About the Cover

From the garden of spectrahedra

This month's cover image was suggested by Cynthia Vinzant's article "WHAT IS… a spectrahedron?" in this issue. The image was produced by Vinzant, but is based on a similar image produced by Pablo Parrilo for the article "Semidefinite representation of the k-ellipse" (*Algorithms in Algebraic Geometry*, 2008) jointly written by him together with Jiawang Nie and Bernd Sturmfels.

We asked Sturmfels to comment on it. He replied:

"Fix three general points in the plane and consider the set of all points (x,y) whose distance sum to the three given points is a certain constant d. This set is a convex curve, known as a 3-ellipse, or an ellipse with three foci. The interior of the 3-ellipse is a spectrahedron, that is, a convex set belonging to the class of objects introduced in Cynthia Vinzant's article. Such higher ellipses and their connections to optimization theory are studied in the article I wrote with Nie and Parrilo.

Spectrahedra are the feasible regions in semidefinite programming. This includes convex polyhedra,which are the feasible regions in linear programming. Both convex polyhedra and spectrahedra can be quite beautiful. While the description of convex polyhedra is based on linear algebra and combinatorics, the boundaries of spectrahedra are usually nonlinear. Their study requires some use of nonlinear algebra and algebraic geometry.

The boundary of the 3-ellipse is an algebraic curve of degree 8. It is represented by a unique (up to scaling) irreducible polynomial $P(x, y, d)$ of degree 8 in three unknowns. The diagram shows the surface $P(x, y, d) = 0$. The convex region seen in the middle is a three-dimensional spectrahedron. Slicing the surface with the horizontal plane at height d gives the planar 3-ellipse for radius d. The other branches of this irreducible surface illustrate the algebraic structure inherent in semidefinite programming."

We wish to thank Vinzant, Sturmfels, and Parrilo for assistance.

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