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A Survey on the Square Peg Problem

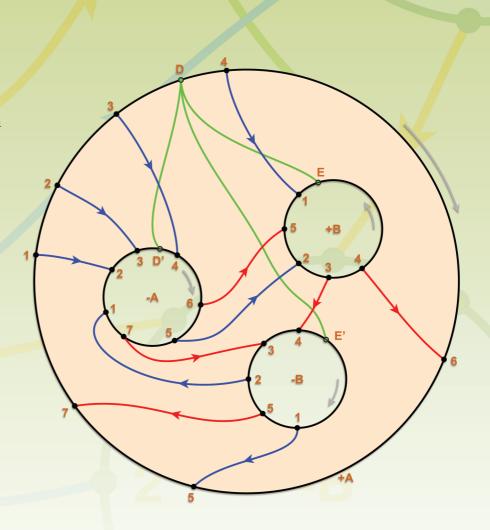
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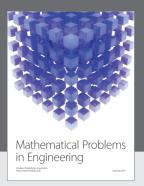
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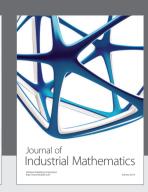




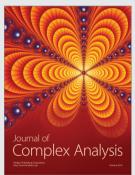














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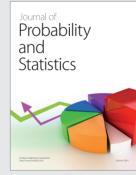




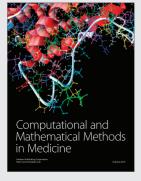






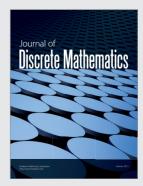


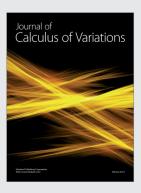














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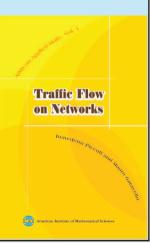
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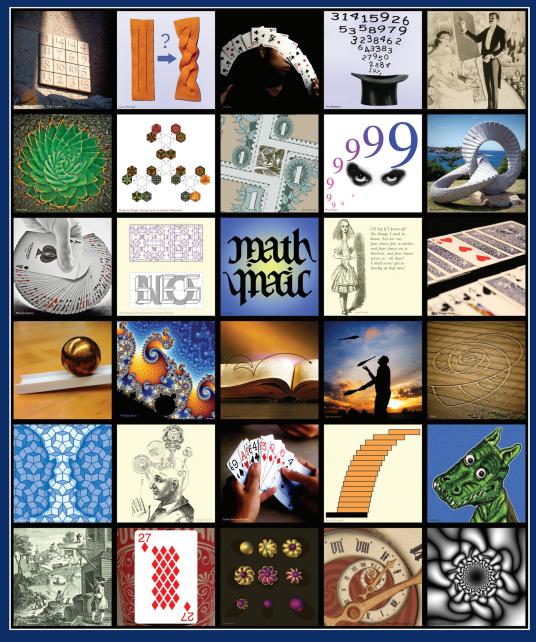
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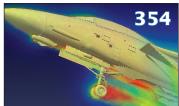
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April brings a blast of fresh air with diverse new Features and Communications. These include a piece on the use of OpenFOAM software in fluid dynamics, an analysis of the square peg problem, a piece on modern industrial mathematics, a study of how well established undergraduate research is in mathematics, and a discussion of whether libraries and open access are becoming irrelevant. Happy reading!

—Steven G. Krantz, Editor

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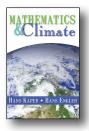
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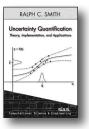
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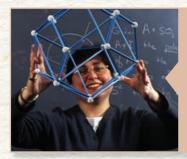
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Opinion

Mathematician, Magician, Mysterian

Nonmathematicians, by definition, do not know mathematics. They also don't know, say, invertebrate paleontology. But we mathematicians can at least hope that the public is aware of the impact of mathematics on their everyday life. Presumably paleontologists have the same ambitions for their subject. Actually, they have it easier: the examination of sedimentary rock samples for invertebrate fossils is a standard part of assessing if the strata from which the samples came are likely to yield oil and gas. Probably the nonpaleontologists among *Notices* readers have a pretty good sense from that one-sentence description of what it means to use paleontology in fossil fuel exploration.

Contrast that with what we would have to explain to give nonmathematicians a sense of the mathematics used in modeling global warming caused by human consumption of fossil fuels, or of searching for patterns of activity in large databases of communications records, or of calculating the monetary value of financial derivatives like credit default swaps. Or imagine explaining the Weil Conjectures. The mathematics that nonmathematicians don't know is only part of the difficulty.

The mathematics they do know—usually the algorithmic techniques of arithmetic and some algebra—leads them to deep misunderstanding of what mathematicians actually do. The arithmetic paradigm—namely, that all problems have routine solutions—causes much of the public to believe that research in mathematics is an oxymoron; the algebra paradigm causes them to believe that mathematical problems are contrived artifices. The latter misapprehension is wonderfully satirized by the lyric "When it's noon on the moon then what time is it here?" in Tom Lehrer's song *That's Mathematics*, a clever account of the ubiquity of mathematics in everyday life. It is safe to say that all mathematicians at one time or another yearn for a way to share with nonmathematicians what it's really like to do mathematics.

Every year JPBM, a joint activity of the mathematics societies, promotes Mathematics Awareness Month to "communicate the power and intrigue in mathematics to a larger audience." Often the theme stresses the power of major applications, for example, the mathematical analysis of climate change. This year, perhaps emphasizing the intrigue, the theme is "Mathematics, Magic, and Mystery".

Some readers may recognize that as the title of a 1956 Dover original by Martin Gardner, late "Mathematical Games" columnist for *Scientific American* and prolific popularizer of mathematics. This is no coincidence: 2014 marks the centennial of Gardner's birth, and Mathematics Awareness Month explicitly intends to celebrate the occasion with "activities that engage a new generation, leading

people to their own magical and mysterious [mathematical] moments." Some of this new generation may become mathematicians themselves. But one can also hope that these mathematical moments will allow a broad public to get a feeling for what mathematical thinking really is.

Andy Magid makes this point, I think correctly, about how mathematical thinking in recreational mathematics is analogous to mathematical thinking in mathematical research in his March 2014 *Notices* review of Gardner's autobiography *Undiluted Hocus-Pocus*, although the review seems to be more about Gardner the Oklahoman than Gardner the mathematician. Gardner, by the way, did not call himself a mathematician, but he may well qualify as "the best friend mathematics ever had," as the Mathematics Awareness Month 2014 theme essay has it. That is, he led a wide public to experience mathematics in the way mathematicians do through his presentations of puzzles, games, and mathematical magic tricks. This probably qualifies him to the title of mathematician after all.

Martin Gardner thought and wrote about much more than mathematics. He was also a noted amateur magician, who carefully observed the magician's code of silence vis-à-vis outsiders, although he did publish (pseudonony-mously) exposés of tricks being passed off as genuine extrasensory perception. Similarly, Gardner also playfully used a pseudonym to publish a tongue-in-cheek negative review of his major work of philosophy, *The Whys of a Philosophical Scrivener*, in the "New York Review of Books" (he later felt that too many readers missed the joke, depressing sales). He even used an anagrammatic pseudonym to include some of his own work in his anthology *Martin Gardner's Favorite Poetic Parodies*; many readers consider those to be some of the highlights of that book.

As Persi Diaconis has pointed out, Gardner towards the end of his life began to describe himself as a mysterian, one who holds a certain scientific/philosophical position about human consciousness. Gardner described this position in his 2007 *Notices* review of Douglas Hofstadter's *I Am a Strange Loop*—thus the third word in the title of this essay, my title being a slight variant of Diaconis's foreword to Gardner's autobiography.

Readers wanting to celebrate Mathematics Awareness Month by entertaining students, colleagues, and friends with a little mathematical magic could do worse than consult Gardner's *Mathematics, Magic, and Mystery*, which is still in print and priced at US\$8.06, which is fifty cents less than its inflation-adjusted cost of one dollar in 1956. Or for some more elaborate and up-to-date material, take a look at Colm Mulcahy's *Mathematical Card Magic: Fifty-Two New Effects*, published in 2013.

—Adam D. Rigny

Editor's Note: "Adam D. Rigny" is the pen name of a mathematician who describes himself as an enthusiastic Martin Gardner fan. He can be reached at notices@ams.org.

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A Survey on the Square Peg Problem

Benjamin Matschke

his is a short survey article on a 103year-old and still open problem in plane geometry, the Square Peg Problem. It is also known as the Inscribed Square Problem and it is due to Otto Toeplitz.

Conjecture 1 (Square Peg Problem, [39]). Every continuous simple closed curve in the plane $y: S^1 \to \mathbb{R}^2$ contains four points that are the vertices of a square.

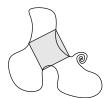


Figure 1. Example for Conjecture 1.

A continuous simple closed curve in the plane is also called a Jordan curve, and it is the same as an injective map from the unit circle into the plane or, equivalently, a topological embedding $S^1 \hookrightarrow \mathbb{R}^2$.

In its full generality Toeplitz's problem is still open. So far it has been solved affirmatively for curves that are "smooth enough" by various authors for varying smoothness conditions; see the next section. All of these proofs are based on the fact that smooth curves inscribe generically an odd number of squares, which can be measured in several topological ways. However, so far none of these methods can be made to work for the general continuous case.

One may think that the general case of the Square Peg Problem can be reduced to the case of smooth curves by approximating a given continuous curve

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Figure 2. We do not require the square to lie fully inside γ ; otherwise there are counterexamples.

y by a sequence of smooth curves y_n : Any y_n inscribes a square Q_n , and by compactness there is a converging subsequence $(Q_{n_k})_k$ whose limit is an inscribed square for the given curve y. However, this limit square is possibly degenerate to a point, and so far there is no argument known that can deal with this problem.

Suppose we could show that any smooth (or equivalently any piecewise linear) curve y that contains in its interior a ball of radius r inscribes a square of side length at least $\sqrt{2}r$ (or at least εr for some constant $\varepsilon > 0$). Then the approximation argument would imply that any continuous curve has the same property. However, it seems that we need more geometric than merely topological ideas to show the existence of *large* inscribed squares.

Other surveys are due to Klee and Wagon [21, Problem 11], Nielsen [30], Denne [4], Karasev [18, 2.6, 4.6], and Pak [32, I.3, I.4]. Jason Cantarella's homepage offers some animations. A Java applet and an extended version of this article are available on my homepage.

In order to raise awareness, let me put 100 euros on each of the Conjectures 1, 8, and 13. That is, you may earn 300 euros in total.

I want to thank the referees for many very useful comments.

History of the Square Peg Problem

The Square Peg Problem first appeared in the literature in the conference report [39] in 1911. Toeplitz gave a talk whose second part had the title "On some problems in topology". The report on that second part is rather short:

b) Ueber einige Aufgaben der Analysis situs.

b) Der Vortragende erzählt von zwei Aufgaben der Analysis Situs, zu denen er gelangt ist, und dann von der folgenden dritten, deren Lösung ihm nur für konvexe Kurven gelungen ist: Auf jeder einfach geschlossenen stetigen Kurve in der Ebene gibt es vier Punkte, welche ein Quadrat bilden. Diskussion: Die Herren Fueter, Speiser, Laemmel, Stäckel, Grossmann.

Here is an English translation:

b) On some problems in topology. [...]

b) The speaker talks about two problems in topology that he obtained, and then about the following third one, whose solution he managed to find only for convex curves: On every simple closed continuous curve in the plane there are four points that form a square. Discussions: Messrs. Fueter, Speiser, Laemmel, Stäckel, Grossmann.

It seems that Toeplitz never published a proof. In 1913 Arnold Emch [6] presented a proof for "smooth enough" convex curves. Two years later Emch [7] published a further proof that requires a weaker smoothness condition. However, he did not note that the special case of smooth convex curves already implies by a limit argument that all convex curves inscribe squares. In a third paper from 1916, Emch [8] proved the Square Peg Problem for curves that are piecewise analytic with only finitely many inflection points and other singularities where the left- and right-side tangents at the finitely many nonsmooth points exist.

Emch states in his second paper [7] that he was not aware of Toeplitz's and his students' work and that the problem was suggested to him by Kempner. From 1906 to 1913 Toeplitz was a postdoc in Göttingen. Aubrey J. Kempner was an English mathematician who finished his Ph.D. with Edmund Landau in Göttingen in 1911. Afterwards he went to the University of Illinois in Urbana-Champaign and stayed there until 1925 according to http://www.maa.org/history/presidents/kempner.html (another biography of Kempner can be found at http://www.findagrave.com/cgi-bin/fg.cgi?page=gr&GRid=13165695, which claims different dates). Emch joined the faculty of the same university in 1911.

I will let the reader decide whether this is enough information on how all these parts fit together and who considered the Square Peg Problem first. It is usually attributed to Toeplitz.

In 1929 Schnirelman proved the Square Peg Problem for a class of curves that is slightly larger than C^2 . An extended version [37] which also corrects some minor errors was published

posthumously in 1944. Guggenheimer [12] states that the extended version still contains errors, which he claims to correct. However, in my point of view, Schnirelman's proof is correct except for some minor errors. His main idea is a bordism argument; below we give some details. Since the transversality machinery was not invented at this time, Schnirelman's proof contains many computations in explicit coordinates. Guggenheimer's main lemma, on the other hand, admits counterexamples; he was not aware that squares can vanish pairwise when one deforms the curve.

Other proofs are due to Hebbert [14] when γ is a quadrilateral, Zindler [43] and Christensen [3] for convex curves, Jerrard [16] for analytic curves, Nielsen-Wright [31] for curves that are symmetric across a line or about a point, Stromquist [38] for locally monotone curves, Vrećica-Živaljević [40] for Stromguist's class of curves, Pak [32] for piecewise linear curves, Sagols-Marín [35], [36] for similar discretizations, Cantarella-Denne-McCleary [2] for curves with bounded total curvature without cusps and for C^1 -curves, Makeev [23] for star-shaped C^2 -curves that intersect every circle in at most 4 points (more generally he proved the Circular Quad Peg Problem 9 for such curves, see below), Matschke [26] for a technical open and dense class of curves and for continuous curves in certain bounded domains. In the next section we shall review some of these special cases in more detail.

Pettersson, Tverberg, and Östergård [33] have the latest result, which uses a computer: They showed that any Jordan curve in the 12×12 square grid inscribes a square whose size is at least $1/\sqrt{2}$ times the size of the largest axis-parallel square that fits into the interior of the curve.

Special Cases

Let us discuss some of the above-mentioned proofs in more detail.

Emch's Proof

Let $\gamma:S^1\hookrightarrow\mathbb{R}^2$ be the given piecewise analytic curve. Fixing a line τ , Emch considers all secants of γ that are parallel to τ and calls the set of all midpoints of these secants the set of *medians* M_τ . Under some genericity assumptions he proves that for two orthogonal lines τ and τ^\perp , M_τ intersects M_{τ^\perp} in an *odd* number of points. Nowadays one could write this down homologically. These intersections correspond to inscribed rhombi, where the two intersecting secants are the two diagonals of the rhombus.

Now he rotates τ continuously by 90 degrees and argues that $M_{\tau} \cap M_{\tau^{\perp}}$ moves continuously, where at finitely many times two intersection points can merge and disappear or two new intersection

points can appear. When τ is rotated by 90 degrees, the one-dimensional family of intersection points closes up to a possibly degenerate union of circle components.

Since $M_{\tau} \cap M_{\tau^{\perp}}$ is odd, Emch argues that an odd number of these components must be $\mathbb{Z}/4\mathbb{Z}$ -invariant, meaning that if $R_1R_2R_3R_4$ is a rhombus in such a component, then $R_2R_3R_4R_1$ must also be in the same component. By the mean value theorem, when moving from $R_1R_2R_3R_4$ to $R_2R_3R_4R_1$ along a component of inscribed rhombi, at some point the diagonals must have equal length. That is, we obtain an inscribed square. This argument also implies that the number of inscribed squares is (generically) odd for Emch's class of curves.

Schnirelman's Proof

Schnirelman solved the Square Peg Problem for a slightly larger class than C^2 using an early bordism argument that yields a very conceptual proof. His idea was that the set of inscribed squares can be described as a preimage, for example, in the following way: Let $\gamma: S^1 \hookrightarrow \mathbb{R}^2$ be the given curve. The space $(S^1)^4$ parameterizes quadrilaterals that are inscribed in γ . We construct a so-called *test-map*,

$$(1) f_{\gamma}: (S^1)^4 \to \mathbb{R}^6,$$

which sends a 4-tuple (x_1, x_2, x_3, x_4) of points on the circle to the mutual distances between $y(x_1), \ldots, y(x_4) \in \mathbb{R}^2$. Let V be the 2-dimensional linear subspace of \mathbb{R}^6 that corresponds to the points where all four edges are of equal length and the two diagonals are of equal length. The preimage $f_{\gamma}^{-1}(V)$ is parameterizing the set of inscribed squares, plus a few "degenerate components". The degenerated components consist of points where $x_1 = x_2 = x_3 = x_4$ —these are the degenerate squares—and more generally of 4-tuples where $x_1 = x_3$ and $x_2 = x_4$.

Now Schnirelman argues as follows: An ellipse inscribes exactly one square up to symmetry. Now deform the ellipse (via some smooth isotopy) into the given curve along other curves γ_t , $t \in [0,1]$. By smoothness these inscribed squares do not come close to the degenerate quadrilaterals during the deformation; that is, they do not shrink to a point. Thus the nondegenerate part of all preimages $f_{\gamma_t}^{-1}(V)$ forms a 1-manifold that connects the solution sets for γ and the ellipse, and since 1-manifolds always have an even number of boundary points, the parities of the number of inscribed squares on γ and on the ellipse coincide.

Thus, any smooth curve inscribes generically an odd number of squares. Here we have swept technical arguments concerning transversality under the rug, which we hope is appreciated by the reader.

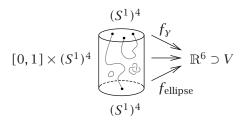


Figure 3. The bordism between the solution sets for γ and the ellipse. To simplify the figure we already modded out the symmetry group of the square and omitted the degenerate components.



Figure 4. Example of a piece of a locally monotone curve. Note that Figure 1 is not locally monotone because of the spiral.

For general curves, it is difficult to separate the degenerate quadrilaterals in $f^{-1}(V)$ from the squares we are interested in. This is the basic reason why the Square Peg Problem could not be solved completely with the current methods.

Stromquist's Criterion

Stromquist's class of curves for which he proved the Square Peg Problem is very beautiful, and it is the second strongest one: A curve $\gamma:S^1 \to \mathbb{R}^2$ is called *locally monotone* if every point $x \in S^1$ admits a neighborhood U and a linear functional $\ell: \mathbb{R}^2 \to \mathbb{R}$ such that $\ell \circ \gamma|_U$ is strictly monotone.

Theorem 2 (Stromquist). Any locally monotone embedding $\gamma: S^1 \to \mathbb{R}^2$ inscribes a square.

In his proof Stromquist also considers the set of inscribed rhombi first.

Fenn's Table Theorem

A beautiful proof for convex curves is due to Fenn [9]. It follows as an immediate corollary from his table theorem.

Theorem 3 (Fenn). Let $f: \mathbb{R}^2 \to \mathbb{R}_{\geq 0}$ be a non-negative function that is zero outside a compact convex disc D and let a > 0 be an arbitrary real number. Then there exists a square in the plane with side length a and whose center point belongs to D such that f takes the same value on the vertices of the square.

As the reader might guess, Fenn's proof basically uses a mod-2 argument showing that the number of such tables is generically odd.

The table theorem implies the Square Peg Problem for convex curves γ by constructing a

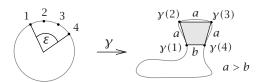


Figure 5. A special trapezoid of size ε .

height function $f: \mathbb{R}^2 \to \mathbb{R}_{\geq 0}$ whose level sets $f^{-1}(x)$ are similar to γ for all x > 0.

Zaks [42] found an analogous "chair theorem", where instead of a square table he considers triangular chairs with a fixed direction. Kronheimer-Kronheimer [22] found conditions on ∂D such that the table/chair can be chosen such that all four/three vertices lie in D: namely, ∂D should not inscribe a square/triangle of a smaller size. More table theorems are due to Meyerson [28].

An Open and Dense Criterion

In [26] the Square Peg Problem was proved for the so far weakest smoothness condition.

Theorem 4. Let $y: S^1 \to \mathbb{R}^2$ be a Jordan curve. Assume that there is $0 < \varepsilon < 2\pi$ such that y contains no (or generically an even number of) special trapezoids of size ε . Then y inscribes a square.

Here an inscribed *special trapezoid* is a 4-tuple of pairwise distinct points $x_1, ..., x_4 \in S^1$ lying clockwise on S^1 such that the points $P_i := y(x_i)$ satisfy

$$||P_1 - P_2|| = ||P_2 - P_3|| = ||P_3 - P_4|| > ||P_4 - P_1||$$
 and

$$||P_1 - P_3|| = ||P_2 - P_4||.$$

The *size* of this special trapezoid is defined as the length of the clockwise arc in S^1 from x_1 to x_4 .

The set of curves without inscribed special trapezoids of a fixed size ε is open and dense in the space of embeddings $S^1 \hookrightarrow X$ with respect to the compact-open topology. This theorem is basically the exact criterion that one obtains by applying equivariant obstruction theory to the test-map (1). Vrećica and Živaljević [40] are the first to apply obstruction theory to the Square Peg Problem, and they proved it for Stromquist's class of locally monotone curves.

An Explicit Open Criterion

All previous criteria on curves for which the Square Peg Problem was proved are defined by local smoothness conditions. The following criterion from [26] is a global one which yields an open set of not necessarily injective curves in $C^0(S^1, \mathbb{R}^2)$ with respect to the C^0 -topology or, equivalently, the compact-open topology.

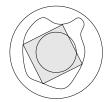


Figure 6. Example for Theorem 5.

Theorem 5. Let A denote the annulus $\{x \in \mathbb{R}^2 \mid 1 \le ||x|| \le 1 + \sqrt{2}\}$. Suppose that $y : S^1 \to A$ is a continuous closed curve in A that is nonzero in $\pi_1(A) = \mathbb{Z}$. Then y inscribes a square of side length at least $\sqrt{2}$.

It is open whether the outer radius $1 + \sqrt{2}$ of A can be increased by some small $\varepsilon > 0$.

The proof idea is very simple: If the annulus A is thin enough, then the set of squares with all vertices in A splits into two connected components: big squares and small squares. A generic curve that represents a generator of $\pi_1(A)$ inscribes an odd number of big squares (and an even number of small squares).

Related Problems

The Number of Inscribed Squares

Popvassilev [34] constructed for any $n \ge 1$ a smooth *convex* curve that has exactly n inscribed squares, every square being counted exactly once and not with multiplicity. All but one of the n squares in his construction are nongeneric. They will disappear immediately after deforming the curve by a suitable C^{∞} -isotopy.

In [26] this author gave the parity of the number of squares on generic smooth *immersed* curves in the plane, which depends not only on the isotopy type of the immersion but also on the intersection angles.

Van Heijst proves in his upcoming master's thesis [15] that any real algebraic curve in \mathbb{R}^2 of degree d inscribes either at most $(d^4-5d^2+4d)/4$ or infinitely many squares. For this he makes use of Bernstein's theorem, which states that the number of common zeros in $(\mathbb{C}^*)^k$ of k generic Laurent polynomials in k variables with prescribed Newton polytopes equals the mixed volume of these polytopes.

Inscribed Triangles

It is not hard to show that any smooth embedding $y: S^1 \to \mathbb{R}^2$ inscribes arbitrary triangles, even if we prescribe where one of the vertices has to sit. Moreover, the set of all such inscribed triangles determines a homology class $\alpha \in H_1(P_3, \mathbb{Z}) = \mathbb{Z}$, where P_3 is the set of 3-tuples of points on y that lie counterclockwise on the curve. The class

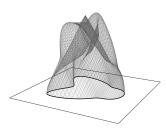


Figure 7. The image of f, a self-intersecting Möbius strip with boundary γ .

 α turns out to be a generator, as one sees from inspecting the situation for the circle.

For continuous curves Nielsen [29] proved the following version of the result:

Theorem 6 (Nielsen). Let T be an arbitrary triangle and $\gamma: S^1 \to \mathbb{R}^2$ an embedded circle. Then there are infinitely many triangles inscribed in γ which are similar to T, and if one fixes a vertex of smallest angle in T, then the set of the corresponding vertices on γ is dense in γ .

Inscribed Rectangles

Instead of squares one may ask whether any embedded circle in the plane inscribes a rectangle. If one does not prescribe the aspect ratio, then the answer is affirmative.

Theorem 7 (Vaughan). Any continuous embedding $y: S^1 \to \mathbb{R}^2$ inscribes a rectangle.

Vaughan's proof, which appeared in Meyerson [28], is very beautiful: $\mathbb{Z}_2 := \mathbb{Z}/2\mathbb{Z}$ acts on the torus $(S^1)^2$ by permuting the coordinates, and the quotient space $(S^1)^2/\mathbb{Z}_2$ is a Möbius strip. The proof of Theorem 7 uses the fact that the map $f: (S^1)^2/\mathbb{Z}_2 \to \mathbb{R}^2 \times \mathbb{R}_{>0}$ given by

$$f(x,y) = ((y(x) + y(y))/2, ||y(x) - y(y)||)$$

must have a double point; otherwise it would extend to an embedding of $\mathbb{R}P^2$ into \mathbb{R}^3 by gluing to that Möbius strip the disc $I \times \{0\}$, where $I \subset \mathbb{R}^2$ is the interior of γ . The double point corresponds to two secants of γ having the same length and the same midpoint. Hence this forms an inscribed rectangle.

If we furthermore prescribe the aspect ratio of the rectangle, then the problem is wide open, even for smooth or piecewise linear curves.

Conjecture 8 (Rectangular Peg Problem). Every C^{∞} embedding $\gamma: S^1 \to \mathbb{R}^2$ contains four points that are the vertices of a rectangle with a prescribed aspect ratio r > 0.

This conjecture is highly interesting, since the standard topological approach does not yield a proof: The equivariant homology class of the solution set, a \mathbb{Z} -valued smooth isotopy invariant

of the curve, turns out to be zero. For example, an ellipse inscribes a positive and a negative rectangle. Stronger topological tools fail as well. It seems again that more geometric ideas are needed.

Equivalently we could state Conjecture 8 for all piecewise linear curves. Proofs exist only for the case r=1, which is the *smooth* Square Peg Problem, for arbitrary r in case the curve is close to an ellipse, see Makeev [23] and Conjecture 9 below; and for $r=\sqrt{3}$ in case the curve is close to convex, see [26].

A proof for the Rectangular Peg Problem was claimed by Griffiths [10], but it contains errors regarding the orientations. Essentially, he calculated that the number of inscribed rectangles of the given aspect ratio counted with appropriate signs and modulo symmetry is 2. However, zero is correct.

Other Inscribed Ouadrilaterals

It is natural to ask what other quadrilaterals can be inscribed into closed curves in the plane. Since the unit circle is a curve, those quadrilaterals must be circular; that is, they must have a circumcircle.

Depending on the class of curves that we look at, the following two conjectures seem reasonable.

Conjecture 9 (Circular Quad Peg Problem). Let Q be a circular quadrilateral. Then any C^{∞} embedding $\gamma: S^1 \to \mathbb{R}^2$ admits an orientation-preserving similarity transformation that maps the vertices of Q into γ .

Makeev [23] proved a first instance of this conjecture, namely, for the case of star-shaped C^2 -curves that intersect every circle in at most 4 points.

Furthermore, Karasev [20] proved that, for any smooth curve and a given Q = ABCD, either this conjecture holds or one can find two inscribed triangles similar to ABC such that the two corresponding fourth vertices D coincide (but D may not lie on y). The proof idea is a beautiful geometric volume argument. It should be stressed that most open problems discussed here are geometric problems rather than topological ones: We understand the basic algebraic topology here quite well but not the restrictions on the topology that the geometry dictates. New geometric ideas such as Karasev's are needed.

Conjecture 10 (Trapezoidal Peg Problem). *Let T be an isosceles trapezoid. Then any piecewise-linear embedding* $y: S^1 \to \mathbb{R}^2$ *inscribes a quadrilateral similar to T.*

The reason for restricting the latter conjecture to isosceles trapezoids, that is, trapezoids with circumcircle, is that all other circular quadrilaterals cannot be inscribed into very thin triangles. This was observed by Pak [32].

Other Inscribed Polygons

For any n-gon P with $n \ge 5$ it is easy to find many curves that do not inscribe P. If we do not require all vertices to lie on γ , then Makeev has some results for circular pentagons; see [25].

Alternatively, we can relax the angle conditions; that is, we require only that the edge ratios are the same as the ones in a given polygon P. Then as for the triangles above, one can show that the set of such n-gons represents the generator of $H_1(P_n; \mathbb{Z}) = \mathbb{Z}$, where P_n is the set of n-tuples of points on γ that lie counterclockwise on the curve; see Meyerson [27], Wu [41], Makeev [25], and Vrećica-Živaljević [40].

Higher Dimensions

In higher dimensions one may ask whether any (n-1)-sphere that is smoothly embedded in \mathbb{R}^n inscribes an n-cube in the sense that all vertices of the cube lie on the sphere. However, most smooth embeddings $S^{n-1} \hookrightarrow \mathbb{R}^n$ do not inscribe an n-cube for $n \geq 3$, in the sense that these embeddings form an open and dense subset of all smooth embeddings in the compact-open topology, a heuristic reason being that the number of equations to fulfill is larger than the degrees of freedom. An explicit example is the boundaries of very thin simplices, as was noted by Kakutani [17] for n=3. Hausel-Makai-Szűcs [13] proved that the boundary of any centrally symmetric convex body in \mathbb{R}^3 inscribes a 3-cube.

If we do not want to require further symmetry on the embedding $S^{n-1} \hookrightarrow \mathbb{R}^n$, then crosspolytopes are more suitable higher analogs of squares: The regular n-dimensional crosspolytope is the convex hull of $\{\pm e_i\}$ where e_i are the standard basis vectors in \mathbb{R}^n .

Theorem 11 (Makeev, Karasev). Let n be an odd prime power. Then every smooth embedding Γ : $S^{n-1} \to \mathbb{R}^n$ contains the vertices of a regular n-dimensional crosspolytope.

The n=3 case was posed as Problem 11.5 in Klee and Wagon [21]. This was answered affirmatively by Makeev [24]. Karasev [19] generalized the proof to arbitrary odd prime powers. Akopyan and Karasev [1] proved the same theorem for n=3 in case Γ is the boundary of a simple polytope by a careful and nontrivial limit argument from the smooth case.

Gromov [11] proved a similar theorem for inscribed simplices.

Theorem 12 (Gromov). Any compact set $S \subset \mathbb{R}^d$ with C^1 -boundary and nonzero Euler characteristic inscribes an arbitrary given simplex up to similarity on its boundary ∂S .



Figure 8. Intuition behind Conjecture 13: Think of a square table for which we want to find a spot on Earth such that all four table legs are at the same height.

Let us finish with the following table problem on the sphere.

Conjecture 13 (Table problem on S^2). Suppose $x_1, ..., x_4 \in S^2 \subset \mathbb{R}^3$ are the vertices of a square that is inscribed in the standard 2-sphere, and let $h: S^2 \to \mathbb{R}$ be a smooth function. Then there exists a rotation $\rho \in SO(3)$ such that $h(\rho(x_1)) = \cdots = h(\rho(x_4))$.

So far this result has been proven only when $x_1, ..., x_4$ lie on a great circle (see Dyson [5]), since this is the only case in which the generic number of solutions is odd. The critical points of h can be thought of as the spots on which you can put an infinitesimally small table.

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OpenFOAM for Computational Fluid Dynamics

Goong Chen, Qingang Xiong, Philip J. Morris, Eric G. Paterson, Alexey Sergeev, and Yi-Ching Wang

Introduction

There is a revolution going on, impacting and transforming how computational mechanics and the associated design and optimization are done: the emergence, availability, and large-scale use of *OpenFOAM* [1]. It belongs to the contemporary *open-source trend* not unlike the roles played by the Linux operating system or the Internet encyclopedia Wikipedia. OpenFOAM is free and is used by thousands of people worldwide in both academic and industrial settings. The acronym OpenFOAM stands for *Open Source Field Operation and Manipulation*.

Computational mathematics and mechanics provide fundamental methods and tools for simulating physical processes. Numerical computation can offer important insights and data that are either

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difficult or expensive to measure or to test experimentally. What is more, numerical computation can simulate supernova explosions and galaxy formations, which cannot be produced in earthbound laboratories. It has been recognized for at least thirty years that *computational science* constitutes a third and independent branch of science on equal footing with *theoretical and experimental sciences*. Cutting across disciplines at the center of computational science is *computational fluid dynamics* (CFD), which makes up the core of OpenFOAM and is the focus of this article.

In the early days of CFD research and development, computer programs ("codes") were primarily developed in universities and national laboratories. Many of these efforts had lifetimes of ten to twenty years and involved numerous Ph.D. students and postdoctoral associates. Fueled by intense Ph.D.level research, those early codes provided the basis of modern CFD knowledge. However, this model of development had several flaws. The constant turnover of personnel in academic research groups created serious continuity problems, especially if the faculty advisor or group leader was not managing the code architecture. Another challenge was that Ph.D. students and postdocs in engineering and mathematics were often self-taught programmers, which meant that most of the codes were suboptimal programs. Those student-written research codes often became notorious as "spaghetti code", which was hard to extend to new physics or new parallel high-performance computing architectures without extraordinary effort. Finally, because such a significant amount of time and financial resources had been invested in the development, those codes were usually proprietary and rarely made available to the public except to those in the extended academic family of the leader.

If the researcher is not a CFD code developer, then most of the time the only alternative is to buy and use commercial CFD software packages. There are now many such CFD packages (see, e.g., those listed in [2], though this list is not exhaustive). License fees for commercial software typically range from US\$10,000 to US\$50,000 per year depending on the "added extras", the number of users, whether multiple licenses are required for parallel computation, and the commercial or academic nature of the license. This is not inexpensive. For a faculty member who doesn't have a research grant or is retired, the cost is generally prohibitive.

As long as the Internet has existed, there has been free and open-source software available to download and share. However, over the past decade, the level of sophistication and quality of open-source software has significantly grown, largely aided by the move to *object-oriented programming* and online version-control repositories (e.g., SourceForge [3], GitHub [4]). As in the early days, much of this software finds its roots in academia and national laboratories.

OpenFOAM was born in the strong British tradition of fluid dynamics research, specifically at The Imperial College, London, which has been a center of CFD research since the 1960s. The original development of OpenFOAM was begun by Prof. David Gosman and Dr. Radd Issa, with principal developers Henry Weller and Dr. Hrvoje Jasak. It was based on the finite volume method (FVM) [5], an idea to use C^{++} and object-oriented programming to develop a syntactical model of equation mimicking (see Box 2) and scalar-vectortensor operations. A large number of Ph.D. students and their theses have contributed to the project. Weller and Jasak founded the company Nabla Ltd., but it was not successful in marketing its product, FOAM (the predecessor of OpenFOAM), and folded in 2004. Weller founded OpenCFD Ltd. in 2004 and released the GNU general public license of OpenFOAM software. OpenFOAM constitutes a C^{++} CFD toolbox for customized numerical solvers (over sixty of them) that can perform simulations of basic CFD, combustion, turbulence modeling, electromagnetics, heat transfer, multiphase flow, stress analysis, and even financial mathematics modeled by the *Black-Scholes equation*. In August 2011, OpenCFD was acquired by Silicon Graphics International (SGI). In September 2012, SGI sold OpenCFD Ltd to the ESI Group.

While OpenFOAM may be the first and most widely adopted open-source computational mechanics software, there indeed are other examples. A few are briefly mentioned here. They include *deal.ii* [6], a finite-element Differential Equations Analysis Library, which originally emerged from

work at the Numerical Methods Group at Universität Heidelberg, Germany, and today it is a global opensource project maintained primarily at Texas A&M University, Clemson University, and Universität Heidelberg and has dozens of contributors and several hundred users scattered around the world.

The Stanford University Unstructured (SU2) [7] suite is an open-source collection of C^{++} -based software tools for performing partial differential equation (PDE) analysis and solving PDE constrained optimization problems. The toolset is designed with computational fluid dynamics and aerodynamic shape optimization in mind, but is extensible to treat arbitrary sets of governing equations such as potential flow, electrodynamics, chemically reacting flows, and many others. SU2 is under active development in the Aerospace Design Lab (ADL) of the Department of Aeronautics and Astronautics at Stanford University and is released under an open-source license.

Méfisto [8], 3D finite element software for numerical solutions of a set of boundary value problems, has been posted by Prof. Alain Perronnet of the Laboratoire Jacques-Louis Lions at the Université Pierre et Marie Curie in Paris, France, who is a long-time collaborator with the first author of this article.

Another open-source software package for CFD or PDEs includes *MFIX* (Multiphase Flows with Interface eXchanges), developed by the National Energy Technology Laboratory (NETL) of the Department of Energy [9], suitable for hydrodynamics, heat transfer, and chemical reactions in fluid-solid systems. It is based on the finite volume method and written in Fortran.

Still more open-source *finite element* softwares such as FEniCS, FreeFem++, etc., can be found in [37]. Nevertheless, most of their primary emphases are not built for the purpose of CFD.

The revenue and survival strategy of the company OpenCFD Ltd. (which has been absorbed into ESI Group), is a "Redhat model" [10] providing support, training, and consulting services. While OpenFOAM is open-source, the development model is a "cathedral" style [11] where code contributions from researchers are not accepted back into the main distribution due to strict control of the code base. For researchers who want to distribute their developments and find other online documentation, there are a community-oriented discussion forum [12], a wiki [13], and an international summer workshop [14].

Now, with the open-source libraries in Open-FOAM, one does not have to spend one's whole career writing CFD codes or be forced to buy commercial softwares. Many other users of OpenFOAM have developed relevant libraries and solvers that are either posted online or may be requested for

```
// define field scalar u and f
volVectorField u, f;
// construct the Laplacian equation and solve it
solve
(
fvm::laplacian(u) == f
);
```

Box 1. OpenFOAM code for the potential equation (1).

free. The number of OpenFOAM users has been steadily increasing. It is now estimated to be of

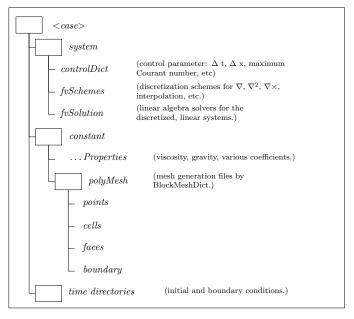


Chart 1. Case directory structure (adapted from

the order of many thousands, with the majority of them being engineers in Europe. But the U.S. is catching up.

A Sketch of How to Use OpenFOAM

For beginners who are enthusiastic about learning how to use OpenFOAM to obtain CFD solutions the best way is to study the many tutorial examples available in [1]. One such tutorial is the lid-driven cavity case [15]. (Such a case will also be computed in Examples 1 and 2.) It provides nearly all information *from start to finish* as to how to use OpenFOAM, including *preprocessing, solving* (i.e., how to run the codes), and *postprocessing*. The tutorial has a couple dozen pages. If the beginner can get some help from an experienced OpenFOAM user, then it usually takes only a few weeks to run a simple OpenFOAM computer program for this problem.

Box 2. OpenFOAM code for the N-S equation (3). Here, as well as in Box 1, equation mimicking is quite obvious. Note that the specifications fvm and fvc are selected by the user from the fvSchemes dictionary in the system dictionary; cf. Chart 1. Here fvm::laplacian means an implicit finite volume discretization for the Laplacian operator, and similarly for fvm::div for the divergence operator. On the other hand, fvc::grad means an explicit finite volume discretization for the gradient operator. The parentheses (,) means a product of the enclosed quantities, including tensor products.

As most readers may not necessarily be interested in running OpenFOAM codes for now, in this section we will mainly give a brief sketch. We first illustrate this for a simple elliptic boundary value problem

(1)
$$\begin{cases} \text{ (i)} & \nabla^2 u(x, y, z) = f(x, y, z) \text{ on } \Omega \subseteq \mathbb{R}^3, \\ \text{ (ii)} & u(x, y, z) = g(x, y, z) \text{ on the boundary } \partial\Omega. \end{cases}$$

In OpenFOAM, one can use a Laplacian solver in the heat transfer library to obtain numerical solutions. By using the C^{++} language, in OpenFOAM (1)(i) is written as in Box 1.

Note that the inhomogeneous Dirichlet boundary condition, given in (1)(ii), will be prescribed elsewhere, in the "time directories" in the Case Directory Structure, as shown in Chart 1. If instead of (1)(ii) we have inhomogeneous Neumann or Robin boundary conditions such as

$$\frac{\partial u(x,y,z)}{\partial n} = g(x,y,z),$$
(2)
$$\frac{\partial u(x,y,z)}{\partial n} + \alpha u(x,y,z) = g(x,y,z) \text{ on } \partial\Omega,$$
they can be experified similarly in the time.

they can be specified similarly in the time directories.

To numerically solve a PDE by using OpenFOAM, a user needs to create a Case Directory Structure as shown in Chart 1. Normally it contains three subdirectories. The user first gives a name for the <case>. The compositions of the various subdirectories are indicated in Chart 1.

Now we look at the core case of this article, the incompressible Navier-Stokes (N-S) equations in

CFD. The governing equations are

(3)
$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) - \mu \nabla^2 \mathbf{u} = -\nabla p + \mathbf{f}(x, y, z, t),$$
(4)
$$\nabla \cdot \mathbf{u} = 0.$$

Note that in (2), **uu** is defined to be the 3×3 matrix

$$uu = [u_i u_i]_{3 \times 3}$$
.

One can specify the given initial and boundary conditions on \mathbf{u} in the "time directories" of Chart 1.

An effective algorithm for solving the coupled system (3) and (4) is the PISO (pressure-implicit with splitting of operators) algorithm of Issa [16]; see also [17]. In OpenFOAM, basically (3) is written as shown in Box 2.

With some details, the PISO algorithm is implemented in OpenFOAM as shown in Box 3. This (largely) takes care of the *equation solving* step.

For *preprocessing* involving *mesh generation*, one can use the utility *blockMesh*, supplied in OpenFOAM, to first generate a rectangular mesh for a cubic domain. The input data consists of coordinates of eight vertices of the cube and numbers of cells in each direction, (n_x, n_y, n_z) . The output is a rectangular mesh containing $n_x \times n_y \times n_z$ cells. In case of more complicated geometry, one can use either the *snappyHexMesh* utility or third-party packages such as Gambit meshing software [18], with subsequent conversion into OpenFOAM format.

Finally, for *postprocessing*, to produce graphical output [19] OpenFOAM uses an open-source, multiplatform data analysis and visualization application called *ParaView* [20]. Alternatively, one can also use third-party commercial products such as *EnSight* [21].

As opposed to a monolithic solver as typically seen in commercial software, *pisoFoam* is one of seventy-six *standard solvers* that are included in the OpenFOAM distribution. These solvers are tailored to specific physics in the broad categories of combustion, compressible flow, discrete methods, electromagnetics, financial, heat transfer, incompressible flow, Lagrangian particle dynamics, multiphase flow, and stress analysis. There are also eighty-plus *standard utilities* for pre- and postprocessing of data, parallel computing, and mesh creation and manipulation. For all of these different programs, the burden is on the user to verify that the implemented physics and models match their needs and intended application.

Turbulence Modeling and Examples

In addition to solvers and utilities, OpenFOAM is distributed with numerous *standard libraries*. These libraries address both numerical algorithms

```
//define field vector fluid velocity u and f, face flux phi,
     and pressure p
volVectorField u, f;
volScalarField p;
surfaceScalarField phi:
//define constant parameter fluid dynamical viscosity nu
scalarField nu:
//construct the fluid velocity equation
fvVectorMatrix UEqn (
fvm::ddt(u) + fvm::div(phi, u) - fvm::laplacian(nu, u) - f)
//solve the momentum equation using explicit pressure
solve (
UEqn == -fvc::grad(p))
//predict the intermediate fluid velocity to calculate face flux
volVectorField rUA = 1.0/UEqn.A();
u = rUA *UEqn.H();
phi = fvc::interpolate(u) & mesh.Sf();
//construct the pressure equation using the constraint
     from continuity equation
fvScalarMatrix pEqn (
fvm::laplacian(rUA,p) == fvc::div(phi))
//correct the fluid velocity by the post-solve pressure
     and update face flux
u = u - rUA*fvc::grad(p);
                                                            (b)
phi = phi - pEqn.flux();
```

Box 3. The OpenFOAM code to solve the N-S equation of incompressible fluid. Note that the codes from lines (a) to (b) implement the PISO algorithm [16], [17].

and basic physics, the latter of which includes thermophysical properties, reaction models, radiation models, chemistry models, liquid properties, and turbulence modeling. For this article, we focus on turbulence modeling due to its universal nature in CFD.

Since exact solutions to the N-S equations are mostly unavailable, we need to rely on numerical methods to find approximate solutions. The computation of such solutions without the introduction of any additional approximations, except those associated with the numerical algorithms is called *Direct Numerical Simulation (DNS)*. From a mathematical viewpoint this would seem to be most logically sound, as mathematicians and many other theoreticians normally desire great purity by being truly faithful to the model of problems under treatment. The great majority of mathematical, numerical analysis papers published are of the DNS type.

However, numerical solutions obtained using DNS are of quite limited usefulness. The reason is that fluids exhibit *turbulent behavior*. Turbulence is characterized by *rapid and irregular fluctuations* in the fluid properties with a wide range of length and time scales. A typical occurrence of turbulence is displayed in Figure 1.

The transition of flow from laminar or smooth to turbulent or irregular is determined by the

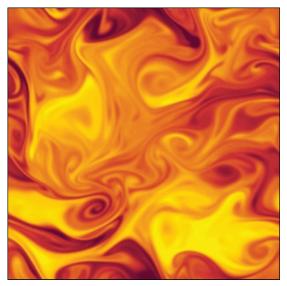


Figure 1. Flow characteristics of turbulence. One can see many swirls (i.e., eddies) of various scales and rapid, irregular fluctuations across the region. High resolutions are required for any digital and numerical processing (and also visual display) of turbulence. This signifies an underlying reason why computation is typically large-scale and challenging for turbulent flow.

(Excerpted from [22], courtesy of Dr. G. Brethouwer of the Department of Mechanics, KTH, Stockholm, Sweden.)

Reynolds number. For example, in a pipe, transition occurs at a Reynolds number of approximately 2300, where the Reynolds number in this case is defined as

(5)
$$Re = \frac{\rho U d}{\mu},$$

where ρ , μ , d, and U are the fluid density, molecular viscosity, pipe diameter, and average velocity, respectively. Beyond Re = 4000 the flow is fully turbulent. The range of length and time scales in a turbulent flow depends on the Reynolds number. Kolmogorov [23] argued that the smallest scales of turbulence should be independent of the largest scales. Dimensional analysis then gives the smallest spatial and time scale, respectively, as

(6)
$$\eta = (\frac{v^3}{\epsilon})^{\frac{1}{4}}, \ \tau_{\eta} = (\frac{v}{\epsilon})^{\frac{1}{2}},$$

where ν and ϵ are the fluid kinematic viscosity and average viscous dissipation rate of turbulent energy per unit mass. For turbulence in equilibrium the rate of viscous dissipation at the smallest scales must equal the rate of supply of energy from the large scales. That is, $\epsilon \sim U^3/L$, where U and L are the largest velocity and length scales of the

turbulence. This gives

(7)
$$\frac{L}{\eta} \sim \left(\frac{UL}{V}\right)^{3/4} = Re^{3/4}.$$

Thus, to simulate all scales of motion in a turbulent flow, the grid size increases as $Re^{9/4}$. Since the Reynolds number in flows of engineering interest are of the order of 10^5 (or much higher in geophysical flows), DNS is of little use in such problems.

To overcome this limitation researchers have resorted to different levels of approximation. This is referred to as turbulence modeling. A comprehensive coverage of turbulent flows and turbulence modeling is given by Pope [24]. The two most widely used approaches are Reynolds-averaged N-S (RANS) simulations and large eddy simulation (LES); cf., e.g., [24], [5, Sections 3.7 and 3.8]. RANS methods are based on the time or ensemble averaged N-S equations. This process results in the appearance of additional terms involving the average of products of the fluctuating velocity, referred to as *Reynolds* stresses. (Additional terms arise in compressible turbulent flows where the density fluctuates.) Equations must be developed to describe the Reynolds stresses, of which there are six independent components. These are generally differential equations. The majority of RANS models are based on the concept of an eddy viscosity. This is a diffusion coefficient, equivalent to the kinematic viscosity of the fluid, that describes the turbulent mixing or diffusion of momentum. It involves the product of a characteristic turbulent velocity and length scale. Two-equation turbulence models, such as the $k - \varepsilon$ and $k - \omega$ models [5, pp. 72-93], provide these scales. Here *k* is the turbulent kinetic energy per unit mass, ω is the specific dissipation rate, and ε was defined earlier. It should be noted that though exact equations can be developed from the equations of motion for these quantities, additional unknown terms arise that must be modeled. Two other RANS approaches should be mentioned. The first is the one-equation Spalart-Allmaras model [5, pp. 89-90]. This involves a differential equation for the eddy viscosity. It was developed specifically for external aerodynamics problems and is not based on modeling terms in the exact equations but on a more general phenomenological approach. The second approach uses *Reynolds stress models*. These involve equations (including modeled terms) for the individual Reynolds stress components.

RANS methods involve empirical models with numerous coefficients that must be specified. In general, these coefficients are valid within a particular class of turbulent flow, for example, wall-bounded or free shear flows. This is because the turbulent mixing is controlled by the large-scale turbulent motions that differ from one class of

Turbulence models	Advantages	Disadvantages		
	Most accurate. Doesn't need	Highly computationally expen-		
DNS	empirical correlations. Capable	sive. Difficult to include accurate		
	of characterizing all the flow	initial and boundary conditions		
	details.	for engineering applications.		
	Capable of capturing the dynam-	Still computationally intensive.		
LES	ics of the dominant eddies in the	Some difficulties in representing		
	system. Relatively more economi-	flow in complex geometries.		
	cal than DNS. More accurate than			
	RANS.			
RANS	Suitable for engineering prob-	Incapable of capturing flow		
	lems. Computational cost is	details. High dependence on		
	modest.	empirical correlations.		
DES	Suitable for engineering prob-	Incapable of capturing flow		
	lems. Captures unsteadiness in	details in near-wall region.		
	separated flows. More generally			
	applicable than RANS.			

Table 1. Comparisons among DNS, LES, RANS, and DES.

flow to another. A more general approach is LES mentioned earlier, which is based on a spatial average of the N-S equations using a box, Gaussian, or spectral cutoff filter. The action of turbulent scales smaller than the cutoff scale is modeled using a *subgrid scale SGS* model. This is usually in the form of an eddy viscosity model that can involve a constant or dynamic coefficient. In the latter case the eddy viscosity or Smagorinsky constant is allowed to vary in space and time and is calculated based on two filterings of the flow variables. Some averaging is generally required for stability. This could be averaging in a homogeneous flow direction or a local spatial average. LES methods are still computationally expensive, though not as much as DNS. This is especially true for wall-bounded turbulent flows, since the "large" scales close to the wall can be very small. An efficient solution to this problem is the use of *hybrid RANS/LES* models. An example is the *Detached Eddy Simulation (DES)*. In DES the turbulence model equations behave as RANS equations in the near-wall region but transition to LES away from the walls. Clearly such solutions cannot simulate the details of the turbulence in the near-wall regions. We refer the reader to Table 1 for a comparison among these basic turbulence models.

OpenFOAM offers all the turbulence modeling methods described here either as standard solvers or libraries for users to simulate turbulence on the proper spatial and temporal scales.

In the following, to compare the simulation capability of the three basic turbulence modeling strategies, i.e., DNS, LES, and RANS, a simple *lid-driven* flow is simulated in both two- (2D) and three-dimension (3D) in Examples 1 and 2. The lid-driven cavity flow [25] is a classical test problem

Length <i>L</i>	0.1 m		
Height <i>H</i>	0.1 m		
Width W	0.1 m		
Kinematic viscosity $ u$	$0.00001 \text{ m}^2\text{s}^{-1}$		
Lid velocity U	1 m/s		
Grid length in RANS, LES and DNS Δx	0.005 m, 0.001 m, 0.0002 m		
Unit time step in RANS, LES and DNS Δt	0.005 s, 0.001 s, 0.0002 s		

Table 2. Physical and geometrical parameters for the lid-driven flow simulation.

for N-S codes and benchmarks. Its geometry and boundary conditions are indicated in Figure 2.

The parameter values of this problem are summarized in Table 2. The turbulent viscosity submodels chosen for RANS and LES are the standard $k - \varepsilon$ model and k-equation eddy-viscosity

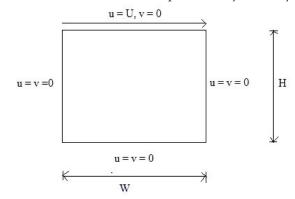
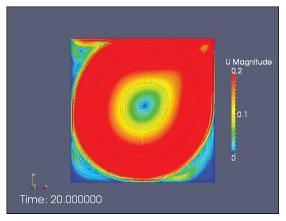
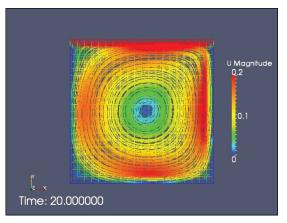


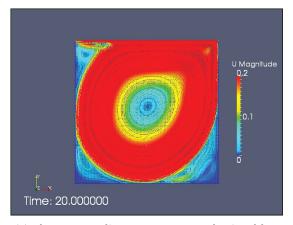
Figure 2. Geometry and boundary conditions for a 2D lid-driven cavity, where (u,v) are the components of flow velocity. The case for 3D is similar. Note that the upper "lid" has a constant horizontal velocity U. Note, however, for the 3D DNS computations, because a huge memory space and CPU time are required, the domain has been reduced to the size of $[0, 0.1] \times [0, 0.1] \times [0, 0.01]$.



(a) Flow streamlines at t=20 sec obtained by DNS [26].

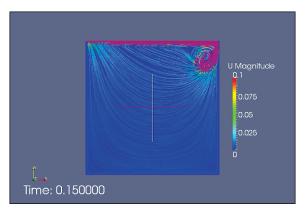


(b) Flow streamlines at t=20 sec obtained by RANS [27].

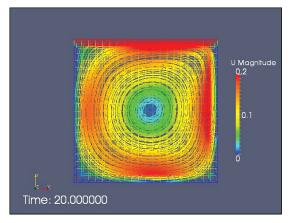


(c) Flow streamlines at t=20 sec obtained by LES [28].

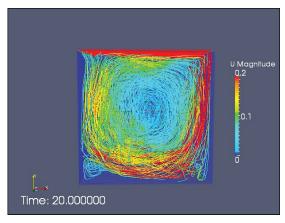
Figure 3. 2D lid-driven flow calculations by OpenFOAM.



(a) Flow field at t=0.15 sec obtained by DNS [26]. (Note that the domain here is $[0, 0.1] \times [0, 0.01]$, not a cube as in subcases (b) and (c).)



(b) Flow field at t=20 sec obtained by RANS [27].



(c) Flow field at t=20 sec obtained by LES [28].

Figure 4. 3D lid-driven flow calculations by OpenFOAM. Note that the snapshot of the DNS case in part (a) is at t=0.15, while those in parts (b) and (c) are at t=20. One can rank the richness of fine features of flow in the order of (a), (c) and (b).

model, respectively, in OpenFOAM [27], [28]. Wall functions [5, pp. 76-78] are applied to turbulent viscosity at all wall types. Computations for Examples 1-3 were run on the Texas A&M Supercomputing Facility's Eos, an IBM iDataPlex Cluster 64-bit Linux with Intel Nehalem processors.

Example 1 (2D lid-driven flow). Graphical results are displayed in Figure 3. The numerical data agree favorably with those in the literature (but we omit the details of comparisons here for lack of space).

Example 2 (3D lid-driven flow). The 3D DNS requires huge resources. Here we used 1024 cores for parallel computing at the TAMU Supercomputing Facility to run this case. It took sixty-four hours to run for the numerical simulation for just 0.15 second. The streamline flow pattern computed by DNS at t=0.15 can be seen in Figure 4, part (a).

For 3D RANS and LES computations, we are able to compute flow fields up to t=20 sec; see their flow patterns in parts (b) and (c) of Figure 4.

To *visualize* the *dynamics* of fluid motion, we have made three short animation videos. The reader can see the dynamic motion of the fluid computed by DNS by clicking on (or pasting) https://www.dropbox.com/s/htoms253d3ckt0n/DNS3Dstreamline2.avi/, while that computed by OpenFOAM RANS, containing two different graphical representations, field and streamlines, can be viewed at https://www.dropbox.com/s/6cwjsdxrmcnud3o/RANS3Dfiledstreamline.wmv/. The counterpart, computed by OpenFOAM LES, can be seen at https://www.dropbox.com/s/6hzz3ct1wljur9n/LES3Dfiledstreamline.wmv/.

Example 3 (Flow field of a Grumman F-14 Tomcat fighter). Examples 1 and 2 involve simple geometry and are intended for easy understanding of simple flow patterns and possible benchmarking. Here we present an example with a more complicated geometry, that of a Grumman F-14 Tomcat [29]. We chose this model for the study of landing gear and airframe noise [30], as the grid-generation for the aircraft body had already been performed and was available on the Internet [31] free of charge. We redacted it in [32].

The aircraft is flying in a headwind of 70 m/sec. The kinematic viscosity of air is chosen to be 1.48×10^{-5} m²/sec. The OpenFOAM solver PimpleFOAM [33] is used. Four processors of TAMU's Supercomputer were used, taking close to four hours of computing time. The flow profile at time=1.2 sec. is plotted in Figure 5.

To visualize the dynamic motion of air flow, go to https://www.dropbox.com/s/uuhnesg2pxy6c3i/fjet-udiff-vol-g.avi/.

We note that numerical results computed here should not be accepted too literally as correct or accurate. As a rule, such results should be subject to the scrutiny of model selection criteria, convergence test and error and multiscales analysis, be validated against experiments (such as wind tunnel data, if available), and be corroborated with those obtained from other numerical schemes.

Concluding Remarks

The "open-source" nature of OpenFOAM is of fundamental importance but is not its *only* beneficial feature. There are many advantages to using OpenFOAM:

- codes are extensible for many customized applications;
- the generality of the various OpenFOAM libraries and solvers empowers the user to solve nearly all CFD problems comprehensively;
- the pre- and postprocessing interfaces are well designed, powerful, and user friendly;
- dynamic meshes (moving grids) can be used and manipulated for dynamically changing geometry;
- object-oriented C⁺⁺ code development strategy makes it convenient for users to incorporate their own submodels.

The *barrier of entry* into OpenFOAM is also quite nontrivial. The website [1] does not provide a systematic user manual for codes. One must learn how to use OpenFOAM through:

- participating in the community-organized international workshop [14];
- attending company-offered tutorial courses, which can easily exceed a couple thousand dollars per person;
- joining online and/or campus Users Groups or CFD communities [12];
- classroom or personal tutorials by experienced users or instructors.

If a prospective user is already familiar with other CFD codes, then the learning curve is usually less steep, as OpenFOAM shares many common features with other codes. Nevertheless, for those mathematicians and engineers not familiar with the C^{++} programming language, they will need to gain familiarity with C^{++} first.

Will the advent and popularity of OpenFOAM and other open-source libraries signal the demise of commercial CFD software? At this time, the answer is firmly negative; however, open-source software does apply pressure on vendors to continue to innovate and evaluate price models, especially with regard to large-scale parallel computing licenses. In our opinion, this answer will also heavily depend upon secondary developers of front- and back-end software to provide graphical-user interfaces [19] and large-scale cloud computing [34], [35], [36].

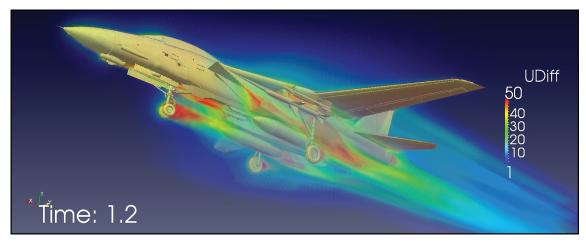


Figure 5. Airflow at t=1.2 of Grumman F-14 Tomcat fighter jet in a headwind of 70 m/sec during takeoff or landing.

After all, for the ultimate health and benefits of the CFD community, it is important to have *diversity* of CFD code developers, whether open source or by commercial vendors.

Only a few years ago the applied mathematicians in this group at Texas A&M would never have imagined that they could compute CFD problems such as those given in Examples 1-3 in this article and beyond. OpenFOAM has really opened up a grand vista for all researchers and empowered them to do CFD and more: general computational mechanics and the affiliated design and optimization problems. This enhances the industrial job prospects for math Ph.D. students up to a par with those of engineering and physics graduates. It also presents numerous opportunities for all researchers to compute, collaborate (in the same computing platform of OpenFOAM), and do interdisciplinary research on complex systems governed by PDEs in nearly every field of science and technology.

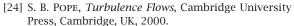
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An Introduction to Modern Industrial Mathematics

C. Sean Bohun

Introduction

When beginning this article one of the most difficult questions I was faced with was to provide a definition of industrial mathematics and to distinguish it from applied mathematics. Applied mathematics is primarily concerned with using mathematics as a tool for analysis. This is quite a broad characterization and encompasses not only what one would typically consider the meat and potatoes of modelling physical phenomena, solving problems and interpreting the results, but also finding connections across disciplines and utilizing the language of mathematics to unify disparate fields of study. Theories and models developed in applied mathematics lie at the interface of a deep understanding of mathematical insight and technique with a specialized knowledge of the particular process being studied. Historically, practical applications have led applied mathematicians to develop models which were then taken on by pure mathematicians, where the mathematics was further developed for its own sake. The field has been responsible in the last few decades for opening up completely new disciplines, including everything from string theory to mathematical

There is a further distinction within the discipline of applied mathematics between those that concern themselves with the mathematical methods and the application of mathematics within other fields. This has led to terms such as "practical mathematics" or "applied science" to describe

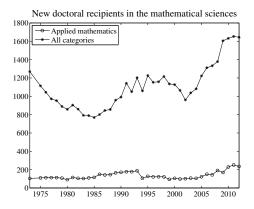
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the latter [7], [11]. In this same vein, a possible definition of industrial mathematics is to view it as a subset of applied mathematics where the focus is on the use and development of mathematics to solve industrial problems. I contend that it is more than this.

Industrial mathematics is concerned with realworld problems which focus on furthering the establishment and dissemination of links between mathematics and the physical world. The term *industry* is meant in a very broad sense consisting of any field with either a commercial or societal benefit, whether it be the optimal design of an electric motor, modelling financial options, or even classifying sociological interactions. The key point to make here is that, despite the inherent complexity of being concerned with real-life problems, industrial mathematics is tasked with laying bare the essential mechanisms at work and effectively disseminating the logical consequences to those outside the field of mathematics, significantly increasing understanding in the process.

This definition reflects the evolution of the contact between the industrial community and the mathematical community. What started as an indirect connection through industry hiring professors as consultants and supporting post-doctoral research positions has grown to a global network of modeling workshops where *direct* contact is made between mathematicians and members from industry [5]. Due in part to this effort, there has been a shift in the thinking of the applied mathematics community and in government over the last couple of decades. As evidenced by the 2012 *SIAM Report of Mathematics in Industry* [19], the role of mathematics in industrial society is not only relevant but required.

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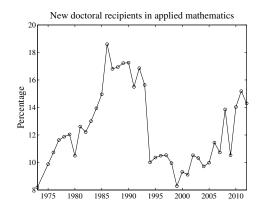


Figure 1. Trends in the number of new doctoral recipients in applied mathematics from 1973 to 2012.

- In 1996 manufacturing accounted for 15.4% of US GDP, while the combination of finance, insurance, and scientific and technical services jointly contributed 12.5%.
- By 2010 the order had reversed, with manufacturing accounting for 11.7% and finance, insurance, and scientific and technical services accounting for 15.9%.

This shift identified by SIAM enhances the fact that the skills attained through the study of applied mathematics are integral to the demands being made by this changing demographic. The SIAM *Careers in Applied Mathematics and Computational Sciences* [18] lists a few of these observations that are repeated below.

- Mathematics has burst the old boundaries that limited what an engineer could design, a scientist could know, or an executive could manage.
- Subtle interactions, masses of data, and complex systems are all within the scope of the tools and ideas of applied mathematics.
- The logical clarity and deductive skills you are being taught in your mathematics classes are exactly the tools required to study these complex real-world problems.

Figure 1 shows data for the last forty years for the U.S., extracted from the annual AMS survey [24]. To the left are the number of new doctoral recipients in applied mathematics compared to those across all of the mathematical sciences, while to the right is the corresponding percentage of recipients in applied mathematics. From 1973 to 1985 the historical record shows a steady increase in the percentage of new graduates in applied mathematics as the total number of graduates in the mathematical sciences decreased, while the number of those in applied mathematics held steady. From 1985 to 1994 the total number of graduates increased, and this trend was matched in the applied mathematics group. During this

time period there was a steady increase in the percentage of unemployed, reaching 10.7% in 1994 [8] and stabilizing in 1995 with a sharp drop in graduates in applied mathematics. This drop was not reflected in the mathematical sciences community as a whole. Since the year 2000 there has been a steady increase in the number of graduates in the mathematical sciences, recently surpassing anything in the historical record, yet the same trend has not been seen in applied mathematics. What we as applied mathematicians need to ask ourselves is why. As I have described above, the current trend in funding for applied mathematics is tied to industrial relationships [1], [9], [13], yet this is not a watershed for applied mathematics. I believe this is primarily a problem of public relations and a lack of a unified message across the discipline to expose potential students to the wonder, excitement, personal sense of accomplishment, and basically fun of being a part of the worldwide effort to apply modern mathematics to real-life problems.

One of the main difficulties of modern applied and industrial mathematics is not relevance or a lack of interesting problems. Personally, I have found that once students begin to have a sense of the applicability and relevancy of this discipline, the problems themselves become the best recruitment tool of all. In fact, the richness of the mathematics found with many of the problems brought to various study groups around the world have formed the backbone of a number of texts of advanced modeling [5], [7], [10], [11].

In the remainder of this report I will consider two case studies in industrial mathematics to give some insight as to the variety of techniques available to the industrial mathematician. One of these problems had its inception as a real-life problem brought to one of the global network of modeling workshops, while the other began with a seemingly innocuous question from an interested student.

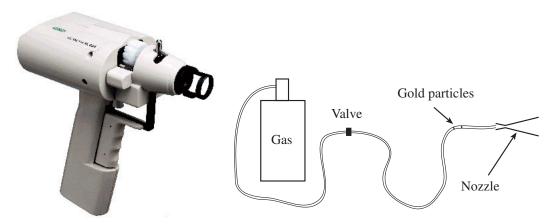


Figure 2. A Helios gas burst gene gun and the underlying gene gun geometry.

The emphasis here is placed on the questions and the process that one goes through when first exposed to any given problem and not on the level of sophistication. In fact, overcomplicating an already complicated problem seems to be a skill that pervades most academic disciplines. If there is a theme in what follows, it is that a deep understanding of simple models as well as their limitations should never be underestimated. What should be clear after reading through the models is that each choice made when modelling has both benefits and disadvantages and the modeller is continually faced with balancing mathematical with practical issues. Effectively navigating through these waters becomes an art as opposed to a science. The first example is taken from the field of biotechnology and is concerned with the design of a gene gun, and my introduction is the following exchange I had with a previous student:

NJ: Dr. Bohun, why do they use helium in a gene gun? Why not just air?

CSB: What's a gene gun?

Let's find out!

Modelling a Gene Gun

A gene gun consists of a tube that is configured to propel DNA-coated microparticles, typically gold, into a wide range of biological samples. There are a number of propulsion methods that have been utilized in the past, including, but not limited to, electric discharge, gunpowder explosions, gas flow and gas burst methods [14], [15], [22]. Figure 2 shows a Helios gas burst gene gun and a diagram of its functionality.

The details of how a gene gun works is a combination of biochemistry and physics. Under certain chemical conditions DNA can become sticky and adhere itself to biologically inert particles, for example, gold. These gold microparticles can act as bullets if they are placed in the path of a

pressure pulse. The gas burst gene gun consists of a high-pressure tank of helium, a shock tube, and a nozzle. By placing the microparticles on the interior wall of the shock tube after the valve, the particles accelerate as the shock wave travels down the tube. They then pass through a nozzle made up of a converging and a diverging section, which makes it possible to achieve high-speed gas molecules without having a very high pressure. Different gases can be used to generate a shock wave inside the shock tube, including helium (He), carbon dioxide (CO₂), and air—mainly nitrogen (N₂). But why is helium used? The modeling may provide an answer to this. In this work, we model the behavior of such a device and search for ways in which to optimize the design as a first step in designing a handheld device using a CO₂ cartridge.

A short pressure pulse imparts a drag force on the microparticles that accelerates them through the nozzle and subsequently into the target. Our specific modeling effort will focus on helium as the accelerating gas but also briefly contrast with carbon dioxide. As for the microparticles, we require a high-density material that does not react with our biological sample. Many materials are available as a delivery particle, including tungsten, gold, polystyrene, borosilicate, and stainless steel [14]. Our model will assume the use of gold, since this has the highest density and is available as essentially spherical particles.

Much of the underlying theory of the operation of a gas-burst gene gun is closely related to the concept of a cold spray whereby an ultrafine metal powder is accelerated to supersonic velocities and deposited on the surface of a target material. The interested reader is directed to the text by Papyrin et al. [16] to further explore this connection. Many of the submodels that are cascaded together in this problem contain material that is well trodden, and the reader will be referred to an appropriate classical text when appropriate.

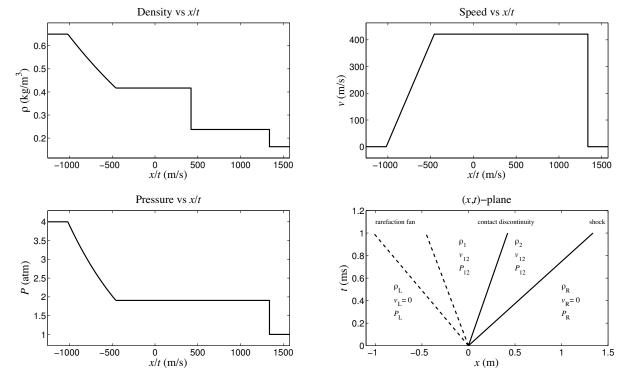


Figure 3. A simulated shock flow of helium gas for an initial uniform temperature of $T_0 = 300$ K. A leading shock in all three variables is followed by a contact discontinuity in the density and finally a trailing rarefaction fan. The bottom right panel shows the structure of the solution in the (x,t)-plane and the piecewise constant values within each of the solution regions.

Shock Tube

The underlying assumptions for the shock tube include:

- the diameter of the tube is much smaller than its length.
- viscosity of the gas is ignored and no heat energy is exchanged within the walls of the tube.
- the tube is considered to be frictionless with respect to the flow of the gas.

The first assumption allows one to reduce the problem to one spatial dimension, whereas the second and third assumptions allow us to assume that the gas flow is *isentropic* (adiabatic and frictionless). Under these assumptions the conservation of mass, momentum, and energy can be expressed as:

(1a)
$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v) = 0, \quad \text{(mass)}$$

(1b)
$$\frac{\partial (\rho v)}{\partial t} + \frac{\partial}{\partial x} (\rho v^2 + P) = 0$$
, (momentum)
(1c) $\frac{\partial E}{\partial t} + \frac{\partial}{\partial x} (v (E + P)) = 0$, (energy)

(1c)
$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x} (v (E + P)) = 0,$$
 (energy)

where ρ , ν , and P respectively denote the density, speed, and pressure of the gas. The variable E denotes the total energy of the gas per unit volume and can be written as the sum of the gas's internal energy and its kinetic energy. In the case of an isentropic gas,

(1d)
$$E = \frac{1}{2}\rho v^2 + \frac{1}{v-1}P,$$

where γ is the ratio of the specific heats. For helium, $y = c_p/c_v = 5/3$.

When the pulse of gas is initiated, we assume that the shock tube is in thermal equilibrium with a room temperature of $T_0 = 300 \text{ K}$ and that the gas is at rest, $v_0 = 0$. Across the valve is a jump of pressure:

(2a)
$$P(x,0) = \begin{cases} P_{L} = 4 \text{ atm}, & x < x_{0}; \\ P_{R} = 1 \text{ atm}, & x > x_{0}, \end{cases}$$

where x_0 is the location of the valve in the tube. There is a corresponding initial jump in the gas density, with pressure and density related through the ideal gas equation (1 atm = 101325 Pa):

$$\rho(x,0) = \frac{P(x,0)}{\mathcal{R}T_0} = \begin{cases} \rho_L = 0.6505 \text{ kg m}^{-3}, & x < x_0, \\ \rho_R = 0.1626 \text{ kg m}^{-3}, & x > x_0, \end{cases}$$

with the ideal gas constant for helium, R = $8.314 \times 10^3 \text{ JK}^{-1} \text{mol}^{-1}/M_W$, $M_W = 4.003 \text{ g mol}^{-1}$, being the molecular weight of the gas. Finally, at the edges of the spatial domain, we assign a zero Neumann boundary condition that is consistent provided no disturbance in any of the variables has time to reach the boundary.

A conservation law together with piecewise constant initial data having discontinuities is known as a Riemann problem, and to solve the system (1), it is written in the form

(3a)
$$\mathbf{u}_t + A\mathbf{u}_X = \mathbf{0}$$
where $\mathbf{u} = (\rho, \rho \nu, E)^{\mathrm{T}}$ and [12], [23]

(3b)
$$A = \begin{pmatrix} 0 & 1 & 0 \\ (\gamma - 3)\frac{v^2}{2} & (3 - \gamma)v & \gamma - 1 \\ (\gamma - 1)\frac{v^3}{2} - (E + P)\frac{v}{\rho} & (E + P)\frac{1}{\rho} - (\gamma - 1)v^2 & \gamma v \end{pmatrix}.$$

The solution is usually presented in terms of the pressure P rather than the less intuitive (for this problem) yet mathematically more convenient total energy E with expression (1d) used to make this exchange.

The described problem is a variant of Sod's [20], and has a well-known exact solution that can be found either in any well-represented gas dynamics text, for example [2], or more recently within the case studies by Danaila et al. [6]. Figure 3 illustrates the structure of the solution, and Table 1 contrasts He and CO₂ where in all cases the initial temperature, pressure differential, and velocity of the gas remain constant: $T_0 = 300 \text{ K}$, $P_L = 4 \text{ atm}$, $P_R = 1 \text{ atm}$, $V_L = V_R = 0$. A shock in all three variables $(\rho, v, P)^T$ propagates at a speed v_S , which is then followed by a contact discontinuity in the density moving at $v_{CD} = v_{12}$ and finally a trailing rarefaction fan in all three variables, the edges of which move at speeds $v_L^{fan} < v_R^{fan}$.

The purpose of modeling the shock wave is to provide initial conditions for the gas entering the nozzle. The values obtained are idealized, since both the friction and viscosity have been ignored, yet they give some initial insight for why helium is preferred. It is clear from the data in Table 1 that, for a fixed pressure differential, the initial shock speed of helium is significantly larger than that of air or carbon dioxide. However, it is not only the speed of the gas particles that determines the behavior of the gold microparticles but the density of the gas as well. Since the density of carbon dioxide is nearly ten times that of helium, any clear advantage of using helium remains elusive.

Nozzle Modelling

At the end of its travel down the shock tube the gas is passed through a nozzle, and this changes its speed, pressure, temperature, and density. The model for the shock tube, together with the nozzle, describes the behavior of the gas along the complete length of the gene gun, and within this

managed stream of gas, the gold microparticles acquire the inertia to be propelled into a biological sample. We begin by illustrating how a nozzle is used to accelerate the gas particles.

An ideal isentropic gas satisfies the additional relationship $P\rho^{-\gamma} = \text{const.}$ and, as a result, the speed of sound is given by

(4)
$$c^2 = \frac{\mathrm{d}P}{\mathrm{d}\rho} = \gamma \frac{P}{\rho} = \gamma \mathcal{R}T$$

using the ideal gas equation of state (2b). For a steady one-dimensional flow through the nozzle, the continuity equation reduces to the hydraulic approximation $\rho vA = \text{const.}$, where A is the cross-sectional area at any point in the nozzle. Expressed in terms of differentials this gives

(5)
$$\frac{\mathrm{d}\rho}{\rho} + \frac{\mathrm{d}\nu}{\nu} + \frac{\mathrm{d}A}{A} = 0.$$

In the same steady one-dimensional limit, using (4), the conservation of momentum reduces to [21]

(6)
$$v dv + \frac{1}{\rho} dP = v dv + \frac{c^2}{\rho} d\rho = 0.$$

Eliminating $d\rho/\rho$ from (5) and (6) and rearranging give the expression

(7)
$$\frac{\mathrm{d}A}{\mathrm{d}v} = \frac{A}{v} \left(M^2 - 1 \right),$$

where the *Mach Number M* = v/c.

Expression (7) embodies the purpose of the nozzle. Incident gas particles approaching the nozzle at subsonic speeds (v < c) are accelerated with the converging portion. If the particles reach the sonic speed (v = c) at the throat of the nozzle, then they can be further accelerated to supersonic speeds (v > c) with the diverging portion. To summarize, the maximum possible exit velocity of the gas particles is attained by having subsonic flow in the converging part of the nozzle and supersonic flow in the diverging part, and this can only be attained by having v = c at the throat.

Continuing the analysis yields a relationship between the Mach number and the cross-sectional area at any point in the nozzle [2], [21]:

(8)
$$\frac{A}{A_*} = \frac{M_*}{M} \left(\frac{1 + \frac{\gamma - 1}{2} M^2}{1 + \frac{\gamma - 1}{2} M_*^2} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}}$$

with A_* and M_* denoting the area and Mach number at the throat of the nozzle. If $M_* < 1$, then the gas speeds up as the nozzle narrows and then simply slows down in the diverging part, since A/A_* can never be less than 1 for a converging/diverging nozzle. For $M_* > 1$, the gas must be entering the nozzle faster than the speed of sound and the converging nozzle actually *slows* the gas, contrary to one's intuition. Only in the case $M_* = 1$, when the speed at the throat of the nozzle matches the speed of sound, can

¹Sod's original problem has the initial conditions $\rho_L = 1$, $P_L = 1$, $V_L = 0$, $\rho_R = 0.125$, $P_R = 0.1$, and $V_R = 0$.

Gas	γ	$M_w (\mathrm{g} \mathrm{mol}^{-1})$	P ₁₂ (atm)	$\rho_1 ({\rm kg} {\rm m}^{-3})$	$\rho_2 ({\rm kg} {\rm m}^{-3})$
He	1.667	4.003	1.905	0.4169	0.2374
CO_2	1.289	44.01	1.941	4.081	2.969
Gas	$v_{12}~({\rm ms^{-1}})$	$v_{\rm L}^{\rm fan}~({ m ms^{-1}})$	$v_{\rm R}^{\rm fan}~({ m ms^{-1}})$	$v_{\rm CD}~({ m ms^{-1}})$	$v_{\rm S}~({\rm ms^{-1}})$
He	421.4	-1019	-457.2	421.4	1338
CO_2	145.6	-270.3	-103.6	145.6	366.2

Table 1. Solution to the Riemann problem for He and CO₂. In all cases $T_0=300$ K, $P_{\rm L}=4$ atm, $P_{\rm R}=1$ atm, $v_{\rm L}=v_{\rm R}=0$ and $\rho_{\rm L}$, $\rho_{\rm R}$ follow from the ideal gas law (2b). The speeds of the initial shock, the contact discontinuity, and the left and right edges of the rarefaction fan are denoted by $v_{\rm S}$, $v_{\rm CD}$, $v_{\rm L}^{\rm fan}$ and $v_{\rm R}^{\rm fan}$ respectively.

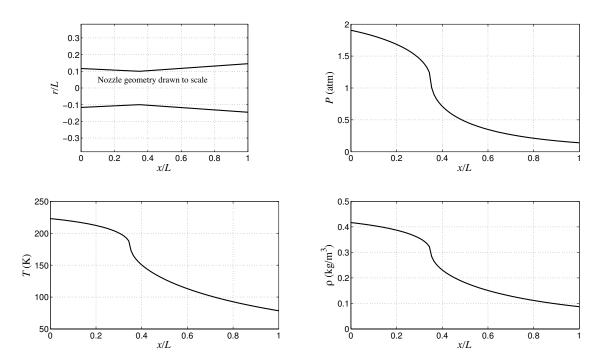


Figure 4. Pressure, temperature, and density for the indicated nozzle design with $L=2\,\mathrm{cm}$ and assuming the flow of helium gas from the shock tube.

the particles transfer from subsonic (M < 1) to supersonic (M > 1) speeds. Subsonic particles enter the nozzle, they speed up to Mach-1 at the throat, and they continue to accelerate as the nozzle diverges.

This increase in speed of the gas is done at the expense of a loss of internal energy. What this means is that the temperature, the pressure, and the density of the gas all drop as it progresses through the nozzle. Assuming that $M_* = 1$, these are given by the expressions [21]

(9)
$$\frac{T}{T_0} = \frac{1 + \frac{\gamma - 1}{2} M_0^2}{1 + \frac{\gamma - 1}{2} M^2},$$
$$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{\gamma}{\gamma - 1}},$$
$$\frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{\frac{1}{\gamma - 1}},$$

where (M_0, T_0, P_0, ρ_0) are the values at the entrance of the nozzle. Connecting the behavior within the nozzle to the results of the shock tube analysis is now simply a matter of matching the nozzle input parameters. That is, $P_0 = P_{12}$, $\rho_0 = \rho_1$, $T_0 = P_{12}/\rho_1\mathcal{R}$ and setting $M_0 = v_{12}/(\gamma \mathcal{R} T_0)^{1/2}$, where the values are taken from Table 1 for the flow *after* the contact discontinuity has passed. For the simulations a simple converging/diverging nozzle is chosen with an entrance angle consistent with the Mach number of the entering gas and an exit angle of four degrees. The resulting flow for a nozzle of length L=2 cm assuming helium is shown in Figure 4.

One of the important design aspects with nozzle design is to ensure that the pressure does not drop too low, or the outside gas will rush into the nozzle, undoing any advantage of making the gas supersonic in the first place. We will not address

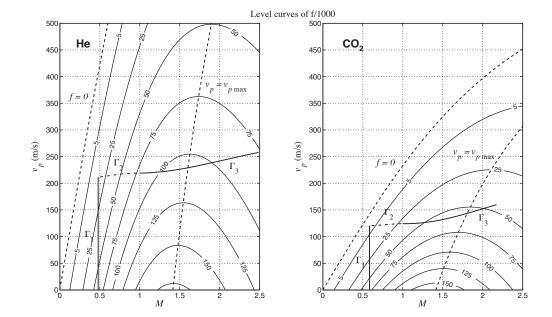


Figure 5. Behavior microparticles when utilizing the simple converging/diverging nozzle depicted in Figure 4. Note that $v_p/c(M) \le M$ for the assumed model of the drag. For this simulation the microparticles are placed 10 cm from the nozzle entrance and the nozzle is 2 cm in length.

this issue of what is referred to as a *normal shock* but refer the reader elsewhere [21].

Drag Force

Having shown how the speed of the gas varies throughout the gene gun apparatus, we can move on to investigating how the gold microparticles are accelerated. By treating the gold particles as tiny spheres that are dragged along by collisions with the gas molecules moving past them and assuming that the microparticles are always moving slower than the gas, we find that the equations of the motion of the gold particles are (10)

$$\frac{\mathrm{d}x_p}{\mathrm{d}t} = v_p, \qquad x_p(0) = 0,$$

$$m_p \frac{\mathrm{d}v_p}{\mathrm{d}t} = \frac{1}{2} C_D A_p \rho (v - v_p)^2, \qquad v_p(0) = 0.$$

This model ignores gravity, since the gold particles are quite small and it is easy to verify that the gravitational force is far outweighed by the force due to the pressure pulse. Quantities m_p , ν_p , and A_p are the mass, speed, and cross-sectional area of the gold particles, C_D is the drag coefficient ($C_D \simeq 1$ if the Reynolds number ≥ 30) [21], and (ν, ρ) are the speed and density of the gas respectively. These are initially obtained from the shock tube model and subsequently changing within the nozzle, as described above. Typical values for the gold

microparticles are [14]

(11a)
$$r_p = 3 \times 10^{-6} \text{ m}, \quad \rho_p = 16800 \text{ kg m}^{-3}$$
 and the combination

(11b)
$$C_D \frac{A_p}{2m_p} = \Gamma_0 = \frac{3}{2r_p \rho_p} \simeq 29.76 \text{ m}^2 \text{ kg}^{-1}$$

so that (10) can be written as

(12a)
$$\frac{\mathrm{d}x_p}{\mathrm{d}t} = v_p, \qquad x_p(0) = 0,$$

$$\frac{\mathrm{d}v_p}{\mathrm{d}t} = \Gamma_0 f(v_p; M, \rho, c), \qquad v_p(0) = 0,$$

for $v_p < Mc$ where

(12b)
$$f(\nu_p; M, \rho, c) = c^2 \left(M - \frac{\nu_p}{c} \right)^2.$$

Figure 5 displays the level curves of $f \times 10^{-3}$ for both He and CO₂. Bounded above by $2c^2$, the admissible domain contains an interior curve along which f attains its maximum value for a fixed value of ν_p . A straightforward calculation gives

(13a)
$$\frac{\partial f}{\partial M} = -y P_0 \left(M - \frac{v_p}{c} \right) \left(M^2 - \frac{v_p}{c} M - 2 \right) \left(1 + \frac{y - 1}{2} M^2 \right)^{\frac{1 - 2y}{y - 1}}$$
 with (13b)

$$c = (\gamma \mathcal{R} T)^{1/2} = (\gamma \mathcal{R} T_0)^{1/2} \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{-1/2},$$

giving this curve as

(13c)
$$v_{p_{max}}(M) = \left(M^2 - 2\right) \frac{c}{M}.$$

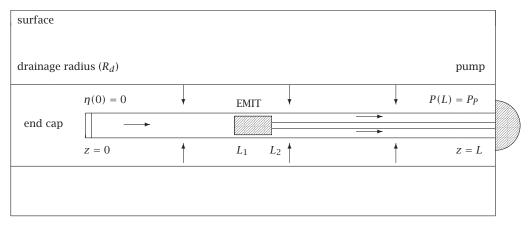


Figure 6. Overall geometry for the horizontal wellbore problem. A horizontal cylindrical well extends from z = 0 to z = L with a pump located at z = L to pull the heated oil out of the rock formation.

In each case the trajectory of the microparticles is computed assuming that they initially lie 10 cm from the nozzle entrance and the nozzle is 2 cm in length. Three distinct regions characterize the behavior.

- 1. Approaching the nozzle, the gas flow is constant and the gold particles accelerate towards the nozzle entrance. In this regime the speed of the microparticles is given by (12) with a fixed Mach number determined by the solution of the shock tube equations. The final speed in this region is determined by the initial distance of the microparticles to the nozzle entrance. Within He the microparticles only achieve 50% (211 m s⁻¹) of the gas flow speed of (421 m s⁻¹), whereas the higher density of CO_2 allows the microparticles to reach 82% (120 m s⁻¹) of the gas flow speed (146 m s⁻¹) over the same distance.
- 2. In the converging section of the nozzle the gas particles rapidly approach M=1, and in both cases of He and CO₂ this occurs over such a short timeframe (He: 32 μ s, CO₂: 57 μ s) that no significant change in microparticle speed occurs.
- 3. In the diverging section the shape of the nozzle can be modified to allow the microparticles to track the curve of maximal drag curve. Again this portion of the nozzle is quickly traversed (He: $55 \mu s$, CO₂: $92 \mu s$) so that only a moderate increase is observed.

With the groundwork that has been laid, a nozzle design that maximizes the exit velocity of the microparticles can be determined. This design will clearly depend on the gas being used. There are other issues as well. First, the gold particles are not uniform in size, and their diameters will have some probability distribution. We may wish to investigate the effect of this fact in the current model. Second, we have only begun the

investigation of contrasting the effect of the use of other gases. And finally, we have not even started to consider a model for stopping the particles and controlling the distribution of where they embed themselves in a target.

Clearly, for validation of the model a set of data needs to be made available and the predictions of the model compared to what is seen experimentally. Let's return to the student's initial question: "Why do they use helium in a gene gun?" The simple answer is the structure of the gas and its nonreactivity. Helium, being a noble gas, will not easily react with a biological sample, but this is not the only reason. The structure of the gas (essentially spheres) gives a value of $\gamma = 5/3 \simeq 1.67$. Carbon dioxide molecules are slightly more complicated, with two oxygen atoms connected to a single carbon atom. The result is a value of $\gamma = 7/5 = 1.4$. The larger value of γ makes helium more efficient in that less energy goes into the internal vibrations of the molecules and consequently there is a larger Mach number for a given area ratio. So now you know!

Horizontal Wellbore

This last case study was brought to the attention of the author at a problem-solving workshop concerned with primarily oil and gas problems. As oil wells age it becomes increasingly difficult to extract oil from them, and they are eventually abandoned when the cost of extraction becomes prohibitive. To increase the production rate for these wells and ultimately increase their lifespan, one possible technique is to somehow heat the trapped oil, allowing it to flow more easily. The heating technique described to the working group was the use of electromagnetic heaters known as EMIT (electromagnetic induction tool) units. For a horizontal well of up to a kilometre in length the practice is to insert several EMIT units at intervals

of about one hundred meters and then power them all from a single cable protected by a steel housing. Figure 6 shows a cut-away section of a well with a single EMIT region. Arrows indicate the flow of oil that seeps into the well from the surrounding rock subsequently removed from the rock formation with a pump located at z = L.

So why do any modelling at all? In this case, the industrialist has an expensive computational fluid dynamics (CFD) code that can be used to simulate the process, but he wants a simplified model that still captures most of the physics. His key requirement is a formulation of the problem that can be solved rapidly so that one can quickly search wide ranges of possible operating parameters. Attempting this with a large commercial CFD software package would become prohibitively expensive. Another application of the model would be financial. Being able to quickly estimate the increase in production by heating the well, one can use the local cost of electrical power to determine the expected profit in a systematic way. As in the previous model, the problem naturally breaks into several subtasks, but in this case they remain intimately coupled throughout the wellbore.

Axial Flow

We start with estimating the amount of oil that will flow into the well over a very short length of time. This is given by an expression originally developed by Peaceman [4], [17] that depends on the drainage radius of the formation; R_d , the outer radius of the well, R_c ; the axial length of the section under consideration; and the pressure in the pipe P(z). The expression for a segment of the well with axial length Δz is

(14)
$$\Delta \eta(z) = \frac{2\pi k (P_R - P(z))}{\mu_0 \ln (R_d/R_c)} \Delta z,$$

where $\Delta \eta$ is measured in m³ s⁻¹, and k, μ_0 are the permeability of the ground and viscosity of the oil respectively. The subscript zero for the viscosity refers to unheated oil, as we have assumed that the fluid seeping into the well is not effectively heated when compared to the fluid flowing along the well. P_R is known as the reservoir pressure and is the minimum suction pressure required to pull oil out of the well. It can vary from one well to the next and increases as the well ages.

Figure 7 illustrates a small section of the well in either the EMIT region or a region with the cable housing. Of course the regions with neither an EMIT nor the housing will not have any blockage. The volumetric flux is given by $\eta(z) = \pi(R_w^2 - R_z^2)\bar{v}$, where \bar{v} is the radially averaged axial velocity, R_w is the inner pipe radius, and R_z depends on whether there is nothing $(R_z = 0)$, an EMIT $(R_z = R_e)$, or the cable housing $(R_z = R_h)$ within the pipe. By

the conservation of mass, the amount of fluid that emerges from this infinitesimal segment must be the sum of the fluid that entered along the well and the fluid that seeped in through the sides. In other words,

(15)
$$\eta(z_* + \Delta z) = \eta(z_*) + \frac{2\pi k (P_R - P(z))}{\mu_0 \ln (R_d/R_c)} \Delta z$$

or, letting $\Delta z \rightarrow 0$,

(16)
$$\frac{\mathrm{d}\eta}{\mathrm{d}z} = \frac{2\pi k (P_R - P(z))}{\mu_0 \ln (R_d/R_c)}, \quad \eta(0) = 0.$$

The condition $\eta(0) = 0$ simply reflects the fact that at the end cap the only flux of oil is that dripping in from the sides.

Axial Pressure

Assuming a steady state solution, the Navier-Stokes equations provide a connection between the fluid flow and the pressure drop applied to the fluid. The axial velocity of the fluid depends on its radial position in the well and for the EMIT and housing regions. By further assuming that the radial flow entering the well from the sides is much smaller than the axial flow (strictly true only away from the sidewall), it then satisfies

(17)
$$\mu \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v}{\partial r} \right) = \frac{dP}{dz}, \quad v(R_w) = 0, \quad v(R_z) = 0,$$

where R_z is either the EMIT or the housing radius with the solution

[18a]

$$v(r) = \frac{1}{4\mu} \frac{\mathrm{d}P}{\mathrm{d}z} \left[r^2 - R_w^2 + \frac{R_w^2 - R_z^2}{\ln(R_w/R_z)} \ln\left(\frac{r}{R_w}\right) \right].$$

For regions where $R_z = 0$ we impose the condition $v(0) < \infty$ to find that

(18b)
$$v(r) = \frac{1}{4\mu} \frac{dP}{dz} (r^2 - R_w^2).$$

Computing the average radial velocity defined as

(19)
$$\bar{v} = \frac{2}{R_w^2 - R_z^2} \int_{R_z}^{R_w} v(r) r \, dr,$$

using $\eta = \pi (R_w^2 - R_z^2) \bar{v}$, and solving for the pressure give

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -\frac{8\mu}{\pi R_w^4} \left[\frac{\ln \lambda}{(1-\lambda^2)^2 + (1-\lambda^4)\ln \lambda} \right] \eta(z),$$

$$P(L) = P_P,$$

where $\lambda = R_z/R_w$ is piecewise constant and P_P is the suction applied by the pump at the surface z = L. Combining this with expression (16) we obtain a two-point boundary value problem

(21a)
$$\frac{d^{2}\eta}{dz^{2}} - y^{2}(z) \frac{\mu}{\mu_{0}} \eta = 0, \qquad \eta(0) = 0, \\ \frac{d\eta}{dz} \bigg|_{L} = \frac{2\pi k (P_{R} - P_{P})}{\mu_{0} \ln(R_{d}/R_{c})},$$

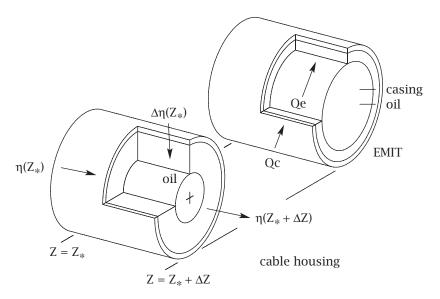


Figure 7. Axial flow within the well illustrating both the fluid flow and the power flux from the casing and the EMIT.

where (21b)
$$\gamma^2 = \frac{16k}{R_w^4 \ln(R_w/R_c)} \frac{\ln \lambda}{(1-\lambda^2)^2 + (1-\lambda^4) \ln \lambda}$$

is piecewise constant, reflecting the placement of EMIT, housing, and free regions.

Heating the Wellbore

How do we deal with the heat that is being generated in the wellbore? Since we know where the oil is flowing, we can determine the heat flux

(22a)
$$\Phi = -k_o \nabla T + \rho C_p \mathbf{v} T,$$

where T is the temperature, \mathbf{v} is the fluid, and k_o , ρ , C_p are respectively the thermal conductivity, density, and heat capacity of the oil. The heat equation can be written in terms of this flux as

(22b)
$$\frac{\partial}{\partial t}(\rho C_p T) + \nabla \cdot \Phi = Q,$$

where Q corresponds to an external heating source. To find an effective axial temperature the flux in each of the four regions (reservoir, casing, well, EMIT) is determined and combined with a radial averaging process. The resulting final expression for the average temperature is obtained in [3], where it is shown to satisfy

$$\rho C_p \frac{\mathrm{d}}{\mathrm{d}z} (\eta(z) T(z)) = \pi \left(Q_c(z) \left(R_c^2 - R_w^2 \right) + Q_e(z) R_e^2 \right),$$

$$T(0) = 0$$

The heat sources Q_c and Q_e have been written as functions of z; if one is not in an EMIT region, these functions are simply zero. As a result, the

Data	Symbol	Value
Outer casing radius	R_c	69.85 mm
Inner casing radius	R_w	63.50 mm
EMIT radius	R_e	50.80 mm
Housing radius	R_h	31.1625 mm
Wellbore length	L	1000 m
Permeability	k	$9.869 \times 10^{-12} \text{ m}^2$
Ambient viscosity	μ_0	15 Pa s
Drainage radius	R_d	100 m
Reservoir pressure	P_R	$5.0 \times 10^{6} \text{ Pa}$
Producing pressure	P_P	$5.0 \times 10^5 Pa$
Fluid heat capacity	ρC_p	$2.8 \times 10^6 \mathrm{J}\mathrm{m}^{-3}\mathrm{K}^{-1}$
Casing power	Q_c	$795.8 \; \text{kW} \text{m}^{-3}$
EMIT power	Q_e	79.58 kW m^{-3}

Table 2. Wellbore, reservoir and thermal properties for a single EMIT region extending from $L_1 = 495$ m to $L_2 = 505$ m.

right-hand side of (23) is piecewise constant and nonzero only where an EMIT is located. In addition, the temperature scale is chosen so that T=0 corresponds to 25°C. Integrating (23) for a single EMIT region extending from L_1 to L_2 gives the result that (24)

$$T(z) = \begin{cases} 0, & 0 \le z < L_1, \\ \frac{\pi\Omega}{\eta(z)} (z - L_1), & L_1 \le z < L_2, \\ \frac{\pi\Omega}{\eta(z)} (L_2 - L_1), & L_2 \le z \le L, \end{cases}$$

$$\Omega = \frac{Q_c (R_c^2 - R_w^2) + Q_e R_e^2}{\rho C_f},$$

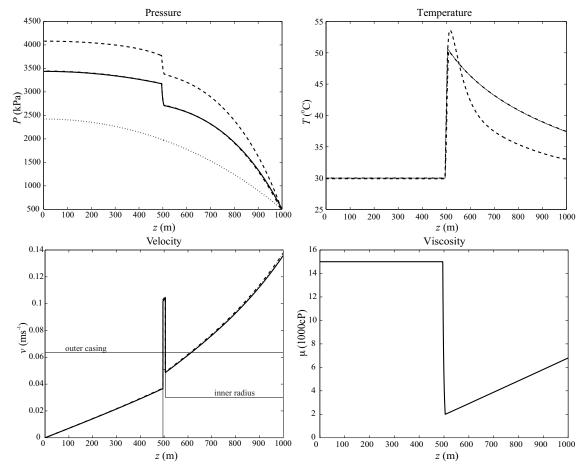


Figure 8. The four figures are the pressure, temperature, velocity, and viscosity as a function of distance along the wellbore. Only the pressure and temperature for the CFD code were available. These are the dashed lines in the respective plots. More than one method was used to solve the simplified model. Where they are distinguishable, the shooting method solution is a solid line, whereas the SOR method is indicated with a dashed dot. A longitudinal section of the wellbore is indicated in the plot of the velocity.

and from this the viscosity μ satisfies the empirical relationship in units of thousands of centipoise² through

(25)
$$\log_{10} \mu(T) = -3.002 + \left(\frac{453.29}{303.5 + T}\right)^{3.5644}$$
,

which closes the model.

Results

To compare the results of the model with the CFD code, a single EMIT unit was placed in the well and the resulting temperature and pressure profiles were calculated. Table 2 lists the various parameters for the model, and Figure 8 shows the resulting pressure, temperature, velocity, and viscosity of the fluid in the well and compares it with the results of the CFD code (dashed lines) in the pressure

and temperature plots. The solid and dashed-dot curves (where discernible) correspond to two different techniques for solving the boundary value problem: namely, a shooting method and an SOR method. Both the pressure and the temperature are reasonably well reproduced considering the relative simplicity of the new model. The simplified model underestimates the production rate at the pump giving $\eta_{\rm CFD}(L)=187.6~{\rm m}^3~{\rm day}^{-1}$ while $\eta(L)=115.2~{\rm m}^3~{\rm day}^{-1}$.

The simplified model reproduces the overall characteristics of the full model described by the CFD code and does this with a comparatively low computational cost. One of the primary benefits of this reduction in the computational cost is that it allows one to run a number of numerical experiments cheaply and onsite in the vicinity of the well itself. The question of determining the optimal placement and number of required EMIT

 $^{^{2}1}$ centipoise = $1 \times 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}$.

regions for a given production rate is explored in Bohun et al. [3].

Final Words

Despite the ever-increasing complexity and crossdisciplinary nature of modern real-life problems and a redoubling of efforts by government and industry to form meaningful collaborative bridges with mathematicians, the historical record shows that such efforts are being achieved only modestly. Industrial mathematicians weave disparate threads of science from many different fields to produce innovative models which are typically utilized outside the discipline, enriching understanding. Leading the reader through two separate case studies, I have attempted to expose some of the inner workings to encourage students and faculty to participate in the global network of modeling workshops. What has not been emphasized is the importance of building and maintaining working relationships with both industry and other disciplines within academia. With collaboration, both academically and institutionally, we stand to increase not only the range of real-world problems that are made available to us, but we also stand to ensure our positive impact on commercial and societal benefits. Personally I take great satisfaction in seeing the technologies I know and love being applied in productive and interesting ways to real-world problems.

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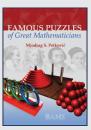
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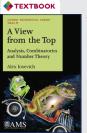
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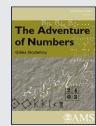
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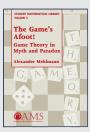
Burkard Polster. Monash University, Clayton, Victoria,

The analyses are elegant, simple, and should be accessible to a

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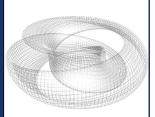
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Book Review

Henri Poincaré. A Scientific Biography

Reviewed by John Stillwell

Henri Poincaré. A Scientific Biography

Jeremy Gray Princeton University Press, 2013 248 pp., US\$16.55 ISBN-13: 978-0691151007

I think I can say, without fear of contradiction, that it takes a brave mathematician to write a scientific biography of Poincaré. It is remarkable that Jeremy Gray has dared to do it and even more remarkable that he has succeeded so brilliantly.

Poincaré was, with the possible exception of Hilbert, the deepest, most prolific, and most versatile mathematician of his time. His collected works fill eleven large volumes, and that does not include several volumes on mathematical physics and another several volumes of essays on science and philosophy for the educated reader. For most people it would be a life's work simply to read his output, let alone understand it well enough to write a clear and absorbing account. We are very fortunate to have this book.

Poincaré is probably best known to modern mathematicians for his contributions to non-Euclidean geometry, his discovery of chaos (in celestial mechanics), and his creation of algebraic topology (in which the "Poincaré conjecture" was the central unsolved problem for almost a century). These topics also belong to the three main areas of Poincaré's research that have been translated into English, and I discuss them further below. But they are merely some highlights, and they cannot be properly understood without knowing how they fit into the big picture of Poincaré's scientific work

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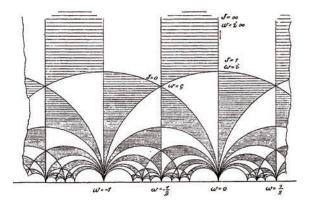


Figure 1. The modular tessellation.

and philosophy. Gray, who has written and edited many books on nineteenth-century mathematics, particularly geometry and complex analysis, is the ideal guide to this big picture.

Non-Euclidean Geometry

How did Poincaré find himself in non-Euclidean geometry? Bolyai and Lobachevskii developed non-Euclidean geometry in the 1820s, and Beltrami put it on a firm foundation (using Riemann's differential geometry) in 1868. So non-Euclidean geometry was already old news, in some sense, when Poincaré began his research in the late 1870s. But in another sense it wasn't. Non-Euclidean geometry was still a fringe topic in the 1870s, and Poincaré brought it into the mainstream by noticing that non-Euclidean geometry was already present in classical mathematics.

Specifically, there are classical functions with "non-Euclidean periodicity", such as the modular function j in the theory of elliptic functions and number theory. The periodicity of the modular function was observed by Gauss, and beautiful

pictures illustrating this periodicity were produced in the late nineteenth century at the instigation of Felix Klein. Figure 1 is one such picture.

The function j(z) repeats its values in the pattern shown: namely, at corresponding points in each of the curved black and white triangles. More precisely, j(z+1)=j(z) (corresponding to the obvious symmetry of the tessellation under horizontal translation), and less obviously j(-1/z)=j(z) (which corresponds to a half turn around the vertex z=i). It turns out that non-Euclidean periodicity is actually more common than the Euclidean periodicity that nineteenth-century mathematicians knew well from their study of elliptic functions.

At least, that is how it looks with hindsight. Poincaré found his way into non-Euclidean geometry by a more roundabout route, partly due to his ignorance of the existing literature (a common occurrence with Poincaré). He began by studying some functions defined by complex differential equations in response to a question posed in an essay competition by his former teacher Charles Hermite. The equations in question were first studied by Lazarus Fuchs, so Poincaré gave the name "Fuchsian" to the equations, to the functions they defined, and later to the symmetry groups he found those functions to possess.

Just as an elliptic function f(z) repeats its value when z is replaced by $z + \omega_1$ or $z + \omega_2$ (where ω_1 and ω_2 are the so-called "periods" of f), Poincaré discovered that a Fuchsian function g(z) repeats its value when z is replaced by $\frac{\alpha z + \beta}{\gamma z + \delta}$ for certain quadruples $\alpha, \beta, \gamma, \delta$ of real numbers. The periodicity of a Fuchsian function can be illustrated by tessellating the upper half-plane by "polygons" whose sides are arcs of semicircles with centers on the real axis. The polygons are mapped onto each other by the transformations $z \mapsto \frac{\alpha z + \beta}{\gamma z + \delta}$.

At some point (Poincaré tells us it was as he was stepping onto an omnibus), he had the epiphany that *such transformations are the same as those of non-Euclidean geometry*. In particular, the polygonal cells tessellating the half-plane are *congruent* in the sense of non-Euclidean geometry, and the transformations mapping one cell onto another are non-Euclidean isometries. Poincaré had in fact rediscovered the *half-plane model* of the non-Euclidean (hyperbolic) plane first found by Beltrami [2], but in a mainstream mathematical context. He also rediscovered Beltrami's related *disk model*.

His non-Euclidean tessellations were also a rediscovery. The modular tessellation was one (though it had not yet been accurately drawn), and so too was a beautiful tessellation of the disk given by Schwarz [19] (see Figure 2) in connection with hypergeometric functions. What was new,

and crucial, was Poincaré's realization of their non-Euclidean nature.

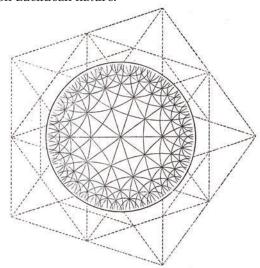


Figure 2. The Schwarz triangle tessellation.

Poincaré was not at first even aware of Schwarz's paper, which led to some awkward repercussions, related by Gray on page 229. In a letter to Klein, Poincaré admitted that he would not have called the functions "Fuchsian" had he known of Schwarz's work. Indeed, the triangle functions and their symmetry groups are now, belatedly, named after Schwarz. But at the time Schwarz got no satisfaction and was said (in a letter from Mittag-Leffler to Poincaré) to be very angry with himself "for having had an important result in his hand and not profiting from it."

Thus Poincaré's first steps in non-Euclidean geometry were mainly a matter of finding a better language to describe known situations: the non-Euclidean interpretation made it possible to describe linear fractional transformations $z\mapsto \frac{\alpha z+\beta}{\gamma z+\delta}$ in traditional geometric terms such as congruence and rigid motions. His next step was more profound: he used this language to describe a situation which, until then, had been incomprehensible. This situation arises when the transformations are of the form $z\mapsto \frac{az+b}{cz+d}$, where a,b,c,d are complex. In this case one again has a tessellation of $\mathbb C$ by curvilinear cells mapped onto each other by the given transformations, but the tessellation can be enormously complicated.

The limit points, where the (Euclidean) size of the cells tends to zero, can form a nondifferentiable curve ("if one can call it a curve," said Poincaré) or other highly complicated sets. Poincaré's inspiration was to view this tessellated plane as the boundary of the *upper half-space*, which turns out to be none other than *non-Euclidean 3-dimensional space*. The upper half-space is nicely tessellated with 3-dimensional cells, bounded by portions of

hemispheres whose centers lie in the plane we started with. Thus the nasty tessellation of the plane can be replaced by the nice tessellation of space, and all is well again. In this way Poincaré was able to make progress with functions that were previously intractable: the so-called *Kleinian functions*, whose values repeat under transformations of the form $z \mapsto \frac{az+b}{cz+d}$. (The name "Kleinian" was mischievously suggested by Poincaré after Klein had complained to him about the name "Fuchsian". It had some basis in Klein's mathematics, but not much.)

English translations of Poincaré's papers on Fuchsian and Kleinian groups may be found in [14].

Celestial Mechanics and Chaos

For most of his career, Poincaré was as much a physicist as a mathematician. He taught courses on mechanics, optics, electromagnetism, thermodynamics, and elasticity, and contributed to the early development of relativity and quantum theory. He was even nominated for the Nobel Prize in physics and garnered a respectable number of votes. But it is probably his contribution to the 3-body problem that is of greatest interest today. Like some of his other groundbreaking work, his results on the 3-body problem were a triumph over initial mistakes.

The story was first uncovered by June Barrow-Green [1], and it is updated in Chapter 4 of Gray's book.

Poincaré's first attempt was an entry in a mathematical prize competition sponsored by King Oscar II of Sweden. The king had a more than amateur interest in mathematics, but the prize questions were probably devised by Poincaré's friend Gösta Mittag-Leffler to ensure Poincaré's participation. At any rate, Poincaré submitted an entry for question 1, on the stability of the solar system, and duly won. His entry was an essay on the 3-body problem, the simplest case where the stability is not obvious.

The judges—Mittag-Leffler, Weierstrass, and Hermite—were agreed that Poincaré should win because of the originality of his results, among them what later became known as the Poincaré return theorem. Nevertheless, they found his essay hard to follow, and Poincaré eventually added ninety-three pages to clarify it. This satisfied them enough to confirm their decision and to go ahead with publication of the essay. Meanwhile, Poincaré had continued thinking about the essay—and he discovered a mistake.

Fortunately, Poincaré was able to fix the error in a month, but not before Mittag-Leffler had frantically recalled all known published copies and asked Poincaré to pay for the printing of a corrected version. This Poincaré willingly did, even though the cost was 50 percent more than the value of the prize. The reason for the panic was that Mittag-Leffler for years had championed Poincaré's work, with its frequent intuitive leaps, over the objections of German mathematicians such as Kronecker. Kronecker was already miffed that he had not been chosen as one of the judges, so if he learned how much Poincaré had been "helped" to win the prize there would be hell to pay. Luckily, he never found out.

Perhaps with some discomfort over his first venture into celestial mechanics, Poincaré decided to write up his ideas properly in the 1890s. The result was the 3-volume *Les Méthodes Nouvelles de la Mécanique Céleste* (1892, 1893, 1899), a monumental work that launched the modern theory of dynamical systems. An English translation [15] was produced on the initiative of NASA. The third volume ends with a glimpse of chaos in the 3-body problem:

One is struck by the complexity of this figure that I am not even attempting to draw. Nothing can give us a better idea of the complexity of the three-body problem and of all the problems of dynamics in general. . . .

(Méthodes Nouvelles, Vol. 3, p. 389)

Before leaving the question of the stability of the solar system, I would like to mention a result that Gray hints at but seemingly forgets to explain. On p. 253 he remarks that the implications of Poincaré's later work

...opened for serious investigation for the first time the idea that Newton's laws might permit a planet to exit the system altogether.

I am guessing that Gray here is referring to the result of Xia [21] that there is a 5-body system in which one of the bodies escapes to infinity in finite time. Xia's result is discussed in the book of Diacu and Holmes [5], a book that Gray does mention.

Algebraic Topology

To appreciate how much Poincaré did for algebraic topology, one needs to review the state of the subject before Poincaré burst onto the scene in 1892.

The topology of compact 2-dimensional manifolds ("closed surfaces") was quite well understood, partly because Riemann had seen the value of orientable surfaces in complex analysis and partly because of the lucky fact that topology is simple for orientable surfaces. It is captured by a single invariant number, the *genus* p (or, equivalently, by the *Euler characteristic*). Riemann [17] described p in terms of *connectivity*—the maximum number of closed cuts that can be made without separating

the surface—and Möbius [7] introduced the normal form of a genus p surface, namely, the "sphere with p handles". Möbius also observed the existence of nonorientable surfaces, such as the Möbius band, but it turns out that they do not greatly complicate the topological classification of surfaces.

Thus by the 1880s the topology of surfaces was well understood and unexpectedly easy. Virtually the only progress in higher-dimensional topology was made by Betti [3] when he generalized Riemann's idea of connectivity to obtain a series of invariants that became known as the *Betti numbers*. A 3-dimensional manifold, for example, has a 2-dimensional connectivity number P_2 , equal to the maximum number of disjoint surfaces in M that fail to separate it. M also has a 1-dimensional connectivity number P_1 , equal to the maximum numbers of closed curves that can lie in M without forming the boundary of a surface.

The dream of topology, at this point, was to find a finite set of invariant numbers that characterize each compact *n*-dimensional manifold up to homeomorphism.

Poincaré [9] struck the first blow against this dream by showing that the Betti numbers do *not* suffice to characterize 3-dimensional manifolds M. He did so with the help of a new kind of invariant, the *fundamental group* $\pi_1(M)$, by showing that certain manifolds M with the same Betti numbers have different fundamental groups.

The fundamental group is an essentially algebraic invariant, because it does not readily reduce to a set of numbers (if at all). What one calculates from M is a set of *generators* and *defining relations* for $\pi_1(M)$, and it is generally hard to tell when two sets of defining relations define the same group. In the case of a surface S of genus ≥ 2 , $\pi_1(S)$ is actually one of the Fuchsian groups introduced by Poincaré in the early 1880s. However, Poincaré at that time interpreted Fuchsian groups geometrically—as symmetry groups of tessellations—and it was Klein [6] who first saw the connection between the group and the topology of the surface obtained by identifying equivalent sides of a tile in the tessellation.

By the 1890s Poincaré had absorbed the topological viewpoint and was ready to extend it to three dimensions. In his first long paper on topology, entitled "Analysis situs" (the name then given to topology), Poincaré [10] constructed several 3-manifolds by identifying sides of polyhedra and calculated their Betti numbers and fundamental groups. They include, in more detail, his 1892 example showing that the fundamental group can distinguish certain 3-manifolds that the Betti numbers cannot. (He also remarks that the Betti numbers can be extracted from π_1 by allowing its generators to commute ("abelianization"), so π_1 is

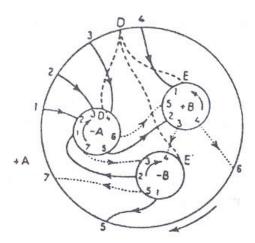


Figure 3. Poincaré's diagram for his homology sphere.

a strictly stronger invariant than the set of Betti numbers.)

Nevertheless, Poincaré did not give up on the Betti numbers. He spent much of "Analysis situs" developing the algebra of homology, proving what we now call *Poincaré duality*, and concluding with a generalization of the Euler polyhedron formula to *n* dimensions.

He made some errors. In 1898 Heegaard pointed out that Poincaré duality was incorrect as it stood. Poincaré [11] responded with a supplement to "Analysis situs", revising his definition of the Betti numbers and formulating his homology theory in a more combinatorial way. He assumed that each manifold could be divided into cells and calculated its Betti numbers from incidence matrices. A second supplement [12] followed when he realized that he had overlooked the presence of *torsion* in the homology of manifolds. The word "torsion", which today appears as much in algebra as in topology, originates here. It reflects Poincaré's view that topological torsion is characteristic of manifolds that are "twisted upon themselves", such as the Möbius band.

Having now found all the invariant numbers that homology had to offer, Poincaré [12] dared to conjecture that *the three-dimensional sphere is the only closed three-dimensional manifold with trivial Betti and torsion numbers.* This was the first, and incorrect, version of the "Poincaré conjecture".

Eventually there were three more supplements, the most important of which is the fifth [13]. In it Poincaré makes two interesting excursions: the first is an analysis of 3-manifolds from the viewpoint of what we now call *Morse theory*; the second is an interesting application of *geometrization* (in this case, the hyperbolic structure of surfaces of genus ≥ 2) to simple curves on surfaces. These excursions lead, in a roundabout way, to one of

Poincaré's greatest discoveries: a 3-manifold *H* with trivial homology but nontrivial fundamental group—the *Poincaré homology sphere*.

The construction of H is based on the diagram in Figure 3, which I include mainly to show how unenlightening it is. Despite the apparent asymmetry of its construction, $\pi_1(H)$ falls out as a group with a 2-to-1 homomorphism onto the icosahedral group A_5 . This shows that $\pi_1(H)$ is nontrivial, but, like A_5 , it has a trivial abelianization. How much of this construction was good luck and how much good management is a mystery. Gray makes a plausible case that Poincaré picked the group first, then tinkered with manifolds until he found one that realized the group. (English translations of Poincaré's topology papers are given in [16].)

We now know that the Poincaré homology sphere can be arrived at by several different symmetric constructions. See, for example, [18]. So perhaps it is more "inevitable" than it seemed at first. We also know now that it is the *only* homology sphere with finite fundamental group.

The immediate effect of Poincaré's discovery was to refute the "Poincaré conjecture" of the second supplement and to amend it to its correct form: the three-dimensional sphere is the only closed three-dimensional manifold with trivial fundamental group. The latter is the conjecture that launched a thousand topology papers, from Tietze [20] and Dehn [4] to Perelman [8]. The early attacks on the conjecture assumed that Poincaré's combinatorial methods in topology and group theory would suffice to settle it. But that hope gradually faded, and in the 1970s Thurston upped the ante to a geometrization conjecture, greatly extending the geometrization long known for surfaces.

In the 1980s Hamilton proposed an approach to geometrization through differential geometry, hoping to approach the nice geometric structures conjectured by Thurston by letting nastier (but easily obtained) geometric structures flow towards nice ones. Hamilton's program was finally carried out by Perelman in 2003 by a tour de force of differential geometry and PDE. Among the mathematicians of Poincaré's era, perhaps only Poincaré himself would have felt at home with such high-powered geometric and analytic equipment.

General Remarks

The three topics above are thoroughly covered in Gray's book, together with their complex web of historical and mathematical connections. There is also much else to enjoy. The book is structured to lead the reader gently into Poincaré's work: first an introduction that could stand alone as a splendid essay on Poincaré, then a chapter on Poincaré's popular science essays, and then a chapter on

Poincaré's career. All of this comes before the chapters on more specialized and difficult topics.

Of course, as a reviewer I read everything in the book, but I believe it would be easy to skip topics according to taste. It is a big book, and there is something for everyone.

I found the description of the French mathematical community and how they differed from the Germans particularly fascinating. Compared with most eminent mathematicians today and with the Germans then, Poincaré was unusually isolated. He had no graduate students and no immediate successors in France. The next generation of French mathematicians (Borel, Lebesgue, Hadamard) had different interests, and the generation after that was almost destroyed in World War I. Bourbaki emerged from the wreckage with a conscious effort to catch up with the Germans, who had forged ahead under the leadership of Klein and Hilbert.

Another interesting thread that runs through the book is Poincaré's interest in physics, particularly his near-discovery of special relativity. Gray shows how Poincaré took many of the right steps, starting from Maxwell's equations and getting as far as introducing the Lorentz group. But he lacked Einstein's physical insight, and the mathematical insight that could have made up for this, Minkowski's space-time, was not yet available. As Gray memorably puts it (p. 378):

For Poincaré...to have grasped the full implications of special relativity he would have had to be not Einstein, but Minkowski.

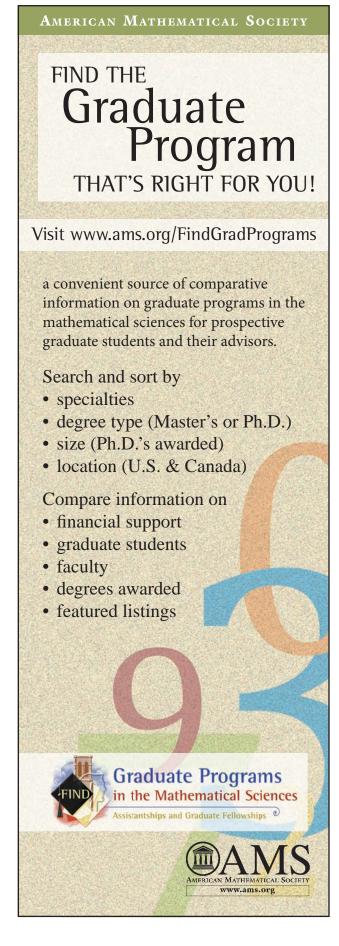
Gray has obviously spent an enormous amount of time immersed in Poincaré's work and has become totally familiar with Poincaré's way of thinking. My only complaint is that occasionally he seems to channel Poincaré only too well, reliving some aspects of Poincaré that are hard for the modern reader to follow. Sometimes complicated geometric arguments are expressed in words when a picture would be clearer; sometimes he is too faithful to Poincaré's notation, as in the topology chapter, where relations in the fundamental group are written additively, even when they are not commutative.

But there is so much excellent exposition in this book that it is easy to skip the occasional difficult formula. I warmly recommend the book to anyone with an interest in the development of modern mathematics. It will surely be the definitive scientific biography of Poincaré for the foreseeable future.

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How Much Undergraduate Research in Mathematics Is Being Done?

John C.D. Diamantopoulos, Michael Dorff, and Shaina Richardson

Undergraduate mathematics students participate in summer Research Experiences for Undergraduates (REUs) and in academic-year undergraduate research. Many undergraduates present research results at professional meetings. Typical outlets include national events such as the Joint Mathematics Meetings (JMM) and regional workshops, like the undergraduate research conferences (RUMCs) sponsored by the Mathematical Association of America and supported by the National Science Foundation. Participants write papers about the results of their research, and some have had these papers published in leading refereed research journals. In a recent article "Undergraduate research in mathematics has come of age" [1], Joe Gallian provides evidence on the tremendous increase in the amount of research being done by undergraduate students. Data about the numbers of students participating in such specific activities can be found. For instance, in 2011 there were sixty-two summer REUs involving roughly 600 undergraduate students, at the 2011 JMM 369 undergraduates presented their research during the undergraduate poster session, and during the 2010-2011 academic year there were 571undergraduate students presenting mathematical talks at the thirty-three RUMCs. Of course, there are more students doing research than attend REUs, present a poster at the JMM, or present a talk at an RUMC. But no one has a ballpark figure on how many undergraduate students are doing research in mathematics each year.

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We decided to address this question by administering a national survey to gather data on how many undergraduate students did original research in mathematics during the 2010–2011 academic year in college and university mathematics departments across the United States. Based on the responses to our survey, we estimate that about 4,500 undergraduate students were engaged in research in 2010–2011.

Procedure

In conducting the survey, we first created a database starting with information about mathematics departments at institutions throughout the United States using the list at http://www.utexas. edu/world/univ/state/. Shaina Richardson, an undergraduate statistics major at Brigham Young University, worked on this survey for a senior project. She visited each institution's website to gather further information for the database.

Next we created a list of questions for the survey. We limited the survey to eight questions—four questions related to the institution and the department and four questions related to undergraduate research—in order to obtain a reasonable response rate, which is about 30 percent for such a survey.

The four questions related to the institution follow:

- (1) In what state does your institution reside?
- (2) What is the highest level of degree your institution offers in mathematics (not including any degrees in mathematics education, statistics, or computer science)?
- (3) How many tenured or tenure-track faculty are in your department?

(4) How many students at your institution are declared mathematical science majors (including mathematics, statistics, actuarial science, and mathematics education, but not including computer science) as of this academic year?

For the four questions related to undergraduate research, there was an issue we needed to address —there is not a universally accepted definition of undergraduate research. Some people consider undergraduate research in mathematics to be a project in which the student explores a topic that is new to him or her but not necessarily to the mathematics community, while others restrict it to a project in which the student investigates an unsolved problem whose result is unknown to the mathematics community. The latter is in line with the definition given by the Council on Undergraduate Research (CUR) (see http:// www.cur.org/about_cur/frequently_asked_ questions_/#2) and by the Mathematical Association of America (MAA) (see http://www.maa.org/ cupm/CUPM-UG-research.pdf), and we decided to go with this. To help avoid confusion among the survey takers, we gave the following definitions at the start of the survey:

Definition of undergraduate research: an investigation by an undergraduate student into an unsolved problem that is likely to result in an original research contribution to the mathematical sciences and may result in a peer-reviewed research publication (typically such investigations require at least 100 hours).

Definition of capstone course/senior project: a culminating project where the student explores a topic new to him or her but which is not original mathematics research as defined above.

With these two definitions we asked the following four questions related to undergraduate research:

- 1. How many students (not limited to mathematical science majors) at your institution are doing undergraduate research in the mathematical sciences in this academic year (fall 2010–spring 2011)?
- 2. How many students at your institution are doing capstone projects/senior projects in the mathematical sciences in this academic year?
- 3. Of the students doing undergraduate research, how many have given at least one presentation (including presenting a talk or poster) at a venue outside of

your department (this includes presentations given at conferences that are sponsored by your department but not attended solely by your department)?

4. Of the students doing undergraduate research, how many have written at least one paper about their current research that has been, or will likely be, submitted for publication?

We created the eight-question survey online through Qualtrics (http://www.qualtrics.com/). From our database we randomly selected roughly an equal number of institutions whose highest degree in mathematics is a bachelor's (153 institutions), master's (155 institutions), and a doctorate (159 institutions) and included a procedure to guarantee that we had representation across the United States. We composed a letter encouraging the department chair or head to participate in the study and sent the email from Michael Dorff as director of the Center for Undergraduate Research in Mathematics (http://curm.byu.edu/). We gave them two weeks to complete the survey and sent out a reminder email half way through.

Survey Results

We received 138 completed or partially completed responses from the 467 institutions sampled (a 29.6 percent response rate, which is typical for such surveys) with thirty-nine out of 153 bachelor's institutions responding, forty-nine out of 155 master's, and fifty out of 159 doctorate institutions and with responses for institutions in fortytwo out of the fifty U.S. states and Washington. D.C. In the case of partially completed responses, we conservatively equated blank responses with the number of students being zero. Also, we have not included data on capstone courses in this article. We are mainly interested in the results about undergraduate research and used the capstone question to help responders differentiate between the two.

From mathematics departments at the thirtynine responding bachelor's institutions, there were 302 faculty members for 1,966 majors of which seventy-five students did an undergraduate research project (working on an unsolved problem that was original to the mathematics community), forty-seven presented their work outside the department, and twenty-four wrote a paper based upon their research. From the forty-nine master's institutions, there were 1.011.5 faculty members for 6,621 majors of which 268 students did an undergraduate research project, 200 presented their work, and ninety-two wrote a research paper. From the fifty doctoral institutions, there were 1,515 faculty members for 11,267 majors of which 574 students did an undergraduate research project,

316 students presented their work, and 139 students wrote a research paper.

Per mathematics department at bachelor's institutions, this averages to 3.8 percent of the majors who worked on undergraduate research, 2.4 percent of the majors who presented their research, and 1.2 percent of the majors who wrote a paper on their research. At master's and doctoral institutions, the averages were, respectively, 4.04 percent and 5.09 percent of the majors who worked on undergraduate research in mathematics, 3.02 percent and 2.80 percent of the majors who presented their research, and 1.39 percent and 1.23 percent of the majors who wrote a paper on their research (see Table 1).

Institution type	% of majors who did under- graduate research in 10-11AY	% of majors who gave a presentation	% of majors who wrote a paper
Bachelor's	3.8 %	2.4%	1.2%
Master's	4%	3%	1.4%
Doctorate	5.1%	2.8%	1.2%
Totals	4.6%	2.8%	1.3%

Table 1. Percentage of students engaged in each category compared to number majoring in mathematical sciences.

If we extrapolate the data to all the mathematics departments in the United States, we get that during the 2010–2011 academic year at bachelor's, master's, and doctoral institutions, respectively, there were approximately (a) 1804, 848, and 1825 undergraduate students who worked on research in mathematics; (b) 1130, 633, and 1005 of these students who presented their research; and (c) 577, 291, and 442 of the students who wrote or co-wrote a paper on their research (see Table 2). It should be noted that there is a possibility of response bias. Specifically, there is a danger that

Institution type	Projected # of students par- ticipating in undergraduate research	Projected # of these students who gave a presentation	Projected # of these students who wrote a paper
Bachelor's	1,804	1,130	577
Master's	848	633	291
Doctorate	1,825	1,005	442
Totals	4477	2768	1310

Table 2. Projected number of students in the United States engaged in each category from fall 2010-spring 2011.

departments with a good track record of producing undergraduate research were more likely to complete this survey, and this could result in an overestimate. From the responses we did receive, there were departments who reported that they were not doing any undergraduate research. We also know there are departments who are doing a significant amount of undergraduate research but because of the random selection were not chosen to participate in the survey. We recommend that any subsequent study might want to make telephone calls to address the question of why some institutions did not participate.

In summary, during the 2010–2011 academic year, about 4.6 percent of the mathematics majors did an undergraduate research project with 60 percent of those students giving a presentation and 28 percent of them writing a paper about their research. Based on the responses to our survey, we estimate that about 4,500 undergraduate students were engaged in research, 2,800 students presented their research, and 1,300 students wrote a paper. These projected numbers are higher

Institution Type	Number of students doing UR per faculty member in the dept.
Bachelors	0.25
Masters	0.26
Doctorate	0.38
Overall %	0.32

Table 3. Number of students doing undergraduate research per faculty member.

than we would have originally thought. Also, we computed the number of students involved in undergraduate research per faculty member in the department (see Table 3). The strong results for doctoral-granting departments may surprise some members of the mathematical community who view undergraduate research as a selling point for smaller schools.

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Descartes's Double Point Method for Tangents: An Old Idea Suggests New Approaches to Calculus

R. Michael Range

Calculus has been around for a long time, evolving into a core requirement for many disciplines. Given the changing and growing clientele, over the past decades there have been extensive and continuing discussions about how to improve the teaching of calculus and the level of understanding of our students. Just remember the "calculus reform" of the 1980s, and—jumping ahead—two recent contributions by K. Stroyan [Str] and F. Quinn [Qui] that have appeared in this column of the *Notices*. Still, the basic *structure* of the introduction to calculus has hardly changed over many years. While the style of presentation has undergone many transformations and—supported by technology graphing and numerical approaches have become more widespread, tangents and derivatives are invariably introduced via the standard approximation process that involves some version of limits. This makes it necessary to study limits, limit theorems, and continuity to lay the foundations for understanding derivatives. Even if discussed in an intuitive and nontechnical form, these topics

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present quite a challenge for the majority of our students. Of course limits and infinite processes cannot be avoided. They are central to the subject, and they are what distinguishes calculus and analysis from algebra, which deals only with finite processes. But are they really necessary at the very beginning?

Limits are a subtle and difficult concept that was properly understood and formalized only about 150 years after the origins of calculus. Furthermore, the typical introductory examples make it difficult to understand what is really going on. For example, calculating tangents to the parabola described by $y = x^2$ at the point (a, a^2) leads to $\lim_{x \to a} \frac{x^2 - a^2}{x - a}$. Since the obvious answer 0/0 is meaningless, one uses algebra to cancel the vanishing x - a, i.e., $\frac{x^2-a^2}{x^2-a^2} = x + a$ for $x \neq a$. Everyone now agrees that the limit is 2a, the result that is obtained by plugging in x = a. Of course the instructor warns that this result, i.e., $\lim_{x\to a} (x+a) = 2a$, requires a proof, since we can't just use x = a in a formula that was derived under the assumption that $x \neq a$. This apparently so simple matter is really quite nontrivial, and it caused a lot of difficulties already in the seventeenth century as calculus was being developed. So it should not surprise us that students still have problems. All other algebraic examples (e.g., x^n , 1/x, \sqrt{x} , and so on) examined in an introduction to calculus

follow the same pattern; i.e., the relevant limit is ultimately obtained by plugging x=a into an algebraic expression. After extensive discussions of limits and limit theorems, this result is eventually justified by the *continuity* of the relevant algebraic expressions. Since continuity—visually reinforced through graphing calculators—is such an intuitively obvious property of all the natural functions encountered by the student, it is not surprising that students find it difficult to understand the *need* for limits at this stage and more often than not forget about limits in the final steps of the calculation of derivatives for standard algebraic functions.

Note that the difficulties with 0/0 disappear if the relevant equation is written in product form $x^2 - a^2 = (x+a)(x-a)$. This latter algebraic identity holds for all x. So how do we justify that the value 2a of the factor q(x) = x + a at x = a is indeed the desired slope of the tangent? As was already recognized by René Descartes (1596-1650), simple algebra readily provides the answer. Given his deep understanding of algebra, it was clear to Descartes that the property that singles out the tangent line through a point *P* on a curve is that it intersects the curve at P with multiplicity greater than one. 1 (See [Des] and [vSc].) Today algebraic geometers are of course well familiar with this definition of the tangent, but it seems that it has been neglected in the teaching of calculus, deferring instead to the definition of tangents as the limiting position of secants. Returning to the parabola, Descartes's idea is implemented as follows. Let $y-a^2=m(x-a)$ be the equation of a line through (a, a^2) with slope m. Its points of intersection with the parabola $y = x^2$ are determined by the solutions of $x^2 - a^2 - m(x - a) = 0$, which factors into

$$(x+a-m)(x-a)=0.$$

The solution x = a has multiplicity 2 if this equation takes the form $(x - a)^2 = 0$; that is, the factor (x + a - m) must also have a zero at x = a. This occurs precisely when m = 2a. It doesn't get simpler than this.²

This elementary technique to identify tangents easily extends to polynomials and rational functions, and with a bit more work also to their local inverses and so on, and ultimately to all functions defined by algebraic expressions. Similarly, all standard differentiation formulas are verified in a straightforward manner. It is quite noteworthy that the chain rule—usually viewed as the deepest rule for differentiation—turns out to be most elementary. To summarize the main conclusion, if a is in the domain of an algebraic function f, a then there exists a factorization

(1)
$$f(x) - f(a) = q(x)(x - a),$$

where q is an algebraic function defined on the domain of f and the value q(a) of q at a is the slope of that unique line through (a, f(a)) that intersects the graph of f with multiplicity greater than one. In other words, the value q(a) is the derivative D(f)(a) of f at a. Full details, and more, may be found in [Ran].

How could this algebraic approach be used in the teaching of calculus?

First of all, in the case of polynomials, the basic factorization (1), the related information about zeroes, and the notion of multiplicity of such zeroes is standard high school material. These methods thus provide a simple solution for the tangent problem for all polynomials, i.e., for a deep problem with a long history that goes back to Greek geometers over 2,000 years ago, and that was one of the principal driving forces in the development of calculus in the seventeenth century. Just one simple additional step gives the corresponding result for all rational functions. All the standard rules of differentiation can easily be obtained in this setting and—if desired—can be extended to root functions and more. Shouldn't high school teachers take a look at this and use it in their algebra classes?

Next, the direct algebraic approach to derivatives avoids the introduction of deep new concepts involving limits and continuity early on in a context where—as we just saw—they clearly are not necessary and may even cause confusion. If the major part of a first course in calculus ends up focusing on the mechanical aspects of differentiation anyway, primarily involving algebraic functions, shouldn't it help the students to be able to do all that without having to worry about limits?

¹Descartes actually was interested in the normal to a curve. For example, he considered circles that intersect an ellipse at a point P, with center on one of the axes, and noted that such a circle is tangential to the ellipse at P precisely when P is a double point of intersection. Algebra allowed him to identify that tangential circle, whose normal at P is then the desired normal to the ellipse. Descartes's expositor F. van Schooten explicitly constructed tangents to a parabola by Descartes's double point method.

²The details were quite a bit more complicated in the seventeenth century, apparently due to the fact that the point-slope form of lines was not used at that time. This required the introduction of a second (distant) point to describe lines. See [Ran] for more details.

 $^{^3}$ Critical values of a function f need to be excluded from the domain of its inverse. The condition $f'(a) \neq 0$ is needed to ensure that the inverse has an appropriate factorization and hence is differentiable at the point b = f(a). For example, the appropriate domain of $g(x) = \sqrt{x}$ in calculus is the open set of positive numbers.

Of course the algebraic method reaches its limits when one considers important non algebraic functions such as exponential and trigonometric functions. However, it still helps, as it provides motivation for and a direct approach to, continuity, thereby suggesting how to handle more general functions. In more detail, since polynomials are trivially bounded on any finite interval, if f (and hence also g) is a polynomial, the factorization (1) implies the estimate

(2)
$$|f(x) - f(a)| \le K|x - a| \text{ for all } x$$
 in an interval centered at a ,

for a suitable constant K. Surely this estimate exhibits the essence of continuity, i.e., that $f(x) \to f(a)$ as $x \to a$, in a precise and much stronger form than necessary. Without any additional work (aside from giving the intuitively obvious property a special name), we thus see that every polynomial is continuous. How does this compare to the typical proof in calculus and analysis texts?

Since the factorization (1) and the *local* boundedness remain correct for all algebraic functions (as indicated earlier, this requires just a bit more work), so does the estimate (2). It therefore follows immediately that all algebraic functions are continuous as well at all points of their domains. We get all this in a precise form without any formal mention of limits. Instead, it is the estimate (2) that follows from the algebraic factorization that *motivates* the idea of continuity and—implicitly—of limits.

By combining the algebraic derivative (obtained via multiplicities) with continuity, one sees that the factor q in the factorization (1) satisfies $q(x) \rightarrow q(a)$ as $x \rightarrow a$; i.e., for $x \neq a$ one obtains

$$\frac{f(x) - f(a)}{x - a} = q(x) \rightarrow q(a) = D(f)(a)$$
$$= f'(a) \text{ as } x \rightarrow a.$$

One therefore recognizes that the exact algebraic derivative can also be captured by an *approximation process*. In other words, the concept of derivative as the limit of certain difference quotients results as the culmination of Descartes's algebraic approach to derivatives. This is the critical new insight that needs to be implemented when studying nonalgebraic functions. Wouldn't it help our students to meet limits only at this point, that is, in a context where they really are needed? For example, as the derivative of $E_2(x) = 2^x$ at 0 cannot be captured by any finite familiar explicit formula, it must be described by an elusive limit 0.69314... that eventually is identified with $\ln 2$.

Finally, the preceding discussion shows that if *a* is a point in the domain of the algebraic function

f, then

(3)
$$f(x) - f(a) = q(x)(x - a)$$
, where q is continuous at a .

Since in this case the factor *q* is algebraic as well, continuity holds even in the strong version given by the estimate (2). However, if *q* is only assumed to be continuous in the most general sense, as expressed by $q(x) \rightarrow q(a)$ as $x \rightarrow a$, then statement (3) is clearly equivalent to the standard definition of differentiability of f at a via limits of difference quotients, with $D(f)(a) = \lim_{x \to a} q(x) = q(a)$. Should the property identified in (3) be used as the primary definition of differentiability? Just try to prove the chain rule based on this definition, and you will recognize one of its major advantages. Furthermore, this formulation relates directly to the fundamental idea that differentiability is equivalent to good local linear approximation: just rearrange (3) in the form

$$f(x) - [f(a) + g(a)(x - a)] = [g(x) - g(a)](x - a).$$

Lastly, this definition generalizes in a most natural way to functions and maps of several variables, allowing for an equally simple proof of the chain rule as in one variable. All of this is hardly new but unfortunately not widely known. In fact, this definition was introduced by C. Carathéodory [Car] already in the middle of the last century, and it has been used successfully by him and other authors in Germany. (See [Ran] for more details and other references.)

I hope that these remarks will convince the reader that there are alternatives to the standard limit-based introduction to calculus. Students in beginning calculus courses and in analysis courses have reacted favorably to this approach. I believe that anyone teaching calculus or involved in writing introductory calculus or analysis texts should think about the questions raised here. As for myself, I have been working on an introduction to calculus that builds upon the ideas discussed here.

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Are Libraries and Open Access Becoming Irrelevant?

Andrew Odlyzko

Open access is a hot topic among mathematicians and other scholars, and so are calls for boycotts of commercial publishers. But the real action in scholarly publishing is elsewhere. The community's reliance on publishers is growing, while that on libraries is decreasing. "Big Deal" packages, in which institutions obtain access to a large bundle of journals, are becoming more comprehensive and are reaching an increasing number of institutions. Books are increasingly also covered by similar bundling deals. This provides an illusion of true open access, in that growing numbers of scholars have seamless access to growing fractions of the material they use.

This process marginalizes libraries and entrenches publishers with their unnecessarily high costs. Few scholars (and few librarians) think of the scholarly information dissemination system as one in which publishers and librarians compete for resources. But that is the reality, and it should be recognized that the reason publishers are winning this competition is that Big Deals are good for society in the near term, as they provide much greater and more egalitarian access to the available literature.

The beneficial effects of Big Deals can be demonstrated using Figure 1. It is based on data from the Association of Research Libraries (ARL). It shows that over the last decade, there has

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Serials in research libraries

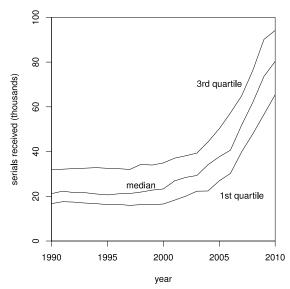


Figure 1. Number of serials available in research libraries from 1990 to 2010. Shows the median as well as the first and third quartiles of the number of serials received by academic members of ARL.

been a dramatic increase in the number of serials available at ARL members. This occurred even though spending on serials was increasing at a slightly lower pace in the 2000–2010 period than in the 1990s. Further statistics and analyses are provided in [3], on which this piece is based. The data and analysis covers just ARL, whose members are most of the large academic libraries in the United States and Canada. It therefore

says nothing about the majority of institutions of higher education in North America, nor about the rest of the world. However, anecdotal evidence suggests they are also benefiting from special pricing, becoming members of consortia, and other measures.

ARL statistics are strictly about quantity, not quality. There are frequent suggestions that the "journal crisis" could be solved by greater selectivity. While quality is important, we have to recognize that historical growth in the volume of publications is continuing and desirable. For the purposes of this brief discussion, I am assuming that more is better, and I leave finer points of this issue aside.

The phenomenon documented by Figure 1 appears to be consistent with personal impressions. Most readers of the *Notices* surely have noticed that a greater fraction of the papers that are available to them at their institutions are available online and are accessible by clicking on the link in MathSciNet reviews. They may also have noticed that a far greater fraction of all papers listed in MathSciNet are available to them than before. This is the result of the Big Deals and the price discrimination they implement. Publishers try to insist on nondisclosure of contract details, but Ted Bergstrom of UC Santa Barbara and his colleagues have used freedom of information requests to pry loose some of this information. As a result we now know, for example, that in 2007 the University of Michigan paid US\$1.96 million for access to all Elsevier journals, while the University of Montana paid just US\$442 thousand. Thus a notorious profit-maximizing capitalist organization was (and is) engaging in a very socialist practice! But that is not surprising. In fact, the only surprising fact is that it took so long for the Big Deals and their discriminatory pricing practices to spread. When marginal costs are low, as they are with electronic-only access to academic journals, the basic economic incentive is to maximize usage by charging each customer what that customer can afford. Therefore Figure 1 shows not only that more journals are available to a typical institution, but that the gaps between the rich and the (relatively) poor have narrowed.

The conclusion is that publishers are indeed correct when they claim that they are providing better service to the academic community, even while their revenues and profits climb. This does not mean, though, that they are providing optimal service.

I started studying electronic publishing two decades ago, stimulated by the issues faced by a small committee of the AMS that I was asked to chair. This effort led to the paper [1] and made me a fervent advocate of open access and of pushing costs of scholarly publishing to a much lower

level. (Open access and publishing costs are often conflated, but while they are not independent, they represent different axes of the publishing space.)

The main failure of [1] was in underestimating the inertia of the academic system. A prediction made there was that little change would be visible in the traditional high-cost journal system for a decade, but that it would likely collapse within two decades and be replaced by a lower-cost open access system, with many of the journals run by scholars themselves. The first part of this prediction has come true, but not the second. An explanation (even if not an adequate excuse) for this was that the paper was written at an industrial research lab, so with an inadequate appreciation of how slowly everything changes in academia. Had I then had my current experience as a professor and university administrator, I would likely have doubled both estimates. However, if current trends persist, the collapse prediction may not come true at all.

The key to understanding what is happening and in particular how the "unsustainable journal price escalation" has been sustained for a long time comes from the observation (made in [1], see also [2]) that most of the costs of the typical academic library in rich countries like the United States are internal. Curating huge collections of paper volumes is a challenge that requires large numbers of professionals, many of them highly trained, as well as expensive physical facilities. Back in the early 1990s, only about 25 percent of the library budget went for outside purchases of books, journals, and other materials. Even today, this proportion is only around 33 percent (see [3] for more details and references). Publishers have been able to increase their revenue and profits by getting a larger piece of the library pie and disintermediating the librarians. But this process can go much further as we move further towards relying on digital formats and continue to reduce usage of printed copies.

Just as Google can handle email for many universities at low cost, so Google, the Internet Archive, JSTOR, and the publishers are able to provide library services in a more efficient centralized way than used to be done. Note that even if the inexpensive open access journal system that I (along with many others) envisaged had come into being, it would still have been true that many traditional functions of libraries and librarians would have become much less significant. While those traditional functions shrink, a variety of new information handling jobs are opening up, as that inevitable growth in volume of information creates new challenges. But librarians have to compete for those jobs with publishers and other agents.



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In my view, the optimal course would have been for librarians and scholars to cooperate in setting up a new system of low-cost open access journals. The resources and organizational skills of the librarians, combined with the subject expertise of the scholars, could have led to a much more effective and efficient system for disseminating research. That would have forced publishers to move into the new functions that are opening up. (As with medical care, there is little likelihood of lowering the costs of the entire scholarly information ecosystem, as the savings in one area are eaten up by new services.) However, publishers suffered from a lower degree of inertia than that of scholars and librarians and have moved faster. They have built a system in which the incentives for scholars to move to true open access are decreasing, as the papers and books those scholars need are just a click away (because they are paid for by the libraries in ways invisible to those scholars). As they include more materials in their Big Deals, especially books, the justification for keeping the expensive traditional library will be diminishing.

Publishers may also be able to adapt to the open access mandates that are increasingly coming from funding agencies and some academic institutions. As long as those mandates can be accommodated with gold open access (where authors or authors' funding agencies or institutions pay, very likely often with money squeezed out of library budgets), publishers might be able to maintain their unnecessarily expensive system. It's not the ideal solution, perhaps, but one that will still be better than what we have now, where much of the valuable information is available in only a handful of libraries in the richest institutions. There will still be opportunities to push for lower costs in basic journal publishing. But for the time being, scholars will likely have to accept the continuation of publishers squeezing librarians out of their functions and resources.

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2014 Steele Prizes

The 2014 AMS Leroy P. Steele Prizes were presented at the 120th Annual Meeting of the AMS in Baltimore, Maryland, in January 2014. The Steele Prizes were awarded to Luis A. Caffarelli, Robert Kohn, and Louis Nirenberg for Seminal Contribution to Research; Dmitri Y. Burago, Yuri D. Burago, and Sergei V. Ivanov for Mathematical Exposition; and Phillip A. Griffiths for Lifetime Achievement.

Seminal Contribution to Research: Luis A. Caffarelli, Robert Kohn, and Louis Nirenberg

Citation

The 2014 Leroy P. Steele Prize for Seminal Contribution to Research is awarded to Luis A. Caffarelli, Robert Kohn, and Louis Nirenberg for their paper "Partial Regularity for Suitable Weak Solutions of the Navier-Stokes Equation", *Communications on Pure and Applied Mathematics* **35** (1982), no. 6, 771-831. This paper was and remains a landmark in the understanding of the behavior of solutions to the Navier-Stokes equations and has been a source of inspiration for a generation of mathematicians.

The Navier–Stokes equations are fundamental to the mathematical understanding of fluid dynamics. The pioneering works of Leray and later Hopf established the existence, globally in time, of the Leray–Hopf *weak* solutions. The arguments of Leray and Hopf left open the possibility that these weak solutions fail to be smooth at a rather sparse closed set of times (of finite 1/2-Hausdorff dimension in time) where uniqueness can potentially be lost.

The work of Caffarelli, Kohn, and Nirenberg was a huge leap forward on this notoriously difficult problem. Building on work of V. Scheffer, Caffarelli, Kohn, and Nirenberg greatly improved Scheffer's results and earlier work of others on the Navier-Stokes equation. They proved the existence of so-called *suitable* weak solutions as well as their partial regularity. They obtain constraints on the size of the singular set in space and time. The singular set has zero measure with respect to a natural one-dimensional Hausdorff measure defined using a parabolic notion of distance. One important consequence is that any two singular points can be separated by a space-time cylinder on the boundaries of which the solution is regular.

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The paper has been a kind of textbook for a whole generation of Navier-Stokes researchers, motivating many of the later developments and simplifications. Though a solution to the global existence of smooth solutions of the Cauchy problems remains open and is now one of the Clay Millennium Problems, the partial regularity theorem of Caffarelli, Kohn, and Nirenberg remains among the sharpest results in this direction.

Biographical Sketches

Luis A. Caffarelli obtained his Master's of Science (1969) and Ph.D. (1972) at the University of Buenos Aires. Since 1996, he has held the Sid Richardson Chair in Mathematics at the Univer-



Luis A. Caffarelli



Robert Kohn

sity of Texas at Austin. Some of his most significant contributions are the regularity of free boundary problems and solutions to nonlinear elliptic partial differential equations, optimal transportation theory, and, more recently, results in the theory of homogenization.

In 1991 he was elected to the National Academy of Sciences. He received the Bôcher Memorial Prize in 1984. He also received the prestigious 2005 Rolf Schock Prize in Mathematics of the Royal Swedish Academy of Sciences, the 2009 Leroy P. Steele Prize for Lifetime Achievement in Mathematics, and the 2012 Wolf Prize.

Robert Vita Kohn graduated from Harvard University in 1974. He got his M.Sc. at the University of Warwick in

1975, then went to Princeton, where he got his Ph.D. in 1979 under the guidance of Fred Almgren. From 1979–1981 he was a visiting member at New York University's Courant Institute of Mathematical Sciences, supported by an NSF Mathematical Sciences Postdoctoral Fellowship. In 1981 he joined the faculty of the Courant Institute, and



Louis Nirenberg

he has remained there since. He has been professor of mathematics at NYU since 1988.

Besides the Navier-Stokes equations, he has worked on many PDE problems, including electric impedance tomography, relaxation of variational problems, blowup of semilinear heat equations, and homogenization. In recent years much of his re-

search has addressed problems from materials science, with particular emphasis on energy-driven pattern formation. His recognitions include an Alfred P. Sloan Foundation Fellowship (1984–1986) and SIAM's Ralph E. Kleinman Prize (1999). He was a Plenary Speaker at ICM 2006 Madrid and an Invited Speaker at ICIAM 2007, and he is both a Fellow of the American Mathematical Society and a SIAM Fellow.

Louis Nirenberg, professor emeritus at Courant Institute, New York University, received his B.Sc. at McGill University in 1945 and his Ph.D. at New York University in 1949. His entire professional career has been at NYU. His research interests are in mathematical analysis, in particular, partial differential equations, complex analysis, differential geometry, and fluid flow. He also has a deep interest in music and cinema.

Joint Response from Luis A. Caffarelli, Robert Kohn, and Louis Nirenberg

It is a great honor to receive this award. The Navier–Stokes equation was originally proposed to describe incompressible viscous flows in moderate regimes. Its study has led over the years to many fundamental and challenging problems, which have motivated the development of deep ideas in many areas of analysis and applied mathematics. Any work on it relies strongly on the deep contributions of many who came before.

In this regard, we particularly want to thank and recognize Vladimir Scheffer for his fundamental contributions to this area. Scheffer was the first to apply partial regularity methods to three-dimensional Navier–Stokes—asking, around 1976, what can be said about the size of the singular set. His papers on partial regularity drew our attention to the problem, and his ideas were crucial to our analysis.

Many of the central issues concerning Navier–Stokes are still open. Deep work continues to be motivated by challenges such as the regularity and uniqueness of solutions, the relationship to turbulence, and the Euler equations, etc.

We did this work at a time when the three of us were at the Courant Institute, benefitting from its wonderful atmosphere. We would like to thank our colleagues at the time for their support.

Finally, we thank the Steele Prize Committee and the American Mathematical Society for choosing us for this award.

Mathematical Exposition: Dmitri Y. Burago, Yuri D. Burago, and Sergei V. Ivanov

Citation

The 2014 Leroy P. Steele Prize for Mathematical Exposition is awarded to Dmitri Y. Burago, Yuri



Dmitri Y. Burago



Yuri D. Burago



Sergei Ivanov

D. Burago, and Sergei A. Ivanov for the book *A Course in Metric Geometry*, Graduate Studies in Mathematics, 33, American Mathematical Society, Providence, RI, 2001, in recognition of excellence in exposition and promotion of fruitful ideas in geometry.

The publishing of this book made available to the mathematical community the emerging ideas and methods of synthetic geometry, initiated by Alexandrov and Gromov. These ideas provided a completely new approach to differential geometry replacing the traditional heavy analytic machinery by a description based on easily accessible, simple geometric axioms that have an immediate appeal to geometric intuition.

An influential contribution through the years, this book provided fundamental tools in connection with geodesically convex spaces, optimal transportation in Alexandrov spaces with curvature bounded below, and has been widely referred to recently in connection to the solution of the Geometrization Conjecture.

This book has clearly left a visible imprint on the landscape of today's geometry. It provides great help to orient students in the introductory studies of synthetic methods and to guide young geometers in their research.

Biographical Sketches

Dmitri (Dima) Burago, a Distinguished Professor at the Pennsylvania State University, received his degree from Leningrad (now St. Petersburg) State University. He moved to the U.S. about twenty years ago. Before that, he was doing lots of crazy things: mathematics, whitewater kayaking, a small zoo at home, kickboxing, and much more. He was teaching school kids and is very proud of his students. He had fantastic teachers, and all he has done in his life is due to their input. In the U.S., he continues doing strange things, including mathematics, Russian literature, painting, and of course, teaching. He has received a prize of the St. Petersburg Mathematical Society and has a Faculty Medal from Penn State. He spoke at the 1998 ICM in Berlin.

Yuri Burago was born in St. Petersburg (formerly Leningrad), Russia, in 1936. He graduated from St. Petersburg State University in 1959 and received his Ph.D. in 1961. His advisers were A. D. Alexandrov and V. A. Zalgaller. Yuri Burago defended his doctoral thesis (the second degree in Russia) in 1969. From 1962 to the present he has been the head of the geometry and topology laboratory. Further, he is a full professor (half position) at St. Petersburg State University. Among Yuri Burago's students are Sergei Buyalo, Grisha Perelman, and Anton Petrunin. Yuri Burago worked in a variety of areas of geometry that include so-called 2-manifolds of bounded curvature, irregular surfaces in Euclidean spaces, Riemannian geometry in the large, Alexandrov spaces, and even the theory of functions in irregular domains. He wrote several books, including Introduction to Riemannian Geometry (only in Russian) and Geometric Inequalities, both jointly with Victor Zalgaller.

Sergei Ivanov was born in St. Petersburg, Russia, in 1972. He received his Ph.D. from St. Petersburg State University in 1996. He spent most of his scientific career at St. Petersburg Department of Steklov Mathematical Institute, where he is currently a principal research fellow. He combines this position with teaching at the Mathematics and Mechanics Department of St. Petersburg State University. He was an invited speaker at the ICM (Hyderabad 2010) and was elected as a corresponding member of the Russian Academy of Sciences in 2011.

Joint Response from Dima Burago, Yuri Burago, and Sergei Ivanov

We are honored and grateful. There are many other books that deserve the prize, so the fact that we are selected makes us humble and speechless.

It took us more than three years to complete the work, and it would take forever without support and encouragement from our colleagues and AMS editors. We are grateful to all the colleagues who supported us and to the people who taught us.

We ourselves represent three generations of Russian geometric tradition: Dmitri's first and primary teacher was Yuri, and in his turn Dmitri taught Sergei.

Our aim when writing the book was to try to bridge the gap between students and the existing literature on the subject. In particular, we kept in mind some of Gromov's works as "bridge destinations". We are happy to know that the book turned out to be useful to many students and researchers.

The list of people without whom this work would be impossible is perhaps too long for this response, especially since there are three of us. We just want to thank all of them!

Lifetime Achievement: Phillip A. Griffiths

The 2014 Leroy Steele Prize for Lifetime Achievement is awarded to Phillip A. Griffiths for his



Phillip A. Griffiths

contributions to our fundamental knowledge in mathematics, particularly algebraic geometry, differential geometry, and differential equations.

It would not be possible in the space of this column to give a detailed description of all of the areas in which Phillip A. Griffiths has made essential and fundamental contributions in mathematics.

Griffiths' work in algebraic geometry has inspired at least two generations of leading mathematicians working in this area, and it will undoubtedly continue to do so long into the future. In differential geometry and differential equations, too, Griffiths has made many fundamental contributions. While his initial interest in these subjects was partly due to their immediate utility in algebraic and complex geometry and partly due to the influence of his postdoctoral mentor, Shiing-shen Chern, Griffiths developed a style and research program that were all his own and that have proved extraordinarily fertile.

Beginning with his beautiful 1974 article, "On Cartan's method of Lie groups and moving frames as applied to uniqueness and existence questions in differential geometry", *Duke Mathematical Journal* 41 (1974), 775–814, he brought to bear classical techniques on a variety of problems in real and complex geometry and laid out a program of applications to period mappings, Nevanlinna theory, integral geometry, and transcendental methods in algebraic geometry. This bore fruit in many papers over the years,

particularly his 1978 paper with Chern, "Abel's theorem and webs", *Jahresberichte der Deutschen Mathematiker Vereinigung* **80** (1978), no. 1–2, 13–110, which presaged many of the recent developments in the recently active area of web geometry, and his 1979 paper with Joseph Harris, "Algebraic geometry and local differential geometry", *Annales Scientifiques de l'École Normale Supérieure* (4) **12** (1979), no. 3, 355–452.

His discovery of and investigations into what are now called the Griffiths infinitesimal period relations on period domains, which are of fundamental importance in moduli problems in algebraic geometry, stimulated his interest in overdetermined systems of differential equations. As a consequence, he led a revitalization of this subject in the 1980s in the form of exterior differential systems. Griffiths applied exterior differential systems to a number of different problems, not just in algebraic or complex differential geometry, but also to attack deep problems in modern differential geometry: rigidity of isometric embeddings in the overdetermined case and local existence of smooth solutions in the determined case in dimension 3, drawing deep results in hyperbolic PDEs (in collaborations with Berger, Bryant, and Yang); geometric formulations of integrability in the calculus of variations and in the geometry of Lax pairs; and treatises on the geometry of conservation laws and variational problems in elliptic, hyperbolic, and parabolic PDEs and exterior differential systems. All of these areas are currently seeing important developments that were stimulated by his work.

Phillip Griffiths' teaching career and research leadership, well measured by the numbers of mentored individuals who have gone on to stellar careers in mathematics and other disciplines, is simply astounding. His expository gifts and his nurturing of mathematical talent have not been reserved for his students alone. Not only has he been generous with his time, but he has written many classic expository papers and books, such as *Principles of Algebraic Geometry* with Joseph Harris, that have remained in print and inspired students of the subject since the 1960s.

A further fundamental characteristic of Phillip Griffiths is his extensive support of mathematics, both personally at the level of research and education and nationally and internationally through committees and boards he has chaired or served on. He has carried on a remarkable research career while serving eight years as Duke University's provost and twelve years as the director of the Institute for Advanced Study, and he currently chairs the Science Initiative Group, whose mission is assisting the development of mathematical training centers in the developing world. His example of service and leadership has inspired many in the mathematics community to emulate him to some

degree, and our mathematical world is much the richer for it.

The Leroy P. Steele Prize for Lifetime Achievement is a further recognition to his dedication, generosity, and inspired leadership that surely fits the fiftieth anniversary of his receiving his Ph.D. from Princeton.

Biographical Sketch

Phillip A. Griffiths is Professor Emeritus in the School of Mathematics at the Institute for Advanced Study, where he was the director from 1991–2003 and a professor from 2004–2009. He was previously provost of Duke University. He has taught mathematics at Duke University, Harvard University, Princeton University, and the University of California Berkeley.

Dr. Griffiths was born in 1938 in Raleigh, North Carolina, and received his Bachelor of Science from Wake Forest University in 1959 and his Ph.D. from Princeton University in 1962. He was a Miller Fellow at UC Berkeley from 1962–1964 and again in 1976.

He is a member of the National Academy of Sciences and the American Philosophical Society and a foreign associate of the Accademia Nazionale dei Lincei, the World Academy of Sciences (TWAS), and the Indian Academy of Sciences. Dr. Griffiths was Chair of the Board on Mathematical Sciences at the National Research Council from 1986 to 1991, a member of the National Science Board from 1991 to 1996, Chair of the Committee on Science, Engineering and Public Policy at NAS/NAE/IOM from 1992 to 1999, Chair of the Program Committee for the International Congress of Mathematicians from 1995 to 1998, Secretary of the International Mathematical Union from 1999 to 2006, and co-chair of the Carnegie-IAS Commission on Mathematics and Science Education from 2007 to 2009. He received the Steele Prize for his paper "Periods of integrals on algebraic manifolds", Bulletin of the American Mathematical Society 7 (1970), 228–296, and more recently the Wolf and Brouwer Prizes.

Dr. Griffiths chairs the Science Initiative Group, an international team of scientists dedicated to building science and engineering capacity in developing countries through innovative programs, including the Millennium Science Initiative (MSI) and the Regional Initiative in Science and Education in Africa (RISE).

$Response\,from\,Phillip\,A.\,Griffiths$

It is a wonderful honor to receive the Leroy P. Steele Prize for Lifetime Achievement. I credit my high school math teacher, Lottie Wilson at the Georgia Military Academy, for sparking my interest in and love for mathematics. From my thesis advisor at Princeton University, Don Spencer, and my postdoctoral mentor at UC Berkeley, S. S. Chern, I learned how to think about math. From my students, collaborators and colleagues I have received far more than I could possibly have given. Finally, my wife, Taffy, our four children, and my

colleagues in other activities have encouraged me to do what I love and have put up with a frequently distracted mathematician. To all of the above, and to the AMS and the selection committee for this award, I owe my deepest gratitude.

About the Prize

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905-1906, and Birkhoff served in that capacity during 1925–1926. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) Mathematical Exposition: for a book or substantial survey or expository research paper; (3) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field or a model of important research. Each Steele Prize carries a cash award of US\$5,000.

The list of previous recipients of the Steele Prize may be found on the AMS website at http://www.ams.org/prizes-awards.

Beginning with the 1994 prize, there has been a five-year cycle of fields for the Seminal Contribution to Research Award. For the 2014 prize, the field was analysis. The Steele Prizes are awarded by the AMS Council acting on the recommendation of a selection committee.

For the 2014 awards, the members of the selection subcommittee for the Seminal Contribution to Research Award were Richard T. Durrett, Jeffrey C. Lagarias, Nikolai Makarov, Tomasz S. Mrowka (Chair), Andrei Okounkov, Gang Tian, Lai-Sang Young, and Efim I. Zelmanov. The members of the selection subcommittee for the Mathematical Exposition Award were Richard T. Durrett, Irene M. Gamba (Chair), Jeffrey C. Lagarias, Nikolai Makarov, Tomasz S. Mrowka, Andrei Okounkov, Gang Tian, Lai-Sang Young, and Efim I. Zelmanov. The members of the selection subcommittee for the Lifetime Achievement Prize were Richard T. Durrett. Irene M. Gamba (Chair), Jeffrey C. Lagarias, Nikolai Makarov, Tomasz S. Mrowka, Andrei Okounkov, Gang Tian, Lai-Sang Young, and Efim I. Zelmanov. -Elaine Kehoe

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The photo of Yuri Burago is courtesy of the St. Petersburg Division of the Steklov Math Institute; the painting of Dmitri Burago is by Dmitri Burago; photo of Luis Caffarelli is courtesy of ICES at the University of Texas Austin; photo of Phillip Griffiths is by Cliff Moore.



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2014 Bôcher Prize



Simon Brendle

SIMON BRENDLE was awarded the 2014 Bôcher Memorial Prize at the 120th Annual Meeting of the AMS in Baltimore, Maryland, in January 2014.

Citation

The 2014 Bôcher Memorial Prize is awarded to Simon Brendle for his outstanding solutions of long-standing problems in geometric analysis, including the solution, with R. Schoen, of the differentiable sphere theorem in "Manifolds with 1/4-pinched curvature are space forms", *Journal of*

the American Mathematical Society **22** (2009), no. 1, 297–307, and the solution of the Lawson Conjecture (to appear in *Acta Mathematica*, 2013). The committee also recognizes Brendle's deep contributions to the study of the Yamabe equation.

Biographical Note

Simon Brendle was born in Tübingen, Germany, in 1981. He received his Ph.D. from Tübingen University in 2001 under the direction of Gerhard Huisken. He has served on the faculty at Princeton University and is currently a professor at Stanford University. He has held visiting professorships at ETH Zürich and at Princeton University. In 2006 he was awarded an Alfred P. Sloan Fellowship, and in 2012 he received the EMS Prize of the European Mathematical Society.

Response from Simon Brendle

I feel very honored to receive the 2014 Bôcher Memorial Prize of the American Mathematical Society. I am grateful to my parents, Helga and Martin Brendle, and my high school mathematics teacher Jakob Nill, who provided me with an excellent education in mathematics. Finally, I am indebted to Richard Hamilton, whose groundbreaking work on the Ricci flow in the 1980s formed the basis for the proof of the Differentiable Sphere Theorem, and

to Gerhard Huisken who, in 1997, introduced the idea of two-point functions which later played an important role in the proof of Lawson's Conjecture.

About the Prize

Established in 1923, the prize honors the memory of Maxime Bôcher (1867–1918), who was the Society's second Colloquium Lecturer in 1896 and who served as AMS president during 1909–1910. Bôcher was also one of the founding editors of *Transactions of the AMS*. The original endowment was contributed by members of the Society. The prize is awarded for a notable paper in analysis published during the preceding six years. To be eligible, the author should be a member of the AMS, or the paper should have been published in a recognized North American journal. The prize is given every three years and carries a cash award of US\$5,000.

The Bôcher Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2014 prize, the members of the selection committee were Ronald R. Coifman, Sun Yung A. Chang, and Gunther A. Uhlmann.

Previous recipients of the Bôcher Prize are G. D. Birkhoff (1923), E. T. Bell (1924), Solomon Lefschetz (1924), J. W. Alexander (1928), Marston Morse (1933), Norbert Wiener (1933), John von Neumann (1938), Jesse Douglas (1943), A. C. Schaeffer and D. C. Spencer (1948), Norman Levinson (1953), Louis Nirenberg (1959), Paul J. Cohen (1964), I. M. Singer (1969), Donald S. Ornstein (1974), Alberto P. Calderón (1979), Luis A. Caffarelli (1984), Richard B. Melrose (1984), Richard M. Schoen (1989), Leon Simon (1994), Demetrios Christodoulou (1999), Sergiu Klainerman (1999), Thomas Wolff (1999), Daniel Tataru (2002), Terence Tao (2002), Fanghua Lin (2002), Frank Merle (2005), and Charles Fefferman, Carlos Kenig, Alberto Bressan (2008), and Assaf Naor and Gunther Uhlmann (2011).

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2014 Cole Prize in Number Theory

YITANG ZHANG, DANIEL GOLDSTON, JÁNOS PINTZ, and CEM Y. YILDIRIM were awarded the 2014 Cole Prizes in Number Theory at the 120th Annual Meeting of the AMS in Baltimore, Maryland, in January 2014.

Citation

The 2014 Frank Nelson Cole Prize in Number Theory is awarded to Yitang Zhang for his work on bounded gaps between primes, and to Daniel



Yitang Zhang



Daniel Goldston

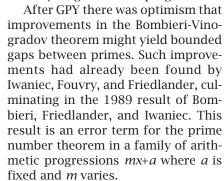
Goldston, János Pintz, and Cem Y. Yıldırım for their work on small gaps between primes. This work appeared in two papers: Yitang Zhang, "Bounded gaps between primes", Annals of Mathematics (2) (to appear); and Daniel Goldston, János Pintz, and Cem Y. Yıldırım, "Primes in tuples. I", Annals of Mathematics (2) 170 (2009), no. 2, 819-862.

One of the oldest problems in number theory is the twin prime conjecture: that there are infinitely many pairs p and q of primes with p-q=2. With the Goldbach conjecture, this is an archetypal problem that motivated the development of sieve theory by Brun, Linnik, Chen, Selberg, Bombieri, Iwaniec, Fried-

lander, Heath-Brown, and many others. Yet the twin prime conjecture seemed far out of reach when in 2005, Goldston, Pintz, and Yıldırım (GPY) showed that for every ϵ >0, there exist infinitely many pairs p and q of distinct primes with $|p-q| < \epsilon \log(p)$. Their very surprising proof of this breakthrough result used standard tools—the Selberg

DOI: http://dx.doi.org/10.1090/noti1110

sieve and the Bombieri-Vinogradov theorem—together with innovative new ideas that are essentially combinatorial. The Bombieri-Vinogradov theorem is an error term for the prime number theorem for primes in an arithmetic progression. It is often used as a substitute for the generalized Riemann hypothesis in arithmetic applications. The Selberg sieve is a flexible tool, and the authors found a new and ingenious way of applying it to obtain their result. Slightly earlier work of Goldston and Yıldırım on this idea played a role in the proof by Green and Tao that primes exist in arbitrarily long arithmetic progressions.





János Pintz



Cem Y. Yıldırım

Yet within a year or two of GPY, no one saw how to use the results of Bombieri, Friedlander, and Iwaniec to prove bounded gaps between primes. So the initial optimism gave way to pessimism. But then, very unexpectedly, Yitang Zhang did find a way, and he proved the striking result that there are infinitely many pairs p,q of distinct primes with $|p-q|<7\times10^7$. Zhang saw that a different modification of the Bombieri-Vinogradov theorem could be used, in which the modulus *m* is constrained, together with a modification of the sieve ideas of Goldston, Pintz, and Yıldırım. Beyond the crucial initial insight, carrying out this plan combines input from several of number theory's most illustrious ideas. For example, the Riemann hypothesis for curves, due to Weil, and that for varieties, due to Deligne, are essential parts for his argument, as is the dispersion method of Linnik, which is used to transport the sieve inequality to the relevant exponential sum bounds.

Photo of D. Goldston by Ryoko Goldston. Photo of Janos Pri László Szelényi.

Biographical Sketch: Yitang Zhang

Yitang Zhang received his B.S. from Peking University and his Ph.D. from Purdue University. Since 1999, he has been a lecturer in the Department of Mathematics and Statistics of the University of New Hampshire. His research is mainly in the field of analytic number theory, in particular the distribution of prime numbers and the distribution of zeros of the zeta function. He received the 2013 Morningside Special Achievement Award in Mathematics from the International Conference of Chinese Mathematicians in July 2013. Recently, he was appointed a full professor by the University of New Hampshire.

Response from Yitang Zhang

I am humbled and honored to have been selected as the corecipient of the Frank Nelson Cole Prize in Number Theory. This honor should really be credited to those who have influenced my work. In particular, it is the work of Bombieri, Fouvry, Friedlander, and Iwaniec on the stronger versions of the Bombieri-Vinogradov theorem that provides indispensable tools for bounding the error terms. I am grateful to the *Annals of Mathematics* for the quick reaction on my submission and to the referees for studying the manuscript thoroughly and making useful comments; I had not expected that the paper would be accepted within a few weeks. Finally, I must thank the people who contributed great help in my academic career: Dr. Perry Tang, Professor Liming Ge, and the late Professor Kenneth Appel.

Biographical Sketch: Daniel Goldston

Daniel Goldston was born on January 4, 1954, in Oakland, California. He attended the University of California Berkeley starting in 1972, receiving his Ph.D. in 1981 under the supervision of R. Sherman Lehman. He worked at the University of Minnesota Duluth for a year before spending the 1982–1983 academic year at the Institute for Advanced Study in Princeton. Since 1983 he has worked at San Jose State University except for semesters spent at the Institute for Advanced Study in 1990, the University of Toronto in 1994, and the Mathematical Sciences Research Institute in 1999.

Response from Daniel Goldston

The mathematical work for which this prize has been awarded was aided by many mathematicians over many years, but I will not attempt to thank them all individually. Let me tell a story which starts in 1999, when, for many of us, the recent stunning work of Yitang Zhang would have seemed less likely than a proof of the Riemann Hypothesis.

Cem and I were both visiting MSRI in Berkeley, and one day he came in and said that rather than working on the paper we were supposed to be writing, he had been trying to work out a triple correlation divisor sum. We started working on this together and, after a few weeks, began to see that it was possible to

work out asymptotic formulas for this type of sum. Over the rest of the term we continued to work on this, at first getting different answers each time we did the calculation but eventually tending toward only one answer. Cem went back to Turkey, but we continued our joint work by email, slowly working out asymptotic formulas for these sums. This was incremental research, the only kind I actually know how to do, where we used standard classical methods and neither knew nor expected any exciting applications. At the time with little kids of ages 0, 2, and 4 in the house, and fragmented times for work, doing these calculations was ideal since they could be interrupted and then easily resumed. Finally, when the kids were 3, 5, and 7 in 2003, Cem and I thought we had made a breakthrough on gaps between primes but, while we received a lot of publicity, this did not help change the fact that the proof was wrong. Math can be a tough business, and while mathematicians often do not have much humility, we all have lots of experience with humiliation. Fortunately, in this case our work was not destined for the wastebasket, and in 2004 Green and Tao found a use for our formulas in their work on arithmetic progressions of primes, and in 2005, with János Pintz, we obtained the GPY method which proved new results on gaps between primes and provided part of the foundation for Zhang's great advance in 2013.

Biographical Sketch: János Pintz

János Pintz received his M.Sc. at the Eötvös Loránd University in Budapest, Hungary, in 1974 and his Ph.D. (so-called candidate's degree) from the Hungarian Academy of Sciences in 1975 under the supervision of Professor Paul Turán. After working for a few years at Eötvös Loránd University, since 1977 he has been a research fellow at the Mathematical Institute of the Hungarian Academy of Sciences, which today is called Alfréd Rényi Mathematical Institute of the Hungarian Academy of Sciences after its founder and first director, Alfréd Rényi. During this period, Pintz was visiting professor at several foreign universities for a few years. His research focuses on prime number theory. In the past twenty years he worked mostly on three of the four famous problems mentioned by Landau more than a hundred years ago in his invited address at the International Congress of Mathematicians in Cambridge in 1912; namely, the Goldbach conjecture, the twin prime conjecture, and large gaps between consecutive primes. He is a member of the Hungarian Academy of Sciences and the Academy of Europe.

Response from János Pintz

I am grateful to the American Mathematical Society and the Selection Committee for the great honor of choosing me as one of the corecipients of the 2014 Frank Nelson Cole Prize in Number Theory. or four decades ago that my famous fellow citizen, Paul Erdős, received this award for his contribution to the elementary proof of the Prime Number Theorem, I could not have imagined that one day I would be honored with the same distinction. This is even more true if one takes a quick look at the list of giants in number theory who received this prize after Paul Erdős in the past sixty years. First I would like to thank my late Professor Paul Turán, who showed me the beauty of primes in his lecture when I was still a first-year undergraduate student at Eötvös University, and who was my advisor until his unduly early death at age sixty-six in 1976. I also would like to thank my friends and colleagues Endre Szemerédi and Gábor Halász, from whom I learned very much in the later stage of my career. My special thanks are due to my coauthors, friends, and corecipients of this prize, Dan Goldston and Cem Yıldırım, for the very friendly and fruitful collaboration during the past years which led to a number of results about small gaps between primes. I also thank my wife and children, the Alfréd Rényi Mathematical Institute of the Hungarian Academy of Sciences, and finally God for a fortunate life, in that I was able to devote most of my time to my work, hobby, and obsession: to think about mathematical problems, especially those connected with the mysteries of primes. In this respect I can quote one of the greatest mathematicians of all times, Leonhard Euler, who once said, "Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate." I am very glad that standing on the shoulders of giants, such as Birch, Bombieri, Deligne, Fouvry, Friedlander, Heath-Brown, Iwaniec, Motohashi, Selberg, and Weil, we, the recipients of the 2014 Frank Nelson Cole Prize, could somewhat reduce the domain of mysteries of primes into which the human mind can never penetrate.

I have to say that when I first learned some three

Biographical Sketch: Cem Y. Yıldırım

Cem Yalçın Yıldırım was born in Bloomington, Indiana, in 1961. He grew up in Ankara, Turkey, and received his B.Sc. degree in physics from Middle East Technical University (METU), Turkey, in 1982, and his Ph.D. from the University of Toronto, Canada, in 1990 under the supervision of John B. Friedlander. Since 2002, he is professor of mathematics at Boğaziçi University, Istanbul, Turkey, studying mostly analytic number theory and classical analysis.

Response from Cem Y. Yıldırım

I am humbled and honored to be one of the recipients of the 2014 Frank Nelson Cole Prize in Number Theory. My education began at home, where I was raised in an intellectually stimulating

environment. I am grateful to my parents for always encouraging me to concentrate upon anything that interested me. I was completely enraptured when I stumbled upon the classic book on number theory by Hardy and Wright in the library of my high school, and from then on I knew what area I wanted to learn most. The Department of Mathematics at METU had an excellent atmosphere for an undergraduate eager to absorb mathematics.

I am grateful to the University of Toronto, where I was granted scholarship throughout the years I pursued my studies toward a dissertation, and to my Ph.D. thesis supervisor, John B. Friedlander, who always gave me his support even well after I graduated. In 1995 I began collaborating with Dan Goldston on his so-called lower bound method while on sabbatical at San Jose State University. Later on, visits to MSRI, MFO, and IAS were very beneficial to me. I am deeply indebted to Dan Goldston and János Pintz for always sharing ideas freely. Our collaboration has been a great learning process for me. I would also like to thank Boğaziçi University for providing enlightened and peaceful working and living conditions.

About the Prize

The Cole Prize in Number Theory is awarded every three years for a notable research memoir in number theory that has appeared during the previous five years. The awarding of this prize alternates with the awarding of the Cole Prize in Algebra, also given every three years.

These prizes were established in 1928 to honor Frank Nelson Cole (1861–1926) on the occasion of his retirement as secretary of the AMS after twenty-five years of service. He also served as editor-in-chief of the *Bulletin* for twenty-one years. The endowment was made by Cole and has received contributions from Society members and from Cole's son, Charles A. Cole. The Cole Prize carries a cash award of US\$5,000.

The Cole Prize in Number Theory is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2014 prize, the members of the selection committee were Daniel Bump, B. H. Gross, and Audrey A. Terras.

Previous recipients of the Cole Prize in Number Theory are H. S. Vandiver (1931), Claude Chevalley (1941), H. B. Mann (1946), Paul Erdős (1951), John T. Tate (1956), Kenkichi Iwasawa (1962), Bernard M. Dwork (1962), James B. Ax and Simon B. Kochen (1967), Wolfgang M. Schmidt (1972), Goro Shimura (1977), Robert P. Langlands (1982), Barry Mazur (1982), Dorian M. Goldfeld (1987), Benedict H. Gross and Don B. Zagier (1987), Karl Rubin (1992), Paul Vojta (1992), Andrew J. Wiles (1997), Henryk Iwaniec (2002), Richard Taylor (2002), Peter Sarnak (2005), Manjul Bhargava (2008), and Chandrashekhar Khare and Jean-Pierre Wintenberger (2011).

-Elaine Kehoe

2014 Doob Prize

CÉDRIC VILLANI was awarded the Joseph L. Doob Prize at the Annual Meeting of the AMS in Baltimore, Maryland, in January 2014.

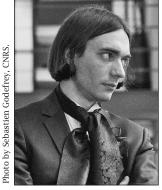
Citation

Optimal Transport, Old and New. Grundlehren der Mathematischen Wissenschaften [Fundamen-

tal Principles of Mathematical Sciences], 338. Springer-Verlag, Berlin, 2009

from fluid mechanics to meteorol-

In 1975 the mathematicians Leonid V. Kantorovich and Tjalling C. Koopmans received the Nobel Prize in Economics "for their contributions to the theory of optimum allocation of resources." Subsequent research on optimal transport has revealed remarkable connections with such varied areas of mathematics as dynamical systems, geometry, and partial differential equations—and also with applications ranging



Cédric Villani

ogy to cosmology.

This book represents a profound rethinking of the subject of optimal transport by one of its leading contributors. The overarching themes are existence, uniqueness, regularity, and stability of optimal transport; and the investigation of Riemannian geometry via optimal transport. Many results appear here in book form for the first time, often in sharper versions than have previously been published. The scope of the volume is breathtaking: the panorama of topics from dynamics, probability, and geometry includes Moser's technique for coupling smooth positive probability measures, Caffarelli's log-concave perturbation theorem, Kantorovich duality, the Wasserstein distance between probability measures, Mather's shortening lemma, the Ma-Trudinger-Wang tensor, a priori estimates for solutions of the Monge-Ampère equation, the Bochner-Weitzenböck-Lichnerowicz formula, the Brunn-Minkowski inequality in nonnegatively curved Riemannian manifolds, the Bakry-Émery theorem, Lichnerowicz's spectral gap inequality, Talagrand inequalities, the measured Gromov-Hausdorff topology, and Ricci curvature bounds on metric spaces.

A pedagogical masterpiece, the book effectively communicates deep ideas while remaining relatively self contained. Engaging historical and bibliographical commentaries further enliven the DOI: http://dx.doi.org/10.1090/noti1112

exposition. J. L. Doob was known for the loving care that he lavished on his books, especially *Classical Potential Theory and Its Probabilistic Counterpart* (like *Optimal Transport*, published in the Springer Grundlehren series). Cédric Villani's readers will recognize a worthy heir to Doob's legacy of outstanding mathematical research exposition.

Biographical Sketch

Born in 1973 in France, Cédric Villani studied mathematics at the École Normale Supérieure in Paris, from 1992 to 1996, and spent four more years as assistant professor there. In 1998 he defended his Ph.D. on the mathematical theory of the Boltzmann equation. Besides his advisor Pierre-Louis Lions (Paris, France), he was much influenced by Yann Brenier (Nice, France), Eric Carlen (Rutgers, USA) and Michel Ledoux (Toulouse, France).

He was professor at the École Normale Supérieure de Lyon from 2000 to 2010 and is now at the Université de Lyon. He has occupied visiting professor positions in Atlanta, Berkeley, and Princeton. Since 2009 he has been the director of the Institut Henri Poincaré in Paris; this eighty-year-old national institute, dedicated to welcoming visiting researchers, is at the very heart of French mathematics.

His work has won him many national and international prizes, in particular the Fields Medal, presented at the 2010 International Congress of Mathematicians in Hyderabad by the President of India. His book *Théoréme vivant* (Broché, 2012)¹ retraces the genesis of the development of the theorem of Landau damping, the subject for which he was awarded the Fields Medal. Since then he has served as a spokesperson for the French mathematical community in media and political circles.

His main research interests are in kinetic theory (Boltzmann and Vlasov equations and their variants) and optimal transport and its applications, a field in which he wrote the two reference books, *Topics in Optimal Transportation* (2003) and *Optimal Transport*, *Old and New* (2008).

Response from Cédric Villani

Books are immaterial children, born out of an intense intellectual experience. Often they acquire a living on their own and impose themselves to you. I never experienced this feeling better than when composing *Optimal Transport*, *Old and New*, which was part of my life for three years. I had

¹ Théorème vivant was reviewed by Jacques Hurtubise in the February 2014 issue of the Notices.

initially planned this book to be a 100-page-long summer school proceeding, and in the end it was a 1000-page-long reference book. This quick growth and change of ambition was the book's decision, not mine. The state of obsession which I arrived at while working on it is almost unparalleled in my personal history. After it was finished, I would often open it at a random page and read it, like a father proudly contemplating his newborn. Fortunately it was not only to his father's taste, since the book has been doing well and has become a classical reference in the field of optimal transport. That it is rewarded with the Doob Prize is a great honor for me; I especially like the reference to Doob, who had the same care for details and presentation as I try to have—always thinking hard about the best way to present and convey messages to the readers' minds, without sacrificing the rigor in the least. But this prize is also for me a mere joy, and the occasion to commemorate what I consider as one of the happy events in my life.

About the Prize

The Doob Prize was established by the AMS in 2003 and endowed in 2005 by Paul and Virginia Halmos in honor of Joseph L. Doob (1910–2004). Paul Halmos (1916–2006) was Doob's first Ph.D. student. Doob received his Ph.D. from Harvard in 1932 and three years later joined the faculty at the University of Illinois, where he remained

until his retirement in 1978. He worked in probability theory and measure theory, served as AMS president in 1963-1964, and received the AMS Steele Prize in 1984 "for his fundamental work in establishing probability as a branch of mathematics and for his continuing profound influence on its development." The Doob Prize recognizes a single, relatively recent, outstanding research book that makes a seminal contribution to the research literature, reflects the highest standards of research exposition, and promises to have a deep and longterm impact in its area. The book must have been published within the six calendar years preceding the year in which it is nominated. Books may be nominated by members of the Society, by members of the selection committee, by members of AMS editorial committees, or by publishers. The prize of US\$5,000 is given every three years.

The Doob Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2014 prize, the members of the selection committee were Harold P. Boas, William Fulton, Philip J. Holmes, Neal I. Koblitz, and John H. McCleary.

The previous recipients of the Doob Prize are William P. Thurston (2005), Enrico Bombieri and Walter Gubler (2008), and Peter Kronheimer and Tomasz Mrowka (2011).

-Elaine Kehoe

2014 Morgan Prize

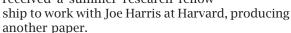
ERIC LARSON was awarded the 2014 AMS-MAA-SIAM Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student at the 120th Annual Meeting of the AMS in Baltimore, Maryland, in January 2014.

Citation

Eric Larson is awarded the 2014 AMS-MAA-SIAM Frank and Brennie Morgan Prize for Outstanding Research by an Undergraduate Student for his truly exceptional record of research. He has so far authored or coauthored eight papers, two as sole author, two with Dmitry Vaintrob, three with Larry Rolen, and one with David Jordan. His papers have appeared in a wide spectrum of research journals, including Advances in Geometry, Bulletin of the London Mathematical Society, Forum Mathematicum, the Journal of Noncommutative Geometry, and Proceedings of the American Mathematical Society.

DOI: http://dx.doi.org/10.1090/noti1114

Eric began his research work while still in high school, working in the REU program at Penn State University under Sergei Tabachnikov and then at the Research Science Institute at the Massachusetts Institute of Technology under Pavel Etinghof. In 2010, after his first year at Harvard, Eric participated in Ken Ono's REU program at the University of Wisconsin. This led to his collaboration with Dmitry Vaintrob. Eric continued in Ono's REU, now at Emory University, in 2011 and again in 2013. His work in this program resulted in five papers. In 2012 Eric received a summer research fellow-



In addition to his stellar research work, Eric also won the Intel Science Talent Search first place prize, took second place in the Siemens competition that same year, and won a gold medal at the



Eric Larson

initially planned this book to be a 100-page-long summer school proceeding, and in the end it was a 1000-page-long reference book. This quick growth and change of ambition was the book's decision, not mine. The state of obsession which I arrived at while working on it is almost unparalleled in my personal history. After it was finished, I would often open it at a random page and read it, like a father proudly contemplating his newborn. Fortunately it was not only to his father's taste, since the book has been doing well and has become a classical reference in the field of optimal transport. That it is rewarded with the Doob Prize is a great honor for me; I especially like the reference to Doob, who had the same care for details and presentation as I try to have—always thinking hard about the best way to present and convey messages to the readers' minds, without sacrificing the rigor in the least. But this prize is also for me a mere joy, and the occasion to commemorate what I consider as one of the happy events in my life.

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-Elaine Kehoe

2014 Morgan Prize

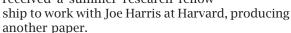
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Eric Larson

International Math Olympiad, all while still a high school student. He was also on Harvard's winning Putnam team in 2011 and a Putnam Fellow in 2012. Eric is one of the most accomplished students of mathematics that the mathematics community has ever seen. In the words of Ken Ono, "Eric is a phenomenon."

Biographical Sketch

Eric Larson is a graduate student at MIT in mathematics. He is from Eugene, Oregon, where, while in elementary school, he discovered his love of mathematics after seeing Euclid's proof of the infinitude of primes. Eric received his bachelor's degree in mathematics from Harvard University, with a secondary in physics. His research interests are concentrated in algebraic geometry and number theory. Currently, he is working on a couple of projects related to the geometry of general curves in projective space.

Response from Eric Larson

I am honored to receive the 2014 Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student, and would like to warmly thank AMS, MAA, and SIAM.

I am also grateful to the many people that have helped me get here. Especially, I would like to thank my research mentors Ken Ono, Joe Harris, and David Zureick-Brown, as well as my family and friends for their support and encouragement.

About the Prize

The Morgan Prize is awarded annually for outstanding research in mathematics by an undergraduate student (or students having submitted joint work). Students in Canada, Mexico, or the United States or its possessions are eligible for consideration for the prize. Established in 1995, the prize was endowed by Mrs. Frank (Brennie) Morgan of Allentown, Pennsylvania, and carries the name of her late husband. The prize is given jointly by the AMS, the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM) and carries a cash award of US\$1,200.

Recipients of the Morgan Prize are chosen by a joint AMS-MAA-SIAM selection committee. For the 2013 prize, the members of the selection committee were Colin C. Adams, Bela Bajnok, Johnny Guzman, Kathleen R. Fowler, Reza Malek Madani, and Susan E. Martonosi.

Previous recipients of the Morgan Prize are Kannan Soundararajan (1995), Manjul Bhargava (1996), Jade Vinson (1997), Daniel Biss (1998), Sean McLaughlin (1999), Jacob Lurie (2000), Ciprian Manolescu (2001), Joshua Greene (2002), Melanie Wood (2003), Reid Barton (2005), Jacob Fox (2006), Daniel Kane (2007), Nathan Kaplan (2008), Aaron Pixton (2009), Scott Duke Kominers (2010), Maria Monks (2011), John Pardon (2012), and Fan Wei (2013).

-Elaine Kehoe

2014 Conant Prize

ALEX KONTOROVICH was awarded the 2014 Levi L. Conant Prize at the 120th Annual Meeting of the AMS in Baltimore, Maryland, in January 2014.

Citation

The 2014 Levi L. Conant Prize is awarded to Alex Kontorovich for his article, "From Apollonius to Zaremba: Local-global phenomena in thin orbits", *Bulletin of the American Mathematical Society* **50** (2013), no. 2, 187–228.

This article introduces us to a new field of number theory that has proven to be extremely fruitful, even in shedding light on some ancient problems. The author illustrates the new ideas by focusing on three problems, which at first glance seem totally unrelated, but each of which is an attractive mixture of algebra and geometry. The first problem (Zaremba's Conjecture) asks whether every integer

DOI: http://dx.doi.org/10.1090/noti1109

is the denominator of a fraction which can be expressed as a continued fraction

$$x = \frac{1}{a_1 + \frac{1}{a_2 + \dots}},$$

where the a^j are constrained to be 1,2,3,4,5. The second problem is more overtly geometric and asks whether all sufficiently large integers (not prohibited by congruence conditions) occur as curvatures in an integral Apollonian gasket, a configuration of circles with many tangencies. Finally, the third problem asks if there are infinitely many primes that occur as hypotenuses in a thin orbit of Pythagorean triples.

Kontorovich masterfully introduces the general reader to these problems and the ways in which they are connected through the concept of orbits of groups of matrices that are of infinite index in a International Math Olympiad, all while still a high school student. He was also on Harvard's winning Putnam team in 2011 and a Putnam Fellow in 2012. Eric is one of the most accomplished students of mathematics that the mathematics community has ever seen. In the words of Ken Ono, "Eric is a phenomenon."

Biographical Sketch

Eric Larson is a graduate student at MIT in mathematics. He is from Eugene, Oregon, where, while in elementary school, he discovered his love of mathematics after seeing Euclid's proof of the infinitude of primes. Eric received his bachelor's degree in mathematics from Harvard University, with a secondary in physics. His research interests are concentrated in algebraic geometry and number theory. Currently, he is working on a couple of projects related to the geometry of general curves in projective space.

Response from Eric Larson

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-Elaine Kehoe

2014 Conant Prize

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Citation

The 2014 Levi L. Conant Prize is awarded to Alex Kontorovich for his article, "From Apollonius to Zaremba: Local-global phenomena in thin orbits", *Bulletin of the American Mathematical Society* **50** (2013), no. 2, 187-228.

This article introduces us to a new field of number theory that has proven to be extremely fruitful, even in shedding light on some ancient problems. The author illustrates the new ideas by focusing on three problems, which at first glance seem totally unrelated, but each of which is an attractive mixture of algebra and geometry. The first problem (Zaremba's Conjecture) asks whether every integer

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Kontorovich masterfully introduces the general reader to these problems and the ways in which they are connected through the concept of orbits of groups of matrices that are of infinite index in a



Alex Kontorovich

group of integral matrices (hence the "thin"). He also introduces the "circle method", a heuristic principle for going from local to global results.

The article is full of pictures, interesting history, and references to a great deal of mathematics, some very classical and some very modern. As the author points out, it is a side of number

theory that borrows freely from *L*-functions, the Langlands program, and classical sieve theory as well as a number of other fields without being subsumed by any of them.

The article dips into some deep mathematics while maintaining a casual flow so that both expert and novice readers alike are entertained throughout this look at ancient problems through a modern lens.

Biographical Sketch

Alex Kontorovich is an assistant professor of mathematics at Yale University. He was born in 1980 in Voronezh, Russia, and grew up in New Jersey after the family emigrated in 1988. He received a B.A. from Princeton in 2002 and a Ph.D. in 2007 from Columbia, advised by Dorian Goldfeld and Peter Sarnak. Following a Tamarkin Assistant Professorship at Brown (2007–2010), he taught at Stony Brook (2010–2011) before moving to Yale. He is the recipient of an NSF Postdoctoral Fellowship, an NSF CAREER Award, and a Sloan Research Fellowship, and has twice been a year-long member at the Institute for Advanced Study. His other joys include music and spending time with his family, wife Amy and son Harry.

Response from Alex Kontorovich

I am deeply humbled and very surprised to receive the 2014 Levi L. Conant Prize from the American Mathematical Society. The idea of writing an expository article had never occurred to me until Andrew Granville planted the seed in my head years ago. Over time, Andrew gently prodded until I finished the task, making innumerable comments and suggestions which drastically improved various drafts along the way (of course his writings were my model of outstanding exposition); a share of the prize belongs to him. Another share belongs to Jean Bourgain: the main theorems discussed in this paper are part of our ongoing collaboration, and I am grateful for his tutelage. There would have been nothing to report had Peter Sarnak not introduced us to Apollonian gaskets, and Curt McMullen to Zaremba's conjecture. I should also like to take this opportunity to thank my many teachers. In addition to the more obvious (advisors and co-authors). these include Yakov Sinai, Eli Stein, John Conway, John Morgan, and Ioannis Karatzas. I greatly enjoyed the opportunity and challenge to collect some aspects of thin groups in this paper; hopefully, it might encourage more people to go into this new field.

About the Prize

The Conant Prize is awarded annually to recognize an outstanding expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years.

Established in 2001, the prize honors the memory of Levi L. Conant (1857–1916), who was a mathematician at Worcester Polytechnic Institute. The prize carries a cash award of US\$1,000.

The Conant Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2013 prize, the members of the selection committee were Thomas F. Banchoff, Brian Conrey, and John F. Oprea.

Previous recipients of the Conant Prize are: Carl Pomerance (2001); Elliott Lieb and Jakob Yngvason (2002); Nicholas Katz and Peter Sarnak (2003); Noam D. Elkies (2004); Allen Knutson and Terence Tao (2005); Ronald M. Solomon (2006); Jeffrey Weeks (2007); J. Brian Conrey, Shlomo Hoory, Nathan Linial, and Avi Wigderson (2008), John W. Morgan (2009), Bryna Kra (2010), David Vogan (2011), Persi Diaconis (2012), and John Baez and John Huerta (2013).

—Elaine Kehoe

2014 Eisenbud Prize for Mathematics and Physics



Gregory W. Moore

GREGORY W. MOORE was awarded the 2014 Leonard Eisenbud Prize for Mathematics and Physics at the 120th Annual Meeting of the AMS in Baltimore, Maryland, in January 2014.

Citation

The 2014 Leonard Eisenbud Prize for Mathematics and Physics is awarded to Gregory W. Moore for his group of works on the structure of four-dimensional supersymmetric theories with extended supersymmetry. His

works on supersymmetric solitons in a variety of contexts—including black holes in supergravity, branes in string theory, and monopoles in gauge theory—have led to an explanation of the wall-crossing phenomena in the BPS spectrum. Moore's research has injected new physical ideas and created new constructions in the mathematical fields of cluster algebras, integrable systems, and hyperkähler geometry.

In particular the following papers are cited:

- 1. Frederik Denef and Gregory W. Moore, "Split states, entropy enigmas, holes and halos", *Journal of High Energy Physics* **129** (2011), no. 11, i, 152 pp.
- 2. Duiliu-Emanuel Diaconescu and Gregory W. Moore, "Crossing the wall: branes versus bundles", *Advances in Theoretical Mathematical Physics* **14** (2010), no. 6, 1621–1650.
- 3. Davide Gaiotto, Gregory W. Moore, and Andrew Neitzke, "Four-dimensional wall-crossing via three-dimensional field theory", *Communications in Mathematical Physics* **299** (2010), no. 1, 163–224.
- 4. Davide Gaiotto, Gregory W. Moore, and Andrew Neitzke, "Wall-crossing, Hitchin systems, and the WKB approximation," *Advances in Mathematics* **234** (2013), 239–403.
- 5. Davide Gaiotto, Gregory W. Moore, and Andrew Neitzke, "Spectral networks", *Annales Henri Poincaré* **14** (2013), no. 7, 1643–1731.

DOI: http://dx.doi.org/10.1090/noti1113

Biographical Sketch

Gregory W. Moore received his A.B. in physics from Princeton University in 1982 and his Ph.D. in physics from Harvard University in 1985. He then joined the Harvard Society of Fellows and in 1987 became a five-year member at the Institute for Advanced Study (IAS) in Princeton. In 1989 he joined the faculty at Yale University. He moved to the Department of Physics and Astronomy at Rutgers University in 2000. He has held visiting professorships at the Kalvi Institute for Theoretical Physics (KITP) in Santa Barbara, California, and at the IAS. The Inspire HEP database lists 170 papers coauthored by Professor Moore on physical mathematics, with an emphasis on geometrical structures in physics. Most notably he has worked on rational conformal field theories (with applications to condensed matter physics), two-dimensional quantum gravity and matrix models, topological field theories, string dualities and D-branes, applications of K-theory to string theory, connections between number theory and supersymmetric black holes, and the properties of BPS states of supersymmetric theories with an emphasis in recent years on their wall-crossing properties and relations to hyperkähler geometry. He is a member of the American Physical Society, the American Mathematical Society, the American Academy of Arts and Sciences, and a general member of the Aspen Center for Physics.

Response from Gregory W. Moore

I am deeply honored, and not a little surprised, to be the sole recipient of the 2014 Leonard Eisenbud Prize for Mathematics and Physics. First and foremost I would like to thank my collaborators, Frederik Denef, Emanuel Diaconescu, Davide Gaiotto, and Andrew Neitzke, for their essential insights and enthusiasm for what turned out to be a very fruitful line of enquiry. I was the senior author only in years—not infrequently it was my collaborators who were leading the charge.

Since the AMS has requested a response to this award, I will use the opportunity to sketch my

viewpoint on how the work mentioned in the citation fits into a broader context and then to conclude even more broadly with some thoughts on the place of physical mathematics in the contemporary relation of the mathematical and physical sciences.

A central theme of the work in the citation is the behavior of four-dimensional theories with N=2supersymmetry. The 1994 breakthrough of Nathan Seiberg and Edward Witten amply demonstrated that quantum field theories with extended supersymmetry constitute a Goldilocks class of theories which are special enough to admit exact nontrivial results on their dynamics but general enough to exhibit a host of nontrivial phenomena in quantum field theory. The promise of the Seiberg-Witten breakthrough is twofold: First, one can make exact statements about how the massless particles in the theory interact at low energies. Second, one can make exact statements about the spectrum of the Hamiltonian for a subsector of the Hilbert space of states called the "BPS subspace". One of the key features of these theories is that the vacuum state is not unique, but rather it is parameterized by a manifold (which carries a special Kähler metric). Thus, an example of the first kind of result is an exact description of the strength of Coulomb's law as a function of the vacuum parameters.

I would guesstimate that there have been well over ten thousand physicist years devoted to the intense investigation of four-dimensional N=2field theories. Nevertheless, the full promise of the Seiberg-Witten breakthrough has not yet been fully realized. Regarding the first kind of result, important and nontrivial insights continue to be uncovered up to the present day in the works of Nikita Nekrasov, Samson Shatashvili, Vasily Pestun, Edward Witten, and a host of others revealing relations to integrable systems and many other things. The papers mentioned in the citation address the second kind of result: deepening our understanding of how to compute the so-called BPS spectrum for ever larger classes of N=2 theories. The key theme in these papers is that, as a function of vacuum parameters, the BPS spectrum can be discontinuous across real codimension 1 loci in the space of vacuum parameters. An important point is that there exists a very beautiful formula which expresses how this spectrum changes. Since a real codimension 1 locus is a wall, the formula is known as a wall-crossing formula. The history of this formula is far too complicated to be explained here, but I will note that it began with a formula of Sergio Cecotti and Cumrun Vafa for the decays of solitons in two-dimensional quantum field theories, and, in addition to my work done in collaboration with Denef, Diaconescu, Gaiotto, and Neitzke, essential insights and breakthroughs were made in the context of pure mathematics and motivated by pure mathematics—by Maxim Kontsevich and Yan Soibelman and separately by Dominic Joyce and Yinan Song in their work on generalized Donaldson-Thomas invariants for Calabi-Yau categories. Research into BPS states continues to be a very active subject.

As indicated in the citation, the investigations into the BPS spectrum have led to a wide variety of unexpected and rich connections to many branches of pure mathematics. Like a beautiful flower which continues to unfold and dazzle, the deeper the probe, the richer the emergent mathematics. In addition to the relations of four-dimensional *N*=2 theories to hyperkähler geometry, cluster algebras, cluster varieties, and integrable systems, several other remarkable links to subjects in pure mathematics have been discovered by many mathematicians and physicists in the past several years. The full list is too long to mention here, but some prominent examples include deep relations to geometric representation theory and nontrivial connections with modular tensor categories and two-dimensional conformal field theory.

In view of the extraordinary richness of the field, one might well wonder if there is some simplifying and unifying viewpoint on all the above connections. Indeed, the following is widely believed by many mathematicians and physicists: A striking prediction of string theory from the mid-1990s (in the hands of Edward Witten, Andrew Strominger, and Nathan Seiberg) is that there is a class of six-dimensional interacting conformal quantum field theories known as the (2,0)-theories. Many of the beautiful connections alluded to above can be traced to the very existence of these theories. On the other hand, these six-dimensional theories have not yet been fully formulated in any systematic way. There is no analog of a statement for nonabelian gauge theory, such as "Make sense of the path integral over connections on a principal bundle weighted by the Yang-Mills action." Indeed the very mention of the (2,0)-theories is greeted by some scientists with an indulgent smile. But many of us take them seriously. An important problem for the future is a deeper understanding and formulation of these theories.

For reviews giving a more extensive explanation of these matters, the reader could consult my review talk at Strings2011 in Uppsala, Sweden, my review talk at the 2012 International Congress on Mathematics and Physics in Aalborg, Denmark, or my 2012 Felix Klein lectures delivered in Bonn, Germany. They are all available on my home page. I would like to stress that there are several viewpoints on this vibrant subject held by several other mathematicians and physicists which are equally if not more valid. For a good example, see the review by Yuji Tachikawa, available on his home page.

Looking further to the future, we should not forget that the very existence of the (2,0)-theory is but a corollary of the existence of string theory.

Work on the fundamental principles underlying string theory has noticeably waned—it seems the community is currently gathering more "data" in the form of examples and solid mathematical truths—but ultimately physical mathematics must return to this grand question.

Finally, I would like to comment on physical mathematics more broadly since the very purpose of the Leonard Eisenbud Prize is to encourage work "that brings mathematics and physics closer together." I think the emergent and very lively field of physical mathematics fits this criterion brilliantly. The use of this term in contrast to the more traditional "mathematical physics" by myself and others is not meant to detract from the magnificent subject of mathematical physics but rather to delineate a smaller subfield characterized by a very distinctive set of questions, goals, and techniques. The questions and goals are often motivated, on the physics side, by quantum gravity, string theory, and supersymmetry, and, on the mathematics side, often involve deep relations to topology, geometry, and even analytic number theory in addition to the more traditional relations of physics to algebra, group theory, and analysis. This is a subject which has not been without its critics. Perhaps the most forceful criticism is that of Arthur Jaffe and Frank Quinn. While these criticisms were very ably answered by Michael Atiyah et al.² and William Thurston,³ the issues raised by Jaffe and Quinn are not without merit and we would do well not to forget them. Nevertheless, given the wide spectrum of astonishing results achieved in physical mathematics in the period since this debate erupted, the overwhelming preponderance of evidence is that the subject has great depth and validity. It is likely to remain an important beacon for progress in mathematics for some time to come.

About the Prize

The Eisenbud Prize was established in 2006 in memory of the mathematical physicist Leonard Eisenbud (1913–2004) by his son and daughter-in-law, David and Monika Eisenbud. Leonard Eisenbud, who was a student of Eugene Wigner, was particularly known for the book *Nuclear Structure* (1958), which he coauthored with Wigner. A friend of Paul Erdős, he once threatened to write a dictionary of "English to Erdős and Erdős to English". He was one of the founders of the Physics Department at the State University of New York, Stony Brook, where he taught from 1957 until his retirement in 1983. His son David was president of

the AMS during 2003–2004. The Eisenbud Prize for Mathematics and Physics honors a work or group of works that brings the two fields closer together. Thus, for example, the prize might be given for a contribution to mathematics inspired by modern developments in physics or for the development of a physical theory exploiting modern mathematics in a novel way. The US\$5,000 prize will be awarded every three years for a work published in the preceding six years.

The Eisenbud Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2014 prize, the members of the selection committee were Elliott H. Lieb, Cumrun Vafa, and Eric G. Zaslow.

Previous recipients of the Eisenbud Prize are Hirosi Ooguri, Andrew Strominger, and Cumrun Vafa (2008), and Herbert Spohn (2011).

—Elaine Kehoe

¹Bull. Amer. Math. Soc. (*N.S.*) **29** (1993) no. 1, 1-13; arXiv:math/9307227 [math.H0].

²Bull. Amer. Math. Soc. (*N.S.*) **30** (1994) no. 2, 178–207; arXiv:math/9404229 [math.HO].

³ Bull. Amer. Math. Soc. (*N.S.*) **30** (1994) no. 2, 161-177; arXiv:math/9404236 [math.HO].

2014 Award for Distinguished Public Service

PHILIP KUTZKO received the 2014 Award for Distinguished Public Service at the 120th Annual Meeting of the AMS in Baltimore, Maryland, in January 2014.

Citation

The American Mathematical Society's 2014 Award for Distinguished Public Service is presented to Phil Kutzko for his leadership of a national effort to increase the number of doctoral degrees in the mathematical sciences earned by students from underrepresented groups. Kutzko was one of several faculty at the Department of Mathematics at the University of Iowa who undertook, in 1995, to increase minority representation in its graduate program. In this role he has served as director of the department's Sloan Foundation Minority Scholarship Program. As a result of this departmental effort, more than twenty-five U.S. citizens of minority backgrounds have earned Ph.D.s in mathematics at the University of Iowa in the period 2001–2013. Kutzko, together with colleagues in the mathematics and statistics departments at the three Iowa Regents universities, founded the National Alliance for Doctoral Studies in the Mathematical Sciences; Kutzko has written the proposals to NSF through which the Alliance is funded and has served as its director from its inception. The Alliance, founded in 2002, has grown to be a community of more than 250 faculty nationally who work closely with math science majors from minority backgrounds together with faculty at twenty-six doctoral granting departments in the mathematical sciences.

Kutzko's area of research is representation theory of *p*-adic groups with applications to the local Langlands program. He has continued to maintain his research program throughout his many years working on behalf of Ph.D. students from underrepresented backgrounds. Indeed, three of his advisees, all of them from minority backgrounds, received their Ph.D.s under his direction in 2012. He is a most worthy recipient of the Distinguished Public Service Award.

DOI: http://dx.doi.org/10.1090/noti1115

Biographical Sketch

Phil Kutzko was born and raised in New York City. He is a product of the New York City public schools, and he attended the City College of New York. He received his M.S. and Ph.D. degrees at the University of Wisconsin. He joined the University of Iowa mathematics faculty in 1974. Kutzko's research area is the representation theory of *p*-adic groups with applications to number theory. He is the author, with Colin Bushnell, of a monograph in the *Annals of Mathematics Studies* and



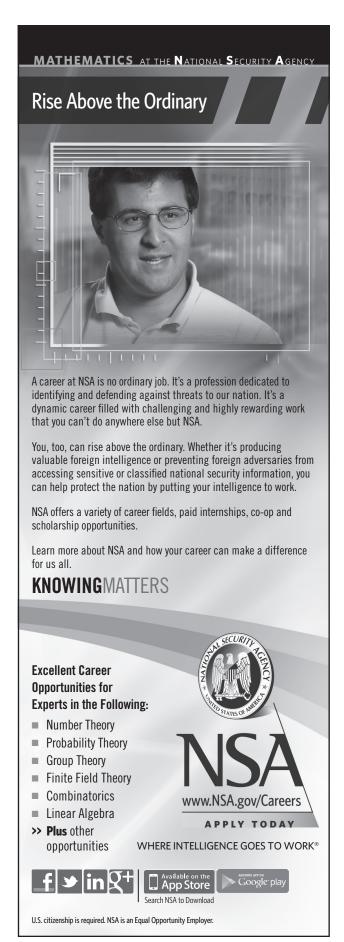
Philip Kutzko

was an invited section speaker at the International Congress of Mathematicians in Berkeley in 1986. He is presently a University of Iowa Collegiate Fellow and a Fellow of the American Association for the Advancement of Science.

Kutzko is honored to have played a part in the University of Iowa Department of Mathematics' activities in minority graduate education and in the extension of these activities to other departments of mathematical sciences, including those of the three Iowa Regents universities. In this context, he directs the departmental Sloan Foundation Minority Ph.D. Program as well as the National Alliance for Doctoral Studies in the Mathematical Sciences, an NSF-funded project that involves mathematical sciences departments at a variety of colleges and universities and whose goal is to increase the number of doctoral degrees in the mathematical sciences awarded to students from backgrounds that are underrepresented in these fields. Kutzko was honored for his work in this area with the 2008 Presidential Award for Excellence in Science, Mathematics and Engineering Mentoring. This award was presented to him by President Obama in a White House ceremony in January 2010.

Response from Philip Kutzko

I am deeply honored to receive the 2014 Award for Distinguished Public Service from the American Mathematical Society and doubly honored when I reflect on those who have preceded me in this hoto by Thomas Jorgensen.



honor. Our initial effort to build diversity into the core of our graduate program at the University of Iowa was greatly aided by the support, and wisdom, of Carlos Castillo Chavez, the 2010 recipient of this award. It is hard to imagine that our department—or our National Alliance—would have had the success that it has had without his advice and leadership. The work that my colleagues and I have done on types and covers for p-adic groups has its origin in the ideas of Roger Howe, the 2006 recipient of this award; it builds on the pioneering work he did in this area. Paul Sally, the 2000 recipient of this award, has provided a whole generation of mathematicians with a model we may emulate—a model for research, a model for mentoring, and, above all, a model for citizenship. My personal and professional debt to him is enormous, and it is a pleasure to acknowledge it on this occasion.

This is a critical time for our profession. It is the nature of the way in which we do mathematics that it is in constant need of renewal, of new ideas and new approaches—of new eyes. This fresh perspective will come, as it always has in the past, from the inclusion of groups that have been previously underrepresented in our profession if we will only open the door and welcome them as they take their place among us. This door was opened for me by my high school and college teachers in New York City, by my professors at the University of Wisconsin, and by the mathematicians in whose steps I follow in accepting this high honor. I am deeply grateful for this recognition of the work that my colleagues and I are doing to open these doors for others.

About the Award

The Award for Distinguished Public Service is presented every two years to a research mathematician who has made a distinguished contribution to the mathematics profession during the preceding five years. The purpose of the award is to encourage and recognize those individuals who contribute their time to public service activities in support of mathematics. The award carries a cash prize of US\$4,000.

The Award for Distinguished Public Service is made by the AMS Council, acting on the recommendation of the selection committee. For the 2014 award, the members of the selection committee were Richard A. Askey, C. Herbert Clemens, Roger E. Howe, William McCallum, and Sylvia M. Wiegand.

Previous recipients of the award are Kenneth M. Hoffman (1990), Harvey B. Keynes (1992), I. M. Singer (1993), D. J. Lewis (1995), Kenneth C. Millett (1998), Paul J. Sally Jr. (2000), Margaret H. Wright (2002), Richard Tapia (2004), Roger Howe (2006), C. Herbert Clemens (2008), Carlos Castillo-Chavez (2010), and William McCallum (2012).

-Elaine Kehoe

Mathematics People

De Lellis and Hairer Awarded Fermat Prize

CAMILLO DE LELLIS of the University of Zurich and MARTIN HAIRER of the University of Warwick have been jointly awarded the 2013 Fermat Prize. De Lellis was honored for his fundamental contributions (in collaboration with László Székelyhidi) to the conjecture of Onsager about dissipative solutions of the Euler equations and for his work in the regularity of minimal surfaces. Hairer was honored for his contributions to the analysis of stochastic partial differential equations, especially for the regularity of their solutions and convergence to the equilibrium. The Fermat Prize is given every two years for research in fields in which Pierre de Fermat made major contributions: statements of variational principles, foundations of probability and analytic geometry, and number theory. The prize carries a cash value of 20,000 euros (approximately US\$27,300).

-Elaine Kehoe

Marshall Scholarships Awarded

Three students in the mathematical sciences have been awarded Marshall Scholarships for 2014. WEI "DAVID" JIA of Stanford University is a student of mathematics, computer science, and poetry; he will study neuroscience at Oxford University. MATTHEW MCMILLAN of Wheaton College, Wheaton, Illinois, will earn his bachelor's degree in mathematics and physics with an honors thesis on embedded contact homology. He plans to take Part III of the Mathematical Tripos at Cambridge and the M.St. in philosophy of physics at Oxford. KIRIN SINHA of the Massachusetts Institute of Technology studies theoretical mathematics and electrical engineering and computer science, with a minor in music. She founded SHINE, a community service organization targeted at encouraging seventh-grade girls to pursue the study of mathematics through a combination of dance and math. She will pursue Part III of the Mathematical Tripos at Cambridge and hopes to expand SHINE internationally.

Marshall Scholarships finance young Americans of high ability to study for a degree in the United Kingdom. Up to

forty scholars are selected each year to study at graduate level at a U.K. institution in any field of study.

-From a Marshall Scholarships announcement

PECASE Awards Announced

Four young scientists whose work involves the mathematical sciences have received Presidential Early Career Awards for Scientists and Engineers (PECASE) from President Obama. Three were nominated by the National Science Foundation (NSF) and one by the Department of Defense (DOD). Those nominated by the NSF, who were among nineteen NSF nominees, are the following. TAMARA MOORE of the University of Minnesota is the STEM Education Center Codirector at the university. Her research is focused on uncovering the most effective tools and techniques that educators can use to inspire students. Tamara's research is based on the idea that teaching STEM subjects in a realistic context will further develop student interest. Benjamin RECHT of the University of Wisconsin, Madison, was recognized for his research in scalable computational tools for large-scale data analysis, statistical signal processing, and machine learning. His work explores the intersections of convex optimization, mathematical statistics, and randomized algorithms. DAVID SAVITT of the University of Arizona was recognized for his work in number theory, specifically Galois representations, modular forms, and p-adic Hodge theory. He is also the deputy director of Canada/USA Mathcamp, a summer program for high school students. The nominee of the DOD is RAMON VAN HANDEL of Princeton University, who was recognized for his work in probability theory, stochastic analysis, ergodic theory, mathematical statistics, information theory, mathematical physics, and applied mathematics.

—Elaine Kehoe

Huang and Zelditch Awarded 2013 Bergman Prize

XIAOJUN HUANG of Rutgers University and STEVE ZEL-DITCH of Northwestern University have been awarded the 2013 Stefan Bergman Prize. Established in 1988, the prize recognizes mathematical accomplishments in the areas of research in which Stefan Bergman worked. Huang and Zelditch will equally share US\$24,564, which is the 2013 income from the Stefan Bergman Trust.

The previous Bergman Prize winners are: David W. Catlin (1989), Steven R. Bell and Ewa Ligocka (1991), Charles Fefferman (1992), Yum Tong Siu (1993), John Erik Fornæss (1994), Harold P. Boas and Emil J. Straube (1995), David E. Barrett and Michael Christ (1997), John P. D'Angelo (1999), Masatake Kuranishi (2000), László Lempert and Sidney Webster (2001), M. Salah Baouendi and Linda Preiss Rothschild (2003), Joseph J. Kohn (2004), Elias M. Stein (2005), Kengo Hirachi (2006), Alexander Nagel and Stephen Wainger (2007–2008), Ngaiming Mok and Duong H. Phong (2009), Gennadi Henkin (2011), and David Jerison and John M. Lee (2012).

On the selection committee for the 2013 prize were Harold P. Boas, Alexander Nagel, and Duong Phong.

Citation: Xiaojun Huang

Xiaojun Huang is recognized for innovative and influential contributions to CR geometry. He has introduced original ideas and powerful techniques to resolve fundamental problems such as the algebraicity of holomorphic maps between algebraic strongly pseudoconvex hypersurfaces in different dimensions, the rigidity of proper holomorphic maps between balls in complex spaces of different dimensions, and a longstanding question of Moser about the moduli space of Bishop surfaces having vanishing Bishop invariant.

Biographical Sketch: Xiaojun Huang

Xiaojun Huang was born on November 1, 1963, in China. He received his bachelor's degree in engineering (in



Xiaojun Huang

the area of aircraft design) in 1983 from Nanjing University of Aeronautics and Astronautics. He received his master's degree in mathematics in 1986 from Wuhan University and served there as a teaching instructor until he came to the U.S.A. in 1989. Huang obtained his Ph.D. from Washington University in 1994. He was an L. E. Dickson Instructor at the University of Chicago from 1994 to 1997.

In 1995/1996 he was appointed as a postdoctoral research fellow at the Mathematical Sciences Research Institute in Berkeley. Since September 1997, he has been on the faculty of the mathematics department of Rutgers University, where he is currently a distinguished professor. Huang has held visiting positions at several institutions including Wuhan University, the Chinese University of Hong Kong, the University of California at San Diego, the University of Rouen in France, and Harvard University. Huang was named an AMS fellow of the class of 2014. His invited mathematical talks include an invited plenary address at the 29th Biannual Brazilian Mathematics Colloquium at the Instituto de Matemática Pura e Aplicada in Brazil in August 2013 and an AMS invited address at the fall 2013 sectional meeting in Philadelphia in October 2013.

Citation: Steve Zelditch

Steve Zelditch is recognized for his ever expanding the horizon of applications of the Bergman kernel. From his semiclassical viewpoint and with his strikingly original vision, he has found deep and diverse relations between the Bergman kernel and many other areas, including complex geometry, probability, and mathematical physics. In the process, he has infused the whole subject of the Bergman kernel with a new vitality.

Biographical Sketch: Steve Zelditch

Steve Zelditch is Wayne and Elizabeth Jones Professor of Mathematics at Northwestern University. He got his



Steve Zelditch

bachelor's degree from Harvard and his Ph.D. from the University of California at Berkeley in 1981. He was Ritt Assistant Professor at Columbia (1981–1985), was at Johns Hopkins from assistant to full professor (1986–2009), and moved to Northwestern in 2010. He was an invited speaker at the International Congress of Mathematicians in Bei-

jing (2004) and has twice been an invited speaker at the International Congress of Mathematical Physics. He gave Current Developments in Mathematics lectures at Harvard in 2009 and an invited AMS national address in 2005. He has been on the editorial boards of *Annales Scientifiques de l'École Normale Supérieure*, the *American Journal of Mathematics*, and the *Journal of Mathematical Physics* and is currently on the editorial boards of the *Communications in Mathematical Physics*, *Analysis & PDE*, and *Journal of Geometric Analysis*.

About the Prize

The Bergman Prize honors the memory of Stefan Bergman, best known for his research in several complex variables, as well as the Bergman projection and the Bergman kernel function that bear his name. A native of Poland, he taught at Stanford University for many years and died in 1977 at the age of eighty-two. He was an AMS member for thirty-five years. When his wife died, the terms of her will stipulated that funds should go toward a special prize in her husband's honor.

The AMS was asked by Wells Fargo Bank of California, the managers of the Bergman Trust, to assemble a committee to select recipients of the prize. In addition the AMS assisted Wells Fargo in interpreting the terms of the will to assure sufficient breadth in the mathematical areas in which the prize may be given. Awards are made every one or two years in the following areas: (1) the theory of the kernel function and its applications in real and complex analysis and (2) function-theoretic methods in the theory of partial differential equations of elliptic type with attention to Bergman's operator method.

-Allyn Jackson

Mathematics Opportunities

Math in Moscow Scholarship Program

The Math in Moscow program at the Independent University of Moscow (IUM) was created in 2001 to provide foreign students (primarily from the United States, Canada, and Europe) with a semester-long, mathematically intensive program of study in the Russian tradition of teaching mathematics, the main feature of which has always been the development of a creative approach to studying mathematics from the outset-the emphasis being on problem solving rather than on memorizing theorems. Indeed, discovering mathematics under the guidance of an experienced teacher is the central principle of the IUM, and the Math in Moscow program emphasizes in-depth understanding of carefully selected material rather than broad surveys of large quantities of material. Even in the treatment of the most traditional subjects, students are helped to explore significant connections with contemporary research topics. The IUM is a small, elite institution of higher learning focusing primarily on mathematics which was founded in 1991 at the initiative of a group of well-known Russian research mathematicians who now compose the Academic Council of the university. Today the IUM is one of the leading mathematical centers in Russia. Most of the Math in Moscow program's teachers are internationally recognized research mathematicians, and all of them have considerable teaching experience in English, typically in the United States or Canada. All instruction is in English.

With funding from the National Science Foundation (NSF), the AMS awards five US\$9,000 scholarships each semester to U.S. students to attend the Math in Moscow program. To be eligible for the scholarships, students must be either U.S. citizens or enrolled at a U.S. institution at the time they attend the Math in Moscow program. Students must apply separately to the IUM's Math in Moscow program and to the AMS Math in Moscow Scholarship program. Undergraduate or graduate mathematics or computer science majors may apply. The deadlines for applications for the scholarship program are April 15, 2014, for the fall 2014 semester and September 15, 2014, for the spring 2015 semester.

Information and application forms for Math in Moscow are available on the Web at http://www.mccme.ru/mathinmoscow or by writing to: Math in Moscow, P.O. Box

524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: mim@mccme.ru. Information and application forms for the AMS scholarships are available on the AMS website at http://www.ams.org/programs/travel-grants/mimoscow or by writing to: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence RI 02904-2294; email student-serv@ams.org.

Call for Proposals for 2015 NSF-CBMS Regional Conferences

The NSF-CBMS Regional Research Conferences in the Mathematical Sciences are a series of five-day conferences, each of which features a distinguished lecturer delivering ten lectures on a topic of important current research in one sharply focused area of the mathematical sciences. The Conference Board of the Mathematical Sciences (CBMS) publicizes the conferences and administers the resulting publications. Support is provided for about thirty participants at each conference. Proposals should address the unique characteristics of the NSF-CBMS conferences, which can be found at http://www.nsf.gov/pubs/2013/nsf13550/nsf13550.htm. The deadline for full proposals is April 25, 2014. See the above website for full information.

-From an NSF announcement

DMS Workforce Program in the Mathematical Sciences

The Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) welcomes proposals for the Workforce Program in the Mathematical Sciences. The long-range goal of the program is increasing the number of well-prepared U.S. citizens, nationals, and permanent residents who successfully pursue careers in the mathematical sciences and in other NSF-supported disciplines. Of primary interest are activities centered on education that broaden participation in the mathematical sciences through research involvement for trainees at the undergraduate through postdoctoral educational levels. The program is particularly interested in activities that

About the cover Poincaré's glue surface

The cover image is an elaboration of one seen in John Stillwell's review of Jeremy Gray's biography of Poincaré. The original is found in Poincaré's 5th Supplement to "Analysis Situs", one of a series of papers founding algebraic topology. Stillwell calls the figure unenlightening, but we suppose that is a matter of perspective. As anyone familiar with Dale Rolfsen's well known book *Knots and Links* will be aware of, it is an example of a Heegaard diagram, a standard tool in 3D topology. Heegaard diagrams are part of the data associated to Heegaard splittings, which remain even now an active topic of research. Alas, it seems that in modern developments few papers on what are called "Heegaard diagrams" actually contain a diagram.

Chapter 9 of Rolfsen's book explains Poincaré's diagram in a series of short exercises. Chapter 6 of Gray's biography contains an impressive account of the 5th Supplement, although one might wish that he had more—and more enlightening—pictures. All we can do here is tell you what exactly you are looking at, and leave detailed interpretation for elsewhere.

The colored region is a surface of genus two that has been cut apart along the circles $\pm A$ and $\pm B$. It is the boundary of a *handlebody* of genus two—i.e., a pair of handles attached to a 3-ball. The blue and red paths represent oriented continuous paths on the original surface, and in Poincaré's construction are places where a second handlebody is attached to form his example of a three-manifold with trivial homology but nontrivial fundamental group. The green paths are used by Poincaré to calculate, at the end of §6 of the 5th Supplement, that fundamental group. It turns out that the fundamenetal group is isomorphic to $SL_2(\mathbb{Z}/5)$, although all that he seems to prove is that the icosahedral group A_5 is a homomorphic image.

There are many ways known nowadays of constructing Poincaré's counterexample to an earlier conjecture he had made. One clear presentation can be found in §6 of the classic topology text by Seifert and Threlfall. For a rather more mysterious picture of Heegaard splittings, take a look at

http://www.its.caltech.edu/~wjiajun/compprog/hfhat/sigma235.html

—Bill Casselman Graphics Editor (notices-covers@ams.org)

Mathematics Opportunities

improve recruitment and retention, educational breadth, and professional development.

The submission period for unsolicited proposals is **May 15-June 15, 2014**. For more information and a list of cognizant program directors, see the website http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=503233.

-From a DMS announcement

NSF Scholarships in Science, Technology, Engineering, and Mathematics

The NSF Scholarships in Science, Technology, Engineering, and Mathematics (S-STEM) program provides institutions with funds for student scholarships to encourage and enable academically talented students demonstrating financial need to enter the STEM workforce or STEM graduate school following completion of an associate, baccalaureate, or graduate degree in fields of science, technology, engineering, or mathematics. Students to be awarded scholarships must demonstrate academic talent and financial need. S-STEM grants may be made for up to five years and provide individual scholarships of up to US\$10,000 per year, depending on financial need. Proposals must be submitted by institutions, which are responsible for selecting the scholarship recipients. The deadline for full proposals is August 12, 2014. For more information, see the website http://www.nsf.gov/ pubs/2012/nsf12529/nsf12529.htm.

-From an NSF announcement

AWM Gweneth Humphreys Award

The Association for Women in Mathematics (AWM) sponsors the Gweneth Humphreys Award to recognize outstanding mentorship activities. This prize will be awarded annually to a mathematics teacher (female or male) who has encouraged female undergraduate students to pursue mathematical careers and/or the study of mathematics at the graduate level. The recipient will receive a cash prize and honorary plaque and will be featured in an article in the AWM newsletter. The award is open to all regardless of nationality and citizenship. Nominees must be living at the time of their nomination.

The deadline for nominations is **April 30, 2014**. For details see www.awm-math.org, telephone: 703-934-0163, or email: awm@awm-math.org.

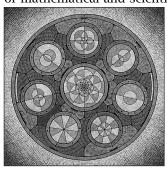
-From an AWM announcement

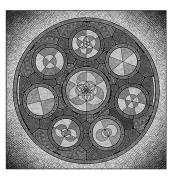
Inside the AMS

2014 Mathematical Art Exhibition Awards

The 2014 Mathematical Art Exhibition Awards were made at the Joint Mathematics Meetings in Baltimore, Maryland, in January "for aesthetically pleasing works that combine mathematics and art." The works were selected from the exhibition of juried works in various media by eighty-six mathematicians and artists from around the world.

"Enigmatic Plan of Inclusion I & II" by Conan Chadbourne was awarded Best Photograph, Painting, or Print. "My work is motivated by a fascination with the occurrence of mathematical and scientific imagery in traditional art





Enigmatic Plan of Inclusion I & II

forms," states Chadbourne in the exhibition catalog. The 24" \times 24" archival inkjet prints "are investigations of the subgroup structure of the icosahedral group (A5). At the center of each image is a graphical representation of A5, as formed by orientation-preserving pairs of reflections in circles and lines in the plane. This is surrounded by similar graphical representations of the seven conjugacy classes of (proper, nontrivial) subgroups of A5, with the trivial group depicted as the space outside of the large circular frame. The interstices between the group images indicate the relationships of inclusion between the different groups, with colors being used to distinguish maxi-

mal subgroup relationships and small graphical markers used to indicate the particular numbers of conjugates involved in each relationship."

"Three-Fold Development" by Robert Fathauer was awarded Best Textile, Sculpture, or Other Medium. "I'm endlessly fascinated by certain aspects of our world, including symmetry, chaos, and infinity. Mathematics allows me to explore these topics in distinctive artworks that I feel are an intriguing blend of complexity and beauty," says Fathauer, a small business owner, puzzle designer, author, and artist. "This $13" \times 13" \times 13"$ ceramic sculpture is based on the first five generations of a fractal curve. The

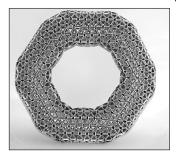


Three-Fold Development

starting point is a circle, and the first iteration produces a three-lobed form. With each iteration, the number of lobes is tripled. The spacing between features is essentially constant throughout a layer, while the threefold symmetric boundary of the curve becomes increasingly complex. A hexagonal version of this curve is found in Benoît Mandelbrot's book *The Fractal Geometry of Na-*

ture. This hyperbolic surface is reminiscent of naturally occurring corals. It was inspired in part by a 3-D-printed model created by Henry Segerman."

"Blue Torus" by Faye E. Goldman received Honorable Mention. "I have been doing origami since elementary



Blue Torus

school," says Goldman. "I was drawn to modular origami by its structure and mathematical properties. The Snapology technique by H. Strobl...has allowed me to dig deeply into the regularity of mathematical shapes finding insight. It has provided insights into mathematical ideas. This $10" \times 10" \times 2.5"$ toroid shape is made from over

2,400 strips of ribbon. It was the first nonconvex shape I've made. I love the fact that there need to be as many heptagons making the negative curvature in the center as there are pentagons around the outside."

The Mathematical Art Exhibition Award was established in 2008 through an endowment provided to the AMS by an anonymous donor who wished to acknowledge those whose works demonstrate the beauty and elegance of mathematics expressed in a visual art form. The awards carry cash prizes of US\$400 for Best Photograph, Painting, or Print; US\$400 for Best Textile, Sculpture, or Other Medium; and US\$200 for Honorable Mention. The Mathematical Art Exhibition of juried works in various media is held at the annual Joint Mathematics Meetings of the American Mathematical Society (AMS) and Mathematical Association of America (MAA).

-Mike Breen and Annette Emerson Public Awareness Officers paoffice@ams.org





2014 CMS Summer Meeting

Delta Winnipeg (Winnipeg Manitoba) June 6 – 9, 2014

Scientific Directors:

Nina Zorboska, Stephen Kirkland (Manitoba)



PRIZE LECTURES

CMS Coxeter-James Prize
CMS Excellence in Teaching Award
CMS Krieger-Nelson Prize Lecture
David Borwein Distinguished Career Award

PUBLIC LECTURES

Barbara Keyfitz (Ohio State)

PLENARY LECTURES

Thomas Ransford (Laval)
Bela Bollobas (Cambridge, Memphis)
James Maynard (Montreal)
Mark Lewis (Alberta)

EDUCATION PLENARY LECTURE

John Mighton (Fields)

HIGHLIGHTS

AARMS-CMS Student Poster Session

Education Sessions

Math Art Exhibit

Contributed Papers

A number of Scientific Sessions

Please see our website for details: www.cms.math.ca/events/summer14

Experience Winnipeg

From the AMS Public Awareness Office

2014 Joint Mathematics Meetings. The Joint Mathematics Meetings in Baltimore drew nearly 6,500 participants. If you attended, we hope you enjoyed the scientific program, exhibits, and making connections with old and new friends. The JMM blog, written by Adriana Salerno, Anna Haesch, and Tyler Clark, is a great place to find out about some of the invited addresses, prizes, sessions, and events and to comment on the blog posts or sessions. Topics include Colin Adams and the Mobius Bandaid Players, Jill Pipher's talk on lattice-based cryptography, AMS Special Session on Analytic Number Theory, mathematical poetry, the Young Mathematicians Network/Project NExT Poster session, The Public Face of Mathematics panel, prizes and awards, and more. See http://blogs.ams.org/ jmm2014/. Those on Twitter can find @JointMath tweets and lots of conversations about the meetings by searching hashtag #JMM14.

Vivek Miglani, pictured with AMS President David Vogan, won the 2014 national *Who Wants to Be a Math-*



ematician contest held at the Joint Meetings. Vivek, a junior at Marjory Stoneman Douglas High School in Florida, earned US\$5,000 for himself and US\$5,000 for the mathematics department at his school. Read more about the contest at http://www.ams.org/programs/students/

wwtbam/jmm2014. (Photo by Sandy Huffaker)

-Mike Breen and Annette Emerson Public Awareness Officers paoffice@ams.org

Deaths of AMS Members

ADELINA GEORGESCU, of Bucharest, Romania, died on May 1, 2010. Born on April 25, 1942, she was a member of the Society for 29 years.

OSCAR E. LANFORD III, of Zurich, Switzerland, died on November 16, 2013. Born on January 6, 1940, he was a member of the Society for 35 years.

PAUL J. SALLY JR., professor, University of Chicago, died on December 29, 2013. Born on January 29, 1933, he was a member of the Society for 57 years.

L. A. SHEPP, professor, Rutgers University, died on April 23, 2013. Born on September 9, 1936, he was a member of the Society for 54 years.

Reference and Book List

The **Reference** section of the Notices is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.

Contacting the Notices

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.wustl.edu in the case of the editor and smf@ams.org in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

March 31, 2014: Nominations for 2014 Graham Wright Award for Distinguished Service of th Canadian Mathematical Society. See the website http://cms.math.ca/Prizes/dis-nom.

March 31, 2014: Nominations for Achievement in Information-Based Complexity Prize. Nominations may be sent to Joseph Traub, email: traub@cs.clumbia.edu. See http://www.cs.columbia.edu/~traub/html/ibc_prize.html.

March 31, 2014: Applications for AMS-Simons Travel Grants. See www.ams.org/programs/travel-grants/AMS-SimonsTG or contact Steven Ferrucci, email: ams-simons@ams.org, telephone: 800-321-4267, ext. 4113.

April 11, 2014: Applications for 2014-2015 Project NEXT Fellowships. See the website http://archives.math.utk.edu/projnext/ or contact Aparna Higgins, Director, at Aparna.Higgins@udayton.edu.

April 15, 2014: Applications for fall 2014 semester of Math in Moscow. See http://www.mccme.ru/ mathinmoscow, or contact: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: mim@mccme.ru. Information and application forms for the AMS scholarships are available on the AMS website at http://www.ams. org/programs/travel-grants/ mimoscow, or contact: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence RI 02904-2294; email student-serv@ams.org.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

AMS Bylaws—November 2013, p. 1358

AMS Email Addresses—February 2014, p. 199

AMS Ethical Guidelines—June/July 2006, p. 701

AMS Officers 2012 and 2013 Updates—May 2013, p. 646

AMS Officers and Committee Members—October 2012, p. 1290

Contact Information for Mathematical Institutes—August 2013, p. 629

Conference Board of the Mathematical Sciences—September 2013, p. 1067

IMU Executive Committee—December 2011, p. 1606

Information for Notices Authors—June/July 2013, p. 776

National Science Board—January 2014, p. 82

NRC Board on Mathematical Sciences and Their Applications— $March\ 2014,\ p.\ 305$

NSF Mathematical and Physical Sciences Advisory Committee—February 2014, p. 202

Program Officers for Federal Funding Agencies—October 2013, p. 1188 (DoD, DoE); December 2012, p. 1585 (NSF Mathematics Education)

Program Officers for NSF Division of Mathematical Sciences—*November 2013, p. 1352*

April 25, 2014: Proposals for 2015 NSF-CBMS Regional Research Conferences in the Mathematical Sciences. See "Mathematics Opportunities" in this issue.

May 1, 2014: Applications for May review for National Academies Research Associateship Programs. See the website http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

May 1, 2014: Applications for AWM Travel Grants and Mathematics Education Research Travel Grants. See https://sites.google.com/site/awmmath/programs/travel-grants; telephone: 703-934-0163; or email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

May 15-June 15, 2014: Proposals for the Workforce Program in the Mathematical Sciences of the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF). See "Mathematics Opportunities" in this issue.

August 1, 2014: Applications for August review for National Academies Research Associateship Programs. See the website http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

September 15, 2014: Applications for spring 2015 semester of Math in Moscow. See http://www.mccme.ru/mathinmoscow, or contact: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: mim@mccme.ru. Information and application forms for the AMS scholarships are available on the AMS website at http://www.ams.org/programs/travel-grants/mimoscow, or contact: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street,

Providence RI 02904-2294; email student-serv@ams.org.

October 1, 2014: Applications for AWM Travel Grants and Mathematics Education Research Travel Grants. See https://sites.google.com/site/awmmath/programs/travel-grants; telephone: 703-934-0163; or email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

November 1, 2014: Applications for November review for National Academies Research Associateship Programs. See the website http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

Book List

The Book List highlights recent books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. Suggestions for books to include on the list may be sent to notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

Algorithms Unlocked, by Thomas H. Cormen. MIT Press, March 2013. ISBN-13: 978-02625-188-02.

Alan M. Turing: Centenary Edition, by Sara Turing. Cambridge University Press, April 2012. ISBN-13: 978-11070-205-80.

Alan Turing: The Enigma, The Centenary Edition, by Andrew Hodges. Princeton University Press, May 2012. ISBN-13: 978-06911-556-47.

Alan Turing: His Work and Impact, edited by S. Barry Cooper and J. van Leeuwen. Elsevier, May 2013. ISBN-13: 978-01238-698-07.

*Alan Turing's Electronic Brain: The Struggle to Build the ACE, the World's Fastest Computer, by B. Jack Copeland et al. Oxford University Press, May 2012. ISBN: 978-0-19-960915-4.

An Accidental Statistician: The Life and Memories of George E. P. Box, by George E. P. Box. Wiley, April 2013. ISBN-13: 978-1-118-40088-3.

A Cabinet of Mathematical Curiosities at Teachers College: David Eugene Smith's Collection, by Diane R. Murray. Docent Press, November 2013. ISBN-13: 978-0-9887449-1-2.

A Calculus of Ideas: A Mathematical Study of Human Thought, by Ulf Grenander. World Scientific, September 2012. ISBN-13: 978-98143-831-89. (Reviewed January 2014.)

Charles S. Peirce on the Logic of Number, by Paul Shields. Docent Press, October 2012. ISBN-13: 978-0-9837004-7-0.

Classic Problems of Probability, by Prakash Gorroochurn. Wiley, May 2012. ISBN-13: 978-1-1180-6325-5. (Reviewed November 2013.)

Computability: Turing, Gödel, Church, and Beyond, edited by B. Jack Copeland, Carl J. Posy, and Oron Shagrir. MIT Press, June 2013. ISBN-13: 978-02620-189-99.

Do I Count?: Stories from Mathematics, by Günter Ziegler (translation of Darf ich Zahlen?: Geschichte aus der Mathematik, Piper Verlag, 2010). CRC Press/A K Peters, July 2013. ISBN-13: 978-1466564916

Girls Get Curves: Geometry Takes Shape, by Danica McKellar. Plume, July 2013. ISBN-13: 978-04522-987-43.

The Gödelian Puzzle Book: Puzzles, Paradoxes and Proofs, by Raymond M. Smullyan. Dover Publications, August 2013. ISBN-13: 978-04864-970-51.

The Golden Ticket: P, NP, and the Search for the Impossible, by Lance Fortnow. Princeton University Press, March 2013. ISBN-13: 978-06911-564-91.

Good Math: A Geek's Guide to the Beauty of Numbers, Logic, and Computation, by Mark C. Chu-Carroll. Pragmatic Bookshelf, July 2013. ISBN-13: 978-19377-853-38.

*Henri Poincaré: A Scientific Biography, by Jeremy Gray. Princeton University Press, November 2012. ISBN-13: 978-06911-527-14. (Reviewed in this issue.)

If A, Then B: How the World Discovered Logic, by Michael Shenefelt and Heidi White. Columbia University Press, June 2013. ISBN-13:978-02311-610-53.

Imagined Civilizations: China, the West, and Their First Encounter, by Roger Hart. Johns Hopkins University Press, July 2013. ISBN-13:978-14214-060-60.

Invisible in the Storm: The Role of Mathematics in Understanding Weather, by Ian Roulstone and John Norbury. Princeton University Press, February 2013. ISBN-13: 978-06911-527-21. (Reviewed September 2013.)

The Logician and the Engineer: How George Boole and Claude Shannon Created the Information Age, by Paul J. Nahin, Princeton University Press, October 2012. ISBN-13: 978-06911-510-07. (Reviewed October 2013.)

*Love and Math: The Heart of Hidden Reality, by Edward Frenkel. Basic Books, October 2013. ISBN-13: 978-04650-507-41.

Magnificent Mistakes in Mathematics, by Alfred S. Posamentier and Ingmar Lehmann. Prometheus Books, August 2013. ISBN-13:978-16161-474-71.

Math in 100 Key Breakthroughs, by Richard Elwes. Quercus, December 2013. ISBN-13: 978-16236-505-44.

Math in Minutes: 200 Key Concepts Explained in an Instant, by Paul Glendinning. Quercus, September 2013. ISBN-13: 978-16236-500-87.

Math on Trial: How Numbers Get Used and Abused in the Courtroom, by Leila Schneps and Coralie Colmez. Basic Books, March 2013. ISBN-13: 978-04650-329-21. (Reviewed August 2013.)

Mathematics in Nineteenth-Century America: The Bowditch Generation, by Todd Timmons. Docent Press, July 2013. ISBN-13: 978-0-9887449-3-6.

Maverick Genius: The Pioneering Odyssey of Freeman Dyson, by Phillip F. Schewe. Thomas Dunne Books, February 2013. ISBN-13:978-03126-423-58.

Meaning in Mathematics, edited by John Polkinghorne. Oxford University Press, July 2011. ISBN-13: 978-01996-050-57. (Reviewed May 2013.)

My Brief History, by Stephen Hawking. Bantam Dell, September 2013. ISBN-13: 978-03455-352-83.

Naming Infinity: A True Story of Religious Mysticism and Mathematical Creativity, by Loren Graham and Jean-Michel Kantor. Belknap Press of Harvard University Press, March 2009. ISBN-13: 978-06740-329-34. (Reviewed January 2014.)

The New York Times Book of Mathematics: More Than 100 Years of Writing by the Numbers, edited by Gina Kolata. Sterling, June 2013. ISBN-13: 978-14027-932-26.

The Noether Theorems: Invariance and Conservation Laws in the Twentieth Century, by Yvette Kosmann-Schwarzbach. Springer, December 2010. ISBN-13: 978-03878-786-76. (Reviewed August 2013.)

Our Mathematical Universe: My Quest for the Ultimate Nature of Reality, by Max Tegmark. Knopf, January 2014. ISBN-13: 978-03075-998-03.

The Outer Limits of Reason: What Science, Mathematics, and Logic Cannot Tell Us, by Noson S. Yanofsky. MIT Press, August 2013. ISBN-13: 978-02620-193-54.

Peirce's Logic of Continuity: A Conceptual and Mathematical Approach, by Fernando Zalamea. Docent Press, December 2012. ISBN-13: 978-0-9837004-9-4.

Perfect Mechanics: Instrument Makers at the Royal Society of London in the Eighteenth Century, by Richard Sorrenson. Docent Press, September 2013. ISBN-13: 978-0-9887449-2-9.

Probably Approximately Correct: Nature's Algorithms for Learning and Prospering in a Complex World, by Leslie Valiant. Basic Books, June 2013. ISBN-13: 978-04650-327-16.

*Quantum Computing since Democritus, by Scott Aaronson. Cambridge University Press, March 2013. ISBN-13: 978-05211-995-68.

Relations between Logic and Mathematics in the Work of Benjamin and Charles S. Peirce, by Allison Walsh. Docent Press, October 2012. ISBN-13: 978-0-9837004-6-3.

Seduced by Logic: Emilie Du Châtelet, Mary Somerville and the Newtonian Revolution, by Robyn Arianrhod. Oxford University Press, September 2012. ISBN-13: 978-01999-316-13. (Reviewed June/July 2013).

The Simpsons and Their Mathematical Secrets, by Simon Singh. Bloomsbury, October 2013. ISBN: 978-14088-353-02.

Sources in the Development of Mathematics: Series and Products from the Fifteenth to the Twenty-first Century, by Ranjan Roy. Cambridge University Press, June 2011. ISBN-

13: 978-05211-147-07. (Reviewed November 2013.)

Strange Attractors (comic book), by Charles Soule, Greg Scott, and Robert Saywitz. Archaia Entertainment, May 2013. ISBN-13: 978-19363-936-26.

Symmetry: A Very Short Introduction, by Ian Stewart. Oxford University Press, July 2013. ISBN-13: 978-01996-519-86.

A Tale of Two Fractals, by A. A. Kirillov. Birkhäuser, May 2013. ISBN-13: 978-08176-838-18.

Théorème vivant, by Cédric Villani (in French). Grasset et Fasquelle, August 2012. ISBN-13: 978-2246798828. (Reviewed February 2014.)

Thinking in Numbers: On Life, Love, Meaning, and Math, by Daniel Tammet. Little, Brown and Company, July 2013. ISBN-13: 978-03161-873-74.

Turbulent Times in Mathematics: The Life of J. C. Fields and the History of the Fields Medal, by Elaine McKinnon Riehm and Frances Hoffman. AMS, November 2011. ISBN-13: 978-0-8218-6914-7.

Turing: Pioneer of the Information Age, by Jack Copeland. Oxford University Press, January 2013. ISBN-13: 978-01996-397-93.

Turing's Cathedral: The Origins of the Digital Universe, by George Dyson. Pantheon/Vintage, December 2012. ISBN-13: 978-14000-759-97.

Undiluted Hocus-Pocus: The Autobiography of Martin Gardner. Princeton University Press, September 2013. ISBN-13: 978-06911-599-11. (Reviewed March 2014.)

Visions of Infinity: The Great Mathematical Problems, by Ian Stewart. Basic Books, March 2013. ISBN-13: 978-04650-224-03.

William Fogg Osgood at Harvard: Agent of a Transformation of Mathematics in the United States, by Diann R. Porter. Docent Press, November 2013. ISBN-13: 978-0-9887449-4-3.

2014 Class of the Fellows of the AMS

Fifty mathematical scientists from around the world have been named Fellows of the American Mathematical Society (AMS) for 2014, the program's second year.

The Fellows of the American Mathematical Society program recognizes members who have made outstanding contributions to the creation, exposition, advancement, communication, and utilization of mathematics. Among the goals of the program are to create an enlarged class of mathematicians recognized by their peers as distinguished for their contributions to the profession and to honor excellence.

The 2014 class of Fellows was honored at a dessert reception held during the Joint Mathematics Meetings in Baltimore, MD. Names of the individuals who are in this year's class, their institutions, and citations appear below.

The nomination period for Fellows is open each year from February 1 to March 31. For additional information about the Fellows program, as well as instructions for making nominations, visit the web page www.ams.org/profession/ams-fellows.

Akram Aldroubi, Vanderbilt University

For contributions to modern harmonic analysis and its applications, and for building bridges between mathematics and other areas of science and engineering.

Stephanie B. Alexander, University of Illinois, Urbana-Champaign For contributions to geometry, for high-quality exposition, and for exceptional teaching of mathematics.

Donald Babbitt, University of California, Los Angeles

For contributions to mathematical physics, for the development of MathSciNet, and for his long service as Publisher of the American Mathematical Society.

Rodrigo Bañuelos, Purdue University

For contributions at the interface between probability and analy-

Hari Bercovici, Indiana University, Bloomington

For contributions to operator theory and to free probability.

Christian Borgs, Microsoft Research

For contributions bringing together analysis, probability theory, graph theory and combinatorics with mathematical statistical physics and rigorous computer science.

Francesco Calegari, Northwestern University

For contributions to number theory and to many aspects of the Langlands program.

Zhen-Qing Chen, University of Washington

For contributions to the potential theory of stable and other jump processes in Euclidean domains.

Tim D. Cochran, Rice University

For contributions to low-dimensional topology, specifically knot and link concordance, and for mentoring numerous junior mathematicians.

John P. D'Angelo, University of Illinois, Urbana-Champaign

For contributions to several complex variables and Cauchy-Riemann geometry, and for his inspiration of students.

Edward G. Effros, University of California, Los Angeles

For contributions to the study of quantized Banach spaces, classification of C*-algebras, and quantum information theory.

Alexandre Eremenko, Purdue University

For contributions to value distribution theory, geometric function theory, and other areas of analysis and complex dynamics. Gregory I. Eskin, University of California, Los Angeles

For contributions to linear partial differential equations and their applications.

Steven C. Ferry, Rutgers, The State University of New Jersey New Brunswick

For contributions to controlled topology, and work on the Novikov conjecture.

Patrick J. Fitzsimmons, University of California, San Diego

For contributions to stochastic analysis and probabilistic potential theory.

Edward Frenkel, University of California, Berkeley

For contributions to representation theory, conformal field theory, affine Lie algebras, and quantum field theory.

Solomon Friedberg, Boston College

For contributions to number theory, representation theory, and automorphic forms, and for the establishment of a new Ph.D. program in mathematics.

Richard J. Gardner, Western Washington University

For contributions to geometric tomography.

Toby Gee, Imperial College

For contributions to Galois representations and automorphic forms.

Steve Gelbart, Weizmann Institute of Science

For contributions to the development and dissemination of the Langlands program.

Jane Piore Gilman, Rutgers, The State University of New Jersey Newark

For contributions to topology and group theory, and for service to her department and the larger community.

Paul G. Goerss, Northwestern University

For contributions to modern homotopy theory through applications of modular forms and algebraic geometry.

Jerome A. Goldstein, University of Memphis

For contributions to partial differential equations and its applications, and to the dissemination of mathematics to a wider public.

Peter M. Gruber, Vienna University of Technology

For contributions to the geometry of numbers and to convex and discrete geometry.

Continued on next page

Michael Handel, Lehman College, The City University of New York For contributions to dynamics on surfaces, pseudo-Anosov maps, and automorphisms of free groups.

Brendan Hassett, Rice University

For contributions to higher-dimensional arithmetic geometry and birational geometry.

Helge Holden, Norwegian University of Science and Technology For contributions to partial differential equations.

Xiaojun Huang, Rutgers, The State University of New Jersey New Brunswick

For contributions to the study of several complex variables, and for service to the community.

Alexander Iosevich, University of Rochester

For contributions to harmonic analysis, analytic number theory, geometric measure theory, and geometric combinatorics.

Srikanth B. Iyengar, University of Nebraska-Lincoln

For contributions to commutative algebra, representation theory, homotopy theory, and algebraic geometry.

Michael J. Larsen, Indiana University, Bloomington

For contributions to group theory, number theory, topology, and algebraic geometry.

Darren Long, University of California, Santa Barbara

For contributions to low-dimensional topology and hyperbolic geometry.

Richard N. Lyons, Rutgers, The State University of New Jersey New Brunswick

For contributions to the classification of the finite simple groups, including the discovery of one of the 26 sporadic finite simple groups.

John C. Meakin, University of Nebraska-Lincoln

For contributions to semigroup theory, and for exemplary service as department chair.

John Willard Milnor, Stony Brook University

For contributions to differential topology, geometric topology, algebraic topology, algebra, and dynamical systems.

Kailash C. Misra, North Carolina State University

For contributions to the representation theory of Kac-Moody Lie algebras and quantum groups, and for service to the mathematical community.

Victor Y. Pan, Graduate Center and Lehman College, The City University of New York

For contributions to the mathematical theory of computation.

Peter Paule, Research Institute for Symbolic Computation, Johannes Kepler University Linz

For contributions to classical combinatorics, computer algebra, and symbolic computation in combinatorics.

Irena Peeva, Cornell University

For contributions to commutative algebra and its applications.

Murray Rosenblatt, University of California, San Diego

For contributions to probability and statistics.

Louis Halle Rowen, Bar-Ilan University

For contributions to noncommutative algebra, and for service to the mathematical community.

K. Peter Russell, McGill University

For contributions to algebraic geometry, for mentoring the next generation of mathematicians, and for professional leadership at the highest levels.

Martin Scharlemann, University of California, Santa Barbara

For contributions to low-dimensional topology and knot theory.

Andreas Seeger, University of Wisconsin, Madison

For contributions to Fourier integral operators, local smoothing, oscillatory integrals, and Fourier multipliers.

Robert J. Vanderbei, Princeton University

For contributions to linear programming and nonlinear optimization problems.

Shouhong Wang, Indiana University, Bloomington

For contributions to geophysical fluid mechanics.

Guofang Wei, University of California, Santa Barbara

For contributions to global Riemannian geometry and its relation with Ricci curvature.

Michael I. Weinstein, Columbia University

For contributions to existence and stability of solitary waves, and nonlinear dispersive wave propagation.

Amie Wilkinson, University of Chicago

For contributions to dynamical systems.

Kevin R. Zumbrun, Indiana University, Bloomington

For contributions to continuum mechanics, shock, and boundary layer theory.

AMERICAN MATHEMATICAL SOCIETY



he selection committees for these prizes request nominations for consideration for the 2015 awards, which will be presented at the Joint Mathematics Meetings in San Antonio, TX in January 2015. Information about these prizes may be found in the November 2013 issue of the *Notices*, pp. 1364-1386, and at www.ams.org/profession/prizes-awards/prizes.

GEORGE DAVID BIRKHOFF PRIZE

The George David Birkhoff Prize is awarded jointly by the AMS and SIAM for an outstanding contribution to applied mathematics in its highest sense. The award was first made in 1968 and now is presented every third year.

RUTH LYTTLE SATTER PRIZE

The Ruth Lyttle Satter Prize is presented every two years in recognition of an outstanding contribution to mathematics research by a woman in the previous six years.

FRANK NELSON COLE PRIZE IN ALGEBRA

The Cole Prize in Algebra, which recognizes a notable paper in algebra published during the preceding six years, is awarded every three years. To be eligible, papers must be either authored by an AMS member or published in a recognized North American journal.

LEVI L. CONANT PRIZE

The Levi L. Conant Prize is presented annually for an outstanding expository paper published in either the *Notices* or the *Bulletin of the American Mathematical Society* during the preceding five years.

ALBERT LEON WHITEMAN MEMORIAL PRIZE

The Albert Leon Whiteman Memorial Prize now is awarded every third year, for notable exposition on the history of mathematics. The ideas expressed and understandings embodied in that exposition should reflect exceptional mathematical scholarship.

www.ams.org

Nominations with supporting information should be submitted using the online form available here: www.ams.org/profession/prizes-awards/nominations. Include a short description of the work that is the basis of the nominations, including complete bibliographic citations. A brief curriculum vitae for the nominee should be included. Those who prefer to submit by postal mail may send nominations to AMS Secretary, Carla Savage, Box 8206, Computer Science Department, North Carolina State University, Raleigh, NC 27695-8206. The nominations will be forwarded by the secretary to the appropriate prize selection committee, which will make final decisions on the awarding of

these prizes.

Deadline for nominations is June 30, 2014.



Frank and Brennie Morgan **AMS-MAA-SIAM Prize**

for Outstanding Research in Mathematics by an Undergraduate Student

The prize is awarded each year to an undergraduate student (or students having submitted joint work) for outstanding research in mathematics. Any student who is an undergraduate in a college or university in the United States or its possessions, or Canada or Mexico, is eligible to be considered for this prize.

The prize recipient's research need not be confined to a single paper; it may be contained in several papers. However, the paper (or papers) to be considered for the prize must be submitted while the student is an undergraduate; they cannot be submitted after the student's graduation. The research paper (or papers) may be submitted for consideration by the student or a nominator. All submissions for the prize must include at least one letter of support from a person, usually a faculty member, familiar with the student's research. Publication of research is not required.

The recipients of the prize are to be selected by a standing joint committee of the AMS, MAA, and SIAM. The decisions of this committee are final. The 2015 prize will be awarded for papers submitted for consideration no later than June 30, 2014, by (or on behalf of) students who were undergraduates in December 2012.





Questions may be directed to:

Barbara T. Faires Secretary Mathematical Association of America Westminster College New Wilmington, PA 16172

Telephone: 724-946-6268 Fax: 724-946-6857

Email: faires@westminster.edu

Nominations and submissions should be sent to:

Morgan Prize Committee c/o Robert J. Daverman, Secretary American Mathematical Society 238 Ayres Hall Department of Mathematics University of Tennessee Knoxville, TN 37996-1320

AMS EXEMPLARY PROGRAM AWARD



The AMS Award for Exemplary Program or Achievement in a Mathematics Department is presented annually to a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of the society. Examples might include a department that runs a notable minority outreach program, a department that has instituted an unusually effective industrial mathematics internship program, a department that has promoted mathematics so successfully that a large fraction of its university's undergraduate population majors in mathematics, or a department that has made some form of innovation in its research support to faculty and/ or graduate students, or which has created a special and innovative environment for some aspect of mathematics research.

The award amount is \$5,000. All departments in North America that offer at least a bachelor's degree in the mathematical sciences are eligible.

The Award Selection Committee requests nominations for this award, which will be announced in Spring 2015. Letters of nomination may be submitted by one or more individuals. Nomination of the writer's own institution is permitted. The letter should describe the specific program(s) for which the department is being nominated as well as the achievements that make the program(s) an outstanding success, and may include any ancillary documents which support the success of the program(s). The letter should not exceed two pages, with supporting documentation not to exceed an additional three pages.

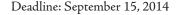
Nominations with supporting information should be submitted to www.ams.org/profession/prizes-awards/nominations. Those who prefer to submit by regular mail may send nominations to the AMS Secretary, Professor Carla D. Savage, North Carolina State University, Department of Computer Science, Campus Box 8206, Raleigh, NC 27695-8206. The nominations will be forwarded by the Secretary to the Prize Selection Committee.

Deadline for nominations is September 15, 2014.

AMERICAN MATHEMATICAL SOCIETY

AMS Award for Mathematics Programs That Make a Difference





This award was established in 2005 in response to a recommendation from the AMS's Committee on the Profession that the AMS compile and publish a series of profiles of programs that:

1. aim to bring more persons from underrepresented backgrounds into some portion of the pipeline beginning at the undergraduate level and leading to advanced degrees in mathematics and professional success, or retain them once in the pipeline;

2. have achieved documentable success in doing so; and

3. are replicable models.

Preference will be given to programs with signifigant participation by underrepresented minorities.

One or two programs are highlighted annually.

Nomination process: Letters of nomination may be submitted by one or more individuals. Nomination of the writer's own institution is permitted. The letter should describe the specific program(s) for which the department is being nominated as well as the achievements that make the program(s) an outstanding success, and may include any ancillary documents which support the success of the program(s). The letter of nomination should not exceed two pages, with supporting documentation not to exceed three more pages. Up to three supporting letters may be included in addition to these five pages.

Send nominations to:
Programs That Make a Difference
c/o Robin Aguiar
American Mathematical Society
201 Charles Street
Providence, RI 02904
or via email to rha@ams.org

Recent Winners:

2013: Nebraska Conference for Undergraduate Women in Mathematics (NCUWM).

2012: Mathematical Sciences Research Institute.

2011: Center for Women in Mathematics, Smith College; Department of Mathematics, North Carolina State University.

2010: Department of Computational and Applied Mathematics (CAAM), Rice University; Summer Program in Quantitative Sciences, Harvard School of Public Health.



www.ams.org

AMERICAN MATHEMATICAL SOCIETY

Mathematical Reviews/MathSciNet Associate Editor

Applications are invited for a full-time position as an Associate Editor of Mathematical Reviews/MathSciNet, to commence as soon as possible after July 1, 2014, preferably before September 1, 2014. The Mathematical Reviews (MR) division of the American Mathematical Society (AMS) is located in Ann Arbor, Michigan, in a beautiful, historic building close to the campus of the University of Michigan. The editors are employees of the AMS; they also enjoy many privileges at the university. At present, the AMS employs over seventy people at Mathematical Reviews, including sixteen mathematical editors. MR's mission is to develop and maintain the MR Database,

from which MathSciNet is produced.

An Associate Editor is responsible for broad areas of the mathematical sciences. Editors select articles and books for coverage, classify these items, determine the type of coverage, assign selected items for review to reviewers, and edit the reviews on their return.

The successful applicant will have mathematical breadth with an interest in current developments, and will be willing to learn new topics in pure and applied mathematics. In particular, the applicant should have expertise in one or more of the following areas: algebra, geometry, number theory, topology. The ability to write well in English is essential. The applicant should normally have several years of relevant academic (or equivalent) experience beyond the Ph.D. Evidence of written scholarship in mathematics is expected. The twelve-month salary will be commensurate with the experience that the applicant brings to the position.

Applications (including a curriculum vitae; bibliography; and the names, addresses, phone numbers, and email addresses of at least three references) should be sent to:

Dr. Graeme Fairweather Executive Editor Mathematical Reviews P. O. Box 8604 Ann Arbor, MI 48107-8604 email: gxf@ams.org
Tel: (734) 996-5257
Fax: (734) 996-2916

URL: www.ams.org/mr-database

Applications received by March 28, 2014, will receive full consideration.

The American Mathematical Society is an Affirmative Action/Equal Opportunity Employer.



Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at http://www.ams.org/cgi-bin/mathcal-submit.pl. The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

April 2014

* 7 PIMS Marsden Memorial Lecture: Mathieu Desbrun, Instituto Nacional de Matematica Pura e Aplicada (IMPA), Rio de Janeiro, Brazil. Description: Geometric discretization for computational modeling This talk will review a number of structure-preserving discretizations of space and time, from discrete counterparts of differential forms and symmetric tensors on surfaces, to finite-dimensional approximation to the diffeomorphism group and its Lie algebra. A variety of applications (from masonry to magnetohydrodynamics) will be used throughout the talk to demonstrate the value of a geometric approach to computations. The Marsden Memorial Lecture Series is dedicated to the memory of Jerrold E Marsden (1942-2010), a world-renowned Canadian applied mathematician.

Information: http://www.pims.math.ca/scientific-event/
140407-pmmlmd.

* 10-12 **39th University of Arkansas Spring Lecture Series in the Mathematical Sciences: Multiparameter Geometry and Analysis,** University of Arkansas, Fayetteville, Arkansas.

Main speaker: Alexander Nagel (University of Wisconsin-Madison). The list of plenary speakers and additional information is available at: http://math.uark.edu/3723.php. The conference is supported by the National Science Foundation and by the University of Arkansas. Funds are available to help defray participants' expenses, though priority in funding goes to graduate students and junior mathematicians. Underrepresented minorities in the mathematical sciences are strongly encouraged to apply. Call for contributed talks: We welcome short contributed talks and poster presentations. In particular we encourage graduate students and recent Ph.D.'s to apply. Information: http://math.uark.edu/3723.php.

* 11–13 **29th Geometry Festival (UPDATE)**, Stony Brook University, Stony Brook, New York.

Description: This year's conference speakers will be: Robert Bryant, Duke University [Colloquium Speaker]; Alice Chang, Princeton Uni-

versity; Mihalis Dafermos, Princeton University; Kenji Fukaya, SCGP and Stony Brook University; Matthew Gursky, Notre Dame University; Robert Haslhofer, New York University; Andre Neves, Imperial College; Song Sun, SCGP and Stony Brook University. The conference will begin late Friday afternoon with a special colloquium by Robert Bryant. Five more talks will be held on Saturday. The conference will then conclude with two final talks on Sunday morning. The conference will take place in the recently opened Simons Center for Geometry and Physics (SCGP). Participants will be accommodated in our new on-campus hotel, located within easy walking distance of the conference site. Our conference banquet will be held at the SCGP on Saturday evening. For more details, visit our web-site: http://www.math.sunysb.edu/geomfest14/. We believe this conference will be of the very greatest interest to mathematicians working in Differential Geometry and related fields. Please mark your calendars!

May 2014

* 8–10 **Student Tropical Algebraic Geometry Seminar**, Yale University, New Haven, Connecticut.

Description: The Student Tropical Algebraic Geometry Seminar is a conference about tropical geometry (broadly defined) and nearby areas. Our target participants are graduate students and early post-doctoral fellows. We hope that most participants will be able to give talks. Our primary objective is for everyone to learn about their colleagues' work, or the areas of research they are interested in. In general, an inherent difficulty in stimulating discussion between mathematicians seems to be that different researchers are accustomed to different approaches, and speak different languages. We aim to promote conversation between people that are fluent, for example, in weighted polyhedral complexes, tropicalization, as well as Berkovich skeleta. We would like to expose the participants to topics of recent research interest, such as the study of limit linear series on curves, moduli spaces, tropical intersection theory, con-

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences

in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

nections to logarithmic geometry, connections to matroid theory, and many others.

Information: http://sites.google.com/site/yalestags/.

* 10-11 **73rd Midwest PDE Seminar**, Northwestern University, Evanston. Illinois.

Description: The Midwest Partial Differential Equations Seminar is a semiannual meeting on recent developments in partial differential equations, which goes back to 1977. Northwestern will host the 73rd meeting of the series. The weekend conference will feature 8 speakers in PDE. Some travel funding is available for participants. Graduate students, recent Ph.D's and members of underrepresented groups are encouraged to apply. Please visit the website for registration and more information.

Information: http://www.math.northwestern.edu/
midwestpde/.

* 11-16 International Conference "Topological and geometric methods in low-dimensional dynamical systems", Moscow Center for Continuous Mathematical Education. 119002, Bolshoy Vlasyevskiy Pereulok 11, Moscow, Russia.

Description: The conference is co-organized by the National Research University Higher School of Economics, Laboratory of Algebraic Geometry and its Applications, Baltic Institute of Mathematics, Aix-Marseille University.

Preliminary list of speakers (TBC): Pablo Aguirre, Laurent Bartholdi, Alexander Blokh, Adam Epstein, Sarah Koch, Lex Oversteegen, Mary Rees, Dierk Schleicher, Nikita Selinger, Katsutoshi Shinohara, Sergei Tabachnikov, Michael Yampolsky.

Information: http://math.hse.ru/en/confmay2014.

* 12-14 Conference in honour of Kenneth Falconer's 60th birthday, Inria-Saclay Ile de France, Saclay, France.

Description: As the name suggests, this is a conference organized for celebrating the 60th birthday of Professor Kenneth Falconer. Professor Falconer has been, and still is, a great source of inspiration for many researchers in fractal geometry. This meeting will feature some of his top co-authors as plenary speakers, as well as a number of his former students as regular speakers. Submissions are welcome provided they have a strong link with one of K. Falconer's main contributions.

Information: http://regularity.saclay.inria.fr/
conference-in-honour-of-kenneth-falconers-60thbirthday.

* 12-16 AIM Workshop: Rational and integral points on higher-dimensional varieties, American Institute of Mathematics, Palo Alto, California.

Description: This workshop, sponsored by AIM, CMI, and the NSF, will be devoted to the study of rational and integral points on algebraic varieties of dimension at least two.

Information: http://www.aimath.org/ARCC/workshops/
ratlhigherdimvar.html.

* 12-16 NSF-CBMS Regional Research Conference in the Mathematical Sciences on Combinatorial Zeta and L-functions, The Sundance Resort, Utah.

Description: The conference will consist primarily of ten lectures given by Wen-Ching Winnie Li, Distinguished Professor of Mathematics at Penn State University. The lectures will be delivered over the five days of the conference and will compare similarities and dissimilarities between combinatorial and number-theoretic zeta functions. Explicit constructions for Ramanujan graphs and expanders will be discussed, highlighting applications of number theory to graph theory. Recent results will also be presented illustrating applications of combinatorics to number theory.

Information: http://math.byu.edu/cbms/.

 * 18–20 Frontier Probability Days 2014, University of Arizona, Tucson, Arizona.

Description: "Frontier Probability Days 2014" (FPD'14) is a regional workshop. Its purpose is to bring together mathematicians, both regionally and globally, who have an interest in probability and its applications.

Information: http://www.math.arizona.edu/~fpd.

* 19–21 Riemannian Geometry and Applications "RIGA" 2014, Technical University of Civil Engineering Bucharest, Romania University of Bucharest, Bucharest, Romania.

Description: This is the 4th Conference Riemannian Geometry and Applications (RIGA), which is devoted to the geometry of Riemannian and pseudo-Riemannian manifolds, submanifold theory, structures on manifolds, complex geometry and contact geometry, Finsler, Lagrange and Hamilton geometries, mathematical modeling in engineering, applications to economics.

Organizing committee: Adela Mihai (Technical University of Civil Engineering Bucharest and University of Bucharest), Ion Mihai (University of Bucharest), Andrei Olteanu, Madalina Stoian (Technical University of Civil Engineering Bucharest).

Scientific Committee: Radu Miron (Romanian Academy), David Blair (Michigan State University), Radu Sarghiuta, Manole Stelian Serbulea (Technical University of Civil Engineering Bucharest), Gheorghe Pitis (Transilvania University of Brasov).

Invited speakers: Bang-Yen Chen (Michigan State University), Leopold Verstraelen (Katholieke Universiteit Leuven), Alfonso Carriazo (University of Seville).

Information: http://stiri.utcb.ro/riga-2014-conferinta-internationala-riemannian-geometry-and-applications-to-engineering-and-economics/.

* 20-21 **2nd International Conference on Advanced Computing, Engineering and Learning Technologies (ICACELT 2014)**, Hotel Le Median, Munich, Germany.

Description: 2nd ICACELT 2014 invites researchers, practitioners and academics to present their research findings, work in progress, case studies and conceptual advances in any branch of the above fields. The conference brings together varied groups of people with different perspectives, experiences and knowledge in one location. It aims to help practitioners find ways of putting research into practice and researchers to gain an understanding of real-world problems, needs and aspirations.

Information: http://www.icacelt.com.

* 20–23 **USA-Uzbekistan Conference on Analysis and Mathematical Physics**, California State University at Fullerton, Fullerton, California. **Description:** The conference aims to stimulate interactions among the U.S. researchers in Analysis and Mathematical Physics and their counterparts in Uzbekistan and other countries, and serve as a catalyst for future collaborations. It will center on the following main themes: dynamical systems, geometric function theory, mathematical physics, operator algebras and several complex variables. A number of significant results have recently been established in these areas, which will be disseminated through the scheduled plenary talks. Parallel sessions will allow for the presentation of a broad spectrum of results in the area through the scheduled invited talks. Poster presentations will afford more junior mathematicians and students the opportunity to speak on their work.

Information: http://nsm.fullerton.edu/usuzcamp.

* 23-26 The Fields workshop on algebraic and geometric invariants of linear algebraic groups and homogeneous spaces, University of Ottawa, Ottawa, Ontario, Canada.

Description: In the last 10–15 years the theory of linear algebraic groups has witnessed an intrusion of the cohomological methods of modern algebraic geometry and algebraic topology. These new methods have led to breakthroughs on a number of classical problems in algebra, which are beyond the reach of earlier purely algebraic techniques. The workshop will focus at the following new emerg-

ing techniques and applications: i) The proof of the Grothendieck-Serre conjecture based on the theory of affine Grassmannians. ii) Computation of generalized equivariant cohomology of projective homogeneous and toric varieties. The workshop is supported by the Fields Institute.

Information: http://www.fields.utoronto.ca/
programs/scientific/13-14/invariants/.

* 26 **2014 Niven Lecture - Bjorn Poonen**, The University of British Columbia, Vancouver, BC V6T 1Z4, Canada.

Description: Undecidability in Number Theory. Hilbert's Tenth Problem asked for an algorithm that, given a multivariable polynomial equation with integer coefficients, would decide whether there exists a solution in integers. Around 1970, Matiyasevich, building on earlier work of Davis, Putnam, and Robinson, showed that no such algorithm exists. However, the answer to the analogous question with integers replaced by rational numbers is still unknown, and there is not even agreement among experts as to what the answer should be. The annual Niven Lecture Series, held at UBC since 2005, is funded in part through a generous bequest from Ivan and Betty Niven to the UBC Mathematics Department.

Information: http://www.pims.math.ca/scientificevent/140526-nlbp.

* 26–28 **Young Women in Probability 2014**, University of Bonn, Germany.

Description: The aim of this workshop is to provide a platform for young female researchers in Probability (Ph.D. students and postdocs), to present their work in an environment which can cultivate collaborations.

Organizers: Loren Coquille (HCM Bonn) and Janna Lierl (HCM, Bonn). **Plenary speakers:** N. Gantert (TU München, Germany), B. de Tilière (Paris 6, France) and A. Winter (Univ. Duisburg-Essen, Germany). We invite young female researchers to submit abstracts for consideration.

Support: Financial support is available.

Information: http://www.iam.uni-bonn.de/ywip2014/.

* 26-30 II International Conference "Geometric Analysis and Its Applications", Volgograd State University, Volgograd, Russia. Description: The conference is dedicated to the memory of Profes-

sor Vladimir Miklyukov. The main aim of the conference is to bring together experts working in the various areas of analysis, geometry and partial differential equations and their applications. Of particular interest will be all aspects of quasiregular mappings, minimal and maximal submanifold theory, non-linear PDEs.

Information: http://www.confgeoman.volsu.ru.

* 26–30 The 8th "International Conference on Topological Algebras and their Applications" (ICTAA 2014), Barceló Capella Beach Resort at the Playa de Villas de Mar Beach, Dominican Republic (about 45 minutes driving from Santo Domingo)

Topics: Of the conference include all areas of mathematics, connected with (preferably general) topological algebras and their applications, including all kinds of topological-algebraic structures as topological linear spaces, topological rings, topological modules, topological groups and semigroups; bornological-algebraic structures such as bornological linear spaces, bornological algebras, bornological groups, bornological rings and modules; algebraic and topological K-theory; topological module bundles, sheaves and others.

Objective: Of the present conference is to bring together experts and young researchers in these fields of mathematics. Please e-mail organizers at katza@stjohns.edu to request the conference announcement.

* 29 PIMS Public Seminar: Jim Gates, The University of British Columbia, Vancouver, BC V6T 1Z4, Canada.

Description: From the Adinkras of Supersymmetry to the Music of Arnold Schoenberg. The concept of supersymmetry, though never observed in Nature, has been one of the primary drivers of

investigations in theoretical physics for several decades. Through all of this time, there have remained questions that are unsolved. This presentation will describe how by looking at such questions one can be led to the `Dodecaphony Technique. of Austrian composer Schoenberg.

Information: http://www.pims.math.ca/scientific-event/140526-ppsjg.

June 2014

* 1-7 Modern Time-Frequency Analysis, Strobl, Austria.

Conference site: Is the Bundesinstitut fuer Erwachsenenbildung (Federal Institute for Continuing Education).

Topics: The topics of the conference include function spaces, time-frequency analysis and Gabor analysis, sampling theory and compressed sensing, pseudodifferential operators and Fourier integral operators, numerical harmonic analysis, abstract harmonic analysis, and applications of harmonic analysis.

Organizers: Hans G. Feichtinger (University of Vienna), Karlheinz Groechenig (University of Vienna) and Thomas Strohmer (University of California at Davis).

Information: http://nuhag.eu/strobl14.

* 4-5 3rd International Conference on Human Computing, Education and Information Management System (ICHCEIMS 2014), The Grace Hotel, Sydney, Australia.

Description: ICHCEIMS 2014 is designed to bring together, researchers active in the two fields (and related sub-fields) to foster an environment conducive to exchanging ideas, information and research. This conference aims to explore and discuss innovative studies of technology and its application in interfaces and welcomes research in progress, case studies, practical demonstrations and workshops in addition to the traditional submission categories.

Information: http://www.hceims.com.

* 5-6 3rd International Conference on Computer Science, Information System and Communication Technology, The Grace Hotel, Sydney, Australia.

Description: 3rd ICCSISCT 2014 is to provide a platform for researchers, engineers, academicians as well as industrial professionals from all over the world to present their research results and development activities in computer Engineering, Information Science and communications. This conference provides opportunities for the delegates to exchange new ideas and application experiences face to face, to establish business or research relations and to find global partners for future collaboration.

Information: http://www.csisct.com.

*6-11 XVI-th International Conference on Geometry, Integrability and Quantization, Sts. Constantine and Elena resort, near Varna, Bulgaria.

Description: This conference is next in a row of previous meetings on Geometry and Mathematical Physics which took place in Bulgaria - Zlatograd (1995) and annual conferences under the same title in Varna (1998-2013). "Geometry" in the title refers to modern differential geometry of real and complex manifolds with some emphasis on curves, sigma models and minimal surface theory; "Integrability" to either the integrability of complex structures or classical dynamical systems of particles, soliton dynamics and hydrodynamical flows presented in geometrical form; and "Quantization" to the transition from classical to quantum mechanics expressed in geometrical terms.

Aim: The overall aim is to bring together experts in Classical and Modern Differential Geometry, Complex Analysis, Mathematical Physics and related fields to assess recent developments in these areas and to stimulate research in related topics.

Principal speakers: Toshiyuki Kobayashi — Visible Actions and Multiplicity-Free Representations.

Information: http://www.bio21.bas.bg/conference.

* 10-13 Nonlinear partial differential equations and stochastic methods, Department of Mathematics and Statistics, Jyväskylä, Finland

Description: The purpose of the meeting is to gather together experts related to nonlinear partial differential equations, the special emphasis being in stochastic methods.

Information: http://www.jyu.fi/maths/en/
pdeandstochastics/.

* 16-18 Eleventh edition of the Advanced Course in Operator Theory and Complex Analysis, Sevilla, Spain.

Invited speakers: Nicola Arcozzi Bologna University (Italy); Alexander Olevskii Tel Aviv University (Israel); Jonathan R. Partington University of Leeds (U.K); Alexander Ulanovskii University of Stavanger (Norway).

Information: You can find further information about the courses at http://congreso.us.es/ceacyto/2014. As well as attending the course, you may also have the opportunity to deliver a contributed talk. Please feel free to pass this information to any colleague who might be interested in our conference.

* 16–20 **12th Workshop on Interactions between Dynamical Systems and Partial Differential Equations**, School of Mathematics and Statistics, Universitat Politècnica de Catalunya, Barcelona, Spain.

Description: EMS Summer School JISD2014 12th Workshop on Interactions between Dynamical Systems and Partial Differential Equations There will be four main courses of six hours each, and also some seminars, communications, and posters.

Courses: The courses will be taught by: Alessio Figalli (The University of Texas at Austin); Konstantin Khanin (University of Toronto); Sylvia Serfaty (Université Pierre et Marie Curie Paris 6); Susanna Terracini (Università di Torino).

Organizers: The JISD2014 are organized by Xavier Cabré, Amadeu Delshams, Maria del Mar Gonzàlez, and Tere M. Seara, from the Universitat Politècnica de Catalunya. with the support of the European Mathematical Society, the Clay Mathematics Institute, the Royal Spanish Mathematical Society, and the Catalan Mathematical Society. Information: http://www.mal.upc.edu/recerca/jisd/jisd2014.

* 16–20 NSF-CBMS Regional Research Conference "Quantum Spin Systems", University of Alabama at Birmingham, Birmingham, Alabama.

Description: Bruno Nachtergaele from the University of California Davis will deliver ten lectures on the mathematical theory of quantum spin systems. Topics covered will range from introductory material to recent results with important applications in statistical mechanics and quantum information theory. The program will also include complementary lectures by other experts in the field as well as tutorials on relevant background. Funding is available through an NSF grant which provides travel and local support for approximately 35 participants, in particular graduate students and recent postgraduates. See the conference website for information on how to apply. **Information:** http://www.uab.edu/cas/mathematics/events/nsf-cbms-conference-2014.

* 22-28 The Fourth International School-Seminar "Nonlinear Analysis and Extremal Problems", Institute of System Dynamics and Control Theory SB RAS, Irkutsk, Russia.

Description: The aim of the school-seminar is to introduce young researchers to some topics of current research in the fields of: nonlinear analysis and its applications, dynamical systems, evolution equations and partial differential equations, calculus of variations and optimal control. The main part of the school-seminar will consist of series of lectures by leading scientists (Z. Artstein (Israel), I. Ekeland (Canada), L. Thibault (France), Yu. S. Ledyaev (USA), V. V. Zhikov (Russia)) in the above fields, and the rest of the time will be devoted to short talks of other participants.

Languages: The working languages of the school-seminar are Russian and English.

Venue: The city of Irkutsk, the venue of the conference, is one of the oldest cities in Siberia which has many interesting tourist attractions. It is located in an immediate vicinity of the famous Lake Baikal. A one day trip to the lake will be also organized.

Information: http://www.idstu.irk.ru/en/nla-2014.

* 22-July 6 9th International Summer School and Conference "Let's Face Chaos through Nonlinear Dynamics", CAMTP, University of Maribor, Maribor, Slovenia, European Union.

Description: The school/conference provides a very solid and broad introduction to nonlinear dynamics and complex systems in the first week, in particular for graduate students of physics, mathematics or other natural sciences and engineering, but also for other researchers at postdoctoral or higher level, or nonexperts, whilst in the second week provides a thorough survey of current research in a series of one-hour invited talks on the conference level, but still with a good introduction for students and nonexperts. The topics covers classical and quantum chaos and complex systems in a very broad interdisciplinary context. The invited lecturers and speakers are among the leading experts in the fields. The participants can present a short report, or posters, or both. The meeting is dedicated to the 65th birthday of Prof. Theo Geisel (Director of the Max-Planck-Institute for Dynamics, Goettingen, Germany) and is under the patronage of the European Academy of Sciences and Arts (Salzburg).

Information: http://www.camtp.uni-mb.si/chaos/2014/.

* 23–26 International Congress in Honour of Professor Ravi P. Agarwal, The Auditorium at the Campus of Uludag University, Bursa, Turkey.

Description: The forthcoming International Congress in Honour of Professor Ravi P. Agarwal is motivated essentially by the remarkable popularity and success of the well-attended International Congresses which were held in August 2010 and August 2012 and also the 24th National Mathematics Symposium in September 2011 under the auspices of Uludag University. Professor Ravi P. Agarwal is one of the few mathematicians in the world that had most contributed to Turkish mathematics and mathematicians which is the main reason for this congress to be held in Bursa. This congress will be organized at Uludag University in the fourth largest city, Bursa in Turkey in honour of Professor Dr. Ravi P. Agarwal. It will cover a wide range of topics of mathematics.

Information: http://agarwal2014.uludag.edu.tr.

* 23–27 **12th Biennial IQSA Meeting Quantum Structures Olomouc 2014**, Palack University Olomouc, Faculty of Science, Olomouc, Czech Republic.

Description: Quantum Structures 2014 Olomouc is a major international conference integrating all fields of quantum mechanics and its applications. It provides an important opportunity for young researchers to disseminate their results and to obtain feedback both from their peers and from senior members of the community. Owing to its interdisciplinary and foundational character, the objective of the conference is to encourage communication between researchers throughout the world whose research is related to quantum structures and their applications in physics, mathematics and philosophy, such as logico-algebraic structures, orthomodular structures, quantum logics, empirical logics, operational structures, quantum mechanics, quantum measurements, quantum computation, quantum information, quantum communication, philosophy of quantum mechanics, quantum probability, interdisciplinary applications of quantum structures, etc.

Information: http://ameql.math.muni.cz/iqsa2014.

* 23–27 Conference on Differential and Difference Equations and Applications 2014 (CDDEA 2014), Jasná, Slovak Republic.

Description: International scientific conference is oriented to the actual research in the field of differential and difference equations

(ordinary differential equations, functional differential equations, partial differential equations, stochastic differential equations, difference equations and dynamic equations on time scales, numerical methods in differential and difference equations) and their applications.

Information: http://fpv.uniza.sk/cddea.

* 23–27 International conference "Dynamical Systems and Their Applications", Institute of Mathematics of National Academy of Sciences of Ukraine, Kyiv, Ukraine.

Description: The internal wealth and beauty of results of dynamical systems theory, and also its exceptional practical importance motivate a growing number of experts in different areas to study dynamical systems. The conference topics cover a wide range of problems, in particular, in topological dynamics, the theory of attractors and chaos, combinatorial and symbolic dynamics, the theory of fractals, infinite-dimensional dynamical systems, and applications. Emphasis is expected to be paid to combinatorial dynamics, that originates from Sharkovsky's well-known theorem on the coexistence of cycles ("Ukrainian Mathematical Journal", 1964) and celebrates its 50th anniversary in 2014.

Information: http://cds2014.imath.kiev.ua/.

* 28-July 2 Conference Board of Mathematical Sciences/National Science Foundation: Mathematical Phylogeny Conference, Winthrop University, Rock Hill, South Carolina.

Description: This conference will feature Dr. Mike Steel who will be giving a series of ten lectures. These lectures will be supplemented by working groups and four additional invited speakers to be announced on the website in the coming weeks. Limited funding will be available to support graduate students, postdocs, beginning researchers and faculty from undergraduate institutions in the southeastern U.S.

Information: http://www.birdnest.org/phylogeny.

July 2014

* 6–10 **CBMS conference on Higher Representation Theory**, North Carolina State University, Raleigh, North Carolina.

Description: The CBMS regional conference will gather main experts working on categorification of Kac-Moody algebras and related topics. Raphael Rouqueir (UCLA) will give a short course on Higher Representation Theory of Kac-Moody Algebras.

Other invited speakers: Jon Brundan (Oregon), Sabin Cautis (USC), Alex Ellis (Oregon), Aaron Lauda (USC), Yiqiang Li (SUNY-Buffalo), Weiqiang Wang (Virginia), and others.

Support: Limited support is available to graduate students and young researchers. For more information, please consult the website. **Information:** http://www.math.ncsu.edu/~jing/conf/CBMS/cbms14.html.

* 14–18 First announcement of the 25th International Workshop of Operator Theory and its Applications (IWOTA 2014), Amsterdam, The Netherlands.

Description: This is the week after the MTNS 2014, which will be held in Groningen. During the conference there will be two special days, one on the occasion of the 80th birthday of Dima Arov, and one on the occasion of the 65th birthday of Leiba Rodman.

Organizing committee: Tanja Eisner, Birgit Jacob, André Ran (chair), Hans Zwart.

Registration: For IWOTA 2014 will be open in February 2014.

Talks/Sessions: We would like to invite you to suggest special sessions and/or submit contributed talks. To suggest a special session, please send a proposal to iwota2014@gmail.com. To submit a contributed talk: 1. Please prepare title and abstract in standard Latex, using the template on the IWOTA 2014 website. 2. Submit title and abstract to iwota2014@gmail.com.

Deadline: For suggestions of special sessions and for submissions of contributed talks is March 1, 2014. We are looking forward to your contribution toward the success of the IWOTA 2014.

Information: Further information can be found on the website: http://www.math.vu.nl/~ran/iwota2014/.

* 23–26 **29th Summer Conference on Topology and its Applications**, College of Staten Island, City Univ. of New York, New York, New York.

Description: Our central goal in this series of annual conferences is to hold a meeting which will promote cross-fertilization of research directions and methods across diverse disciplines. We accomplish this with special sessions (each with 2 dozen 30-minute talks) in half a dozen of these specialties, and having a plenary talk associated with each session.

Topic, Head Organizer, Speaker: Asymmetry and its Applications, Ralph Kopperman, Jimmie Lawson; Continua with Applications, Judy Kennedy, Tobias Jaeger; Foliations, Laminations, Attractors, Krystyna Kuperberg, Pawel Walczak; Geometric Topology, Ilya Kofman, Jeremy Kahn; Set-theoretic and General Topology, Alan Dow, Michael Hrusak; Topology in Functional Analysis, Jerzy Kakol, Grzegorz Plebanek. Information: http://csivc.csi.cuny.edu/sumtopo2014.

* 28-August 1 XXIII Escola de Álgebra (Brazilian Algebra Meeting), Universidade Estadual de Maringá, Maringá, Paraná/Brazil.

Description: Escola de Álgebra is a biannual meeting of algebraists and it is the largest event in Brazil entirely devoted to Algebra and related topics. Its main objective is to provide an opportunity for researchers and students to exchange ideas, to communicate and discuss research findings in all branches of Algebra. The activities of the Escola de The Escola de Álgebra will include plenary talks, invited lectures, thematic sessions, mini-courses, communications and posters.

Information: http://www.dma.uem.br/algebra2014/.

* 28-August 8 Poisson 2014 - International Conference and School on Poisson Geometry in Mathematics and Physics, University of Illinois at Urbana-Champaign, Champaign, Illinois.

Description: The conference will be preceded by a Summer School on Poisson Geometry, from July 28 to August 1, aimed at preparing students and young researchers for the conference. For more information (list of speakers, lecturers, etc), please consult the meetings web page. For junior participants, we plan to offer several merit-based fellowships to provide financial assistance towards living and/or travel expenses. The conference Poisson 2014 is the 9th event of a series of biennial meetings that have previously taken place in Warsaw (1998), Luminy (2000), Lisbon (2002), Luxembourg (2004), Tokyo (2006), Lausanne (2008), Rio de Janeiro (2010) and Utrecht (2012).

Information: http://www.math.illinois.edu/
Poisson2014/.

August 2014

* 4-8 Kazhdan-Lusztig theory and Soergel bimodules, University of Oregon, Eugene, Oregon.

Description: The goal of this workshop will be to first get a solid handle on Soergel bimodules and the philosophy of algebraic categorification, and then understand the recent proof of the Soergel conjecture in the paper "The Hodge theory of Soergel bimodules", by Ben Elias and Geordie Williamson. The workshop will be led by Ben Elias, and will consist of a combination of lectures and problem sessions.

 $\textbf{Information:} \ \texttt{http://pages.uoregon.edu/njp/kl.html}.$

*6-10 International Conference on K-Theory and Related Topics, Chinese Academy of Sciences, Beijing, China.

Description: The aim of this conference (a satellite conference of the 2014 ICM) is to bring together researchers active in K-theory and related areas of mathematics where K-theory plays an important role, including algebraic geometry, number theory, classical-like

groups, and noncommutative geometry. Specific topics wlll include algebraic cycles, A^1 -homotopy theory, derived and triangulated categories, motivic cohomology, KK-theory, and cyclic (co)homology. The K-theory Prize(s) of the K-Theory Foundation will be awarded for the first time at this conference.

Information: http://www.ktheorybeijing.org/.

* 19-21 Advances in Applied Mathematics and Mathematical Physics, Yildiz Technical University, Istanbul, Turkey.

Goal: Of the conference is to bring together active researchers from Mathematics and Physics to showcase their state-of-the-art research results and hopefully to forge new cross-disciplinary interactions among the participants. The conference provides a unique opportunity for in-depth technical discussions and exchange of ideas in Mathematics and Mathematical Physics, as well as explores the potential of their applications in natural and social sciences, engineering and technology and industry and finance. The objectives of this conference are to: provide a forum for researchers, educators, students, contributors, users of mathematical knowledge and industries to exchange ideas and communicate and discuss research findings and new advancement in mathematics and statistics.

Information: http://icaammp.naturalspublishing.com.

*25-29 Research School on Algebraic Lie Theory, University of Glasgow, Glasgow, Scotland.

Description: The LMS-Clay Institute research school is aimed at Ph.D. students and early postdocs wanting to learn about topics in algebraic Lie theory and representation theory that are currently of great interest. The three main courses are: 1) Rational Cherednik Algebras (Iain Gordon, Edinburgh). 2) Quiver Hecke Algebras (Andrew Mathas, Sydney). 3) Categorification in Lie Theory (Catharina Stroppel, Bonn). These lecture courses will be supplemented by tutorial sessions.

Information: http://www.maths.gla.ac.uk/~gbellamy/
summer/.

* 25–30 **19th International Summer School on Global Analysis and its Applications** - **"Symmetries"**, Lednice, Czech Republic Chateau Hotel, Lednice, Czech Republic.

Description: The 19th Summer School on Global Analysis has the title "Symmetries". It will be held in the heart of the UNESCO land-scape Lednice. The programme of the school consists of the courses, devoted by foundations and delivered by recognized specialists in the field. To support the dissemination of current scientific results, the workshop of oral presentations and commented poster session are organized.

Information: http://www.lepageri.eu/summer-school; http://www.hotellednice.cz/.

* 25-September 5 NATO Advanced Study Institute: Arithmetic of Hyperelliptic Curves and Cryptography, University for Information Science and Technology "St. Paul the Apostle", Ohrid, Macedonia. Description: The goal of this Advanced Study Institute is to provide training and expertise to mathematicians, computer scientists, and engineers from NATO and partner countries in the area of arithmetic of hyperelliptic curves in developing and maintaining crypto systems based on hyperelliptic cryptography.

Directors: Tony Shaska, Eustrat Zhupa. **Information:** http://nato.risat.org.

September 2014

* 3-5 **4th IMA Numerical Linear Algebra and Optimisation**, University of Birmingham, Birmingham, United Kingdom.

Description: The success of modern methods for large-scale optimisation is heavily dependent on the use of effective tools of numerical linear algebra. On the other hand, many problems in numerical linear algebra lead to linear, nonlinear or semidefinite optimisation problems. The purpose of the conference is to bring together researchers from both communities and to find and communicate points and topics of common interest.

Information: http://www.ima.org.uk/conferences/
conferences_calendar/4th_ima_conference_on_
numerical_linear_algebra_and_optimisation.cfm.

* 11-13 Second International Conference on Analysis and Applied Mathematics (ICAAM 2014), M. Auezov South Kazakhstan State University, Shymkent, Kazakhstan.

Aim: To bring mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. In mathematics, the developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why we plan to found the conference series to provide a forum for researchers and scientists to communicate their recent developments and to present their original results in various fields of analysis and applied mathematics.

Information: http://www.icaam-online.org/index/.

* 15-19 AIM Workshop: Generalized persistence and applications, American Institute of Mathematics, Palo Alto, California.

Description: This workshop, sponsored by AIM and the NSF, will be devoted to generalizations of persistent homology with a particular emphasis on finding calculable algebraic invariants useful for applications.

Information: http://aimath.org/workshops/upcoming/
persistence.

* 17–20 Third International Conference of Numerical Analysis and Approximation Theory (NAAT2014), Babes - Bolyai University, Faculty of Mathematics and Computer Science, Department of Mathematics. Clui-Napoca. Romania.

Description: The conference is an opportunity for meeting and sharing ideas among researchers whose interest lies in function approximation, linear approximation processes, numerical analysis, statistics, stochastic processes.

Confirmed keynote speakers: Francesco Altomare (University of Bari, Italy), Francisco - Javier Muoz - Delgado (University of Jaen, Spain), Gradimir Milovanović; (Mathematical Institute of the Serbian Academy of Sciences and Arts, Serbia), Maria Neuss - Radu (University of Erlangen-Nürenberg, Germany), Gregory M. Nielson (Arizona State University, USA), Iuliu Sorin Pop (Eindhoven University of Technology, Netherlands), Björn Schmalfuss (Friedrich-Schiller-University, Jena, Germany).

Information: http://naat.math.ubbcluj.ro/.

October 2014

* 13-17 AIM Workshop: Positivity, graphical models, and modeling of complex multivariate dependencies, American Institute of Mathematics, Palo Alto, California.

Description: This workshop, sponsored by AIM and the NSF, will be devoted to studying functions that preserve Loewner properties on (distinguished submanifolds of) the cone of positive semidefinite matrices.

Information: http://aimath.org/workshops/upcoming/
modelmultivar.

* 22-24 International Conference in Modeling Health Advances **2014**, UC Berkeley, San Francisco Bay Area, California.

Description: The purpose of this conference is to bring all the people working in the area of epidemiology under one roof and encourage mutual interaction. The conference ICMHA'14 is held under the World Congress on Engineering and Computer Science WCECS 2014. The WCECS 2014 is organized by the International Association of Engineers (IAENG), a non-profit international association for the engineers and the computer scientists. The congress has the focus on the frontier topics in the theoretical and applied engineering and computer science subjects. The last IAENG conference has attracted more than five hundred participants from over 30 countries. All submitted papers will be under peer review and accepted papers will be published in the conference proceeding (ISBN: 978-988-19252-0-6).

The abstracts will be indexed and available at major academic databases. The accepted papers will also be considered for publication in the special issues of the journal Engineering Letters.

Information: http://www.iaeng.org/WCECS2014/
TCMHA2014.html.

* 27–31 **AlM Workshop: Configuration spaces of linkages**, American Institute of Mathematics, Palo Alto, California.

Description: This workshop, sponsored by AIM and the NSF, will be devoted to the mathematical study of configuration spaces of linkages consisting of rigid bars connected by revolute joints embedded in an ambient space of fixed dimension.

Information: http://aimath.org/workshops/upcoming/ linkages.

November 2014

* 3-7 AIM Workshop: Combinatorics and complexity of Kronecker coefficients, American Institute of Mathematics, Palo Alto, California.

Description: This workshop, sponsored by AIM and the NSF, will be devoted to the study of Kronecker coefficients which describe the decomposition of tensor products of irreducible representations of a symmetric group into irreducible representations.

Information: http://aimath.org/workshops/upcoming/
kroncoeff.

December 2014

* 8-12 **8th Australia - New Zealand Mathematics Convention**, University of Melbourne, Melbourne, Australia.

Description: The Australia - New Zealand Mathematics Convention is held every six years. It is the combined meeting of the Australian and New Zealand Mathematical Societies and it will also include the 2014 annual meeting of ANZAMP - the Australian and New Zealand Association of Mathematical Physics.

Information: http://www.austms2014.ms.unimelb.edu.au.

*19-21 International Conference on Current Developments in Mathematics and Mathematical Sciences (ICCDMMS-2014), Calcutta Mathematical Society, AE-374, Sector-1, Salt Lake City, Kolkata-700064 West Bengal, India.

Description: The main objective of ICCDMMS-2014 is to promote mathematical research and to focus the recent advances in mathematics and mathematical sciences along with their applications. The conference aims to provide an ideal platform for the young researchers throughout the world to interact with senior scientists, to exchange their views and ideas and to initiate possible scientific collaboration in different domains.

Talks: The programme will consist of plenary invited talks as well as contributed presentations. Thrust areas of ICCDMMS-2014: i) Algebra, ii) Analysis, iii) Geometry and Topology, iv) Continuum Mechanics, v) Mathematical and Theoretical Physics, vi) Financial Mathematics, vii) Complex Phenomena and Dynamical Systems, viii) Space Research and Atmospheric Sciences, ix) Stochastic Processes, x) Numerical Techniques and Simulations, xi) Soft Computing, xii) Number Theory, xiii) Information Theory, xiv) General Relativity and Cosmology.

Information: Contact email: cmsconf@gmail.com; Contact No.:
+91-33-2337 8882; http://www.calmathsoc.org/.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

February 2016

* 15-June 17 **Melt in the Mantle**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom.

Description: The Earth's mantle is almost entirely solid, but on geological timescales it convects vigorously, the well-known surface expression of this being plate tectonics. Although the basic thermodynamics of melt generation in these settings is well understood, how the melt is transported to the surface is not, despite several decades of work on the problem. Sophisticated mathematical techniques are needed to map an understanding of physics at the smallest scales to plate-tectonic scales. Seismology offers a way to image melt in the mantle, but development of new tools in inverse theory is required to extract that information. Models are cast as a series of coupled non-linear PDEs, which require advanced numerical techniques to solve. This programme will bring together a broad spectrum of mathematicians and solid Earth scientists to tackle these and other challenges in the area. Several workshops will take place during the programme. For full details please see www.newton.ac.uk/events.html. Information: http://www.newton.ac.uk/programmes/MIM/.

July 2016

* 11-December 21 **Theoretical Foundations for Statistical Network Analysis**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom.

Description: The core of this 6-month programme is understanding and quantifying mathematical structure in network models. Networks are ubiquitous in modern science and society. In fact, whenever we observe entities and relationships between them, we have network data. The behaviour of almost all networks, natural or engineered, physical or information-based, involves a strong component of randomness and is typically not fully or directly observed. Considerable open challenges remain in proving properties both of generative mechanisms for such networks, as well as of methods for inference. This motivates the development of theoretical foundations for statistical network analysis. Several workshops will take place during the programme, including an opening, midterm and closing workshop, as well as a Satellite Meeting and an Open for Business industry day. For full details please see http://www.newton.ac.uk/events.html.

Information: http://www.newton.ac.uk/programmes/SNA/.

New Publications Offered by the AMS

To subscribe to email notification of new AMS publications, please go to http://www.ams.org/bookstore-email.

Geometry and Topology



Introduction to 3-Manifolds

Jennifer Schultens, *University of California*, *Davis*, *CA*

This book grew out of a graduate course on 3-manifolds and is intended for a mathematically experienced audience that is new to low-dimensional topology.

The exposition begins with the definition of a manifold, explores possible additional structures on manifolds, discusses the classification of surfaces, introduces key foundational results for 3-manifolds, and provides an overview of knot theory. It then continues with more specialized topics by briefly considering triangulations of 3-manifolds, normal surface theory, and Heegaard splittings. The book finishes with a discussion of topics relevant to viewing 3-manifolds via the curve complex.

With about 250 figures and more than 200 exercises, this book can serve as an excellent overview and starting point for the study of 3-manifolds.

Contents: Perspectives on manifolds; Surfaces; 3-manifolds; Knots and links in 3-manifolds; Triangulated 3-manifolds; Heegaard splittings; Further topics; General position; Morse functions; Bibliography; Index.

Graduate Studies in Mathematics, Volume 151

June 2014, approximately 289 pages, Hardcover, ISBN: 978-1-4704-1020-9, LC 2013046541, 2010 *Mathematics Subject Classification:* 57N05, 57N10, 57N16, 57N40, 57N50, 57N75, 57Q15, 57Q25, 57Q40, 57Q45, **AMS members US\$60**, List US\$75, Order code GSM/151

New AMS-Distributed Publications

Algebra and Algebraic Geometry



Transformations Birationnelles de Petit Degré

Dominique Cerveau, *Université de Rennes 1, France*, and **Julie Déserti**, *Université Paris 7, France*

Since the end of the 19th century, we have known that each birational map of the complex projective plane is the product of a

finite number of quadratic birational maps of the projective plane. This has motivated the authors' work, which essentially deals with these quadratic maps.

The authors establish algebraic properties such as the classification of one parameter groups of quadratic birational maps or the smoothness of the set of quadratic birational maps in the set of rational maps. The authors prove that a finite number of generic quadratic birational maps generates a free group. They show that if f is a quadratic birational map or an automorphism of the projective plane, the normal subgroup generated by f is the full group of birational maps of the projective plane, which implies that this group is perfect.

The authors study some dynamical properties: following an idea of Guillot, they translate some invariants for foliations; in particular, they obtain that if two generic quadratic birational maps are birationally conjugate, then they are conjugate by an automorphism of the projective plane. The authors are also interested in invariant objects: curves, foliations, fibrations. They study birational maps of degree 3 and, by considering the different possible configurations of the exceptional curves, they give the "classification" of these maps and can deduce from it that the set of the birational maps of degree 3 exactly is irreducible, and is, in fact, rationally connected.

New AMS-Distributed Publications

This item will also be of interest to those working in analysis.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Transformations rationnelles et birationnelles quadratiques; Germes de flots birationnels quadratiques; Transformations rationnelles, feuilletages, conjugaison dans les $\sum i$; Quelques propriétés dynamiques des transformations birationnelles quadratiques; Propriétés algébriques du groupe de Cremona; Transformations birationnelles de degré 3; Index; Index des notations; Bibliographie.

Cours Spécialisés—Collection SMF, Number 19

November 2013, 223 pages, Softcover, ISBN: 978-2-85629-770-4, 2010 *Mathematics Subject Classification*: 14E07, 14E05, 37F10, 37F50, **AMS members US\$86.40**, List US\$108, Order code COSP/19

Analysis



Metric Spaces, Convexity and Nonpositive Curvature

Second Edition

Athanase Papadopoulos, Université de Strasbourg, France

This book is about metric spaces of nonpositive curvature in the sense of Busemann, that is, metric spaces whose

distance function satisfies a convexity condition. The book also contains a systematic introduction to metric geometry, as well as a detailed presentation of some facets of convexity theory that are useful in the study of nonpositive curvature.

The concepts and the techniques are illustrated by many examples, in particular from hyperbolic geometry, Hilbert geometry and Teichmüller theory.

For the second edition, some corrections and a few additions have been made, and the bibliography has been updated.

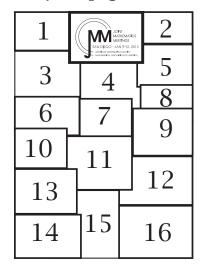
A publication of the European Mathematical Society. Distributed within the Americas by the American Mathematical Society.

Contents: Introduction: Some historical markers; Lengths of paths in metric spaces; Length spaces and geodesic spaces; Maps between metric spaces; Distances; Convexity in vector spaces; Convex functions; Strictly convex normed vector spaces; Busemann spaces; Locally convex spaces; Asymptotic rays and the visual boundary; Isometries; Busemann functions, co-rays and horospheres; Bibliography; Index.

IRMA Lectures in Mathematics and Theoretical Physics, Volume 6

December 2013, 320 pages, Softcover, ISBN: 978-3-03719-132-3, 2010 *Mathematics Subject Classification*: 26-01, 30F25, 30F45, 30F60, 32G15, 32Q45, 51-01, 51K05, 51K10, 51M09, 51M10, 51F99, 52-01, 52A07, 52A41, 53-01, 53C70, 54-01, 54E35, **AMS members US\$46.40**, List US\$58, Order code EMSILMTP/6.R

2014 Baltimore, MD Joint Mathematics Meetings Photo Key for page 344



- 1. AMS Booth in Exhibits area.
- 2. Employment Center.
- 3. Contestants in the Who Wants to Be a Mathematician game.
- 4. AMS banquet.
- 5. Attending a session.
- Karen Vogtmann's talk, "The Geometry of Outer Space", Current Events Bulletin.
- 7. 2014 AMS Fellows dessert reception.
- 8. Networking.
- Jill Pipher, MAA Invited Address, "The Mathematics of Lattice-Based Cryptography".
- 10. Art exhibit.
- 11. Andrew Blake, AMS Gibbs Lecture, "Machines that See, Powered by Probability".
- 12. Email Center.
- 13. Mime and mathematician Tim Chartier.
- 14. WWTBAM winner Vivek Miglani with AMS Executive Director Don McClure and President David Vogan.
- 15. Cédric Villani, winner of Joseph L. Doob Prize.
- 16. Cole Prize winner János Pintz (left), Steele Prize winner Louis Nirenberg (center) and Steele Prize winner Luis Caffarelli (right).

Classified Advertisements

Positions available, items for sale, services available, and more

MARYLAND

JOHNS HOPKINS UNIVERSITY Department of Applied Mathematics and Statistics

Department of Mathematics Bloomberg Distinguished Professor

Johns Hopkins University invites applications for a Bloomberg Distinguished Professorship in the area of the Mathematical Foundations of Data Intensive Computation and Inference. This position is one of 50 new Bloomberg Distinguished Professorships designated for outstanding scholars at the associate or full professor rank who carry out interdisciplinary research and teaching in areas identified for significant growth at the university. The position will include joint tenure in the Department of Applied Mathematics and Statistics in the Whiting School of Engineering and the Department of Mathematics in the Krieger School of Arts and Sciences. The holder of this Bloomberg Distinguished Professorship will participate in the research and teaching activities of both departments and would devote 50% of his/her effort to each department. Applicants should possess distinguished records of achievement in research and teaching in areas of mathematics and statistics applicable to the representation and analysis of large data sets. Applicants should submit a cover letter, curriculum vitae and a list of publications to bdp. mathdata@jhu.edu. Review of applications will begin on February 28, 2014, and will continue until the position is filled. Johns Hopkins University is committed to enhancing the diversity of its faculty and encourages applications from women and minorities. The Johns Hopkins University is an Affirmative Action/Equal Opportunity Employer.

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CHILE

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE

Departamento de Matemáticas

The Department of Mathematics invites applications for two tenure-track positions at the Assistant Professor level beginning either March or August 2015. Applicants should have a Ph.D. in mathematics, proven research potential either in pure or applied mathematics, and a strong commitment to teaching and research. The regular teaching load for assistant professors consists of three one-semester courses per year, reduced to two courses during the first two years. The annual salary will be approximately US\$48,000. A startup grant of US\$15,000 to be used during the first three years will be available to support research activities.

Please send a letter indicating your main research interests, potential collaborators in our department (http://www.mat.puc.cl), a detailed curriculum vitae, and three letters of recommendation to:

Monica Musso Departamento de Matemáticas Pontificia Universidad Católica de Chile Av. Vicuña Mackenna 4860 Santiago, Chile; fax: (56-2) 552-5916; email: mmusso@mat.puc.cl

For full consideration, complete application materials must arrive by June 30, 2014.

000019

FOR SALE

Books

3231 mathematics books, 503 computer science books, 375 physics books, 508 cosmology books. contact: cjm@ix.netcom.com.

000018

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2014 rate is \$3.50 per word with a minimum two-line headline. No discounts for multiple ads or the same ad in consecutive issues. For an additional \$10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: May 2014 issue–March 3, 2014; June/July 2014 issue–April 29, 2014; August 2014 issue–May 29, 2014; September 2014 issue–June

 $30,2014; October\ 2014$ issue–July 29, 2014; November 2014 issue–September 4, 2014

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to classad@ams.org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the Notices. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See http://www.ams.org/meetings/. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the <code>Notices</code> as noted below for each meeting.

Knoxville, Tennessee

University of Tennessee, Knoxville

March 21-23, 2014

Friday - Sunday

Meeting #1097

Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of Notices: January 2014

Program first available on AMS website: February 6, 2014

Program issue of electronic Notices: March 2014

Issue of Abstracts: Volume 35, Issue 2

Deadlines

For organizers: Expired For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Maria Chudnovsky, Columbia University, *Coloring graphs with forbidden induced subgraphs* (Erdős Memorial Lecture).

Ilse C.F. Ipsen, North Carolina State University, *Introduction to randomized matrix algorithms*.

Daniel Krashen, University of Georgia, *Algebraic structures, topology, and the arithmetic of fields.*

Suresh Venapally, Emory University, *Quadratic forms and Galois cohomology*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Methods in Graph Theory and Combinatorics, Felix Lazebnik, University of Delaware, Andrew Woldar, Villanova University, and Bangteng Xu, Eastern Kentucky University.

Arithmetic of Algebraic Curves, **Lubjana Beshaj**, Oakland University, **Caleb Shor**, Western New England University, and **Andreas Malmendier**, Colby College.

Commutative Ring Theory (in honor of the retirement of David E. Dobbs), **David Anderson**, University of Tennessee, Knoxville, and **Jay Shapiro**, George Mason University.

Completely Integrable Systems and Dispersive Nonlinear Equations, Robert Buckingham, University of Cincinnati, and Peter Perry, University of Kentucky.

Complex Analysis, Probability, and Metric Geometry, Matthew Badger, Stony Brook University, Jim Gill, St. Louis University, and Joan Lind, University of Tennessee, Knoxville.

Discontinuous Galerkin Finite Element Methods for Partial Differential Equations, Xiaobing Feng and Ohannes Karakashian, University of Tennessee, Knoxville, and Yulong Xing, University of Tennessee, Knoxville, and Oak Ridge National Laboratory.

Diversity of Modeling and Optimal Control: A Celebration of Suzanne Lenhart's 60th Birthday, Wandi Ding, Middle Tennessee State University, and Renee Fister, Murrray State University.

Fractal Geometry and Ergodic Theory, Mrinal Kanti Roychowdhury, University of Texas Pan American.

Galois Cohomology and the Brauer Group, **Ben Antieau**, University of Washington, **Daniel Krashen**, University of Georgia, and **Suresh Venapally**, Emory University.

Geometric Topology, **Craig Guilbault**, University of Wisconsin-Milwaukee, and **Steve Ferry**, Rutgers University.

Geometric Topology and Number Theory, Eriko Hironaka and Kathleen Petersen, Florida State University.

Geometric and Algebraic Combinatorics, Benjamin Braun and Carl Lee, University of Kentucky.

Geometric and Combinatorial Methods in Lie Theory, Amber Russell and William Graham, University of Georgia.

Graph Theory, **Chris Stephens**, **Dong Ye**, and **Xiaoya Zha**, Middle Tennessee State University.

Harmonic Analysis and Nonlinear Partial Differential Equations, Jochen Denzler, Michael W. Frazier, Tuoc Phan, and Grozdena Todorova, University of Tennessee, Knoxville.

Invariant Subspaces of Function Spaces, Catherine Beneteau, University of South Florida, Alberto A. Condori, Florida Gulf Coast University, Constanze Liaw, Baylor University, and Bill Ross, University of Richmond.

Mathematical Modeling of the Within- and Between-Host Dynamics of Infectious Diseases, Megan Powell, University of St. Francis, and Judy Day and Vitaly Ganusov, University of Tennessee, Knoxville.

Mathematical Physics and Spectral Theory, Roger Nichols, The University of Tennessee at Chattanooga, and Günter Stolz, University of Alabama at Birmingham.

Metric Geometry and Topology, **Catherine Searle**, Oregon State University, **Jay Wilkins**, University of Connecticut, and **Conrad Plaut**, University of Tennessee, Knoxville.

Nonlinear Partial Differential Equations in the Applied Sciences, Lorena Bociu, North Carolina State University, Ciprian Gal, Florida International University, and Daniel Toundykov, University of Nebraska-Lincoln.

Recent Development on Hyperbolic Conservation Laws, Geng Chen, Ronghua Pan, and Weizhe Zhang, Georgia Tech.

Scientific Computing, Numerical Analysis, and Mathematical Modeling, Vasilios Alexiades, Xiaobing Feng, and Steven Wise, University of Tennessee, Knoxville.

Singularities and Physics, **Mboyo Esole**, Harvard University, and **Paolo Aluffi**, Florida State University.

Stochastic Processes and Related Topics, Jan Rosinski and Jie Xiong, University of Tennessee, Knoxville.

von Neumann Algebras and Free Probability, Remus Nicoara, University of Tennessee, Knoxville, and Arnaud Brothier, Vanderbilt University.

Baltimore, Maryland

University of Maryland, Baltimore County

March 29-30, 2014

Saturday - Sunday

Meeting #1098

Eastern Section

Associate secretary: Steven H. Weintraub Announcement issue of *Notices*: January 2014 Program first available on AMS website: February 26, 2014 Program issue of electronic *Notices*: March 2014 Issue of *Abstracts*: Volume 35, Issue 2

Deadlines

For organizers: Expired For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Maria Gordina, University of Connecticut, Stochastic analysis and geometric functional inequalities.

L. Mahadevan, Harvard University, *Shape: Mathematics, physics, and biology.*

Nimish A. Shah, The Ohio State University, Homogeneous dynamics and its interactions with number theory.

Daniel T. Wise, McGill University, *Cube complexes*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Data Assimilation Applied to Controlled Systems, Damon McDougall, University of Texas at Austin, and Richard Moore, New Jersey Institute of Technology.

Difference Equations and Applications, Steven Miller, Williams College, and Michael Radin, Rochester Institute of Technology.

Discrete Geometry in Crystallography, Egon Schulte, Northeastern University, and Marjorie Senechal, Smith College.

Harmonic Analysis and Its Applications, Susanna Dann, University of Missouri, Azita Mayeli, Queensborough College, City University of New York, and Gestur Olafsson, Louisana State University.

Interaction between Complex and Geometric Analysis, Peng Wu, Cornell University, and Yuan Yuan, Syracuse University.

Invariants in Low-Dimensional Topology, Jennifer Hom, Columbia University, and **Tye Lidman**, University of Texas at Austin.

Knots and Applications, **Louis Kauffman**, University of Illinois at Chicago, **Samuel Lomonaco**, University of

Maryland, Baltimore County, and **Jozef Przytycki**, George Washington University.

Low-dimensional Topology and Group Theory, David Futer, Temple University, and Daniel Wise, McGill University.

Mathematical Biology, **Jonathan Bell** and **Brad Peercy**, University of Maryland Baltimore County.

Mathematical Finance, **Agostino Capponi**, John Hopkins University.

Mechanics and Control, **Jinglai Shen**, University of Maryland Baltimore County, and **Dmitry Zenkov**, North Carolina State University.

Novel Developments in Tomography and Applications, Alexander Katsevich, Alexandru Tamasan, and Alexander Tovbis, University of Central Florida.

Open Problems in Stochastic Analysis and Related Fields, **Masha Gordina**, University of Connecticut, and **Tai Melcher**, University of Virginia.

Optimization and Related Topics, M. Seetharama Gowda, Osman Guler, Florian Potra, and Jinlai Shen, University of Maryland at Baltimore County.

Substitution and Tiling Dynamical Systems, Natalie Priebe Frank, Vassar College, and E. Arthur Robinson Jr., George Washington University.

Theory and Applications of Differential Equations on Graphs, Jonathan Bell, University of Maryland Baltimore County, and Sergei Avdonin, University of Alaska Fairbanks.

Undergraduate Research and its Impact on Students and Faculty, Matthias Gobbert and Nagaraj Neerchal, University of Maryland, Baltimore County, and Padmanabhan Seshaiyer, George Mason University.

Albuquerque, New Mexico

University of New Mexico

April 5-6, 2014

Saturday - Sunday

Meeting #1099

Western Section

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: January 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: April 2014
Issue of *Abstracts*: Volume 35, Issue 2

Deadlines

For organizers: Expired For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Anton Gorodetski, University of California Irvine, *Hyperbolic dynamics and spectral properties of one-dimensional quasicrystals*.

Fan Chung Graham, University of California, San Diego, *Some problems and results in spectral graph theory.*

Adrian Ioana, University of California, San Diego, *Rigidity for von Neumann algebras and ergodic group actions*.

Karen Smith, University of Michigan, Ann Arbor, *The power of characteristic p.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Analysis and Topology in Special Geometries, Charles Boyer, Daniele Grandini, and Dimiter Vassilev, University of New Mexico.

Arithmetic and Differential Algebraic Geometry, Alexandru Buium, University of New Mexico, Taylor Dupuy, University of California, Los Angeles, and Lance Edward Miller, University of Arkansas.

Commutative Algebra, **Daniel J. Hernandez**, University of Utah, **Karen E. Smith**, University of Michigan, and **Emily E. Witt**, University of Minnesota.

Descriptive Set Theory and its Applications, Alexander Kechris, California Institute of Technology, and Christian Rosendal, University of Illinois, Chicago.

Flat Dynamics, Jayadev Athreya, University of Illinois, Urbana-Champaign, Robert Niemeyer, University of New Mexico, Albuquerque, Richard E. Schwartz, Brown University, and Sergei Tabachnikov, The Pennsylvania State University.

Harmonic Analysis and Dispersive Equations, Matthew Blair, University of New Mexico, and Jason Metcalfe, University of North Carolina.

Harmonic Analysis and Its Applications, Jens Gerlach Christensen, Colgate University, and Joseph Lakey and Nicholas Michalowski, New Mexico State University.

Harmonic Analysis and Operator Theory (in memory of Cora Sadosky), Laura De Carli, Florida International University, Alex Stokolos, Georgia Southern University, and Wilfredo Urbina, Roosevelt University.

Hyperbolic Dynamics, Dynamically Defined Fractals, and Applications, Anton Gorodetski, University of California Irvine.

Interactions in Commutative Algebra, Louiza Fouli and Bruce Olberding, New Mexico State University, and Janet Vassilev, University of New Mexico.

Mathematical Finance, **Indranil SenGupta**, North Dakota State University.

Modeling Complex Social Processes Within and Across Levels of Analysis, **Simon DeDeo**, Indiana University, and **Richard Niemeyer**, University of Colorado, Denver.

Nonlinear Waves and Singularities in Water Waves, Optics and Plasmas, Alexander O. Korotkevich and Pavel Lushnikov, University of New Mexico, Albuquerque.

Partial Differential Equations in Materials Science, Lia Bronsard, McMaster University, and Tiziana Giorgi, New Mexico State University.

Physical Knots, honoring the retirement of Jonathan K. Simon, **Greg Buck**, St. Anselm College, and **Eric Rawdon**, University of St. Thomas.

Progress in Noncommutative Analysis, **Anna Skripka**, University of New Mexico, and **Tao Mei**, Wayne State University.

Spectral Theory, **Milivoje Lukic**, Rice University, and **Maxim Zinchenko**, University of New Mexico.

Stochastic Processes in Noncommutative Probability, Michael Anshelevich, Texas A&M University, and Todd Kemp, University of California San Diego.

Stochastics and PDEs, Juraj Földes, Institute for Mathematics and Its Applications, Nathan Glatt-Holtz, Institute for Mathematics and Its Applications and Virginia Tech, and Geordie Richards, Institute for Mathematics and Its Applications and University of Rochester.

The Common Core and University Mathematics Instruction, Justin Boyle, Michael Nakamaye, and Kristin Umland, University of New Mexico.

The Inverse Problem and Other Mathematical Methods Applied in Physics and Related Sciences, Hanna Makaruk, Los Alamos National Laboratory, and Robert Owczarek, University of New Mexico and Enfitek, Inc.

Topics in Spectral Geometry and Global Analysis, Ivan Avramidi, New Mexico Institute of Mining and Technology, and Klaus Kirsten, Baylor University.

Weighted Norm Inequalities and Related Topics, Oleksandra Beznosova, Baylor University, David Cruz-Uribe, Trinity College, and Cristina Pereyra, University of New Mexico.

Lubbock, Texas

Texas Tech University

April 11-13, 2014

Friday - Sunday

Meeting #1100

Central Section

Associate secretary: Georgia Benkart

Announcement issue of Notices: January 2014

Program first available on AMS website: February 27, 2014

Program issue of electronic Notices: April 2014

Issue of *Abstracts*: Volume 35, Issue 2

Deadlines

For organizers: Expired For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Nir Avni, Northwestern University, *To be announced*.

Alessio Figalli, University of Texas, *To be announced*. **Jean-Luc Thiffeault**, University of Wisconsin-Madison, *To be announced*.

Rachel Ward, University of Texas at Austin, *To be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Geometry, **David Weinberg**, Texas Tech University.

Analysis and Applications of Dynamic Equations on Time Scales, **Heidi Berger**, Simpson College, and **Raegan Higgins**, Texas Tech University.

Applications of Special Functions in Combinatorics and Analysis, Atul Dixit, Tulane University, and Timothy Huber, University of Texas Pan American.

Approximation Theory in Signal Processing, **Rachel Ward**, University of Texas at Austin, and **Rayan Saab**, University of California San Diego.

Complex Function Theory and Special Functions, Roger W. Barnard and Kent Pearce, Texas Tech University, Kendall Richards, Southwestern University, and Alex Solynin and Brock Williams, Texas Tech University.

Developments from PASI 2012: Commutative Algebra and Interactions with Related Disciplines, Kenneth Chan, University of Washington, and Jack Jeffries, University of Utah.

Differential Algebra and Galois Theory, Lourdes Juan and Arne Ledet, Texas Tech University, Andy R. Magid, University of Oklahoma, and Michael F. Singer, North Carolina State University.

Fractal Geometry and Dynamical Systems, Mrinal Kanti Roychowdhury, The University of Texas-Pan American.

Geometry and Geometric Analysis, Lance Drager and Jeffrey M. Lee, Texas Tech University.

Homological Methods in Algebra, Lars W. Christensen, Texas Tech University, Hamid Rahmati, Miami University, and Janet Striuli, Fairfield University.

Hysteresis and Multi-rate Processes, **Ram Iyer**, Texas Tech University.

Interactions between Commutative Algebra and Algebraic Geometry, Brian Harbourne and Alexandra Seceleanu, University of Nebraska-Lincoln.

Issues Regarding the Recruitment and Retention of Women and Minorities in Mathematics, James Valles Jr., Prairie View A&M University, and Doug Scheib, Saint Mary-of-the-Woods College.

Lie Groups, **Benjamin Harris**, **Hongyu He**, and **Gestur Ólafsson**, Louisiana State University.

Linear Operators in Representation Theory and in Applications, **Markus Schmidmeier**, Florida Atlantic University, and **Gordana Todorov**, Northeastern University.

Mathematical Models of Infectious Diseases in Plants, Animals and Humans, Linda Allen, Texas Tech University, and Vrushal Bokil, Oregon State University. *Navier-Stokes Equations and Fluid Dynamics*, **Radu Dascaliuc**, Oregon State University, and **Luan Hoang**, Texas Tech University.

Noncommutative Algebra, Deformations, and Hochschild Cohomology, Anne Shepler, University of North Texas, and Sarah Witherspoon, Texas A&M University.

Numerical Methods for Systems of Partial Differential Equations, JaEun Ku, Oklahoma State University, and Young Ju Lee, Texas State University.

Optimal Control Problems from Neuron Ensembles, Genomics and Mechanics, Bijoy K. Ghosh and Clyde F. Martin, Texas Tech University.

Qualitative Theory for Non-linear Parabolic and Elliptic Equations, **Akif Ibragimov**, Texas Tech University, and **Peter Polacik**, University of Minnesota.

Recent Advancements in Differential Geometry and Integrable PDEs, and Their Applications to Cell Biology and Mechanical Systems, Giorgio Bornia, Akif Ibragimov, and Magdalena Toda, Texas Tech University.

Recent Advances in the Applications of Nonstandard Finite Difference Schemes, Ronald E. Mickens, Clark Atlanta University, and Lih-Ing W. Roeger, Texas Tech University.

Recent Developments in Number Theory, **Dermot Mc-Carthy** and **Chris Monico**, Texas Tech University.

Statistics on Manifolds, **Leif Ellingson**, Texas Tech University.

Topology and Physics, **Razvan Gelca** and **Alastair Hamilton**, Texas Tech University.

Undergraduate Research, **Jerry Dwyer**, **Levi Johnson**, **Jessica Spott**, and **Brock Williams**, Texas Tech University.

Tel Aviv, Israel

Bar-Ilan University, Ramat-Gan and Tel-Aviv University, Ramat-Aviv

June 16-19, 2014

Monday - Thursday

Meeting #1101

The Second Joint International Meeting between the AMS and the Israel Mathematical Union.

Associate secretary: Michel L. Lapidus

Announcement issue of Notices: January 2014

Program first available on AMS website: To be announced Program issue of electronic *Notices*: To be announced Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses

Ian Agol, University of California, Berkeley, *3-manifolds* and cube complexes.

Gil Kalai, Hebrew University, *Influence, thresholds, and noise sensitivity.*

Michael Larsen, Indiana University, *Borel's theorem on word maps and some recent variants.*

Andrei Okounkov, Columbia University, *The M-theory index*.

Leonid Polterovich, Tel-Aviv University, *Symplectic topology: from dynamics to quantization*.

Tamar Zeigler, Technion, Israel Institute of Technology, *Patterns in primes and dynamics on nilmanifolds.*

Special Sessions

Additive Number Theory, **Melvyn B. Nathanson**, City University of New York, and **Yonutz V. Stanchescu**, Afeka Tel Aviv Academic College of Engineering.

Algebraic Groups, Division Algebras and Galois Cohomology, Andrei Rapinchuk, University of Virginia, and Louis H. Rowen and Uzi Vishne, Bar Ilan University.

Applications of Algebra to Cryptography, **David Garber**, Holon Institute of Technology, and **Delaram Kahrobaei**, City University of New York Graduate Center.

Asymptotic Geometric Analysis, Shiri Artstein and Boaz Klar'tag, Tel Aviv University, and Sasha Sodin, Princeton University.

Combinatorial Games, **Aviezri Fraenkel**, Weizmann University, **Richard Nowakowski**, Dalhousie University, Canada, **Thane Plambeck**, Counterwave Inc., and **Aaron Siegel**, Twitter.

Combinatorics, Gil Kalai, Hebrew University of Jerusalem

Dynamics and Number Theory, **Alex Kontorovich**, Yale University.

Field Arithmetic, **David Harbater**, University of Pennsylvania, and **Moshe Jarden**, Tel Aviv University.

Financial Mathematics, Jean-Pierre Fouque, University of California, and Eli Merzbach and Malka Schaps, Bar Ilan University.

Geometric Group Theory and Low-Dimensional Topology, **Ian Agol**, University of California, Berkeley, and **Zlil Sela**, Hebrew University.

Geometry and Dynamics, **Yaron Ostrover**, Tel Aviv University.

History of Mathematics, **Leo Corry**, Tel Aviv University, **Michael N. Fried**, Ben Gurion University, and **Victor Katz**, University of District of Columbia.

Mirror Symmetry and Representation Theory, **Roman Bezrukavnikov**, Massachusetts Institute of Technology, and **David Kazhdan**, Hebrew University.

Nonlinear Analysis and Optimization, Boris Morduk-hovich, Wayne State University, and Simeon Reich and Alexander Zaslavski, Technion Israel Institute of Technology.

PDEs: Modeling Theory and Numerics, **Edriss S. Titi**, University of California, Irvine.

Qualitative and Analytic Theory of ODE's, **Andrei Gabrielov**, Purdue University, and **Yossef Yomdin**, Weizmann Institute of Science.

Quasigroups, Loops and Applications, **Tuval Foguel**, Western Carolina University.

Random Matrix Theory, Brendan Farrell, California Institute of Technology, Mark Rudelson, University of Michigan, and Ofer Zeitouni, Weizmann Institute of Science.

Recent Trends in History and Philosophy of Mathematics, Misha Katz, Bar Ilan University, and David Sherry, Northern Arizona University.

Teaching with Mathematical Habits in Mind, **Theodore Eisenberg**, Ben Gurion University, **Davida Fishman**, California State University, San Bernardino, and **Jennifer Lewis**, Wayne State University.

The Mathematics of Menahem M. Schiffer, **Peter L. Duren**, University of Michigan, and **Lawrence Zalcman**, Bar Ilan University.

Topological Graph Theory and Map Symmetries, **Jonathan Gross**, Columbia University, and **Toufik Mansour**, University of Haifa.

Eau Claire, Wisconsin

University of Wisconsin-Eau Claire

September 20-21, 2014

Saturday - Sunday

Meeting #1102

Central Section

Associate secretary: Georgia Benkart Announcement issue of *Notices*: June 2014 Program first available on AMS website: August 7, 2014 Program issue of electronic *Notices*: September 2014 Issue of *Abstracts*: Volume 35, Issue 3

Deadlines

For organizers: March 20, 2014 For abstracts: July 29, 2014

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Matthew Kahle, Ohio State University, *To be announced.*Markus Keel, University of Minnesota, *To be announced.*Svitlana Mayboroda, University of Minnesota, *To be announced.*

Dylan Thurston, Indiana University, *To be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Combinatorics (Code: SS 8A), **Pavlo Pylyavskyy**, **Victor Reiner**, and **Dennis Stanton**, University of Minnesota.

Cohomology and Representation Theory of Groups and Related Structures (Code: SS 6A), Christopher Bendel, University of Wisconsin-Stout, and Christopher Drupieski, De Paul University.

Commutative Ring Theory (Code: SS 3A), **Michael Axtell**, University of St. Thomas, and **Joe Stickles**, Millikin University.

Directions in Commutative Algebra: Past, Present and Future (Code: SS 1A), Joseph P. Brennan, University of Central Florida, and Robert M. Fossum, University of Illinois at Urbana-Champaign.

Graph and Hypergraph Theory (Code: SS 7A), **Sergei Bezrukov**, University of Wisconsin-Superior, **Dalibor Froncek**, University of Minnesota Duluth, and **Xiaofeng Gu**, **Uwe Leck**, and **Steven Rosenberg**, University of Wisconsin-Superior.

Lie Algebras and Representation Theory (Code: SS 5A), **Michael Lau**, Université Laval, **Ian Musson**, University of Wisconsin, Milwaukee, and **Matthew Ondrus**, Weber State University.

New Trends in Toric Varieties (Code: SS 4A), Christine Berkesch Zamaere, University of Minnesota, Daniel Erman, University of Wisconsin-Madison, and Hal Schenck, University of Illinois Urbana-Champaign.

von Neumann Algebras and Related Fields (Code: SS 2A), **Stephen Avsec** and **Ken Dykema**, Texas A&M University.

Halifax, Canada

Dalhousie University

October 18-19, 2014

Saturday - Sunday

Meeting #1103

Eastern Section

Associate secretary: Steven H. Weintraub Announcement issue of *Notices*: August 2014 Program first available on AMS website: September 5, 2014 Program issue of electronic *Notices*: October 2014 Issue of *Abstracts*: Volume 35, Issue 3

Deadlines

For organizers: March 18, 2014 For abstracts: August 19, 2014

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

François Bergeron, Université du Québec à Montréal, *Title to be announced*.

Sourav Chatterjee, New York University, *Title to be announced*.

William M. Goldman, University of Maryland, *Title to be announced*.

Sujatha Ramdorai, University of British Columbia, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Commutative Algebra and Its Interactions with Algebraic Geometry (Code: SS 2A), Susan Marie Cooper, Central Michigan University, Sara Faridi, Dalhousie University, and William Traves, U.S. Naval Academy.

Sampling Theory (Code: SS 4A), **John J. Benedetto**, University of Maryland, **Jean-Pierre Gabardo**, McMaster University, and **Ozgur Yilmaz**, University of British Columbia.

Special Functions and Their Applications. (Code: SS 3A), **Mourad E. H. Ismail**, University of Central Florida, and **Nasser Saad**, University of Prince Edward Island.

p-adic Methods in Arithmetic (Code: SS 1A), **Henri Darmon**, McGill University, **Adrian Iovita**, Concordia University, and **Sujatha Ramdorai**, University of British Columbia.

San Francisco, California

San Francisco State University

October 25-26, 2014

Saturday - Sunday

Meeting #1104

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of Notices: August 2014

Program first available on AMS website: September 11, 2014

Program issue of electronic *Notices*: October 2014 Issue of *Abstracts*: Volume 35, Issue 4

Deadlines

For organizers: March 25, 2014 For abstracts: September 3, 2014

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Kai Behrend, University of British Columbia, Vancouver, Canada, *Title to be announced*.

Kiran S. Kedlaya, University of California, San Diego, *Title to be announced*.

Julia Pevtsova, University of Washington, Seattle, *Title to be announced*.

Burt Totaro, University of California, Los Angeles, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Geometry (Code: SS 1A), **Renzo Cavalieri**, Colorado State University, **Noah Giansiracusa**, University of California, Berkeley, and **Burt Totaro**, University of California, Los Angeles.

Categorical Methods in Representation Theory (Code: SS 4A), Eric Friedlander, University of Southern California, Srikanth Iyengar, University of Nebraska, Lincoln, and Julia Pevtsova, University of Washington.

Geometry of Submanifolds (Code: SS 3A), **Yun Myung Oh**, Andrews University, **Bogdan D. Suceava**, California State University, Fullerton, and **Mihaela B. Vajiac**, Chapman University.

Polyhedral Number Theory (Code: SS 2A), **Matthias Beck**, San Francisco State University, **Martin Henk**, Universität Magdeburg, and **Joseph Gubeladze**, San Francisco State University.

Recent Progress in Geometric Analysis (Code: SS 5A), David Bao, San Francisco State University, and Ovidiu Munteanu, University of Connecticut.

Greensboro, North Carolina

University of North Carolina, Greensboro

November 8-9, 2014

Saturday - Sunday

Meeting #1105

Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of Notices: August 2014

Program first available on AMS website: September 25,

Program issue of electronic *Notices*: November 2014 Issue of *Abstracts*: Volume 35, Issue 4

Deadlines

For organizers: April 8, 2014 For abstracts: September 16, 2014

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Susanne Brenner, Louisiana State Unviersity, *Title to be announced*.

Skip Garibaldi, Emory University, *Title to be announced*. **Stavros Garoufaldis**, Georgia Institute of Technology, *Title to be announced*.

James Sneyd, University of Auckland, *Title to be announced* (AMS-NZMS Maclaurin Lecture).

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Difference Equations and Applications (Code: SS 1A), Michael A. Radin, Rochester Institute of Technology, and Youssef Raffoul, University of Dayton.

Recent Advances in Numerical Methods for Fluid Flow Problems (Code: SS 2A), Leo Rebholz, Clemson University, and Zhu Wang, University of South Carolina.

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10-13, 2015

Saturday - Tuesday

Meeting #1106

Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub Announcement issue of *Notices*: October 2014 Program first available on AMS website: To be announced Program issue of electronic *Notices*: January 2015 Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014 For abstracts: To be announced

Washington, District of Columbia

Georgetown University

March 7-8, 2015

Saturday - Sunday Eastern Section

Associate secretary: Steven H. Weintraub Announcement issue of *Notices*: To be announced Program first available on AMS website: To be announced Program issue of electronic *Notices*: To be announced Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 7, 2014 For abstracts: To be announced

East Lansing, Michigan

Michigan State University

March 13-15, 2015

Friday - Sunday Central Section

Associate secretary: Georgia Benkart Announcement issue of *Notices*: To be announced Program first available on AMS website: To be announced Program issue of electronic *Notices*: To be announced Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 26, 2014 For abstracts: January 20, 2015

Huntsville, Alabama

University of Alabama in Huntsville

March 27-29, 2015

Friday - Sunday Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: To be announced Program first available on AMS website: To be announced Program issue of electronic *Notices*: To be announced Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 20, 2014 For abstracts: To be announced

Las Vegas, Nevada

University of Nevada, Las Vegas

April 18-19, 2015

Saturday - Sunday Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: To be announced Program first available on AMS website: To be announced Program issue of electronic *Notices*: To be announced

Issue of Abstracts: To be announced

Deadlines

For organizers: September 18, 2014 For abstracts: To be announced

Porto, Portugal

University of Porto

June 10-13, 2015

Wednesday - Saturday

First Joint International Meeting involving the American Mathematical Societry (AMS), the European Mathematical Society (EMS), and the Sociedade de Portuguesa Matematica (SPM).

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: To be announced Program first available on AMS website: To be announced Program issue of electronic *Notices*: To be announced Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced For abstracts: To be announced

Chicago, Illinois

Loyola University Chicago

October 3-4, 2015

Saturday - Sunday Central Section

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: To be announced Program first available on AMS website: To be announced

Program issue of electronic *Notices*: October 2015

Issue of Abstracts: To be announced

Deadlines

For organizers: March 10, 2015 For abstracts: To be announced

Memphis, Tennessee

University of Memphis

October 17-18, 2015

Saturday - Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced

Deadlines

For organizers: March 17, 2015 For abstracts: August 18, 2015

Issue of *Abstracts*: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Computational Analysis (Code: SS 1A), **George Anastassiou**, University of Memphis.

Fullerton, California

California State University, Fullerton

October 24-25, 2015

Saturday - Sunday Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: To be announced Program first available on AMS website: To be announced Program issue of electronic *Notices*: October 2015

Issue of Abstracts: To be announced

Deadlines

For organizers: March 27, 2015 For abstracts: To be announced

New Brunswick, New Jersey

Rutgers University

November 14-15, 2015

Saturday - Sunday Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: To be announced Program first available on AMS website: To be announced Program issue of electronic Notices: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6-9, 2016

Wednesday - Saturday

Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus

Announcement issue of Notices: October 2015

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: January 2016

Issue of Abstracts: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015 For abstracts: To be announced

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4-7, 2017

Wednesday - Saturday

Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the

winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Brian D. Boe

Announcement issue of *Notices*: October 2016

Program first available on AMS website: To be announced Program issue of electronic *Notices*: January 2017

Issue of Abstracts: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016 For abstracts: To be announced

Charleston, South Carolina

College of Charleston

March 10-12, 2017

Friday - Sunday

Southeastern Section

Associate secretary: Brian D. Boe

Announcement issue of Notices: To be announced Program first available on AMS website: To be announced Program issue of electronic Notices: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: November 10, 2016 For abstracts: To be announced

Pullman, Washington

Washington State University

April 22-23, 2017

Saturday - Sunday Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: To be announced Program first available on AMS website: To be announced Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, e-mail: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at www.ams.org/meetings/.**

Meetings:

March 13-15

March 27-29

April 18-19

June 10-13

October 3-4

October 17-18

October 24-25

November 14-15

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March 21-23	Knoxville, Tennessee	p. 437
March 29-30	Baltimore, Maryland	p. 438
April 5-6	Albuquerque, New Mexico	p. 439
April 11-13	Lubbock, Texas	p. 440
June 16-19	Tel Aviv, Israel	p. 441
September 20-21	Eau Claire, Wisconsin	p. 442
October 18-19	Halifax, Canada	p. 442
October 25-26	San Francisco, California	p. 443
November 8-9	Greensboro, North Carolina	p. 443
2015		
January 10-13	San Antonio, Texas	p.444
	Annual Meeting	444
March 7–8	Washington, DC	p. 444

East Lansing, Michigan

Huntsville, Alabama

Memphis, Tennessee

Fullerton, California

New Brunswick, New Jersey

Las Vegas, Nevada

Porto, Portugal

Chicago, Illinois

2016 January 6-9	Seattle, Washington Annual Meeting	p. 446
2017		
January 4–7	Atlanta, Georgia Annual Meeting	p. 446
March 10-12 April 22-23	Charleston, South Carolina Pullman, Washington	p. 446 p. 446
-		-

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 99 in the January 2014 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LATEX is necessary to submit an electronic form, although those who use LATEX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LATEX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

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Handbook of Financial Data and Risk Information

Edited by

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Risk has always been central to finance, and managing risk depends critically on information. As evidenced by recent events, the need has never been greater for skills, systems and methodologies to manage risk information in financial markets. Authored by leading figures in risk management and analysis, this handbook serves as a unique and comprehensive reference for the technical, operational, regulatory and political issues in collecting, measuring and managing financial data. It is targeted towards a wide range of audiences, from financial industry practitioners and regulators responsible for implementing risk management systems, to system integrators and software firms helping to improve such systems.

Volume I: Principles and Context

Volume I examines the business and regulatory context that makes risk information so important. A vast set of techniques and processes have grown up over time, and without an understanding of the broader forces at work, it is all too easy to get lost in the details.

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Volume II: Software and Data

Volume 2 describes a structural and operational framework for managing a financial risk data repository. As experience accumulates on managing modern risk systems, the knowledge base of practical lessons grows. Understanding these issues and leading practices may mean the difference between failed and successful implementations of risk systems.

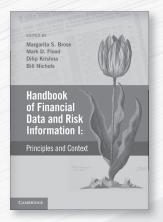
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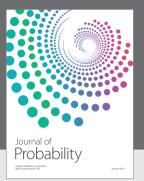
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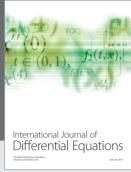


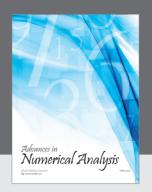


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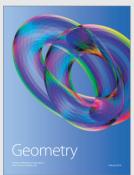








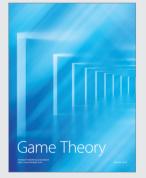


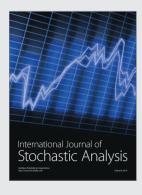




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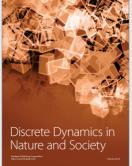


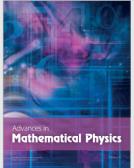


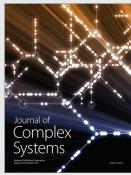






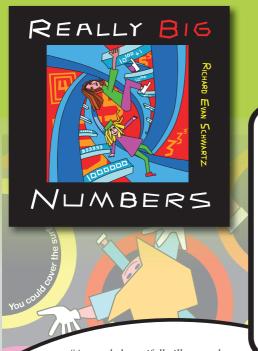






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Richard Evan Schwartz



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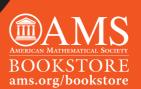
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