[Contemporary Pure] Math Is Far Less Than the Sum of Its [Too Numerous] Parts

In a newspaper article entitled "Math is more than the sum of its parts" (*New York Daily News*, July 8, 2012), the great pure mathematician Edward Frenkel, along with mathematics educator Ronald Ross, preached the importance of math, apropos of the announcement of the discovery of the Higgs boson.

What Frenkel and Ross did not tell us is that the "math" that led to the discovery of the Higgs boson is not their kind of (pure-and-rigorous) math, but the much more effective, and efficient, nonrigorous mathematics practiced by theoretical physicists called quantum field theory. This highly successful (and precise!) mathematical theory would not be considered mathematics by most members of the American Mathematical Society, since it is completely nonrigorous.

The detection of the Higgs boson probably also involved many hours of heavy-duty computer calculations, very far afield from esoterica most pure mathematicians hold dear, such as Frenkel's own research in the Langlands program. Ironically, (pure) mathematicians are much more indebted to theoretical physicists than vice versa (e.g., Seiberg-Witten and quantum groups). From physics, mathematicians have absorbed fresh ideas with which to pursue their often very beautiful, but completely useless, game.

It is common for pure mathematicians to praise the RSA algorithm. Let me remind you that the "safety" of RSA is only conjectural (from the pedantic standpoint of pure mathematicians). It is possible (but very unlikely!) that tomorrow an assistant professor of computer science (not math!), together with two undergrads, will find a fast algorithm for integer factorization. The rest of the math behind the clever RSA algorithm goes back to Euler. Establishing RSA does not require mathematical arcana such as the Langlands program. And the RSA algorithm would be *just as useful* if it had only an "empirical" proof.

The reason so many mathematically talented students are so turned off from math is that once they go to university, even the science and engineering students are taught by professional mathematicians, whose rigid, pedantic, "rigor-or-nothing" philosophy is imposed on the courses, at least in part.

Communication in mathematics is, even at the "highest" level of conference talks, highly dysfunctional. Highly specialized specialists who attempt to communicate their subject to a "general mathematical audience", just read their highly technical, usually very dry, preprepared laptop presentations, and (almost) no one has any clue. Indeed, pure math has gotten so splintered that very few people see the mathematical forest. Most can barely understand their own trees.

One example is the AMS Colloquium Lecture series at the Joint Mathematics Meetings. No doubt some of these three-hour lecture series have been very good. But too often they are delivered by talented mathematicians who do not even attempt to make the lectures accessible to a general mathematical audience. Rather, they give highly technical talks with completely unrealistic expectations about the background of the audience.

Mathematics is so useful because physical scientists and engineers have the good sense to largely ignore the "religious" fanaticism of professional mathematicians and their insistence on so-called rigor, which in many cases is misplaced and hypocritical, since it is based on "axioms" that are completely fictional, i.e., those that involve the so-called infinity.

The purpose of mathematical research should be the increase of mathematical knowledge, broadly defined. We should not be tied up with the antiquated notions of alleged "rigor". A new philosophy of and attitude toward mathematics is developing, called "experimental math" (though it is derided by most of my colleagues; I often hear the phrase, "It's only experimental math"). Experimental math should trickle down to all levels of education, from professional math meetings, via grad school, all the way to kindergarten. Should that happen, Wigner's "unreasonable effectiveness of math in science" would be all the more effective!

Let's start right now! A modest beginning would be to have every math major undergrad take a course in experimental mathematics.

Please don't misunderstand me. Personally, I love (quite a few) rigorous proofs, and it's okay for anyone who loves them to look for them in his or her spare time. However, for the research and teaching that we get paid for, we should adopt a much more open-minded attitude to mathematical truth similar to the standards of the "hard" physical sciences. We need to abandon our fanatical insistence on "rigorous" proofs.

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