MOOCs and the Future of Mathematics

Calculus pedagogy and online courses are well-trod fields of war. I write with care, given the wide array of opinions strongly held.

In 2012 I created a calculus MOOC (massive open online course) for Penn's partnership with Coursera. The course ran in the spring of 2013 and again in summer 2013. You are welcome to view the materials for this course (and other free courses) on Coursera's website.

Contrary to apocalyptic fears, this is not the end of calculus instruction. Rather, this and other open-access online courses herald a time of experimentation and rapid improvement in how we communicate mathematics to the world.

I volunteered to build this MOOC to give a proof-of-concept for a different approach to calculus and calculus education. Over the course of a year, I designed, drew, animated, and recorded a complete second-semester calculus course. Two dedicated graduate students and two postdocs helped generate assessments and write a free Wiki-based text. The work was difficult; the impact on grateful learners across the world redeemed that difficulty in full.

Before turning to the question of MOOCs and their future, I wish to note that MOOCs are more than a translation of an existing course to a video format: they represent a novel means of rapid innovation in how we present mathematics. A few particulars of my MOOC should illustrate this point.

1) The video lectures are hand drawn and animated. Students gain a visually dynamic understanding of series, surfaces, moments, and more. Many find it fun, an adjective not normally applied to calculus texts.

2) The ordering is novel (apologies to Euler), beginning with Taylor series. The initial emphasis on their mechanics draws in those students of AP calculus who grasp derivatives without depth. The course then turns to limits, derivatives, integrals, and applications, drawing conceptual insight from Taylor expansions throughout. Sequences and a more rigorous approach to series are part of "discrete calculus"—recapitulating calculus for functions with discrete input and analog output (i.e., sequences).

3) Connections to deeper ideas in mathematics and applications are repeatedly drawn. The nonthreatening nature of comic-book-style animations lets me foreshadow commutative diagrams, generating functions, and network topology without overwhelming students. Asymptotic notation (big-O) is used throughout, and an emphasis on contemporary applications empowers students for future courses in the sciences.

Some will view these changes as refreshing; others will find them misguided. Whatever the value of this particular offering, it demonstrates the degree of innovation possible with MOOCs. The rise of low-cost platforms for putting our courses online is a liberation from the publishing bureaucracy that has made the calculus education of today look almost identical to that of the 1980s and earlier (in every respect save price).

This course is one example of the creative possibilities for MOOCs. The reader can imagine what could be done with higher-level material. The potential advances arising from modular cross-linked content, collaborative development, and improvement based on data-mining student performance are profound. Though only a handful of MOOCs in mathematics have been developed, the optimism surrounding this platform is justified.

There are many open questions concerning MOOCs: what do they portend for our profession? our students? our community?

I wish to sidestep the more hyperbolic criticisms (such as meaningless completion rates or conspiracy theories of venture-capitalist plots) and instead address realistic risks and rewards. MOOCs certainly have the potential to do harm: harm to our students if regulations or laziness lead to entertaining pabulum catering to the lowest common denominator, harm to faculty if we are robbed of our freedom of creative expression in the classroom. However, the potential returns, especially to those without ready access to the best schools, are so compelling as to demand experimentation and investment.

The future of MOOCs is uncertain. I have learned this past year from my inaccuracy in predicting six months ahead that it is folly to predict a decade hence. I have also learned from the last century of mathematics research (not infrequently funded by the U.S. Department of Defense) that an innovation that can be imagined as being turned to evil almost always ends up being turned to an unimagined good. I am optimistic about MOOCs, especially in mathematics.

"What will MOOCs change?" is not the ultimate question. "Why do mathematicians exist?" is the critical question that we must answer—persuasively—with skillful exposition to all those we serve: students, parents, scientists, deans, and lawmakers. If we do not provide a comprehensible and compelling answer, others will answer the question for us. If we do not build high-quality expositions that demonstrate why mathematics is to be loved, others will build expositions communicating that mathematics is to be feared or endured. To us is given the gift of seeing the beauty and art in our discipline, a beauty that is not grasped without great effort. The rise of MOOCs is our profession's moment of opportunity to communicate its truths skillfully and artfully and to promote our core insights and modes of thought to an eager worldwide audience through this medium of rapid innovation.

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MOOCs (massive open online courses) are causing a revolution in higher education today. What will be the impact of this revolution on mathematics teaching in colleges and universities? The *Notices* invites short pieces (800 words or less) on the subject of MOOCs in mathematics. Please send contributions to notices-mooc@ams.org.

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