Letters to the Editor

Is There a Better Format for the Presentation of Mathematical Subjects?

The ongoing concern with mathematics education that was expressed by two articles in the November 2013 issue of the *Notices*, makes me wonder if part of the problem might not be the format in which mathematics is presented—a format that goes back to Euclid, some 300 years b.c.e. For years, in my own studies, I have relied on a different format that has proven to be far more efficient for learning and problem solving. The format is based on several ideas from computer science: object-oriented programming (task orientation), separating the What from the How, and structured programming. In brief: each subject is conceived as involving a set of "entities." For example, in high school algebra, one of these entities is "equation." Associated with each entity is a template (the same for all entities), which is a list consisting of: definition of entity, ways of representing entity, common tasks performed on the entity, types of the entity, theorems pertaining to the entity, closely related entities. Each item in the list is then followed by a reference in the student's notes and/or in the textbook, to details on the item.

Thus in the case of the entity "equation," the list of common tasks includes: convert an equation into polynomial form, determine the type of an equation (linear, quadratic, etc.), solve an equation, add a term to both sides of an equation, multiply both sides of an equation by a term, divide both sides of an equation by a term.

Another characteristic of this format is the writing down of procedures to perform the more difficult tasks. The goal here is to have something that can be looked up and rapidly re-used days, weeks, months, years after the procedure was first learned. (It is not enough to more or less know how to do most integrals in a calculus course: the goal is to write down a procedure (it is known that no algorithm exists).)

Another characteristic is that all, or most, proofs are written in structured proof format (analogous to structured program format), which makes the devising of proofs, and the understanding of existing proofs, much more rapid.

The format can be applied to all subjects from primary school up to at least all the advanced mathematics subjects that I am familiar with. It makes all mathematical subjects look "the same." Primary school students can be introduced to it by being asked to consider all the tasks associated with, for example, a bicycle, or an iPad, or a TV set.

The format is not an alternative to the traditional textbook and classroom format, but in my experience it is a great enhancement to it.

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Ludwig van Beethoven and the Metronome

I read with great interest the article about the metronome ["Was something wrong with Beethoven's metronome?", by Sture Forsén, Harry B. Gray, L. K. Olof Lindgren, and Shirley B. Gray, *Notices*, October 2013], which presents an analysis of what happens if the weights are not in proper position and discusses the big question mark left behind by Beethoven's metronome markings. The article is very enjoyable and informative.

Nevertheless, there is one significant omission that devalues the article.

While reading the article, I was curious to see what the final conclusion would be. After all, Beethoven gave a metronome number not only for the first movement of the *Hammerklaviersonate*, but for *all* movements. And *all* the markings—including the one for the slow movement, that is, the third movement—are too fast. (At least, every musician would agree on this. There is a benchmark recording of the Beethoven sonatas by Friedrich Gulda; in the accompanying text he says, concerning the *Hammerklaviersonate*, that he intended to follow Beethoven's markings as much as possible; but even he plays the first movement a bit slower than Beethoven's indication—which is still incredibly fast!—and the same is true for the other movements, except perhaps for the second.)

The analysis in the article shows that—depending on whether the bottom weight is too low or too high—fast tempos are made faster (respectively, slower) and slow tempos are made slower (respectively, faster). I therefore wondered how this relates to the above-mentioned fact, namely, that *all* of Beethoven's metronome indications in the *Hammerklaviersonate* are too fast. The indication for the third movement (Adagio sostenuto) is: eighth = 92. Is this a slow tempo, a medium tempo? This should have been addressed, as should have the indications in the other movements. Otherwise, the theory as presented stands on weak grounds.

Finally, I can offer the following anecdote that my piano professor at the Vienna University of Music and Performing Arts (when I was a pianist in a previous life) loved to tell: it concerns Igor Stravinsky, who on some occasion was asked by a journalist how fast he would would want a certain piece of his to be played. Stravinsky thought for a moment, and then indicated a tempo. My teacher found it very amusing that the tempo was completely different from the metronome indication that Stravinsky had given for the piece.

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Pseudo-Education Marches On

In the October 2013 *Notices* article "Teaching mathematics with women in mind," Professors Deshler and Burroughs wrote the following under the heading "What Are We Teaching Our Students": "In recent years a focus on conceptual understanding has led to a curriculum reform movement in mathematics across all school levels, a movement that is focused on conceptual understanding rather than procedural understanding of mathematics."

Unfortunately, they failed to expose that the assorted "reforms" of the past twenty-four years, which I have followed closely, have simply enhanced the continuing mathematics pseudo-education of American students. One example of this grim reality is the brief essay written by one of my students in 1995, which is posted at: `http://mathforum.org/kb/message.jspa?messageID=1461554`.

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Adjuncts and Teaching

Catching up on old *Notices*, I read Prof. Reys's article ["Getting evidence-based teaching practices into mathematics departments: Blueprint or fantasy?," by Robert Reys, *Notices*, August 2013] with some interest, noticing that the article does not address a major issue, and that is the increasing burden of teaching being born by badly paid and overworked adjuncts, many of whom are quite talented, but who operate outside the main life of the department and who often do not have the time nor access to resources that would allow them to seriously rethink their teaching methodology.

Any serious efforts to change the culture will have to include them, one hopes with serious improvements in their remuneration and working conditions.

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AMS in Arabic Means "Yesterday"

In response to "[Contemporary pure] math is far less than the sum of its [too numerous] parts," by Doron Zeilberger, *Notices*, December 2013:

Opposing "rigorous" mathematical proof to "field" (experimental) mathematics as ethically/socially top down vs. bottom up is a cultural practice with little taxonomic value. Pure math can be as usefully defined as mathematics having conceptual distance or having no immediate application, except for other mathematicians. The proof as purity paradigm is gold standard/virginity testing stuff. It is a cultural practice, an old meme, not a paradigm that exactly gives flight to the imagination. Mathematical proof is "pure" when it extends proof/analysis/logic, and is "field" mathematics when it connects mathematical disciplines, and is "applied" when it merely verifies. Verification proof is mathematics accounting.

Field mathematics—the pure mathematics casually located adjunct to proofy math and frequently accompanied by and ambiguated with applied mathematics—doesn't need much defense, and using QFT [quantum field theory] to do so seems to invite criticism. (If I had to make an ignorant over-arching comment on QFT based on conceptual distance of the title, I would dismiss it as magnitudinal incrementalism that was probably visible twenty or thirty years ago. Pure mathematics might have a go at this.)

Partially outsourcing mathematics to machines is a done deal but it is annoying because of known limitations (100 years of quantum physics without quantum computers) and the implied administrative/power relations overhead. If they weren't dumb, metered, and politicized and more of them had names like "scratchpad" instead of "megalyth-o-tron" they would probably be better received. How many filters does a person need to pass through before qualifying to use one? Similarly, "John Henry" would probably not be a good name for a highly politically, commercially, or bureaucratically leveraged research computer, which pretty much includes all of them. A concern that their use is in equal measure "insipid" as "intrepid" reflects a certain amount of self-understanding. We are sometimes so, why not somewhat they? Fair allocation minimally requires excess flop/hrs. (or quantum equivalent). What are the benefits to managing an unused supercomputer (probably similar to pet ownership)?

What I liked about Doron's letter was that it was all over the place and unabashedly wrong. That is called self-expression. Thank you.

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Outdoing the Soviets

Amusingly, and be it intentionally or not, the December 2013 issue of the *Notices* has on page 1431 an item by Doron Zeilberger on how mathematics should allegedly be, while on pages 1448-58, another item by Christopher Hollings on the vagaries of past Soviet ideology in mathematics. During their about seven decades of ideological rampages, the Soviets got to the conclusion, see (4) on top of page 1455, that: "Nevertheless, the growth of practical applications should not hinder work in abstract areas of mathematics." As for Zeilberger, he is—more than two decades after the pitiful collapse of the Soviets, who managed to go down the drain without one single bullet being fired—trying to delight us with some "radical" views of how mathematics should be, views which are, to put it mildly, incomparably more raw, primitive, and one-sided than those of the so shamefully and utterly failed and fallen Soviets. Such a strange contrast is, of course, one of the assumed individual privileges in democracy. And as those familiar with Systems Theory may know, the more complex an entity, the more its various instances may spread across a wider spectrum. And we humans are, beyond our bodies, by far the most complex entities known to us on Planet Earth. Well, Zeilberger either knows this or not, or likes it or not, but he does manage to prove how wide the spectrum is across which we do indeed happen to spread.

By the way, the Soviets, among others, fell because of considerably less raw and primitive views than those of a Zeilberger.

Democracies do not seem to fall because of types like Zeilberger.

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The Purpose of Rigor

Regarding Doron Zeilberger's opinion piece in the December 2013 issue of the *Notices*: The purpose of mathematical rigor is not so that mathematicians can feel superior to physicists, although this may be a fringe benefit. Rather, the purpose of rigor is to know what is actually true. After Cauchy "proved" in 1821 that the limit of a sequence of continuous functions is necessarily continuous, Abel showed that "this theorem admits exceptions." Many theorems in the physics literature likewise admit exceptions. These results are not meaningless or worthless, but they do challenge mathematicians to find the version that holds without exception. Rigorous mathematics is not the only kind, but it does have a valuable place.

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Reply from Zeilberger

Nothing is absolutely certain in this world, and a traditional rigorous proof gives you only the illusion of absolute certainty, since, until now, with a few exceptions (most notably the Four Color Theorem and Kepler's conjecture), such proofs were done entirely by humans, who are notoriously unreliable.

Most of the "crises" of mathematics were illusionary, assuming the fictional infinity and the "continuous" "real" numbers. They are akin to crises in religion where good guys suffer and God is not behaving as he (or she or it) should, and to millions of pages of scholastic drivel.

Traditional "rigorous proof" is yet another religious dogma, which did some good for a long time (as did the belief in God). Of course, it is not surprising that people can get deeply offended when someone denies the existence of their "God".

But the God of (alleged!) rigorous proof is dead (well, not yet, but it should be!), and we should allow diversity. Rigorous proofs should still be tolerated, but they should lose their dominance, and the A*nnals of Mathematics* should mostly accept articles with mathematics that has only semi-rigorous or non-rigorous proofs (of course, aided by our much more powerful and superior silicon brethren), because this way the horizon of mathematical knowledge (and mathematical insight!), broadly defined, would grow exponentially wider.

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Further Remarks on a Quartic Algorithm

As noted in Dan Jurca's October 2013 *Notices* letter, when approximating $\sqrt{S}$ starting from some initial guess x_0, the default form of the Bakhshali algorithm (*Notices* August 2013, page 845) is indeed less computationally efficient than simply performing two steps of the (Newton-) Heron algorithm

$$x_{n+1}=(x_n+S/x_n)/2.$$

However, a little algebra shows that a single step of the Bakhshali method can be written as

$$x_{n+1} = \frac{2(x_n^2+S)^2-(x_n^2-S)^2}{4x_n(x_n^2+S)},$$

which is still quartically convergent, but now involves only one division. Since division is the major bottleneck—taking between five and twelve times longer than multiplication on modern computer processors [1] [2]—this single-step Bakhshali method remains computationally competitive with two steps of Heron's method.

Incidentally, the Bakhshali algorithm can be derived by applying the multiplicity-corrected Newton's method to the function

$$f(x) = \frac{(x^2-S)^3}{3x^2+S}$$

or moreover to $\sqrt[3]{f(x)}$ or to $f(x)^n$ for integers $n>0$. However, just as with Heron's method, the algorithm itself came a millennium or so before Newton's method was around to provide such post-hoc "derivations" of these inspirational historical formulae.

[1] http://www.intel.com/content/www/us/en/architecture-and-technology/64-ia-32-architectures-optimization-manual.html

[2] http://developer.amd.com/resources/documentation-articles/developer-guides-manuals/

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