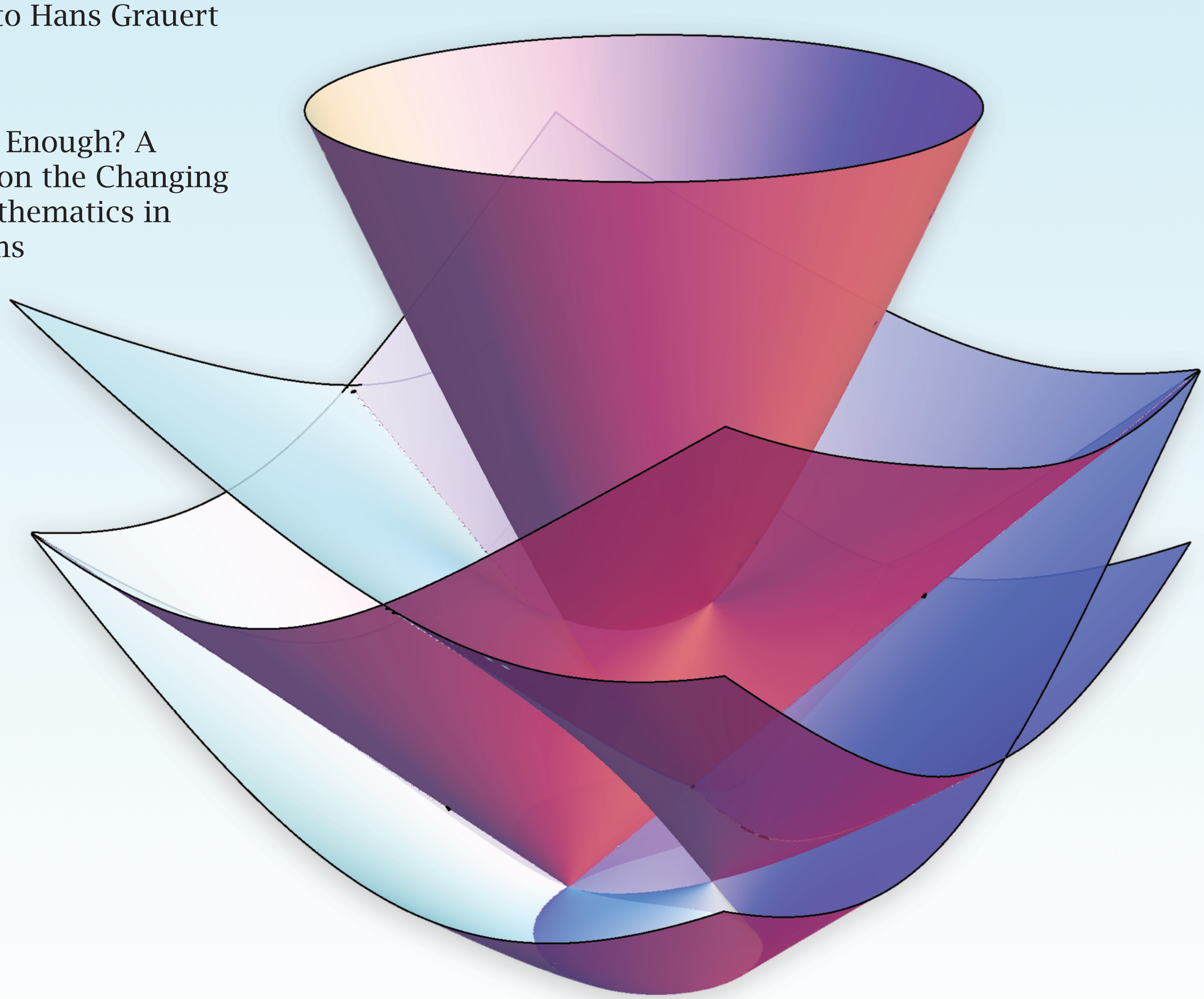
ISSN 0002-9920 (print)
ISSN 1088-9477 (online)

Notices

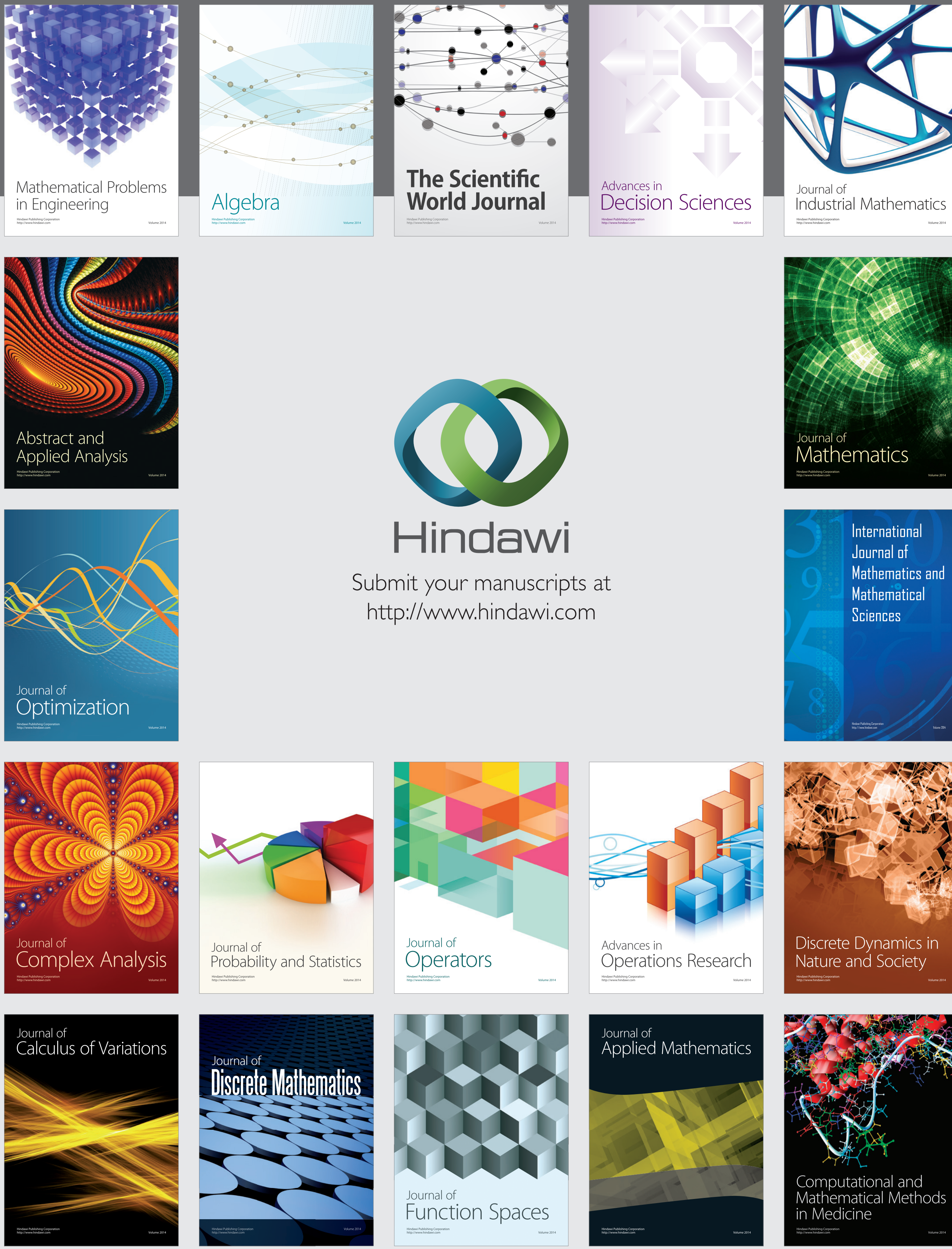
of the American Mathematical Society

May 2014 **Volume 61, Number 5**



AMERICAN MATHEMATICAL SOCIETY

About the cover: From the garden of spectrahedra *(see page 531)*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The connection between mathematics and art goes back thousands of years. Mathematics has been used in the design of Gothic cathedrals, Rose windows, oriental rugs, mosaics and tilings. Geometric forms were fundamental to the cubists and many abstract expressionists, and award-winning sculptors have used topology as the basis for their pieces. Dutch artist M.C. Escher represented infinity, Möbius bands, tessellations, deformations, reflections, Platonic solids, spirals, symmetry, and the hyperbolic plane in his works.
Mathematicians and artists continue to create stunning works in all media and to explore the visualization of mathematics--origami, computer-generated landscapes, tesselations, fractals, anamorphic art, and more.
A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.
—G. H. Hardy, A Mathematician's Apology
Fractal Pancakes
Simon Beck's Snow Patterns
2012 Mathematical Art Exhibition
Erica Rollings Glass Works
All my life I have vacillated between mathematics and art and find I am happiest when doing both. It's generally acknowledged that math and music are closely related in human developmental processes. I guess it boils down to basic communication. Math and music are languages, and art is a visual means of communication. My medium of choice is glass, and my favorite designs are mathematical and usually the ones that nature presents in both anatomical and botanical spheres of life. --- Erica Rollings Glass Works (www.ericarollings.net)
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You are viewing page 1
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"Fractal Tessellation of Spirals," by Robert Fathauer (Tesselations, Phoenix, AZ)
Dear Bill,
Here's one of the e-postcards from the site.
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Bridges: Mathematical Connections in Art, Music, and Science
M.C. Escher: the Official Website
Images and Mathematics, MathArchives
The Institute for Figuring
Kalendar, by Herwig Hauser
The KnotPlot Site
Mathematical Imagery by Jos Leys
Mathematics Museum (Japan)
Visual Mathematics Journal
ARTICLES & RESOURCES
Art & Music, MathArchives
Geometry in Art & Architecture, by Paul Calter (Dartmouth College)
Harmony and Proportion, by John Boyd-Brent
International Society of the Arts, Mathematics and Architecture
Journal of Mathematics and the Arts
Mathematics and Art, the April 2003 Feature Column by Joe Malkevitch
Maths and Art: the whistlestop tour, by Lewis Dartnell
Mathematics and Art, (The theme for Mathematics Awareness Month in 2003)
Viewpoints: Mathematics and Art, by Annalisa Crannell (Franklin & Marshall College) and Marc Frantz (Indiana University)
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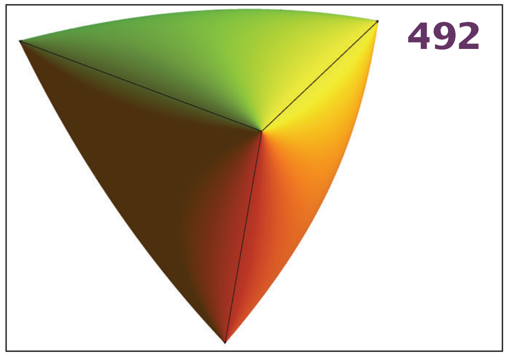
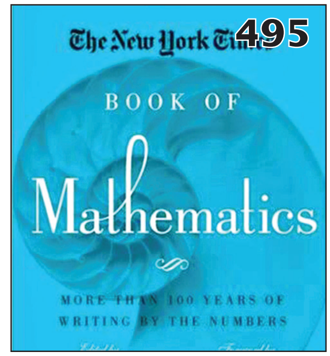
Notices
of the American Mathematical Society

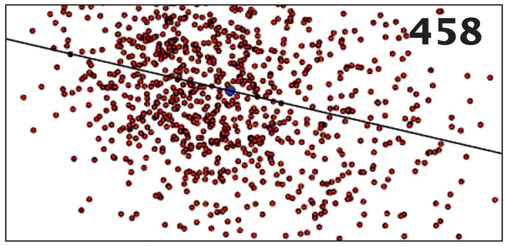
May 2014

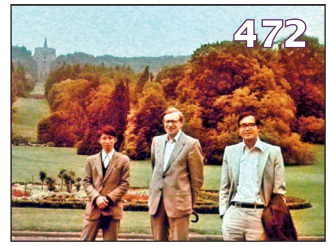
Communications

Commentary









Our May issue features articles that reflect the diversity and richness of mathematics. There is a piece on financial charlatanism (one of the first *Notices* articles on the mathematics of finance), one on the perils and pulchritude of big data, and an exploration (from the educational point of view) of the difference between hedgehogs and foxes. Finally we have a memorial article for the distinguished mathematician Hans Grauert.

—*Steven G. Krantz, Editor*

Features

Notices
of the American Mathematical Society

SUBSCRIPTION INFORMATION: Subscription prices for Volume 61 (2014) are US$569 list; US$455.20 institutional member; US$341.40 individual member; US$512.10 corporate member. (The subscription price for members is included in the annual dues.) A late charge of 10% of the subscription price will be imposed upon orders received from nonmembers after January 1 of the subscription year. Add for postage: Surface delivery outside the United States and India—US$27; in India—US$40; expedited delivery to destinations in North America—US$35; elsewhere—US$120. Subscriptions and orders for AMS publications should be addressed to the American Mathematical Society, P.O. Box 845904, Boston, MA 02284-5904 USA. All orders must be prepaid.

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[*Notices of the American Mathematical Society* (ISSN 0002-9920) is published monthly except bimonthly in June/July by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2294 USA, GST No. 12189 2046 RT****. Periodicals postage paid at Providence, RI, and additional mailing offices. POSTMASTER: Send address change notices to *Notices of the American Mathematical Society,* P.O. Box 6248, Providence, RI 02940-6248 USA.] Publication here of the Society's street address and the other information in brackets above is a technical requirement of the U.S. Postal Service. Tel: 401-455-4000, email: notices@ams.org.

Departments

From the AMS Secretary

Letters to the Editor

Unhiding History

I read with a large degree of pleasure Alexey Glutsyuk's review of *Naming Infinity: A True Story of Religious Mysticism and Mathematical Creativity* by Loren Graham and Jean-Michel Kantor [*Notices*, January 2014]. The book is a serious attempt to look at both the mathematical and nonmathematical contexts in which mathematics is created and does an excellent job of both aspects of its task. Similarly, the review did an excellent job of discussing most of the major themes of the book. But the reviewer, in describing the relationship of Alexandrov and Kolmogorov as "friends and collaborators" does a disservice to Graham and Kantor, and to the unsuspecting reader of the review. Graham and Kantor make a very clear case that Alexandrov and Kolmogorov (and, also, Alexandrov and Urysohn) were lovers, and that the tenuous position of homosexuals in Russian society (which, sadly, continues to this day) shaped at least some of their political behavior, in particular Alexandrov's and Kolmogorov's denunciations of Luzin and Solzhenitsyn. If Glutsyuk has evidence to challenge Graham's and Kantor's claims, he should mention it. But simply avoiding the issue avoids one of the major themes in the book, and continues to hide an aspect of history which is too often hidden.

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(Received February 10, 2014)

Roger Howe in Perspective

We write in response to the thought-provoking article by Eileen Pollack, "Why Are There Still So Few Women in Science?", which appeared in *The New York Times* in October 2013. Pollack—one of the first women to earn a B.S. in physics at Yale—subsequently left science entirely and is now Professor of Creative Writing at Michigan. We are glad to see how Pollack's article has reinvigorated the public discussion of women in science.

At the same time we're concerned that the article paints a potentially misleading portrait of Roger Howe, Professor of Mathematics at Yale and supervisor of Pollack's undergraduate thesis. Pollack's article recounts that Howe neither praised that thesis nor encouraged her to pursue a Ph.D. Indeed, it was only years later that Pollack learned that he had considered her thesis work "exceptional". In view of Pollack's central theme—"The most powerful determinant of whether a woman goes on in science might be whether anyone encourages her to go on"—it's easy for readers to see Howe as a villain in Pollack's personal story.

But historical perspective is important here. When Pollack first met Howe in the 1970s, it was early in Howe's career—and very early in Yale's transition to undergraduate coeducation. Pollack's article leaps over the intervening thirty-five years of Howe's career, in which he has been celebrated not only for his research but also for his long-term involvement with K–12 education. He received the 2006 AMS Award for Distinguished Public Service.

Our own experiences with Howe contrast sharply with Pollack's. Howe played a positive role in our development as mathematicians, and made the Yale Mathematics Department a friendly and welcoming place when we earned our Ph.D.'s there in the 1980s and 1990s. Two of us (Nahmod and Wu) had mathematical interests quite different from Howe's, but we took several topics courses with him. When we were newly arrived from abroad, Roger invited us to celebrate Thanksgiving with his family. He made several such gestures that carried a deep and lasting message of inclusiveness and encouragement. One of us (Kim) was Howe's Ph.D. student and confidently asserts that she would not have had a mathematical career without him.

We've contacted several other mathematicians, male and female, who interacted with Howe at Yale early in their careers. Their experiences complement ours. For example, Gail Ratcliff of East Carolina University, Howe's first female Ph.D. student, considers Howe "a good friend" thirty years later. Hadi Salmasian of the University of Ottawa, a male Ph.D. student of Howe, recalls that, while working on his dissertation, Howe explained to him with great excitement the works of two former Yale Ph.D. female students. Miriam Logan of Bowdoin College earned her Ph.D. with Howe after five years of meeting with him for "four–five days each week". If that's not dedication and encouragement, she says, "I do not know what is."

We hope that this note helps to provide a more complete portrait of Roger Howe, by describing the support he's provided to young scientists, especially women, over the course of his long and varied career.

A longer version of this note is at http://web.mit.edu/juleekim/www/Roger_Howe.pdf.

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(Received January 30, 2014)

Bibliometric Indices and Competition

Bibliometric indices are tools. I totally agree that they cannot establish or rank the quality of theorems. Still, they do indicate something, even if they do it roughly: citation indices indicate impact on the scientific community. As in many human problems, the danger lies in extreme attitudes: one is a blind use of the indices—completely disregarding a qualitative analysis—the other is their substantial neglect. The latter, for instance, is the policy recently followed in Italy by the committee for the habilitation to full professor in the Geometry and Algebra sector, in some cases, invoking documents by the European Mathematical Society and the International Mathematical Union. Everything is officially documented—in Italian—at `https://abilitazione.cineca.it/ministero.php/public/elencodomande/settore/01%252FA2/fascia/1`.

One outcome was that a mathematician I know well (XY say) was refused the habilitation. This failure looks paradoxical to me: on the one hand it is based on an "insufficient impact on the research of the area," on the other, XY's indices show the opposite. The H-index and the contemporary H-index—based on Scopus and ISI and supplied to the committee by the Ministry itself—place XY near the top of the list of candidates. The paradox is solved by the explicit admission that the committee, at least in this case, focused on MathSciNet. Beyond the fact that this choice still gives XY an H-index not lower than the ones of most of the candidates that obtained the habilitation, this approach raises some important questions. Can we disregard the impact of our research outside our own community? Wouldn't such disregard be antithetical to a widespread trend towards interdisciplinarity well represented, e.g., in the National Research Council document `http://www.nap.edu/catalog.php?record_id=15269`? I think that a sound scientific judgment should consider all citation data. As the IMU puts it, it is information which should not be hidden but illuminated (`http://www.mathunion.org/fileadmin/IMU/Report/CitationStatistics.pdf`, page 5). In my opinion, without a clear and consistent position about this issue, every statement of our community claiming the interdisciplinary role of mathematics runs a real risk of being perceived as unreliable or empty.

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(Received February 10, 2014

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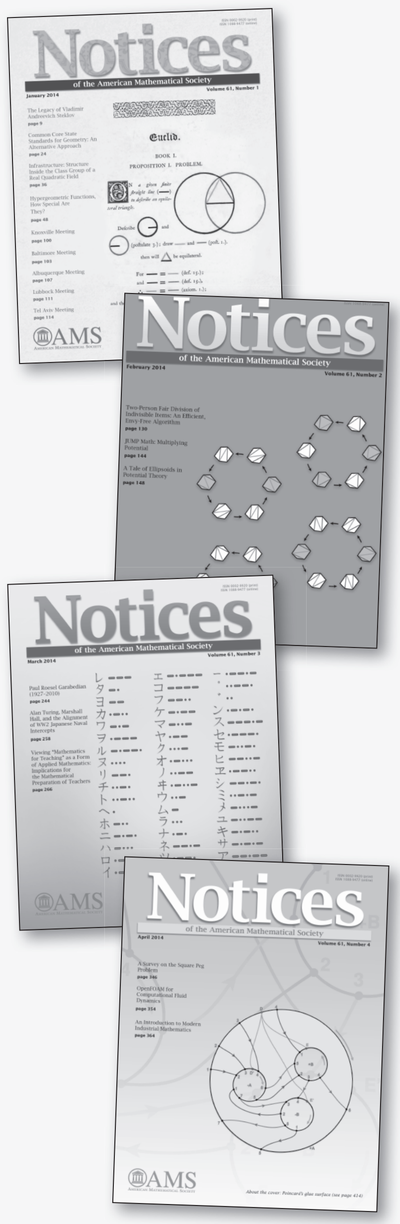
Call for Applications & Nominations

Chief Editor of the *Notices*



Applications and nominations are invited for the position of Editor of the *Notices of the American Mathematical Society*, to commence with the January 2016 issue. The Society seeks an individual with strong mathematical research experience, broad mathematical interests, and a commitment to communicating mathematics to a diverse audience and at a wide range of levels. The applicant must demonstrate excellent written communication skills.

The Editor has editorial responsibility for a major portion of the *Notices* within broad guidelines. The goal of the *Notices* is to serve all mathematicians by providing a lively and informative magazine containing exposition about mathematics and mathematicians, and information about the profession and the Society.

The Editor is assisted by a board of Associate Editors, nominated by the Editor, who help to fashion the contents of the *Notices* and solicit material for publication. Some writing, and all publication support, will be provided by AMS staff in Providence. The Editor will operate from his or her home institution with part-time secretarial support. In order to begin working on the January 2016 issue, some editorial work would begin in early 2015.

Nominations and applications (including curriculum vitae) should be sent to the Executive Director, Donald E. McClure, at exdir@ams.org. Confidential inquiries may also be sent directly to the Executive Director or to any other member of the search committee (Dan Abramovich, Amber Puha, Carla Savage, or David Vogan).

To receive full consideration, nominations and applications should be sent on or before August 31, 2014.

Pseudo-Mathematics and Financial Charlatanism: The Effects of Backtest Overfitting on Out-of-Sample Performance

David H. Bailey, Jonathan M. Borwein, Marcos López de Prado, and Qiji Jim Zhu

> Another thing I must point out is that you cannot prove a vague theory wrong. [...] Also, if the process of computing the consequences is indefinite, then with a little skill any experimental result can be made to look like the expected consequences.
>
> —Richard Feynman [1964]

Introduction

A *backtest* is a historical simulation of an algorithmic investment strategy. Among other things, it computes the series of profits and losses that such strategy would have generated had that algorithm been run over that time period. Popular performance statistics, such as the *Sharpe ratio* or the *Information ratio*, are used to quantify the backtested strategy's return on risk. Investors typically study those backtest statistics and then allocate capital to the best performing scheme.

Regarding the measured performance of a backtested strategy, we have to distinguish between two very different readings: *in-sample* (IS) and *out-of-sample* (OOS). The IS performance is the one simulated over the sample used in the design of the strategy (also known as "learning period" or "training set" in the machine-learning literature). The OOS performance is simulated over a sample not used in the design of the strategy (a.k.a. "testing set"). A backtest is *realistic* when the IS performance is consistent with the OOS performance.

When an investor receives a promising backtest from a researcher or portfolio manager, one of her key problems is to assess how realistic that simulation is. This is because, given any financial series, it is relatively simple to *overfit* an investment strategy so that it performs well IS.

Overfitting is a concept borrowed from machine learning and denotes the situation when a model targets particular observations rather than a general structure. For example, a researcher could design a trading system based on some parameters that target the removal of specific recommendations that she knows led to losses IS (a practice known as "data snooping"). After a few iterations, the researcher will come up with "optimal parameters", which profit from features that are present in that particular sample but may well be rare in the population.

Recent computational advances allow investment managers to methodically search through thousands or even millions of potential options for a profitable investment strategy. In many instances, that search involves a pseudo-mathematical argument which is spuriously validated through a backtest. For example, consider a time series of daily prices for a stock X. For every day in the sample, we can compute one average price of that stock using the previous m observations $\bar{x}_m$ and another average price using the previous n observations $\bar{x}_n$, where $m < n$. A popular investment strategy called "crossing moving averages" consists of owning X whenever $\bar{x}_m > \bar{x}_n$. Indeed, since the sample size determines a limited number of parameter combinations that m and n can adopt,

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DOI: http://dx.doi.org/10.1090/noti1105

it is relatively easy to determine the pair (m, n) that maximizes the backtest's performance. There are hundreds of such popular strategies, marketed to unsuspecting lay investors as mathematically sound and empirically tested.

In the context of econometric models several procedures have been proposed to determine overfit in White [27], Romano et al. [23], and Harvey et al. [9]. These methods propose to adjust the p-values of estimated regression coefficients to account for the multiplicity of trials. These approaches are valuable for dealing with trading rules based on an econometric specification.

The machine-learning literature has devoted significant effort to studying the problem of overfitting. The proposed methods typically are not applicable to investment problems for multiple reasons. First, these methods often require explicit point forecasts and confidence bands over a defined event horizon in order to evaluate the explanatory power or quality of the prediction (e.g., "E-mini S&P500 is forecasted to be around 1,600 with a one-standard deviation of 5 index points at Friday's close"). Very few investment strategies yield such explicit forecasts; instead, they provide qualitative recommendations (e.g., "buy" or "strong buy") over an undefined period until another such forecast is generated, with random frequency. For instance, trading systems, like the crossing of moving averages explained earlier, generate buy and sell recommendations with little or no indication as to forecasted values, confidence in a particular recommendation, or expected holding period.

Second, even if a particular investment strategy relies on such a forecasting equation, other components of the investment strategy may have been overfitted, including entry thresholds, risk sizing, profit taking, stop-loss, cost of capital, and so on. In other words, there are many ways to overfit an investment strategy other than simply tuning the forecasting equation. Third, regression overfitting methods are parametric and involve a number of assumptions regarding the underlying data which may not be easily ascertainable. Fourth, some methods do not control for the number of trials attempted.

To illustrate this point, suppose that a researcher is given a finite sample and told that she needs to come up with a strategy with an SR (Sharpe Ratio, a popular measure of performance in the presence of risk) above 2.0, based on a forecasting equation for which the AIC statistic (Akaike Information Criterion, a standard of the regularization method) rejects the null hypothesis of overfitting with a 95 percent confidence level (i.e., a false positive rate of 5 percent). After only twenty trials, the researcher is expected to find one specification that passes the AIC criterion. The researcher will quickly be able to present a specification that not only (falsely) passes the AIC test but also gives an SR above 2.0. The problem is, AIC's assessment did not take into account the hundreds of other trials that the researcher neglected to mention. For these reasons, commonly used regression overfitting methods are poorly equipped to deal with backtest overfitting.

Although there are many academic studies that claim to have identified profitable investment strategies, their reported results are almost always based on IS statistics. Only exceptionally do we find an academic study that applies the "hold-out" method or some other procedure to evaluate performance OOS. Harvey, Liu, and Zhu [10] argue that there are hundreds of papers supposedly identifying hundreds of factors with explanatory power over future stock returns. They echo Ioannidis [13] in concluding that "most claimed research findings are likely false." Factor models are only the tip of the iceberg.[1] The reader is probably familiar with many publications solely discussing IS performance.

This situation is, quite frankly, depressing, particularly because academic researchers are expected to recognize the dangers and practice of overfitting. One common criticism, of course, is the credibility problem of "holding-out" when the researcher had access to the full sample anyway. Leinweber and Sisk [15] present a meritorious exception. They proposed an investment strategy in a conference and announced that six months later they would publish the results with the pure (yet to be observed) OOS data. They called this approach "model sequestration", which is an extreme variation of "hold-out".

Our Intentions

In this paper we shall show that it takes a relatively small number of trials to identify an investment strategy with a spuriously high backtested performance. We also compute the *minimum backtest length* (MinBTL) that an investor should require given the number of trials attempted. Although in our examples we always choose the Sharpe ratio to evaluate performance, our methodology can be applied to any other performance measure.

We believe our framework to be helpful to the academic and investment communities by providing a benchmark methodology to assess the reliability of a backtested performance. We would

[1] *We invite the reader to read specific instances of pseudo-mathematical financial advice at this website:* `http://www.m-a-f-f-i-a.org/`. *Also, Edesses (2007) provides numerous examples.*

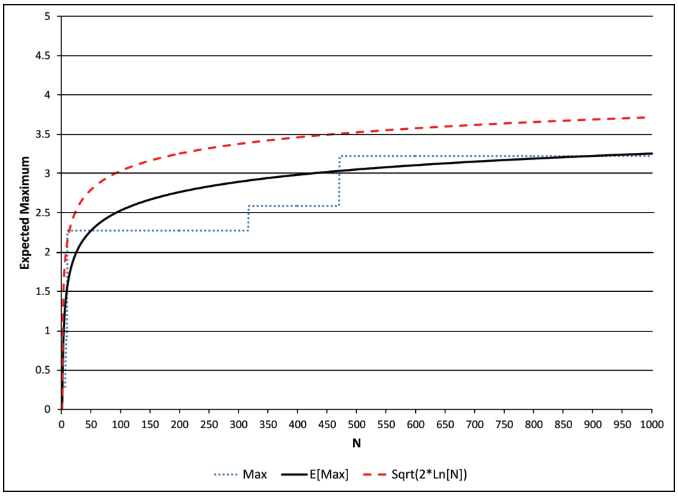

Figure 1. Overfitting a backtest's results as the number of trials grows.

Figure 1 provides a graphical representation of Proposition 1. The blue (dotted) line shows the maximum of a particular set of N independent random numbers, each following a Standard Normal distribution. The black (continuous) line is the expected value of the maximum of that set of N random numbers. The red (dashed) line is an upper bound estimate of that maximum. The implication is that it is relatively easy to wrongly select a strategy on the basis of a maximum Sharpe ratio when displayed IS.

feel sufficiently rewarded in our efforts if at least this paper succeeded in drawing the attention of the mathematical community regarding the widespread proliferation of journal publications, many of them claiming profitable investment strategies on the sole basis of IS performance. This is perhaps understandable in business circles, but a higher standard is and should be expected from an academic forum.

We would also like to raise the question of whether mathematical scientists should continue to tolerate the proliferation of investment products that are misleadingly marketed as mathematically sound. In the recent words of Sir Andrew Wiles,

> One has to be aware now that mathematics can be misused and that we have to protect its good name. [29]

We encourage the reader to search the Internet for terms such as "stochastic oscillators", "Fibonacci ratios", "cycles", "Elliot wave", "Golden ratio", "parabolic SAR", "pivot point", "momentum", and others in the context of finance. Although such terms clearly evoke precise mathematical concepts, in fact in almost all cases their usage is scientifically unsound.

Historically, scientists have led the way in exposing those who utilize pseudoscience to extract a commercial benefit. As early as the eighteenth century, physicists exposed the nonsense of astrologers. Yet mathematicians in the twenty-first century have remained disappointingly silent with regard to those in the investment community who, knowingly or not, misuse mathematical techniques such as probability theory, statistics, and stochastic calculus. Our silence is consent, making us accomplices in these abuses.

The rest of our study is organized as follows: The section "Backtest Overfitting" introduces the problem in a more formal way. The section "Minimum Backtest Length (MinBTL)" defines the concept of Minimum Backtest Length (MinBTL). The section "Model Complexity" argues how model complexity leads to backtest overfitting. The section "Overfitting in Absence of Compensation Effects" analyzes overfitting in the absence of compensation effects. The section "Overfitting in Presence of Compensation Effects" studies overfitting in the presence of compensation effects. The section "Is Backtest Overfitting a Fraud?" exposes how backtest overfitting can be used to commit fraud. The section "A Practical Application" presents a typical example of backtest overfitting. The section "Conclusions" lists our conclusions. The mathematical appendices supply proofs of the propositions presented throughout the paper.

Backtest Overfitting

The design of an investment strategy usually begins with a *prior* or *belief* that a certain pattern may help forecast the future value of a financial variable. For example, if a researcher recognizes a lead-lag effect between various tenor bonds in a yield curve, she could design a strategy that bets on a reversion towards equilibrium values. This model might take the form of a cointegration equation, a vector-error correction model, or a system of stochastic differential equations, just to name a few. The number of possible model configurations (or trials) is enormous, and naturally the researcher would like to select the one that maximizes the performance of the strategy. Practitioners often rely on *historical simulations* (also called backtests) to discover the optimal specification of an investment strategy. The researcher will evaluate, among other variables, what are the optimal sample sizes, signal update frequency, entry and profit-taking thresholds, risk sizing, stop losses, maximum holding periods, etc.

The *Sharpe ratio* is a statistic that evaluates an investment manager's or strategy's performance on the basis of a sample of past returns. It is defined as the ratio between average excess returns (in excess of the rate of return paid by a risk-free asset, such as a government note) and the standard deviation of the same returns. Intuitively, this can be interpreted as a "return on risk" (or as William Sharpe put it, "return on variability"). But the standard deviation of excess returns may be a misleading measure of variability when returns follow asymmetric or fat-tailed distributions or when returns are not independent or identically distributed. Suppose that a strategy's excess returns (or risk premiums), r_t, are independent and identically distributed (IID) following a Normal law:

$$r_t \sim \mathcal{N}(\mu, \sigma^2), \tag{1}$$

where $\mathcal{N}$ represents a Normal distribution with mean μ and variance σ^2. The *annualized* Sharpe ratio (SR) can be computed as

$$SR = \frac{\mu}{\sigma}\sqrt{q}, \tag{2}$$

where q is the number of returns per year (see Lo [17] for a detailed derivation of this expression). Sharpe ratios are typically expressed in annual terms in order to allow for the comparison of strategies that trade with different frequency. The great majority of financial models are built upon the IID Normal assumption, which may explain why the Sharpe ratio has become the most popular statistic for evaluating an investment's performance.

Since μ, σ are usually unknown, the true value SR cannot be known for certain. Instead, we can estimate the Sharpe ratio as $\widehat{SR} = \frac{\hat{\mu}}{\hat{\sigma}}\sqrt{q}$, where $\hat{\mu}$ and $\hat{\sigma}$ are the sample mean and sample standard deviation. The inevitable consequence is that SR calculations are likely to be the subject of substantial estimation errors (see Bailey and López de Prado [2] for a confidence band and an extension of the concept of Sharpe Ratio beyond the IID Normal assumption).

From Lo [17] we know that the distribution of the estimated annualized Sharpe ratio $\widehat{SR}$ converges asymptotically (as $y \to \infty$) to

$$\widehat{SR} \xrightarrow{a} \mathcal{N}\left[SR, \frac{1 + \frac{SR^2}{2q}}{y}\right], \tag{3}$$

where y is the number of years used to estimate $\widehat{SR}$.[2] As y increases without bound, the probability distribution of $\widehat{SR}$ approaches a Normal distribution with mean SR and variance $\frac{1+\frac{SR^2}{2q}}{y}$. For a sufficiently large y, (3) provides an approximation of the distribution of $\widehat{SR}$.

[2]*Most performance statistics assume IID Normal returns and so are normally distributed. In the case of the Sharpe ratio, several authors have proved that its asymptotic distribution follows a Normal law even when the returns are not IID Normal. The same result applies to the Information Ratio. The only requirement is that the returns be ergodic. We refer the interested reader to Bailey and López de Prado [2].*

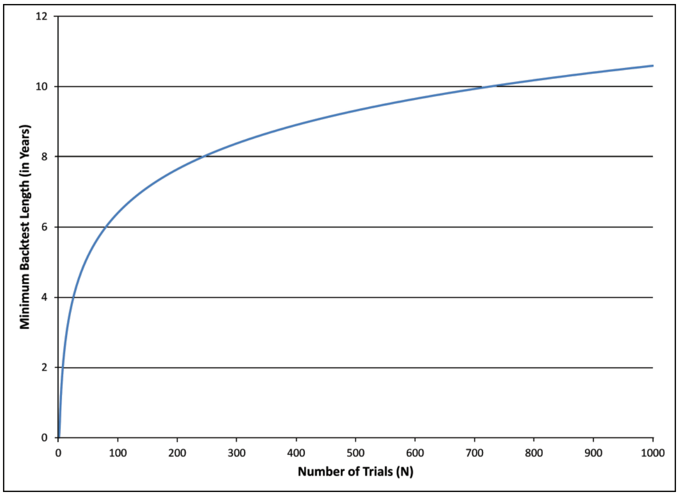

Figure 2. Minimum Backtest Length needed to avoid overfitting, as a function of the number of trials.

Figure 2 shows the tradeoff between the number of trials (N) and the minimum backtest length (MinBTL) needed to prevent skill-less strategies to be generated with a Sharpe ratio IS of 1. For instance, if only five years of data are available, no more than forty-five independent model configurations should be tried. For that number of trials, the expected maximum SR IS is 1, whereas the expected SR OOS is 0. After trying only seven independent strategy configurations, the expected maximum SR IS is 1 for a two-year long backtest, while the expected SR OOS is 0. The implication is that a backtest which does not report the number of trials N used to identify the selected configuration makes it impossible to assess the risk of overfitting.

Even for a small number N of trials, it is relatively easy to find a strategy with a high Sharpe ratio IS but which also delivers a null Sharpe ratio OOS. To illustrate this point, consider N strategies with $T = yq$ returns distributed according to a Normal law with mean excess returns μ and with standard deviation σ. Suppose that we would like to select the strategy with optimal $\widehat{SR}$ IS, based on one year of observations. A risk we face is choosing a strategy with a high Sharpe ratio IS but zero Sharpe ratio OOS. So we ask the question, *how high is the expected maximum Sharpe ratio IS among a set of strategy configurations where the true Sharpe ratio is zero?*

Bailey and López de Prado [2] derived an estimate of the *Minimum Track Record Length (MinTRL)* needed to reject the hypothesis that an estimated Sharpe ratio is below a certain threshold

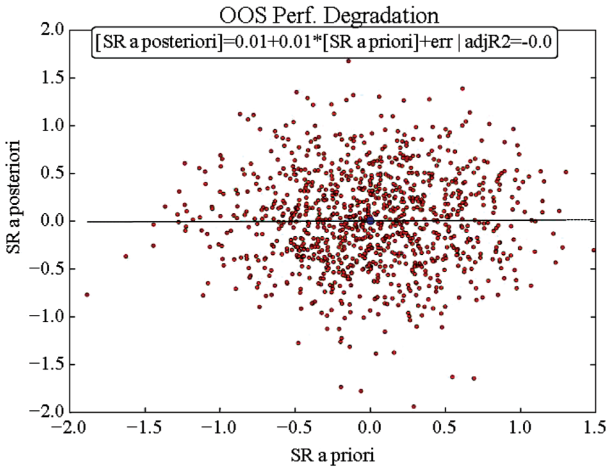

Figure 3. Performance IS vs. OOS before introducing strategy selection.

Figure 3 shows the relation between SR IS (x-axis) and SR OOS (y-axis) for $\mu = 0, \sigma = 1, N = 1000, T = 1000$. *Because the process follows a random walk, the scatter plot has a circular shape centered at the point* $(0,0)$. *This illustrates the fact that, in absence of compensation effects, overfitting the IS performance (x-axis) has no bearing on the OOS performance (y-axis), which remains around zero.*

(let's say zero). MinTRL was developed to evaluate a strategy's track record (a single realized path, $N = 1$). The question we are asking now is different, because we are interested in the backtest length needed to avoid selecting a skill-less strategy among N alternative specifications. In other words, in this article we are concerned with overfitting prevention when comparing multiple strategies, not in evaluating the statistical significance of a single Sharpe ratio estimate. Next, we will derive the analogue to MinTRL in the context of overfitting, which we will call *Minimum Backtest Length (MinBTL)*, since it specifically addresses the problem of backtest overfitting.

From (3), if $\mu = 0$ and $y = 1$, then $\widehat{SR} \xrightarrow{a} \mathcal{N}(0,1)$. Note that because $SR = 0$, increasing q does not reduce the variance of the distribution. The proof of the following proposition is left for the appendix.

Proposition 1. *Given a sample of IID random variables,* $x_n \sim Z$, $n = 1, \ldots, N$, *where* Z *is the CDF of the Standard Normal distribution, the expected maximum of that sample,* $E[\max_N] = E[\max\{x_n\}]$, *can be approximated for a large* N *as*

$$E[\max_N] \approx (1-\gamma)Z^{-1}\left[1 - \frac{1}{N}\right] + \gamma Z^{-1}\left[1 - \frac{1}{N}e^{-1}\right] \quad (4)$$

where $\gamma \approx 0.5772156649\ldots$ *is the Euler-Mascheroni constant and* $N \gg 1$.

An upper bound to (4) is $\sqrt{2\ln[N]}$.[3] Figure 1 plots, for various values of N (x-axis), the expected Sharpe ratio of the optimal strategy IS. For example, if the researcher tries only $N = 10$ alternative configurations of an investment strategy, she is expected to find a strategy with a Sharpe ratio IS of 1.57 despite the fact that all strategies are expected to deliver a Sharpe ratio of zero OOS (including the "optimal" one selected IS).

Proposition 1 has important implications. As the researcher tries a growing number of strategy configurations, there will be a nonnull probability of selecting IS a strategy with null expected performance OOS. Because the hold-out method does not take into account the number of trials attempted before selecting a model, it cannot assess the representativeness of a backtest.

Minimum Backtest Length (MinBTL)

Let us consider now the case that $\mu = 0$ but $y \neq 1$. Then, we can still apply Proposition 1 by rescaling the expected maximum by the standard deviation of the annualized Sharpe ratio, $y^{-1/2}$. Thus, the researcher is expected to find an "optimal" strategy with an IS annualized Sharpe ratio of

$$E[\max_N] \approx y^{-1/2}\left((1-\gamma)Z^{-1}\left[1 - \frac{1}{N}\right] + \gamma Z^{-1}\left[1 - \frac{1}{N}e^{-1}\right]\right). \quad (5)$$

Equation (5) says that the more independent the configurations a researcher tries (N), the more likely she is to overfit, and therefore the higher the acceptance threshold should be for the backtested result to be trusted. This situation can be partially mitigated by increasing the sample size (y). By solving (5) for y, we obtain the following statement.

Theorem 2. *The Minimum Backtest Length (MinBTL, in years) needed to avoid selecting a strategy with an IS Sharpe ratio of* $\overline{E[\max_N]}$ *among N independent strategies with an expected OOS Sharpe ratio of zero is*

$$MinBTL \approx \left(\frac{(1-\gamma)Z^{-1}\left[1 - \frac{1}{N}\right] + \gamma Z^{-1}\left[1 - \frac{1}{N}e^{-1}\right]}{\overline{E[\max_N]}}\right)^2 < \frac{2\ln[N]}{\overline{E[\max_N]}^2}. \quad (6)$$

Equation (6) tells us that MinBTL must grow as the researcher tries more independent model configurations (N) in order to keep constant the expected maximum Sharpe ratio at a given level

[3] *See Example 3.5.4 of Embrechts et al.* [5] *for a detailed treatment of the derivation of upper bounds on the maximum of a Normal distribution.*

$\overline{E[\max_N]}$. Figure 2 shows how many years of backtest length (MinBTL) are needed so that $E[\max_N]$ is fixed at 1. For instance, if only five years of data are available, no more than forty-five independent model configurations should be tried or we are almost guaranteed to produce strategies with an annualized Sharpe ratio IS of 1 but an expected Sharpe ratio OOS of zero. Note that Proposition 1 assumed the N trials to be independent, which leads to a quite conservative estimate. If the trials performed were not independent, the number of independent trials N involved could be derived using a dimension-reduction procedure, such as Principal Component Analysis.

We will examine this tradeoff between N and T in greater depth later in the paper without requiring such a strong assumption, but MinBTL gives us a first glance at how easy it is to overfit by merely trying alternative model configurations. As an approximation, the reader may find it helpful to remember the upper bound to the minimum backtest length (in years), $MinBTL < \frac{2\ln[N]}{\overline{E[\max_N]}^2}$.

Of course, a backtest may be overfit even if it is computed on a sample greater than MinBTL. From that perspective, MinBTL should be considered a necessary, nonsufficient condition to avoid overfitting. We leave to Bailey et al. [1] the derivation of a more precise measure of backtest overfitting.

Model Complexity

How does the previous result relate to model complexity? Consider a one-parameter model that may adopt two possible values (like a switch that generates a random sequence of trades) on a sample of T observations. Overfitting will be difficult, because $N = 2$. Let's say that we make the model more complex by adding four more parameters so that the total number of parameters becomes 5, i.e., $N = 2^5 = 32$. Having thirty-two independent sequences of random trades greatly increases the possibility of overfitting.

While a greater N makes overfitting easier, it makes perfectly fitting harder. Modern supercomputers can only perform around 2^{50} raw computations per second, or less than 2^{58} raw computations per year. Even if a trial could be reduced to a raw computation, searching $N = 2^{100}$ will take us 2^{42} supercomputer-years of computation (assuming a 1 Pflop/s system, capable of 10^{15} floating-point operations per second). Hence, a skill-less brute force search is certainly impossible. While it is hard to perfectly fit a complex skill-less strategy, Proposition 1 shows that there is no need for that. Without perfectly fitting a strategy or making it overcomplex, a researcher can achieve high Sharpe ratios. A relatively simple strategy with just seven binomial independent parameters offers $N = 2^7 = 128$ trials, with an expected maximum Sharpe ratio above 2.6 (see Figure 1).

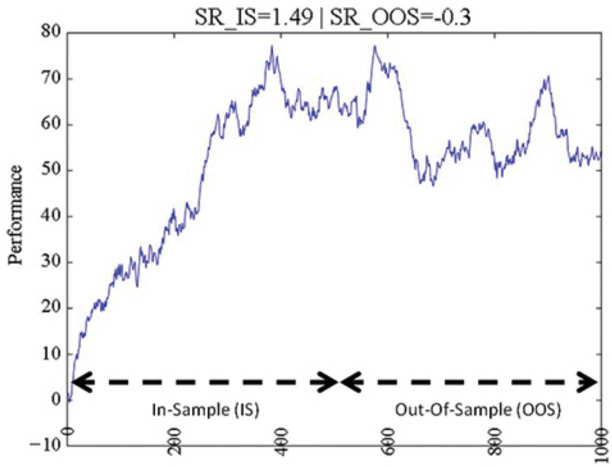

Figure 4. Performance IS vs. performance OOS for one path after introducing strategy selection.

Figure 4 provides a graphical representation of what happens when we select the random walk with highest SR IS. The performance of the first half was optimized IS, and the performance of the second half is what the investor receives OOS. The good news is, in the absence of memory, there is no reason to expect overfitting to induce negative performance.

We suspect, however, that backtested strategies that significantly beat the market typically rely on some combination of valid insight, boosted by some degree of overfitting. Since believing in such an artificially enhanced high-performance strategy will often also lead to over leveraging, such overfitting is still very damaging. Most Technical Analysis strategies rely on filters, which are sets of conditions that trigger trading actions, like the random switches exemplified earlier. Accordingly, extra caution is warranted to guard against overfitting in using Technical Analysis strategies, as well as in complex nonparametric modeling tools, such as Neural Networks and Kernel Estimators.

Here is a key concept that investors generally miss:

> *A researcher that does not report the number of trials N used to identify the selected backtest configuration makes it impossible to assess the risk of overfitting.*

Because N is almost never reported, the magnitude of overfitting in published backtests is unknown. It is not hard to overfit a backtest (indeed, the previous theorem shows that it is hard *not to*), so we suspect that a large proportion of backtests published in academic journals may be misleading. The

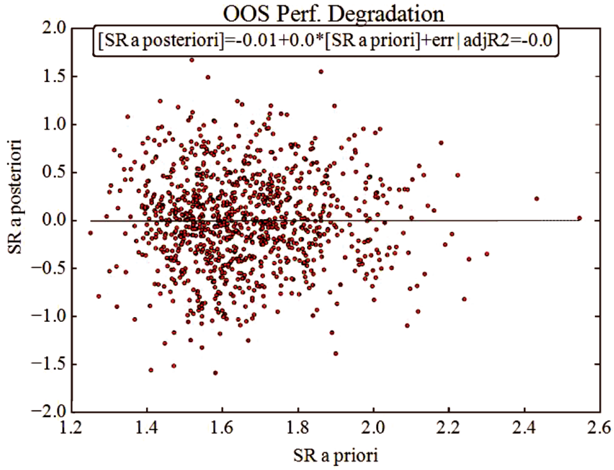

Figure 5. Performance degradation after introducing strategy selection in absence of compensation effects.

Figure 5 illustrates what happens once we add a "model selection" procedure. Now the SR IS ranges from 1.2 *to* 2.6*, and it is centered around* 1.7*. Although the backtest for the selected model generates the expectation of a* 1.7 *SR, the expected SR OOS is unchanged and lies around* 0*.*

situation is not likely to be better among practitioners.

In our experience, overfitting is pathological within the financial industry, where proprietary and commercial software is developed to estimate the combination of parameters that best fits (or, more precisely, overfits) the data. These tools allow the user to add filters without ever reporting how such additions increase the probability of backtest overfitting. Institutional players are not immune to this pitfall. Large mutual fund groups typically discontinue and replace poorly performing funds, introducing *survivorship* and selection bias. While the motivation of this practice may be entirely innocent, the effect is the same as that of hiding experiments and inflating expectations.

We are not implying that those technical analysts, quantitative researchers, or fund managers are "snake oil salesmen". Most likely they most genuinely believe that the backtested results are legitimate or that adjusted fund offerings better represent future performance. Hedge fund managers are often unaware that most backtests presented to them by researchers and analysts may be useless, and so they unknowingly package faulty investment propositions into products. One goal of this paper is to make investors, practitioners, and academics aware of the futility of considering backtest without controlling for the probability of overfitting.

Overfitting in Absence of Compensation Effects

Regardless of how realistic the prior being tested is, there is always a combination of parameters that is optimal. In fact, even if the prior is false, the researcher is very likely to identify a combination of parameters that happens to deliver an outstanding performance IS. But because the prior is false, OOS performance will almost certainly underperform the backtest's results. As we have described, this phenomenon, by which IS results tend to outperform the OOS results, is called overfitting. It occurs because a sufficiently large number of parameters are able to target specific data points—say by chance buying just before a rally and shorting a position just before a sell-off—rather than triggering trades according to the prior.

To illustrate this point, suppose we generate N Gaussian random walks by drawing from a Standard Normal distribution, each walk having a size T. Each performance path m_τ can be obtained as a cumulative sum of Gaussian draws

$$\Delta m_\tau = \mu + \sigma \varepsilon_\tau, \tag{7}$$

where the *random shocks* ε_τ are IID distributed $\varepsilon_\tau \sim Z, \tau = 1, \dots, T$. Suppose that each path has been generated by a particular combination of parameters, backtested by a researcher. Without loss of generality, assume that $\mu = 0, \sigma = 1$, and $T = 1000$, covering a period of one year (with about four observations per trading day). We divide these paths into two disjoint samples of equal size, 500, and call the first one IS and the second one OOS.

At the moment of choosing a particular parameter combination as optimal, the researcher had access to the IS series, not the OOS. For each model configuration, we may compute the Sharpe ratio of the series IS and compare it with the Sharpe ratio of the series OOS. Figure 3 shows the resulting scatter plot. The p-values associated with the intercept and the IS performance (SR a priori) are respectively 0.6261 and 0.7469.

The problem of overfitting arises when the researcher uses the IS performance (backtest) to choose a particular model configuration, with the expectation that configurations that performed well in the past will continue to do so in the future. This would be a correct assumption if the parameter configurations were associated with a truthful prior, but this is clearly not the case of the simulation above, which is the result of Gaussian random walks without trend ($\mu = 0$).

Figure 4 shows what happens when we select the model configuration associated with the random walk with highest Sharpe ratio IS. The performance of the first half was optimized IS, and the performance of the second half is what the investor receives OOS. The good news is that under these conditions, there is no reason to expect overfitting to induce negative performance. This is illustrated in Figure 5, which shows how the optimization causes the expected performance IS to range between 1.2 and 2.6, while the OOS performance will range between -1.5 and 1.5 (i.e., around μ, which in this case is zero). The p-values associated with the intercept and the IS performance (SR a priori) are respectively 0.2146 and 0.2131. Selecting an optimal model IS had no bearing on the performance OOS, which simply equals the zero mean of the process. A positive mean ($\mu > 0$) would lead to positive expected performance OOS, but such performance would nevertheless be inferior to the one observed IS.

Overfitting in Presence of Compensation Effects

Multiple causes create compensation effects in practice, such as overcrowded investment opportunities, major corrections, economic cycles, reversal of financial flows, structural breaks, bubble bursts, etc. Optimizing a strategy's parameters (i.e., choosing the model configuration that maximizes the strategy's performance IS) does not necessarily lead to improved performance (compared to not optimizing) OOS, yet again leading to overfitting.

In some instances, when the strategy's performance series lacks memory, overfitting leads to no improvement in performance OOS. However, the presence of memory in a strategy's performance series induces a compensation effect, which increases the chances for that strategy to be selected IS, only to underperform the rest OOS. Under those circumstances, IS backtest optimization is in fact detrimental to OOS performance.[4]

Global Constraint

Unfortunately, overfitting rarely has the neutral implications discussed in the previous section. Our previous example was purposely chosen to exhibit a globally unconditional behavior. As a result, the OOS data had no memory of what occurred IS. Centering each path to match a mean μ removes one degree of freedom:

$$\overline{\Delta m_\tau} = \Delta m_\tau + \mu - \frac{1}{T}\sum_{\tau=1}^{T} \Delta m_\tau. \tag{8}$$

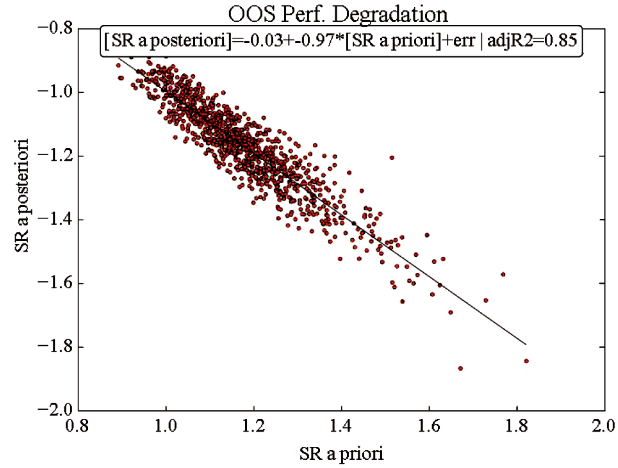

Figure 6. Performance degradation as a result of strategy selection under compensation effects (global constraint).

Figure 6 shows that adding a single global constraint causes the OOS performance to be negative even though the underlying process was trendless. Also, a strongly negative linear relation between performance IS and OOS arises, indicating that the more we optimize IS, the worse the OOS performance of the strategy.

We may rerun the same Monte Carlo experiment as before, this time on the recentered variables $\overline{\Delta m_\tau}$. Somewhat scarily, adding this single global constraint causes the OOS performance to be negative even though the underlying process was trendless. Moreover, a strongly negative linear relation between performance IS and OOS arises, indicating that the more we optimize IS, the worse the OOS performance. Figure 6 displays this disturbing pattern. The p-values associated with the intercept and the IS performance (SR a priori) are respectively 0.5005 and 0, indicating that the negative linear relation between IS and OOS Sharpe ratios is statistically significant.

The following proposition is proven in the appendix.

Proposition 3. *Given two alternative configurations (A and B) of the same model, where $\sigma_{IS}^A = \sigma_{OOS}^A = \sigma_{IS}^B = \sigma_{OOS}^B$ imposing a global constraint $\mu^A = \mu^B$, implies that*

$$SR_{IS}^A > SR_{IS}^B \Leftrightarrow SR_{OOS}^A < SR_{OOS}^B. \tag{9}$$

[4] *Bailey et al.* [1] *propose a method to determine the degree to which a particular backtest may have been compromised by the risk of overfitting.*

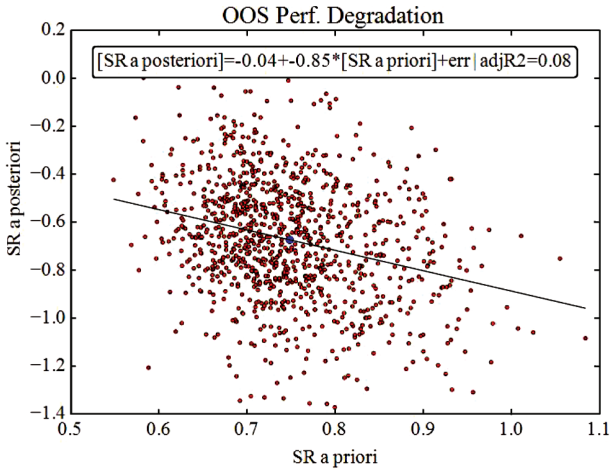

Figure 7. Performance degradation as a result of strategy selection under compensation effects (first-order serial correlation).

Figure 7 illustrates that a serially correlated performance introduces another form of compensation effects, just as we saw in the case of a global constraint. For example, if $\varphi = 0.995$, it takes about 138 observations to recover half of the deviation from the equilibrium. We have rerun the previous Monte Carlo experiment, this time on an autoregressive process with $\mu = 0, \sigma = 1, \varphi = 0.995$, and have plotted the pairs of performance IS vs. OOS.

Recentering a series is one way to introduce memory into a process, because some data points will now compensate for the extreme outcomes from other data points. By optimizing a backtest, the researcher selects a model configuration that spuriously works well IS and consequently is likely to generate losses OOS.

Serial Dependence

Imposing a global constraint is not the only situation in which overfitting actually is detrimental. To cite another (less restrictive) example, the same effect happens if the performance series is serially conditioned, such as a first-order autoregressive process,

$$\Delta m_\tau = (1-\varphi)\mu + (\varphi - 1)\varphi m_{\tau-1} + \sigma\varepsilon_\tau \tag{10}$$

or, analogously,

$$m_\tau = (1-\varphi)\mu + \varphi m_{\tau-1} + \sigma\varepsilon_\tau, \tag{11}$$

where the random shocks are again IID distributed as $\varepsilon_\tau \sim Z$. The following proposition is proven in the appendix. The number of observations that it takes for a process to reduce its divergence from the long-run equilibrium by half is known as the *half-life period*, or simply *half-life* (a familiar physical concept introduced by Ernest Rutherford in 1907).

Proposition 4. *The half-life period of a first-order autoregressive process with autoregressive coefficient $\varphi \in (0,1)$ occurs at*

$$\tau = -\frac{\ln[2]}{\ln[\varphi]}. \tag{12}$$

For example, if $\varphi = 0.995$, it takes about 138 observations to retrace half of the deviation from the equilibrium. This introduces another form of compensation effect, just as we saw in the case of a global constraint. If we rerun the previous Monte Carlo experiment, this time for the autoregressive process with $\mu = 0, \sigma = 1, \varphi = 0.995$, and plot the pairs of performance IS vs. OOS, we obtain Figure 7.

The p-values associated with the intercept and the IS performance (SR a priori) are respectively 0.4513 and 0, confirming that the negative linear relation between IS and OOS Sharpe ratios is again statistically significant. Such serial correlation is a well-known statistical feature, present in the performance of most hedge fund strategies. Proposition 5 is proved in the appendix.

Proposition 5. *Given two alternative configurations (A and B) of the same model, where $\sigma^A_{IS} = \sigma^A_{OOS} = \sigma^B_{IS} = \sigma^B_{OOS}$ and the performance series follows the same first-order autoregressive stationary process,*

$$SR^A_{IS} > SR^B_{IS} \Leftrightarrow SR^A_{OOS} < SR^B_{OOS}. \tag{13}$$

Proposition 5 reaches the same conclusion as Proposition 3 (a compensation effect) without requiring a global constraint.

Is Backtest Overfitting a Fraud?

Consider an investment manager who emails his stock market forecast for the next month to $2^n x$ prospective investors, where x and n are positive integers. To half of them he predicts that markets will go up, and to the other half that markets will go down. After the month passes, he drops from his list the names to which he sent the incorrect forecast, and sends a new forecast to the remaining $2^{n-1}x$ names. He repeats the same procedure n times, after which only x names remain. These x investors have witnessed n consecutive infallible forecasts and may be extremely tempted to give this investment manager all of their savings. Of course, this is a fraudulent scheme based on random screening: The investment manager is hiding the fact that for every one of the x successful witnesses, he has tried 2^n unsuccessful ones (see Harris [8, p. 473] for a similar example).

To avoid falling for this psychologically compelling fraud, a potential investor needs to consider the economic cost associated with manufacturing the successful experiments, and require the investment manager to produce a number n for which the scheme is uneconomic. One caveat is, even if n is too large for a skill-less investment manager, it may be too low for a mediocre investment manager who uses this scheme to inflate his skills.

Not reporting the number of trials (N) involved in identifying a successful backtest is a similar kind of fraud. The investment manager only publicizes the model that works but says nothing about all the failed attempts, which as we have seen can greatly increase the probability of backtest overfitting.

An analogous situation occurs in medical research, where drugs are tested by treating hundreds or thousands of patients; however, only the best outcomes are publicized. The reality is that the selected outcomes may have healed in spite of (rather than thanks to) the treatment or due to a placebo effect (recall Proposition 1). Such behavior is unscientific—not to mention dangerous and expensive—and has led to the launch of the *alltrials.net* project, which demands that all results (positive and negative) for every experiment are made publicly available. A step forward in this direction is the recent announcement by Johnson & Johnson that it plans to open all of its clinical test results to the public [14]. For a related discussion of reproducibility in the context of mathematical computing, see Stodden et al. [25].

Hiding trials appears to be standard procedure in financial research and financial journals. As an aggravating factor, we know from the section "Overfitting in Presence of Compensation Effects" that backtest overfitting typically has a detrimental effect on future performance due to the compensation effects present in financial series. Indeed, the customary disclaimer "past performance is not an indicator of future results" is too optimistic in the context for backtest overfitting. When investment advisers do not control for backtest overfitting, good backtest performance is an indicator of negative future results.

A Practical Application

Institutional asset managers follow certain investment procedures on a regular basis, such as rebalancing the duration of a fixed income portfolio (PIMCO); rolling holdings on commodities (Goldman Sachs, AIG, JP Morgan, Morgan Stanley); investing or divesting as new funds flow at the end of the month (Fidelity, BlackRock); participating in the regular U.S. Treasury Auctions (all major investment banks); delevering in anticipation of payroll, FOMC or GDP releases; tax-driven effects around the end of the year and mid-April; positioning for electoral cycles, etc. There are a large number of instances where asset managers will engage in somewhat predictable actions on a regular basis. It should come as no surprise that a very popular

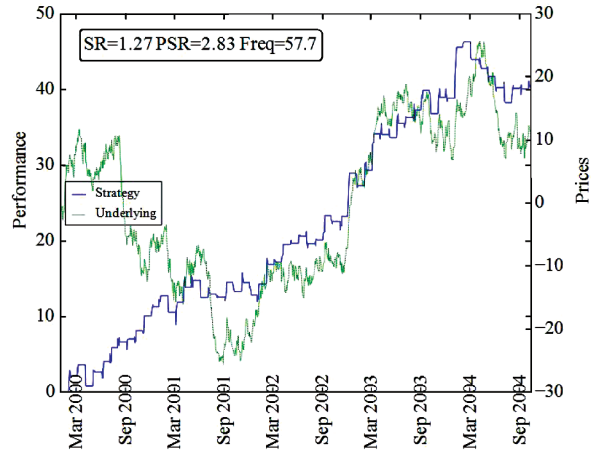

Figure 8. Backtested performance of a seasonal strategy (Example 6).

We have generated a time series of 1000 daily prices (about four years) following a random walk. The PSR-Stat of the optimal model configuration is 2.83*, which implies a less-than 1 percent probability that the true Sharpe ratio is below* 0*. Consequently, we have been able to identify a plausible seasonal strategy with an SR of* 1.27 *despite the fact that no true seasonal effect exists.*

investment strategy among hedge funds is to profit from such seasonal effects.

For example, a type of question often asked by hedge fund managers follows the form: "Is there a time interval every [ ] when I would have made money on a regular basis?" You may replace the blank space with a word like day, week, month, quarter, auction, nonfarm payroll (NFP) release, European Central Bank (ECB) announcement, presidential election year, ... The variations are as abundant as they are inventive. Doyle and Chen [4] study the "weekday effect" and conclude that it appears to "wander".

The problem with this line of questioning is that there is always a time interval that is arbitrarily "optimal" regardless of the cause. The answer to one such question is the title of a very popular investment classic, *Do Not Sell Stocks on Monday*, by Hirsch [12]. The same author wrote an almanac for stock traders that reached its forty-fifth edition in 2012 and he is also a proponent of the "Santa Claus Rally", the quadrennial political/stock market cycle, and investing during the "Best Six Consecutive Months" of the year, November through April. While these findings may indeed be caused by some underlying seasonal effect, it is easy to demonstrate that any random data contains similar patterns. The discovery of a pattern IS typically has no bearing OOS, yet again is a result of overfitting. Running such experiments without controlling for the probability of backtest overfitting will lead the researcher to spurious claims. OOS performance will disappoint, and the reason will not be that "the market has found out the seasonal effect and arbitraged away the strategy's profits." Rather, the effect was never there; instead, it was just a random pattern that gave rise to an overfitted trading rule. We will illustrate this point with an example.

Example 6. Suppose that we would like to identify the optimal monthly trading rule given four customary parameters: Entry_day, Holding_period, Stop_loss, and Side. *Side* defines whether we will hold long or short positions on a monthly basis. *Entry_day* determines the business day of the month when we enter a position. *Holding_period* gives the number of days that the position is held. *Stop_loss* determines the size of the loss (as a multiple of the series' volatility) that triggers an exit for that month's position. For example, we could explore all nodes that span the set $\{1, \ldots, 22\}$ for *Entry_day*, the set $\{1, \ldots, 20\}$ for *Holding_period*, the set $\{0, \ldots, 10\}$ for *Stop_loss*, and $\{-1, 1\}$ for *Side*. The parameter combinations involved form a four-dimensional mesh of 8,800 elements. The optimal parameter combination can be discovered by computing the performance derived by each node.

First, we generated a time series of 1,000 daily prices (about four years), following a random walk. Figure 8 plots the random series, as well as the performance associated with the optimal parameter combination: Entry_day = 11, Holding_period = 4, Stop_loss = -1 and Side = 1. The annualized Sharpe ratio is 1.27.

Given the elevated Sharpe ratio, we could conclude that this strategy's performance is significantly greater than zero for any confidence level. Indeed, the *PSR-Stat* is 2.83, which implies a less than 1 percent probability that the true Sharpe ratio is below 0.[5] Several studies in the practitioners and academic literature report similar results, which are conveniently justified with some ex-post explanation ("the posterior gives rise to a prior"). What this analysis misses is an evaluation of the probability that this backtest has been overfit to the data, which is the subject of Bailey et al. [1].

In this practical application we have illustrated how simple it is to produce overfit backtests when answering common investment questions, such as the presence of seasonal effects. We refer the reader to the appendix section "Reproducing the Results in Example 6" for the implementation of this experiment in the Python language. Similar experiments can be designed to demonstrate overfitting in the context of other effects, such as trend following, momentum, mean-reversion, event-driven effects, etc. Given the facility with which elevated Sharpe ratios can be manufactured IS, the reader would be well advised to remain highly suspicious of backtests and of researchers who fail to report the number of trials attempted.

Conclusions

While the literature on regression overfitting is extensive, we believe that this is the first study to discuss the issue of overfitting on the subject of investment simulations (backtests) and its negative effect on OOS performance. On the subject of regression overfitting, the great Enrico Fermi once remarked (Mayer et al. [20]):

> I remember my friend Johnny von Neumann used to say, with four parameters I can fit an elephant, and with five I can make him wiggle his trunk.

The same principle applies to backtesting, with some interesting peculiarities. We have shown that backtest overfitting is difficult indeed to avoid. Any perseverant researcher will always be able to find a backtest with a desired Sharpe ratio regardless of the sample length requested. Model complexity is only one way that backtest overfitting is facilitated. Given that most published backtests do not report the number of trials attempted, many of them may be overfitted. In that case, if an investor allocates capital to them, performance will vary: *It will be around zero if the process has no memory, but it may be significantly negative if the process has memory.* The standard warning that *"past performance is not an indicator of future results"* understates the risks associated with investing on overfit backtests. When financial advisors do not control for overfitting, positive backtested performance will often be followed by negative investment results.

We have derived the expected maximum Sharpe ratio as a function of the number of trials (N) and sample length. This has allowed us to determine the *Minimum Backtest Length* (MinBTL) needed to avoid selecting a strategy with a given IS Sharpe ratio among N trials with an expected OOS Sharpe ratio of zero. Our conclusion is that the more trials a financial analyst executes, the greater should be the IS Sharpe ratio demanded by the potential investor.

We strongly suspect that such backtest overfitting is a large part of the reason why so many algorithmic or systematic hedge funds do not live up to the elevated expectations generated by their managers.

[5] *The Probabilistic Sharpe Ratio (or PSR) is an extension to the SR. Nonnormality increases the error of the variance estimator, and PSR takes that into consideration when determining whether an SR estimate is statistically significant. See Bailey and López de Prado* [2] *for details.*

We would feel sufficiently rewarded in our efforts if this paper succeeds in drawing the attention of the mathematical community to the widespread proliferation of journal publications, many of them claiming profitable investment strategies on the sole basis of in-sample performance. This is understandable in business circles, but a higher standard is and should be expected from an academic forum.

A depressing parallel can be drawn between today's financial academic research and the situation denounced by economist and Nobel Laureate Wassily Leontief writing in *Science* (see Leontief [16]):

> A dismal performance
>
> … "What economists revealed most clearly was the extent to which their profession lags intellectually." This editorial comment by the leading economic weekly (on the 1981 annual proceedings of the American Economic Association) says, essentially, that the "king is naked." But no one taking part in the elaborate and solemn procession of contemporary U.S. academic economics seems to know it, and those who do don't dare speak up.
>
> […]
>
> [E]conometricians fit algebraic functions of all possible shapes to essentially the same sets of data without being able to advance, in any perceptible way, a systematic understanding of the structure and the operations of a real economic system.
>
> […]
>
> That state is likely to be maintained as long as tenured members of leading economics departments continue to exercise tight control over the training, promotion, and research activities of their younger faculty members and, by means of peer review, of the senior members as well.

We hope that our distinguished colleagues will follow this humble attempt with ever-deeper and more convincing analysis. We did not write this paper to settle a discussion. On the contrary, our wish is to ignite a dialogue among mathematicians and a reflection among investors and regulators. We should do well also to heed Newton's comment after he lost heavily in the South Seas bubble; see [21]:

> For those who had realized big losses or gains, the mania redistributed wealth. The largest honest fortune was made by Thomas Guy, a stationer turned philanthropist, who owned £54,000 of South Sea stock in April 1720 and sold it over the following six weeks for £234,000. Sir Isaac Newton, scientist, master of the mint, and a certifiably rational man, fared less well. He sold his £7,000 of stock in April for a profit of 100 percent. But something induced him to reenter the market at the top, and he lost £20,000. "I can calculate the motions of the heavenly bodies," he said, "but not the madness of people."

Appendices

Proof of Proposition 1

Embrechts et al. [5, pp. 138–147] show that the maximum value (or last order statistic) in a sample of independent random variables following an exponential distribution converges asymptotically to a Gumbel distribution. As a particular case, the Gumbel distribution covers the Maximum Domain of Attraction of the Gaussian distribution, and therefore it can be used to estimate the expected value of the maximum of several independent random Gaussian variables.

To see how, suppose there is a sample of IID random variables, $z_n \sim Z, n = 1, \dots, N$, where Z is the CDF of the Standard Normal distribution. To derive an approximation for the sample maximum, $\max_n = \max\{z_n\}$, we apply the Fisher-Tippett-Gnedenko theorem to the Gaussian distribution and obtain that

$$\lim_{N\to\infty} Prob\left[\frac{\max_N - \alpha}{\beta} \le x\right] = G[x], \tag{14}$$

where

- $G[x] = e^{-e^{-x}}$ is the CDF for the Standard Gumbel distribution.
- $\alpha = Z^{-1}\left[1 - \frac{1}{N}\right]$, $\beta = Z^{-1}\left[1 - \frac{1}{N}e^{-1}\right] - \alpha$, and Z^{-1} corresponds to the inverse of the Standard Normal's CDF.

The normalizing constants (α, β) are derived in Resnick [22] and Embrechts et al. [5]. The limit of the expectation of the normalized maxima from a distribution in the Gumbel Maximum Domain of Attraction (see Proposition 2.1(iii) in Resnick [22]) is

$$\lim_{N\to\infty} E\left[\frac{\max_N - \alpha}{\beta}\right] = \gamma, \tag{15}$$

where γ is the Euler-Mascheroni constant, $\gamma \approx 0.5772156649\dots$. Hence, for N sufficiently large, the mean of the sample maximum of standard normally distributed random variables can be approximated by

$$E[\max_N] \approx \alpha + \gamma\beta = (1-\gamma)Z^{-1}\left[1 - \frac{1}{N}\right] + \gamma Z^{-1}\left[1 - \frac{1}{N}e^{-1}\right] \tag{16}$$

where $N > 1$.

Proof of Proposition 3

Suppose there are two random samples (A and B) of the same process $\{\Delta m_\tau\}$, where A and B are of equal size and have means and standard deviations $\mu^A, \mu^B, \sigma^A, \sigma^B$. A fraction δ of each sample is called IS, and the remainder is called OOS, where for simplicity we have assumed that $\sigma_{IS}^A = \sigma_{OOS}^A = \sigma_{IS}^B = \sigma_{OOS}^B$. We would like to understand the implications of a global constraint $\mu^A = \mu^B$.

First, we note that $\mu^A = \delta\mu_{IS}^A + (1-\delta)\mu_{OOS}^A$ and $\mu^B = \delta\mu_{IS}^B + (1-\delta)\mu_{OOS}^B$. Then, $\mu_{IS}^A > \mu_{OOS}^A \Leftrightarrow \mu_{IS}^A > \mu^A \Leftrightarrow \mu_{OOS}^A < \mu^A$. Likewise, $\mu_{IS}^B > \mu_{OOS}^B \Leftrightarrow \mu_{IS}^B > \mu^B \Leftrightarrow \mu_{OOS}^B < \mu^B$.

Second, because of the global constraint $\mu^A = \mu^B$, $\mu_{IS}^A + \frac{(1-\delta)}{\delta}\mu_{OOS}^A = \mu_{IS}^B + \frac{(1-\delta)}{\delta}\mu_{OOS}^B$ and $\mu_{IS}^A - \mu_{IS}^B = \frac{(1-\delta)}{\delta}(\mu_{OOS}^B - \mu_{OOS}^A)$. Then, $\mu_{IS}^A > \mu_{IS}^B \Leftrightarrow \mu_{OOS}^A < \mu_{OOS}^B$. We can divide this expression by $\sigma_{IS}^A > 0$, with the implication that

$$SR_{IS}^A > SR_{IS}^B \Leftrightarrow SR_{OOS}^A < SR_{OOS}^B \tag{17}$$

where we have denoted $SR_{IS}^A = \frac{\mu_{IS}^A}{\sigma_{IS}^A}$, etc. Note that we did not have to assume that Δm_τ is IID, thanks to our assumption of equal standard deviations. The same conclusion can be reached without assuming equality of standard deviations; however, the proof would be longer but no more revealing (the point of this proposition is the implication of global constraints).

Proof of Proposition 4

This proposition computes the half-life of a first-order autoregressive process. Suppose there is a random variable m_τ that takes values of a sequence of observations $\tau \in \{1, \dots, \infty\}$, where

$$m_\tau = (1-\varphi)\mu + \varphi m_{\tau-1} + \sigma\varepsilon_\tau \tag{18}$$

such that the random shocks are IID distributed as $\varepsilon_\tau \sim N(0,1)$. Then

$$\lim_{\tau\to\infty} E_0[m_\tau] = \mu$$

if and only if $\varphi \in (-1,1)$. In particular, from Bailey and López de Prado [3] we know that the expected value of this process at a particular observation τ is

$$E_0[m_\tau] = \mu(1-\varphi^\tau) + \varphi^\tau m_0. \tag{19}$$

Suppose that the process is initialized or reset at some value $m_0 \neq \mu$. We ask the question, how many observations must pass before

$$E_0[m_\tau] = \frac{\mu + m_0}{2}? \tag{20}$$

Inserting (20) into (19) and solving for τ, we obtain

$$\tau = -\frac{\ln[2]}{\ln[\varphi]}, \tag{21}$$

which implies the additional constraint that $\varphi \in (0,1)$.

Proof of Proposition 5

Suppose that we draw two samples (A and B) of a first-order autoregressive process and generate two subsamples of each. The first subsample is called IS and is comprised of $\tau = 1, \dots, \delta T$, and the second subsample is called OOS and is comprised of $\tau = \delta T + 1, \dots, T$, with $\delta \in (0,1)$ and T an integer multiple of δ. For simplicity, let us assume that $\sigma_{IS}^A = \sigma_{OOS}^A = \sigma_{IS}^B = \sigma_{OOS}^B$. From Proposition 4, (18), we obtain

$$E_{\delta T}[m_T] - m_{\delta T} = (1-\varphi^T)(\mu - m_{\delta T}). \tag{22}$$

Because $1-\varphi^T > 0$, $\sigma_{IS}^A = \sigma_{IS}^B$, $SR_{IS}^A > SR_{IS}^B \Leftrightarrow m_{\delta T}^A > m_{\delta T}^B$. This means that the OOS of A begins with a seed that is greater than the seed that initializes the OOS of B. Therefore, $m_{\delta T}^A > m_{\delta T}^B \Leftrightarrow E_{\delta T}[m_T^A] - m_{\delta T}^A < E_{\delta T}[m_T^B] - m_{\delta T}^B$. Because $\sigma_{IS}^B = \sigma_{OOS}^B$, we conclude that

$$SR_{IS}^A > SR_{IS}^B \Leftrightarrow SR_{OOS}^A < SR_{OOS}^B \tag{23}$$

Reproducing the Results in Example 6

Python code implementing the experiment described in "A Practical Application" can be found at `http://www.quantresearch.info/Software.htm` and at `http://www.financial-math.org/software/`.

Acknowledgments

We are indebted to the editor and two anonymous referees who peer-reviewed this article for the *Notices of the American Mathematical Society*. We are also grateful to Tony Anagnostakis (Moore Capital), Marco Avellaneda (Courant Institute, NYU), Peter Carr (Morgan Stanley, NYU), Paul Embrechts (ETH Zürich), Matthew D. Foreman (University of California, Irvine), Jeffrey Lange (Guggenheim Partners), Attilio Meucci (KKR, NYU), Natalia Nolde (University of British Columbia ad ETH Zürich), and Riccardo Rebonato (PIMCO, University of Oxford). The opinions expressed in this article are the authors' and they do not necessarily reflect the views of the Lawrence Berkeley National Laboratory, Guggenheim Partners, or any other organization. No particular investment or course of action is recommended.

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A Tribute to Hans Grauert

Alan Huckleberry and Thomas Peternell, coordinating editors

Hans Grauert, one of the most creative, prolific, and scientifically influential mathematicians in the second half of the last century, was born on February 8, 1930, in Haren, a small town in northern Germany near the Dutch border. In the summer of 1949, he enrolled in the University of Mainz. One semester later he changed to Münster, where he soon met his lifelong friend and collaborator Reinhold Remmert. Heinrich Behnke, who had already built an internationally recognized research group in complex analysis in the 1930s, was the leading figure in Münster at this time. In particular, due to his connections to Henri Cartan and Heinz Hopf, after the war Behnke was able to keep Münster on the mathematical map. Karl Stein was already a well-established professor in the group and Friedrich Hirzebruch was a rising star. Grauert had entered, probably by chance, one of the most stimulating mathematical atmospheres in postwar Germany.

Profiting from the Münster complex analysis climate and an extended visit to Zürich at the invitation of Beno Eckmann, Grauert received his Ph.D. in Münster in 1954. His dissertation opened new connections between Kähler geometry and Stein theory. At the age of twenty-seven he obtained his habilitation with a series of papers on what is now known as the Oka-Grauert principle.

After spending the academic year 1957/1958 in Princeton at the Institute for Advanced Study and a semester at the IHES in Paris, in 1959 Grauert was named to the Gauss chair in Göttingen and became the successor of C. L. Siegel. Despite receiving offers from various distinguished universities, he remained in Göttingen for his entire academic life. Among numerous honors, he was an invited speaker at the ICM in 1958, 1962 (plenary lecture), and 1966.

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DOI: http://dx.doi.org/10.1090/noti1103

Hans Grauert, 1960.

Grauert's contributions to mathematics incorporate all aspects of higher-dimensional complex analysis. Apart from his remarkable habilitation thesis where he proved the equivalence of the topological and holomorphic categories of various types of fiber bundles over Stein spaces, his works include

- foundational results on complex spaces and Stein theory (with Remmert),
- the direct image theorem and consequences in moduli theory,
- the solution to the Levi problem in the smooth case,
- pseudoconvex spaces and their applications in analytic and algebraic geometry,
- deformation theory, e.g., existence of versal deformations,
- vector bundles on projective spaces.

Grauert, Stein, and Remmert, 1957. Three men in a boat at the Arbeitstagung.

Three fundamental research monographs were jointly published with Remmert, and with various coauthors he published textbooks in the areas of real and complex analysis.

Grauert was an extremely successful "Akademischer Lehrer" in the classical German sense. In particular, he guided more than forty Ph.D. students, many of whom have continued the lines of research of their mentor. Two of them, Ingo Lieb and Günter Trautmann, together with Daniel Barlet, Jean-Pierre Demailly, Takeo Ohsawa, and Yum-Tong Siu, have contributed to this homage to Hans Grauert.

After a long illness, Hans Grauert passed away in September of 2011 at the age of 81. His mathematical influence will last forever.

Yum-Tong Siu

Hans Grauert in Memoriam

The very first time I was introduced to the name and work of Professor Hans Grauert was at the seminar of Professor Solomon Bochner at Princeton. At the beginning of my first year at Princeton as a graduate student, I participated in Bochner's seminar, in which newcomers were assigned some papers to read and present. My assignment was Grauert's 1955 *Mathematische Annalen* paper on the characterization of holomorphically complete complex spaces. It was my first experience presenting a paper at such a seminar. I spent a great deal of time reading the paper and reorganizing it in the standard format of numbered definitions, lemmas, propositions, theorems, and corollaries in what I considered to be a better logical order of interdependence.

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Only a few minutes into my presentation, Bochner stopped me and said that the way I presented it was definitely not how Grauert originally organized his paper. He went on to point out that Grauert was a real master and a master's own presentation contains the very valuable information of the master's thought process, which would be lost in a reorganization of the paper's contents. He illustrated his point by saying that when you play the music of a master composer, you are not supposed to substitute for it a variation of your own.

Needless to say, given all the effort and time I put into reorganizing Grauert's paper, I was very disappointed by Bochner's reaction. On the other hand, Bochner's comments served as a useful guide in my learning process from then on. While working on my Ph.D. thesis under the guidance of Professor Robert C. Gunning at Princeton, I spent a great deal of time poring over the original papers of Hans Grauert and avoided simply reading their expositions by other mathematicians. For me and the entire generation of mathematicians in several complex variables, Grauert's results plus all the explicit and implicit ideas in Grauert's papers are goldmines to explore and work on.

In the late 1950s and early 1960s, Grauert launched the new and richest phase of the theory of several complex variables on three fronts: (i) the Levi problem, (ii) the Oka principle for fiber bundles, and (iii) the direct image theorem.

For the Levi problem Grauert introduced his exceedingly ingenious bumping technique to construct global holomorphic functions. How to construct global holomorphic functions on abstractly defined complex manifolds and spaces is one of the most fundamental problems in complex geometry. Grauert's solution of the Levi problem reduces the construction of global holomorphic functions to the existence of strictly plurisubharmonic exhaustion functions. Prior to Grauert's solution of the Levi problem, the only abstractly defined complex manifolds for which a general method of constructing global holomorphic functions is available are noncompact Riemann surfaces. It is the method of minimizing Dirichlet integrals on Riemann surfaces, which is motivated by electrostatic potentials. Grauert's bumping technique of constructing holomorphic functions was a breakthrough of fundamental importance. Currently, besides Grauert's bumping technique, for construction of holomorphic functions there is also available the technique of L^2 estimates of $\bar\partial$ due to Morrey, Andreotti-Vesentini, Kohn, Hörmander, and others. However, for situations of singular spaces and sections of coherent sheaves, Grauert's method remains the only directly applicable tool.

Grauert and Siu at Princeton, 1979.

Grauert and Siu, 2000. Seventieth birthday in Göttingen.

As an offshoot of his solution of the Levi problem by his bumping technique, Grauert observed that the total space of a negative line bundle over a compact complex space is strictly pseudoconvex. With this observation Grauert established a link between his bumping technique of constructing holomorphic functions and the vanishing and embedding theorems of Kodaira in algebraic geometry, thereby extending the latter to the setting of singular spaces. Such a connection also led to the development of his important theory of modifications and exceptional analytic sets.

Later in 1962 in joint papers with Aldo Andreotti, Grauert generalized his bumping technique to yield finiteness theorems for p-pseudoconvex and q-pseudoconcave spaces and coherent sheaves with cohomological codimension conditions and to construct compactifications of pseudoconcave ends.

The Oka principle is the general philosophical framework concerning the equivalence between topological and holomorphic objects over Stein spaces. Grauert developed Runge approximation techniques for functions with values in holomorphic fiber bundles to prove the equivalence between topological and holomorphic fiber bundles over Stein spaces. Grauert's approximation technique was later applied by others to yield important results on embedding and immersion dimensions and the number of defining equations for submanifolds of Stein manifolds. To date, Grauert's equivalence and results from generalizations of his approximation techniques have remained the best rigorously proved cases of the Oka principle.

The direct image theorem of Grauert gives the coherence of the direct images of coherent sheaves under proper holomorphic maps. Grauert proved it by introducing a smoothing method to relate the norm estimates of cochains in two different Stein coverings for the same cohomology class in order to guarantee the convergence of power series whose coefficients are inductively defined elements of Fréchet spaces. Grauert's smoothing method resembles the smoothing process used in the Nash-Moser implicit function theorem in Nash's 1956 *Annals of Mathematics* paper and Moser's 1966 *Annali della Scuola Normale Superiore di Pisa* paper. Although later in the early 1970s Forster-Knorr and Kiehl-Verdier gave simpler proofs of Grauert's direct image theorem, yet even in the simpler proofs there is still some analogue of Grauert's key smoothing process.

While at Göttingen, Grauert made Göttingen the mecca of several complex variables. The golden foundational period of the modern theory of several complex variables started with the work of the schools of Behnke, Oka, Cartan, Serre, Stein, Remmert, Andreotti, and many others and culminated in the phenomenal contributions of Grauert in the late 1950s and early 1960s. In Göttingen, between 1960 and 1996 Grauert produced forty Ph.D. students. Through visitor programs in Göttingen and international conferences Grauert mentored an entire generation of young mathematicians working on the theory of several complex variables. He set the research agenda in the field by trailblazing in a broad range of new directions, such as non-Archimedean function theory, integral formulas, deformation of singularities, low-rank vector bundles, and many others.

The first time I met Professor Grauert in person was in the several complex variables conference in Maryland in 1969. Since then he has invited me to visit Göttingen a number of times. Grauert was a man of very few words. Yet his concise, to-the-point professional comments carried great impact. Grauert was very kind, affable, and generous. Whenever he was in the company of young mathematicians, he always inspirationally shared without any hesitation his mathematical ideas with anybody within earshot. Jointly with Remmert and

Kawai, Grauert, Chandrasekharan, Kuranishi, 1960. International Colloquium on Function Theory, Tata Institute in Bombay.

Stein, for many years Grauert organized workshops in several complex variables in Oberwolfach. I still recall with nostalgic fondness those casual after-dinner conversations in the lounge area outside the library. While sipping German white wine priced at ten marks a bottle, I and other junior mathematicians sat in a circle around Grauert and other senior mathematicians, attentively taking in Grauert's remarks about his mathematical ideas and viewpoints.

During all my visits to Göttingen, I benefited immensely from the many stimulating mathematical discussions in his office and in walks with him around Göttingen. I especially enjoyed my visit in 1993, when Grauert was compiling his own selected papers and invited me to Göttingen to keep him up to date concerning the impact of his work on the field. We conversed daily about his panoramic views of mathematics in general and about his own work. His recounting of the circumstances and his thought processes which led to his results were to me a real eye-opener and a sumptuous intellectual feast.

My conversations with Grauert rarely strayed far from the subject of mathematics. In one of my visits to Göttingen, he took me to the Gauss tower near Göttingen and we had lunch in the restaurant there, surrounded by brass instruments devised and used by Gauss. He went into a long discourse about Gauss and, in particular, about how a committee, which was charged with the responsibility of singling out one contribution of Gauss to put on the ten mark bill honoring Gauss, eventually came up with the choice of the Gaussian distribution in probability. He then told me that when he was a student he was very interested in physics. He began to focus his attention on mathematics only after he accidentally broke a galvanometer in a physics laboratory and was shown his way out of the laboratory. It was the good fortune of mathematics that the accidental breaking of a lowly galvanometer at that time led later to a complete change of the landscape of the modern theory of several complex variables.

Grauert was an intellectual giant. Those of us who had the good fortune of coming under his tutelage are not only awed by his brilliant mind as a mathematician but inspired, too, by his personal qualities. He was truly a shining example of integrity and magnanimity.

Takeo Ohsawa

Thesis, Smiles, and Beyond

It was in January 1980 when I went to Göttingen to see Grauert. I really wanted to meet him in person because I was strongly impressed by his masterpieces. In Grauert's papers I witnessed how excellent ideas work, feeling the strength of simplicity of arguments. Not to mention the great achievements, including the solution of the Levi problem on complex manifolds (see [G3]), I highly appreciated his thesis [G1] on the domains with complete Kähler metrics because it contained a result contrary to my naive guess.

In order to realize the desired trip from Kyoto to Göttingen, I had to write a letter to Grauert to get his approval for the application to the Alexander von Humboldt Foundation. In that letter I enclosed my master's thesis showing that a nonsingular rational curve embedded in a complex manifold admits a holomorphically convex neighborhood whenever the normal bundle is seminegative. (This was an answer to a problem which A. Fujiki had given to me after I finished reading [G5].) To my great pleasure I received a positive answer. In the letter Grauert simply and strongly wrote, "I will be glad if you can come." As a result, it was possible for me to spend sixteen months in Göttingen. I happily remember that Grauert welcomed me into his office with a distinctive smile and a few days later introduced me as a student of Shigeo Nakano (1923–1997) to the members of the seminar including Michael Schneider (1947–1997). It was good for me that everybody there seemed to know the Akizuki-Nakano vanishing theorem.

Among my pleasant experiences in Göttingen following this good start, let me recall an incident by which Grauert influenced my direction of research. Shortly before the summer break, Yum-Tong Siu visited us for several weeks. On Sundays Grauert took Siu and me in his Mercedes to the suburbs. Around noon on a sunny day, we saw the mountain on which Gauss measured relatively big triangles. The next day Siu gave a seminar talk. It was on the compactification of complete Kähler manifolds

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(see [SY]). I remember that he used the word "spike" to describe the shape of the ends of manifolds. At the end of the talk, Grauert made a remark that it was still an open problem whether or not complete Kähler domains with C^1-smooth boundary are locally pseudoconvex. I was happy to hear this because I had already submitted a paper solving it affirmatively (see [O]). Why not capture this wonderful chance! So, after hearing Siu say that all that he knew was Grauert's affirmative solution [G1] for domains with real analytic boundary, I proposed to explain why C^1-smoothness suffices. Allowed by Grauert, I did it in ten or fifteen minutes because I could skip some part of the L^2-theory. Grauert seemed to be convinced and applauded me by knocking on the table, with a contented smile. All of this happened in a lecture room where very large portraits of Klein and Hilbert were hung.

Encouraged by this experience, I tried to optimize the method of extending holomorphic functions with L^2-growth conditions and eventually arrived at a result [OT] which recently turned out to be useful for many purposes. In the selected papers of Grauert [G9] with his own commentary, [O] is referred to but [OT] is not. So I wish to supplement his commentary on [G1] by adding a remark: The idea of understanding the role of pseudoconvexity via geometry of Kähler manifolds yielded an effective extension technique in several complex variables.

Ohsawa, Grauert, and Siu at Kassel-Wilhelmshöhe, 1980.

Jean-Pierre Demailly

Hans Grauert and the Foundation of Modern Complex Analysis

Hans Grauert was born in 1930 in Haren, a town in Niedersachsen, Germany. He passed away on September 4, 2011, leaving the mathematical world with an extraordinary legacy in complex analysis and analytic geometry. I would like to share here a few recollections of my encounters with him during the last decades and some connections of his work with my own research. I first met Hans Grauert at the end of the 1970s on the occasion of a conference in Paris and still remember very well a discussion we had then. I was a young student at that time, and on the detour of a naive question I had raised, he had to explain the concept of a meromorphic map $X \to Y$ between complex manifolds…. Of course, I had already been somewhat acquainted with Grauert's major contributions to the theory of analytic spaces [GR1], [G5] and the Levi problem [G1], [G3]—and as a consequence was very impressed to exchange a few words with him.

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In 1986, Hans Grauert had heard more of my work, and he invited me to spend a couple of weeks in Göttingen; my stay actually took place in November 1986. At that time, Grauert was interested in the study of Kobayashi hyperbolicity, especially in view of his recently published paper "Hyperbolicity of the complement of plane curves" [GP], in collaboration with his daughter Ulrike Grauert-Peternell. He raised on the occasion a number of tough questions about hyperbolicity, and at that time I could not even think of any possible attempt to investigate them. Anyway, the discussion was to have a profound influence on my thinking years later—I will give a few more details on this below. It was also around that period that Grauert had received a private copy of Grothendieck's writing "Récoltes et semailles" [Gro2], a very personal account of an important period of Grothendieck's mathematical life. Grauert had already been in close contact with Grothendieck at the end of the 1950s, culminating with their work on the direct image theorem for coherent sheaves (in the algebraic and analytic settings, [G3] and [G4], respectively). Although he was obviously not at all targeted, I remember that Grauert was a bit upset about some of the controversial sentences contained in Grothendieck's testimony….

Coming back to Kobayashi hyperbolicity theory, Grauert introduced in [G8] the important concept of a jet metric, following previous work by Green and Griffiths [GG] on jet differentials. If X is a projective nonsingular variety, let us consider J_kX to be the bundle of k-jets of holomorphic curves $f : (\mathbb{C}, 0) \to X$, together with the $\mathbb{C}^*$-action $(\lambda \cdot f)(t) = f(\lambda t)$ obtained by reparameterizing the curve with a linear change of parameter. One can consider the projectivized jet bundle $X_k := J_kX/\mathbb{C}^*$ whose fibers are weighted projective spaces and the corresponding $\mathcal{O}_{X_k}(1)$ tautological sheaf. A *k-jet metric* is then just a hermitian metric on that sheaf; in other words, this is a nonnegative function

Hans Grauert, 1990.

$\rho : J_kX \to \mathbb{R}^+$ such that $\rho(\lambda \cdot f) = |\lambda|\,\rho(f)$, in Grauert's own notation. By looking at all holomorphic curves $f : \Delta \to X$ on the unit disk possessing a prescribed k-jet at 0 up to the $\mathbb{C}^*$-action and trying to maximize the multiple, one defines in a canonical way a k-jet metric $\rho_{k,\mathrm{can}}$ which is just the k-jet analogue of the Kobayashi infinitesimal metric in the case $k = 1$. Grauert realized that, under a suitable negativity assumption for the curvature of the k-jet metric ρ, the Ahlfors-Schwarz lemma would imply the Kobayashi hyperbolicity of X; he then asked what type of curvature estimates the k-jet metric $\rho_{k,\mathrm{can}}$ should satisfy. This question, which was further explained to me by Grauert's younger collaborators Gerd Dethloff and Siegmund Kosarew, is still unsolved at present. In fact, it is convenient to introduce a variant of these jet bundles (replacing the $\mathbb{C}^*$-action by the group $\mathbb{G}_k$ of k-jets of biholomorphisms $\varphi : (\mathbb{C}, 0) \to (\mathbb{C}, 0)$; cf. [D2]), and then the conjecture is expressed by saying that X *is Kobayashi hyperbolic if and only if there exists a k-jet metric with strictly negative curvature (in a suitable sense), with poles contained in the set of k-jets that are singular at the origin, for all $k \geq k_0$ large enough.* In this statement, it can be shown, starting with hyperbolic complex surfaces, that k_0 may have to be taken arbitrarily large. In the positive direction, it can be derived from a recent result of [D3] that a projective manifold of general type always possesses a negatively curved k-jet metric for k large if one forgets about the demands on the set of poles, thus proving only some sort of weak generic hyperbolicity of X. This is done by studying the cohomology of the bundles of jet differentials and inferring from that that every entire holomorphic curve $f : \mathbb{C} \to X$ has to satisfy global algebraic differential equations $P(f; f', \ldots, f^{(k)}) = 0$; in fact f has to satisfy a large number of them when k increases.

Another foundational result is the Grauert-Riemenschneider theorem [GRi]: *if X is a projective or Moishezon manifold, then $H^q(X, K_X \otimes L) = 0$ for every $q \geq 1$ and every semiample line bundle $L \to X$ of maximal Kodaira dimension.* The result is often used in its relative form, stating that *if $\mu : X \to Y$ is a projective birational morphism over some base Y, then all higher direct images $R^q\mu_*\mathcal{O}(K_X)$ vanish ($q \geq 1$).* In the same paper, Grauert and Riemenschneider conjectured that a compact analytic space is Moishezon if and only if it carries an almost positive coherent sheaf of rank 1; cf. also [R1]. Pursuing these ideas, Oswald Riemenschneider solved the Kähler case in [R2] by showing that a compact Kähler manifold carrying a line bundle whose curvature is semipositive and strictly positive at one point is actually Moishezon. The general case (removing the Kähler assumption) was finally settled by Yum-Tong Siu in [S1], using very clever bounds on Čech cohomology classes and their harmonic counterparts. These results served as the main motivation and as a strong guide for the discovery of holomorphic Morse inequalities in [D1] (probably the reason for Grauert's interest in my work in 1986 and, incidentally, also one of the main ingredients for the above-mentioned result of [D3]). These inequalities can be stated as follows: *for every compact complex manifold X and every holomorphic line bundle L, we have as $k \to +\infty$ the asymptotic estimate of cohomology groups*

$$(*)\quad \frac{n!}{k^n}\sum_{j=0}^{q}(-1)^{q-j}h^j(X, L^{\otimes k}) \leq \int_{X(u,\leq q)}(-1)^q u^n + o(1), \quad n = \dim_{\mathbb{C}} X,$$

where u is a smooth closed $(1,1)$-form in $c_1(L)$ and $X(u, \leq q)$ is the open set of points $x \in X$ where $u(x)$ is nondegenerate with at most q negative eigenvalues. Assuming $\int_{X(u,\leq 1)} u^n > 0$, the estimate implies that $L^{\otimes k}$ has many sections, hence that L is big and that the base manifold X is Moishezon. Using a slight improvement due to L. Bonavero [B] ("singular holomorphic Morse inequalities"), one concludes that *a compact complex manifold is Moishezon if and only if it carries a holomorphic line bundle possessing a singular hermitian metric h with analytic singularities such that the curvature current $u = \Theta_{L,h} \in c_1(L)$ satisfies $\int_{X(u,\leq 1)\smallsetminus Z} u^n > 0$, on the complement of the set of poles $Z \subset X$.* This characterization strengthens Siu's solution of the Grauert-Riemenschneider conjecture.

Among Grauert's other fundamental contributions, the Andreotti-Grauert theorem [AG] stands out as one of the most important finiteness theorems of analytic geometry. Let us recall that *a complex n-dimensional complex manifold X is said to be q-convex (resp. q-complete) if X possesses a smooth exhaustion function φ such that the Levi form $i\partial\overline{\partial}\varphi$ has at least $n - q + 1$ strictly positive* *eigenvalues on the complement $X \setminus K$ of a compact set* (*resp. on X itself*); an appropriate definition can also be given for arbitrary complex spaces. Along with many other results, [AG] proves that *the cohomology groups $H^j(X, \mathcal{F})$ of a coherent analytic sheaf $\mathcal{F}$ on X are finite dimensional (resp. vanish) if X is q-convex (resp. q-complete) and $j \geq q$*. This finiteness statement implies a very interesting corollary also in the compact case. Let (L, h) be a hermitian line bundle on a compact complex manifold X such that the curvature form $\Theta_{L,h} = -\frac{i}{2\pi}\partial\overline{\partial}\log h$ has at least $n - q + 1$ positive eigenvalues at every point. Then the total space of the dual line bundle L^* is q-convex, and one can easily derive from this that $H^j(X, L^{\otimes k}) = 0$ for $j \geq q$ and k large enough. The holomorphic Morse inequalities would yield here the related (but somewhat less precise) result that $h^j(X, L^{\otimes k}) = o(k^n)$ for $j \geq q$, whenever $\Theta_{L,h}$ has at least $n - q + 1$ semipositive eigenvalues at every point. An important unsolved question is whether a converse of the Andreotti-Grauert theorem holds true: *assuming that X (resp. L) is cohomologically q-convex, in the sense that the relevant cohomology groups $H^j(X, \mathcal{F})$ are finite dimensional* (*resp. vanish, resp. $H^j(X, L^{\otimes k}) = 0$, $k \gg 0$*) *for $j \geq q$, does it follow that X is q-convex* (*resp. q-complete*)*; resp. does it follow that L possesses a hermitian metric h with the required signature*? The latter question has been analyzed in more depth in [DPS]. In general, the answer is unknown except in the strictly pseudoconvex case ($q = 1$). Very recently, a partial converse was settled for line bundles L over compact complex surfaces ($n = \dim_{\mathbb{C}} X = 2$), in the form of an *asymptotic Morse equality*

$$(**)\quad \limsup_{k\to+\infty} \frac{n!}{k^n} \sum_{j=0}^{q} (-1)^{q-j} h^j(X, L^{\otimes k}) = \inf_{u \in c_1(L)} \int_{X(u,\leq q)} (-1)^q u^n, \qquad 0 \leq q \leq n$$

(cf. [D4]), the case of complex curves ($n = 1$) also being easy to check. It would be interesting to know whether such a result holds true when $n \geq 3$.

Hans Grauert has contributed many more fundamental results than those briefly discussed here. He can be considered as one of the founders of modern analytic geometry, and his achievements will certainly provide very strong guidelines for future research in the field. I have been deeply influenced by the research directions he initiated and also strongly encouraged by the role he played in the recognition of my work as a member of the Mathematisches Institut and of the Akademie der Wissenschaften zu Göttingen.

Daniel Barlet

In Memory of Hans Grauert

Throughout the years I have been impressed by the dynamism and quality of the German school of several complex variables under the influence of H. Grauert and R. Remmert. As a permanent invited guest to most conferences organized by this group, in Oberwolfach or elsewhere, for almost forty years, I have had the privilege of following and admiring the mathematical guidance of Hans Grauert in this school. Furthermore, it is evident that he also had a strong influence on our group in Nancy (among many others).

It was not so easy having mathematical discussions with Hans Grauert, and it was often difficult to follow him when he explained his works during these conferences. I have the feeling that his way of thinking about mathematics was not the usual way of ordinary people like me. I confess that it was more enlightening for me to think about his beautiful results and methods by myself than trying to follow him during these talks, and, in some sense I think it was the same for him in that he preferred to follow results of other mathematicians in his own way. But it is clear that from his *special* point of view and way of looking at mathematics, he was able to see and to understand many beautiful results that ordinary people cannot see. This insight would have been useless if he had not also had great strength for solving the technical problems arising along the way. But maybe I'm just giving a description of what an exceptional mathematician is.

Günther Trautmann

On Grauert's Early Days in Göttingen

When I entered Göttingen University in 1960 to study mathematics, I was supposed, as was the standard of this time, to follow a course in "Infinitesimalrechnung" and "Analytische Geometrie". The first was offered by the thirty-year-old Hans Grauert, the second by the then sixty-seven-year-old Kurt Reidemeister. While the first one was extremely formal for us, in the second we experienced an explanation of ideas seemingly without convergence to concrete results, a tremendous but fruitful contrast in teaching. Beforehand, older students had frightened me by saying that the lecture

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of Grauert would formally follow the pattern "definition, proposition, proof" without comment. This was true, but, to the contrary, many of my fellow students and I appreciated this style because we received the complete formalism and a secure and steady way to higher results and thereby learned what it meant to be precise in doing our own work. What the warning of the older students did not contain was that by the careful organization of his lectures Grauert approached the contents in a very efficient and short way, e.g., Stokes's theorem for differential forms at the end of the first year. Later I learned that he was much less formal in his mathematical thinking.

In this first year I was not aware that the young Hans Grauert had already published several of the most important papers in complex analysis at that time, e.g., the work on the Oka principle for holomorphic vector bundles [G2], the solution of the Levi problem, and the famous theorem of the coherence of direct images under proper mappings [G4]. Here I cannot refrain from making a few comments on some of these papers that had an enormous impact in algebraic geometry and were guidelines for me.

Some Mathematical Landmarks of Grauert

The paper [G4] is certainly his most famous and his most technically complicated. It was called "Blaue Arbeit" because of the cover of the IHES offprint and because some of its proofs seemed mysterious to us. In fact, it took another decade before improved versions of the proof and generalizations appeared in common papers of Knut Knorr and Otto Forster and of Reinhardt Kiehl and Jean-Louis Verdier. Previous theorems on special cases for proper holomorphic mappings then appeared as corollaries and, as Alan Huckleberry puts it in [H], "one cannot think of working in global complex geometry without the availability of this result." The typical method in this paper is the construction of shrinking coverings together with estimates of operators between spaces of Čech cocycles which occurred in the unwrapping of the problem and which were linked in a complicated induction process. Such estimates were also used in the important paper [AG] with Aldo Andreotti on the finiteness of cohomology groups for concave and convex complex spaces. My own work, some of it with coauthors, on the extension of coherent analytic sheaves was inspired by this paper, as was the later work of Yum-Tong Siu on the coherence of direct images under concave-convex mappings.

In [G5] Hans Grauert found the remarkable link between pseudoconvexity and contractibility of divisors. His criterion for contractibility, expressed in terms of negative definite matrices, has become one of the important theorems

Grauert, Stein, and Remmert, 1990.

in the theory of complex surfaces. Another landmark in complex algebraic geometry was the Grauert-Riemenschneider vanishing theorem, [GRi], which had a lasting influence on further works on vanishing results.

In a later period Grauert's papers [G6] and [G7] in deformation theory became as important and pioneering as his previous ones, proving the existence of versal deformations. In [G6] he developed his method of reduction of analytic ideals in order to prove convergence of formal deformations, a method used later on in many other papers on deformations of holomorphic vector bundles, coherent analytic sheaves, and more general analytic objects. His method even works nowadays in computer algebra systems in order to calculate deformations. The existence of versal deformations for the global case of compact complex spaces was proved in [G7] in competition with similar efforts of Adrian Douady.

In the mid-1970s the question of the existence of small rank indecomposable vector bundles in projective *n*-space arose, which is considered as one of the most interesting questions in projective algebraic geometry. The Grauert-Mülich theorem on the behavior of jumping lines of stable vector bundles is one of the cornerstones for the classification of such bundles. Grauert was very interested in classifying algebraic vector bundles on higher-dimensional projective spaces. Many questions, even for rank two vector bundles, remain open today. I remember one of the last conversations with Grauert about that problem, where we were betting, for lack of a well-founded conjecture, whether and which vector bundles of low rank would occur or not.

The Göttingen Seminar

Many of his former students may remember Grauert's lecture on algebraic topology in his third

Grauert's seventieth birthday celebration in Göttingen. Hans-Heinrich Voigt, president of the Göttingen Academy of Sciences, Trautmann, and Grauert.

year at Göttingen, which at this time was revolutionary at Göttingen and marked the beginning of a new mathematical branch of the Göttingen institute, the branch of complex analysis with its "Oberseminar". It was my good fortune to have been in this seminar at this time. In this seminar we all learned about and gave talks on sheaf theory and pseudoconvexity. Shortly thereafter the group doubled when Reinhold Remmert joined Göttingen. This seminar can be considered the Göttingen continuation of the "Sturm und Drang" period in complex analysis described by Remmert in [R]. The seminar took place at 5 p.m. every Monday. Each Monday at 3 p.m. Grauert was present in his office for "Sprechstunde", where we could ask him about special problems concerning our projects for diploma and doctoral theses or talks. I remember one occasion when I told him about texts and papers I was reading or planned on reading. He said, "Lesen Sie nicht so viel, vieles Lesen macht dumm" (don't read so much; much reading makes stupid). It was always his intention to make us think on our own and push things through. A mathematician should develop his own imagination on the material he is working on. Another time when I could not settle a partial problem for my doctoral thesis he was sitting at his desk for about five (for me endless) minutes without saying a word, thinking of a solution and then told me to look at a Koszul complex, and indeed this was the way out. This mirrors the way he promoted and encouraged his students.

In the stimulating atmosphere of Göttingen's complex analysis we also appreciated the lectures of Reinhold Remmert on commutative algebra and the precise lectures of Emeritus Carl Ludwig Siegel on automorphic and modular functions. This lecture was held parallel to Grauert's lecture on complex analysis and was later published in the three volumes [S]. The contrast between a classical viewpoint in mathematics and the modern language of ringed spaces and functors became apparent in the controversies between Siegel and Grauert which enriched our studies.

At this time Grauert and Remmert had already written the larger part of the manuscripts of the books *Stein Spaces* and *Coherent Analytic Sheaves* which we used for the talks. There were many distinguished guests, among them Heisuke Hironaka talking about his work on desingularization before it was completed. So the time of the first half of the 1960s was the beginning of a very fruitful period of complex analysis in Göttingen. Many of Grauert's doctoral students later became university professors. The continuously successful complex analysis group of Hans Grauert and his later colleagues Michael Schneider and Hubert Flenner led to the establishment of a DFG- "Sonderforschungsbereich" for the Mathematical Institute in Göttingen in the 1980s. Its activities had a large and long-lasting impact in complex and algebraic geometry.

Ingo Lieb

Hans Grauert: Teacher Extraordinary

Hans Grauert is rightly considered as one of the leading mathematicians of the last half-century. His research achievements have made him known worldwide and have initiated substantial further developments in complex analysis and algebraic geometry. In the following lines I will describe a different, less spectacular, part of his work: his teaching. Through his students this has led to further significant contributions to mathematics. Let me first go back to the time of my studies at Göttingen and look at Grauert…

Through the Eyes of a Student

The schedule of my second year of mathematics at Göttingen University (1959) contained the theory of functions of a complex variable. A young professor —just some eight years older than his students— who had come from Münster as the successor to Carl Ludwig Siegel, took charge of this course: Hans Grauert. I vividly remember the lecture and the lecturer: a slender, tall, young, and even younger looking, lecturer, giving the impression of great personal modesty, even shyness, very few motivating remarks, but absolute completeness of the mathematical arguments, never skipping any details; rarely had I understood a mathematical

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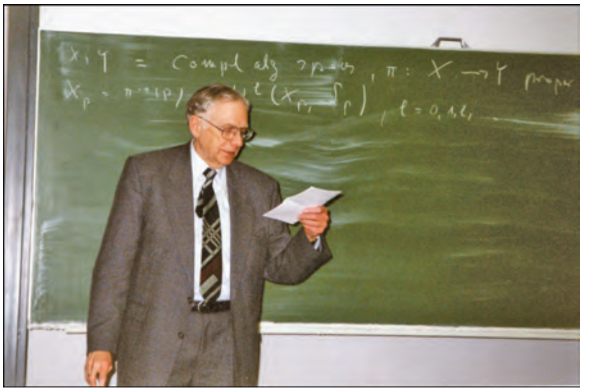
theory so directly or had been so impressed by its beauty. These characteristic features of Grauert's teaching style did not change very much in the course of the years. True, he exuded quite naturally more self-assurance in later years; moreover, he usually gave more motivating hints in his later lectures, usually very profound remarks, but in all his lectures he insisted on logical precision and complete arguments (the phrases *trivial* or *as can be easily seen* never came up in his lectures), and he always stepped behind the mathematical content, trying to make mathematics speak for itself. This is all the more remarkable as there would have been many instances where he might have quite rightly mentioned his own work, but I saw this only much later. (See "With the Eyes of a Coworker.")

The Göttingen Oberseminar

Starting around 1962, Grauert looked for students whom he could lead into mathematical research. To this end, he established his Oberseminar (graduate seminar), which he directed from 1962 until his retirement in 1995, at times jointly with Reinhold Remmert, Egbert Brieskorn, Michael Schneider, et al. Among the first members of this seminar were Günter Trautmann, Wolfgang Fischer, Ingo Lieb, Klas Diederich, Enrique Ramirez, Helmut Reckziegel, Rolf Richberg, Oswald Riemenschneider, Gudrun Kalmbach, all of whom became university teachers. Every single talk in that seminar was—without us clearly realizing it—a talk at the borderline of actual knowledge, and we, the students, were allowed to participate. The central mathematical topics in those days were the theory of Stein spaces, deformation theory, and analytic sheaf theory. We learned all these things from typed notes, which gave Grauert's and Remmert's own views of Stein theory and sheaf theory. These notes crystallized some years later in the famous three volumes of Grundlehren [GR2], [GR3], [GR4]. In addition, the seminar offered an opportunity to organize visiting lectures: H. Hironaka, F. Norguet, A. Douady, R. Wells, A. Andreotti, and many other leading researchers gave us talks.

Work in the Oberseminar has led altogether to forty-five Ph.D. theses supervised by Grauert, and in addition a considerable number of theses were supervised by his colleagues Remmert et al. The problems that Grauert suggested involved the whole area of complex analysis, from deformations of complex spaces to the geometry and analysis of domains of holomorphy and the boundary behavior of holomorphic maps. The overriding goal invariably aimed at substantial progress on major open areas of the field. He frequently gave indications of possible solutions generally in very concise, sometimes almost cryptic, formulation.



Beginning of a lecture. Grauert with his famous little piece of paper.

One was well advised to take each of his words seriously. His remarks were always founded on his previous deep preliminary reflections on the task in question. Sometimes one understood his hints only after one's own solution to the problem, just to see that his ideas had been right! Work on a problem posed by Grauert opened up, for a young, well-motivated mathematician, new fruitful fields of research, and no one was happier than Grauert when one of his students had successfully worked out his original ideas. Let me now see him…

With the Eyes of a Coworker

Hans Grauert loved to work jointly with his students. He wrote a joint paper with H. Reckziegel, when the latter was still an undergraduate [GRe]. More examples of this research in pairs stemming from Grauert's circle of ideas are joint papers with W. Fischer, I. Lieb, Michael Commichau, O. Riemenschneider [GFi], [GL], [CG], [GRi]. He particularly relied on his coauthors for his work on monographs and textbooks [GFL], [GFr]; Klaus Fritzsche, W. Fischer, I. Lieb, O. Riemenschneider [GR2] should be mentioned in this context. The books were usually based on preceding lectures, for which Grauert had detailed scripts written. This brings me back to my remarks in the first section: even standard chapters of the teaching material were freshly thought through and occasionally substantially transformed. For example, we find in his textbooks or lecture notes a new approach to Lebesgue integration (which easily carries over to functions with values in topological vector spaces but has been made accessible to first year undergraduates); a fairly general version of Stokes's theorem, probably valid even for semianalytic sets; a presentation of Maxwell's equations in the language of differential forms and currents clearly bringing to the foreground the invariance properties of the system (all this worked out for second year students!) [GFL]; an apparently

Hans Grauert in Bonn, 2000.

new construction of the completion of topological vector spaces [G10]; new proofs for the regularity of elliptic boundary value problems [G11]. In addition, the choice of topics, the choice between different options to build up and present a theory, shows Grauert's trademark: *ex ungue leonem*. The picture is completed by his monographs and textbooks on complex analysis [10], [13] in which many of his research results have found their place. So, Grauert would have had plenty of reason to mention himself in his lectures, but to my knowledge he never did it.

A last remark in this context: like most professors, Hans Grauert had to examine large numbers of students. In the German system this is mostly done in oral examinations. On these occasions he was a very mild and indulgent interview partner, as are, as I believe, most great scientists. As a recorder in these examinations, I have experienced this again and again.

Looking back…

Through the Eyes of a Colleague

The experiences with Hans Grauert that I have been describing here go back to the period between 1959 and 1971, when I was Grauert's student and coworker. This was a time when Grauert's amazing scientific productivity (starting around 1954) was at its peak. I did not notice this as a student, but later on as his colleague at Münster and Bonn I appreciated it and was deeply impressed. Research and teaching are in Grauert's work intimately intertwined. *Einheit von Forschung und Lehre* (unity of research and teaching) is Humboldt's university ideal, an ideal which seems quite unworldly in our time of impact factors, citation indexes, fights for and counting of grants, evaluations, professional formation, etc. Grauert realized this unity in his life: it is the same desire to know and the same urge for a profound understanding that characterizes his impressive research work and his lectures and textbooks.

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Is Big Data Enough? A Reflection on the Changing Role of Mathematics in Applications

Domenico Napoletani, Marco Panza, and Daniele C. Struppa

The advent of computers, and especially high-performance computers, has had a dramatic impact on the way in which mathematics is done and even more on how mathematics is applied, as demonstrated by the growth of computational mathematics as well as what goes under the name of "experimental mathematics", to which a journal is now devoted. More importantly, computers are now used to perform highly complex computations in order to apply mathematical models to a variety of empirical problems that could never be attacked otherwise. It is in this way that mathematics is often applied in different branches of biology (think of genomics and proteomics) as well as to social sciences and in what now goes under the generic term of "big data".

High-performance computing has also brought changes in the way we think about mathematics: its power, its methods, and the way in which it can be used to solve problems. For this reason, we would like to do something unusual in a mathematical journal by bringing the mathematical community into one of the discussions that are taking place among philosophers of mathematics and of science. While many mathematicians are weary of asking broad methodological questions because the danger of being vague and unclear is all too real, we also think that we must have this discussion concerning the modalities in which mathematics is applied if we don't want to be trapped by our own assumptions and miss out on more fruitful approaches to understanding reality. This is a discussion about the ways computational mathematics has not only changed our approach to science but even our way of understanding a phenomenon.

We believe it is possible to identify four methodological motifs that are closely related to each other and whose elucidation may make explicit the approach to problem solving in data analysis and statistical learning. By using short and evocative labels, we could say that these motifs concern, respectively, the *microarray paradigm*, the *preeminence of historical phenomena*, the *conceptualization of developmental processes*, and the *principle of forcing*, which we have introduced and partially expanded upon in [23], [24], [25], also in response to some acute commentaries in [14], [20]. Here we would like to simply focus on their rationale and potential significance in clarifying the underlying trends of quantitative, data-driven science.

The first methodological motif, the microarray paradigm, is derived by considerations that have become quite important in modern molecular biology. For example, DNA microarray technology (see for example [2]) allows the scientist to capture and visually represent, on an extremely large array of microscopic sites, information on the expression level of short strands of messenger RNA (mRNA), extracted and amplified from a given cell population. When the large amount of data derived from DNA microarrays is coupled with simple clustering techniques, it is often sufficient not only to differentiate cell populations from distinct tumors but also to determine the outcome of a given therapy without the need for an understanding, in each tumor population, of the actual role of each of the mRNA strands whose expression level is quantified by the microarrays.

Thus in [23] we have spoken of the microarray paradigm to indicate how much modern data analysis is supported by the belief that sufficiently large data collected from a phenomenon will allow

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DOI: http://dx.doi.org/10.1090/noti1102

one to answer any question about the phenomenon itself if treated with appropriate methods and assisted by powerful enough algorithms. Significant methods in statistical learning theory that embody the microarray paradigm include clustering techniques in their traditional hierarchical forms [31], but especially in their more unstructured versions, such as the affinity propagation method recently introduced in [11]. This exploits a completely local passage of information among data points to obtain fast and accurate splitting into clusters, whose best representative, the "exemplar", is also identified. Boosting [10] is another striking technique, perhaps the most transparent embodiment of the microarray paradigm, where a large number of classifiers, slightly better than random guesses, are combined and boosted to build a new classifier that is much more accurate than its components (cf. [13, Chapter 10]). Finally, we mention here the recent field of "nonlinear manifold learning", a collection of methods to find low-dimensional objects preserving some type of local, neighborhood structure in unstructured, high-dimensional data (see [30], [28] and [15, Chapter 16]).

As we can see from these examples, within the microarray paradigm answers are found through a process of automatic fitting of the data to models that do not carry any structural understanding beyond the actual solution of the problem itself. This is a distinctive lack of knowledge that we wanted to emphasize in [23] by speaking of "agnostic science". The overly optimistic belief in the assumptions of the microarray paradigm was popularized in a recent cover article in *Scientific American* [32], where the author discusses how the collection of unprecedented amounts of data could allow the construction of a computing model to predict the future, and even earlier in the opinion piece of Chris Anderson in *Wired* magazine [1] which envisioned a future of atheoretical, automatic science.

Our objective in [23] was less concerned with stating that such a paradigm has gained ground or why this has happened than in uncovering its structure, and the assumptions it depends upon. Indeed, physicists often work and approach their science from a very different point of view. Regardless of how complex nature may be, there is a belief that a fundamental explanation can be found and that a theoretical, explicative model can be reached. So, for example, Newtonian dynamics is considered a fundamentally successful description of the universe and an archetype of the way science should look irrespective of the fact that within its framework a solution of a basic question such as the three-body problem is, effectively, not computable. And yet these shortcomings have not led physicists to abandon Newtonian dynamics: within nonrelativistic velocities and as long as the bodies we study are not too small, Newtonian mechanics is still (rightly) considered a perfectly good model of reality—indeed, one of the most successful and enduring.

If it is not the lack of computability (as in the three-body problem) or the lack of precision (as happens for the relativistic effects that Newtonian dynamics fails to account for or predict), then what does require the agnostic approach embodied in the microarray paradigm? We suggest that such an approach is required when the phenomena we want to deal with are historical in a sense that needs some explanation: at least in an intuitive sense, we call "historical" those phenomena whose development can only be constrained locally (in time and/or space) by (potentially multiple) optimization processes acting on subsets of variables and in such a way that the functions to be optimized change over long periods of time. It appears that many of the sciences which have been less receptive to the classical process of mathematization concern this sort of phenomena. Biology, economics, the social sciences in general—all are examples of disciplines that study phenomena whose development seems to be historical in the sense suggested above. Our second broad methodological motif is thus the preeminence of historical phenomena in contemporary science.

Fitness landscapes, introduced by Sewall Wright in [34], best exemplify and motivate the definition of historical phenomena: the evolution along such landscapes is partially shaped by local optimization constraints. But, crucially, these constraints, change their nature dramatically over time so that such evolution is not likely to be fully described by any single global optimization process. The selective pressure due to varying environmental conditions on the genotypes of a given population is an example where a local search for an optimal phenotype is subject to constantly varying constraints. Recalling that alternate versions of the same gene are called "alleles", the basic idea of fitness landscapes is to represent a population as a point in a high-dimensional space determined by its average allele frequencies and to represent the average fitness of the population by the value of a corresponding function. The graph of the function generates a multidimensional surface (which is just the fitness landscape), and the potential evolution of the population is described by the local maximization of the fitness on the graph. Note that the landscape itself will slowly change when the evolution of several competing populations (not necessarily from the same species) is considered at once, since the fitness of one of them will affect the environment of the other (see [12] for a recent overview of these issues).

Coming back to the general definition of historical phenomena, we can assume that the time scale at which there is a switching from one local optimization process to another in the development of such phenomena is much longer than the time scale of the optimization processes themselves. Again, fitness landscapes could provide a motivation for this assumption, since the environmental conditions and the shape of the landscape change slowly with respect to the change of the position of the individual genotype points ([12, page 1613]). Note also that not all variables are likely to be subject to selective local optimization at the same time.

Now it is possible—indeed it happens often—that specific questions about a historical phenomenon can be reduced to simple models that require knowledge of only a few key measured quantities. But the models intended to answer each individual question are not only usually data driven but often unable to answer other relevant questions on the same phenomenon. And the methods exemplifying the microarray paradigm that we mentioned earlier certainly lead to temporary models that have this characteristic. This inability to gain a global understanding is not surprising if we think about the randomness and relative independence of the selective optimization processes that shape individual characteristics of the current state of a historical phenomenon.

It may be that a full, structured understanding that potentially leads to answers for a wide enough class of questions about a typical historical phenomenon would require knowledge of its entire history or the development of a proxy, a data-driven model that is likely as complex, and opaque, as the phenomenon itself, a perspective reminiscent of incompressibility ideas in Kolmogorov complexity [21] but in the context of a whole class of suitable questions. The methodological danger is that the flood of data generated by our innumerable measuring devices may convince us that data is enough; that there is nothing beyond the microarray paradigm; and that opaque, enormous, data-driven models are the privileged way to approach phenomena, even though they become so similar to the famous map of Borges [4], which was useless, since it was as big as the geography it was supposed to describe.

So the problem arises of how we can gain meaningful understanding of historical phenomena given the tremendous potential variability of their developmental processes. Indeed, several techniques in mathematical modeling already show the usefulness of a partial historical modeling of key variables, evolving in time, describing a phenomenon; we think here of nonlinear time series analysis methods [17], where modeling of random and/or nonlinear processes includes knowledge of its past states at several time points, or of forecasting methods where ensembles of initial conditions are used to best predict the future state of complex systems, such as the ensemble Kalman filter [16], [9], or the even less structured particle filters method [19]. All these methods show a distinct awareness of the relevance of modeling incidental, historical developmental processes. And we should not forget that many successful heuristic methods for optimization are directly inspired by the idea of fitness landscapes. One example is the field of evolutionary algorithms [8], modeling optimization of a function as a form of reproduction of solutions, which allows tentative solutions to the optimization problem to generate new ones, through mutation and genetic crossover, therefore allowing unexpected changes to their fitness, seen as the evaluation of the function to be optimized at the tentative solution. Another example is particle swarm optimization [18], where the search for optimal solutions to a problem is seen as a collective process in which individual tentative solutions are changed in time not only by their tendency to improve their current fitness (in the fitness landscape generated by the function to be optimized) but also by their tendency to retain close contact with the best-known tentative solution.

Still, all these methods work more at the operational level, where model classes have already been chosen to solve a certain problem. They do not constitute, taken individually, a coherent, conceptual shift in the way to approach historical phenomena and their complexity. We can try to imagine more radical ways in which such complexity can be tackled by looking at existing attempts within biology to conceptualize rules of development. In [22], for example, a general "principle of biological inertia" is introduced to give a broad conceptual basis to the multiform expression of default dynamics in developmental biology. Roughly stated, the principle asserts that without external disturbances or internal (genetic) control, there is a "local self-perpetuation of cell-level dynamics" ([22, page 119]). Among the many embodiments of this principle in the context of embryo development, biological inertia would correspond to stating that embryos have a tendency to reproduce spatially the same basic structure indefinitely. This embodiment of biological inertia could be seen in the context of historical processes as due to local optimization functions that try to maximize replicates, or spatial distribution, of a basic template. However, in a real system we cannot expect this inertial behavior of developmental systems to be indefinitely exact ([22, page 123]), therefore the local optimization process will break down in the timeframe of full development, and a complex, inhomogeneous organism may arise.

Note the subtle and unconventional use of ideas from physics where only analogy is at work: the principle of biological inertia parallels the principle of inertia in mechanics but only at the very general level of establishing the equivalent for biological systems of a state of rest or dynamical invariance. At the same time, what is essential in the idea of physical inertia is retained, and its usefulness is seen in the power of conceptualizing the developmental process itself rather than in the ability to identify and predict the final outcome of the process. This logically rigorous and yet informal use of ideas from mathematics and physics to define principles that partially govern biological processes might become the standard for a proper structuring of historical sciences exactly because individual historical phenomena do not seem amenable to a compact explanation of their structure and only the development that led to their current state may be open to meaningful theorization. This is a shift advocated by our third methodological motif, the conceptualization of developmental processes.

Indeed, the theory of evolution could be seen as the archetypical example of this motif. Such theory, in itself, is not mathematical and does not allow quantitative predictions, but it has its internal logical structure and provides a conceptual scaffolding that has inspired biology and specific mathematical models since its inception. As an aside, we believe that our point of view is in line with the recent commentary by Wilson [33], who suggested that profound ideas in science need not be mathematically profound; in this perspective, mathematization is an overflow of the richness and potential of an idea, not a condition of its power.

What is crucial for historical sciences is that the conceptualization of a developmental process may not even lead to a structural, full understanding of the state of the resulting phenomenon. The phenomenon may forever remain hidden to our understanding, and the microarray paradigm will stand as the only way to find quantitative answers to most problems we will pose about it. Conceptualization at best can provide the ground, the language, on which to develop data-driven, agnostic methods to solve problems. This acknowledgment does not hinder, however, the tremendous potential of finding ways to use mathematical structures to solve problems about historical phenomena.

To reflect on this potential, it may be useful to change our focus. Up to now our discourse moved from the microarray paradigm to an attempt to characterize the types of phenomena that most likely will require its application. But we could also reverse this viewpoint, taking for granted the applicability of the microarray paradigm (i.e., that we have enough data) and trying to understand, operationally, how to apply it.

One way to do that is of course to give free rein to statistical learning, data-driven techniques. But here we want to discuss a more general organizational motif: the principle of *forcing*,[1] which we introduced in [23]. In that paper we suggested that several disparate techniques developed to apply sophisticated mathematics to empirical problems can be brought together under a common methodological viewpoint: the idea of forcing mathematical ideas and methods on the data. More precisely, by forcing we mean the application to a problem of powerful techniques—such as multiscale methods as applied to image processing and numerical solutions of differential equations [5], [6], continuity and functional data analysis as applied to regularization and statistical analysis [26], [27], or topological graph analysis as applied to classification of molecular structures [3]. This is not on the basis of a previous evaluation that these techniques fit with the relevant phenomenon because of its specific nature but rather on the basis of a priori confidence in the power and flexibility of these techniques [23] and using the large amount of available measurements to adapt the technique to the phenomenon. This is an approach that contrasts with the usual idealization process in modeling, where there is a progressive stripping away of details from the phenomenon to reach a simplified image of the same, which is eventually amenable to analytical treatment by a mathematical technique (whose usefulness is often suggested by the idealization process itself).

The example of functional data analysis is perhaps the simplest instance of forcing and the most telling: a phenomenon may be discrete, and yet, if there are enough data (microarray paradigm), we can force regularization to be able to treat the data as if they were continuous or even smooth and therefore access the full machinery of analysis. A more recent example of forcing, along the same general trust of functional data analysis, is diffusion geometry [7], [29]. The organizational principle of this theory can be seen, at a very general level, as the belief that, regardless of whether the data available in the empirical problem are categorical and/or discrete, it is useful to define a notion of geometrical manifold associated to the data, because in this way the whole apparatus of functional analysis on manifolds can be adapted

[1] *We named this principle "forcing" in* [23] *because this term is strongly evocative of the general methodological approaches it pertains to. It has, however, no significant relation to the method introduced by Paul Cohen, now of common use in set theory, that goes by the same name.*

and used to solve the problem itself. For example, the review [7] describes how scientific journals can be seen as points on a low-dimensional manifold, built by first associating each article to a vector of the frequencies of preselected words in the article itself and then by projecting the cloud of points associated to the whole set of articles onto the directions of maximal variance. We refer to [23], [7] and to the relevant primary literature [29] for more details.

To be sure, forcing may be viewed as a coarse and willful attempt to use mathematics for a specific purpose, but are there limits to the applicability of forcing? Is it plausible to think that, for sufficiently large and diverse data sets, any mathematical structure can be forced on them in a computationally efficient way to solve problems?

While we do not have answers to these questions, the weakening of the relation between individual models and phenomena should make us think differently about the role of mathematics in science. We suggested above that historical phenomena may not be amenable to uniform, structured reduction to simple models and that in these cases a data analysis approach to each problem instance is most likely the best that can be expected when approaching them, whether by forcing or by standard statistical learning and classification techniques. And these data-driven applications of mathematics lead us to an intriguing version of the famous question of Wigner about the unreasonable effectiveness of mathematics: how can classification methods, which are essentially function fitting on data, be so successful at predicting phenomena?

Our suggestion is that the four methodological motifs we've highlighted may give us a context for asking such questions and for a comprehensive reflection on the ways modern data science is so effective at problem solving. We are also encouraged by the interdependence of the motifs: the notion of historical phenomena and the principle of forcing seem pertinent only in the context of large data sets and therefore are deeply dependent on the microarray paradigm And defining historical phenomena naturally shifts the emphasis from phenomena's states to the developmental processes that generate them.

Surely science may have become agnostic, and data analysis methods are often incapable of providing understanding of phenomena—they only give answers to our problems. But this does not imply that its methods should not be amenable to understanding. On the contrary, it is exactly the absence of understanding of phenomena that brings urgency to a widely shared methodological and epistemological reflection of the scientific community. Mathematics may not necessarily work for historical sciences the way it did for physics, but that does not mean that it has to reduce itself to blind computations, and principles such as biological inertia show that it is possible to gain deep insight into the rules of this historical development.

Therefore this is, in the end, an invitation to mathematicians to approach biology and other historical sciences on their own terms, a process that frustrates superficial knowledge of each field and challenges us, if we want to be relevant, not so much to be interdisciplinary as to be scientifically bilingual. We may discover that what is essential in a field and the true linchpin of its conceptualization is often very different from what we deem profound or interesting in our own mathematical disciplines.

Acknowledgments

We thank the three anonymous referees—a mathematician, a philosopher, and a biologist—for their thoughtful and very helpful comments.

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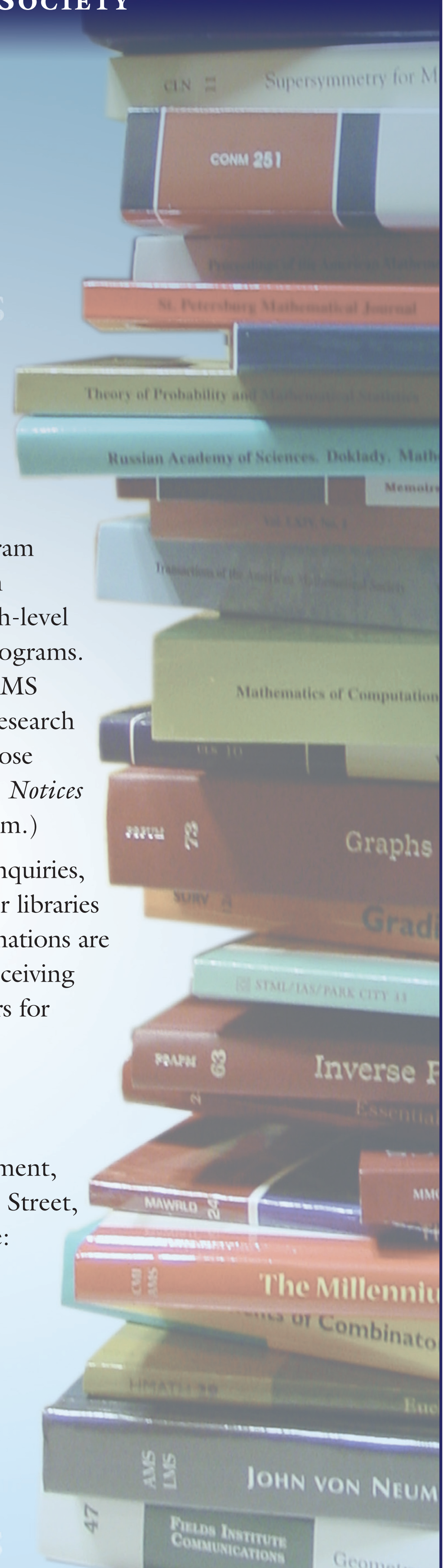

WHAT IS...

a Spectrahedron?

Cynthia Vinzant

A spectrahedron is a convex set that appears in a range of applications. Introduced in [3], the name joins “spectra”, evoking the eigenvalues of a matrix, with “hedron”, suggesting that spectrahedra generalize convex polyhedra.

First we need to recall some linear algebra. All the eigenvalues of a real symmetric matrix are real, and if these eigenvalues are all nonnegative then the matrix is *positive semidefinite*. The set of positive semidefinite matrices is a convex cone in the vector space of real symmetric matrices.

A *spectrahedron* is the intersection of an affine linear space with this convex cone of matrices. An n-dimensional affine linear space of real symmetric matrices can be parameterized by

$$A(x) = A_0 + x_1 A_1 + \cdots + x_n A_n$$

as $x = (x_1, \ldots, x_n)$ ranges over $\mathbb{R}^n$, where $A_0, \ldots, A_n$ are real symmetric matrices. This identifies our spectrahedron with the set of x in $\mathbb{R}^n$ for which the matrix $A(x)$ is positive semidefinite. This condition, denoted $A(x) \succeq 0$, is commonly known as a *linear matrix inequality*.

For example, we can write the cylinder

$$\{(x, y, z) \in \mathbb{R}^3 \ : \ x^2 + y^2 \leq 1, \ -1 \leq z \leq 1\}$$

as a spectrahedron. To do this, parameterize a 3-dimensional affine space of 4×4 matrices by

$$\begin{pmatrix} 1+x & y & 0 & 0 \\ y & 1-x & 0 & 0 \\ 0 & 0 & 1+z & 0 \\ 0 & 0 & 0 & 1-z \end{pmatrix}.$$

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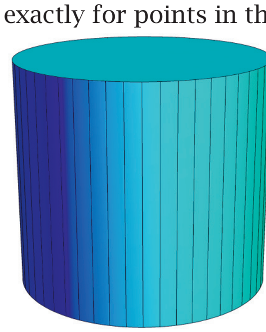
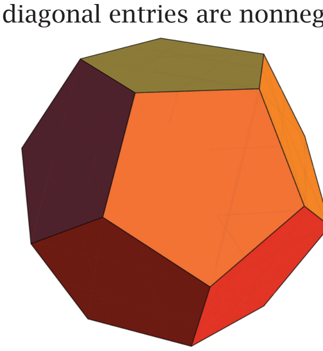
DOI: http://dx.doi.org/10.1090/noti1116

This matrix is clearly positive definite at the point $(x, y, z) = (0, 0, 0)$. In fact, it is positive semidefinite exactly for points in the cylinder.



This matrix has rank four at points in the interior of the cylinder, rank three at most points on the boundary, and rank two at points on the two circles on the top and bottom. Here we start to see the connection between the geometry of spectrahedra and rank. The boundary is “more pointy” at matrices of lower rank.

Another example is a *polyhedron*, which is the intersection of the nonnegative orthant with an affine linear space. Any polyhedron is a spectrahedron parameterized by diagonal matrices since a diagonal matrix is positive semidefinite exactly when the diagonal entries are nonnegative.



Like polyhedra, spectrahedra have faces cut out by tangent hyperplanes, but they may have infinitely many. For example, one can imagine rolling a cylinder on the floor along the 1-dimensional family of its edges.

This brings us to an important motivation for studying spectrahedra: optimization. The problem of maximizing a linear function over a polyhedron is a *linear program*. Generalizing polyhedra to spectrahedra leads to *semidefinite programming*, the problem of maximizing a linear function over a spectrahedron. Semidefinite programming problems can be solved numerically in polynomial time using *interior-point methods* and form a broad and powerful tool in optimization.

Angles, Statistics, and Graphs

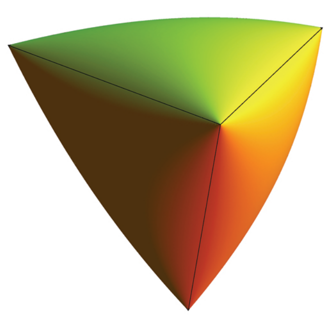
Semidefinite programming has been used to relax many "hard" optimization problems, allowing one to find a bound on the true solution. This approach has been most successful in cases where the geometry of the underlying spectrahedron reveals the bounds to be close to the true answer.

For a flavor of these applications, consider the spectrahedron (displayed below) of 3×3 matrices with 1's along the diagonal:

$$\left\{ (x,y,z) \in \mathbb{R}^3 : \begin{pmatrix} 1 & x & y \\ x & 1 & z \\ y & z & 1 \end{pmatrix} \text{ is positive semidefinite} \right\}.$$

This spectrahedron consists of points $(x, y, z) = (\cos(\alpha), \cos(\beta), \cos(\gamma))$ where α, β, γ are the pairwise angles between three length-one vectors in $\mathbb{R}^3$. To see this, note that we can factor any positive semidefinite matrix A as a real matrix times its transpose, $A = VV^T$. The entries of A are then the inner products of the row vectors of V.

The four rank-one matrices on this spectrahedron occur exactly when these row vectors lie on a common line. They correspond to the four ways of partitioning the three vectors into two sets.



This *elliptope* appears in statistics as the set of correlation matrices and in the remarkable Goemans-Williamson semidefinite relaxation for finding the maximal cut of a graph (see [2]).

This spectrahedron sticks out at its rank-one matrices, meaning that a random linear function often (but not always) achieves its maximum at one of these points. This is good news for the many applications that favor low-rank matrices.

Sums of Squares and Moments

Another important application of semidefinite programming is to polynomial optimization [1, Chapter 3]. For example, one can bound (from below) the global minimum of a multivariate polynomial $p(x)$ by the maximum value of λ in $\mathbb{R}$ such that the polynomial $p(x) - \lambda$ can be written as a *sum of squares* of real polynomials. (Sums of squares are guaranteed to be globally nonnegative!) The expressions of a polynomial as a sum of squares form a spectrahedron, and finding this λ is a semidefinite programming problem.

For example, take the univariate polynomial $p(t) = t^4 + t^2 + 1$. For any choice of the parameter a in $\mathbb{R}$ we can write our polynomial as

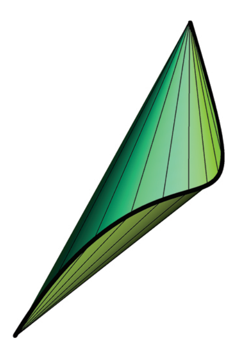
$$p(t) = \begin{pmatrix} 1 & t & t^2 \end{pmatrix} \begin{pmatrix} 1 & 0 & a \\ 0 & 1-2a & 0 \\ a & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ t \\ t^2 \end{pmatrix}.$$

When this 3×3 matrix is positive semidefinite, it gives a representation of $p(t)$ as a sum of squares. Indeed, if it has rank r, we can write it as a sum of r rank-one matrices $\sum_{i=1}^r v_i v_i^T$. Multiplying both sides by $(1, t, t^2)$, we then write $p(t)$ as the sum of squares $\sum_{i=1}^r ((1, t, t^2) \cdot v_i)^2$.

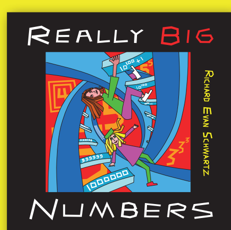
Here the spectrahedron is a line segment parameterized by $a \in [-1, 1/2]$. Its two rank-two end points correspond to the two representations of $p(t)$ as a sum of two squares:

$$(t^2 - 1)^2 + (\sqrt{3}t)^2 \text{ and } (t^2 + 1/2)^2 + (\sqrt{3}/2)^2.$$

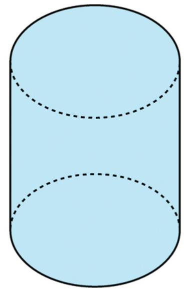
This idea extends to relaxations for optimization of a multivariate polynomial over any set defined by polynomial equalities and inequalities.



Dual to this theory is the study of *moments*, which come with their own spectrahedra. The convex hull of the curve $\{(t, t^2, t^3) : t \in [-1, 1]\}$ (a spectrahedron) is an example shown above.



A Nonexample



To finish, let us return to the question of what a spectrahedron is, giving a nonexample. Projecting our original cylinder onto the plane $x + 2z = 0$ results in the convex hull of two ellipses. This convex set is not a spectrahedron! A matrix is positive semidefinite exactly when its diagonal minors are nonnegative. Hence any spectrahedron is cut out by finitely many polynomial inequalities. However, the projection cannot be written this way. This shows that, unlike polyhedra, the class of spectrahedra is not closed under taking projections.

Spectrahedral Conclusions

The study of spectrahedra brings together optimization, convexity, real algebraic geometry, statistics, and combinatorics, among other areas. There are effective computer programs like `cvx` and `YALMIP` (both for `MATLAB`) that work with spectrahedra and solve semidefinite programming problems.

Spectrahedra are beautiful convex bodies and fundamental objects in optimization and matrix theory. By understanding the geometry of spectrahedra, we can fully explore the potential of semidefinite programming and its applications.

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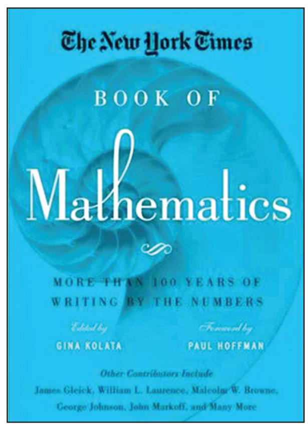
Book Review

The New York Times Book of Mathematics

Reviewed by Ian Stewart

The New York Times Book of Mathematics
Edited by Gina Kolata
Sterling, 2013
US$24.95, 496 pages
ISBN-13: 978-1402793226



Mathematicians often complain that their subject is neglected in the mass media, although when it does get reported there always seem to be a few of us who find it impossible to resist the urge to complain bitterly about inaccuracies and "hype", which usually seems to mean the promotion of the area concerned instead of their own. As this sumptuous volume demonstrates, neither complaint can sensibly be directed at the *New York Times* [*NYT*]. For more than a century this high-quality newspaper has done sterling work in the service of our profession, the public, and the cause of science journalism.

This collection contains more than a hundred *NYT* articles on mathematics and its applications of all kinds, dating back to 1892 and written by over thirty authors—some journalists, some mathematicians, some neither. Together they provide an unusual overview of our subject and how it is perceived by everyone else. It is an eclectic collection, ranging from an anonymous piece on life insurance at the end of the nineteenth century to Grigori Perelman's proof of the Poincaré Conjecture. The editor has resisted the temptation to place the articles in chronological order, grouping them into seven major areas and then following an order that helps to tell a story—which is, after all, what journalism is about.

This choice makes the book far more accessible to the general reader, though I'm tempted to photocopy the contents list and shuffle the articles back into the order of publication. Reading them in that order would give a dramatic perception of how our subject has changed over the last 120 years; however, this comes over anyway. Paul Hoffman's foreword and Gina Kolata's introduction provide concise and helpful overviews of such questions, and the first group of topics addresses general issues of what mathematics is and what it's for. That said, this is clearly a book that you can dip into fairly randomly, and I'm going to spend much of this review doing just that. Along the way, I'll say a little about how the selected articles illuminate broader "philosophical" issues—just as the whim takes me.

Roughly speaking, the seven sections deal with the nature of mathematics, chance, famous problems, chaos, cryptography, computers, and biography. Immediately we get a glimpse of which areas of mathematics are deemed newsworthy or, perhaps, just appealed to the editor. Large regions of the mathematical landscape are untouched—only passing reference to Fourier transforms (though the use of wavelets for image compression is mentioned), nothing on symmetry except the E8 "size of Manhattan" calculation. The only mathematical

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DOI: http://dx.doi.org/10.1090/noti1117

biology is a short piece on the flocking of birds from 1987. I'm not complaining about what's not been included; just posting a mild warning notice for potential readers. What *has* been included is worthy, valuable, and fascinating, fully justifying the book's existence.

I'll jump in with the third section, on famous problems, solved and unsolved. This covers what by now is familiar ground to most mathematicians: the Poincaré conjecture, the four-color problem, the Goldbach conjecture, Fermat's Last Theorem [FLT], and a flurry of less celebrated but still important questions such as optimal packing of tetrahedra and Steiner trees. In its treatment of Fermat's Last Theorem, the *NYT* offered its readers glimpses of mathematics in the making. "Mathematics expert may soon resolve a 350-year problem…" You'd be justified in assuming this was advance notice of Andrew Wiles's work, but actually the piece is about an abortive attempt by Yoichi Miyaoka in 1988. It reminds us that mathematical research does not consist of triumphs alone; it involves many unsuccessful attempts. The article is suitably cautious: "Nobody has the right to have an opinion yet." The subsequent article, "Fermat's theorem solved? Not this time" provides the dénouement.

A month later, the *NYT* is reporting what experts in number theory had been quietly saying for about a decade: the road to a solution might well be opened up by the theory of elliptic curves. Five years later "At last, shout of Eureka!" told the world of Wiles's Cambridge announcement, confirming that belief. This was followed up by a longer description of what Wiles did—"you use Hilbert irreducibility and the Čebotarev density theorem … to produce a noncuspidal rational point"—well done, James Gleick! Handled so deftly that he gets away with it, while readers gain a subliminal insight that this stuff is difficult and advanced, yet follows some kind of coherent mathematical narrative. Except… "Flaw is found in math proof, but repairs are under way." Then "Fermat's puzzle is still not quite Q.E.D." And finally, "How a gap in the Fermat proof was bridged."

This is what real research is like. A few steps forward, a step or two back. With luck and a following wind, you may actually reach your destination. But should any of this have been reported? Is it "premature" to inform the press of major new developments as they are happening, and before the community has peer-reviewed them? Well, Wiles kept everything secret until he was confident he was right, and his announcement was low-key: nothing terribly wrong there. But, gosh, well… there was a gap. These things happen. Wiles was lucky, Miyaoka not. The plain fact is that, even in the early 1990s, there is no way that the media would *not* have got wind of a possible solution of FLT. The best we can do, as a profession, is to help them tell the story accurately as it unfolds. In every other field of human activity, this kind of thing happens on a daily basis. I've lost count of how many times a cosmologist has announced that dark matter has been detected definitively, only for another one to shoot the whole thing down in flames.

If keeping the lid on Miyaoka's ideas and Wiles's discovery was difficult two decades ago, it's totally impossible in this age of Twitter and Facebook. We may privately wish the world were otherwise, with discoveries being made public only when they have become definitive—but it's not. Indeed, in most areas outside mathematics something can be definitive for decades and then turn out to be totally wrong. The payoff from this kind of warts-and-all disclosure is almost entirely positive: the public experiences the excitement of serious new mathematics, and sees *new things happening* as they occur. They discover that we are actually *doing things*. It may be "unfortunate" when sometimes the excitement fizzles out, but a week later the only people fretting about it are a few anally retentive specialists. Get a life.

James Gleick features prominently in the book, because he writes so much and so well about our subject, so it is hardly surprising that chaos and related topics get a section to themselves. The area is, actually, rather important. Remarkably, the *NYT* spotted this coming in 1938—in a way. It reported Norbert Wiener's work on Brownian motion, revealing statistical regularities in random movements. But it also mentioned George Birkhoff's approach to chaotic events, which later led Stephen Smale to the horseshoe, a key step in understanding that deterministic chaotic dynamics has its own patterns. This section is an eclectic mix of various topics from nonlinear dynamics—chaos, catastrophes, complexity, stochastics—ending with an obituary of Benoît Mandelbrot and a summary of the development of fractals.

Much of this area was a cause of controversy when the media got hold of it; some still is. "Here is a mathematician's nightmare that I heard in the 1980s," Gleick writes. "[A] mathematician dreamed that Mandelbrot died and God spoke: 'You know, there really was something to that Mandelbrot.'" Today's mathematicians mostly think that God had a point, though a few reckon we are still living the nightmare. Perhaps if they read *Science* and *Nature* regularly they would realize how thoroughly fractals now pervade numerous areas of science.

A third major theme, one that future historians of mathematics will have to come to grips with, is the remarkable interplay between mathematics and computing. Once again the *NYT* is on the case early; once again, the story is not what we might expect. In 1927 it carried a short article on Vannevar Bush's "thinking machine"—more properly, the "product integraph", an analog device for solving differential equations. By 1947 we are hearing of two "giant electronic brains" being established by the National Bureau of Standards, successors to ENIAC and similar beasts. "A third computing machine might be built in the near future to speed up the calculations of the Census Bureau." In the same year, ENIAC is being "converted so that it can handle without resetting all types of mathematical problems… Seventeen per cent of the machine's time is now lost changing the set-up by resetting switches and pulling plugs…the latest change-over [is based on] a new mathematical approach by Dr. John von Neumann."

A huge piece from 1967 describes switching circuits, Boolean algebra, information theory, and the latest high-speed memory device: ferrite cores. By 1997 the story has moved on to Intel's supercomputer with 9,072 Pentium processors. The last item in this section features Vinay Deolalikar's attempt to prove that P does not equal NP—which is left hanging at its unresolved state in 2010. On May 2, 2013, a *New Yorker* blog carried a posting called "A most profound math problem" [`http://www.newyorker.com/online/blogs/elements/2013/05/a-most-profound-math-problem.html`], which told us how "Computer scientists and mathematicians went at Deolalikar's proof with the ferocity of sharks in the presence of blood." That is perhaps a trifle unfair, since there were good reasons all along to be suspicious of the method, even though much of the work was of high quality. At any rate "It wasn't long before Deolalikar's paper was thoroughly discredited." The blog concludes with these wise words: "all of mathematics rests on a fundamental hubris, a belief that we can order what Wallace Stevens calls 'a slovenly wilderness'. It is a necessary confidence, yet we are not always rewarded for it."

Mathematics is not just about theorems and proofs, nor even about methods and uses: it is driven by *people*. One of the less desirable features of newsworthiness is that the most interesting people tend to range from mild eccentrics to the seriously deranged. I doubt I could convince the *NYT* to carry a story about a happily (and long-) married father of two who drives to the office five days a week, does competent but not outstanding research into specialist areas of nonlinear dynamics, has been based at the same university for his entire career, and whose main hobby is reading books. I might *just* get them to feature one about a mathematician who traded his undergraduate lecturing for popular science and broadcasting, writes science fiction, has collaborated with Terry Pratchett (if you've not heard of this guy, he writes humorous fantasy bestsellers, set on a flat world supported by giant elephants on the back of a space-faring turtle, where magic works), appeared in *Nature* wearing a wizard's outfit, and once brought a live tiger into a lecture room full of children.

You'll guess that both of them are the same person—me. Even the second one pales into insignificance compared to the remarkable mathematicians that actually *have* been featured in the *NYT*. Included here: Paul Erdős, Terry Tao, Grigori Perelman, John Horton Conway, Claude Shannon, Srinivasa Ramanujan, Kurt Gödel, Andrew Wiles, and Leonard Adleman—all brilliant and thoroughly deserving of public recognition. I'm not sure exactly what impression nonmathematical readers will get from these articles, and I'm absolutely sure that it won't be the father of two who drives to the office five days a week, but I'm also sure that it will be very interesting indeed. And, in the end, that's what media exposure is for. Not to tell the full story in every detail and complete precision, but rather to show the world that we are committed, innovative, useful—and real people. Thank, you, *New York Times*, for having done just that for more than a hundred years. Thank you, Ms. Kolata, for collecting so much gripping and informative material together and arranging it so well.

Please keep it up.

AMERICAN MATHEMATICAL SOCIETY

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1. aim to bring more persons from underrepresented backgrounds into some portion of the pipeline beginning at the undergraduate level and leading to advanced degrees in mathematics and professional success, or retain them once in the pipeline;
2. have achieved documentable success in doing so; and
3. are replicable models.

Preference will be given to programs with signifigant participation by underrepresented minorities.

One or two programs are highlighted annually.

Nomination process: Letters of nomination may be submitted by one or more individuals. Nomination of the writer's own institution is permitted. The letter should describe the specific program(s) for which the department is being nominated as well as the achievements that make the program(s) an outstanding success, and may include any ancillary documents which support the success of the program(s). The letter of nomination should not exceed two pages, with supporting documentation not to exceed three more pages. Up to three supporting letters may be included in addition to these five pages.

Send nominations to:
Programs That Make a Difference
c/o Associate Executive Director, Meetings and Professional Services
American Mathematical Society
201 Charles Street
Providence, RI 02904
or via email to AED-MPS@ams.org

Recent Winners:

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Rice University Summer Institute of Statistics.

2013: Nebraska Conference for Undergraduate Women in Mathematics (NCUWM).

2012: Mathematical Sciences Research Institute.

2011: Center for Women in Mathematics, Smith College;
Department of Mathematics, North Carolina State University.

2010: Department of Compuational and Applied Mathematics (CAAM), Rice University;
Summer Program in Quantitative Sciences, Harvard School of Public Health.

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Sarnak Awarded 2014 Wolf Prize

The 2014 Wolf Prize in Mathematics has been awarded to Peter Sarnak of Princeton University and the Institute for Advanced Study "for his deep contributions to analysis, number theory, geometry, and combinatorics."

Description of the Prizewinner's Work

Following is the citation from the Wolf Foundation.

Peter Sarnak is a mathematician covering an extremely broad spectrum with a far-reaching vision. He has influenced the development of several mathematical fields, often by uncovering deep and unsuspected connections. In analysis, he investigated eigenfunctions of quantum mechanical Hamiltonians which correspond to chaotic classical dynamical systems in a series of fundamental papers. He formulated and supported the quantum unique ergodicity conjecture, asserting that all eigenfunctions of the Laplacian on negatively curved manifolds are uniformly distributed in phase space. Sarnak's introduction of tools from number theory into this domain allowed him to obtain results which had seemed out of reach and paved the way for much further progress, in particular the recent works of E. Lindenstrauss and N. Anantharaman. In his work on *L*-functions (jointly with Z. Rudnick) the relationship of contemporary research on automorphic forms to random matrix theory and the Riemann hypothesis is brought to a new level by the computation of higher correlation functions of the Riemann zeros. This is a major step forward in the exploration of the link between random matrix theory and the statistical properties of zeros of the Riemann zeta function going back to H. Montgomery and A. Odlyzko. In 1999 it culminated in the fundamental work, jointly with N. Katz, on the statistical properties of low-lying zeros of families of *L*-functions. Sarnak's work (with A. Lubotzky and R. Philips) on Ramanujan graphs had a huge impact on combinatorics and computer science. Here again he used deep results in number theory to make surprising and important advances in another discipline.

By his insights and his readiness to share ideas he has inspired the work of students and fellow researchers in many areas of mathematics.

Biographical Sketch

Peter Sarnak was born in Johannesburg, South Africa, in 1953. He received his Ph.D. from Stanford University in 1980 under the direction of Paul Cohen. He has held positions at the Courant Institute of Mathematical Sciences, New York University (1980–1983 and 2001–2005); Stanford University (1984–1991); Princeton University (1991–1999 and 2002–present); and the Institute for Advanced Study (member, 1999–2002, 2005–2007; professor, 2007–present). He is currently the Eugene Higgins Professor of Mathematics at Princeton University, as well as a professor at the Institute for Advanced Study in Princeton.

Courtesy of, and with the express consent of, the IAS. Photo by Cliff Moore.

Peter Sarnak

Sarnak held an Alfred P. Sloan Foundation Fellowship from 1983 to 1985. He was chosen as a Presidential Young Investigator (1985–1990). He was awarded the Polya Prize of the Society of Industrial and Applied Mathematics (SIAM) in 1998 and the Ostrowski Prize in 2001. In 2003, Sarnak and his coauthor Nicholas Katz received the AMS Conant Prize for their article "Zeroes of zeta functions and symmetry", which appeared in the *Bulletin of the AMS* **36** (1999), 1–26. Sarnak also received the AMS Cole Prize in Number Theory in 2005 for his fundamental contributions to number theory and, in particular, his book *Random Matrices, Frobenius Eigenvalues and Monodromy*, written jointly with Katz. He was awarded the Lester Ford Prize of the Mathematical Association of America (MAA) in 2012. He has been elected to the American Academy of Arts and Sciences (1991), the National Academy of Sciences (2002), and the American Philosophical Society (2008). He was elected a fellow of the Royal Society of London in 2002.

About the Prize

The Wolf Prize carries a cash award of US$100,000. The science prizes are given annually in the areas of agriculture, chemistry, mathematics, medicine, and physics. Laureates receive their awards from the president of the State of Israel in a special ceremony at the Knesset building (Israel's parliament) in Jerusalem. The list of previous recipients of the Wolf Prize in Mathematics is available on the website of the Wolf Foundation, `http://www.wolffund.org.il`.

—*Elaine Kehoe*

DOI: http://dx.doi.org/10.1090/noti1122

Faltings Awarded Faisal Prize

Gerd Faltings

Gerd Faltings of the Max-Planck Institute and the University of Bonn, has been awarded the King Faisal Prize for Science for 2014 by the Faisal Foundation. Mathematics was the field selected for the award this year.

According to the prize citations, Faltings "has made groundbreaking contributions to algebraic geometry and number theory. His work combines ingenuity, vision, and technical power. He has introduced stunning new tools and techniques which are now constantly used in modern mathematics. His deep insights into the p-adic cohomology of algebraic varieties have been crucial to modern developments in number theory. His work on moduli spaces of abelian varieties has had great influence on arithmetic algebraic geometry. He has introduced new geometric ideas and techniques in the theory of Diophantine approximation, leading to his proof of Lang's conjecture on rational points of abelian varieties and to a far-reaching generalization of the subspace theorem. Faltings has also made important contributions to the theory of vector bundles on algebraic curves with his proof of the Verlinde formula."

Gerd Faltings was born in Gelsenkirchen, Germany, in 1954. He studied mathematics and physics at the University of Münster and received his Ph.D. in mathematics there in 1978, as well as the habilitation in mathematics in 1981. He has been an assistant professor at the University of Münster, a professor at the University of Wuppertal (1982–1984), and a professor at Princeton University (1985–1996). He is currently a director of the Max-Planck Institute for Mathematics in Bonn.

DOI: http://dx.doi.org/10.1090/noti1118

Faltings was awarded the Fields Medal in 1986, primarily for his proof of the Mordell conjecture. In 1922 Louis Mordell had conjectured that a system of algebraic equations with rational coefficients that defines an algebraic curve of genus greater than or equal to 2 (a surface with two or more "holes") has only a finite number of rational solutions that have no common factors. By proving this, Faltings showed that $xn + yn = zn$ could have only a finite number of solutions in integers for $n > 2$, which was a major breakthrough in proving Fermat's Last Theorem that this equation has no natural number solutions for $n > 2$.

Faltings was the recipient of a Guggenheim Fellowship for Natural Sciences, United States and Canada, in 1988. In 1996, he received the Gottfried Wilhelm Leibniz Prize of the Deutsche Forschungsgemeinschaft, the highest honor awarded in German research.

Faltings's publications include *Rational Points* (1984), *Degeneration of Abelian Varieties* (with Ching-Li Chai) (1990), and *Lectures on the Arithmetic Riemann-Roch Theorem* (1992).

The King Faisal International Prize is awarded to "scientists and scholars whose research results in significant advances in specific areas that benefit humanity." The prize consists of a certificate handwritten in Arabic calligraphy summarizing the laureate's work, a commemorative twenty-four carat gold medal uniquely cast for each prize, and a cash award of 750,000 Saudi riyal (approximately US$200,000). The science prize is given alternately in mathematics, chemistry, physics, and biology.

—Elaine Kehoe

Meeting the Challenges of Improved Post-Secondary Education in the Mathematical Sciences

David M. Bressoud, Eric M. Friedlander, and C. David Levermore

In 2012, the President's Council of Advisors on Science and Technology (PCAST), which advises President Barack Obama, issued a report, *Engage to Excel*, that raised many concerns about the teaching of the mathematical sciences in the first two years in our colleges and universities. On behalf of the Joint Policy Board for Mathematics (JPBM)—the umbrella organization of the American Mathematical Society (AMS), the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM)—David Bressoud, Eric Friedlander, and David Levermore, with support from the leadership of all four societies, crafted a response. After participation in many discussions within our professional societies, JPBM, and PCAST, we are guided in formulating the following statement. We are calling upon the entire mathematical sciences community to achieve much needed change in undergraduate education in the mathematical sciences. This statement is appearing in *AMS Notices*, *MAA Focus*, *SIAM News*, and *Amstat News*.

The mathematical sciences play a foundational and crosscutting role in enabling substantial advances across a broad array of fields: medicine, engineering, technology, biology, chemistry, computer science, social sciences, and others. The delivery of excellent post-secondary mathematics education is essential to the present and future wellbeing of our nation and its citizens.

Whereas research in the mathematical sciences is flourishing, with dramatic advances regularly occurring in core mathematics and in applications, mathematics education needs **immediate attention**. We focus on the needs of students in two-year colleges, four-year colleges, and universities. Mathematics education is a critical component of all undergraduate Science, Technology, Engineering, or Mathematics (STEM) degrees and plays a key role in educating the next generation of leaders in our increasingly technological, data-driven, and scientific society.

The President's Council of Advisors on Science and Technology (PCAST) presented many challenges to the mathematics community as it addressed the needs of post-secondary mathematics education in its 2012 report *Engage to Excel*.[1] Answering these challenges will require collaboration among all of the scientific disciplines that are working to prepare the STEM workforce of the future. We acknowledge many of the shortcomings highlighted by the report. The *wake-up call* delivered by this PCAST report has underscored the immediacy of the need for intensive, broad-scale efforts to address these problems. Whereas efforts by a great many in the mathematical sciences community predate PCAST's report, now more than ever we need a broad community-wide effort to implement innovation in all of our college and university educational programs.

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DOI: http://dx.doi.org/10.1090/noti1106

[1] `www.whitehouse.gov/sites/default/files/microsites/ostp/pcast-engage-to-excel-final_feb.pdf`

What are these challenges, why is this an especially critical time for the mathematical sciences community, what efforts are underway to meet them, and what do we ask of the mathematical community?

Among the challenges we face is the need to find new ways to educate students who are poorly prepared for post-secondary mathematics. This includes new teaching methodologies and technology, as well as changes in curricula at all levels. We must do more to adapt the mathematics we teach to the career needs of the students we teach. We must pursue cooperation ever more energetically with mathematics-intensive disciplines. For emphasis, we rephrase these challenges as explicit questions. How should mathematics educators improve developmental education in order to enable students to aspire to STEM careers? What methods of placement and advising best help students navigate through a STEM curriculum? How should mathematical scientists in colleges and universities augment their cooperative efforts with "partner disciplines" to best serve the needs of students needing basic university mathematics? How should mathematical sciences departments reshape their curricula to suit the needs of a well-educated workforce in the twenty-first century? How can technology be best used to serve educational needs?

We are at a critical juncture. Members of the academic mathematical sciences community should recognize that *change is coming rapidly* in their world. There is great pressure to reduce costs in order to relieve state budgets and student debt; this pressure will translate to "efficiencies" and new measures of effective teaching. Numerous agencies are identifying mathematics courses as a stumbling block for success in undergraduate programs leading to a STEM degree. Increasing numbers of students coming to colleges and universities seek STEM careers that require post-secondary mathematics, yet many of these are poorly prepared. There is much demand to make mathematics education directly relevant to STEM careers.

The mathematical sciences themselves are changing as the needs of big data and the challenges of modeling complex systems reveal the limits of traditional curricula. The National Research Council report *The Mathematical Sciences in 2025*[2] eloquently describes the opportunities and challenges of this shifting landscape. This well-received report can serve as one foundation for the change that is needed, providing a springboard for initiatives in mathematics education that more closely intertwine the learning of mathematics with the appreciation of its applications.

We mention a few of the national efforts underway to address these challenges. There are many, many efforts at individual institutions that we hope will be shaped into more coherent efforts as well. For two-year colleges, the *New Mathways Project*,[3] *Statway*,[4] and *Quantway*[5] programs are assisting under-prepared students. *Project NExT*[6] is now past its twentieth year of introducing new faculty to effective strategies for teaching. MAA's national study of Calculus has identified characteristics of successful programs.[7] *Modeling Across the Curriculum*[8] is working to embed computational learning and exposure to modeling and simulation in early STEM courses. CAUSE,[9] which grew out of an ASA initiative, provides resources, professional development, outreach, and research for the needs of modern undergraduate statistics education. At research universities, there is a new program of the Association of American Universities to implement more "evidence based" teaching practices and improve the quality of teaching and learning.[10] The INGenIoUS project[11] is a joint effort of AMS, ASA, MAA, and SIAM to develop strategies for future investments in training at the graduate and undergraduate levels. Carnegie Corporation of New York and the Sloan Foundation are supporting a broad-ranging initiative entitled *Transforming Post Secondary Education in Mathematics*.[12] These efforts are steps in the right direction, but much remains to be done.

We call upon all mathematical scientists in academia to renew their focus on post-secondary mathematics education. We challenge department chairs to incentivize innovation for the sake of their students and the health of our discipline. We encourage mathematics faculty to reach out to colleagues in mathematics-intensive disciplines in order to heighten the relevance of their courses to the careers of their students. And we urge departments as a whole to investigate with an open mind new teaching methodologies and technologies, keeping in mind the need to retain and motivate students.

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[2] www.nap.edu/catalog.php?record_id=15269

[3] www.utdanacenter.org/higher-education/new-mathways-project

[4] www.carnegiefoundation.org/statway

[5] www.carnegiefoundation.org/quantway

[6] archives.math.utk.edu/projnext/

[7] www.maa.org/cspcc

[8] connect.siam.org/siam-nsf-workshop-on-modeling-across-the-curriculum

[9] *Consortium for the Advancement of Undergraduate Statistics Education*, www.causeweb.org

[10] *AAU Undergraduate STEM Education Initiative*, www.aau.edu/policy/article.aspx?id=12588

[11] *Investing in the Next Generation through Innovative and Outstanding Strategies*, www.ingeniousmathstat.org

[12] tpsemath.org

The Chinese Hedgehog and the American Fox: An Invitation to Dialogue

Mark Saul

> The fox knows many things, but the hedgehog knows one big thing.
>
> –Archilochus

The title refers originally to a fragment by an obscure ancient Greek poet, made famous in an essay by Isaiah Berlin. In this essay, Berlin distinguishes two types of thinkers. Briefly, "hedgehogs" are people who concentrate their efforts on a single set of ideas, while "foxes" are those who bring a variety of ideas, from widely different areas, to bear on their work.

In Berlin's own words:

> …the words can be made to yield a sense in which they mark one of the deepest differences which divide writers and thinkers, and, it may be, human beings in general. For there exists a great chasm between those, on one side, who relate everything to a single central vision, one system, less or more coherent, or articulate, in terms of which they understand, think and feel—a single, universal, organising principle in terms of which alone all that they are and say has significance—and, on the other side, those who pursue many ends, often unrelated and even contradictory, connected, if at all, only in some *de facto* way, for some psychological or physiological cause, related by no moral or aesthetic principle. These last lead lives, perform acts and entertain ideas that are centrifugal rather than centripetal; their thought is scattered or diffused, moving on many levels, seizing upon the essence of a vast variety of experiences and object for what they are in themselves, without, consciously or unconsciously, seeking to fit them into, or exclude them from, any one unchanging, all-embracing, sometimes self-contradictory and incomplete, at times fanatical, unitary inner vision. The first kind of intellectual and artistic personality belongs to the hedgehogs, the second to the foxes…we may, without too much fear of contradiction, say that, in this sense, Dante belongs to the first category, Shakespeare to the second; Plato, Lucretius, Pascal, Hegel, Dostoevsky, Nietzsche, Ibsen, Proust, are, in varying degrees hedgehogs; Herodotus, Aristotle, Montaigne, Erasmus, Moliere, Goethe, Pushkin, Balzac, Joyce are foxes.[1]

While Berlin's immediate purpose was literary criticism—he used it to examine Leo Tolstoy's ideas about history—his metaphor has influenced writers in the history of ideas in a wide range of areas.

This note applies Berlin's idea to the situation described by Liping Ma[2] with regard to American-style and Chinese-style elementary mathematics curriculum documents. Ma argues that the very structure of American curricula, as laid out in these documents, leads to flaws in the system of education. But, seen through the lens of Berlin, one

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DOI: http://dx.doi.org/10.1090/noti1108

[1]*Isaiah Berlin,* The Hedgehog and the Fox. An Essay on Tolstoy's View of History. *Elephant Paperbacks, Ivan R. Dee, Publisher, Chicago, 1993. Originally published 1953 by George Weidenfeld and Nicolson, Ltd. No place of publication listed, pages 3–4.*

[2]*Liping Ma, "A critique of the structure of U.S. elementary school mathematics",* Notices of the American Mathematical Society*, Vol. 60, No. 10, Nov. 2013, pp. 1282–1296.*

can say that there are corresponding flaws in the Chinese system. And, I argue, a synthesis of the positive aspects of both will achieve more than either of us has achieved separately.

For what follows, it is important to note that Berlin—and most other writers—do not see any value judgment in the division into hedgehogs and foxes. Berlin himself (above) names important hedgehogs and foxes, and it would be difficult to say that one set of these thinkers is in any way better or more important than the other. The same list shows that Berlin's classification does not correlate with the field in which the thinker is working. Any field can profit from either sort of contribution.

From Berlin's point of view, we can characterize the Chinese curricula described by Ma as largely hedgehog-like, while American thinking is much more fox-like. If, like Berlin, we withdraw from the notion of judgment, we can distinguish strong and weak points in either way of thinking. And, if we look for ways to synthesize these viewpoints, we may be able to strengthen both efforts.

Before commenting on this possibility, I note that the criterion of "coherence" used by Ma to judge curricula is a problematic one. For example, we could construct a marvelously coherent curriculum centered around the goal of learning the English names of rational numbers between, say, 10^{-24} and 10^{24}. Children would be taught to recite these numbers correctly, complete with Latin or Greek prefixes, and using the word "and" in the correct places. The skill would be easily demonstrable, both on a written test and at the dinner table. And, in the age of teraflops and nanoseconds, a case could be made that it is useful. But central?

Less flippantly, we might construct another marvelously coherent curriculum around mastery of a certain set of arithmetic algorithms, termed "standard". (Whether or not this term is appropriate is here irrelevant.) Students would learn to multiply three-digit numbers fluently and accurately, would perform long division in an approved format, and would subtract the bottom number from the top, using algorithms tailor-made and honed by historical forces for computation in the base ten system, itself a product of long intellectual evolution.

These skills would be easily demonstrable on a written test, if not at the dinner table. Are they useful, in this age of ubiquitous hand-held calculators? We can debate that question at another time. The point here is that the answer to that question does not depend on the coherence of the curriculum, but on values external to the curriculum. Values are cultural, not mathematical. Values, like taste or morality, are not subject to proof (the central criterion of mathematics) or refutation (the central criterion of science). So whether or not a curriculum is useful is not a question answered by looking at internal coherence.

The Structure of American Curricula: Where Is the Center?

Let us set aside these comments for a moment, and accept the notion of coherence as central to the discussion of a curriculum. It is my view that Ma's note makes the hedgehog-like assumption that arithmetic is central to elementary mathematics, then demonstrates the coherence of the Chinese curriculum she discusses. But if another assumption is made about the central concept for a curriculum, then the coherence may vanish, or may reduce to a foolish consistency. I will argue below that Ma's comments flirt with this danger.

To illustrate why I question the assumption of the centrality of arithmetic in elementary education, let us consider some third-grade students. (These sketches are in fact drawn from life.) Student A can add two-digit numbers in the standard way: 46+35=81. She can do this consistently, accurately, and fluently, and can do the same for three- and four-digit numbers. Student B understands what addition means, but has trouble remembering to "carry the one", or makes similar errors in computation. So he may write: 46+35=71. Student C has his own idiosyncratic way of adding two two-digit numbers (for example, he might reason that 47+47 is six less than 100 because 50 +50=100, and so on). Perhaps it is not as efficient as student A's. And perhaps it doesn't quite generalize to three-digit numbers. But it serves him for many two-digit numbers, and he clearly knows what he's doing, having invented the algorithm.

On the other hand, student A makes mistakes like adding 46 degrees in New York to 35 degrees in Chicago, getting 81 degrees, which student B does not make. Or student A says that the perimeter of a rectangle with adjacent sides of lengths 46 and 35 is 81, while student B, says it is 142 (recalling his error in the last paragraph). That is, student A makes errors in logic while student B makes errors in computation. And student C makes errors in neither, but has to re-think the problem when he must add three-digit numbers with paper and pencil.

The assumption we make here is that the errors made by student A are much more significant than those made by student B or C. The empirical evidence for making this assumption, drawn from my own classroom experience and that of colleagues, is that the errors of student B or C are much more easily remediated than those of student A.

It follows from this assumption that the central goal of elementary mathematics education—and perhaps of secondary mathematics education—is an understanding of logic (not formal or symbolic logic, but the intuitive and essential idea of a chain of implications). The study of arithmetic algorithms, when well taught, then becomes a tool for the delivery of what I see as a deeper skill: reasoning from one statement to another, a skill that cannot be duplicated, or even aided, by the use of a calculator.

An Alternative View of American Curricula

Using the alternative assumption of the centrality of logic in elementary curriculum, we can reexamine some of Ma's points.

Most of Ma's discussion is an examination of syllabi, and standards documents, rather than about live classrooms. The hedgehog-like assumption is made right at the start of the discussion, in noting that older American textbooks of arithmetic took Euclid's Elements as the standard for a unified exposition of their subject, and that more recently, Chinese texts, among others, have continued and refined this tradition.

But it is clear, from this very choice of models, that there is something deeper than simply arithmetic which is the standard. Euclid, too, was in some ways a hedgehog-like thinker. He has a wonderfully articulated paradigm of mathematical truth, using geometry as the central topic. His rigorous treatment of arithmetic (including the arithmetic of irrational numbers) and number theory was based more or less entirely on geometric intuition, as was his treatise on optics that has come down to us[3] (but not as part of the *Elements*). His algebra remained two-or three-dimensional because it, too, is expressed geometrically. The solution of equations of higher degree, and an efficient algebraic notation, were never developed by the Greeks.

The fact that one can base a development of arithmetic on the same methods Euclid used for geometry suggests that Euclid's basic concept is deeper. It consists in the idea of an axiomatic system, or the underlying concept of implication: one statement or set of statements implying others. If we take this wider view of what Euclid is about, indeed of what mathematics is about, Ma's analysis looks somewhat different.

The Chinese, as described by Ma, can be now seen as using an exposition of arithmetic as an example of an axiomatic system. On the pedagogical level, the Chinese elementary curriculum uses statements from arithmetic to build a notion of implication. A student who knows the "compensation law for addition" that Ma mentions[4] can reason that if 8+8=16, then 9+7 must also equal 16. That if, well taught, the Chinese curriculum delivers not simply the facts and algorithms of arithmetic, but also ways to reason about numbers and statements about numbers. Poorly taught, of course, it deteriorates into recitation and memorization.

Likewise, the American curriculum—as described by Ma—can now be seen differently. Placing implication at the center of elementary mathematics, we can read American curricula and standards as delivering this same central concept, but using a variety of examples: arithmetic, geometric, statistical, probabilistic, and so on. Viewed this way, the American "strand" structure pointed out by Ma becomes a flexible asset, and not a wild liability.

Has this asset been exploited in published American curricula? I would argue that the answer is "sometimes". But I would likewise argue that the Chinese curriculum "sometimes" uses arithmetic to deliver more profound mathematical ideas. Evidence for this is lacking in Ma's work (she concentrates, here and elsewhere, on successful instances of Chinese teaching, and in fact those are probably the ones we can learn the most from). My own work in China with gifted students indicates otherwise: that they are often poised to think out of the box, but must be invited. That is, I have found that when Chinese students are faced with new and unusual problems, they often balk, intellectually. They try to think where they have seen the problem before. Sometimes they apply familiar techniques or patterns of solution inappropriately. However (my work is almost entirely with gifted students), when invited to invent their own solutions, they respond quickly, and often succeed brilliantly. The point is not that they cannot think outside the box. The point is that their curriculum does not invite this. Rather, it cultivates a deep and reliable mastery of a specific set of techniques.

Chinese colleagues have commented on this phenomenon. One said, "I don't worry about my [Chinese] students passing a test. But I want to give them opportunities to do something new." I have often been asked, by Chinese colleagues, "How can I introduce creativity into my classroom?" This is itself a hedgehog-like question, as if there were an algorithm for inducing creativity. Colleagues I've worked with in China have often questioned exactly the highly structured curriculum that Ma describes. Perhaps the most striking comment on this level is about Chinese Nobel prize winners in the sciences. A list of these will show that none of them did their work in Chinese institutions.[5] Some might see this as a narrow comment on graduate education and research institutions, but the implication that many Chinese educators draw is that they need to look at how the twig is bent.

I might add that I have found this phenomenon in work with American students as well. They too,

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[3] *See* `http://www.math.cornell.edu/~web1600/Terrell_OpticsOfEuclid.pdf` *(accessed September 2013).*

[4] *Ma,* Op. cit., *page 1286.*

[5] *See* `http://en.wikipedia.org/wiki/List_of_Chinese_Nobel_laureates`, *accessed September 2013. One laureate is listed as affiliated with the University of Hong Kong. All the others worked in American or European institutions. Two winners in peace and literature were educated in China.*

often need to be invited to think outside the box that their curriculum has put them in. This is partly a consequence of the very nature of curriculum: a set of skills and concepts to be mastered. The striking aspects of my work with Chinese (and Japanese, and Taiwanese, and Malaysian) students are two: the occurrence of the phenomenon in very able students, and the alacrity with which they accept the invitation to break outside the box created by their training.

Ma brings up another point in her work, one not directly connected to her central thesis. We can look at curriculum in yet another way (examine yet another strand!): the way in which it is perceived by the student. This aspect of curriculum is not discussed by Ma. It is perhaps the most fox-like way of looking at education, in that it has, as of now, the least structure. As Ma points out, the structure of mathematics has been worked out over thousands of years. On the other hand, we are just beginning to formulate and test theories of learning, either of mathematics or of other subjects. And we have almost no theories of teaching, which cannot be directly inferred either from theories of learning or from the structure of mathematics.

Ma mentions the fact that many of the American curriculum documents she reviews include strands that are not properly mathematics, involving attitudes towards mathematics, "number sense", or problem solving. Should these in fact be part of a curriculum document? This is a difficult judgment call. The question becomes: is teaching and learning mathematics informed solely, or even centrally, by the nature of mathematics itself? I do not wish to answer, or assume an answer to, this question here—it is much too involved. But I would argue that a fox-like way of thinking about curriculum would include at least a consideration of these other factors, even if merely to reject them or distinguish them from mathematics "proper".

The Critique

Ma's article goes beyond a description and characterization of specific curriculum documents. She levels three charges against the "strand" structure of the American curriculum: instability, lack of accumulation of knowledge (of teaching and learning), and incoherence among concepts. In this next section—the least constructive in this essay—I argue that (a) many of these charges vanish if we look at curriculum as logic-centered, rather than arithmetic-centered, and (b) many of these same negative characteristics can be found in the "core structured" arithmetic-centered curriculum, as Ma describes it. We look at these charges one by one.

Instability

Ma asserts that the "strands structure" leads to instability in the curriculum. From the fox's point of view, we might just as well say that it leads to flexibility(!). As we find new ways to approach the basic logical structure of mathematics, a "strands structure" will allow us to incorporate these new approaches easily into classroom technique. For example, the American curriculum has looked at probability, statistics, and computer algorithms to find ways to approach the idea of implication.

It is important to understand that we are talking here about Ma's concept of structure, and not about implementation of structure. Perhaps this or that American curriculum has not been successful in flexibly introducing new ideas, or has introduced them inaccurately. The argument against "strands", against fox-like thinking, would have to be that it actually implies failure in introducing new ideas. But the argument against the Chinese curriculum could equally be made that it prevents—not just makes difficult—the introduction of new approaches. The American structure has doors, where the Chinese structure—as described by Ma—has walls.

Accumulation of Knowledge

In fact, American curriculum writers do accumulate knowledge. We now know better than we did thirty years ago how to teach combinatorics in elementary school. We have had a number of successes in introducing concepts from computer science into elementary education. Is this mathematics? Do these topics relate to the drawing of logical inferences? I would argue that the relationships are certainly there, but are still being worked out. That is, knowledge is still being accumulated.

And, I would argue, the multi-strand nature of American curriculum encourages the accumulation of knowledge. A more open structure to curriculum allows us to introduce new materials in ways that are consistent with the old. Whether a student draws implications from a statement in arithmetic, in geometry, in statistics, or whether they use arithmetic equations or a computer programming language, becomes a detail of implementation, rather than a source of confusion.

Incoherence

We can say the same about what Ma terms "incoherence". No matter how we structure our curriculum, we need teachers who are trained to see how mathematics coheres. A narrow focus on arithmetic may easily lead to an incoherent view among teachers. Ma herself mentions that only 10% of her sample of Chinese teachers really had a profound understanding of arithmetic.[6] This statistic seems to indicate that concentrating on arithmetic leads to an incoherent view of mathematics for most of the teachers exposed to it.

Incoherence in a single-strand curriculum can occur on the student level as well. We have mentioned

[6] *Ma,* Op. cit., *page 1295.*

that a well-taught arithmetic-centered curriculum may be capable of delivering the concepts of implication and logic. (Whether this transfer is easy or difficult to implement is an empirical question, not examined by Ma.) But let us take a case where a student has successfully learned about logic from a study of arithmetic. Such a student may or may not see that its application to counting problems is part of mathematics. Or that we can harness the same sort of logic to a study of geometry. That is, a narrow focus on arithmetic can create a discontinuity in the student's experience, just the fault that Ma attributes to a "strand" structure of curriculum.

Implementation and Synthesis

Ma's central point, as I read it, is that a strand structure of American curriculum, whatever its strengths, encourages poor implementation. I see an equal but opposite danger with a core-structured curriculum. Especially in America, with our emphasis on testing and accountability, the temptation will be strong to teach students the mechanics of arithmetic without its meaning. And this is certainly possible. There is no evidence that a mastery of standard algorithms or of the notion of place value delivers to students power in logical deduction, any more than there is evidence that strong deductive powers allow students fluency in arithmetic. A faulty delivery of either style of curriculum will not serve students well, and there is no "teacher-proof" way of designing a curriculum.

And here is where the notion of a synthesis becomes useful. My feeling is that we must look at both types of thinkers, hedgehogs and foxes, and find ways to use both. For example, we can find foxes to design curricula with many strands, then hedgehogs to polish each strand. A conscious effort to fit them together would effect a synthesis of the two views.

Ma herself points out that many Chinese are open to working this way: they are looking at American curricula to see what they might learn. Where Ma bemoans this phenomenon, I celebrate it. Reciprocally, the success of her book *Knowing and Teaching Elementary Mathematics* is one indication that some of us are also open to new insights. Let us hope that this sort of synthesis can be accomplished successfully.

I would like to thank Susan Addington, Judy Roitman, Douglas Clements, Yvonne Lai, and Peter Shiue for their input into this article.

Marc Yor (1949–2014)

Jean-François Le Gall and Jim Pitman

Marc Yor, one of the most distinguished probabilists in the world in recent decades, died suddenly on January 9, 2014, near his home in St. Chéron, France, at the age of sixty-four. He was born on July 24, 1949, in Brétigny-sur-Orge, France. After studying at the École normale supérieure de Cachan, with thesis work under the supervision of Pierre Priouret, he quickly became a researcher at the French Centre National de la Recherche Scientifique (CNRS), then in 1981 a professor at the Université Pierre et Marie Curie, where he remained until his retirement on January 1, 2014.

Marc Yor is world renowned as a prolific researcher in the theory of probability and stochastic processes. He wrote over 400 research articles and ten research monographs. Most of the research articles and several of the monographs were written jointly with one or more coauthors from a list of over 100 collaborators from all over the world (see `http://zbmath.org/authors/?q=ai:yor.marc`) including many of the most prominent probabilists of the era. During the 1980s and 1990s, Marc Yor largely took over from Paul-André Meyer the mantle of responsibility for development of research in probability in France. He was an influential editor of the Séminaire de Probabilités, founded by Meyer in 1967, over a span of twenty five years. In this capacity he set a new tone for the Séminaire as a diverse compendium of contemporary research in probability, with a focus on work done in France, but also welcoming contributions from abroad.

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First appeared in the IMS Bulletin, *February 15, 2014* (`http://bulletin.imstat.org/2014/02/marc-yor-1949%E2%80%932014/`). *Article by Jean-François Le Gall and Jim Pitman, with thanks to Jean Bertoin for his advice.*

DOI: http://dx.doi.org/10.1090/noti1128

He also did extensive editorial work for several major probability journals, including the *Annals of Probability* and *Probability Surveys*. He played an irreplaceable role in welcoming the best mathematics students interested in probability and engaging them in research, advising over thirty theses during his career as a university professor. A large number of these students went on to be researchers at the CNRS or professors in France and other countries. Without him, the recent successes of the French probability school, most notably the Fields Medal of his grandstudent Wendelin Werner in 2006, would most likely never have been achieved.

The research work of Marc Yor covered many aspects of the modern theory of probability, but he became most celebrated internationally for his applications of stochastic calculus. Stochastic calculus was born from the work of the great Japanese mathematician Kiyoshi Itô in the 1940s. It was developed by other mathematicians in the 1960s, most notably in France under the influence of Meyer, in what has become known as the Strasbourg School. But it was the work of Marc Yor which most fully demonstrated the potential of this mathematical tool. In his hands, stochastic calculus became a powerful tool for computing features of the probability laws associated with all kinds of stochastic processes. Marc Yor became acknowledged internationally as an extraordinarily talented user of stochastic calculus in combination with other powerful tools of analysis including Fourier and Laplace transforms, which he deployed in virtuoso displays of computational technique.

Among stochastic processes, the one he cherished the most was Brownian motion. Following the works of Paul Lévy, for whom he held a great admiration, Marc Yor wrote a number of famous articles about Brownian motion, which have inspired and continue to inspire generations of researchers. His deep study of local time processes of Brownian motion and continuous semimartingales, the Ray-Knight theorems and related properties of Bessel processes, and the windings of planar Brownian motion, stand out in this respect. His work on these topics made him well known in all corners of the world, where he was always eager to travel—both for short meetings and for longer stays—to share his ideas and mathematical discoveries. Other topics which he developed extensively include enlargement of filtrations, intersection local times, exponential functionals of Brownian motion and Lévy processes, and penalizations.

In addition to his hundreds of articles, Marc Yor was the author of a number of research monographs. Among these, his monograph with Daniel Revuz *Continuous Martingales and Brownian Motion*, based on the DEA courses given by Marc Yor in the early 1980s, is by far the most well known. This work has been phenomenally successful for a mathematical research monograph. It provided great exercises for training the next generation of probabilists, both in France and many other countries, as well as a basis for applications in financial mathematics. In the last part of his career, Marc Yor was interested in this domain, not because of any particular attraction to finance (he was later concerned about the responsibility of mathematicians in the financial crisis), but because he saw a vast field of application for the techniques of stochastic calculus which he had mastered so well.

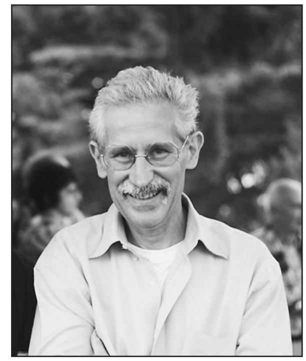
Photo by Murad Taqqu.

Marc Yor

Throughout his career, Marc Yor's work exemplified the highest standards of scholarship and respect for the history of probability and related fields. He wrote extensively and organized meetings around the history of probability, especially the work of Bachelier, Kolmogorov, Doeblin, Doob, Lévy, Itô, and Meyer.

Marc Yor received a number of scientific distinctions: the Humboldt Prize, the Montyon Prize of the French Academy of Sciences in 1986, and the Ordre National du Mérite by the French Republic. He was elected correspondent to the French Academy of Sciences in 1997, then member in 2003. He was also a senior member of the Institut Universitaire de France since 2004.

Two words which best decribe the scientific personality of Marc Yor are without doubt enthusiasm and generosity. Enthusiasm, because he knew so well how to communicate his taste for research and to share the joy of discovery of new theorems and formulas. Generosity, because he helped so many young researchers, publishing with them numerous research articles which everyone knew he had essentially written himself, but for which he was always happy to share the credit.

Beyond mathematics, Marc Yor was a talented soccer player, who played competitively for many years, then coached youth teams in his community with the same dedication he showed in the training of mathematicians. He was devoted to his family, and is survived by his wife Carmel, son Serge, and two daughters Kathleen and Geraldine.

Happy 91st, Cathleen Synge Morawetz

Allyn Jackson

Photo from AMS archives.

Cathleen Synge Morawetz

Cathleen Synge Morawetz is a legendary and beloved figure in mathematics. Renowned for her striking work in analysis, she has been an inspiration to many in the field, particularly to women, as well as a leader in the AMS and in the scientific profession more broadly. Her 91st birthday, which occurs in May of this year, is a fitting time to pay tribute to her life.

Her great-uncle was John Millington Synge, the Irish playwright best known for *Playboy of the Western World*, which caused riots when it was first performed in 1907. Her father was John Lighton Synge, an Irish mathematician who had a long career at the University of Toronto; her mother also had some training in mathematics. When Cathleen Morawetz received the AMS Steele Prize for Lifetime Achievement in 2004, she credited her father with instilling the ideal of intellectual achievement and her mother with instilling ambition, which, Morawetz commented, was "at the time very unladylike."

On the various occasions when Morawetz has discussed her long career[1], she has put the emphasis not so much on herself but on the people around her who helped her succeed. Among these was Cecilia Krieger, who fled Poland during World War I and earned a Ph.D. in mathematics at the University of Toronto, later becoming a professor there. At a crucial moment in Cathleen's final year as a mathematics major at Toronto, Krieger encouraged Cathleen to attend graduate school and promised to find funding.

Krieger came through on the promise, and Cathleen became a graduate student at New York University. There were few women in the doctoral program, but the atmosphere was supportive, partly because of the influence of Richard Courant: He had been a strong mentor for Emmy Noether in Göttingen and continued to encourage women in mathematics after he had emigrated to New York. One of Morawetz's first serious mathematical undertakings was the editing of the now-classic book *Supersonic Flow and Shock Waves* (1948) by Courant and Kurt Friedrichs. Morawetz said that, despite her junior status, she had standing with the two professors because she could correct their English. Editing this book was one of her formative experiences in mathematics.

Courant was forceful and outgoing, while Friedrichs was shy and a bit withdrawn. Morawetz worked well with both. She flourished in the stimulating atmosphere of what later came to be called the Courant Institute of Mathematical Sciences, which drew no borders between pure and applied mathematics. She was hooked by problems of transonic flow through contact with Friedrichs, who became her thesis advisor, and with Lipman Bers. The support of Courant was crucial as Morawetz followed a very nonstandard career path while she bore and raised four children. She earned her Ph.D. in 1951 and, after a postdoc at the Massachusetts Institute of Technology, she returned to the Courant Institute as a faculty

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[1] *See in particular the video interviews and excellent article, by Siobhan Roberts, on the "Science Lives" website of the Simons Foundation,* `http://www.simonsfoundation.org/category/features/science-lives/`.

DOI: http://dx.doi.org/10.1090/noti1123

member in 1955 and remained there for the rest of her career. She continued to draw inspiration from those around her at the Courant Institute, learning about problems in magnetohydrodynamics from Harold Grad and about wave propagation from Joseph Keller.

Asked whether she is a problem solver or a theory builder, Morawetz replied, “I am an applied mathematician who proves theorems to solve problems.” She has taste both for the mathematics and for the choice of problems from other areas. One gets a sense of this taste from two of her expository articles. The first is the writeup of her 1981 AMS Gibbs Lecture, “The mathematical approach to the sonic barrier” (*Bulletin of the AMS*, March 1982). The article provides clear, down-to-earth explanations of the physics and engineering of air foil design, with a mathematician’s viewpoint of trying to understand what’s going on at the deepest level and to nail down the details with proof. The other article is her Retiring AMS Presidential Address “Mathematics to the rescue” (*Notices*, January 1999). This piece puts the spotlight on a few outstanding problems—weather prediction, molecular structure, and transonic flow—and throws the mathematical aspects into sharp relief. Morawetz moves seamlessly from nontechnical and insightful presentations of the physics, engineering, and mathematics to entertaining asides about the people involved. Both articles give a sense of her deep knowledge, as well as her immense charm and humor.

Morawetz is the first and only woman to have received the AMS Steele Prize for Lifetime Achievement (2004); the same can be said for her receiving the AMS-SIAM Birkhoff Prize in Applied Mathematics (2006). She is also the first and only woman mathematician to have received the U.S. National Medal of Science (1998). In addition to a term as AMS president (1995–1996), Morawetz has served the Society in various capacities, including terms on the Board of Trustees and the Council. This long association with the AMS played a part in her decision, in celebration of her 90th birthday last year, to make a major donation to the Society. The gift from her and her husband, Herbert Morawetz, significantly increased the size of the long-under-funded Oswald Veblen Prize Fund, bringing it on a par with other AMS prize funds.

Veblen was a good friend of Morawetz’s father, John Lighton Synge. How this friendship began is recounted in Synge’s article “For the 100th birthday of the American Mathematical Society”, which appeared in *A Century of Mathematics in America: Part 1*, edited by Peter Duren (AMS, 1988). The article is a written version of a talk Synge gave at the AMS Centennial Celebration in 1988. In the article, he recalls an AMS meeting he attended in December 1921 in Toronto. He had come to Toronto from Dublin the year before and found few colleagues with mathematical interests similar to his own. His encounter with Veblen at that AMS meeting and the kindness and consideration Veblen showed were important to Synge as he made his way in mathematics in a new land.

At the time Synge wrote the article, he was 91 years old, the same age his daughter is now. One hears in his article an echo of the lively intellect and warmth of Cathleen Synge Morawetz. For those qualities and for her many contributions to mathematics and to the profession, she has evoked great fondness in the mathematical community. Happy 91st, Cathleen!

Listening to Teachers

Priscilla Bremser

In the first *Doceamus* column [12], Herbert Wilf wrote, "Our quality of education will improve when we have more teachers who are knowledgeable in their subject and who enjoy doing it and talking about it." A year later, Solomon Garfunkel's "The Common Core Standards—Educational reform and us" [4] mentions teachers in just one paragraph. Ten months after that, Irwin Kra offered "(Math) Teachers are the key" [5]. Absent from any of those columns are the voices of teachers themselves. While it is gratifying to see mathematicians paying close attention to K–12 education, we will be better able to contribute constructively to that enterprise if we simply listen to teachers now and then.

Last summer, Ms. K. mentioned that she would be teaching fourth grade in the fall, but prefers middle school students, a cohort that would challenge many of us. I'm still amazed by the volatile energy my sons and their friends presented at that age. As it turns out, though, that abundant adolescent energy is the attraction for Ms. K.; she finds it invigorating. For another teacher, "This is our last chance to get the math right before high school." A third says, "What these kids are thinking is closer to the surface. They're more willing to show enthusiasm." This brings me years back to my twins' eighth-grade classroom. During an enrichment activity I was leading before school, a boy blurted out, "Oh! That's a parabola! I love parabolas!" Not one of my college students has ever said that.

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DOI: http://dx.doi.org/10.1090/noti1107

My point is not to convert mathematicians into middle-school educators, but rather to suggest that teachers' stories can challenge our assumptions about mathematics education and how it might be improved. We've all heard conjectures, stated as facts, from colleagues concerning how arithmetic is taught, why some of our students can't do algebra, and so on; I've made a few myself. As we consider test scores [8], [11] and critiques of testing [7], [10], follow the implementation of the Common Core State Standards [2], and scan recent work [3] on how children learn mathematics, we can round out our understanding by hearing directly from people doing the real work. It is anecdotal information, so we need to use it carefully. But it might keep us from rushing to judgment. At the very least it can help shape the questions we ask.

For example, a seventh-grade teacher plans his lessons with the high school curriculum in mind. It's a small middle school, and in one class he has kids bouncing out of their seats with enthusiasm for repeating decimals alongside two who are functioning at a second-grade level and trying not to get noticed. Is there anything in this teacher's training or professional development that could support his efforts to reach all of his students? Is there time in his day to consult with colleagues? Does his principal understand the mathematics well enough to recognize the challenge he faces? (Not necessarily, I've learned.)

The usual setting for my conversations with teachers is the Vermont Mathematics Initiative, a master's program at the University of Vermont for practicing teachers. I have been a VMI instructor since 2007, including three summer sessions in the Cincinnati offshoot. But one doesn't need a formal commitment to engage in meaningful discussions with K–12 educators.[1] I have stayed in touch with former students teaching mathematics in public, charter, and private schools in the U.S. and in the Peace Corps. I also give math talks in Vermont schools, which sometimes includes lunch (if only for 32 minutes) with teachers. Provocative stories grow out of seemingly ordinary exchanges.

In a school district far from Vermont, teachers have strict guidelines that specify weekly content, and students take standardized tests each month. One Friday, a teacher was confident that she had completed that month's material, and took the time to go back to a previous topic that had been rushed. The "Math Coach" visiting her class that day went directly to the principal and reported the teacher for deviating from procedure. I fervently hope that this story is not typical, but it does serve as a reminder that public school teachers have far less autonomy than I do. There are natural reasons for that, of course, but surely some decisions are better left to teachers. What would an appropriate level of teacher autonomy look like?

Closer to home, a kindergarten teacher near the Canadian border has some entering students who cannot count. They reel off numbers, but they have no sense of order. This story gets filed in the "Reality Check" folder; when we say we have clear expectations for teachers, are those expectations reasonable in context? Also in this file is a middle-school teacher in a crumbling Southern city. During parent-teacher conferences, parents nod and smile and promise to reinforce the homework agreement and get their kids to school on time, but there's often no follow-through; a few parents actually enable truancy. How can teachers form productive partnerships with parents whose own school experiences were largely negative?

Early one June morning in Cincinnati, two teachers are discussing the death of a student in an isolated low-income neighborhood. A few months later, it's hunting season in Vermont, and one high school has an honor roll of sorts in the front hall, listing students' deer kills. A few of their families rely on that venison in the freezer to get through the winter. When we compare our kids to those in Finland and Canada, countries with lower Gini indices than ours, how do we tease out the strong correlation between family income and performance on standardized tests [6]? Can we realistically expect to improve educational outcomes, in mathematics or any other area, for all children without addressing poverty in the United States?

Then there's the "Omnes Docemus"—"We All Teach"—folder. Some stories capture the joy of watching a student catch on to a new idea, be it subtraction or quotient groups. Others note the infiltration into our classrooms of a stubborn cultural resistance to the very subject we teach. We who teach mathematics also share a constant tension between the breadth of material we expect to cover and the depth to which we want our students to understand it. Can mathematicians who choose active involvement in schools build on such commonalities?[2]

Not so fast, you say—there are differences between schoolteachers and college professors. For one thing, we know more mathematics than they do. This is true, and teachers know that it's true; sadly, their discomfort can get in the way of a good conversation. Fortunately for me, the VMI environment entails substantive mathematical discourse with teachers. While I'm in their company, though, I also want to hear about the things they know better. What do you do on the first day with Somali teenagers who know only basic arithmetic and no English? How do you keep an immature but mathematically brilliant fourth-grader engaged? What techniques have you found for channeling that middle-school energy toward mathematics?

Herbert Wilf and Irwin Kra bring our attention to a crucial issue; a student's teacher is the single in-school variable with the most influence on learning [9].[3] Kra's conclusion: "To put good math teachers in the classroom, society must expect more from them, must pay them more, and must give them the support and respect they deserve." Agreed. But as we devise strategies to do so, we would do well to listen to the ones who are already there. We'll get a more detailed picture of why current and future teachers deserve our respect, and more focused proposals for supporting them.

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2014 Award for Impact on the Teaching and Learning of Mathematics

PAUL J. SALLY JR. is the recipient of the inaugural AMS Award for Impact on the Teaching and Learning of Mathematics. Sadly, Paul passed away unexpectedly on December 30, 2013. But news about the award reached him shortly before his death, and he took great pleasure in knowing that he was the first recipient of a prestigious award.

Citation

Paul Joseph Sally Jr. has combined cutting-edge research with a parallel life-long commitment to education at all levels—precollege, undergraduate, and graduate. As director of undergraduate studies at the University of Chicago for over thirty years, he was a leading voice in the undergraduate program and was seen by many as "a legendary math prof". But what makes Paul Sally the ideal recipient of the inaugural Award for Impact on the Teaching and Learning of Mathematics is his work, which began in the 1960s and continued unabated to the day of his death, in which he established outreach programs for precollege teachers and precollege students.

Important outreach programs that he founded include Seminars for Elementary Specialists and Mathematics Educators (SESAME) that provides State of Illinois middle school mathematics endorsement for teachers in the Chicago public schools, and the Young Scholars Program (YSP) for talented high school students from less than privileged backgrounds. He also cofounded and served as director of the University of Chicago School Mathematics Program (UCSMP), spearheaded the founding of the Chicago Algebra Initiative, and led the mathematics component of Chicago's Urban Teacher Education Program (UTEP).

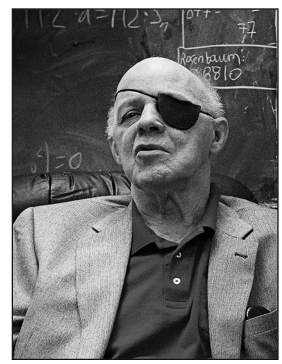
Not only did he establish programs in Chicago, but he was also instrumental in their expansion to other sites.

One of the purposes of the Award for Impact on the Teaching and Learning of Mathematics is to encourage research mathematicians to be actively engaged in replicable activities that improve mathematics learning and teaching, especially at the precollege level and in the first two undergraduate years. Paul Sally's career is a testament to the achievement of that goal and makes him an eminently excellent choice to be the inaugural recipient of this award.

DOI: http://dx.doi.org/10.1090/noti1124

About Paul J. Sally Jr.

When Paul J. Sally Jr. died on December 30, 2013, at the age of eighty, the mathematical community mourned the loss of a beloved figure who contributed to the field in many ways, particularly through his teaching, and who did so with an enthusiasm that was infectious. Anyone who ever met Paul could not have helped being struck by the figure he presented: a six-foot-three-inch tall man, who later in his life was bald, had a black eye patch, and had two prosthetic legs. He lost his eyesight and his legs to Type 1 diabetes that had afflicted him since his teenage years. Those losses seemed a hindrance neither to his commanding yet cheerful personality nor to his bottomless energy. He demanded and received total devotion to mathematics from those he worked with,and in his later years was prone to provide the explanation, "because it is impossible to give a blind double amputee excuses for shirking." This last quotation is lifted from an unsigned memorial posted on the home page of the Mathematics Department at the University of Chicago, but Paul was heard saying this many times himself.



Courtesy of the University of Chicago. Photo by Jason Smith.

Paul J. Sally Jr.

Born in 1933 in Boston, Paul never lost the accent of his hometown. When he visited MIT in 2007, the *Boston Globe* ran an article about this colorful hometown hero. The article recounted a story Paul told that was set in the Joint Mathematics Meetings. One of Paul's colleagues told Paul that he knew who was getting an award but he refused to tell Paul who it was. So Paul enlisted the help of Philip Kutzko of the University of Iowa to dangle the person from a hotel balcony until he did tell. The story is apocryphal, but Paul milked it for all the humor he could get. "It was only the third floor, but if we had dropped him, it would have been serious," he told the *Globe*. Later the article quoted Kutzko: "[Paul Sally is] unique because he's this big powerful man, but his hallmark is that he nurtures people. The jokes are funny, but the reason his students and colleagues love him is that he's been there for people."

Paul received his bachelor's (1954) and master's (1956) degrees from Boston College. After teaching in Boston high schools and at Boston College—and also driving a cab and a moving truck while playing plenty of basketball (he was an All-Scholastic basketball player in high school and played varsity basketball at Boston College)—he entered the mathematics doctoral program at Brandeis University. While in graduate school, he married fellow graduate student Judith Donovan, now Judith D. Sally, Professor Emeritus of Mathematics at Northwestern University. Paul earned a Ph.D. in 1965 under the direction of Ray Kunze and then joined the faculty at the University of Chicago, where he remained for the rest of his career. In 1967–1968, he was a member at the Institute for Advanced Study. There he came into contact with Harish Chandra, who made a deep impression and stimulated some of Paul's best mathematical work, including his papers with Joseph Shalika on $SL(2)$, which ushered in an era of rapid development in harmonic analysis on reductive p-adic groups. Paul had three other year-long stays at the IAS and, along with Murray Gerstenhaber, helped to found AMIAS (the Association of Members of the IAS).

Paul had thirty-seven publications in MathSciNet, and he supervised the Ph.D. theses of nineteen students. His noteworthy research accomplishments were intertwined with his main love: teaching. Arnold Ross, founder of the celebrated program at Ohio State University for mathematically talented high school students, once said that outstanding teachers are "people who respect struggles with ideas and have gone through that themselves and were victors." This was true of Paul. In fact, one of his favorite mottoes was "To teach math, you better know some." (Paul was a great admirer of Ross and in 1996 established an endowment that today helps to fund the AMS Arnold Ross Lecture series.) In 1988, Paul launched his own Young Scholars Program at the University of Chicago for seventh to twelfth grade students. By that time he had had a great deal of experience working with students and teachers in the Chicago public schools. In 1983, the University of Chicago School Mathematics Project (UCSMP) was founded, and Paul was named director. He served in that post until 1987, when he became more involved in the staff development activities of UCSMP.

In 1992, he started SESAME (Seminars for Elementary Specialists and Mathematics Educators), which has since reached over 600 teachers from 125 schools. Many aspects of the programs he ran were emulated by other programs around the country. Paul had a long association with the AMS and served the Society in several capacities: He was a member of the AMS Council and the Board of Trustees (in each case for a total of ten years), and he was on numerous committees, including the Long Range Planning Committee, the Committee on Science Policy, the Committee on Education, and the Nominating Committee. In 2013, he was among those named in the initial class of Fellows of the AMS. His other honors include the AMS Distinguished Public Service Award (2000) and the Haimo Award for Distinguished University Teaching from the Mathematical Association of America (2002).

When Paul received the AMS Distinguished Public Service Award, the citation praised his "longitudinal mentoring of students." With his characteristic irreverent humor, Paul wrote in his response that he was very honored to receive the award "and as soon as I figure out what 'longitudinal mentoring' means, I'll feel really good." What would he have written in response to receiving the AMS Impact Award? This we cannot know. We will have to content ourselves with memories of this unique individual who contributed so much to mathematics, to education, and to the people whose lives he touched.

There are several excellent sources of information about Paul Sally on the Web, including a profile called "Sally marks the spot", by Carrie Golus, *University of Chicago Magazine*, `http://magazine.uchicago.edu/0856/features/sally.shtml`; and the video interviews with Paul Sally on the Science Lives website of the Simons Foundation, `https://www.simonsfoundation.org/science_lives_video/paul-sally/`.

About the Award

The AMS Award for Impact on the Teaching and Learning of Mathematics was established by the AMS Committee on Education (COE) in 2013. The Award is given annually to a mathematician (or group of mathematicians) who has made significant contributions of lasting value to mathematics education. Priorities of the award include recognition of (a) accomplished mathematicians who have worked directly with precollege teachers to enhance teachers' impact on mathematics achievement for all students or (b) sustainable and replicable contributions by mathematicians to improving the mathematics education of students in the first two years of college. The US$1,000 award is given annually. The recipient is selected by the COE. The endowment fund that supports the award was established in 2012 by a contribution from Kenneth I. and Mary Lou Gross in honor of their daughters, Laura and Karen. The award is presented by the AMS COE acting on the recommendation of a selection subcommittee. For the inaugural award, the members of the subcommittee were Arthur Benjamin, Irwin Kra (chair), and Kay Somers.

—Kenneth I. Gross
University of Vermont

—Allyn Jackson
Notices *Deputy Editor*

2014 Award for an Exemplary Program or Achievement in a Mathematics Department

The Award for an Exemplary Program or Achievement in a Mathematics Department was established by the AMS Council in 2004 and was given for the first time in 2006. The purpose is to recognize a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. Departments of mathematical sciences in North America that offer at least a bachelor's degree in mathematical sciences are eligible. Through the generous support of an anonymous donor, the award carries a cash prize of US$5,000.

The award is presented by the AMS Council acting on the recommendation of a selection committee. For the 2014 award, the members of the selection committee were: Aloysius Helminck, Monica Jackson, Suzanne Marie Lenhart, J. D. Phillips, and Francis Edward Su (chair).

The previous recipients of the award are Harvey Mudd College (2006), the University of California, Los Angeles (2007), the University of Iowa (2008), the University of Nebraska, Lincoln (2009), North Carolina State University (2010), the Math Center at the University of Arizona (2011), Bryn Mawr College (2012), and the University of Texas at Arlington (2013).

The recipient of the 2014 Award for an Exemplary Program or Achievement in a Mathematics Department is the Department of Mathematics and Statistics at Williams College.

Citation

The American Mathematical Society is pleased to recognize Williams College with the 2014 Award for an Exemplary Program or Achievement in a Mathematics Department. The Williams department has demonstrated excellence in providing exceptional teaching and research experiences for its students, as well as those in the wider mathematical community.

The department has made quality teaching a backbone of its program, and it shows in their success in attracting students to mathematics. Although there is no college mathematics or statistics requirement at Williams, eighty-four percent of students in recent graduating classes have completed a course in mathematics or statistics, including forty-eight percent who have taken Multivariable Calculus. Participation in statistics courses has grown from fifteen percent of the 1994 graduating class to sixty percent of recent classes. About one of every eight Williams students majors in mathematics (sixty-seven majors, a significant increase since six in 1985). Of eighteen full-time faculty in mathematics and statistics, six have been recognized with national teaching awards from the Mathematical Association of America. Many of the faculty have published books aimed at making mathematics lively and accessible at the beginning and advanced undergraduate level, and these books have had a wide impact on students across the country.

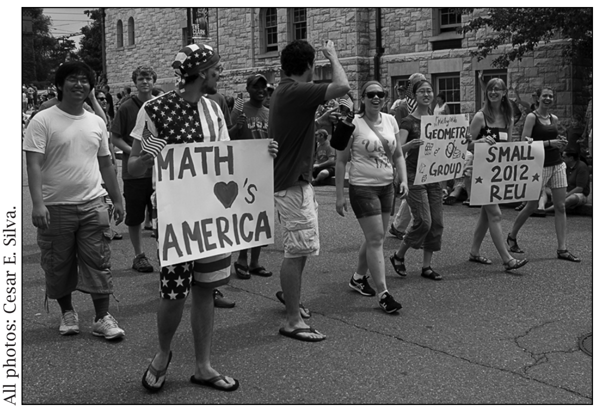
Undergraduate research in mathematics has also thrived at Williams College. Their REU program (the "SMALL" program) has been successful in helping undergraduates from across the nation discover the excitement and challenges of pursuing original research. The 400 alumni have produced over 100 journal publications. Many women REU alumni have won the Alice T. Schafer Prize for their research. Faculty members have maintained notable research programs and have offered an impressive variety of advanced courses as well as senior thesis research projects. The department is one of the four founding members of the Hudson River Undergraduate Mathematics Conference (HRUMC), an annual meeting that draws 500 students from a wide variety of New England schools to present their research. Since its inception in 1994, HRUMC has inspired a number of similar conferences across the country.

In addition, the department has fostered a vibrant community by organizing math-centered social activities, monthly dinners for faculty and students, a public lecture series, and outreach to high school students and teachers. To encourage mathematical communication, faculty and students participate in a colloquium series in which every Williams math major must prepare a talk on a new topic. For the many ways in which the Williams College Department of Mathematics and Statistics has had impact on their students as well as the broader community, we are happy to present the AMS Award for an Exemplary Program or Achievement in a Mathematics Department to Williams College.

DOI: http://dx.doi.org/10.1090/noti1125

Creative Streak: Williams College Math Department Wins National AMS Award

Allyn Jackson



All photos: Cesar E. Silva.

SMALL at 4th of July Parade, 2012.

It was 1988, and Colin Adams was in his third year on the faculty at Williams College. He donned a gaudy green suit plus cowboy boots, mustered a Texas accent, and gave a lecture in his department. In the lecture, “Real Estate in Hyperbolic Space: Investment Opportunities for the Next Millennium”, he played an unprincipled salesman called Mel Slugbate, who was hawking hyperbolic plots to those chary about investing in traditional real estate. It was the first time Adams had ever put on a costume to give a lecture. He was a little nervous. “I was a junior faculty member,” he recalled. “I thought, ‘This is the end of my career right here’.” As it turned out, his department was very supportive of this unconventional way of introducing ideas about hyperbolic space. Since then, Mel Slugbate has made many appearances in mathematics departments and at meetings, and Adams has developed a thriving sideline in mathematical humor. The now-classic “Great π-e Debate”, which he performs with his Williams colleague Thomas Garrity, is available on DVD.

One big reason Adams’s risky venture worked is the open and creative atmosphere of the Department of Mathematics and Statistics at Williams College. The department hires extraordinarily talented people and gives them a great deal of freedom in deciding how best to meet their dual responsibilities in teaching and research. This diverse group of dynamic individuals work together on an enormous variety of programs and activities, and they do so in a harmonious way, keeping their focus trained on the big picture: Communicating, doing, and enjoying mathematics. For their wide-ranging accomplishments and for being an inspiration to mathematics departments around the nation, Williams College is receiving the 2014 AMS Award for an Exemplary Program or Achievement in a Mathematics Department.

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DOI: http://dx.doi.org/10.1090/noti1126

Serving All Students Well

Within the mathematical community, the most famous math major from Williams College is no doubt Curtis McMullen, who earned his bachelor’s degree in 1980 and was awarded the Fields Medal in 1998. But outside the mathematical community, the best-known Williams math major might be Bethany McLean, who finished a double-major degree in math and English in 1992. In 1995, she got a job as a fact-checker at *Fortune* and eventually started writing stories for the magazine. On March 5, 2001, *Fortune* published her bombshell story, “Is Enron Overpriced?”, the first report in a major magazine that raised questions about Enron’s accounting. People at Enron tried to dissuade McLean from digging into the details, telling her she didn’t really understand the numbers. “But she had understood very hard mathematics problems, so she believed she could understand the Enron numbers,” said Cesar Silva, who has been at Williams since 1984. As a math major, McLean had also learned tenacity, which helped her to pursue the Enron story at a time when others believed the company to be above reproach. Today McLean is a visiting scholar at New York University, writes for *Vanity Fair* and Reuters, and often appears on television news programs (see sidebar on page 522).

Around the time McMullen got his degree, Williams had perhaps half a dozen senior majors. By McLean's time, the number had risen to between twenty and thirty. "When we hit forty majors a year, we thought, okay, it's stabilizing," recalled Susan Loepp, who joined the department in 1996. This year, the department has sixty-seven senior majors, about twelve percent of the senior class (nationally, about one percent of seniors are math majors). The department continues to serve well those like McMullen, who are bound for graduate school in mathematics, while also reaching out to students like McLean, who have other aspirations. For both kinds of students, the department tries to spark and stoke a love of mathematics.

The department did not set out to increase the number of math majors. Rather, its goal has been to increase student learning and enjoyment of mathematics. One turning point for the department came about twenty years ago, when it changed its calculus sequence. At the time, the course crammed single- and multi-variable calculus into a single year. Students had "quite a time with it," recalled Frank Morgan, who has been in the department since 1988. Freshmen coming to campus would hear from other students, "Don't take calculus." The department made the course more approachable by rearranging the material to run over three semesters. The course's reputation changed overnight, and now freshmen hear, "You *have* to take calculus."

And they do: Eighty-four percent of Williams students take a course in mathematics or statistics, and about sixty percent complete multivariable calculus. Only a few of these students intend to major in mathematics, but after taking a couple of introductory courses, they are hooked. The department's statistics courses are also highly popular: Sixty percent of the graduating class has taken a statistics course, compared to fifteen percent a decade ago. "We like having a lot of students," Loepp commented. "It adds an energy to the department."

Once the students are hooked on mathematics, the department keeps them hooked by delivering outstanding teaching and offering a wide variety of activities that build a sense of community. Williams has a long tradition of excellent mathematics teaching. These days, in order to be hired at Williams, Morgan said, one's application must have a document containing a statement like, "This is the best teacher I have ever known." Many Williams mathematicians have won teaching awards at the local or national level, and teaching quality as measured in student surveys has risen over the years. When faculty come up for tenure, the quality of their teaching is compared to that of the tenured

SMALL 2013.

faculty, creating a climate in which continual improvement is the norm.

Using what Adams calls an "n-pronged approach", where n is a fairly large integer, the department has created a range of student activities. There are weekly problem-solving dinners, monthly dinners for students and faculty, and an ice cream social in which new students get informal course advising from older students, all the while enjoying ice cream sundaes. There is a weekly math puzzle night, and a monthly "Math Conundrum": A mathematics problem with an unusual twist is posted on the departmental webpage, and students can win prizes for solving it. Faculty member Steven Miller runs a "Math Riddles" website that gets 4,000 hits a month. Students have the opportunity to attend conferences, including the Joint Mathematics Meetings and the Hudson River Undergraduate Mathematics Conference, of which the Williams department is one of the founders. The many activities contribute to students' enjoyment of mathematics and make the department a friendly, sociable place to be. Students often comment on the high level of activity in the department.

Another major activity in the department is the series of colloquia given by senior math majors. The colloquium talk is a requirement for the degree. Each math major works intensively with a faculty member to choose and study a topic and to prepare the lecture. Faculty routinely listen to two or more practice runs of the talk before the student is ready. Students often identify the colloquium lecture as one of their most valuable educational experiences. With so many math majors, scheduling all the colloquia is a logistical challenge, and the department has sometimes had to resort to running talks in parallel sessions. The colloquia are well attended, often with friends and family in the audience, and they create a buzz on campus. "Everyone has a friend who gives a math colloquium," Morgan said.

A group of Williams math majors, mostly seniors, in 2010.

SMALL Program, Large Impact

One day during his first summer at Williams, Morgan attended a science lunch talk and encountered a hundred undergraduates who were working on research projects. "I thought, What a marvelous thing," he recalled. "I knew then that mathematics had to be a part of it." People understood how undergraduates could contribute to research in a lab science—but to mathematics research? This was 1988, and today's proliferation of Research Experiences for Undergraduates (REU) programs in mathematics had not yet occurred, so there were not many exemplars to follow. But Adams and Silva had done a couple of research projects with mathematics students, and they had a sense of what was possible. That same summer Silva, Morgan, Adams, William Lenhart, and David B. Levine submitted a proposal to the National Science Foundation to start an undergraduate research program named SMALL, an acronym of their last names.

Now more than twenty-five years old, SMALL is one of the best known and most successful REUs. It runs for nine weeks over the summer and brings in about thirty students, some of them from Williams, some from other schools around the nation, and even some from abroad. Each SMALL faculty member assembles a group of undergraduate students, chooses a problem for them to work on, and provides guidance and advice as the students collaborate. Many of the groups have published in standard research journals. The students have done research in a wide variety of topics, including knot theory, minimal surfaces, number theory, symmetry groups, combinatorics, graph theory, computational geometry, dynamics and ergodic theory, neural networks, Bayesian statistics, and commutative algebra.

How do the SMALL faculty come up with appropriate problems for the students? Partly it is a matter of experience, and partly it is a matter of keeping one's antennae up, Adams noted. For example, when listening to lectures or talking to people at meetings, "you look for problems where you don't need a lot of background to make progress, or that can be broken down into cases or boiled down to a simpler situation," Adams explained. When one is thinking like this, "it just takes off."

Because SMALL is a well-established program with an excellent reputation, it gets very strong applicants. "It's so much fun for us to work with these students at an early point in their careers," Adams said. Today sixty SMALL alumni are in the process of getting their Ph.D.'s in mathematics, and many others have finished their degrees and hold faculty, postdoctoral, and visiting positions in a variety of institutions. Two of the best-known SMALL alumni are Michael Hutchings of the University of California, Berkeley, and Jeff Brock of Brown University. Several SMALL participants have received distinctions such as the Schafer Prize of the Association for Women in Mathematics and the Morgan Prize for Outstanding Research by an Undergraduate, which is sponsored jointly by the AMS, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics (Morgan's mother endowed the latter prize, which is named after her and his late father).

Susan Loepp served as principal investigator on the NSF grant for SMALL from 2009 to 2013 and directed the program for three of those years. For her SMALL groups, she used to choose the students with the strongest records. "Now, I take those I think will work the hardest," she said. She and the other SMALL faculty have also made a concerted effort in recent years to recruit more students from underrepresented minorities, in particular by forging ties with programs like the Math Alliance and its Field of Dreams conference. Before 2009, SMALL usually had one or two students from underrepresented groups every few years. Today, it is unusual for SMALL to have a year without several such students. About one-third of SMALL participants have been women. "I try to be really supportive of underrepresented students here and elsewhere," said Loepp. "We need to encourage them and let them know they're good even if they think they're not."

Hiring Outstanding People

Loepp grew up in the Midwest and had never heard of Williams until she was on the job market after getting her Ph.D. Many people recommended she apply to Williams, so she did. "I came out and saw all the fantastic things the department was doing," she said. The department clearly valued both teaching and research—and excelled in both. "The faculty were writing books, advising undergraduates, teaching innovative classes," she said. "No one in the department, after getting tenure, was slacking off. Some did even more. That was the kind of energetic department I wanted to be in." Not wanting to be a "trailblazer", as she put it, Loepp was glad that the department already had two women. Today five of the department's fifteen faculty are women.

One of the main reasons for the success of the Williams department is its approach to hiring. "We say, let's hire the best person in terms of teaching and research and not worry about area," Adams explained. "So our pool is maybe eight times as large as at other places. That has served us well." One exception is statistics, where the department made targeted hires in the belief that statistics is one of the most important needs for the college. In 1992, the department hired Richard De Veaux, who had been at Princeton University. "That was a huge hire," Adams remarked. "He is someone who organizes things and moves them forward." The department now has four statisticians, two of them women, and enough faculty to support its new statistics major, established last year.

The department hires serious researchers and expects them to produce. In decisions about hiring and promotion, research and teaching quality are weighted close to equally. The research must consist of published articles in good journals; little credit is given for writing a textbook. (The faculty has nevertheless written a range of textbooks, some of which have been used widely in other institutions and have led to successful teaching innovations.) Faculty in the department are indeed very active in research. Since 2000, they have been awarded a total of twelve National Science Foundation research grants and have published on average a total of nineteen journal articles per year. A weekly department-wide faculty seminar provides a venue to discuss current research, and this seminar has sometimes led to collaborations. The Williams mathematicians and statisticians are in demand as speakers outside the college, delivering each year nearly 200 talks at conferences and other institutions.

Research in the department is closely tied to teaching. "At Williams, we get really strong, smart students," Adams said. "To be a successful teacher here you have to be involved in research and excited about it. The students expect that." The department offers a range of advanced courses in areas such as hyperbolic three-manifolds, transcendental number theory, analytic number theory, Riemannian geometry, ergodic theory, algebraic geometry, and Fuchsian groups. These courses are similar in level to graduate courses. "I couldn't teach such courses without doing research in the area," Adams said. Math majors have the option of writing a senior thesis, and faculty need to be involved in research in order to be able to identify suitable thesis topics.

SMALL 20th anniversary conference.

A department with so many high-powered personalities could easily become fractured. But the Williams department has not. "Somehow, we have gotten faculty who are very dedicated to the department," Loepp noted. "There are no political groups. We have opinionated people, but when we vote, we don't vote according to who we like. We vote on the issue." Sometimes discussions in department meetings become heated, but afterwards, everyone has lunch together. And they don't bicker over the small stuff. A department can easily get embroiled in arguments over, for example, whether calculus should be taught by lecturing, by using "reform" methods, by using computers, and so on. But, Morgan noted, if the focus is on whether students are learning, and faculty are free to use whatever teaching methods they personally find effective, "What's there to argue about?" Whether students are happily learning "is all we care about," he said. "That's all we had to agree on. That is the shared vision."

Building an Academic Community

Olga (Ollie) R. Beaver, a beloved member of the Williams department and recipient of the 1992 Louise Hay Award of the Association for Women in Mathematics, died in 2012. In a memorial tribute to Beaver, posted on the department website, Loepp recounted a story from her first weeks at Williams, when Beaver was department chair. Loepp was heading to a reception for women faculty and stopped by Beaver's office to ask if she was going to the "chick event". "I was immediately horrified that I had mistakenly used the possibly offensive word 'chick' in the presence of my department chair," Loepp wrote. "As I stood paralyzed, trying to figure out how to get out of this one, without missing a beat, Ollie leaned over to me and said, 'Some of us are hens.'"

This anecdote captures Beaver's inimitable charm. An enormously popular teacher, she co-founded the college's new Summer Science Program

(SSP) in 1987, serving as director of the program for ten years and teaching in it until her death. SSP identifies Williams freshmen who express an interest in mathematics and science and who are from underrepresented minorities or are the first in their families to attend college (Williams' policy of need-blind admissions and full need for all American students has brought more such students to campus in recent years). SSP is not a remedial program; the SSP participants are talented, high-achieving individuals.

However, some of them come to Williams with the idea that they have to "go it alone". "The main thing SSP does is develop a sense of academic community," Morgan explained. Over the course of the five-week program, the SSP students are encouraged to ask questions of the instructors and to work together with their peers when they get stuck. "This spirit has been transmitted to other, non-SSP students," Morgan wrote in a letter nominating Beaver for the Hay Award. "Now there are successful role models; students help each other. Word gets around that people care here." The majority of SSP participants opt for science-related majors, and a large number enroll in a mathematics course. Some have gone on to complete the mathematics major.

Through the SSP, Beaver touched the lives of around 500 students, conveying a welcoming and encouraging message and transmitting to them her ethic of hard work and perseverance. Silva, who is originally from Peru, has also taught in the SSP since the mid-1990s; other faculty, including Loepp, have taught in it as well. Calling the program "fantastic", Loepp nevertheless acknowledges that progress in bringing underrepresented students into mathematics has been slow. "It's hard," she said. "We are not the best at it. But we are making progress, and I hope we can continue."

Everyone Loves Math

The faculty of the Williams College Department of Mathematics and Statistics are almost unbelievably energetic. How does the department inspire the faculty to work so hard? "We love what we're doing," Morgan said. "Mathematics is an honor and a thrill to be part of. People come here excited to take their place in what we are doing." With its open, creative atmosphere, the department provides fertile ground for new ideas for pursuing its main goal: communicating and enjoying mathematics. For his part, Silva hopes the department will continue to spread its message: "Everyone loves math, even if they don't know it yet."

Digging into the Numbers

Bethany McLean at SMALL 20th anniversary conference.

Bethany McLean, whose article in Fortune *magazine was the first to report on the Enron fraud, received her bachelor's degree in mathematics from Williams in 1992 (see main article). Asked about how studying mathematics influenced her career, she made the following remarks.*

Math was easy for me until I hit abstract algebra. And then it became impossible. I struggled for Bs and definitely was not going to go on to graduate school. The really great thing about the Williams math department is that they didn't discourage people like me from still majoring in math—in fact, they encouraged it. I think the department has long had the view that they want more students than those who are going to go on to get Ph.D.'s. Math is a relevant degree for so many careers!

I gave a talk at the college a few years ago entitled "Why math made me a better journalist". Part of my argument was that math taught me inescapable logic. I am not naturally a tough, confrontational person. But when A doesn't lead to B, I dig in. I can't get around it any more than I could skip a step in a proof. That's made me ask questions until I get answers.

I also think that it's good to do something in college that isn't easy for you. Math was humbling for me. But I learned what it was like to have to work at something that didn't come easily. That's an incredibly important lesson for any career.

2014 JPBM Communications Award

The 2014 Communications Award of the Joint Policy Board for Mathematics (JPBM) was presented at the Joint Mathematics Meetings in Baltimore, Maryland, in January 2014.

The JPBM Communications Award is presented annually to reward and encourage journalists and other communicators who, on a sustained basis, bring mathematical ideas and information to nonmathematical audiences. JPBM represents the American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. The award carries a cash prize of US$1,000.

Previous recipients of the JPBM Communications Award are: James Gleick (1988), Hugh Whitemore (1990), Ivars Peterson (1991), Joel Schneider (1993), Martin Gardner (1994), Gina Kolata (1996), Philip J. Davis (1997), Constance Reid (1998), Ian Stewart (1999), John Lynch and Simon Singh (special award, 1999), Sylvia Nasar (2000), Keith J. Devlin (2001), Claire and Helaman Ferguson (2002), Robert Osserman (2003), Barry Cipra (2005), Roger Penrose (2006), Steven H. Strogatz (2007), Carl Bialik (2008), George Csicsery (2009), Marcus du Sautoy (2010), Nicolas Falacci and Cheryl Heuton (2011), Dana Mackenzie (2012), and John Allen Paulos (2013).

Citation

The 2014 JPBM Communications Award was presented to DANICA MCKELLAR, an actress (*The Wonder Years, The West Wing*), a published mathematician (while earning her bachelor's degree in mathematics at the University of California Los Angeles), an advocate for mathematics education, and a *New York Times* best-selling author. Her books, blog, and public appearances have encouraged countless middle and high school students, especially girls, to be more interested in mathematics.

Photo by Cathryn Farnsworth.

Danica McKellar

Her books include *Math Doesn't Suck: How to Survive Middle-School Math Without Losing Your Mind or Breaking a Nail* (2007), *Kiss My Math: Showing Pre-Algebra Who's Boss* (2008), *Hot x: Algebra Exposed* (2010), and *Girls Get Curves: Geometry Takes Shape* (2012). Their brilliant presentation of mathematical concepts in ways that relate to young girls have attracted a huge audience that includes both boys and girls. The first three books made the *New York Times* Best Sellers list. Links to all of their webpages are found at `http://www.danicamckellar.com/math-books/`. Her blog *Math & More*, also found on her website, reaches out to the same audience with mathematical puzzles, links to her appearances at book signings, and broadcast promotions. Over the past seven years she may have inspired more young people to embrace mathematics than anyone else.

— *JPBM announcement*

DOI: http://dx.doi.org/10.1090/noti1121

MAA Awards Given in Baltimore

At the Joint Mathematics Meetings in Baltimore, Maryland, in January 2014, the Mathematical Association of America awarded several prizes.

Gung and Hu Award for Distinguished Service

The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award made by the MAA. It honors distinguished contributions to mathematics and mathematical education in one particular aspect or many, whether in a short period or over a career.

The 2014 Gung and Hu Award was presented to Joan Leitzel for her farsighted work of creating programs to decrease the need for remediation in colleges and for her leadership on the national level. Her distinguished career began at Ohio State University, where she led visionary projects that formed parts of a coherent plan to reverse the need for remediation in Ohio.

By 1974, Ohio State, an open-door university, had twice the enrollment in remedial mathematics that it had had ten years earlier. Leitzel, with Bert Waits and Frank Demana, attacked this problem by developing a curriculum for a remedial course that was not a repeat of the typical high school course. The course was centered around problem solving, while including computation and symbolic manipulation. They wanted students to be able to confront more realistic problems and explore situations numerically, relieving classes from focusing on arithmetic. All of this sounds rather routine now, but it certainly was not in 1974 when Leitzel, Waits, and Demana were designing their program. The projects were far ahead of their time in that they collected data and were evidence based.

A second step in the program was teacher preparation. From 1976 to 1979, Leitzel was co-director (with James Schultz) of the NSF-funded project Testing Alternatives in the Mathematics Preparation of Elementary Teachers. As part of this project, they organized three different course sequences for preservice elementary teachers. They concluded that "highly integrated content-methods instruction provided no measurable advantage for students but that coordination of separate courses in content and methods served to improve students' performance in both." Another step was the early placement testing for high school juniors, followed by an appropriate twelfth-grade course to prepare for college mathematics. One problem revealed by early placement testing was that many high school juniors had very limited algebra skills and no appropriate course for them to take in their senior year. With funding from the Battelle Foundation, Leitzel and Demana undertook the project A Numerical Problem-Solving Course for Underprepared College-Intending Seniors. The course they developed took a highly numerical approach to algebra and geometry and very successfully brought students up to competency at the level of intermediate algebra. A final step was the project A Numerical Problem-Solving Approach to Variables and Functions in Pre-algebra for Grades 7 and 8, led by Leitzel, together with Frank Demana and Alan Osborn.

These early articulation programs provided instructional materials and served as models later copied by many universities that now have early placement testing for high school students and a more coordinated mathematical transition from high school to college. One final piece was the course Algebra for Adults, developed by Leitzel with Suzanne Damarin. Data documented that the course was successful in preparing students for engineering calculus and business calculus.

Leitzel joined the Department of Mathematics at Ohio State University in 1965, just after receiving her Ph.D. in mathematics from Indiana University. At Ohio State she served both as a faculty member and as associate provost. She received the Ohio State Distinguished Teaching Award (1982) and the Distinguished Service Award (2002). From 1990 to 1992 she served at the National Science Foundation as the division director for materials development, research, and informal science education. In 1992, she moved to the University of Nebraska-Lincoln, where she served as senior vice chancellor for academic affairs and provost. From

DOI: http://dx.doi.org/10.1090/noti1120

1995 to 1998, she served as vice chair and chair of the Board of Directors of the American Association for Higher Education (AAHE). She also served on the governing board of the National Association of State Universities and Land Grant Colleges and on the American Council of Education's Committee on Leadership and Institutional Effectiveness.

In 1996 Leitzel was appointed president of the University of New Hampshire. She served six years and on April 19, 2002, was awarded the Charles Holmes Pettee Medal in recognition of her contributions to the university. During this period the university experienced unprecedented change. Leitzel developed a new institutional strategic plan, worked tirelessly to raise the level of excellence in academic programs, implemented new and exemplary financial and fiscal management policies, guided the most ambitious capital campaign in university history, and coordinated key renovations and expansions of university facilities. The Joan and James Leitzel Center for Mathematics, Science, and Engineering Education was established in 2002 in honor of Leitzel and her late husband, James R. C. Leitzel. From 2000 to 2004 she chaired the Mathematical Sciences Education Board of the National Research Council, and in 2012 she completed a two-year term as chair of the Conference Board of the Mathematical Sciences (CBMS).

Following retirement from the presidency at the University of New Hampshire, Leitzel returned to Ohio and plunged into administrative duties directly connected to mathematics. She worked with the Ohio Department of Education to design and launch their statewide Mathematics Initiative, which was an extension of her earlier work in Ohio to coordinate mathematics education across the levels of instruction. In 2005 the Ohio Resource Center thanked her for her vision that placed the state of Ohio at the forefront in moving mathematics education from a reliance on remediation to a focus on intervention. Leitzel continues to be a member of the Ohio Mathematics and Science Coalition. She chairs the Ohio Board of Regents' Mathematics Steering Committee and the national advisory board for NebraskaMATH. She is a member of the national advisory board for the Association of Public and Land Grant Universities' Mathematics Teacher Education Partnership. In 2008–2009 she returned to Ohio State for another year and led the restructuring of Arts and Sciences, which resulted in replacing five separate colleges with one large unified college. She has served on many MAA committees, including the Strategic Planning and Design Committee in 2004–2005, which she chaired.

Euler Book Prize

The Euler Book Prize is given to the author(s) of an outstanding book about mathematics. Mathematical monographs at the undergraduate level, histories, biographies, works of mathematical fiction, and anthologies are among those types of books eligible for the prize. The prize was given for the first time in 2007, the 300th anniversary of the birth of Leonhard Euler.

The Euler Prize for 2014 was awarded to Steven Strogatz of Cornell University for *The Joy of x: A Guided Tour of Math, from One to Infinity* (Houghton Mifflin Harcourt, Boston, 2012).

From the one-sentence chapter descriptions in the table of contents to the extensive endnotes with comments and guidance for further reading (many on the Internet), *The Joy of x* is a masterpiece of expository writing.

The book is directed to the millions of readers who claim they never really understood what the mathematics they studied was all about, for whom math was a series of techniques to be mastered for no apparent reason. In succinct chapters the book revisits grade school arithmetic, high school algebra and geometry, and selected topics from undergraduate mathematics. Professor Strogatz writes, "Instead of worrying about the details of these subjects, we have the luxury of concentrating on their most beautiful, important, and far-reaching ideas." This explains the book in a nutshell: to show that these subjects have a beautiful side, a playful side, a mysterious side, and a practical side even in our present-day cyberculture.

But there is so much more to this book, making it highly recommended for every mathematician. The forty-five pages of Notes at the end of the book offer mathematical arguments and sketches of proofs, fresh ideas and interesting anecdotes, and annotated references to online and printed resources, including relevant articles and books published by the Mathematical Association of America. Pedagogically, the book is a model of how mathematics can be presented to a general audience in an appealing and humanizing way. Each chapter begins with an anecdote or story that connects with the reader and rivets attention. The focus is on the ideas, their simplicity, power, and universality. We are impelled to think, but also to feel, to appreciate, to understand, to connect, to relate, to play, to enjoy.

One of the most pleasant aspects of reading the book is the feeling that you're connecting with a real person. Professor Strogatz doesn't take himself too seriously; he exhibits a sense of humor and empathizes with students suffering from information/technique overload. Throughout the book he demonstrates the value of visual thinking, common sense, and making educated guesses over trying to remember anxiety-producing prescriptions for solving problems. This lesson alone makes the book invaluable.

In addition to the school mathematics previously mentioned, the book offers overviews of other subjects of contemporary interest: statistics, probability, Markov chains, number theory, group theory, topology, differential geometry, infinite series, infinity. These topics are introduced and presented in the same compelling manner: relevant, personal, important, far-reaching, fun. *The Joy of x* is a joy to read.

Steven Strogatz is the Jacob Gould Schurman Professor of Applied Mathematics at Cornell University. He studied at Princeton, Cambridge, and Harvard and taught at MIT before moving to Cornell in 1994. His research interests include nonlinear dynamics and complex networks applied to physics, biology, and social science. He also blogs about math for the *New York Times* and has been a frequent guest on RadioLab. A SIAM Fellow and member of the American Academy of Arts and Sciences, he received the JPBM Communications Award in 2007. He is the author of *Nonlinear Dynamics and Chaos, Sync*, and *The Calculus of Friendship*.

Chauvenet Prize

The Chauvenet Prize recognizes a member or members of the MAA for the writing of an outstanding expository article. First awarded in 1925, the prize is named for William Chauvenet, who was a professor of mathematics at the United States Naval Academy.

The 2014 Chauvenet Prize has been awarded to Ravi Vakil of Stanford University for his article "The mathematics of doodling", *American Mathematical Monthly* **118** (2011), no. 2, 116–129. In the article Vakil takes us on an engrossing mathematical journey initiated by this simple exercise. Along the way we learn about the radius r neighborhood $Nr(X)$ of a set X in the plane and how $Nr(X)$ becomes more disklike as r increases. We see how the perimeter of $Nr(X)$ is related to the area of X, first when X is a convex polygon, then when X is any convex set, then when X is arbitrary. We see how the winding number and the Euler characteristic account for the changes in the resulting formulas. We move to three dimensions and encounter Hilbert's Third Problem and the Dehn invariant and to n dimensions and meet other dissection invariants. Finally, our tour culminates in a brief visit to the moduli space of curves. Vakil's elegant yet ever-friendly exposition provides a wonderful framework for this clinic in conjecture, proof, and generalization. The article is an enticing illustration of how mathematical curiosity can lead us from gentle musings to sophisticated, interconnected, and deep ideas.

Ravi Vakil is professor of mathematics and the Packard University Fellow at Stanford University. He is an algebraic geometer whose work touches on topology, string theory, applied mathematics, combinatorics, number theory, and more. He was a four-time Putnam Fellow while at the University of Toronto. He received his Ph.D. from Harvard and taught at Princeton and the Massachusetts Institute of Technology before moving to Stanford. He has received the Dean's Award for Distinguished Teaching, the American Mathematical Society Centennial Fellowship, the Terman Fellowship, a Sloan Research Fellowship, the NSF CAREER grant, and the Presidential Early Career Award for Scientists and Engineers. He has also received the Coxeter–James Prize from the Canadian Mathematical Society and the André-Aisenstadt Prize. He was the 2009 Hedrick Lecturer at MathFest and is currently an MAA Pólya Lecturer. He is a director of the entity running the website MathOverflow and the director of a potential new school in San Francisco called the "Proof School". He works extensively with talented younger mathematicians at all levels, from high school through recent Ph.D.'s.

Beckenbach Book Prize

The Beckenbach Book Prize is named for Edwin Beckenbach, a long-time leader in the publications program of the MAA. The prize is intended to recognize the author(s) of a distinguished, innovative book published by the MAA and to encourage the writing of such books. The award is not given on a regularly scheduled basis. To be considered for the Beckenbach Prize a book must have been published during the five years preceding the award.

The 2014 prize has been awarded to Judith Grabiner of Pitzer College for *A Historian Looks Back: The Calculus as Algebra and Selected Writings* (MAA Spectrum, 2010).

The title of this book is both a tautology and a fitting description of its contents. Historians, by definition, look back. They survey, describe, and interpret that which has come before. In her introductory remarks, Grabiner reminds us of this as she writes, "Mathematics is incredibly rich and mathematicians have been unpredictably ingenious. Therefore the history of mathematics is not rationally reconstructible. It must be the subject of empirical investigation."

Such investigation lies at the heart of this book. But the title also suggests that Grabiner, in giving us a selection of prior writings, is looking back across a distinguished career of mathematical exposition. The book begins with an extended treatment of Lagrange's endeavor to reduce the calculus to algebra. Grabiner argues that he thereby advanced the journey toward rigor in analysis in work that, if not ultimately successful, contributed to Cauchy's triumph a generation later.

In subsequent articles, she examines the ideas of Descartes, Maclaurin, and others, even as she describes the evolution of such mathematical concepts as the limit and the derivative. Because so many pieces are available in this single volume, the reader can discover fascinating interconnections across the history of mathematics. Throughout, Grabiner's scholarship is first rate, and she moves the story along in a fashion that is as informative as it is engaging. And those who know the author's broad interests will not be surprised to encounter Leonardo da Vinci, David Hume, and Walt Whitman—among many others—on the pages of this remarkable book.

Judith V. Grabiner is the Flora Sanborn Pitzer Professor of Mathematics at Pitzer College, one of the Claremont Colleges in California. She is the author of *The Origins of Cauchy's Rigorous Calculus* (MIT Press, 1981) and *The Calculus as Algebra: J.-L. Lagrange, 1736–1813* (Garland Press, 1990). She also is the author of a Teaching Company DVD course called "Mathematics, Philosophy, and the 'Real World'". Grabiner was recently named to the first class of Fellows of the American Mathematical Society. Besides having written many articles about the history of mathematics and history of science and having won several Lester Ford and Allendoerfer awards from the Mathematical Association of America, she received the Deborah and Franklin Tepper Haimo Award for Distinguished College or University Teaching from the MAA in 2003.

David P. Robbins Prize

This prize was established in memory of David P. Robbins by members of his family. Robbins, who died in 2003, received his Ph.D. in 1970 from MIT. He was a long-time member of the Institute for Defense Analysis Center for Communication Research and a prolific mathematician whose work (much of it classified) was in discrete mathematics. The prize is for a paper that reports on novel research in algebra, combinatorics, or discrete mathematics; has a significant experimental component; and is on a topic which is broadly accessible. The paper shall provide a simple statement of the problem and clear exposition of the work. This prize is awarded every three years.

The 2014 Robbins Prize was awarded to Frederick V. Henle and James M. Henle for "Squaring the plane", *American Mathematical Monthly* **115** (2008), no. 1, 3–12.

The problem is simple. You are supplied with infinitely many square tiles, but they all have different sizes; in fact there is exactly one n-by-n square for each positive integer n. Your task is to use these squares to tile the plane, no overlaps or gaps allowed, and you must use all of the squares. A traditional tiling uses many congruent copies of the same tile or a few tiles. Now, no two tiles are alike.

Inspired by a paper of William Tutte in 1950 showing that a square can be tiled by finitely many different smaller squares, Solomon Golomb (recent winner of the U.S. National Medal of Science) posed the question in 1975 of whether the plane can be tiled with the integer squares. Shortly after, it was picked up by Martin Gardner in his *Scientific American* column, and several partial results appeared in the intervening years.

With only meager progress, some began to think such a tiling was not possible. But it is.

This delightful article gives a complete description of a tiling of the plane using one square of each integral side. The argument itself does not bring in any "big guns" to settle the problem; rather, it uses "big ingenuity", which is always preferable. As such, the paper is completely accessible to undergraduates. The article closes with a number of intriguing open problems; we hope that this award will help call attention to them.

Frederick V. Henle received his baccalaureate from Harvard University in 1992 and his master's degree from Dartmouth College in 1997. He has taught mathematics and computer science at Mercersburg Academy, played in the first violin section of the Maryland Symphony Orchestra, and is now a lead developer at athenahealth, Inc. Work on this paper and subsequent papers with his father, Jim, has been both personally and professionally fulfilling and an experience that he hopes one day to share with his children.

James M. Henle earned his baccalaureate degree from Dartmouth College in 1968 and his Ph.D. from MIT in 1976. Early in his career he taught at the University of the Philippines and at Burgundy Farm Country Day School, but for most of his professional life he has been a member of the faculty at Smith College. He credits his mathematical awakening and development to his high school teacher Richard Jameson; to Dartmouth logician Donald Kreider; to his thesis advisor, Gene Kleinberg; to his Smith colleagues Marjorie Senechal and Joe O'Rourke; to the columns of Martin Gardner; and, most importantly, to his brother, Michael Henle. He counts over two dozen collaborators on his research papers. The most frequent have been his academic siblings Carlos Di Prisco, Arthur Apter, and Bill Zwicker; the most significant is his son, Fred.

Haimo Awards for Teaching

The Deborah and Franklin Tepper Haimo Awards for Distinguished College or University Teaching were established in 1991. These awards honor college or university teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has been shown to have had influence beyond their own institutions.

The 2014 Haimo Awards were presented to Carl Lee (University of Kentucky), Gavin LaRose (University of Michigan), and Andrew Bennett (Kansas State University). The following is taken from their prize citations and biographical information.

Carl Lee is an outstanding teacher who has made substantial contributions to student learning at all university levels. He has developed and taught innovative general education courses and has inspired mathematics majors with upper-level undergraduate courses. He has supervised Ph.D. students and has mentored faculty colleagues. He is a leader at improving mathematics education throughout the state of Kentucky and across the United States.

Lee's students commend him for his impeccable lectures, for brilliantly crafted examples that illustrate subtleties and connections among branches of mathematics, and for devising assignments that challenge and enlighten students. He has developed and taught a successful and popular general education course at the University of Kentucky titled Introduction to Contemporary Mathematics. He also taught a freshman seminar course that introduced students from across the university to his research area of polyhedra. Students in this course impressively demonstrated their understanding and careful thinking about the topic by creating striking visual images of polyhedra.

Lee has also inspired mathematics majors with his upper-level course offerings and independent studies. At the graduate level, Lee has supervised the Ph.D. dissertations of thirteen students, most of whom now hold faculty positions at a variety of institutions and credit Lee with their enthusiasm for and commitment to teaching mathematics effectively.

Lee's efforts at preparing preservice and in-service teachers of mathematics are remarkable. Much of his time and talent at the University of Kentucky have been invested in developing, teaching, and disseminating courses in geometry and in mathematical problem solving for prospective teachers of middle school mathematics. Moreover, Lee has been a driving force throughout Kentucky and the Appalachian region for improving mathematics and science education at K–12 levels. He has served as a principal contributor on several large, multi-institution-funded projects that have fundamentally and profoundly influenced the teaching and learning of mathematics for tens of thousands of students across the Appalachian region.

Lee grew up in an extended family of academics. One of his earliest memories of his love of mathematics was in second grade, when his mother taught him how to multiply with a slide rule. As he grew older, he devoured his father's recreational math books, encountering flexagons, polyhedra, stitchings of conic sections, and many more lifelong friends. Gardner, Steinhaus, Ball and Coxeter, and Cundy and Rollett were his silent mentors who complemented his wonderful public school teachers in Baltimore County. He couldn't find the polyhedra in college (Yale) but learned where they were lurking in graduate school (Cornell, 1981, Applied Mathematics), and now he surrounds himself (sometimes physically) with higher-dimensional ones. He was welcomed by the Department of Mathematics at the University of Kentucky in 1980, where he has found a supportive environment for his interests in discovering, teaching, learning, and playing with mathematics. He was an IBM Postdoctoral Research Fellow, an Alexander von Humboldt Fellow, a recipient of a Provost's Outstanding Teaching Award, and is presently a Chellgren Endowed Professor, continuing investigations into polyhedral and discrete geometry while engaged in mathematics education and outreach projects.

Gavin LaRose is an exceptional and passionate teacher who instills in students a desire to study and learn mathematics. He is also an instructional technology guru who provides invaluable help to his colleagues for using technology effectively in teaching mathematics. He is as committed to teaching as to mathematics itself, leading him to inspire colleagues at his own institution and across the country to improve their teaching of undergraduate mathematics.

Student after student from LaRose's classes attest with admiration to his excitement about mathematics, his enthusiasm for teaching, and his commitment to helping students succeed in learning mathematics. Moving testimonials about LaRose's inspiring teaching come not only from his students majoring in mathematics but also from students in a variety of fields that make use of mathematics. Many of these students commend LaRose's enthusiasm for stimulating their excitement about mathematics.

Colleagues point out that LaRose has a large and positive impact not only on his own students but on virtually all undergraduates at the University of Michigan by virtue of his work with instructional technology. For example, nearly all students take an online mathematics placement exam developed and maintained by LaRose that plays an important role in students' orientation and advising experiences. Students also experience his handiwork with instructional technology as they interact with online assessment tools and with review/tutorial modules that he not only created but continually maintains and improves. His colleagues say that they hope to retire before LaRose does, because his instructional technology tools are so important in their teaching and so effortless to use. Moreover, and most tellingly, his colleagues point out that LaRose devotes so much of his time and talent to developing these tools for one reason only: to help students learn.

Another of LaRose's important contributions, both at his own institution and nationally, concerns preparing and mentoring instructors of undergraduate mathematics. He is involved with all aspects of his department's extensive training and mentoring program for new faculty, offering insights and support that benefit instructors and course coordinators to the ultimate benefit of all of their students. This mentoring activity extends far beyond the University of Michigan, as LaRose has served on the MAA Project NExT (New Experiences in Teaching) leadership team in various capacities, most recently as associate director. In this role LaRose conveyed information and ideas to a new generation of college professors about how to teach undergraduate mathematics effectively.

LaRose is jointly appointed as a lecturer and instructional technology manager in the Department of Mathematics at the University of Michigan. He received his B.A. from Grinnell College and his Ph.D. from Northwestern University, where he worked in the applied mathematics program. He worked at Nebraska Wesleyan University for six years before moving to Michigan. His research interests are in the mathematical modeling of real-world systems and, informally, in understanding the impact of pedagogical strategies (and technological tools) on student learning. Among a handful of other publications, he is a coauthor of the book *Writing Projects for Mathematics Courses (Crushed Clowns and Coffee to Go)*. He was a Project NExT Fellow in 1994–1995 and served as an associate codirector and subsequently associate director of Project NExT between 1997 and 2012.

Andrew Bennett excels in the teaching of mathematics in the broadest sense and at all levels: K–12, undergraduate, graduate, and postdoctoral. His teaching prowess is not limited by classroom walls, and his influence on teaching and learning extends far beyond the considerable impact on students at Kansas State University.

Bennett is an outstanding teacher who has taught and left his mark on nearly every course in the undergraduate mathematics curriculum at Kansas State, from elementary service courses to specialized courses for mathematics majors. He has been a pioneer and driving force in the use of technology in mathematics classrooms at Kansas State and beyond. At his home institution he has successfully developed Web-based and online homework systems for a variety of mathematics courses, initiated a Studio College Algebra course that emphasizes hands-on activities and computer work and that has shown improved retention rates compared to traditional offerings, and introduced computer lab components into courses in differential equations back in the 1980s when such innovations had few precedents. At the regional and national levels he has organized conferences, sessions, and panel discussions that have helped to shed light on how mathematicians can use technology as an effective teaching tool.

Six undergraduate students who went on to receive Goldwater Scholarships conducted research under Bennett's supervision. He has also advised master's and Ph.D. students in diverse areas, such as harmonic analysis, probability theory, and mathematics education. Bennett has also served as a mentor for ten postdoctoral fellows.

As founder and director of the Center for Quantitative Education at Kansas State, Bennett has improved instruction across campus by using data-mining techniques to study how students interact with online homework systems to gain insight into student learning. Bennett is one of the first in the world to use such sophisticated methodology to study and improve student learning, research for which he has been recognized with substantial grant funding.

He has also led remarkable outreach efforts to improve K–12 education in mathematics. These include conducting workshops for K–8 teachers, funded by the U.S. Department of Education, that involve teachers taking challenging mathematical content courses that model effective pedagogy while also taking a mathematical pedagogy course that considers how to incorporate such content into the classroom.

Andrew Bennett earned a B.S. in mathematics from Colorado State University in 1981 and received his Ph.D. in mathematics from Princeton University in 1985. After a postdoctoral appointment at the University of Texas at Austin, he came to Kansas State University in 1988, where he has been ever since. He has received over $19 million in extramural funding for work with teachers, faculty, students, and research in mathematics and science education. He was the founding director of the Center for Quantitative Education at Kansas State University and is currently serving as head of the mathematics department. He has served a term on the MAA Board of Governors and has also served as chair of the MAA Subcommittee on Curriculum Renewal and the First Two Years (CRAFTY). He has been married to his high school sweetheart for thirty-one years, and they have two children, a daughter (A.B. in math from Chicago) and a son (B.S. in math from Michigan). His hobbies include ballroom dance and sleeping in front of sports on TV.

Certificates for Meritorious Service

Each year the MAA presents Certificates for Meritorious Service for service at the national level or to a section of the MAA. Those honored in 2014 are: Lowell Beineke (Indiana University-Purdue University), Indiana Section; Stan Chadick (Northwestern State University of Louisiana; Louisiana School for Math, Science, and the Arts), Louisiana-Mississippi Section; Aparna Higgins (Project NexT, University of Dayton), Ohio Section; Tina Straley (Kennesaw State University), Southeastern Section.

—MAA announcement

AWM Awards Given in Baltimore

The Association for Women in Mathematics (AWM) presented several awards at the Joint Mathematics Meetings in Baltimore, Maryland, in January 2014.

Schafer Prize

The Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman was established in 1990. The prize is named in honor of Alice T. Schafer, one of the founders of AWM and one of its past presidents. Schafer passed away in September of 2009.

The 2014 Schafer Prize was presented to Sarah Peluse, a senior mathematics major at the University of Chicago. She is hailed by the faculty there as one of the "top five undergraduates in forty-nine years". Peluse transferred to the University of Chicago in 2011 from Lake Forest College and has gone on to take a rigorous curriculum of advanced mathematics courses. In one reading course, she gave a "seminar-quality presentation at the board" each week, skillfully fielding questions on extensions and applications of the material and discussing current research. She is currently working as a research assistant to a faculty member in the area of model theory.

Peluse attended an REU at Williams College; her work there resulted in a talk and poster at the Joint Mathematics Meetings in 2012. She also attended an REU in number theory at Emory University in 2012 and 2013 and was recognized as a "true star". At Emory, she worked on problems concerning lacunary q-series, irreducible representations of $SU(n)$ which have prime power degree, and zeros of Eichler integrals of cusp forms. This work has resulted in one published article and others submitted for publication.

Peluse is described as having impressive creativity and the capability to obtain deep understanding of sophisticated material on her own. Peluse's recommendation letters praise not only her "impressive talent" but also her motivation, saying that she is a "ferocious worker" who "has a drive…only observed in a few top people". She is viewed as a "future superstar".

In addition, Morgan Opie of the University of Massachusetts, Amherst, was honored as runner-up for the Schafer Prize. Shiyu (Jing Jing) Li of the University of California Berkeley and Jesse Zhang of the Massachusetts Institute of Technology were awarded honorable mentions.

DOI: http://dx.doi.org/10.1090/noti1119

Louise Hay Award

Established in 1991, the Louise Hay Award for Contributions to Mathematics Education recognizes outstanding achievements in any area of mathematics education. Louise Hay was widely recognized for her contributions to mathematical logic and her devotion to students.

The 2014 award was presented to Sybilla Beckmann, Josiah Meigs Distinguished Teaching Professor of Mathematics at the University of Georgia, in recognition of her vision, persistence, and leadership in enhancing the teaching and learning of mathematics in this country and beyond. Her work is based on her insight that sustainable improvement in mathematics education can only occur when the mathematical culture in the schools and the universities is "built on respect for the innate mathematical abilities that are the birthright of every student." She has worked to energize every link of this chain, from the daily challenges that teachers face in their classrooms to the highest levels of the national discussions of K–12 education.

Beckmann has made substantial contributions to Galois theory. She began her career as a Gibbs Instructor at Yale University and has been at the University of Georgia since 1988. More bravely, she taught sixth grade for a year and volunteered at another elementary school where she "started a math revolution." Her redesigned mathematics courses for prospective elementary teachers led to her highly regarded and widely adopted textbook, and she created the Mathematicians Educating Future Teachers program. She was a writer of the NCTM's Curriculum Focal Points for PreKindergarten through Grade Eight and two supplemental books. She played a significant role in writing the Common Core State Standards in Mathematics and was the lead writer on the elementary grades for the *Mathematical Education of Teachers II.*

M. Gweneth Humphreys Award for Mentorship of Undergraduate Women in Mathematics

This award is named for M. Gweneth Humphreys (1911–2006). Humphreys graduated with honors in mathematics from the University of British

Columbia in 1932, earning the prestigious Governor General's Gold Medal at graduation. After receiving her master's degree from Smith College in 1933, Humphreys earned her Ph.D. at age twenty-three from the University of Chicago in 1935. She taught mathematics to women for her entire career. This award, funded by contributions from her former students and colleagues at Randolph-Macon Woman's College, recognizes her commitment to and her profound influence on undergraduate students of mathematics.

The 2014 award was presented to WILLIAM YSLAS VÉLEZ of the University of Arizona. He is legendary for his ability to encourage women to study mathematics and pursue mathematical careers. Particularly impressive is his success in instilling confidence in first-generation and minority students who are often struggling to overcome expectations based on culture and gender. At an early stage, Vélez identifies and recruits students he believes would benefit from taking more math courses. Numerous women describe how he met with them their first days on campus and got them thinking about degree and career paths. Others gratefully express how he completely changed their academic horizon when he pulled them aside and urged them to consider graduate studies in mathematics. Many appreciate how he listened carefully to their interests and guided them to attain well-matched research experiences. He challenges his students to step out of their comfort zones so they can achieve greater success. One former student writes: "I catch myself encouraging others to obtain an education and specifically that they should consider a degree in mathematics.… I have experienced firsthand how much impact one person alone can have on a student's academic and professional life, and I hope to be to other students what Dr. Vélez was to me."

AWM Service Award

Two women were presented with the AWM Service Award, which recognizes individuals for helping to promote and support women in mathematics through exceptional voluntary service to the AWM. The award is given annually to one or two AWM members in recognition of their extensive time and effort devoted to AWM activities during the previous seven years. The 2014 awardees are TAI MELCHER of the University of Virginia and KATHARINE OTT of the University of Kentucky. They were recognized for their service as principal investigators on the NSF Sonia Kovalevsky Day grant, as well as for organizing AWM activities at the 2012 USA Science and Engineering Festival.

— AWM announcement

About the Cover

From the garden of spectrahedra

This month's cover image was suggested by Cynthia Vinzant's article "WHAT IS… a spectrahedron?" in this issue. The image was produced by Vinzant, but is based on a similar image produced by Pablo Parrilo for the article "Semidefinite representation of the k-ellipse" (*Algorithms in Algebraic Geometry*, 2008) jointly written by him together with Jiawang Nie and Bernd Sturmfels.

We asked Sturmfels to comment on it. He replied:

"Fix three general points in the plane and consider the set of all points (x,y) whose distance sum to the three given points is a certain constant d. This set is a convex curve, known as a 3-ellipse, or an ellipse with three foci. The interior of the 3-ellipse is a spectrahedron, that is, a convex set belonging to the class of objects introduced in Cynthia Vinzant's article. Such higher ellipses and their connections to optimization theory are studied in the article I wrote with Nie and Parrilo.

Spectrahedra are the feasible regions in semidefinite programming. This includes convex polyhedra,which are the feasible regions in linear programming. Both convex polyhedra and spectrahedra can be quite beautiful. While the description of convex polyhedra is based on linear algebra and combinatorics, the boundaries of spectrahedra are usually nonlinear. Their study requires some use of nonlinear algebra and algebraic geometry.

The boundary of the 3-ellipse is an algebraic curve of degree 8. It is represented by a unique (up to scaling) irreducible polynomial $P(x, y, d)$ of degree 8 in three unknowns. The diagram shows the surface $P(x, y, d) = 0$. The convex region seen in the middle is a three-dimensional spectrahedron. Slicing the surface with the horizontal plane at height d gives the planar 3-ellipse for radius d. The other branches of this irreducible surface illustrate the algebraic structure inherent in semidefinite programming."

We wish to thank Vinzant, Sturmfels, and Parrilo for assistance.

—Bill Casselman
Graphics Editor
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2014 Mathematics Programs That Make a Difference

Each year, the AMS Committee on the Profession (CoProf) selects outstanding programs to be designated as Mathematics Programs That Make a Difference. For 2014 CoProf selected two programs: the Carleton College Summer Mathematics Program (SMP) and Rice University Summer Institute of Statistics (RUSIS)[1].

Citation: Carleton College SMP

Be it resolved that the American Mathematical Society and its Committee on the Profession recognize the Carleton College Summer Mathematics Program (SMP) for its significant efforts to encourage women to continue in the study of mathematics.

Since 1995 the Carleton College Summer Mathematics Program for undergraduate women has been working to increase the probability that the participants will attempt and achieve advanced degrees in mathematics. To date, fifty-six SMP alumnae have earned Ph.D.'s and seventy-one are currently enrolled in graduate programs in the mathematical sciences. The summer program consists of four weeks of intense coursework, seminars, and group discussions all aimed to challenge the participants and provide them with a network of peers and mentors. The program has expanded to include many activities for its extensive network of alumnae. Each summer there is a three-day reunion conference and several recent Ph.D.'s are invited back for two to three weeks to work intensively on their own research programs while serving as mentors to that summer's participants. Through all of their activities, the organizers of SMP have been able to form an impressive vertically integrated network of support and mentoring for and by the members of the SMP community. Alumnae of the program remark on the profound and lasting impact their participation in SMP has had on their careers. They are active, engaged mathematicians who also recognize the importance of mentoring other women in the field.

The AMS commends the Summer Mathematics Program for its high level of commitment and successful efforts to improve diversity in the profession of mathematics in the United States.

About Carleton College SMP

"Even today, I regularly feel the effects of the four weeks I spent in Northfield, Minnesota, seventeen years ago," Suzanne Boyd, Associate Professor of Mathematical Sciences, University of Wisconsin-Milwaukee, commented in her letter of support nominating the Carleton Summer Mathematics Program for Women (SMP) for the AMS Programs That Make a Difference award. In 1995, during the summer between her sophomore and junior years of college, Boyd was a participant in the first year of the Carleton program. "During those four weeks,'' Boyd summarized, "I found my research area (dynamical systems) and became connected to a network of peers—women in mathematics. That network has continued to grow." That "network" now includes more than 300 other women who have benefited from similar experiences at SMP.

Nearly twenty years ago, Carleton Professors Deanna Haunsperger and Stephen Kennedy launched SMP as a summer opportunity to advance undergraduate women with an interest in mathematics. In 2013 SMP celebrated its seventeenth anniversary. In the intervening years, SMP has welcomed 308 "official" undergraduate participants along with dozens of other women serving as instructors, TAs, speakers, and other guests. Of the 308 summer participants, sixty-two now hold Ph.D.'s (while many are still currently enrolled as undergraduate and graduate students). Based on past trends, Haunsperger and Kennedy project that, out of the eighteen students who take part each summer, eight will go on to finish a Ph.D. in mathematics. That is a striking statistic in light of national trends (see, for example, "Women in science, technology, engineering and math (STEM): A fact sheet" by Kristine De Welde, Sandra Laursen, and Heather Thiry, Sociologists for Women in Society, 2007.)

How do they do it? SMP invites women who have completed one or two years of undergraduate work to the Carleton College campus each summer for four weeks of classes and activities. The SMP students love math, but when they first arrive at the program, most have no understanding of what it means to be a mathematician, let alone what it would take to become one. During the program,

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[1] *This program has now moved to the University of Nevada at Reno and has been renamed Research for Undergraduates Summer Institute of Statistics at the University of Nevada at Reno,* `RUSIS@UNR`.

DOI: http://dx.doi.org/10.1090/noti1129

the directors, faculty, teaching assistants, and mathematicians in residence provide detailed information about exactly what the students need to do over the next few years in order to prepare for graduate school and succeed in completing a Ph.D. in mathematics.

The students take two intense courses to prepare them for the rigors of graduate study. These classes are designed to (i) introduce a student to an area of mathematics that she would probably not find at her home institution, (ii) provide an intensely challenging experience, and (iii) offer a meaningful opportunity for collaborative learning. Women faculty with strong research programs and distinguished teaching records offer these courses that motivate students to learn advanced mathematics. Over the years, SMP class topics have included Morse theory, coding theory, game theory, fuzzy logic, low-dimensional dynamics, knots, and topology.

Beyond the classroom instruction, SMP students engage in a variety of social activities (ranging from home-cooked dinners to an excursion to the Mall of America in nearby Minneapolis), attend biweekly colloquia on a variety of topics in mathematics, and hear panels on practical topics for navigating a career in mathematics. Mary Ellen Rudin gave an SMP colloquium, for example. (Imagine that? An eighteen-year-old learning about topology for the first time in her life from Mary Ellen Rudin!) Joe Gallian, who has run an REU program at the University of Minnesota-Duluth since 1977, is the lone male speaker on the program and often offers his insights on REUs. SMP makes a point to include colloquia on applied mathematics and the history of mathematics. Panel topics include "Making the most of your mathematics major," "Applying, surviving and succeeding at graduate school," and "Nonacademic careers in mathematics." Consequently, SMP provides students with an idea of how to take their immediate next step with finesse and with a vision for what is possible in the long run for a career in mathematics.

The four weeks at Carleton are just the beginning of a rich experience in mathematics for these young women. SMP is structured to bring together alums of the program at different stages of their education and development as mathematicians. Alums who are now in graduate school serve as teaching assistants in the program, while those with Ph.D.'s participate in SMP as speakers and panelists at the annual summer reunion conference (cleverly known as the SMPosium) or as Mathematicians in Residence (MiRs). These former SMP students serve as mentors and role models for the current students. They help current students imagine themselves several years down the road to becoming mathematicians. In this way, the program has maximum impact on its current students, while simultaneously continuing to mentor, advise, and offer opportunities to its former students. By living, working, and playing closely together, students create bonds with each other, with SMP alums, the SMP instructors, and the SMP directors. These relationships continue to foster a supportive community that former SMP participants can rely on throughout their careers. More recently, SMP added a day-long Graduate Education Mentoring workshop (GEM) during the Joint Mathematics Meetings each year for alumnae currently enrolled in Ph.D. programs. Several senior women members of the SMP community (past instructors or visitors, some of the older Ph.D. alums) organize discussions and listen to and provide constructive critiques of talks by graduate student participants.

Karen Lange, assistant professor of mathematics at Wellesley College, views the Carleton SMP as "one of the few constant sources of support throughout my mathematical career, from my undergraduate days to the present." She attended SMP in 1999, the summer after her freshman year of college. "At that point," as Lange put it, "I was far from committed to a career in mathematics. I loved the course work and thrived in the enthusiastic and supportive atmosphere; I left certain I wanted to continue studying mathematics. The program opened my eyes to other resources and opportunities, which I then took advantage of throughout my college years. I cannot emphasize enough how important the SMP network, which is continually growing and being strengthened, has been to my career. After completing a Ph.D. at the University of Chicago and an NSF postdoctoral fellowship at the University of Notre Dame, I am now in a tenure-track position at Wellesley College. At conferences I regularly run into many SMP alumnae. When I am in need of any kind of advice, whether related to a job search, work-life balance, building a research program, or teaching, I have the network to turn to, and similarly, the network makes me available to support and encourage others. SMP has an incredible impact on those who have been lucky enough to be a part of it.''

—Della Dumbaugh,
University of Richmond

Citation: RUSIS

Be it resolved that the American Mathematical Society and its Committee on the Profession recognize the Rice University Summer Institute of Statistics (RUSIS) for its significant efforts to encourage underrepresented minorities and women to continue in the study of mathematics.

The mission of the Rice University Summer Institute of Statistics is to train and mentor underrepresented minority undergraduate students, in particular those students without easy access to career experiences at their own institutions. The fundamental aim of the program is to encourage more students to pursue graduate degrees in mathematical and statistical sciences and to facilitate their transition into research. Since RUSIS began in 2001, 83 percent of the participants who have graduated from college have gone on to pursue graduate degrees. This is particularly remarkable because, upon entrance into the program, very few students report considering graduate school as an option for their future. As the alumnae of the program report, RUSIS has a tremendous impact on their professional lives by providing them with a challenging research experience coupled with intensive conversations about their professional and academic futures. Additionally, it is making a profound impact on the numbers of underrepresented minorities in the mathematical sciences. Over the eleven years of the program, 61 percent of the participants have been underrepresented minorities and 53 percent have been female. Three minority participants have received Ph.D.'s and another twenty-five are currently enrolled in Ph.D. programs; many others have completed Master's degrees. As the first REU in statistics, RUSIS has served as a model program for others to emulate, both by encouraging undergraduates to pursue graduate studies in the mathematical sciences and for increasing the numbers of underrepresented minorities and women in mathematics and, in particular, statistics.

The AMS commends the Rice University Summer Institute of Statistics for its high level of commitment and successful efforts to improve diversity in the profession of mathematics in the United States.

About RUSIS

Today, the need for professionals with high-level quantitative training is increasing in all areas, from medicine to engineering to communications. For work in such areas, a doctorate in statistics opens doors to a broad variety of excellent professional opportunities; yet, many students with an aptitude for studying statistics are unaware of the great opportunities the field offers.

The Rice University Summer Institute of Statistics (RUSIS) stimulates students' interest in statistics by exposing them to actual research problems in the field and by providing guidance and encouragement to pursue a Ph.D. in the mathematical sciences. Established in 2003 by Javier Rojo at Rice University, the program has now moved to the University of Nevada at Reno, where Rojo has taken a position as chair of the mathematics and statistics department. The program has been renamed Research for Undergraduates Summer Institute of Statistics at the University of Nevada at Reno, `RUSIS@UNR`.

Rojo spent seventeen years at the University of Texas at El Paso, where he founded a math club for undergraduates and worked hard to recruit students from underrepresented groups. In 2001, he moved to Rice University, and two years later he launched RUSIS, with funding from the National Science Foundation and the National Security Agency. As the country's first Research Experiences for Undergraduates program focusing on the field of statistics, RUSIS has been phenomenally successful: Among the RUSIS participants who have now graduated from college, 83 percent have gone on to graduate school. This high percentage is especially remarkable because questionnaires filled out by students at the start of the program indicate that, before participating in RUSIS, only a few had considered graduate school.

Initially, RUSIS had seventeen to nineteen participants each year, and for the past three years it has had twelve students. The program runs for ten intense weeks over the summer. To be admitted to RUSIS, students must typically have done well in a set of prerequisite courses in mathematics. But the program also keeps the door open to accepting students who do not quite meet the academic requirements but who exhibit the kind of perseverance and creativity that signal potential for success in graduate school.

Because the students typically come to RUSIS with varied backgrounds, the mornings of the first three weeks of the program are devoted to an intensive course in probability, stochastic processes, statistical inference, and survival analysis. In the afternoon, students are given a course in computation, which takes place in a computer lab. Here the students gain a working knowledge of computer software needed for their later work in the program. Throughout this period, students are closely supervised and mentored by faculty, who provide intellectual challenges and encouragement, as well as practical information about how to apply to and succeed in graduate school.

During the last six weeks of the program, students form groups and participate in at least one research project in which they analyze data, run computer simulations, develop algorithms, and, when appropriate, engage in theoretical work. With help from postdoctoral associates and graduate students, RUSIS mentors work closely with the students to provide background material specific to the projects, as well as research direction necessary as the projects evolve.

Most of the projects focus on understanding, developing, and assessing the merit of new methodologies in the areas of multivariate survival analysis, multivariate extreme value theory, analysis of microarray data, analysis of massive data sets, and other biomedical and statistical problems. These areas present a wealth of interesting questions, many of which are accessible to undergraduates. Problems of current interest are used to motivate the students and serve as a point of departure for the research projects. Students are expected to prepare presentations about their work, for delivery in the final week of RUSIS and sometimes in national meetings. When the work is of sufficient merit, students have written research papers for submission to professional journals.

Throughout the program, RUSIS draws on centers of research and engineering in the Houston area—such as the MD Anderson Cancer Center, the Michael E. DeBakey Department of Surgery at the Baylor College of Medicine, and the University of Texas Health Sciences Center at Houston—to put students in contact with individuals who are doing exciting work. Researchers from these centers come to RUSIS to discuss with the students such topics as survival analysis applications in cancer research, liver transplants, and other health-related and environmental applications. Students pay a visit to the Anderson Cancer Center, and they also spend a day touring NASA's Johnson Space Center, where they can operate the flight simulators and visit Mission Control.

In the tenth week, the six-person RUSIS advisory committee, consisting of mathematical scientists from a variety of colleges and universities, visits the program. Students present their work to the committee and interact with the committee members. In addition, the committee members deliver short lectures highlighting how statistics plays a role in their disciplines and providing new perspectives on science and engineering and on career opportunities. Students often comment that these lectures are inspirational. In addition, the students meet with the advisory committee, without RUSIS personnel in attendance, to provide feedback on ways to improve the program.

While open to all students, RUSIS places special emphasis on recruiting students from groups traditionally underrepresented in mathematics and statistics. In this regard, RUSIS has been very successful: Of the 177 RUSIS participants to date, 61 percent are from underrepresented minorities, and 53 percent are female. Of the minority RUSIS alumni, three have obtained Ph.D. degrees and ten have obtained M.S. degrees. Twenty-five minority RUSIS alumni are currently in Ph.D. programs, and many of these are within one or two years of completing their doctorates.

RUSIS has also had a positive impact on the statistics department at Rice University. Before 2001, when RUSIS started, only one student from an underrepresented group had received a Ph.D. in the department. Since 2001, the department has awarded the Ph.D. to eleven individuals who are from underrepresented groups. The success, energy, and excitement engendered by RUSIS has created a change in the departmental culture.

Of course, the real success of RUSIS lies in the way it has transformed the individual lives of the participants. Several RUSIS alumni wrote letters in support of the nomination of RUSIS to receive the Mathematics Programs That Make a Difference award. One of them is Raymundo Navarrette, a RUSIS alumnus who is now a graduate student at the University of Michigan. Navarrette is a U.S. citizen who grew up with working class parents in Mexico. Within his family background, his obtaining a bachelor's degree at the University of Arizona was already a huge accomplishment. Graduate school had not occurred to him until he attended RUSIS. "Through brilliant and very sincere talks given by the RUSIS director Dr. Javier Rojo and other invited speakers, I was convinced that a doctorate degree would benefit me intellectually and economically and would allow me to do more for my community," Navarrette wrote. "Additionally, the RUSIS program never failed to increase my appetite for more advanced knowledge and to inspire me to dream big."

— *Allyn Jackson,*
Notices *Deputy Editor*

About the Award

CoProf created the Mathematics Programs That Make a Difference designation in 2005 as a way to bring recognition to outstanding programs that successfully address the issue of underrepresented groups in mathematics. Each year CoProf identifies one or two exemplary programs that:

1. aim to bring more individuals from underrepresented minority backgrounds into some portion of the pipeline beginning at the undergraduate level and leading to advanced degrees in mathematics and professional success, or retain them in the pipeline;
2. have achieved documentable success in doing so; and
3. are replicable models.

Previously designated Mathematics Programs That Make a Difference are: the graduate program at the University of Iowa and the Summer Institute in Mathematics for Undergraduates/Research Experience for Undergraduates at Universidad de Puerto Rico, Humacao (2006); Enhancing Diversity in Graduate Education (EDGE) and the Mathematical Theoretical Biology Institute (2007); the Mathematics Summer Program in Research and Learning (Math SPIRAL) at the University of Maryland and the Summer Undergraduate Mathematical Science Research Institute at Miami University (Ohio) (2008); the Department of Statistics at North Carolina State University and the Department of Mathematics at the University of Mississippi (2009); the Department of Computational and Applied Mathematics at Rice University and the Summer Program in Quantitative Sciences, Harvard School of Public Health (2010); the Center for Women in Mathematics at Smith College and the Department of Mathematics at North Carolina State University (2011); the Mathematical Sciences Research Institute (2012); and the Nebraska Conference for Undergraduate Women in Mathematics (2013).

Mathematics People

Bourgade and Corwin Awarded Davidson Prize

Paul Bourgade of the Institute for Advanced Study and the University of Cambridge and Ivan Corwin of Columbia University and the Massachusetts Institute of Technology have been awarded the 2014 Rollo Davidson Prize. Bourgade was honored for "remarkable new results in random matrix theory and related topics," and Corwin was selected for "outstanding achievements in the area of stochastic growth processes and their relation to integrable systems." The Rollo Davidson Trust was founded in 1975 and awards the annual prize to young mathematicians working in the field of probability.

—From a Rollo Davidson Trust announcement

Strogatz Receives AAAS Award

Steven Strogatz of Cornell University has been chosen as the recipient of the 2013 Public Engagement with Science Award of the American Association for the Advancement of Science (AAAS). He was recognized for "his exceptional commitment to and passion for conveying the beauty and importance of mathematics to the general public." According to the prize citation, Strogatz "has contributed widely to the popularization and public understanding of mathematics through his newspaper articles, books, radio and television appearances, documentaries and public lectures." He was awarded the 2014 Euler Book Prize of the Mathematical Association of America (MAA) for his latest book, *The Joy of x*.

—From an AAAS announcement

Landman Awarded 2014 ANZIAM Medal

Kerry Landman of the University of Melbourne has been awarded the ANZIAM (Australia and New Zealand Industrial and Applied Mathematics) Medal for 2014 for her "fundamental and significant contributions to the understanding of physical and biological processes across a remarkably broad spectrum of applications." According to the prize citation, "she has successfully exploited a wide variety of mathematical techniques, and has pioneered their use in numerous disparate applications." She has worked in the areas of mathematical modeling of biological cell invasion, mathematical study of patterns on growing domains, discrete cellular automata models, and industrial applications of mathematics.

The ANZIAM medal is awarded on the basis of research achievements, of activities enhancing applied or industrial mathematics or both, and of contributions to ANZIAM.

—From an ANZIAM announcement

Bobkov Receives Humboldt Award

Sergey Bobkov of the University of Bielefeld, Germany, has been awarded a Humboldt Research Award. This award is conferred in recognition of lifetime achievements in research. Award winners are invited to carry out research projects of their own choice for up to one year in cooperation with specialist colleagues in Germany. The amount of the award is 60,000 euros (approximately US$82,400).

—From a Humboldt Foundation announcement

de Vries Receives CMS Teaching Award

Gerda de Vries of the University of Alberta has been named the recipient of the 2014 Excellence in Teaching Award of the Canadian Mathematical Society (CMS). The award recognizes sustained and distinguished contributions in mathematics teaching at the undergraduate level at a Canadian postsecondary education institution.

From a CMS announcement

Sloan Research Fellowships Awarded

The Alfred P. Sloan Foundation has announced the names of the recipients of the 2014 Sloan Research Fellowships. Each year the foundation awards fellowships in the fields of mathematics, chemistry, computational and evolutionary molecular biology, computer science, economics, neuroscience, physics, and ocean sciences. Grants of US$50,000 for a two-year period are administered by each fellow's institution. Once chosen, fellows are free to pursue whatever lines of inquiry most interest them, and they are permitted to employ fellowship funds in a wide variety of ways to further their research aims.

Following are the names and institutions of the 2014 awardees in mathematics: Nir Avni, Northwestern

University; Nayantara Bhatnagar, University of Delaware; Maksym Fedorchuk, Boston College; Jonathan Hauenstein, North Carolina State University; Kai-Wen Lan, University of Minnesota; Lionel Levine, Cornell University; Ivan Loseu, Northeastern University; Maryanthe Malliaris, University of Chicago; Amir Mohammadi, University of Texas, Austin; Aaron Naber, Northwestern University; Deanna Needell, Claremont McKenna College; Michael J. Neilan, University of Pittsburgh; Benoit Pausader, Princeton University; Charles Smart, Massachusetts Institute of Technology; Jared Speck, Massachusetts Institute of Technology; Samuel Stechmann, University of Wisconsin, Madison; Song Sun, Stony Brook University; Benjamin Webster, University of Virginia; Jared Weinstein, Boston University; and Jun Yin, University of Wisconsin, Madison.

—From a Sloan Foundation announcement

Clay Research Fellows

The Clay Mathematics Institute is pleased to announce that June Huh, Miguel Walsh, and Alex Wright have been appointed Clay Research Fellows.

June Huh will receive his Ph.D. in 2014 from the University of Michigan under the supervision of Mircea Mustaţă. He applies algebraic geometry and singularity theory to problems in combinatorics and other areas. His recent interests include singularities of projective hypersurfaces, positivity of Chern classes of Schubert varieties, and connections between realizability problems in algebraic geometry and combinatorial geometry. June has been appointed as a Clay Research Fellow for a term of five years beginning July 1, 2014.

Miguel Walsh was born in Buenos Aires, Argentina. He received his “Licenciatura” degree in 2010 from Universidad de Buenos Aires and his Ph.D. from the same institution in 2012, under the supervision of Román Sasyk. During this period he held a CONICET doctoral fellowship. He is currently based at the University of Oxford. His research so far has focused on inverse problems in arithmetic combinatorics, the limiting behavior of ergodic averages and the estimation of rational points on curves. Miguel has been appointed as a Clay Research Fellow for a term of four years beginning July 1, 2014.

Alex Wright will receive his Ph.D. in 2014 from the University of Chicago under the supervision of Alex Eskin. His recent work concerns dynamics on moduli spaces and special families of algebraic curves that arise in this context. His interests include dynamics, geometry, and especially ergodic theory on homogenous spaces and Teichmüller theory. Alex received his B.Math. from the University of Waterloo in 2008. Alex has been appointed as a Clay Research Fellow for a term of five years beginning July 1, 2014.

For more information, visit `www.claymath.org`.

—From a Clay Mathematics Institute announcement

National Academy of Engineering Elections

The National Academy of Engineering (NAE) has elected sixty-seven new members and eleven foreign associates. Following are the new members whose work involves the mathematical sciences.

Jan P. Allebach, Purdue University, “for development of algorithms for digital image half-toning for imaging and printing”; Daniel E. Atkins III, University of Michigan, Ann Arbor, “for leadership in development of radix algorithms and cybertechnical collaborative systems”; James K. Baker, Dragon Systems, Maitland, Florida, “for introducing hidden Markov models to speech processing and applications to commercial speech-recognition systems”; Harrison H. Barrett, University of Arizona, Tucson, “for contributions to the physical and statistical foundations and applications of radiological and nuclear medical imaging”; Paul F. Boulos, Innovyze, Broomfield, Colorado, “for contributions to theory and practice of computational hydraulics simulation technology for water infrastructure”; Tony F. Chan, Hong Kong University of Science and Technology, “for numerical techniques applied to image processing and scientific computing, and for providing engineering leadership at the national and international levels”; Brenda L. Dietrich, Business Analytics Software, IBM, “for contributions to engineering algorithms, frameworks, and tools to solve complex business problems”; Thomas F. Edgar, University of Texas, Austin, “for contributions to mathematical modeling, optimization, and automatic control of chemical and microelectronics processes, and for professional leadership”; Gregory L. Fenves, University of Texas, Austin, “for contributions to computational modeling, creation of open source software for earthquake engineering analysis, and academic leadership”; J. Karl Hedrick, University of California, Berkeley, “for analysis and control methods for nonlinear systems with application to practical problems”; Michael Luby, Qualcomm Inc., Berkeley, California, “for contributions to coding theory, including the inception of rateless codes”; James J. Riley, University of Washington, Seattle, “for contributions in analysis, modeling, and computations of transitioning and turbulent phenomena”; Robert E. Schapire, Princeton University, “for contributions to machine learning through invention and development of boosting algorithms”; and Jery R. Stedinger, Cornell University, “for statistical methods for flood risk assessment and optimizational methods for hydropower system management”. Elected as foreign members were David Harel, Weizmann Institute of Science, Rehovot, Israel, “for invention of statecharts and contributions to the logic of programming” and Kurt Mehlhorn, Max-Planck Institute for Informatics, Saarbruecken, Germany, “for contributions to algorithm design and the development of the LEDA software library”.

—From an NAE announcement

Mathematics Opportunities

Math for America Fellowships

Math for America (MfA) is a nonprofit organization with a mission to improve mathematics education in U.S. public secondary schools by recruiting, training, and retaining outstanding mathematics teachers and leaders. MfA offers fellowships for new and experienced teachers and school leaders. The MfA Master Teacher Fellowship is a four-year program that rewards outstanding experienced public secondary school mathematics and science teachers. Master Teacher Fellowships are available in Berkeley, Boston, New York City, Utah, and Washington, DC. The application deadline for Berkeley, Boston, and Utah is **April 27, 2014**. The New York City deadlines are **April 27, 2014** (priority deadline) and **July 20, 2014** (regular deadline). The deadline for Washington, DC, is **May 30, 2014**. The Math for America Early Career Fellowship is awarded to public secondary school mathematics teachers early in their careers. MfA Early Career Fellows exhibit outstanding potential, a dedication to professional development, and an interest in collaboration with the Math for America community. The program provides professional support and growth opportunities for new teachers. The MfA Early Career Fellowship requires a commitment of four years. Applications are being accepted for the Early Career Fellowship in New York City. The deadline is **April 27, 2014**. For more information and to apply, see `http://www.mathforamerica.org/web/guest/apply`.

—From an MfA announcement

NSF Postdoctoral Research Fellowships

The National Science Foundation (NSF) awards Mathematical Sciences Postdoctoral Research Fellowships (MSPRF) for appropriate research in areas of the mathematical sciences, including applications to other disciplines. Awardees are permitted to choose research environments that will have maximal impact on their future scientific development. Awards are made in the form of either Research Fellowships or Research Instructorships. The Research Fellowship option provides full-time support for any eighteen academic-year months in a three-year period in intervals not shorter than three consecutive months. The Research Instructorship option provides either two academic years of full-time support or one academic year of full-time and two academic years of half-time support. Under both options the award includes six summer months; however, no more than two summer months of support may be received in any calendar year. Under both options, the stipend support for twenty-four months (eighteen academic-year months plus six summer months) will be provided within a forty-eight-month period. The deadline for proposals is **October 15, 2014**. See `http://www.nsf.gov/pubs/2012/nsf12496/nsf12496.htm`.

—From an NSF announcement

Mentoring through Critical Transition Points in the Mathematical Sciences

The National Science Foundation (NSF) Mentoring through Critical Transition Points in the Mathematical Sciences (MCTP) program provides funds for the training of U.S. students and postdoctoral researchers in the mathematical sciences. Proposals are solicited from departments of the mathematical sciences to support projects that are intended to improve training at critical transition points in the educational careers of students and junior researchers. MCTP awards are intended to support training programs that have strong potential to increase the number of well-prepared U.S. citizens, nationals, and permanent residents who pursue careers in the mathematical sciences and in other NSF-supported disciplines. The deadline for full proposals is **June 3, 2014**. For more information see `http://www.nsf.gov/pubs/2011/nsf11542/nsf11542.htm`.

—From an NSF announcement

Research Training Groups in the Mathematical Sciences

The National Science Foundation (NSF) Research Training Groups in the Mathematical Sciences (RTG) program provides funds for the training of U.S. students and postdoctoral researchers in the mathematical sciences. Proposals are solicited from groups of researchers based in a subarea of the mathematical sciences or linked by a multidisciplinary theme to support training at educational levels from undergraduate to postdoctoral within that focus. RTG awards are intended to support training programs that have strong potential to increase the number of well-prepared U.S. citizens, nationals, and permanent residents who pursue careers in the mathematical sciences and in other NSF-supported disciplines. The deadline for full proposals is **June 3, 2014**. For more information see `http://www.nsf.gov/pubs/2011/nsf11540/nsf11540.htm`.

—From an NSF announcement

International Mathematics Competition for University Students

The Twenty-first International Mathematics Competition (IMC) for University Students will be held July 29–August 4, 2014, at American University in Blagoevgrad, Bulgaria. Participating universities are invited to send several students and one teacher; individual students are welcome. Students completing their first, second, third, or fourth years of university education are eligible. The competition will consist of two sessions of five hours each. Problems will come from the fields of algebra, analysis (real and complex), geometry, and combinatorics. The working language will be English. See the website `http://www.imc-math.org.uk/` or contact John Jayne, University College London, Gower Street, London WC1E 6BT, United Kingdom; telephone: +44 (0)77 40304010; email: `j.jayne@ucl.ac.uk`; or Chrisina Jayne, Computing Department, Coventry University; email: `chrisina.jayne@gmail.com`.

—John Jayne
University College London

NSF-CBMS Regional Conferences 2014

With funding from the National Science Foundation (NSF), the Conference Board of the Mathematical Sciences (CBMS) will hold eight NSF-CBMS Regional Research Conferences during the summer of 2014. These conferences are intended to stimulate interest and activity in mathematical research. Each five-day conference features a distinguished lecturer who delivers ten lectures on a topic of important current research in one sharply focused area of the mathematical sciences. The lecturer subsequently prepares an expository monograph based on these lectures.

Support for about thirty participants will be provided for each conference. Both established researchers and interested newcomers, including postdoctoral fellows and graduate students, are invited to attend. Information about an individual conference may be obtained by contacting the conference organizer. The conferences to be held in 2014 follow.

May 12–16, 2014: Combinatorial Zeta and *L*-functions. Wen-Ching Winnie Li, lecturer. Sundance Resort, Utah. Organizers: Jasbir S. Chahal, 801-422-2271, `jasbir@math.byu.edu`; and Michael D. Barrus, 801-422-2336, `barrus@math.byu.edu`. Conference website: `math.byu.edu/cbms`.

May 27–31, 2014: Inverse Scattering and Transmission Eigenvalues. David Colton, lecturer. University of Texas at Arlington. Organizer: Tuncay Aktosun, 817-272-1545, `aktosun@uta.edu`. Conference website: `fermat.uta.edu/cbms2014`.

June 10–15, 2014: Mathematical Foundations of Transformation Optics. Allan Greenleaf, lecturer. Howard University. Organizers: M. F. Mahmood, 202-806-6295, `mmahmood@howard.edu`; and Anjan Biswas, `abiswas@desu.edu`. Conference website: `www.coas.howard.edu/mathematics/cbms2014.html`.

June 16–20, 2014: Quantum Spin Systems. Bruno Nachtergaele, lecturer. University of Alabama at Birmingham. Organizers: Shannon Starr, 205-934-8577, `slstarr@uab.edu`; Paul H. Jung, 205-934-5266, `pjung@uab.edu`; and Gunter Stolz, 205-934-2154, `stolz@uab.edu`. Conference website: `www.uab.edu/cas/mathematics/events/nsf-cbms-conference-2014`.

June 23–27, 2014: Fast Direct Solvers for Elliptic PDEs. Gunnar Martinsson, lecturer. Dartmouth College. Organizers: Alex H. Barnett, 603-646-3178, `ahb@math.dartmouth.edu`; Min Hyung Cho, 603-646-9847, `Min.H.Cho@dartmouth.edu`; Adrianna Gillman, 603-646-2293, `adrianna.gillman@dartmouth.edu`; and Leslie F. Greengard, 212-998-3306, `greengard@courant.nyu.edu`. Conference website: `www.math.dartmouth.edu/~fastdirect/`.

June 28–July 2, 2014: Mathematical Phylogeny Conference. Mike Steel, lecturer. Winthrop University. Organizers: Joe Rusinko, 803-323-4643, `rusinkoj@winthrop.edu`; and Trent Kull, 803-323-2211, `kullt@winthrop.edu`. Conference website: `www.birdnest.org/phylogeny/`.

July 6–10, 2014: Higher Representation Theory. Raphael Rouquier, lecturer. North Carolina State University. Organizer: Naihuan Jing, 919-513-3584, `jing@unity.ncsu.edu`. Conference website: `www.math.ncsu.edu/~jing/conf/CBMS/cbms14.html`.

July 21–25, 2014: Problems of PDEs Related to Fluids. Peter Constantin, lecturer. Oklahoma State University. Organizer: Jiahong Wu, 405-744-5788, `jiahong@math.okstate.edu`. Conference website: `www.math.okstate.edu/nfs-cbms_constantin`.

—From a CBMS announcement

Inside the AMS

From the AMS Public Awareness Office

Mathematics Events at the 2014 AAAS Meeting. The AMS hosted an exhibit at the annual meeting in Chicago that featured materials on the role mathematics plays in bringing solutions to global problems (the theme of the meeting). Steven Strogatz, Jacob Gould Schurman Professor of Applied Mathematics at Cornell University, received the 2014 AAAS Award for Public Engagement with Science for "his exceptional commitment to and passion for conveying the beauty and importance of mathematics to the general public." A report on some of the mathematics sessions as well as a slideshow are at `http://www.ams.org/meetings/aaas2014`.

Left to right: AAAS Executive Director Alan Leshner, Steve Strogatz, AAAS President Phillip A. Sharp.

Selected Highlights of the 2014 Joint Mathematics Meetings. See slideshows of Invited Addresses, Sessions and Panels; Prizes and Awards session and reception; Exhibits and Sponsors; Mathematical Art Exhibition and Awards; Graduate School Fair; Employment Center; and AMS Activities at JMM, including the AMS Publications Exhibit, national *Who Wants to Be a Mathematician?* game, AMS Fellows Reception, and the "Celebrating Connection and Collaboration" AMS Dinner, where over 200 enjoyed seeing old friends, making new friends, entering the raffle, having their photos taken, and hearing remarks of AMS President David Vogan and Executive Director Donald McClure, at `http://www.ams.org/meetings/national/jmm14-highlights`.

—Annette Emerson and Mike Breen
AMS Public Awareness Officers
`paoffice@ams.org`

AMS Hosts Congressional Briefing

The AMS hosted a congressional briefing presented by Mark L. Green, professor emeritus at the University of California Los Angeles, held on December 11, 2013, in Washington, DC. His presentation, titled "How Math Fuels the Knowledge Economy", discussed how the role of the mathematical sciences has expanded dramatically in recent years. He illustrated this expansion by providing examples drawn from the dynamic and rapidly growing driver of American prosperity, the knowledge economy—including Internet startups, biotech, film production, computer games, and individualized medicine.

Professor Green's presentation was based on results from a recent National Academies/National Research Council study, "Fueling Innovation and Discovery: The Mathematical Sciences in 2025". The report describes the many contributions of the mathematical sciences to our technologically advanced society and how they permeate virtually every aspect of the modern world.

The AMS holds annual congressional briefings as a means to communicate information to policymakers. Speakers discuss the importance of mathematics research and present their work in layman's terms to congressional staff as a way to inform members of Congress of how mathematics impacts today's important issues.

—Anita Benjamin
AMS Washington Office

Deaths of AMS Members

Carl Faith, of Princeton, New Jersey, died on January 12, 2014. Born on April 28, 1927, he was a member of the Society for 57 years.

Heini Halberstam, professor, University of Illinois, died on January 25, 2014. Born on September 11, 1926, he was a member of the Society for 58 years.

David O. Lomen, professor, University of Arizona, died on November 15, 2013. Born on May 11, 1937, he was a member of the Society for 45 years.

Samuel L. Marateck, of Long Beach, New York, died on January 14, 2014. Born on September 28, 1939, he was a member of the Society for 23 years.

Jiri Jan Sichler, professor, University of Manitoba, died on November 26, 2013. Born on December 30, 1941, he was a member of the Society for 42 years.

Clinton Curtis Williams, of Oakland, Michigan, died on December 3, 2013. Born on January 29, 1936, he was a member of the Society for 22 years.

Reference and Book List

The ***Reference*** *section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.wustl.edu` in the case of the editor and `smf@ams.org` in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

April 25, 2014: Proposals for 2015 NSF-CBMS Regional Research Conferences in the Mathematical Sciences. See `http://www.nsf.gov/pubs/2013/nsf13550/nsf13550.htm`.

April 27, 2014: Applications for Berkeley, Boston, and Utah Master Teacher Fellowships of Math for America (MfA); priority deadline for New York City Master Teacher Fellowship; applications for New York City Early Career Fellowship. See "Mathematics Opportunities" in this issue.

April 30, 2014: Nominations for AWM Gweneth Humphreys Award. See `www.awm-math.org`, telephone: 703-934-0163, or email: `awm@awm-math.org`.

May 1, August 1, November 1, 2014: Applications for May, August, and November reviews, respectively, for National Academies Research Associateship Programs. See the website `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2760; fax: 202-334-2759; email: `rap@nas.edu`.

May 1, October 1, 2014: Applications for AWM Travel Grants and Mathematics Education Research Travel Grants. See `https://sites.google.com/site/awmmath/programs/travel-grants`; telephone: 703-934-0163; email: `awm@awm-math.org`; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

May 15–June 15, 2014: Proposals for the Workforce Program in the Mathematical Sciences of the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF). See http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=503233.

May 30, 2014: Applications for Washington, DC, Master Teacher Fellowships of Math for America (MfA). See "Mathematics Opportunities" in this issue.

June 3, 2014: Full proposals for NSF Mentoring through Critical Transition Points in the Mathematical Sciences. See "Mathematics Opportunities" in this issue.

June 3, 2014: Full proposals for NSF Research Training Groups in the Mathematical Sciences. See "Mathematics Opportunities" in this issue.

July 20, 2014: Applications for New York City Master Teacher Fellowships of Math for America (MfA). See "Mathematics Opportunities" in this issue.

August 12, 2014: Full proposals for NSF Scholarships in Science, Technology, Engineering, and Mathematics (STEM). See http://www.nsf.gov/pubs/2012/nsf12529/nsf12529.htm.

September 15, 2014: Applications for spring 2015 semester of Math in Moscow. See http://www.mccme.ru/mathinmoscow or contact: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: mim@mccme.ru. Information and application forms for the AMS scholarships are available on the AMS website at http://www.ams.org/programs/travel-grants/mimoscow or contact: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2294; email: student-serv@ams.org.

October 15, 2014: Proposals for NSF Postdoctoral Research Fellowships. See "Mathematics Opportunities" in this issue.

Book List

The Book List highlights recent books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. Suggestions for books to include on the list may be sent to notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

Alan M. Turing: Centenary Edition, by Sara Turing. Cambridge University Press, April 2012. ISBN-13: 978-11070-205-80.

Alan Turing: The Enigma, The Centenary Edition, by Andrew Hodges. Princeton University Press, May 2012. ISBN-13: 978-06911-556-47.

Alan Turing: His Work and Impact, edited by S. Barry Cooper and J. van Leeuwen. Elsevier, May 2013. ISBN-13: 978-01238-698-07.

Alan Turing's Electronic Brain: The Struggle to Build the ACE, the World's Fastest Computer, by B. Jack Copeland et al. Oxford University Press, May 2012. ISBN-13: 978-0-19-960915-4.

Algorithms Unlocked, by Thomas H. Cormen. MIT Press, March 2013. ISBN-13: 978-02625-188-02.

An Accidental Statistician: The Life and Memories of George E. P. Box, by George E. P. Box. Wiley, April 2013. ISBN-13: 978-1-118-40088-3.

A Cabinet of Mathematical Curiosities at Teachers College: David Eugene Smith's Collection, by Diane R. Murray. Docent Press, November 2013. ISBN-13: 978-0-9887449-1-2.

A Calculus of Ideas: A Mathematical Study of Human Thought, by Ulf Grenander. World Scientific, September 2012. ISBN-13: 978-98143-831-89. (Reviewed January 2014.)

Classic Problems of Probability, by Prakash Gorroochurn. Wiley, May 2012. ISBN-13: 978-1-1180-6325-5. (Reviewed November 2013.)

Computability: Turing, Gödel, Church, and Beyond, edited by B. Jack Copeland, Carl J. Posy, and Oron Shagrir. MIT Press, June 2013. ISBN-13: 978-02620-189-99.

Do I Count?: Stories from Mathematics, by Günter Ziegler (translation of *Darf ich Zahlen?: Geschichte aus der Mathematik,* Piper Verlag, 2010). CRC Press/A K Peters, July 2013. ISBN-13: 978-1466564916

**Enlightening Symbols: A Short History of Mathematical Notation and Its Hidden Powers,* by Joseph Mazur. Princeton University Press, March 2014. ISBN-13: 978-06911-546-33.

**Four Lives: A Celebration of Raymond Smullyan,* edited by Jason Rosenhouse. Dover Publications, February 2014. ISBN-13: 978-04864-906-70.

**Fractals: A Very Short Introduction,* by Kenneth Falconer. Oxford University Press, December 2013. ISBN-13: 978-01996-759-82.

Girls Get Curves: Geometry Takes Shape, by Danica McKellar. Plume, July 2013. ISBN-13: 978-04522-987-43.

The Gödelian Puzzle Book: Puzzles, Paradoxes and Proofs, by Raymond M. Smullyan. Dover Publications, August 2013. ISBN-13: 978-04864-970-51.

Good Math: A Geek's Guide to the Beauty of Numbers, Logic, and Computation, by Mark C. Chu-Carroll. Pragmatic Bookshelf, July 2013. ISBN-13: 978-19377-853-38.

Heavenly Mathematics: The Forgotten Art of Spherical Trigonometry, by Glen Van Brummelen. Princeton University Press, December 2012. ISBN-13: 978-06911-489-22.

Henri Poincaré: A Scientific Biography, by Jeremy Gray. Princeton University Press, November 2012. ISBN-13: 978-06911-527-14. (Reviewed April 2014.)

If A, Then B: How the World Discovered Logic, by Michael Shenefelt and Heidi White. Columbia University Press, June 2013. ISBN-13:978-02311-610-53.

Imagined Civilizations: China, the West, and Their First Encounter, by Roger Hart. Johns Hopkins University Press, July 2013. ISBN-13:978-14214-060-60.

**The Improbability Principle: Why Coincidences, Miracles, and Rare Events Happen Every Day,* by David J. Hand. Scientific American/Farrar, Straus and Giroux, February 2014. ISBN-13: *978-03741-753-44.*

Invisible in the Storm: The Role of Mathematics in Understanding Weather, by Ian Roulstone and John Norbury. Princeton University Press, February 2013. ISBN-13: 978-06911-527-21. (Reviewed September 2013.)

**Jane Austen, Game Theorist,* by Michael Suk-Young Chwe. Princeton University Press, April 2013. ISBN-13: 978-06911-557-60.

**The Logic of Infinity*, by Barnaby Sheppard Cambridge University Press, May 2014. ISBN-13: 978-11076-786-68.

The Logician and the Engineer: How George Boole and Claude Shannon Created the Information Age, by Paul J. Nahin, Princeton University Press, October 2012. ISBN-13: 978-06911-510-07. (Reviewed October 2013.)

Love and Math: The Heart of Hidden Reality, by Edward Frenkel. Basic Books, October 2013. ISBN-13: 978-04650-507-41.

Magnificent Mistakes in Mathematics, by Alfred S. Posamentier and Ingmar Lehmann. Prometheus Books, August 2013. ISBN-13: 978-16161-474-71.

The Math Book: From Pythagoras to the 57th Dimension, 250 Milestones in the History of Mathematics, by Clifford A. Pickover. Sterling. February, 2012. ISBN-13: 978-14027-882-91.

**Mathematics: An Illustrated History of Numbers*, edited by Tom Jackson. Shelter Harbor Press, October 2012. ISBN-13: 978-09853-230-42.

Mathematics in Nineteenth-Century America: The Bowditch Generation, by Todd Timmons. Docent Press, July 2013. ISBN-13: 978-0-9887449-3-6.

**Mathematics of the Transcendental: Onto-logy and being-there*, by Alain Badiou (translated by A. J. Bartlett and Alex Ling). Bloomsbury Academic, March 2014. ISBN-13: 978-14411-892-40.

Math in Minutes: 200 Key Concepts Explained in an Instant, by Paul Glendinning. Quercus, September 2013. ISBN-13: 978-16236-500-87.

Math in 100 Key Breakthroughs, by Richard Elwes. Quercus, December 2013. ISBN-13: 978-16236-505-44.

Math Is Murder, by Robert C. Brigham. iUniverse, March, 2012. ISBN-13 978-14697-972-81.

Math on Trial: How Numbers Get Used and Abused in the Courtroom, by Leila Schneps and Coralie Colmez. Basic Books, March 2013. ISBN-13: 978-04650-329-21. (Reviewed August 2013.)

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You, our members and friends, make this work possible through your generosity. The Society celebrated its 125th Anniversary in 2013, and in concordance, many of you directed your philanthropic resources toward the AMS. Your donations promoted excellence in research via the Centennial Fellowship and other prizes and awards; they ensured that young mathematicians participated in camps that fed their eagerness for mathematics; they supported graduate students as they took crucial steps early in their careers and they enabled access to MathSciNet across the globe, assisting mathematicians in developing countries. On behalf of all these beneficiaries and those working to support these programs, I thank you.

The Society's work was lifted by substantial individual gifts this year. D. Andrew Beal increased the prize related to the Beal Conjecture to $1 million dollars; the spendable income funds the Paul Erdős Memorial Lecture and other key AMS programs. The Epsilon Fund was aided by Adrian Banner with royalties from his book sales, as well as by Thomas Savage. Aurellia Sobczyk made a gift in memory of her husband, Andrew. Cathleen Synge Morawetz and Herbert Morawetz bolstered the Oswald Veblen Prize in Geometry. The AMS received a legacy gift from the late mathematician Kathleen Baxter. The Society also recognizes Robert and Maria Steinberg's longtime support of the AMS, and we acknowledge with sadness the passing of Maria in 2013.

Every donor builds connections between legacies of scholars and mathematical thought. By giving, you are indelibly connected to the field, its history, its future and to members of the mathematical community everywhere. Passion for mathematics drives us to do great things. Thank you for your gifts that make so much possible.

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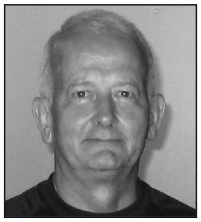

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I'm elated…I like that math is fun and there are so many problems to solve…so many interesting concepts. There's so much that it represents… it's a universal art.

—Anton Karpovich, Who Wants to Be a Mathematician regional winner

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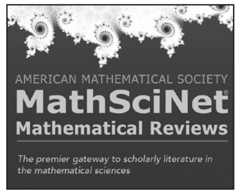

MathSciNet permits mathematicians of developing countries to be informed at every moment of recent developments in the different fields of mathematics.

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Director of PROTaSM,
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This report reflects contributions received January 1, 2013, through December 31, 2013. We apologize for any errors in the listing. Please do not hesitate to bring discrepancies to our attention by calling AMS Development at 401.455.4111 or emailing `development@ams.org`.

American Mathematical Society

Call for NOMINATIONS

The selection committees for these prizes request nominations for consideration for the 2015 awards, which will be presented at the Joint Mathematics Meetings in San Antonio, TX in January 2015. Information about these prizes may be found in the November 2013 issue of the *Notices*, pp. 1364-1386, and at www.ams.org/profession/prizes-awards/prizes.

GEORGE DAVID BIRKHOFF PRIZE

The George David Birkhoff Prize is awarded jointly by the AMS and SIAM for an outstanding contribution to applied mathematics in its highest sense. The award was first made in 1968 and now is presented every third year.

RUTH LYTTLE SATTER PRIZE

The Ruth Lyttle Satter Prize is presented every two years in recognition of an outstanding contribution to mathematics research by a woman in the previous six years.

FRANK NELSON COLE PRIZE IN ALGEBRA

The Cole Prize in Algebra, which recognizes a notable paper in algebra published during the preceding six years, is awarded every three years. To be eligible, papers must be either authored by an AMS member or published in a recognized North American journal.

LEVI L. CONANT PRIZE

The Levi L. Conant Prize is presented annually for an outstanding expository paper published in either the *Notices* or the *Bulletin of the American Mathematical Society* during the preceding five years.

ALBERT LEON WHITEMAN MEMORIAL PRIZE

The Albert Leon Whiteman Memorial Prize now is awarded every third year, for notable exposition on the history of mathematics. The ideas expressed and understandings embodied in that exposition should reflect exceptional mathematical scholarship.

Nominations with supporting information should be submitted using the online form available here: www.ams.org/profession/prizes-awards/nominations. Include a short description of the work that is the basis of the nominations, including complete bibliographic citations. A brief curriculum vitae for the nominee should be included. Those who prefer to submit by postal mail may send nominations to AMS Secretary, Carla Savage, Box 8206, Computer Science Department, North Carolina State University, Raleigh, NC 27695-8206. The nominations will be forwarded by the secretary to the appropriate prize selection committee, which will make final decisions on the awarding of these prizes.

Deadline for nominations is June 30, 2014.

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Call for Nominations

2015

Frank and Brennie Morgan AMS-MAA-SIAM Prize

for Outstanding Research in Mathematics by an Undergraduate Student

The prize is awarded each year to an undergraduate student (or students having submitted joint work) for outstanding research in mathematics. Any student who is an undergraduate in a college or university in the United States or its possessions, or Canada or Mexico, is eligible to be considered for this prize.

The prize recipient's research need not be confined to a single paper; it may be contained in several papers. However, the paper (or papers) to be considered for the prize must be submitted while the student is an undergraduate; they cannot be submitted after the student's graduation. The research paper (or papers) may be submitted for consideration by the student or a nominator. All submissions for the prize must include at least one letter of support from a person, usually a faculty member, familiar with the student's research. Publication of research is not required.

The recipients of the prize are to be selected by a standing joint committee of the AMS, MAA, and SIAM. The decisions of this committee are final. The 2015 prize will be awarded for papers submitted for consideration no later than June 30, 2014, by (or on behalf of) students who were undergraduates in December 2012.

Questions may be directed to:

Barbara T. Faires
Secretary
Mathematical Association of America
Westminster College
New Wilmington, PA 16172

Telephone: 724-946-6268
Fax: 724-946-6857
Email: faires@westminster.edu

Nominations and submissions should be sent to:

Morgan Prize Committee
c/o Carla Savage, Secretary
Box 8206
Computer Science Department
North Carolina State University
Raleigh, NC 27695-8206

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at http://www.ams.org/cgi-bin/mathcal-submit.pl. The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

May 2014

* 5–9 **Hamiltonian Perturbation Theory: Separatrix Splitting, Theory and Applications**, Centro di Ricerca Matematica Ennio De Giorgi, Piazza dei Cavalieri 3, Pisa, Italy.

Main topics: Separatrix splitting for asymptotic manifolds: synthesis of various definitions, relationship with variational methods and weak KAM theory. Analytical methods numerical methods, examples. Applications to Arnold diffusion, non-integrability, entropy estimates, destruction of KAM tori. The workshop will start on Monday at 9:30 a.m. and end on Friday at 12:30 a.m.

Invited speakers* (to be confirmed): Marie Claude Arnaud, Patrick Bernard*, Ugo Bessi, Abed Bounemoura, Oriol Castejon, Thierry Combot, Gonzalo Contreras, Rafael De La Llave, Amadeu Delshams, Jacques Fejoz, Vassili Gelfreich, Marian Gidea, Marcel Guardia, Thomas Levy, Pierre Lochak, Vito Mandorino, Jean-Pierre Marco, Pau Martin, Richard Moeckel*, Lara Sabbagh, Tere Seara, Carles Simo, Alfonso Sorrentino, Dmitry Treschev*.

Registration: No registration fee. All participants must register their attendance online: http://www.crm.sns.it/event/294/registration.html.

Information: http://www.crm.sns.it/event/294/.

* 5–9 **Workshop on Theory and Practice of Secure Multiparty Computation**, Aarhus University, Aarhus, Denmark.

Description: Secure Multiparty Computation is a powerful cryptographic notion that — in theory — can solve virtually any cryptographic protocol problem. In recent years the technology has been used in practice and holds great promise for future applications. A similar — very successful — workshop was held in Aarhus in 2012. MPC technology can be used to implement, for instance, voting, auctions, procurement and benchmarking with better security, in particular without anyone having to reveal his private data to anyone else. This workshop brings together people in both theory and practice of the field, and we are convinced that this will prove very productive. The workshop is open for all participants, the target audience is Ph.D. students, postdocs and faculty members. The tutorials will not assume any background in secure computation, but will assume basic background in foundations of Cryptography.

Information: http://cfem.au.dk/events/mpc-2014/.

* 11–16 **International Conference "Topological and geometric methods in low-dimensional dynamical systems"**, Moscow Center for Continuous Mathematical Education. 119002, Bolshoy Vlasyevskiy Pereulok 11, Moscow, Russia.

Description: The conference is co-organized by the National Research University Higher School of Economics, Laboratory of Algebraic Geometry and its Applications, Baltic Institute of Mathematics, Aix-Marseille University.

Preliminary list of speakers (TBC): Pablo Aguirre, Laurent Bartholdi, Alexander Blokh, Adam Epstein, Sarah Koch, Lex Oversteegen, Mary Rees, Dierk Schleicher, Nikita Selinger, Katsutoshi Shinohara, Sergei Tabachnikov, Michael Yampolsky.

Information: http://math.hse.ru/en/confmay2014.

* 14–15 **The 7th International Conference of Iranian Operation Research Society**, Semnan University, Semnan, Iran.

Goal: To bring scholars in the field of operations research together to share their recent researches in the related topics. OR2014 (International Conference of Iranian Operations Research Society http://www.OR2014.semnan.ac.ir) has an open call for papers. The organizers of the conference are pleased to invite you to participate in this annual conference by submitting a paper reflecting your current research work.

Information: http://or2014.semnan.ac.ir/index.php?slc_lang=en&sid=1.

* 19–23 **School on the geometry and physics of moduli spaces**, Residencia La Cristalera, Miraflores de la Sierra, Madrid, Spain.

Description: The school will consist of 4 introductory courses on several active fields of current research in the theory of moduli

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This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

spaces and their interplay with geometry, topology and theoretical physics.
Program: Sergei Gukov (Caltech): From Higgs bundles to knots and 3-manifolds; Nigel Hitchin (Oxford): Spectral curves; Franis Labourie (Orsay): Cyclic surfaces and Hitchin components; Peter Topping (Warwick): The Teichmueller harmonic map flow and minimal immersions. This activity is organized within the ICMAT Research Term on the Geometry and Physics of Moduli Spaces.
Organizers: Luis Alvarez-Consul (ICMAT, Madrid), Tomas L. Gomez (ICMAT, Madrid).
Sponsors: ICMAT Severo Ochoa Programme, GEAR Network–Geometric Structures and Representation Varieties, MODULI–Indo European Collaboration on Moduli Spaces.
Registration deadline: March 20th, 2014.
Information: http://www.icmat.es/RT/GPMODULI/school.php.

* 24 **ALGECOM 10**, University of Illinois, Urbana-Champaign, Illinois.
Description: Algebra, Geometry and Combinatorics Day (AlGeCom) is a one day, informal meeting of mathematicians from the University of Illinois, Purdue University, IUPUI, Loyola University Chicago, and nearby universities, with interests in algebra, geometry and combinatorics (widely interpreted).
Information: http://sites.google.com/site/algecomday/algecom10.

* 24–28 **Ramsey Theory Conference**, University of Denver, Denver, Colorado.
Aim: Of this conference is to bring together students and researchers from around the world in the field of Ramsey Theory. The focus is on structural and infinitary Ramsey theory and applications to other fields of mathematics, including Banach spaces, Boolean algebras, Set Theory, and Topological Dynamics. The conference will consist of a large number of plenary talks and contributed talks of 25 minutes.
Language: The official language of the conference is English.
Information: http://portfolio.du.edu/ramsey/.

* 26–30 **Algebraic Topology — Methods, Computation and Science 6 (ATMCS6)**, PIMS, University of British Columbia, Vancouver, Canada.
Description: Applied and computational topology refers to the adaptation of topological ideas and techniques to study problems in science and engineering. A particular focus is on using invariants and methods of algebraic topology to understand large high-dimensional data sets. The further development of topological techniques for use in applications and the creation of new areas of application in the subject are amongst the goals of this workshop. The workshop will bring together leading researchers in this emerging discipline as well as providing an opportunity for young mathematicians to get involved in it. In past years, the ATMCS conference has been very successful in providing a forum for cutting-edge research to be disseminated; attendance tends to represent a broad swath of the diverse research community which works in this area.
Information: http://www.pims.math.ca/scientific-event/140526-atmcs.

* 26–June 6 **Stochastic Equations for Complex Systems: Theory and Applications**, University of Wyoming, Laramie, Wyoming.
Description: This program will expose participants to recent developments in stochastic analysis and its application to complex systems. Topics include the theory of SDEs, Malliavin calculus, multiscale methods, Monte Carlo methods and molecular dynamics.
Information: http://www.uwyo.edu/bessaih/rmmc.html.

* 29–31 **GAP 2014: Geometry and Physics (Geometry and Supersymmetry)**, Pacific Institute for Mathematical Sciences (PIMS), Vancouver, British Columbia, Canada.
Theme: This year's theme is: geometry and supersymmetry. Every year, the GAP conference focuses on a current hot research topic at the interface between geometry and physics. The conference usually consists of 4 or 5 mini-courses on different aspects of the chosen theme.
Organizers: Charles Doran (Alberta), Marco Gualtieri (Toronto), Tristan Hübsch (Howard), Spiro Karigiannis (Waterloo), Ruxandra Moraru (Waterloo), and McKenzie Wang (McMaster).
Mini-courses: Will be given by the following speakers: Chris Brav (Institute for Advanced Study), Ron Donagi (University of Pennsylvania), José Figueroa-O'Farrill (University of Edinburgh), Daniel Waldram (Imperial College London). In addition, an introductory lecture and a special public lecture will be given by: James Gates (University of Maryland).
Information: http://www.math.uwaterloo.ca/~gap.

June 2014

* 5–7 (UPDATE) **Number Theory at Illinois: A Conference in Honor of the Batemans and Heini Halberstam**, University of Illinois, Urbana, Illinois.
Description: A Number Theory Conference in memory of Paul and Felice Bateman will be held at the University of Illinois. The Batemans were long-time members of the faculty and Paul was department head for 14 years. Paul was a member of the American Mathematical Society for 71 years and among his other services, was a Trustee of the AMS. This meeting continues a long tradition of number theory conferences at Illinois.
Invited talks: There will be twenty invited talks as well as opportunities for contributed talks. These will cover a broad spectrum of number theory, representing Paul's many interests. A banquet will be held on June 6. There will be a refereed proceedings volume of conference talks. The conference will be preceded by the Midwest Number Theory Conference for Graduate Students, June 3–4, 2014 (which is being announced separately).
Information: http://www.math.illinois.edu/nt2014.

* 11–13 **Karatekin Mathematics Days 2014**, Karatekin University, Çankiri, Turkey.
Description: The main purpose of the symposium is to provide a platform for mathematicians from all around the world to present their recent works, share innovations, new ideas and methods. In addition, it also aims to construct new opportunities of collaboration or to improve collaboration among local and international participants by bringing together mathematicians and experts from various fields. The scientific program covers the full spectrum of topics in Mathematics, with diverse and eminent participants sharing their knowledge and experience.
Information: http://kmd.karatekin.edu.tr.

* 11–15 **5th Cornell Conference on Analysis, Probability, and Mathematical Physics on Fractals**, Cornell University, Ithaca, New York.
Description: The purpose of this conference, held every three years, is to bring together mathematicians who are already working in the area of analysis and probability on fractals with students and researchers from related areas. The conference is partially supported by the NSF grant DMS-1361934. Financial support will be available to a limited number of participants to cover the cost of housing in Cornell single dormitory rooms and partially support other travel expenses. Students and junior researchers from underrepresented groups in STEM are particularly encouraged to apply for travel funding.
Information: http://www.math.cornell.edu/~fractals/.

* 23 **One-Day Conference on Geometry and Statistics**, Department of Mathematical Sciences, University of Bath, Bath, United Kingdom.
Description: The aim of this one-day conference is to attract mathematicians and in particular, Geometers, to work together with Statisticians in this area by highlighting interesting and powerful applications of Geometry to Statistics. The conference takes a very broad view of the interplay between geometry (differential, algebraic, convex, etc) and statistics which includes: the model

functional space approach; information geometry; which endows probability densities with appropriate differential geometric structures and from there develops method to analyse data; the data geometry approach which gives geometries to the space of the data and for which prominent examples are the computational geometry based algorithms used in robust statistics; the geometric approach to enhance/understand modern Monte Carlo sampling techniques such as Hamiltonian Monte Carlo.
Information: `http://people.bath.ac.uk/kai21/conference.html`.

* 23–27 **Mini-courses in Mathematical Analysis 2014**, University of Padova, Padova, Italy.
Description: Following a long-standing tradition, the University of Padova, Italy, is organizing the meeting "Mini-courses in Mathematical Analysis 2014". The meeting will take place at "Torre Archimede", a new building of the University of Padova in the city center, during the week 23-27 June 2014. The program consists of four lecture courses delivered by invited speakers and a limited number of short communications. The meeting aims at introducing the participants to important current research fields in Mathematical Analysis. The meeting is particularly indicated not only to graduate students, postdocs and young researchers but also to well-established experts in Mathematical Analysis. In association with ISAAC — International Society for Analysis, its Applications and Computation.
Information: `http://minicourses.dmsa.unipd.it/`.

* 24–28 **Flint: One City — 100 Years Under Variability**, Kettering University, 1700 University Ave., Flint, Michigan.
Description: This international conference is being organized by Kettering University to celebrate the IYS 2013 and the 175th anniversary of the American Statistical Association. The main focus of this conference will be on the Statisticals Methods and Studies of Historical Data. Participants may use any data of their choice. Data about the City of Flint, MI, consisting of up to 100 years of demographic, health, labor, census, and crime records will be summarized in advance of the conference and made available to the participants. Conference sessions will start with extended presentations of the statistical achievements and perspectives in the discussed area, followed by several talks on current results.
Information: `http://bulldogs.kettering.edu/fisc/`.

* 30–July 11 **Advanced School on PDEs in Geometry and Physics**, University of Science and Technology of China, Hefei, People's Republic of China.
Description: ICTP, Trieste, along with USTC, Hefei, are jointly organizing an advanced school to be held at USTC, Hefei, directed by C. Arezzo, J. Li and G. Tian.
Focus: The main focus of the school will be on various aspects of elliptic and parabolic equations on real and complex manifolds aimed primarily at graduate students and young researchers working in geometric analysis. Six-hour lecture courses will be given by the following lecturers: H.-J. Hein (Universite' de Nantes, France); Y. Rubinstein (University of Maryland, U.S.A.); J. Streets (University of California at Irvine, USA.) Three-hour lecture courses will be given by Q. Zhang (University of California at Riverside, U.S.A.); Z. L. Zhang (Capital Normal University, Beijing, People's Republic of China). A number of advanced one-hour seminars will be given during the school by leading researchers.
Information: `http://agenda.ictp.it/smr.php?2625`.

July 2014

* 3–7 **CMMSE 2014: 14th International Conference Computational and Mathematical Methods in Science and Engineering**, Costa Ballena, Cadiz, Spain.
Call for Presentations: The 2014 Conference on Mathematical Methods in Science and Engineering Conference (CMMSE-2014), is the 14th of this conference series.
Aim: CMMSE2014 aims to be a unifying, cross-cutting, interdisciplinary gathering, where specialists can have exposure to diverse fields and participate in special sessions different from but still close to their own interests.
Description: Standard presentation: 30 minutes in length. –Special Sessions: Minimum of 4-5 presentations. –Minisymposium more than 14 papers. Special session & minisymposium chairs can act as plenary speakers. Special session & minisymposium chairs can act as editors of the proceedings. Special session & mini-symposium chairs can act as editors of the journals special issues produced.
Information: `http://gsii.usal.es/~CMMSE/`.

* 14–17 **International Workshop on Neurodynamics (NDy14)**, CIEM, Castro-Urdiales, Spain.
Description: Neuroscience is nowadays one of the most collaborative and active scientific research fields as it has been increasingly involving the participation of experts from other disciplines. The aim of this Workshop on Neurodynamics (NDy'14) is to present an overview of successful achievements in this rapidly developing collaborative field by putting together different types of applications of nonlinear dynamics (geometrical tools, numerical methods, computational schemes,...) to problems in neuroscience (mononeuronal dynamics, network activity, cognitive problems,...). Emphasis will be put on experimental findings seeking for theoretical explanations, and so this meeting is focused on using mathematics as the primary tool for elucidating the fundamental mechanisms responsible for experimentally observed behavior in the applied neurosciences. The final goal is spreading together mathematical methodology and neuroscience, being Mathematical Neuroscience the generic topic for NDy'14.
Information: `http://cody.unizar.es/events/neurodynamics14`.

* 28–August 1 **The 2014 Brock International Conference on Groups, Rings and Group Rings**, Brock University, St. Catharines, Ontario, Canada.
Description: Recent advances in several areas, including algebras with polynomial identities, graded algebras and Lie algebras, coding theory, and the search for set-theoretic solutions of the Yang-Baxter equations have shed light on the importance of techniques developed initially for group rings. The Brock International Conference on Groups, Rings and Group Rings will bring international groups of researchers in group rings together with those in areas with close connections to group rings to present and discuss the latest advances in these areas in the beautiful setting of the Niagara region. Online Registration and further information is now available at the conference website.
Information: `http://www.fields.utoronto.ca/programs/scientific/14-15/grouprings/`.

August 2014

* 11–15 **ALTENCOA6-2014**, Departamento de matemáticas y Estadistica, Universidad de Nariño, San Juan de Pasto, Colombia.
Description: During these days we will celebrate 10 years from the creation of the ALTENCOA meetings and 15 years of the foundation of the research group ALTENUA. In this version the meeting will have the following areas: Algebra (Hernan Giraldo, Univ. de Antioquia, Medellin); Number theory (Carlos Trujillo, Univ. del Cauca, Popayán); Combinatorics (Carolina Benedetti, Michigan State Univ.); Applications (Oscar Moreno, Gauss Research Laboratory, Inc., Puerto Rico); Solving problems (Luis Fernando Cáceres, Univ. de Puerto Rico, Campus Mayagüez).
Information: `http://dematyes.udenar.edu.co/altencoa6/` or send email to `altencoa62014@gmail.com`.

* 12–14 **Third Seminar on Algebra and its Applications**, University of Mohaghegh Ardabili, Ardabil, Iran.
Description: The seminar continues the tradition of "workshop on Algebra and its Applications" and "Second Seminar on Algebra and

its Applications" which was held at the University of Mohaghegh Ardabili in 2010 and 2012. We would like to invite you to participate, present a paper or a workshop in this seminar whose main topics are all areas of algebra and applications of algebra. The seminar will take place at the Faculty of Mathematical Sciences of the University of Mohaghegh Ardabili which is located in the main campus of the University. After the seminar, selected papers will be published in the special issue of *Journal of Hyperstructures* according to the policies of the journal.

Information: http://www.uma.ac.ir/index.php?sid=44&slc_lang=en.

* 12–16 **13th International Conference on p-adic Functional Analysis**, University of Paderborn, Paderborn, Germany.

Description: The conference is directed to researchers in p-adic functional analysis and related areas, like non-archimedean analysis and non-archimedean dynamical systems. It is part of a series of biennial international conferences on p-adic functional analysis.

Scientific board: J. Aguayo Garrido, A. Escassut, A. K. Katsaras, A. N. Kochubei, H. Ochsenius, M. C. Perez Garcia, K. Shamseddine, W. H. Schikhof.

Local organizer: Helge Glöckner.

Information: http://www2.math.uni-paderborn.de/index.php?id=17337.

* 26–30 **The 10th William Rowan Hamilton Geometry and Topology Workshop on Homological Invariants in Low-dimensional Topology and Geometry**, The Hamilton Mathematics Institute, Trinity College Dublin, Ireland.

Description: The workshop consists of a two day mini-course August 26–27, followed by a three day lecture series, August 28–30, 2014. This year's minicourse will be given by Jacob Rasmussen (Cambridge) and Liam Watson (Glasgow) on "Floer homology and low-dimensional topology". The minicourse will run August 26–27 and consist of a two-day series of lectures and discussions. The target audience is graduate students and junior researchers. The lecture series will be August 28–30, speakers to be announced. The workshop is co-sponsored by Boston College, the HMI and the NSF. Funding is available for graduate students and junior researchers wishing to attend the minicourse and workshop. A limited amount of funding is also available for junior and senior researchers wishing to attend the workshop. If you would like to request travel support you can do so on the workshop website.

Information: http://www.hamilton.tcd.ie/events/gt/gt2014.htm.

September 2014

* 1–5 **International School on Mathematical Epidemiology-ISME 2014**, Strathmore University, Nairobi, Kenya.

Description: ISME 2014 is the first school of annual series of international graduate schools on Mathematical Modelling in Biology and Medicine organized by CARMS of Strathmore University, Kenya. The school will include lectures on mathematical epidemiology, and one of the most important aspects will be projects for groups of 4.6 participants, mixing scientific backgrounds and levels of experience, and focusing on real-world problems around which participants develop and analyze models. It will also incorporate several lectures on public-health topics with focus on those relevant to other events such as global spread, indigenous population's health, vector-borne diseases and integration of surveillance, statistical data analysis and dynamical modelling and simulations.

Information: http://www.strathmore.edu/carms.

* 8–11 **CICAM 7, Seventh China-Italy Colloquium on Applied Mathematics**, Palermo, Italy.

Description: The Italian-Chinese Congress of Applied Mathematics was born in consequence of a long scientific collaboration between mathematicians at the University of Napoli, Catania, Bologna, Palermo and Torino and some universities of China. Over the years, there were two editions of the Conference in Napoli, three in China (Xian, Chonqing, Shanghai) and one in Catania. One of the main purposes of the conference is to strengthen and develop scientific cooperation between the research groups of Applied Mathematics, Physics and Biomathematics operating in Italy and China.

Information: http://www.math.unipa.it/~cicam7.

* 8–12 **Workshop on Special Geometric Structures in Mathematics and Physics**, University of Hamburg, Hamburg, Germany.

Description: This 5-day workshop aims to bring together researchers in various fields of differential geometry with theoretical physicists. Each speaker will give two talks, one of which will be of a more introductory nature, thus making the workshop accessible also for Ph.D. students.

Information: http://www.math.uni-hamburg.de/sgstructures/.

* 10–12 **IMA Conference on Mathematical Modelling of Fluid Systems**, University of Bath, United Kingdom.

Description: As the most versatile medium for transmitting signals and power, fluids (gas or liquid) have wide usage in industry. Fluid systems are used in machine tool applications, vehicle control systems, where high power to weight ratio, accuracy and quick response are required. Fluid systems for industrial processes often involve networks consisting of tanks, pipes, orifices, valves, pumps and other flow devices. It is important to develop a systematic method to mathematically model different types of fluid systems for safe and optimal operations.

Information: http://www.ima.org.uk/conferences/conferences_calendar/ima_conference_on_mathematical_modelling_of_fluid_systems.html

* 29–October 3 **AIM Workshop: Quantum curves, Hitchin systems, and the Eynard-Orantin theory**, American Institute of Mathematics, Palo Alto, California.

Description: This workshop, sponsored by AIM and the NSF, will be devoted to establishing a mathematical theory of quantum curves.

Information: http://aimath.org/workshops/upcoming/quantumcurves.

November 2014

* 13–15 **SIAM Conference on Financial Mathematics and Engineering (FM14)**, The Palmer House, A Hilton Hotel, Chicago, Illinois.

Description: The Activity Group on Financial Mathematics and Engineering focuses on research and practice in financial mathematics, computation, and engineering. Its goals are to foster collaborations among mathematical scientists, statisticians, computer scientists, computational scientists, and researchers and practitioners in finance and economics, and to foster collaborations in the use of mathematical and computational tools in quantitative finance in the public and private sector. The activity group promotes and facilitates the development of financial mathematics and engineering as an academic discipline.

Information: http://www.siam.org/meetings/fm14/.

* 14–17 **Conference on Mathematics and its Applications-2014**, Kuwait University, Kuwait City, Kuwait.

Description: The Conference on Mathematics and its Applications-2014 (CMA-2014) is broad-based spanning Algebra, Analysis, Discrete Mathematics, and Inverse Problems and Imaging.

Aim: Is to highlight the latest advances in different areas of mathematics and emphasis their applications to other disciplines. The conference will gather renowned scholars in mathematics from around the world and provide a forum to exchange ideas and discuss research findings, as well as to form new inter-disciplinary connections among the participants.

Information: http://cma2014.science.ku.edu.kw.

* 17–21 **AIM Workshop: Bounded gaps between primes**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will focus on the remarkable progress made in the last year on gaps between prime numbers.
Information: http://aimath.org/workshops/upcoming/primegaps2.

* 26–29 **International Congress on Music and Mathematics**, University of Guadalajara, Puerto Vallarta, Mexico.
Description: In the context of the 20th anniversary of the University Center for Exact Sciences and Engineering (CUCEI-UdeG, Mexico), and the 40th anniversary of the National Center for Music Research, Documentation and Information (CENIDIM.INBA, Mexico), this Congress will focus on the relationship between music and mathematics, both applied and pure, understood as systems, techniques, technologies, theories, and creative work. International and interdisciplinary contributions are highly appreciated. The Congress will examine the essentials of analogous thought and its meaning and functioning in the broadest sense of .abstract forms in music. A wider view on music and mathematics will be also considered. The venue will bring together scholars, researchers, students and artists from many disciplines, converging within the announced topics. We welcome innovative and unexpected proposals on topics that address diverse aspects of music an mathematics.
Information: http://icmm.cucei.udg.mx/.

December 2014

* 1–5 **International Conference on Applied Mathematics — in honour of Professor Roderick S. C. Wong's 70th Birthday**, City University of Hong Kong, Tat Chee Avenue, Kowloon Tong, Hong Kong.
Description: The objectives of the conference are to review and discuss some of the latest trends in various fields of applied mathematics. In particular, with a special emphasis on asymptotic and special functions, partial differential equations, computational mathematics, approximation theory, mathematical physics, mathematical biology and financial mathematics. The conference is dedicated to Professor Roderick in recognition of his mathematical achievements and his contributions in the mathematical society. During the conference, the William Benter Prize in Applied Mathematics 2014 will be awarded, and the recipient will give a plenary lecture. The aim of the Prize is to recognize outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, finance and engineering applications.
Information: http://www6.cityu.edu.hk/rcms/icam2014/index.htm.

* 8–12 **AIM Workshop: Transversality in contact homology**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will bring together specialists in symplectic and contact topology with the goals of clarifying the gaps in current arguments concerning the definition of contact homology and of moving forward to fill these gaps and build precise foundations for the cylindrical, linearized, local, and possibly other versions of contact homology.
Information: http://aimath.org/workshops/upcoming/transcontacthom.

* 19–21 **2014 Fourth International Conference on Emerging Applications of Information Technology (EAIT 2014)**, Indian Statistical Institute, Kolkata, India.
Description: Encouraged by the earlier responses and keeping the tradition CSI Kolkata Chapter is organizing EAIT 2014 during Dec 19-21, 2014. The event will comprise of Pre-Conference Tutorials, plenary sessions, invited lectures by eminent speakers of international repute, session papers and panel discussions. Original unpublished contributions are solicited for presentation at EAIT 2014. Papers cannot be submitted in parallel to any other conference or journal.
Topics: Of interest include but are not limited to: Image Processing, Computer Vision and Pattern Recognition; Machine Learning, Data Mining and Computational Life Sciences; Management of Data including Big Data and Analytics; Distributed and Mobile Systems including Grid and Cloud infrastructure; Information Security and Privacy. Please check conference website for more details and updates.
Information: http://sites.google.com/site/csieait/.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

May 2015

* 28–31 **3rd International Conference on "Applied Mathematics & Approximation Theory-AMAT 2015"**, Ankara, Turkey.
Description: All subareas and topics of Applied Mathematics and Approximation Theory are welcome.
Plenary Speakers: George A. Anastassiou (University of Memphis, USA); Jerry L. Bona (University Illinois at Chicago, USA); Alexander Goncharov (Bilkent University, Turkey); Weimin Han (University of Iowa, USA); Varga Kalantarov (Ko University, Turkey); Gitta Kutyniok (Technische Universitt, Germany); Choonkil Park (Hanyang University, South Korea); Tamaz Vashakmadze (Tbilisi State University, Georgia).
Organizers: George A. Anastassiou (University of Memphis, USA) and Oktay Duman (TOBB Economics and Technology University, Ankara, Turkey).
Contact: email: amat2015@etu.edu.tr; amat2015conference@gmail.com.
Information: http://amat2015.etu.edu.tr/.

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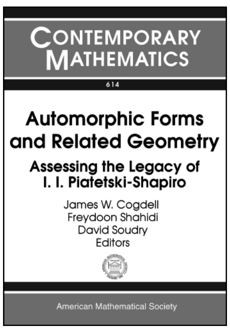
Algebra and Algebraic Geometry



Automorphic Forms and Related Geometry

Assessing the Legacy of I. I. Piatetski-Shapiro

James W. Cogdell, *Ohio State University, Columbus, OH*, **Freydoon Shahidi**, *Purdue University, West Lafayette, IN*, and **David Soudry**, *Tel Aviv University, Israel*, Editors

This volume contains the proceedings of the conference Automorphic Forms and Related Geometry: Assessing the Legacy of I. I. Piatetski-Shapiro, held from April 23–27, 2012, at Yale University, New Haven, CT.

Ilya I. Piatetski-Shapiro, who passed away on February 21, 2009, was a leading figure in the theory of automorphic forms. The conference attempted both to summarize and consolidate the progress that was made during Piatetski-Shapiro's lifetime by him and a substantial group of his co-workers, and to promote future work by identifying fruitful directions of further investigation. It was organized around several themes that reflected Piatetski-Shapiro's main foci of work and that have promise for future development: functoriality and converse theorems; local and global L-functions and their periods; p-adic L-functions and arithmetic geometry; complex geometry; and analytic number theory. In each area, there were talks to review the current state of affairs with special attention to Piatetski-Shapiro's contributions, and other talks to report on current work and to outline promising avenues for continued progress.

The contents of this volume reflect most of the talks that were presented at the conference as well as a few additional contributions. They all represent various aspects of the legacy of Piatetski-Shapiro.

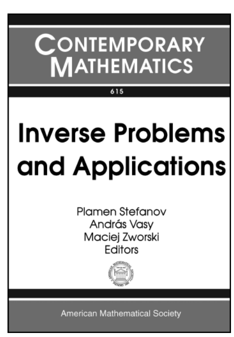
This item will also be of interest to those working in number theory.

Contents: J. Arthur, On parameters for the group $SO(2n)$; **J. W. Cogdell**, Piateski-Shapiro's work on converse theorems; **S. Gelbart**, **S. D. Miller**, **A. Panchishkin**, and **F. Shahidi**, A p-adic integral for the reciprocal of L-functions; **S. Gindikin**, Harmonic analysis on symmetric spaces as complex analysis; **M. Harris**, Testing rationality of coherent cohomology of Shimura varieties; **H. Hida**, Hecke fields of Hilbert modular analytic families; **R. Howe** and **S. T. Lee**, Structure of holomorphic unitary representations: The case of $U_{2,2}$; **H. Jacquet**, Mellin transform of Whittaker functions; **D. Jiang**, Automorphic integral transforms for classical groups I: Endoscopy correspondences; **J.-L. Kim**, An inductive formula for ε-factors; **E. Lapid** and **Z. Mao**, On a new functional equation for local integrals; **C. Mœglin**, Paquets stables des séries discrètes accessibles par endoscopie tordue; leur paramètre de Langlands; **N. B. Châu**, On a certain sum of automorphic L-functions; **A. Panchishkin**, Analytic constructions of p-adic L-functions and Eisenstein series; **J. W. Cogdell**, **F. Shahidi**, and **T.-L. Tsai**, On stability of root numbers; **C. Skinner**, Cap forms, Eisenstein series, and some arithmetic applications; **D. Soudry**, Automorphic descent: An outgrowth from Piatetski-Shapiro's vision; **M. Friedman** and **M. Teicher**, On the singularities of branch curves of $K3$ surfaces and applications.

Contemporary Mathematics, Volume 614

April 2014, 441 pages, Softcover, ISBN: 978-0-8218-9394-4, LC 2013039849, 2010 *Mathematics Subject Classification:* 11Fxx, 22Exx, 14Jxx, **AMS members US$108.80**, List US$136, Order code CONM/614

Differential Equations



Inverse Problems and Applications

Plamen Stefanov, *Purdue University, West Lafayette, IN*, **András Vasy**, *Stanford University, CA*, and **Maciej Zworski**, *University of California, Berkeley, CA*, Editors

This volume contains the proceedings of two conferences on Inverse Problems and Applications, held in 2012, to celebrate the work of Gunther Uhlmann. The first conference was held at the University of California, Irvine, from June 18–22, 2012, and the second was held at Zhejiang University, Hangzhou, China, from September 17–21, 2012.

The topics covered include inverse problems in medical imaging, scattering theory, geometry and image processing, and the mathematical theory of cloaking, as well as methods related to inverse problems.

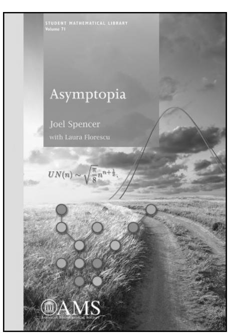
This item will also be of interest to those working in applications.

Contents: H. Ammari, **G. Ciraolo**, **H. Kang**, **H. Lee**, and **G. W. Milton**, Spectral theory of a Neumann-Poincaré-type operator and analysis of cloaking by anomalous localized resonance II; **G. Bal**, Hybrid inverse problems and redundant systems of partial differential equations; **G. Bao**, **K. Huang**, **P. Li**, and **H. Zhao**, A direct imaging method for inverse scattering using the generalized Foldy-Lax formulation; **F. Cakoni**, **D. Colton**, and **X. Meng**, The inverse scattering problem for a penetrable cavity with internal measurements; **E. Chung**, **C. Y. Lam**, and **J. Qian**, A Neumann series based method for photoacoustic tomography on irregular domains; **S. J. Hamilton** and **S. Siltanen**, Nonlinear inversion from partial EIT data: Computational experiments; **V. Isakov**, **S. Nagayasu**, **G. Uhlmann**, and **J.-N. Wang**, Increasing stability of the inverse boundary value problem for the Schrödinger equation; **H. Isozaki**, **Y. Kurylev**, and **M. Lassas**, Recent progress of inverse scattering theory on non-compact manifolds; **A. Jollivet** and **V. Sharafutdinov**, On an inverse problem for the Steklov spectrum of a Riemannian surface; **C. Kenig** and **M. Salo**, Recent progress in the Calderón problem with partial data; **M. Lassas** and **L. Oksanen**, Local reconstruction of a Riemannian manifold from a restriction of the hyperbolic Dirichlet-to-Neumann operator; **J. Li**, **H. Liu**, and **H. Sun**, Damping mechanisms for regularized transformation-acoustics cloaking; **S. Moskow** and **J. C. Schotland**, Hybrid inverse problem for porous media; **J. Qian**, **C. Yang**, **A. Schirotzek**, **F. Maia**, and **S. Marchesini**, Efficient algorithms for ptychographic phase retrieval; **S. Zelditch**, Matrix elements of Fourier integral operators.

Contemporary Mathematics, Volume 615

May 2014, 309 pages, Softcover, ISBN: 978-1-4704-1079-7, LC 2013039861, 2010 *Mathematics Subject Classification:* 35R30, **AMS members US$81.60**, List US$102, Order code CONM/615

Discrete Mathematics and Combinatorics



Asymptopia

Joel Spencer, *New York University, NY*
with Laura Florescu, *New York University, NY*

Asymptotics in one form or another are part of the landscape for every mathematician. The objective of this book is to present the ideas of how to approach asymptotic problems that arise in discrete mathematics, analysis of algorithms, and number theory. A broad range of topics is covered, including distribution of prime integers, Erdős Magic, random graphs, Ramsey numbers, and asymptotic geometry.

The author is a disciple of Paul Erdős, who taught him about Asymptopia. Primes less than n, graphs with v vertices, random walks of t steps—Erdős was fascinated by the limiting behavior as the variables approached, but never reached, infinity. Asymptotics is very much an art. The various functions $n \ln n$, n^2, $\frac{\ln n}{n}$, $\sqrt{\ln n}$, $\frac{1}{n \ln n}$ all have distinct personalities. Erdős knew these functions as personal friends. It is the author's hope that these insights may be passed on, that the reader may similarly feel which function has the right temperament for a given task. This book is aimed at strong undergraduates, though it is also suitable for particularly good high school students or for graduates wanting to learn some basic techniques.

Asymptopia is a beautiful world. Enjoy!

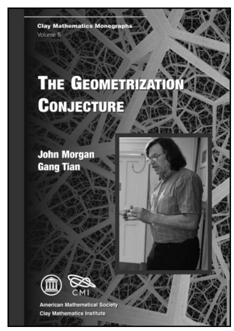
This item will also be of interest to those working in analysis and probability and statistics.

Contents: An infinity of primes; Stirling's formula; Big Oh, little Oh and all that; Integration in Asymptopia; From integrals to sums; Asymptotics of binomial coefficients $\binom{n}{k}$; Unicyclic graphs; Ramsey numbers; Large deviations; Primes; Asymptotic geometry; Algorithms; Potpourri; Really Big Numbers!; Bibliography; Index.

Student Mathematical Library, Volume 71

June 2014, approximately 195 pages, Softcover, ISBN: 978-1-4704-0904-3, LC 2013049249, 2010 *Mathematics Subject Classification:* 05-01, 05A16; 05C80, 68W40, 11A41, 60C05, **AMS members US$31.20**, List US$39, Order code STML/71

Geometry and Topology



The Geometrization Conjecture

John Morgan, *Simons Center for Geometry and Physics, Stony Brook University, NY*, and **Gang Tian**, *Princeton University, NJ, and Peking University, Beijing, China*

This book gives a complete proof of the geometrization conjecture, which describes all compact 3-manifolds in terms of geometric pieces, i.e., 3-manifolds with locally homogeneous metrics of finite volume. The method is to understand the limits as time goes to infinity of Ricci flow with surgery. The first half of the book is devoted to showing that these limits divide naturally along incompressible tori into pieces on which the metric is converging smoothly to hyperbolic metrics and pieces that are locally more and more volume collapsed. The second half of the book is devoted to showing that the latter pieces are themselves geometric. This is established by showing that the Gromov-Hausdorff limits of sequences of more and more locally volume collapsed 3-manifolds are Alexandrov spaces of dimension at most 2 and then classifying these Alexandrov spaces.

In the course of proving the geometrization conjecture, the authors provide an overview of the main results about Ricci flows with surgery on 3-dimensional manifolds, introducing the reader to this difficult material. The book also includes an elementary introduction to Gromov-Hausdorff limits and to the basics of the theory of Alexandrov spaces. In addition, a complete picture of the local structure of Alexandrov surfaces is developed. All of these important topics are of independent interest.

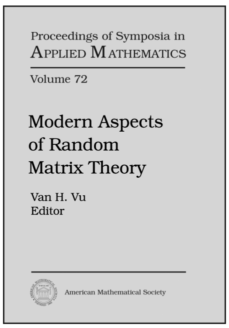
Titles in this series are co-published with the Clay Mathematics Institute (Cambridge, MA).

Contents: Introduction; *Geometric and analytic results for Ricci flow with surgery:* Ricci flow with surgery; Limits as $t \to \infty$; Local results valid for large time; Proofs of the three propositions; *Locally volume collapsed 3-manifolds:* Introduction to part II; The collapsing theorem; Overview of the rest of the argument; Basics of Gromov-Hausdorff convergence; Basics of Alexandrov spaces; 2-dimensional Alexandrov spaces; 3-dimensional analogues; The global result; *The equivariant case:* The equivariant case; Bibliography; Glossary of symbols; Index.

Clay Mathematics Monographs, Volume 5

May 2014, 291 pages, Hardcover, ISBN: 978-0-8218-5201-9, LC 2013045837, 2010 *Mathematics Subject Classification:* 53C21, 53C23, 53C30, 53C44, 53C45, 57M40, 57M60, **AMS members US$64.80**, List US$81, Order code CMIM/5

Probability and Statistics



Modern Aspects of Random Matrix Theory

Van H. Vu, *Yale University, New Haven, CT*, Editor

The theory of random matrices is an amazingly rich topic in mathematics. Random matrices play a fundamental role in various areas such as statistics, mathematical physics, combinatorics, theoretical computer science, number theory and numerical analysis.

This volume is based on lectures delivered at the 2013 AMS Short Course on Random Matrices, held January 6–7, 2013 in San Diego, California.

Included are surveys by leading researchers in the field, written in introductory style, aiming to provide the reader a quick and intuitive overview of this fascinating and rapidly developing topic. These surveys contain many major recent developments, such as progress on universality conjectures, connections between random matrices and free probability, numerical algebra, combinatorics and high-dimensional geometry, together with several novel methods and a variety of open questions.

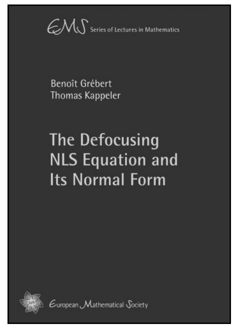
Contents: **C. Bordenave** and **D. Chafaï**, Lecture notes on the circular law; **A. Guionnet**, Free probability and random matrices; **A. Edelman**, **B. D. Sutton**, and **Y. Wang**, Random matrix theory, numerical computation and applications; **M. Rudelson**, Recent developments in non-asymptotic theory of random matrices; **T. Tao** and **V. Vu**, Random matrices: The universality phenomenon for Wigner ensembles; Index.

Proceedings of Symposia in Applied Mathematics, Volume 72

June 2014, 176 pages, Hardcover, ISBN: 978-0-8218-9471-2, LC 2013051063, 2010 *Mathematics Subject Classification:* 11C20, 60B20, 15B52, 05D40, 60H25, 62-07, **AMS members US$44.80**, List US$56, Order code PSAPM/72

New AMS-Distributed Publications

Differential Equations



The Defocusing NLS Equation and Its Normal Form

Benoît Grébert, *University of Nantes, France*, and **Thomas Kappeler**, *University of Zurich, Switzerland*

The theme of this monograph is the nonlinear Schrödinger equation. This equation models slowly varying wave envelopes in dispersive media and arises in various physical systems such as water waves, plasma physics, solid state physics and nonlinear optics. More specifically, this book treats the defocusing nonlinear Schrödinger (dNLS) equation on the circle with a dynamical systems viewpoint. By developing the normal form theory, it is shown that this equation is an integrable partial differential equation in the strongest possible sense. In particular, all solutions of the dNLS equation on the circle are periodic, quasi-periodic or almost-periodic in time and Hamiltonian perturbations of this equation can be studied near solutions far away from the equilibrium.

The book is intended not only for specialists working at the intersection of integrable PDEs and dynamical systems but also for researchers farther away from these fields as well as for graduate students. It is written in a modular fashion; each of its chapters and appendices can be read independently of each other.

This item will also be of interest to those working in analysis.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Zakharov-Shabat operators; Spectra; Liouville coordinates; Birkhoff coordinates; Appendices; References; Index; Notations.

EMS Series of Lectures in Mathematics, Volume 18

March 2014, 176 pages, Softcover, ISBN: 978-3-03719-131-6, 2010 *Mathematics Subject Classification:* 35Q55, 37K15, 37K10, 34L40, 34L20, **AMS members US$30.40**, List US$38, Order code EMSSERLEC/18

Classified Advertisements

Positions available, items for sale, services available, and more

CALIFORNIA

CALIFORNIA INSTITUTE OF TECHNOLOGY
Division of Physics, Mathematics, and Astronomy

The Division of Physics, Mathematics, and Astronomy at the California Institute of Technology invites applications for a tenure-track or tenured position in Geometry, Topology and related fields. At the tenure-track level, the term of the initial appointment is normally four years (with a possible extension to as many as seven years). Appointment is contingent upon completion of the Ph.D. Applicants should apply online at `mathjobs.org`. Please include curriculum vitae, list of publications, description of research, and at least three letters of recommendation. We are seeking highly qualified applicants who are committed to a career in research and teaching.

Caltech is an Affirmative Action/Equal Opportunity Employer. Women and disadvantaged minorities are encouraged to apply.

000020

MINNESOTA

THE INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS
Accepting Board of Governors Nominations

The Institute for Mathematics and its Applications (IMA) is now accepting nominations for its Board of Governors. Applicants may either self-nominate or they may be nominated by others.

The IMA's board consists of 15 distinguished members from academia, industry, and government. The board is the principal governing body of the IMA. Incoming members of the board will serve a five-year term, beginning on January 1, 2015.

The role of the board is twofold: first, to provide oversight and advice on matters of institute management, development, and institutional relationships. And second, board members play an active scientific role in planning and developing annual program themes as well as identifying lead program organizers. The board meets for two days annually and subcommittees meet several times annually by conference call.

Submission of Nominations Prospective applicants should submit an application via the online form at `www.ima.umn.edu/bog`. All nominations will be reviewed by the Nominations Committee. Applicants will be notified of the committee's decision no later than December 1, 2014.

Closing Date Nominations are due no later than July 31, 2014.

Questions? Contact IMA Director Fadil Santosa (`santosa@ima.umn.edu`), or Dana Randall (`randall@cc.gatech.edu`), chair, IMA Board of Governors.

000021

CHILE

PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
Departamento de Matemáticas

The Department of Mathematics invites applications for two tenure-track positions at the Assistant Professor level beginning either March or August 2015. Applicants should have a Ph.D. in Mathematics, proven research potential either in pure or applied mathematics, and a strong commitment to teaching and research. The regular teaching load for assistant professors consists of three one-semester courses per year, reduced to two courses during the first two years. The annual salary will be approximately US$48,000. A startup grant of US$15,000 to be used during the first three years will be available to support research activities.

Please send a letter indicating your main research interests, potential collaborators in our department (`http://www.mat.puc.cl`), a detailed curriculum vitae, and three letters of recommendation to:

Monica Musso
Departamento de Matemáticas
Pontificia Universidad Católica de Chile Av.
Vicuña Mackenna 4860
Santiago, Chile;
fax: (56-2) 552-5916;
email: `mmusso@mat.puc.cl`

For full consideration, complete application materials must arrive by June 30, 2014.

000019

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2014 rate is $3.50 per word with a minimum two-line headline. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: June/July 2014 issue–April 29, 2014; August 2014 issue–May 29, 2014; September 2014 issue–June 30, 2014; October 2014 issue–July 29, 2014; November 2014 issue–September 4, 2014; December 2014 issue–September 30, 2014.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Tel Aviv, Israel

Bar-Ilan University, Ramat-Gan and Tel-Aviv University, Ramat-Aviv

June 16–19, 2014

Monday – Thursday

Meeting #1101

The Second Joint International Meeting between the AMS and the Israel Mathematical Union.
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: January 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Ian Agol, University of California, Berkeley, *3-manifolds and cube complexes.*

Gil Kalai, Hebrew University, *Influence, thresholds, and noise sensitivity.*

Michael Larsen, Indiana University, *Borel's theorem on word maps and some recent variants.*

Andrei Okounkov, Columbia Unversity, *The M-theory index.*

Leonid Polterovich, Tel-Aviv University, *Symplectic topology: From dynamics to quantization.*

Tamar Zeigler, Technion, Israel Institute of Technology, *Patterns in primes and dynamics on nilmanifolds.*

Special Sessions

Additive Number Theory, **Melvyn B. Nathanson**, City University of New York, and **Yonutz V. Stanchescu**, Afeka Tel Aviv Academic College of Engineering.

Algebraic Groups, Division Algebras and Galois Cohomology, **Andrei Rapinchuk**, University of Virginia, and **Louis H. Rowen** and **Uzi Vishne**, Bar Ilan University.

Applications of Algebra to Cryptography, **David Garber**, Holon Institute of Technology, and **Delaram Kahrobaei**, City University of New York Graduate Center.

Asymptotic Geometric Analysis, **Shiri Artstein** and **Boaz Klar'tag**, Tel Aviv University, and **Sasha Sodin**, Princeton University.

Combinatorial Games, **Aviezri Fraenkel**, Weizmann University, **Richard Nowakowski**, Dalhousie University, Canada, **Thane Plambeck**, Counterwave Inc., and **Aaron Siegel**, Twitter.

Combinatorics, **Gil Kalai**, Hebrew University of Jerusalem.

Dynamics and Number Theory, **Alex Kontorovich**, Yale University.

Field Arithmetic, **David Harbater**, University of Pennsylvania, and **Moshe Jarden**, Tel Aviv University.

Financial Mathematics, **Jean-Pierre Fouque**, University of California, and **Eli Merzbach** and **Malka Schaps**, Bar Ilan University.

Geometric Group Theory and Low-Dimensional Topology, **Ian Agol**, University of California, Berkeley, and **Zlil Sela**, Hebrew University.

Geometry and Dynamics, **Yaron Ostrover**, Tel Aviv University.

History of Mathematics, **Leo Corry**, Tel Aviv University, **Michael N. Fried**, Ben Gurion University, and **Victor Katz**, University of District of Columbia.

Mirror Symmetry and Representation Theory, **Roman Bezrukavnikov**, Massachusetts Institute of Technology, and **David Kazhdan**, Hebrew University.

Nonlinear Analysis and Optimization, **Boris Mordukhovich**, Wayne State University, and **Simeon Reich** and **Alexander Zaslavski**, Technion Israel Institute of Technology.

PDEs: Modeling Theory and Numerics, **Edriss S. Titi**, University of California, Irvine.

Qualitative and Analytic Theory of ODE's, **Andrei Gabrielov**, Purdue University, and **Yossef Yomdin**, Weizmann Institute of Science.

Quasigroups, Loops and Applications, **Tuval Foguel**, Western Carolina University.

Random Matrix Theory, **Brendan Farrell**, California Institute of Technology, **Mark Rudelson**, University of Michigan, and **Ofer Zeitouni**, Weizmann Institute of Science.

Recent Trends in History and Philosophy of Mathematics, **Misha Katz**, Bar Ilan University, and **David Sherry**, Northern Arizona University.

Teaching with Mathematical Habits in Mind, **Theodore Eisenberg**, Ben Gurion University, **Davida Fishman**, California State University, San Bernardino, and **Jennifer Lewis**, Wayne State University.

The Mathematics of Menahem M. Schiffer, **Peter L. Duren**, University of Michigan, and **Lawrence Zalcman**, Bar Ilan University.

Topological Graph Theory and Map Symmetries, **Jonathan Gross**, Columbia University, and **Toufik Mansour**, University of Haifa.

Eau Claire, Wisconsin

University of Wisconsin-Eau Claire

September 20–21, 2014

Saturday – Sunday

Meeting #1102

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: June 2014
Program first available on AMS website: August 7, 2014
Program issue of electronic *Notices*: September 2014
Issue of *Abstracts*: Volume 35, Issue 3

Deadlines

For organizers: Expired
For abstracts: July 29, 2014

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Matthew Kahle, Ohio State University, *To be announced.*

Markus Keel, University of Minnesota, *To be announced.*

Svitlana Mayboroda, University of Minnesota, *To be announced.*

Dylan Thurston, Indiana University, *To be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Algebraic Combinatorics (Code: SS 8A), **Pavlo Pylyavskyy**, **Victor Reiner**, and **Dennis Stanton**, University of Minnesota.

Cohomology and Representation Theory of Groups and Related Structures (Code: SS 6A), **Christopher Bendel**, University of Wisconsin-Stout, and **Christopher Drupieski**, De Paul University.

Commutative Ring Theory (Code: SS 3A), **Michael Axtell**, University of St. Thomas, and **Joe Stickles**, Millikin University.

Directions in Commutative Algebra: Past, Present and Future (Code: SS 1A), **Joseph P. Brennan**, University of Central Florida, and **Robert M. Fossum**, University of Illinois at Urbana-Champaign.

Graph and Hypergraph Theory (Code: SS 7A), **Sergei Bezrukov**, University of Wisconsin-Superior, **Dalibor Froncek**, University of Minnesota Duluth, and **Xiaofeng Gu**, **Uwe Leck**, and **Steven Rosenberg**, University of Wisconsin-Superior.

Knot Concordance and 4-Manifolds (Code: SS 9A), **Christopher W. Davis**, University of Wisconsin-Eau Claire, **Taylor Martin**, Sam Houston State University, and **Carolyn Otto**, University of Wisconsin-Eau Claire.

Lie Algebras and Representation Theory (Code: SS 5A), **Michael Lau**, Université Laval, **Ian Musson**, University of Wisconsin, Milwaukee, and **Matthew Ondrus**, Weber State University.

New Trends in Toric Varieties (Code: SS 4A), **Christine Berkesch Zamaere**, University of Minnesota, **Daniel Erman**, University of Wisconsin-Madison, and **Hal Schenck**, University of Illinois Urbana-Champaign.

Problem Solving in Extremal Combinatorics and Combinatorial Geometry (Code: SS 10A), **Jeremy Alm**, Illinois College, and **Jacob Manske**, Epic.

Von Neumann Algebras and Related Fields (Code: SS 2A), **Stephen Avsec** and **Ken Dykema**, Texas A&M University.

Halifax, Canada

Dalhousie University

October 18–19, 2014

Saturday – Sunday

Meeting #1103

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: August 2014
Program first available on AMS website: September 5, 2014
Program issue of electronic *Notices*: October 2014
Issue of *Abstracts*: Volume 35, Issue 3

Deadlines

For organizers: Expired
For abstracts: August 19, 2014

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

François Bergeron, Université du Québec à Montréal, *Title to be announced.*

Sourav Chatterjee, New York University, *Title to be announced.*

William M. Goldman, University of Maryland, *Title to be announced.*

Sujatha Ramdorai, University of British Columbia, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Commutative Algebra and Its Interactions with Algebraic Geometry (Code: SS 2A), **Susan Marie Cooper**, Central Michigan University, **Sara Faridi**, Dalhousie University, and **William Traves**, U.S. Naval Academy.

New Directions in Category Theory (Code: SS 5A), **Pieter Hofstra**, University of Ottawa, and **Dorette Pronk**, Dalhousie University.

Sampling Theory (Code: SS 4A), **John J. Benedetto**, University of Maryland, **Jean-Pierre Gabardo**, McMaster University, and **Ozgur Yilmaz**, University of British Columbia.

Special Functions and Their Applications. (Code: SS 3A), **Mourad E. H. Ismail**, University of Central Florida, and **Nasser Saad**, University of Prince Edward Island.

p-adic Methods in Arithmetic (Code: SS 1A), **Henri Darmon**, McGill University, **Adrian Iovita**, Concordia University, and **Sujatha Ramdorai**, University of British Columbia.

San Francisco, California

San Francisco State University

October 25–26, 2014

Saturday – Sunday

Meeting #1104

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2014
Program first available on AMS website: September 11, 2014
Program issue of electronic *Notices*: October 2014
Issue of *Abstracts*: Volume 35, Issue 4

Deadlines

For organizers: Expired
For abstracts: September 3, 2014

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Kai Behrend, University of British Columbia, Vancouver, Canada, *Title to be announced.*

Kiran S. Kedlaya, University of California, San Diego, *Title to be announced.*

Julia Pevtsova, University of Washington, Seattle, *Title to be announced.*

Burt Totaro, University of California, Los Angeles, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Geometry (Code: SS 1A), **Renzo Cavalieri**, Colorado State University, **Noah Giansiracusa**, University of California, Berkeley, and **Burt Totaro**, University of California, Los Angeles.

Categorical Methods in Representation Theory (Code: SS 4A), **Eric Friedlander**, University of Southern California, **Srikanth Iyengar**, University of Nebraska, Lincoln, and **Julia Pevtsova**, University of Washington.

Geometry of Submanifolds (Code: SS 3A), **Yun Myung Oh**, Andrews University, **Bogdan D. Suceava**, California State University, Fullerton, and **Mihaela B. Vajiac**, Chapman University.

Nonlinear Partial Differential Equations (Code: SS 6A), **Nathan Glatt-Holtz**, Virginia Tech, and **Geordie Richards**, University of Rochester.

Polyhedral Number Theory (Code: SS 2A), **Matthias Beck**, San Francisco State University, **Martin Henk**, Universität Magdeburg, and **Joseph Gubeladze**, San Francisco State University.

Recent Progress in Geometric Analysis (Code: SS 5A), **David Bao**, San Francisco State University, and **Ovidiu Munteanu**, University of Connecticut.

Greensboro, North Carolina

University of North Carolina, Greensboro

November 8–9, 2014

Saturday – Sunday

Meeting #1105

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: August 2014
Program first available on AMS website: September 25, 2014
Program issue of electronic *Notices*: November 2014
Issue of *Abstracts*: Volume 35, Issue 4

Deadlines

For organizers: Expired
For abstracts: September 16, 2014

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Susanne Brenner, Louisiana State Unive rsity, *Title to be announced.*

Skip Garibaldi, Emory University, *Title to be announced.*

Stavros Garoufaldis, Georgia Institute of Technology, *Title to be announced.*

James Sneyd, University of Auckland, *Title to be announced* (AMS-NZMS Maclaurin Lecture).

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Difference Equations and Applications (Code: SS 1A), **Michael A. Radin**, Rochester Institute of Technology, and **Youssef Raffoul**, University of Dayton.

Recent Advances in Numerical Methods for Fluid Flow Problems (Code: SS 2A), **Leo Rebholz**, Clemson University, and **Zhu Wang**, University of South Carolina.

Recent Developments in Graph Theory and Hypergraph Theory (Code: SS 3A), **David Galvin**, University of Notre Dame, and **Clifford Smyth**, University of North Carolina Greensboro.

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday

Meeting #1106

Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015
Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: Expired
For abstracts: To be announced

Washington, District of Columbia

Georgetown University

March 7–8, 2015

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 7, 2014
For abstracts: To be announced

East Lansing, Michigan

Michigan State University

March 13–15, 2015
Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 26, 2014
For abstracts: January 20, 2015

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Fractional Calculus and Nonlocal Operators (Code: SS 1A), **Mark M. Meerschaert** and **Russell Schwab**, Michigan State University.

Random Fields and Long Range Dependence (Code: SS 2A), **Mark M. Meerschaert** and **Yimin Zhao**, Michigan State University.

Huntsville, Alabama

University of Alabama in Huntsville

March 27–29, 2015
Friday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 20, 2014
For abstracts: To be announced

Las Vegas, Nevada

University of Nevada, Las Vegas

April 18–19, 2015
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 18, 2014
For abstracts: To be announced

Porto, Portugal

University of Porto

June 10–13, 2015
Wednesday – Saturday
First Joint International Meeting involving the American Mathematical Societry (AMS), the European Mathematical Society (EMS), and the Sociedade de Portuguesa Matematica (SPM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For abstracts: To be announced

Chicago, Illinois

Loyola University Chicago

October 3–4, 2015
Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: October 2015
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 10, 2015
For abstracts: To be announced

Memphis, Tennessee

University of Memphis

October 17–18, 2015

Saturday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 17, 2015
For abstracts: August 18, 2015
The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Computational Analysis (Code: SS 1A), **George Anastassiou**, University of Memphis.

Fullerton, California

California State University, Fullerton

October 24–25, 2015

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: October 2015
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 27, 2015
For abstracts: To be announced

New Brunswick, New Jersey

Rutgers University

November 14–15, 2015

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday
Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For abstracts: To be announced

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday
Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For abstracts: To be announced

Charleston, South Carolina

College of Charleston

March 10–12, 2017

Friday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: November 10, 2016
For abstracts: To be announced

Pullman, Washington

Washington State University

April 22–23, 2017

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday
Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2017
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 16–19, 2019

Wednesday – Saturday
Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM)and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2018
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2018
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Central Section: **Georgia Benkart**, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

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Southeastern Section: **Brian D. Boe,** Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, e-mail: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/.

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 99 in the January 2014 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

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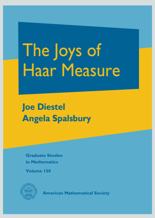

The Joys of Haar Measure

Joe Diestel, *Kent State University, OH*, and **Angela Spalsbury**, *Youngstown State University, OH*

The aim of this book is to present invariant measures on topological groups, progressing from special cases to the more general. Presenting existence proofs in special cases, such as compact metrizable groups, highlights how the added assumptions give insight into just what the Haar measure is like; tools from different aspects of analysis and/or combinatorics demonstrate the diverse views afforded the subject.

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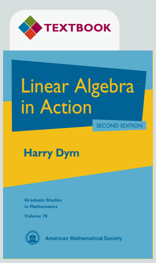

Linear Algebra in Action

Second Edition

Harry Dym, *Weizmann Institute of Science, Rehovot, Israel*

This new edition, augmented with a number of new revisions, introductory sessions, and exercises, provides a user-friendly introduction to the basic and advanced techniques of linear algebra from the point of view of a working analyst.

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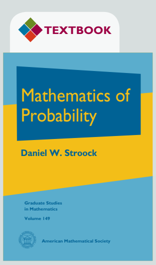

Mathematics of Probability

Daniel W. Stroock, *Massachusetts Institute of Technology, Cambridge, MA*

The book is a self-contained introduction to probability theory and the measure theory, covering the basics of modern probability theory.

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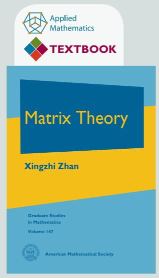
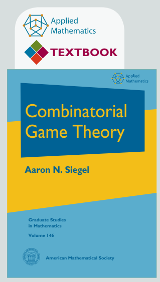


Matrix Theory

Xingzhi Zhan, *East China Normal University, Shanghai, China*

A clear and concise presentation of modern perspectives of matrix theory, which provides a thorough treatment of topics not found in standard textbooks.

Graduate Studies in Mathematics, Volume 147; 2013; 264 pages; Hardcover; ISBN: 978-0-8218-9491-0; List US$65; AMS members US$52; Order code GSM/147



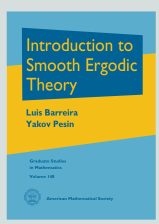
Combinatorial Game Theory

Aaron N. Siegel, *San Francisco, CA*

For those wishing to know about combinatorial games in depth this is the book to read. ... Aaron Siegel is currently the strongest researcher in the field and has been involved with many of the central developments. In this book, he has brought them together. ... [He] has kept the tone of the book light and infused it with history, anecdotes, and important observations making it an entertaining as well as an educational read.

—Richard Nowakowski, MAA Reviews

Graduate Studies in Mathematics, Volume 146; 2013; 523 pages; Hardcover; ISBN: 978-0-8218-5190-6; List US$89; AMS members US$71.20; Order code GSM/146



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Luis Barreira, *Instituto Superior Técnico, Lisbon, Portugal*, and **Yakov Pesin**, *Pennsylvania State University, State College, PA*

This book is the first comprehensive introduction to smooth ergodic theory. It is aimed at anyone who wants to acquire a working knowledge of smooth ergodic theory and to learn how to use its tools.

Graduate Studies in Mathematics, Volume 148; 2013; 277 pages; Hardcover; ISBN: 978-0-8218-9853-6; List US$65; AMS members US$52; Order code GSM/148

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