About the cover

Poincaré's glue surface

The cover image is an elaboration of one seen in John Stillwell’s review of Jeremy Gray’s biography of Poincaré. The original is found in Poincaré’s 5th Supplement to “Analysis Situs”, one of a series of papers founding algebraic topology. Stillwell calls the figure unenlightening, but we suppose that is a matter of perspective. As anyone familiar with Dale Rolfsen’s well known book *Knots and Links* will be aware of, it is an example of a Heegaard diagram, a standard tool in 3D topology. Heegaard diagrams are part of the data associated to Heegaard splittings, which remain even now an active topic of research. Alas, it seems that in modern developments few papers on what are called “Heegaard diagrams” actually contain a diagram.

Chapter 9 of Rolfsen’s book explains Poincaré’s diagram in a series of short exercises. Chapter 6 of Gray’s biography contains an impressive account of the 5th Supplement, although one might wish that he had more—and more enlightening—pictures. All we can do here is tell you what exactly you are looking at, and leave detailed interpretation for elsewhere.

The colored region is a surface of genus two that has been cut apart along the circles ±A and ±B. It is the boundary of a *handlebody* of genus two—i.e., a pair of handles attached to a 3-ball. The blue and red paths represent oriented continuous paths on the original surface, and in Poincaré’s construction are places where a second handlebody is attached to form his example of a three-manifold with trivial homology but nontrivial fundamental group. The green paths are used by Poincaré to calculate, at the end of §6 of the 5th Supplement, that fundamental group. It turns out that the fundamental group is isomorphic to $SL_2(\mathbb{Z}/5)$, although all that he seems to prove is that the icosahedral group $A_5$ is a homomorphic image.

There are many ways known nowadays of constructing Poincaré’s counterexample to an earlier conjecture he had made. One clear presentation can be found in §6 of the classic topology text by Seifert and Threlfall. For a rather more mysterious picture of Heegaard splittings, take a look at

[http://www.its.caltech.edu/~wjiajun/compprog/hfhat/sigma235.html](http://www.its.caltech.edu/~wjiajun/compprog/hfhat/sigma235.html)

—Bill Casselman

*Graphics Editor*

(notices-covers@ams.org)