering in my ear as I've read the *Treatise*. For example, when Maxwell describes Ampère as "the Newton of electricity", I missed something crucial about the tone of voice. Although Maxwell is a genuine admirer of both Newton and Ampère, there's also an undertone of irony here, calling attention to Ampère's pomposity. Simpson supplies the missing smiley ;-) at the end of the sentence.

In a more extensive example, Simpson devotes several pages to Green's Theorem interpreted as a "figure of thought", a rhetorical device for expressing ideas about fields and flows. The very notion of viewing an equation as a rhetorical tool struck me as novel and provocative; more to the point, Simpson's discussion helped me understand the key role of this principle in Maxwell's thinking. The version of Green's Theorem at issue here establishes an identity between an integral over a two-dimensional surface and an integral over the surrounding volume. Maxwell observes that the theorem relates the energy per unit area on the surface of an electrically charged body to the energy per unit volume, $W$, throughout space. Simpson elaborates:

> Formally, this is just a number, an alternate way of computing $W$. But if we take the identity seriously—with all its rhetorical

> force—it becomes a source of new insight: it suggests that the energy in question need not be thought of as existing on the charged surface. Instead, the energy $W$ may actually be distributed over all space, in a very real *electrostatic field*.

I'm still not sure how to distinguish Green's Theorem as a "figure of thought" from Green's Theorem as a mathematical fact, but Simpson's commentary does illuminate Maxwell's use of the idea.

I wish Simpson had pursued this kind of argument a little further and looked into other developments in mathematical notation and methodology that had a bearing on Maxwell's work. Consider the four equations that we now call Maxwell's equations—the ones that appear on nerdy tee shirts:

$$\nabla \cdot \mathbf{D} = \rho,$$
$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

Maxwell never wrote that particular set of equations; they were given their modern form by Josiah Willard Gibbs after Maxwell's death. It's a curious anomaly of history that Maxwell had sophisticated tools for dealing with operators (in the sense of functions applied to functions) and the apparatus of partial differential equations, but he had no proper representation for vectors in three-dimensional Euclidean space. He had to adapt William Rowan Hamilton's quaternions to the purpose, considering one element of a quaternion to be a scalar magnitude and the other three elements as $x$, $y$, and $z$ vector components. In many cases Maxwell gave up and listed trios of componentwise equations.

### The Future of Math-crit

Will analysis of literary tropes be the next fashion craze in scientific and mathematical writing? I think that's unlikely, if only because so few potential authors are knowledgeable of and interested in both subjects. (Indeed, Simpson might be the unique practitioner of this art.) Looking at it through the other end of the telescope, I doubt that poetics is the most rewarding approach to the scientific and mathematical literature for most readers.

But the method does have its attractions. As a reader of scientific and mathematical prose, I welcome any development that might encourage authors to give greater thought and care to questions of expository technique. In other words, I'd like less badly written good mathematics. As a writer who tries to communicate scientific and mathematical ideas to a wider audience, I am gratified to see serious attention paid to questions at the heart of my own craft, such as how best to engage and motivate readers. More broadly, any practice that brings even a few representatives of the "other culture" to the mathematical literature has got to be a good thing. If it also leads readers back to the well-known but seldom-read classics (as it did me), so much the better.