
About the Cover

From L. E. J. Brouwer's *Zur Analysis Situs*

The cover image was suggested by the review in this issue of a biography of the Dutch mathematician L. E. J. Brouwer. The original diagram was one of two inserted as separate sheets into Brouwer's article "Zur Analysis Situs" in the 1910 volume of the *Mathematischen Annalen*. As far as we can tell, it was the first colored diagram to be published in any mathematical research journal, and is apparently the only colored diagram ever to have been published in the *Annalen*.

Brouwer is probably best known for his strong stance on constructive proofs, but of course he is also well known for his theorem on fixed-points. He was in fact one of the founders of modern topology. "Zur Analysis Situs" was his response to some prior work of the mathematician Schoenflies, who was at that time considered the world expert on the topology of point sets in the plane. But Brouwer had found Schoenflies's work to contain serious errors. Basically, what he pointed out was that Schoenflies had not been rigorous enough, and that the topological structure of point sets in the plane was often far from intuitive.

The figure on the cover served in his paper to illustrate several of his objections. It exhibited among other things Brouwer's construction of the first known *indecomposable continuum*—that is to say, a compact, connected subset of the plane that could not be expressed as the union of two proper closed connected subsets. This set is constructed as the intersection of a sequence of annuli. Each one is obtained from the previous one by removing two disjoint sets from the previous one. In the cover image, the construction is at the third stage. The annulus is the light region, the interior of the annulus is the region hatched in red, and its exterior is hatched in black. On the cover, Brouwer's diagram has been extended to include a larger background area hatched in black.

We find, as Schoenflies also found, Brouwer's explanation of his figure a bit hard to follow. His results were soon incorporated into the literature of topology, but rather different and simpler diagrams replaced his. The clearest explanation of Brouwer's diagram itself that we have found is the survey "A brief historical view of continuum theory" in the 2006 volume of *Topology and its Applications*, by W. T. Ingram. He interprets the intersection of the sequence of annuli as an inverse limit of circles, mapping onto each other by wrapping around roughly one and one-half times.

The controversy between Brouwer and Schoenflies, as well as subsequent history, is dealt with in §4.6 of the biography of Brouwer under review.

The color in Brouwer's diagram gave rise to some technical difficulties and thus correspondence among him, Hilbert, and Korteweg. Because of the color, his diagrams could not be printed on the pages of the article itself, and had to be inserted. Brouwer made the original diagram by hand at a size of about 60 cm by 70 cm, and this had to be reduced in size for insertion. It seems, from the evidence of a letter from Brouwer to Hilbert, that the journal's first response was to ask Brouwer to redraw his figures at a

smaller scale. Brouwer objected strongly to this, insisting that it be only a last resort. He pointed out that in Amsterdam the printers could reduce and reproduce the diagrams by a photographic process, and asked with some asperity why German printers couldn't do the same. He even offered to pay for this himself. We do not know who did eventually pay.

Incidentally, the process he referred to was heliotypy, about which the Internet tells us much. It is a variation of the better known collotype process, and was extensively used for fast, inexpensive, high-quality reproduction of photographs. We are not sure how the printers handled the two colors.

We wish to thank Dirk van Dalen, the author of the biography under review, for supplying us with information about the publication of the diagram, and particularly for translating into English a letter, originally in Dutch, from Brouwer to Korteweg. He tells us that most of Brouwer's personal archives were unfortunately destroyed in a fire, so that what is extant concerning the production of the diagrams is sadly incomplete.

Also sadly, in the course of time many libraries have lost the colored inserts of Brouwer's article. It is slightly depressing to see that copies of this volume of the *Annalen* scanned for Internet archives do not include the inserts, either. (But then this is often true of inserted figures in old science books.) The photograph on the cover was made from the copy of the *Annalen* at the Institute for Advanced Study in Princeton, to which we are grateful.

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