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Notices

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February 2015

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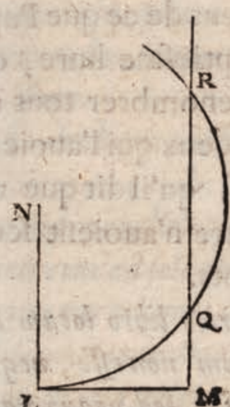
LIVRE PREMIER. 303

angle, iusques a O, en sorte qu'N O soit esgale a N L, la toute O M est z la ligne cherchée. Et elle s'exprime en cete sorte

$$z \infty \tfrac{1}{2} a + \sqrt{\tfrac{1}{4} aa + bb}.$$

Que si iay $yy \infty -- ay + bb$, & qu'y soit la quantité qu'il faut trouuer, ie fais le mesme triangle rectangle N L M, & de sa baze M N i'oste N P esgale a N L, & le reste P M est y la racine cherchée. De façon que iay $y \infty -- \tfrac{1}{2} a + \sqrt{\tfrac{1}{4} aa + bb}$. Et tout de mesme si i'auois $x^4 \infty -- a x^2 + b^2$. P M seroit x^2. & i'aurois $x \infty \sqrt{-- \tfrac{1}{2} a + \sqrt{\tfrac{1}{4} aa + bb}}$: & ainsi des autres.



Enfin si i'ay

$$z^2 \infty az -- bb:$$

ie fais N L esgale à $\tfrac{1}{2} a$, & L M esgale à b cõme deuãt, puis, au lieu de ioindre les poins M N, ie tire M Q R parallele a L N. & du centre N par L ayant descrit vn cercle qui la couppe aux poins Q & R, la ligne cherchée z est M Q, oubiẽ M R, car en ce cas elle s'exprime en deux façons, a sçauoir $z \infty \tfrac{1}{2} a + \sqrt{\tfrac{1}{4} aa -- bb}$, & $z \infty \tfrac{1}{2} a -- \sqrt{\tfrac{1}{4} aa -- bb}$.

Et si le cercle, qui ayant son centre au point N, passe par le point L, ne couppe ny ne touche la ligne droite M Q R, il n'y a aucune racine en l'Equation, de façon qu'on peut assurer que la construction du problesme proposé est impossible.

Au-

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About the cover: x, y, and z. (see page 147)

AMERICAN MATHEMATICAL SOCIETY

AMS Centennial Fellowship Fund

Fostering outstanding mathematical research

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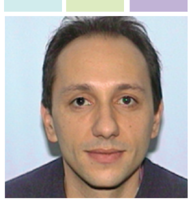

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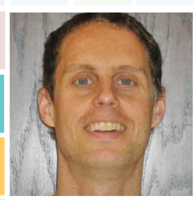

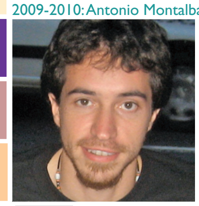

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The American Mathematical Society presents

The 2015 AMS Einstein Public Lecture in Mathematics

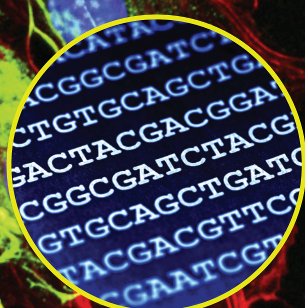
Simon Tavaré

Director, *Cancer Research UK Cambridge Institute,*
and Professor, *Department of Applied Mathematics and Theoretical Physics, University of Cambridge*

Cancer by the Numbers

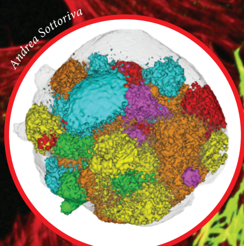
Saturday, March 7 at 5:00 p.m.

Lohrfink Auditorium in the Rafik B. Hariri Building,
Georgetown University • *Reception to follow*



The mathematical sciences have contributed substantially to our understanding of the way cancer evolves. Cancer is a disease of the genome, so the focus of this lecture will be on mutations in DNA and how they inform us about tumor evolution. We will discuss "tumor heterogeneity," the DNA sequence variation observed between tumors and within them, and what this tells us about progression, treatment, and relapse. Along the way we will illustrate some of the underlying mathematics that have helped in this endeavor.

The Einstein Lecture is part of the Spring 2015 AMS Eastern Sectional Meeting (March 7–8) at Georgetown University.

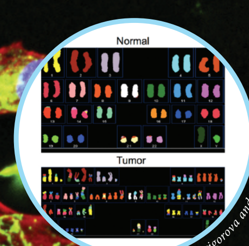


Andrea Sottoriva

For more information:

www.ams.org/meetings/sectional/2225_events.html

www.ams.org/meetings/sectional/2225_program.html



Courtesy of Mira Grigorova and Paul Edwards

Georgetown University

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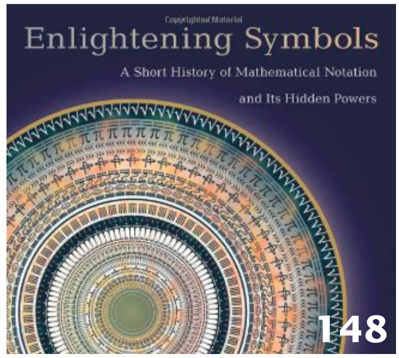
Notices
of the American Mathematical Society

February 2015

Communications

Commentary



From the frigid depths of winter we offer mathematical solace:

We begin this February 2015 issue with the first part of a two-part centenary-celebration feature on the life of Paul Erdős. Next we have intriguing discussions of the relationship between the National Security Agency and the mathematics profession. There are interviews with outgoing AMS President David Vogan and with Abel Prize Laureate Yakov Sinai.

We present a description of the characteristics of successful calculus programs, offer a *Scripta Manent* on one professor's account of editing an electronic journal, and a *Doceamus* treatment of how we can use history to more effectively teach mathematics.

We hope that these fascinating readings will help to sustain the *Notices* reader until spring. *—Steven G. Krantz, Editor*

Features

Notices

of the American Mathematical Society

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Departments

From the AMS Secretary


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calculus
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Time to Talk about Gender Equality in Math?

Gender equality is always a hot topic, especially in the area of financial compensation. I recently was reminded of the importance of this issue when I interviewed the new chief executive officer of Microsoft, Satya Nadella, at the annual Grace Hopper Celebration of Women in Computing. Satya's response to "What advice would you give to women who feel uncomfortable in asking for a raise?" suggested that people should trust the system and that not asking for raises but simply doing good work would eventually result in rewards. Satya has since publicly apologized for those remarks. He said that his advice at the conference was "just plain wrong," despite being advice from his mentors that he has followed in his own career. He reinforced that this advice underestimated exclusion and bias—conscious and unconscious—that can hold people back in their careers. And, in fact, he has told all Microsoft employees that the company will work harder to make progress on diversity and inclusion at the core of Microsoft's culture in the areas of equal pay for equal work and equal opportunity for equal work, will recruit more diverse employees, and will expand training on how to foster an inclusive culture. This is real progress and action by Satya and Microsoft.

One of the positive outcomes of this recent experience is that it has sparked a valuable discussion about the importance of addressing gender equity issues with respect to pay, promotions, and access to resources. Despite numerous studies indicating that women are paid significantly less than men in most professions, the disparities remain. Among the hundreds of emails I received after the interview, one was from a female mathematician who wished that the mathematics community would have a serious discussion about gender equity issues in our profession. I'm hoping this "Opinion" piece might contribute to that discussion.

A very interesting article titled "Academic women in science, a changing landscape" was published recently by Stephen Ceci, et al. in *Psychological Science*. The article reviewed many recent studies on why there are so few female majors and faculty members in mathematically intensive areas of science and engineering and explores factors that might account for the difference in average pay received by men and women. I have personally been in situations where my discomfort in asking for a raise or negotiating a salary resulted in my being underpaid.

Am I the only mathematician who finds it easy to argue for raises, promotions, recognition, and resources for the people in my department but finds it difficult to ask for them for myself? Are there others who feel frustrated because they are always the only person who asks why there are so few women on lists of invited speakers or participants in prestigious events?


Courtesy of the Anita Borg Institute

Satya Nadella with Maria Klawe at the Grace Hopper Celebration 2014.

And what about the high school teachers who still tell girls (but not boys) in their math and science classes they will have trouble with the more challenging concepts? There are many individuals and organizations who work on this every day, but isn't it time for all of us to work on it?

Satya said that when he became CEO, the advice he got was to be bold and be right. This advice I support, and I challenge all of us to apply this to our gender equality discussions and actions.

—Maria Klawe
Harvey Mudd College
`klawe@hmc.edu`

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Letters to the Editor

The AMS Must Justify Its Support of the NSA

Roger Schlafly (letters, *Notices*, November 2014) accuses mathematicians of an "overwrought" and "over-excited" response to the recently-revealed activities of the National Security Agency (NSA). So, let us look at some cold facts. In 2011, the NSA explicitly stated its goal of universal surveillance, describing its "posture" as "collect it all", "know it all", "exploit it all". The same year, the NSA's close British partner GCHQ said it was intercepting over fifty billion communication events per day. In 2012, a single NSA program celebrated its *trillionth* metadata record.

On encryption: the NSA's 2013 budget request sought funds to "Insert vulnerabilities into commercial encryption systems". The NSA described its secret program Sentry Raven as "work[ing] with specific US commercial entities…to modify US manufactured encryptions systems to make them exploitable for SIGINT [signals intelligence]". The aim is clear: that no two human beings shall be able to communicate digitally without the NSA being able to listen.

Schlafly is, at least, correct in noting that outrage at the intelligence agencies' abuse of surveillance powers is nothing new: from the FBI's bugging of Martin Luther King and subsequent attempt to blackmail him into suicide, to the 2011 extrajudicial killing of an American child by CIA drone strike (a program to which the NSA supplies surveillance data). He is justified in worrying about the data held by Google, Facebook, etc., but he writes as if concern over that and state surveillance were mutually exclusive, which of course they are not; and much of that data is harvested by the NSA's PRISM program anyway.

Further, his comparison with 1970s technology distracts from the awesome invasive power of today's Internet. As the NSA's former general counsel Stewart Baker said, "metadata absolutely tells you everything about somebody's life". Former NSA director Michael Hayden agreed, adding "we kill people based on metadata".

By collaborating with the NSA, the AMS sends a strong political message: that it is proud to support the NSA's work and welcomes it into the mathematical community. It is just as surely a political position as withdrawing cooperation would be. Many members are vigorously opposed to much of what the NSA does; indeed, when the *Notices* set out to organize the series "Mathematicians discuss the Snowden revelations", its editors could not find *anyone* to write in the NSA's defense. (And when they finally did, it was a longtime NSA employee.)

How does the AMS leadership justify its continued cooperation with the NSA? Is it certain it has the backing of the membership? And what exactly *would* the NSA have to do in order for the AMS to declare "Enough: this partnership brings mathematicians into disrepute"?

(A fully referenced version of this letter is available at `www.maths.ed.ac.uk/~tl/a`.)

—*Tom Leinster*
University of Edinburgh
`tom.leinster@ed.ac.uk`

(Received October 23, 2014)

Difference between the NSA and Google

In his June 28, 2014, letter to the *Notices* [November 2014 issue], Roger Schlafly claims that he does not see a distinction between the dangers posed by the massive collection of data by commercial companies like Google and the collection of data by the NSA. Perhaps that is because he is also unable to see a distinction between public and covert oversight. No doubt the practices of Google are a real danger, but commercial companies are subject to regulations and can be brought before open courts whose judges are appointed by an elected president and have to be approved by the Senate. The regulations governing the NSA are classified, and the NSA is answerable only to a closed court whose judges are appointed, without further review, by a man who himself was appointed by a president who believed that one can defeat terror by declaring a war on it. Maybe these distinctions seem trivial to Dr. Schlafly, but even he should be able to understand why somebody like Alexander Beilinson, who grew up in a country where all courts were secret, does not.

—*Daniel W. Stroock*
Professor emeritus
Massachusetts Institute of Technology
`dws@math.mit.edu`

(Received November 14, 2014)

Reflections on Paul Erdős on His Birth Centenary

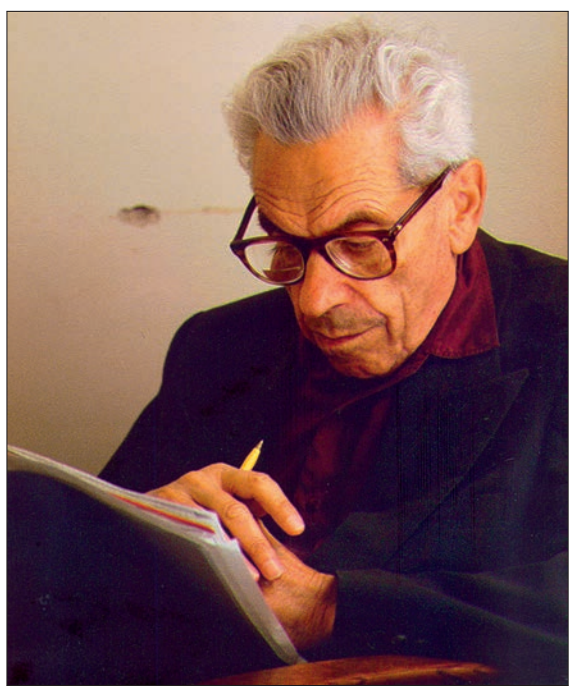
Krishnaswami Alladi and Steven Krantz, Coordinating Editors

This is Part I of a two-part feature on Paul Erdős following his centennial. There are eleven articles by leading experts who have reflected on the remarkable life, contributions, and influence of this towering figure of twentieth century mathematics. Here in Part I we have contributions from Krishnaswami Alladi and Steven Krantz, László Lovász and Vera T. Sós, Ronald Graham and Joel Spencer, Jean-Pierre Kahane, and Mel Nathanson.
Part II will contain articles by Noga Alon, Dan Goldston, András Sárközy, József Szabados, Gérald Tenenbaum, and Stephan Garcia and Amy Shoemaker.

Krishnaswami Alladi and Steven Krantz

One of the Most Influential Mathematicians of Our Time

The 100th birth anniversary of the great Hungarian mathematician Paul Erdős was celebrated in Budapest in July 2013 with an international conference that attracted about 750 participants. Erdős was one of the most influential mathematicians of the twentieth century for a variety of reasons. He made fundamental and pioneering contributions in several fields of mathematics, such as number theory, combinatorics, graph theory, analysis, geometry, and set theory. He was perhaps the most prolific mathematician in history after Euler, with more than 1,500 papers, but what was most interesting about this was that more than half of these were joint papers and many of his collaborators were very young. Thus through the fundamental ideas in his papers and through these collaborations, he influenced several generations of mathematicians and molded the careers of many. He was the greatest problem proposer in history,



Courtesy of Harold Diamond

Paul Erdős working in his office at the Hungarian Academy of Sciences, Budapest.

Krishnaswami Alladi is professor of mathematics at the University of Florida. His email address is `alladik@ufl.edu`.

Steven Krantz is professor of mathematics at Washington University in St. Louis. His email address is `sk@math.wustl.edu`.

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Photo courtesy of Krishnaswami Alladi

An emblem of Erdős made for the Erdős Memorial Conference in Budapest, Hungary, July 1999.

and for many of his problems he offered prize money depending on their difficulty. These problems have shaped the development of several areas, and most have remained unsolved. The lectures at the Erdős Centennial Conference demonstrated that the influence of Paul Erdős on the growth of mathematics remains strong. Here we have gathered eleven articles by leading mathematicians on various aspects of Erdős's work and his influence on current research. In our article we make some personal reflections, touch upon some of Erdős's work in number theory, and describe ways in which his legacy has been honored.

Like all great geniuses, Erdős had his idiosyncrasies. But even in a mathematical world used to the peculiarities of its luminaries, Erdős was a most unusual phenomenon. There are hundreds of Erdős stories that are fondly recalled at various mathematical gatherings, and this feature has a good sampling of such recollections.

Like many of the greatest mathematicians in history, Erdős made his entry into the world of research very early in his life and in a grand manner. His very first paper [4] was a proof of Bertrand's postulate, which states that for any $n \geq 1$, there is always a prime number in the interval $[n, 2n]$. The Russian mathematician Chebychev was the first to prove this, but Erdős's proof utilizing the study of the prime factors of the middle binomial coefficient is so elegant and clever that it is this proof that is given in all textbooks on number theory. News of Erdős's proof spread like wildfire and was accompanied by a rhyme:"Chebychev said it and I say it again, there is always a prime between n and $2n$." During many of his lectures, and in particular in an article entitled "Ramanujan and I" [7], published on the occasion of the Ramanujan Centennial, Erdős has pointed out that there are similarities between his proof of Bertrand's postulate and Ramanujan's, but he was not aware of Ramanujan's proof when he wrote his first paper in 1932.

Erdős has often jokingly said that God has a book of the most beautiful proofs of the most important theorems. His desire was to glance through this book of God (after death)! It should be pointed out that Erdős was an atheist and referred to God as the Supreme Fascist. But in the case of the proofs, he admitted that God possessed this wonderful Book. Martin Aigner and Gunter Ziegler have brought out a publication [1] entitled *Proofs from The Book* containing the most beautiful proofs of important theorems in various branches of mathematics. There are several proofs of Erdős in this book, since his proofs are extremely clever, elegant, and elementary.

A famous problem of Erdős which is still unsolved and for which he has offered US$3,000 is the following: If $\{a_n\}$ is a sequence of increasing positive integers such that $\sum \frac{1}{a_n}$ is divergent, then prove that the sequence $\{a_n\}$ contains arbitrarily long arithmetic progressions. An important special case of this problem when the $\{a_n\}$ is the sequence of primes has been settled, and this is the celebrated Green-Tao theorem [11]. In 1936 Erdős and Turán conjectured that if $\{a_n\}$ has upper positive density, then $\{a_n\}$ contains arbitrarily long arithmetic progressions. Erdős offered US$1,000 for the resolution of this conjecture, which was proved by Szemerédi [20].

Prime numbers were among Erdős's favorite topics of investigation. The prime number theorem, which states that the number of primes up to x is asymptotic to $x/\log x$, implies that the average gap between the nth prime p_n and the next one is asymptotic to $\log n$. Two questions immediately arise: (i) Can the ratio $r_n = (p_{n+1} - p_n)/\log n$ be arbitrarily large? (ii) How small can this ratio be infinitely often? The unsolved prime twins conjecture is the extreme solution to (ii). The recent sensational result of Zhang [23] on bounded gaps between primes shows that the ratio is $O(1/\log n)$ infinitely often (there is more about this in Goldston's article in Part II of this feature. See also Note Added in Proof to this article). The famous US$10,000 problem of Erdős concerns (i). Westzynthius [22] in 1931 showed that the ratio r_n can be arbitrarily large. Subsequently, the Scottish mathematician Rankin [17] was able to show more precisely that there exists a constant c such that

$$r_n > \frac{c.\log\log n.\log\log\log\log n}{(\log\log\log n)^2} \quad \text{infinitely often.}$$

Back in 1936, Erdős [5] had established a similar result but without the logloglogloglogn factor in the numerator. The Erdős US$10,000 problem is to prove or disprove that the constant c can be chosen arbitrarily large. See "Note Added in Proof" in this article. Ron Graham, who was the financial caretaker of Erdős for many years, has a pot of money left over to give out the prizes when the problems are solved.

Among Erdős's many fundamental contributions it is universally agreed that the two most important are (i) his joint paper with Mark Kac [9], which ushered in the subject of probabilistic number theory, and (ii) his elementary proof of the prime number theorem along with Atle Selberg.

The first proof of the prime number theorem was given toward the end of the nineteenth century simultaneously by Hadamard and de la Vallee Poussin by utilizing the properties of the zeta function $\zeta(s)$ as a function of the complex variable s as envisioned by Riemann. The method of this proof shows that the prime number theorem is equivalent to the assertion that $\zeta(1+it) \neq 0$ for real t. This led to the belief that any proof of the prime number theorem had to rely on the theory of functions of a complex variable. The noted British mathematician G. H. Hardy challenged the world to produce an "elementary" proof of the prime number theorem, namely, a proof that uses only properties of real numbers. He proclaimed that if such an elementary proof is found, then the books would have to be rewritten, since it would change our view of how the subject hangs together. In 1949 Erdős and Selberg created a sensation by producing such an elementary proof. The elementary proof was actually found by them jointly by starting with a fundamental lemma of Selberg, but for various reasons (that we shall not get into here) they had a misunderstanding and decided to write separate papers. Selberg's paper [19] appeared in the *Annals of Mathematics*; Erdős published his paper [6] in the *Proceedings of the National Academy of Sciences*.

In contrast, the Erdős-Kac collaboration [9] was a happy story. To understand this in context, we mention that in 1917 Hardy and Ramanujan [13], who did the first serious investigation of $\nu(n)$, the number of prime factors of n, showed that the average order of $\nu(n)$ is asymptotically $\log\log n$. They noted that $\nu(n)$ also has *normal order* $\log\log n$. This means that, for every $\epsilon > 0$, $\nu(n)/\log\log n$ is almost always between $1-\epsilon$ and $1+\epsilon$. They also showed that it makes no difference whether the prime factors of n are counted distinctly or with multiplicity. In 1934, Paul Turán, another great Hungarian mathematician who was Erdős's close friend, gave a simpler proof [21] of the Hardy-Ramanujan results by computing an upper bound for the second moment of $\nu(n)$ with mean $\log\log n$ and noted that a similar second moment estimate could be given for certain additive functions, namely, functions $f(n)$ which like $\nu(n)$ satisfy $f(mn) = f(m) + f(n)$ when m and n are relatively prime. The Hardy-Ramanujan and Turán results gave a hint of probabilistic underpinnings, but this became clear only later. In 1939 the great probabilist Mark Kac was giving a lecture at Princeton outlining various applications of probability. He constructed a model for the study of additive functions and conjectured that the distribution of a wide class of additive functions about their average order and with an appropriately defined variance would be Gaussian. Erdős, who was in the audience, perked up. He realized that Kac's conjecture could be proved using the Brun sieve, a topic in which Erdős was a master. He spoke to Kac after the lecture and they proved the conjecture together. To quote Erdős (see [7]): "Neither of us completely understood what the other was doing, but we realized that our joint effort will give the theorem, and to be a little impudent and conceited, probabilistic number theory was born! This collaboration is a good example to show that two brains can be better than one, since neither of us could have done the work alone." Subsequently Kubilius [15] extended the ideas and methods of Turán and of Erdős-Kac to treat the non-Gaussian cases as well. The subject of probabilistic number theory, ushered in by Erdős and Kac, is an active field of research today (see the two-volume book of Elliott [3]).

Courtesy of Krishnaswami Alladi

Paul Erdős and Ernst Straus on the campus of UC Santa Barbara during the West Coast Number Theory Conference, December 1978.

Another significant example of a paper of Erdős that led to a major field of study is his work with Rényi on random graphs [10]. There are numerous other ideas of Erdős that have had, and continue to have, widespread influence. The articles that follow describe many developments whose origins can be traced back to Erdős.

Many great mathematicians have written papers that have influenced the development of the subject and created new fields of study. What sets Erdős apart from all these luminaries is the

Courtesy of Krishnaswami Alladi

Paul Erdős lecturing at the Institute of Mathematical Sciences in Madras, India, in January 1984. Krishna Alladi is in the audience in the first row.

manner in which he collaborated with hundreds of mathematicians junior and senior and profoundly influenced them. It was his life's mission to seek out talented youngsters and mentor them in the study of mathematics. He jokingly said that he sought young collaborators, because he believed that he would be alive if at least half his collaborators were alive! The two of us will briefly share our experience in collaborating with Erdős.

Alladi's First Meeting and Collaboration with Erdős

As an undergraduate student in Madras, India, I was working on number theory on my own and read several introductory books. I noticed that there was practically nothing in these books on the sum of the prime factors of an integer. So I defined $A(n)$ to be the sum of the prime factors of an integer (summed with multiplicity) and observed it had two very nice properties: (i) $A(mn) = A(m) + A(n)$, for all m, n, and (ii) the number of solutions to $A(n) = m$ is the number of partitions of m into primes. Thus I was convinced that $A(n)$ was worthy of a closer study, and I investigated many of its properties. I was advised that I should contact Erdős to evaluate this work. Thus I sent a letter to the Hungarian Academy of Sciences, requesting that it be forwarded to Erdős. Within three weeks I received a response from him saying that soon he would be at the Indian Statistical Institute in Calcutta in December 1974 to speak at a conference for the 100th birth anniversary of its founder, Professor Mahalanobis. He enquired whether I could come to Calcutta. I responded saying that my paper on this topic was accepted for presentation at that conference, but I could not go due to my university exams. I said that my father, who will be giving one of the main lectures on probability at that conference, will be presenting my work in a special session talk. Erdős attended my father's presentation of my work. He went up to him and said, "While I am pleased to meet you, I would be happier to meet your son." He then told my father that he was scheduled to fly to Sydney from Calcutta a week later, but he was willing to reroute his journey, fly via Madras to Sydney to meet me. This gesture by a great mathematician to journey out of the way to meet a student speaks volumes about his passion to spot and encourage young mathematicians.

I went to the Madras airport to meet Erdős. Needless to say I was very nervous. He put me at ease by saying hello and immediately afterward the following: "Do you know my poem about Madras:

This to the city of Madras
the home of the curry and the dhal,
where Iyers speak only to Iyengars,
and Iyengars speak only to God"?

I said I did not know this poem. He said this is modeled along a similar well-known poem about Boston:

This to the city of Boston
the home of the bean and the cod,
where Lowells speak only to Cabots,
and Cabots speak only to God.

The Iyers and Iyengars are two Brahmin castes of the Hindu religion. The great Nobel laureate physicist Sir C. V. Raman was an Iyer, and Srinivasa Ramanujan was an Iyengar.

He spent three days in Madras, and I was with him all the time. When I told him that I had calculated the average order of $A(n)$ to be asymptotically $\pi^2 n/6 \log n$ but could not determine its normal order, he said that $A(n)$ will not have a normal order because $A(n)$ is dominated by $P_1(n)$, the largest prime factor of n, and $P_1(n)$ does not have a normal order. Then I asked him if $P_1(n)$ is subtracted from $A(n)$, would the remaining sum be dominated by $P_2(n)$, the second largest prime factor on n, and so on? He said that this is very likely and that my question was very nice. So this led to our very first paper [2], in which we showed among other things that if $P_k(n)$ denotes the kth largest prime factor of n, then $A(n) - P_1(n) - \cdots - P_{k-1}(n)$ and $P_k(n)$ have the same average (asymptotically), namely, $c_k n^{1/k}/log^k n$, where c_k is a rational multiple of $\zeta(1 + 1/k)$. All in all, I wrote five joint papers with him.

In December 1974 I had applied for admission to graduate schools in America. Erdős said he would write exactly one letter for me regarding my graduate admission. So while in Madras, he wrote a letter to Ernst Straus, his long-time friend at UCLA. By the end of January 1975, I received a Chancellors Fellowship from UCLA for my PhD, and I went there to work under Straus. Thus my contact with Erdős profoundly influenced my career. He corresponded with me from 1975 until his death in 1996 and supported my mathematical and professional progress. Over the years my family and I had the pleasure of hosting him in India, Florida, and in almost every place where I went for an extended visit.

Krantz's Collaboration with Erdős

In the early 1980s I had a big fight with my colleague Tory Parsons. We were very fond of each other and did not really want to fight, so we ended up kissing and making up. We did so by talking about mathematics. And we ended up writing a little paper about a covering lemma. The following summer, Tory gave a talk on our paper at a conference in Europe. In the middle of the talk Erdős jumped up and shouted, "But don't you realize that your result will allow us to prove the following theorems?" And Chris Godsil jumped up and said, "And you can also prove these other theorems." Next thing we knew we had a four-way paper going [8], and three of us earned an Erdős number of one. This just goes to show that even fighting can have fruitful results.

Honoring the Legacy of Erdős

In the last quarter-century, books and movies on the remarkable life of Paul Erdős have been produced. The most well-known book is by Paul Hoffman, entitled *The Man Who Loved Only Numbers* [14], and it appeared in 1998 after Erdős passed away. The equally charming book entitled *My Brain Is Open* by Bruce Schechter [18] also appeared in 1998. Erdős loved to discuss mathematics with almost anyone, and whenever he was ready for discussion, he would say, "Go ahead, my brain is open." Hence the title of the second book. The story of Erdős rerouting his travel to meet Alladi is described in this book in the opening chapter, entitled "Traveling", thanks to Ron Graham, who drew the attention of the author, Bruce Schechter, to this story. Both of these books are for the general public and convey the greatness of Erdős as a man and as a mathematician.

To complement these books is a nice documentary film called *n Is a Number* by Paul Csicsery, which has been shown at several major conferences. Many top mathematicians who worked closely with Erdős and collaborated with him are interviewed, and there are many lovely clips of Erdős in discussion with mathematicians around the world.

For Erdős's seventieth birth anniversary, *Combinatorica*, a journal he founded, brought out a special volume in his honor. Three years after Erdős passed away, a conference entitled "Paul Erdős and his mathematics" was held at the Hungarian Academy of Sciences, Budapest, from July 4 to 11, 1999. A two-volume book [12] under the same title was published comprised of mathematical papers and reminiscences by several of the main speakers at that conference. More than five hundred mathematicians attended that conference, and some of them contributed papers to the memorial volume of *Combinatorica*.

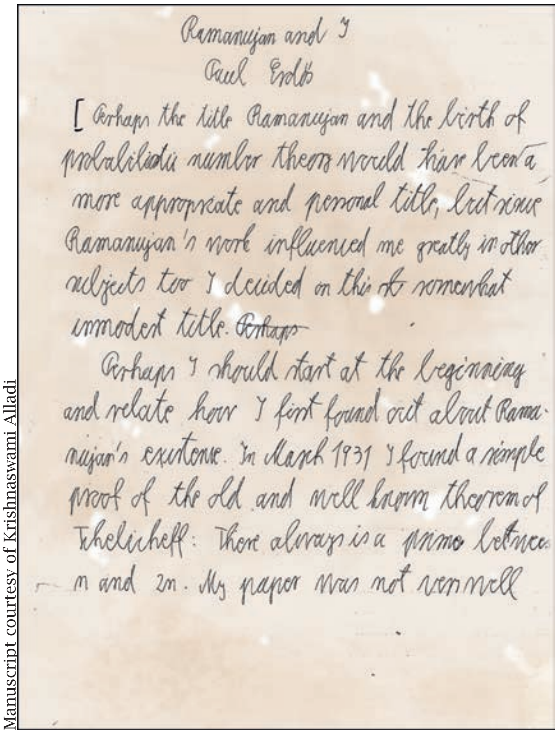
In 1999 the American Mathematical Society started the Erdős Memorial Lectures with support from a fund created by Mr. Beal, a Dallas banker and mathematics enthusiast. This lecture is delivered annually at one of its meetings. A year earlier, in 1998, at the University of Florida, where Erdős visited every spring, the annual Erdős Colloquium was launched during Alladi's term as chair. Ron Graham gave the first Erdős Colloquium in Florida as well as the first Erdős Memorial Lecture for the AMS. One year earlier, in 1997, Memphis State University (where Erdős was an adjunct professor since 1975) launched the Erdős Memorial Lectures, the first of which was delivered by Vera T. Sós.

Erdős, like Ramanujan, was such an unusual personality that articles about him have appeared in magazines such as the *New Yorker*, *Discover*, and the like. When he died, both the *New York Times* and the *London Times* published substantial obituaries.

Awards and Distinctions

Erdős received many prizes and much recognition for his monumental contributions. We mention just a few. He was awarded the 1951 Cole Prize of the AMS for his many fundamental papers and specifically for his "Elementary proof of the prime number theorem". In 1983 he was the recipient of the Wolf Prize for his lifelong contributions. He was also given an Honorary Doctorate by Cambridge University in 1991. He was elected a Member of the US National Academy of Sciences and also a Foreign Member of the Royal Society. He was elected as a Member of the Hungarian Academy of Sciences in 1956. But these laurels rested lightly on his shoulders. He always gave away the prize money he received for a good mathematical cause. When interviewed for the documentary *n Is a Number*, Erdős said that he would trade all his awards for a nice theorem and its proof. To him what was important was to prove and conjecture.

An academician must be judged not only by the quality and significance of his contributions but also by the work of his direct disciples and many others he influenced through his ideas. In 1998 Tim Gowers of Cambridge University was awarded the Fields Medal. Gowers was in a sense a grand student of Erdős, because Gowers received his PhD under the direction of Bela Bollobás, who was a disciple


Ramanujan and I
Paul Erdős
[Perhaps the title Ramanujan and the birth of probabilistic number theory would have been a more appropriate and personal title, but since Ramanujan's work influenced me greatly in other subjects too I decided on this somewhat immodest title. ~~Perhaps~~
Perhaps I should start at the beginning and relate how I first found out about Ramanujan's existence. In March 1931 I found a simple proof of the old and well known theorem of Tchebicheff: There always is a prime between n and 2n. My paper was not very well

Manuscript courtesy of Krishnaswami Alladi

First page of the handwritten manuscript "Ramanujan and I" by Erdős, which he wrote for the Ramanujan Centennial.

of Erdős. The Fields Medal to Gowers was a major recognition by the international mathematical community of the importance of combinatorial methods and the kind of mathematics that Erdős pursued. This was followed by the 2006 Fields Medal to Terence Tao for contributions to many areas of mathematics, especially to the field of additive combinatorics. The award of the 2012 Abel Prize to E. Szemerédi, a protege of Erdős, is yet another recognition for Erdős-type mathematics and for the Hungarian mathematical tradition. Even though Erdős started out in number theory and made several pioneering contributions to that area, in later years his attention was mostly directed towards combinatorics, graph theory, and discrete mathematics, and he strongly influenced the development of these fields. If Erdős were alive today, he would be the happiest person to see that combinatorics and discrete mathematics have been given their due place in mathematics.

References

[1] M. Aigner and G. M. Zeigler, *Proofs from The Book*, Springer, Berlin, 1998.

[2] K. Alladi and P. Erdős, On an additive arithmetic function, *Pacific J. Math.* **71** (1977), 275–294.

[3] P. D. T. A. Elliott, *Probabilistic Number Theory. I, II*, Springer, New York, 1980.

[4] P. Erdős, Beweis eines Satzes von Tscebyschef, *Acta Sci. Math. (Szeged)* **5** (1930–1932), 194–198.

[5] ______, On the difference between consecutive primes, *Quart. J. Math.*, Oxford Ser., **6** (1935), 124–128.

[6] ______, On a new method in elementary number theory which leads to an elementary proof of the prime number theorem, *Proc. Nat'l. Acad. Sci. USA* **35** (1949), 374–384.

[7] ______, Ramanujan and I, in *Number Theory, Madras 1987, Proc. Ramanujan Centenary Conf.* (K. Alladi, ed.), Springer Lecture Notes in Math.,1395, 1989, pp. 1–20.

[8] P. Erdős, C. Godsil, S. G. Krantz, and T. D. Parsons, Intersection graphs for families of balls in R^n, *Eur. J. Combin.* **9** (1998), 501–505.

[9] P. Erdős and M. Kac, The Gaussian law of errors in the theory of additive number theoretic functions, *Amer. J. Math.* **62** (1940), 738–742.

[10] P. Erdős and A. Rényi, On the evolution of random graphs, *Magyar Tud. Akad. Mat. Kutató Int. Kozl.* **5** (1960), 17–61.

[11] B. Green and T. Tao, The primes contain arbitrarily long arithmetic progressions, *Annals of Math.* (2) **167** (2008), 481–547.

[12] G. Halasz, L. Lovász, M. Simonovits, and V. T. Sós, eds., *Paul Erdős and his mathematics. I, II*, Bolyai Society Math. Studies, 11, Springer, Berlin, 2002.

[13] G. H. Hardy and S. Ramanujan, On the normal number of prime factors of a positive integer n, *Quart. J. Math.* (Oxford) **48** (1917), 76–92.

[14] P. Hoffman, *The Man Who Loved Only Numbers*, Hyperion, New York, 1998.

[15] J. Kubilius, Probabilistic methods in the theory of numbers, *Uspehi Math. Nauk.* **11** (1956), 31–66.

[16] L. Lovász, I. Z. Ruzsa, and V. T. Sós, eds., *Erdős Centennial*, Bolyai Society Math. Studies, 25, Springer, Berlin, 2013.

[17] R. A. Rankin, The difference between consecutive prime numbers, *J. London Math. Soc.* **13** (1938), 242–247.

[18] B. Schechter, *My Brain Is Open—The Mathematical Journeys of Paul Erdős*, Simon and Schuster, New York, 1998.

[19] A. Selberg, An elementary proof of the prime number theorem, *Annals of Math.* (2) **50** (1949), 305–313.

[20] E. Szemerédi, On sets of integers containing no k elements in arithmetic progression, *Acta Arith.* **27** (1975), 199–245.

[21] P. Turán, On a theorem of Hardy and Ramanujan, *J. London Math. Soc.* **9** (1934), 274–276.

[22] E. Westzynthius, Über die Verteilung der Zahlen, die zu den n ersten Primzahlen teilerfremd sind, *Comm. Phys. Math. Helsingfors* **25** (1931), 1–37.

[23] Y. Zhang, Bounded gaps between primes, *Annals of Math.* (2) **179** (2014), 1121–1174.

Note Added in Proof

Very recently there has been tremendous progress on both the small gap and large gap problems. Maynard [25] has stunned the world by showing that the gap between primes can be made made ≤ 600 infinitely often. Following Zhang's bounded gap theorem, the Polymath Project led by Tao had achieved a bound of 4,680 vastly improving Zhang's bound of seventy million, but Maynard has reduced this even further. Maynard is awarded the 2014

SASTRA Ramanujan Prize for this achievement and other results. See [27] for the latest results on the bounded gap problem. In the last few months, Ford-Green-Konyagin-Tao [24] and Maynard [26] have announced a solution to the Erdős $10,000 problem by showing that the constant in Rankin's lower bound can be made arbitrarily large. The methods in [24] and [26] are different.

References

[24] K. Ford, B. Green, S. Konyagin and T. Tao, *Large gaps between consecutive prime numbers*, preprint, http://arxiv.org/abs/1408.4505.

[25] J. Maynard, Small gaps between primes, *Annals of Math.* (to appear).

[26] ———, *Large gaps between primes*, arXiv1409.5110v1[math.NT].

[27] D. H. J., Polymath: The "Bounded Gaps Between Primes" Polymath Project, *Newsletter EMS* **94** (2014), 13–23.

László Lovász and Vera T. Sós

Erdős Centennial

It would be impossible to discuss the tremendous work of Paul Erdős especially in such a short article. All we can do is to contribute some impressions and ideas about the nature of his work, flavored by some quotations from letters of Erdős and by a few personal impressions and experiences. Several "mathematical," and not only mathematical, biographies of Erdős were written, among these we mention here two thorough ones written by Babai [2] and Bollobás [6]. His work was treated in depth in a number of volumes containing expert articles [20], [21], [22], [24], and even on the pages of these *Notices* [4].

The idea of the present issue of the *Notices* arose in connection with the Erdős Centennial Conference we organized in summer 2013. To be precise, we organized three Paul Erdős conferences in Budapest: The first took place in 1996, one day after his funeral. At that one-day meeting, our goal was to give an immediate short survey of his oeuvre, a demonstration of his unique role in mathematics in the past seven decades. The second conference took place three years later, in 1999, when our primary aim was to cover as much as possible the full scope and richness of his mathematics and its impact. The third conference was in 2013 to celebrate the hundredth anniversary of his birth. The intention of this third one was to give a panorama of the monumental development originating in his mathematics, of the wide-ranging influence of his work, and to give some indication of possible trends in the future. The success of the conference surpassed all our expectations: more than twice as many mathematicians participated as we first expected, illustrating the tremendous interest in his work and its proceeds.

László Lovász is professor of computer science at Eötvös Lorand University. His email address is lovasz@cs.elte.hu.

Vera T. Sós is professor of mathematics at the Alfred Rényi Institute of Mathematics, The Hungarian Academy of Sciences. Her email address is sos@renyi.hu.

Courtesy of Vera Sós

Paul Erdős and Paul Turán were great mathematicians, close friends, and partners in several important collaborations.

Trying to collect a few thoughts about the character of Erdős's mathematics, our starting point could be what he wrote in a letter that included a scientific biography written by him in the late 1970s:

> To finish this short outline of my scientific biography, I observe that most of my papers contain some type of combinatorial reasoning and most of them contain unsolved problems.

Indeed, a special trait all across his work was his unparalleled power of formulating and posing problems and conjectures. He had a special sense for asking just the right questions: how else can we explain that many of his innocent-looking problems have opened up new areas, in some cases after several decades? He wrote the first "problem paper" in 1956 [9], which contained six problems. After more than half a century, in spite of the many important results and methods initiated by this paper, none of these six problems is completely solved.

In one of his letters from 1979 he wrote:

> I am writing a paper with the title: 'Combinatorial problems I would most like to see solved'—the subjective title is better because then I do not have to write about my opinion on the importance of the problems.

While Erdős often just asked a problem in a very compact form without mentioning any reasons, these problems were not as spontaneous as it would seem. An example: In a letter to Paul Turán in 1938 he formulated a conjecture about the maximum number of k-element subsets of an n-element set with the property that any two of them intersect in at least r points. He mentioned that with the help of Ko he could prove the case $r = 1$, and he added the remark: "The theorem could have beautiful applications in number theory." The proof was published only in 1961 in a famous paper by Erdős, Ko, and Rado [15], but possible applications in number theory are not mentioned in the paper.

In its simplest form, the Erdős-Ko-Rado Theorem says that to get the largest number of k-subsets of an n-set ($n \geq 2k$) that mutually intersect, one should take all k-subsets containing a given element. When reading such a statement, one realizes that many similar problems can be raised about subsets of a finite set, and then, depending on one's temperament, one might escape, or one might be challenged by, the fact that such basic questions are unsolved. Luckily, Erdős and several others felt the challenge, and over a relatively short period a wealth of basic questions in extremal set theory were answered. The theorems of Sperner (about sets not containing each other), Erdős-de Bruijn (about sets, any two intersecting in exactly one element), Erdős-Rado (about sets among which no three mutually have the same intersection) and Kruskal-Katona (about k-sets covering the least number of r-sets) are not only standard theorems in combinatorics textbooks, but they have very important applications in geometry, number theory, computer science, and elsewhere. These problems, which arise in a very simple and natural way, are often quite difficult to solve, and in some cases a complete solution is still missing after decades of intensive research.

Another characteristic of his mathematics was that very often his questions and proofs reveal deep relationships between different areas in mathematics. Even though he was never directly involved in computer science, he had an essential influence on it, mostly through extremal set theory and the probabilistic method. These connections could not have been foreseen, except perhaps by Erdős himself. (About this aspect of his work see Babai [3].) Using his own words from the late seventies:

> I am basically a pure mathematician and had little contact with applied mathematics, I expect that my paper with Rényi on the evolution of random graphs will be used in several branches in science—Rényi planned to work in this direction but was prevented by his untimely death. Graham, Szemerédi and I [16] have a paper on problems raised by computer scientist but I am not competent enough to judge their importance for applications.

Erdős started out as a number theorist, and number theory remained present in his mathematics all the time. There are several survey articles dealing with his work on the theory of primes, equidistribution, diophantine approximation, additive and multiplicative number theory, and many more. Discovering the combinatorial nature of some of his early number theory problems led him to general questions in combinatorics and in graph theory. He writes in his above-quoted "scientific biography":

> My main subjects are: number theory (a subject which interested me since early childhood when I learned from my father Euclid's proof that the number of primes is infinite), combinatorial analysis, set theory, probability, geometry and various branches of analysis.

His work in set theory often arose from combinatorics as infinite versions of finite problems. His problems and results in geometry and algebra also have a combinatorial flavor. He was the driving force behind the development of large areas of modern combinatorics, including extremal graph theory and extremal set theory. Since combinatorics is the best-known area of his work (which is due, at least in part, to the fact that this was the focus of his work in his later years), we will not go into the details of these results.

Another area that Erdős introduced into several branches of mathematics is probability. The interaction with probability is a very hot topic in number theory, combinatorics, computer science, and other areas, and the pioneering work of Erdős is present all over this work. We could talk about four different ways in which he contributed to this field.

1. He studied problems in pure probability theory (often with a combinatorial flavor but belonging to mainstream probability), like random walks or the Law of Iterated Logarithm. As to this last work, let us quote Bollobás [6]: "There are very few people who have contributed more to the fundamental theorems in probability theory than Paul Erdős."

2. Starting with problems in number theory, he showed how to exploit the random-like behavior of different structures. Let's quote his own words [13]:

> Heuristic probability arguments can often be used to make plausible but often hopeless conjectures on primes and on other branches of number theory.

The deliberate and systematic application of probability theory to number theory started with the celebrated Erdős-Kac theorem [14]. For a detailed review of the story of this theorem, see the article of Alladi-Krantz in this issue of the *Notices*. Erdős himself wrote about the formation of the Erdős-Kac theorem several times; let's quote from [10]:

> I conjectured that the convergence of the three series is both necessary and sufficient for the existence of the distribution function (of an additive function) but this I could not prove due to my gaps of knowledge in Probability Theory.…After the lecture (of Kac) we got together…and thus with a little impudence we would say, that probabilistic number theory was born.

Elliott writes in his book [8] about this theorem:

> This result, of immediate appeal, was the archetype of many results to follow. It firmly established the application of the theory of probability to the study of fairly wide class of additive and multiplicative functions.

3. Perhaps most important of Erdős's achievements is the "probabilistic method," the use of probability to prove the existence of certain objects without explicitly constructing them (and whose explicit construction is sometimes still open sixty years later). This issue of the *Notices* contains other papers that describe this fundamental method and its applications, and we can also refer to the books of Alon and Spencer [1] and Erdős and Spencer [19].

4. The Erdős-Rényi theory of random graphs is the first major example of the investigation of random structures. To be precise, random sets of integers, random polynomials, random matrices, and other random structures were considered before by several mathematicians (including Erdős and Rényi themselves), but random graphs were the first where a comprehensive theory arose that showed how basic properties of these graphs are different from their deterministic counterparts. Several books have been written about random graphs [5], [23]. The Erdős-Rényi random graphs serve as basic examples in the recent explosion of random graph models for many real-life networks (like the Internet and social networks), where the understanding and explanation of the differences from this basic model is the main goal.

Courtesy of Vera Sós

Paul Erdős with his mother, who travelled with him around the world until her death in 1971.

Analysis, in particular approximation, interpolation, polynomials, complex functions, and infinite series, were also in the foreground of his research from the thirties through the sixties. His analytic power can be felt in his papers all along. It is best to quote Paul Turán, who was an early collaborator of Erdős and wrote a detailed survey on Erdős's work on the occasion of his fiftieth birthday [27]. (This became an important source for many later articles on Erdős.) Out of the several topics in analysis which Turán discussed in this paper, let's quote what he wrote about the application of probability in analysis:

"The application of probabilistic methods runs right through the whole oeuvre of Erdős and this holds for his works in analysis as well. In this connection I have in my mind especially three of his papers, the first of which was published in 1956 in the *Proc. London Math. Soc.* with Offord [17], the second in 1959 in the *Michigan Math. J.* with Dvoretzky [7], the third will be published with Rényi in the volume to be issued to celebrate the 75th birthday of György Pólya [18]. In the first they showed that if $\varepsilon_\nu = \pm 1$, then the 2^n equations

$$1 + \varepsilon_1 x + \cdots + \varepsilon_n x^n = 0$$

have, with at most $o(2^n/\sqrt{\log\log n})$ exceptions,

$$\frac{2}{\pi}\log n + o\left(\log^{\frac{2}{3}} n \log\log n\right)$$

real roots each.

"The second gives an *existence proof* of the nice theorem that there exists a power series $\sum_0^\infty \frac{e^{i\alpha_n}}{\sqrt{n}} z^n$ with real α_n that diverges on the *whole* unit circle (that this can be achieved excluding a set

Courtesy of Vera Sós

(From l to r) George Graetzer, Paul Erdős, Paul Turán and Alfred Renyi.

of measure zero was known). In the third they solve an old problem of Zygmund in connection with a theorem of N. Wiener. This theorem of Wiener states (in a weakened form) that if the series

$$\sum_{\nu}(a_\nu \cos l_\nu x + b_\nu \sin l_\nu x),$$

where the l_ν's are positive integers satisfying

$$\lim_{\nu\to\infty}(l_{\nu+1} - l_\nu) = \infty$$

is Abel-summable in an arbitrarily small interval (a, b) to a function $f(x)$ belonging to L_2, when we have $\sum(a_\nu^2 + b_\nu^2) < \infty$, hence the series is the Fourier series of a function belonging to L_2 on, the whole $[0, 2\pi]$, thus $f(x)$ has an extension to $[0, 2\pi]$ that is in L_2 and whose Fourier series is the given series. Ingham, Zygmund and Marcinkiewicz and the author of these lines gave much simpler proofs of this theorem than the original; some twenty years ago Zygmund raised the question whether the theorem can be extended to a class L_q with $q > 2$ in the place of L_2. Now Erdős and Rényi with probabilistic methods showed for every $q > 2$ the *existence* of a trigonometric series satisfying the above lacunarity condition that is summable to a function continuous in (a, b) for every $0 < a < b < 2\pi$ and still the series is not the Fourier series of any function belonging to L_q on $[0, 2\pi]$."

Erdős was always very supportive of young people. In the 1960s, when the Cold War began to melt and he started to spend more time in Budapest, he would often sit in the lobby of his hotel all day, with students and young researchers coming and going, discussing their new results, and learning about new developments and new problems from all over the world. One of us (the first author) was lucky enough, as a high school student, to have the opportunity to stay there and take part in these discussions. The effect of these discussions on how to look at mathematics, research, colleagues, science, and the world has lasted a lifetime.

From this experience, and in general from the attitude of Erdős towards open problems, conjectures, dissemination of ideas and collaboration, his basic (probably unstated) philosophy can be distilled: he believed in total openness in research, where the goal is to advance knowledge, and we all work together to achieve it.

Let me (the second author) also mention my first and last meeting with Paul Erdős—the beginning and the end of almost fifty years of acquaintance and more than three decades of collaboration, partly in several hundreds of letters. I met Erdős the first time in 1948, when he returned to Hungary after a break of ten years. My high school teacher, Tibor Gallai, one of Erdős's best friends, introduced me to him. I cannot recall the particulars of our conversation, but I am sure he asked mathematical questions, as he usually did when meeting young people interested in mathematics. However, I remember that because of a long break his visit had a special significance. Let me say a few words about this.

Erdős and Gallai were members of the now legendary "Anonymus group."[1] The members of this group met regularly during their university years at the *Statue of Anonymus* in City Park in Budapest. Lifelong friendships were formed between them, and their meetings had a deep impact on their professional lives as well.

Arranged by Mordell, Erdős spent the years 1934–38 in Manchester. During this period he returned to Hungary quite regularly three times a year for shorter visits. In 1938 he decided to leave Hungary, with its adverse and deteriorating political situation. He had to leave his family, he had to leave his friends. Then came the war years; Erdős returned to Budapest only ten years later to see his mother and his friends. This was the occasion when, in December 1948, I met Erdős for the first time.

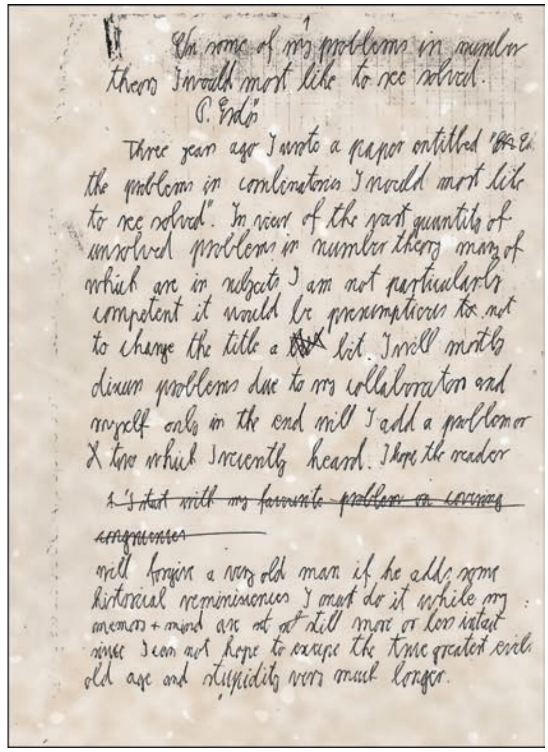
In September 1996 we both attended a graph theory conference in Warsaw. Our plan was to go from Warsaw, together with András Sárközy, to Vilnius to participate in a number theory conference the following week. On the morning of Wednesday, September 18, he gave his very last problem lecture. The last problem he mentioned was a problem of Hajnal (and perhaps himself). He got stuck, started again, and this was repeated two

[1] *László Alpár (1914–1991), Pál Erdős (1913–1996), János Erőds, (1912–1944), Ervin Feldheim (1912–1944), Géza Grünwald (1913–1944), Tibor Grünwald (Gallai) (1912–1992), Eszter Klein (1910–1975), Dezső Lázár (1913–1943), György Szekeres (1911–1975), Pál Turán (1910–1976), Márta Wachsberger (Sved) (1911–2005), Endre Weiszfeld (1913–1976).*

more times. After the third attempt, he put down the chalk and finished the talk. The audience broke out in applause, and he responded, "Thank you. I know this is meant as a consolation!" There was an excursion the same afternoon, which he skipped, partly because of the cold weather. Instead of that, the rest of the day became the last hours we spent together, switching between topics and problems perhaps more often than at other times. Paul Erdős passed away on Friday, September 20 [26].

Erdős's brilliant mathematical thinking, pure character, helpful and sympathizing nature; his quest for truth in science, politics, everyday life—these are what motivated his untiring, relentless activity and creativity until his last days. His personality is perhaps evoked by the simple lines he wrote one morning in 1976:

> It is six in the morning, the house is still asleep, I am listening to lovely music, while writing and conjecturing.[2]


On some of my problems in number theory I would most like to see solved.
P. Erdős
Three years ago I wrote a paper entitled "On the problems in combinatorics I would most like to see solved". In view of the vast quantity of unsolved problems in number theory many of which are in subjects I am not particularly competent it would be presumptious not to change the title a bit. I will mostly discuss problems due to my collaborators and myself only in the end will I add a problem or two which I recently heard. I hope the reader ~~I start with my favourite problem on covering congruences~~ will forgive a very old man if he adds some historical reminiscences I must do it while my memory + mind are still more or less intact since I can not hope to escape the two greatest evils old age and stupidity very much longer.

Manuscript courtesy of Krishnaswami Alladi

Erdős contribution to the Proceedings of the Conference on Number Theory in his honor for his 70th birthday held in Ootacamund, India in January 1984, and referred to in the Lovász-Sós article.

References

[1] N. Alon and J. Spencer, *The Probabilistic Method*, Wiley–Interscience, 2000.

[2] L. Babai, In and Out of Hungary, Paul Erdős, his friends, and times, in *Combinatorics: Paul Erdős is Eighty*, János Bolyai Mathematical Society, 1993.

[3] ______, Paul Erdős (1913–1996). His influence on the theory of computing, *Proc. 29th STOC*, ACM Press, 1997, pp. 383–401.

[4] L. Babai, C. Pomerance, and P. Vértesi, The mathematics of Paul Erdős, *Notices of the AMS* **45** (1998), 19–31.

[5] B. Bollobás, *Random Graphs*, Academic Press, 1985.

[6] B. Bollobás, To prove and conjecture: Paul Erdős and his mathematics, *Amer. Math. Monthly* **105** (1998), 209–237.

[7] A. Dvoretzky and P. Erdős, Divergence of random power series, *Michigan Math. J.* **6** (1959), 343–347.

[8] Peter D. T. A. Elliott, *Probabilistic Number Theory I-II*, Grundlehren der Math. Wissenschaften, 239–240, Springer-Verlag, Berlin-New York, 1980.

[9] P. Erdős, Problems and results on additive number theory, in *Colloque sur la Thèorie des Nombres, Bruxelles, 1955*, George Thone, Liège; Masson and Cie, Paris, 1956, pp. 127–137.

[10] ______, On some of my favourite theorems, in D. Miklós, V. T. Sós, T. Szőnyi (eds.), *Combinatorics, Paul Erdős Is Eighty*, Bolyai Society Mathematical Studies, 2, János Bolyai Mathematical Society, Budapest, 1996, pp. 97–132.

[11] ______, On the combinatorial problems which I would most like to see solved, *Combinatorica* **1** (1981), 25–42.

[12] ______, On some of my problems in number theory I would most like to see solved, *Number theory (Ootacamund, 1984)* K. Alladi (ed.), Lecture Notes in Math., 1122, Springer, Berlin, 1985, pp. 74–84.

[13] ______, Some applications of probability methods to number theory. Successes and limitations, in Capocelli, R. (ed.), *Sequences (Naples-Positano, 1988)*, Springer, New York, 1990, pp. 182–194.

[14] P. Erdős and M. Kac, The Gaussian law of errors in the theory of additive number theoretic functions, *Amer. J. Math.* **62** (1940), 738–742

[15] P. Erdős, Chao Ko, and R. Rado, Intersection theorems for systems of finite sets, *Quart. J. Math.*, Oxford Ser. **12** (1961), 313–320.

[16] P. Erdős, R. L. Graham, and E. Szemerédi, On sparse graphs with dense long paths, *Computers and Mathematics with Applications*, Pergamon, Oxford, 1976, pp. 365–369.

[17] P. Erdős and A. C. Offord, On the number of real roots of a random algebraic equation, *Proc. London Math. Soc. (3)* **6** (1956), 139–160.

[18] P. Erdős and A. Rényi, On a problem of A. Zygmund, *Studies in Mathematical Analysis and Related Topics*, Stanford Univ. Press, Stanford, California, 1962, pp. 110–116.

[19] P. Erdős and J. Spencer, Probabilistic methods in combinatorics, *Probability and Mathematical Statistics*, Academic Press, New York-London, 1974.

[20] R. L. Graham and J. Nešetřil (eds.), *The Mathematics of Paul Erdős. I–II*, Springer-Verlag, New York, 1996.

[21] R. L. Graham, J. Nešetřil, and S. Butler (eds.), *The Mathematics of Paul Erdős.* I–II (second edition), Springer-Verlag, New York, 2013.

[22] G. Halász, L. Lovász, M. Simonovits, and V. T. Sós (eds.), *Paul Erdős and His Mathematics. I–II*, Bolyai Society Mathematical Studies, 11, János Bolyai

[2] *The original one, in Hungarian: "Reggel hat van s a ház még alszik, szép zenét hallgatok, s közben írok és sejtek."*

Mathematical Society, Budapest; Springer-Verlag, Berlin, (2002).
[23] S. Janson, T. Łuczak, and A. Ruczynski, *Random Graphs*, Wiley, 2000.
[24] L. Lovász, I. Z. Ruzsa, and V. T. Sós (eds.), *Erdős Centennial*, Bolyai Society Mathematical Studies, 25, János Bolyai Mathematical Society, Budapest; Springer-Verlag, Berlin, 2013.
[25] V. T. Sós, Turbulent years: Erdős in his correspondence with Turán from 1934 to 1940, in *Paul Erdős and His Mathematics*, Bolyai Society Mathematical Studies, 11, János Bolyai Mathematical Society, Budapest; Springer-Verlag, Berlin, 2002, pp. 85–146.
[26] ______, Paul Erdős, 1913–1996, *Aequationes Math.* **54** (1997), no. 3, 205–220.
[27] P. Turán, Pál Erdős is fifty, in *Paul Erdős and His Mathematics*, Bolyai Society Mathematical Studies, 11, János Bolyai Mathematical Society, Budapest; Springer-Verlag, Berlin, 2002, pp. 55–83. (Translation from Hungarian: Turán Pál, Erdős Pál 50 éves, *Mat. Lapok* **14** (1963), 1–28.)

Ronald L. Graham and Joel Spencer

Ramsey Theory and the Probabilistic Method

Ramsey Theory was a lifelong interest of Paul Erdős. It began [11] in the winter of 1931–32. George Szekeres recalled:

> We had a very close circle of young mathematicians, foremost among them Erdős, Turán and Gallai; friendships were forged which became the most lasting that I have ever known and which outlived the upheavals of the thirties, a vicious world war and our scattering to the four corners of the world. I [...] often joined the mathematicians at weekend excursions in the charming hill country around Budapest and (in the summer) at open air meetings on the benches of the city park.

Szekeres, Esther Klein, and Erdős attacked an unusual geometric problem: Is it true that for every k there exists an n so that given any n points in the plane, no three collinear, some k of them form a convex k-gon? Szekeres, in finding a proof of this conjecture, actually proved Ramsey's Theorem, which none of the three knew about at the time.

The mantra for Ramsey Theory is "Complete disorder is impossible." Let s, r, k be positive integers. Then, Ramsey showed, for n sufficiently large (dependent on s, r, k), the following holds: Let Ω have size n. Take *any* partition of the s-element subsets of Ω into r colors. Then there will be a k-element set $S \subset \Omega$ which is monochromatic, in the sense that *all* of its s-element subsets are the same color. In the important special case $s = 2$ one may think of an r-coloring of the edges of the complete graph K_n. While Erdős was not the originator of Ramsey Theory, he was its chief proponent, with conjectures and theorems in myriad directions that truly turned Ramsey's Theorem into Ramsey Theory.

A natural question arose: Just how big does n need to be? We'll restrict here to $s = 2$, though the other cases are also important. The Ramsey function $r(k)$ is the least n such that if the edges of the complete graph K_n are red/blue colored, then there will necessarily be a monochromatic K_k. The proof of Szekeres worked for $n = \binom{2k-2}{k-1}$ so that, thinking asymptotically, $r(k) < (4 + o(1))^k$. In 1947 Erdős published a three-page paper [3] in the *Bulletin of the AMS* that had a profound effect on both the Probabilistic Method and on Ramsey Theory.

Theorem. *Let n, k satisfy*

$$\binom{n}{k} 2^{1-\binom{k}{2}} < 1.$$

Then $r(k) > n$. That is, there exists *a two-coloring of the edges of K_n such that there is no monochromatic K_k.*

Today, for those in the area, the proof is two words: Color Randomly! Consider a random coloring of the edges. For each of the $\binom{n}{k}$ sets S of k vertices there is a probability 2^{1-m}, $m = \binom{k}{2}$, that the m edges are all colored the same. The probability of a disjunction is at most the sum of the probabilities, and so the disjunction has probability strictly less than one. Thus with positive probability the coloring is as desired. But (this part is sometimes called Erdős Magic) if there were no such coloring, then the probability would be zero, so, reversing, the coloring absolutely positively must exist.

Asymptotic analysis (from Erdős's paper) gives $r(k) > (\sqrt{2} + o(1))^k$. There have been some improvements in both the upper and lower bounds, most notably by David Conlon, but only for lower-order terms. The gap between $\sqrt{2}$ and 4 has not moved since 1947 and is a central question in the field.

In 1950 [7], with Richard Rado, Erdős began the area of canonical Ramsey Theory. Let S be an ordered set. They gave four special colorings of the pairs of S: They could all have the same color; they could all have different colors; the color of $\{x, y\}$ with $x < y$ could be different for different x and the same for the same x; the color of $\{x, y\}$ with $x < y$

Ronald L. Graham is professor of computer science and engineering at the University of California at San Diego. His email address is graham@ucsd.edu.

Joel Spencer is professor of mathematics and computer science at New York University. His email address is spencer@cims.nyu.edu.

could be different for different y and the same for the same y. These they called the canonical colorings of the pairs of S. Fix k and consider any coloring (no restriction on the number of colors) of the pairs of Ω with $\Omega = \{1, \ldots, n\}$. Then, for n sufficiently large they showed that there must be a k-set $S \subset \Omega$ on which the coloring is canonical (there were similar results for triples, etc.).

Generalizing $r(k)$ the Ramsey function $r(l,k)$ is the least n such that if the edges of the complete graph K_n are red/blue colored, then there will necessarily be either a red K_l or a blue K_k. The special case $l = 3$ was a lifelong fascination of Erdős. Associating red/blue with whether or not an edge is in the graph, $r(3,k) \leq n$ means that every triangle-free graph on n vertices must contain an independent set of size k. The argument of Szekeres from 1931–3 showed that $n = \binom{k+1}{2}$ has this property. In 1957 Erdős gave an intriguing geometric construction showing $R(3,k) > k^{1+c}$ for a small constant c. He returned [4] to the problem in 1961 with a probabilistic tour de force. He considered a random graph on n vertices with adjacency probability $p = \epsilon n^{-1/2}$, ϵ a small constant. Such a graph G will have many triangles. But then Erdős ordered the edges of G and considered them sequentially. He rejected an edge if it would form a triangle, thus tautologically forming a triangle-free subgraph H. Erdős then employed an array of probabilistic techniques that would be impressive even today, but that they were done so early is simply amazing. With them he showed that with high probability the graph H would not have an independent set of size $k = c\sqrt{n}\ln n$. Reversing the variables, this gave $r(3,k) > c_1 k^2 \ln^{-2} k$. For forty years, while alternate proofs of this result were given, the asymptotics of this lower bound were unchanged. In the meantime, Ajtai, Komlós, and Szemerédi improved the upper bound to $R(3,k) = O(k^2/\ln k)$. In 1995 [10] (with Peter Winkler and Steve Suen) Erdős returned once again to the lower bound. This time, rather than use an arbitrary ordering, they looked at a random ordering. They applied what we would call today a random greedy algorithm:[3] the $\binom{n}{2}$ potential edges were ordered randomly; an edge was rejected when it would form a triangle with previously accepted edges. Their analysis only improved the constant c_1 of Erdős's 1961 paper. But it opened the door, and soon after Jeong Han Kim [12], using a minor modification of the random greedy algorithm, improved the lower bound to $r(3,k) = \Omega(k^2/\ln k)$, thus resolving the asymptotics of $r(3,k)$ up to a constant factor. Indeed, Tom Bohman in 2009 showed that Erdős's random greedy algorithm

[3]*While Erdős's work has been tremendously influential in the analysis of algorithms, he himself never used that language.*

(From l to r) Fan Chung, Vera Sós, and Paul Erdős in Budapest.

did give Kim's result, although his proof involved elaborate modern methods. However, to illustrate our lack of knowledge here, the best current bounds on $r(4,n)$ are only

$$\frac{cn^{\frac{5}{2}}}{\log^2 n} < r(4,n) < \frac{c'n^3}{\log^2 n}$$

for suitable positive constants c, c'. In particular, we don't even know the correct exponent of n!

In 1956 Erdős [8], with Richard Rado, expanded the study of Ramsey's Theorem to infinite cardinals. Let a set Ω have cardinality α. Let the pairs $\{x,y\} \subset \Omega$ be split into a finite number of classes. What is the largest β such that there exists a set $S \subset \Omega$ of cardinality β which is monochromatic? There are many surprises. For example, assuming the Axiom of Choice, when α is the continuum there is a two-coloring of the pairs so that no monochromatic set S has more than countable size. Erdős also explored Ramsey's Theorem on countable ordinals. Here is a representative beautiful result of Jean Larson: Let the pairs on ω^ω be colored red and blue. Then there exists either a red triangle or a blue set S of ordinal type ω^ω.

Coloring to avoid a monochromatic set (or proving that this cannot be done) was another problem that fascinated Paul Erdős over many decades. In 1963 he began [5] the study of perhaps the purest form of the problem. Let A_i, $1 \leq i \leq m$, be m sets in some universe Ω, each with n elements. No assumption about the size of Ω nor the intersection patterns is made. It is convenient to parametrize $m = k2^{n-1}$. Erdős showed that if $k < 1$, then the family is two-colorable; that is, there exists a two-coloring of the vertices of Ω such that no set is monochromatic. The proof today: Color Randomly! Let $m(n)$ be the largest value of m such that every family could be two-colored. With this notation $m(n) \geq 2^n - 1$, Erdős immediately asked for the asymptotics of $m(n)$. In 1964 he showed $m(n) \leq cn^2 2^n$ by taking random sets in an appropriately chosen Ω. This upper bound has not been improved. In 2000 Radhakrishnan and Srinivasan colored an arbitrary family with $k = c\sqrt{n/\ln n}$ by first randomly coloring and then applying an ingenious recoloring algorithm. This result was recently duplicated independently by Cherkashin and by Kozik. Kozik's algorithm is the simplest: Order the vertices randomly. Color a vertex blue unless it would create a blue set; then color it red. Random greedy! Uncle Paul would have been pleased.

Motivated by the difficulty of finding $r(l,k)$, it was natural to extend the class of desired monochromatic objects to include *all* graphs and not just complete graphs. Thus, in the simplest case, for two given graphs G and H, we define the Ramsey number $r(G,H)$ to be the least integer r (guaranteed to exist by Ramsey's Theorem) so that in any red/blue coloring of the edges of the complete graph K_r, there must always be formed either a red copy of G or a blue copy of H. This generalization has proved to be very fruitful, with literally many hundreds of papers dealing with these questions since they were first raised in the early 1970s. (Erdős himself was an author of more than fifty of them!)

One of the earliest (simple) results in this area was the following:

Theorem. $r(G,H) \geq (\chi(G)-1)(c(H)-1)$*, where* $\chi(G)$ *denotes the chromatic number of* G *and* $c(H)$ *denotes the cardinality of the largest connected component of* H*.*

An immediate consequence of this is the following elegant result:

Theorem. $r(T_m, K_n) = (m-1)(n-1)+1$*, where* T_m *denotes a tree on* m *vertices.*

Let us use the usual "arrow" notation $H \to (G,G)$ to denote the fact that any two-coloring of the vertices of the graph H always produces a monochromatic copy of the graph G. Further, denote by $\mathbf{C}(\mathbf{G})$ the class of graphs H for which $H \to (G,G)$ but such that for any proper subgraph $H' \subseteq H$, it is **not** true that $H' \to (G,G)$. Such graphs H are called *Ramsey-minimal* for G. An interesting unsolved problem is to determine those graphs G for which $\mathbf{C}(\mathbf{G})$ is infinite. It is known that this is the case, for example, when G is 3-connected or G has chromatic number at least 3 or when G is a forest which is not the union of stars. A nice conjecture involving $\mathbf{C}(\mathbf{G})$ is the following: If $\mathbf{C}(\mathbf{G})$ is finite and G' is formed from G by adding disjoint edges, then $\mathbf{C}(\mathbf{G}')$ is also finite.

From the simplest arrow relation $K_6 \to (K_3,K_3)$ (the celebrated "party" problem), it follows that any graph H containing K_6 as a subgraph also satisfies $H \to (K_3,K_3)$. Erdős and Hajnal already asked in the 1960s for an example of the graph H *not* containing K_6 as a subgraph for which $H \to (K_3,K_3)$. It was shown that the smallest such graph has eight vertices and is formed by removing a 5-cycle from K_8. A much more challenging question was to find such a graph which had no K_4 as a subgraph. This was finally settled by a brilliant construction of Jon Folkman. Unfortunately, his example had more than $10^{10^{10}}$ vertices. This prompted Erdős to offer a prize for the first example of a "Folkman" graph with fewer than 10^{10} vertices, a prize that one of the authors (JS) was proud to claim. Unfazed, the other author (RG) then offered a reward of \$100 to show that there was a Folkman graph with fewer than 10^6 vertices. This remained unresolved until 2007 when L. Lu constructed a Folkman graph with only 9,697 vertices. The next year this bound was lowered to 941 by Dudek and Rödl. Both of these results used techniques from spectral graph theory. The current record (in 2012) is 786 and is held by Lange, Radziszowski, and Xu. There is some evidence that the best possible bound is below 100 (and RG offers \$100 for a proof or disproof of this).

Some forty years ago, Erdős and Graham conjectured that in some sense, the complete graph had the largest Ramsey number among all graphs with the same number of edges. Since it is known that $r(K_n) > 2^{\frac{n}{2}}$ and K_n has $\binom{n}{2}$ edges, it was then conjectured that there is an absolute constant c such that for any graph G with m edges, $r(G) < 2^{c\sqrt{m}}$. This was finally proven [13] in 2011 by Sudakov. However, the (somewhat) related conjecture that for any graph H with chromatic number n, $r(H) \geq r(K_n)$ is still unresolved.

We say that a graph H is *d-degenerate* if every subgraph has a vertex of degree at most d. Equivalently, there is an ordering of the vertices of H such that each vertex has at most d edges "to the left." A conjecture of Erdős and Stephan Burr from the 1970s has been quite influential:

Burr-Erdős Conjecture. For each positive integer d, there is a constant $c(d)$ such that every d-degenerate graph H satisfies $r(H) < c(d)n$.

In other words, d-degenerate graphs (fixing d) have *linearly* growing Ramsey numbers. While this conjecture is still unsettled, it has served as a focal point for a variety of results related to it. The best result in this direction up to now is the recent striking result of Fox and Sudakov which shows that there is a constant $c(d)$ so that any d-degenerate graph H satisfies $r(H) < 2^{c(d)\sqrt{\log n}}n$. (Close, but no cigar!)

In fact, there are quite a few of the very first questions raised by Erdős which are still unanswered. One of the nicest is the following. Denote by $r(G;k)$ the least integer r such that if the edges of K_r are k-colored, then a monochromatic copy of G will always be formed. (Again, the existence of $r(G;k)$ follows from Ramsey's Theorem.) What is the true order of growth for $r(C_3;k)$ as k tends to infinity (where C_t denotes a cycle with t edges)? The best current bounds available are essentially $2^k < r(C_3;k) < 2(k+2)!$. Is it true that for some constant A, we have $r(C_3;k) < A^k$? Is it true that $r(C_5;k) > r(C_3;k)$? Can one show that $r(C_{2m+1};k) > r(C_3;k)$ for fixed m and k large? Our lack of knowledge here is painfully obvious.

Another direction in Ramsey Theory pioneered by Erdős and the authors (together with P. Montgomery, B. Rothschild, and E. Straus) in a series of papers [6] published some forty years ago had a geometrical flavor. Let's call a (finite) point configuration $C \subseteq \mathbf{E}^m$ *Ramsey* if for every positive integer r there is a number $N = N(r)$ such that for any r-coloring of the points in $\mathbf{E}^N$, there is always formed a *homothetic* copy of C which is monochromatic (i.e., a set C' obtained by some Euclidean motion of C). It is known that the Cartesian product of Ramsey sets is Ramsey. Since any 2-point set is Ramsey, then any subset of the vertices of a rectangular parallelepiped is Ramsey. On the other hand, it can be shown that any Ramsey configuration must lie on the surface of some Euclidean sphere. Thus, the collinear set $T = \{0,1,2\}$ of the three vertices of the degenerate triangle T is not Ramsey. In fact, it is a nice exercise to show if the points $\overline{x} \in \mathbf{E}^n$ are four-colored by color $(\overline{x}) = \lfloor \|\overline{x}\|^2 \rfloor \pmod 4$, then there is no monochromatic copy of T. It is not known if this is possible using only three colors. A long-standing conjecture is the following.

Conjecture A. A configuration is Ramsey if and only if it is spherical.

Recently, an alternative conjecture has been proposed by Leader, Russell, and Walters. Let us call a configuration C *transitive* if it has a transitive symmetry group. Furthermore, let us say that a configuration C' is *subtransitive* if it is a subset of a transitive configuration. Leader et al. noted that all configurations which had been shown to be Ramsey are in fact subtransitive. Consequently, they conjectured:

Conjecture B. A configuration is Ramsey if and only if it is subtransitive.

Interestingly, neither of these conjectures implies the other. Conjecture A implies that all finite subsets of a circle are Ramsey. On the other hand, it has been shown that almost all 4-point subsets of a circle are not subtransitive. The strongest results on this problem so far are due to Kříž, who showed that C is Ramsey if it has a solvable transitive group of symmetries. Thus, the set of vertices of a regular n-gon is Ramsey.

Courtesy of Ron Graham

(From l to r) Paul Erdős, Ron Graham, and Fan Chung in Hakone, Japan, in 1986.

We close with several tantalizing (simple!) Euclidean Ramsey conjectures.

Conjecture. In any 2-coloring of the points in the plane, every 3-point configuration must occur monochromatically, with the possible exception of the set of three vertices of some fixed equilateral triangle.

It is easy to two-color $\mathbf{E}^2$ by half-open alternating red/blue strips of width $\frac{\sqrt{3}}{2}$ which avoids monochromatic copies of the three vertices of a unit equilateral triangle.

On the other hand, we have:

Conjecture. For any 3-point configuration C_3 of the plane, there is some three-coloring of the points of the plane which contains no monochromatic copy of C_3.

Clearly, there is a lot more to be done before we have a complete understanding of these questions. Now that Uncle Paul can read proofs from THE BOOK, we are sure that he is annoyed for not having seen the answers while he was still with us!

References

As general references we give our own books [1], [2], [9]. From the more than 1,500 papers of Paul Erdős we select a handful that we feel are particularly noteworthy:

References

[1] N. Alon and J. Spencer, *The Probabilistic Method*, Wiley, 1992.

[2] F. Chung and R. L. Graham, *Erdős on Graphs, His Legacy of Unsolved Problems*, A K Peters, 1998.

[3] P. Erdős, Some remarks on the theory of graphs, *Bull. Amer. Math. Soc.* **53** (1947), 292–294.

[4] ———, Graph theory and probability II, *Canad. J. Math.* **13** (1961), 346–352.
[5] ———, On a combinatorial problem I, *Nordisk. Mat. Tidskr.* **11** (1963), 5–10.
[6] P. Erdős, R. L. Graham, P. Montgomery, B. L. Rothschild, J. H. Spencer, and E. G. Straus, Euclidean Ramsey theorems, I., *J. Combinatorial Theory Ser.* A **14** (1973), 341–363.
[7] P. Erdős and R. Rado, A combinatorial theorem, *J. London Math. Soc.* **25** (1950), 249–255.
[8] ———, A partition calculus in set theory, *Bull. Amer. Math. Soc.* **62** (1956), 427–489.
[9] R. L. Graham, B. L. Rothschild, and J. Spencer, *Ramsey Theory*, Wiley, 1980.
[10] P. Erdős, S. Suen, and P. Winkler, On the size of a random maximal graph, *Random Structures & Algorithms* **6** (1995), 309–315.
[11] P. Erdős and G. Szekeres, A combinatorial problem in geometry, *Compositio Math* **2** (1935), 463–470.
[12] J. H. Kim, The Ramsey number $R(3,t)$ has order of magnitude $t^2/\ln t$, *Random Structures & Algorithms* **7** (1995), 173–207.
[13] B. Sudakov, A conjecture of Erdős on graph Ramsey numbers, *Advances in Mathematics* **227** (2011), 601–609.

Jean-Pierre Kahane

Bernoulli Convolutions and Self-similar Measures after Erdős

A Personal hors d'øeuvre

I never collaborated with Paul Erdős and didn't meet him very frequently. However, I regarded him, and believe he regarded me, as a friend. Several hundreds of mathematicians all around the world may feel the same—anyway it is a strong personal feeling of mine. I wasn't his closest friend by far. His closest French friend was Jean-Louis Nicolas, who is much younger and entertained Paul in Limoges and Lyons. Jean-Louis collaborated with Paul and wrote beautiful papers about him [N1], [N2], [N3]. As a detail let me mention their common interest in highly composite numbers, a term coined by Ramanujan for numbers like 60 or 5040 that have more divisors than any smaller number [E1], [EN], [N4]; my own interest was in the implicit occurrence of the notion in Plato's utopia: 5040 is the best number of citizens in a city because it is highly divisible (*Laws*, 771c). Through Nicolas my Erdős number is 2.

I met Paul Erdős for the first time in Amsterdam in 1954 at the International Congress of Mathematicians. He was a famous mathematician, a famous nonwinner of the Fields Medal in 1950. He was not a member of the Hungarian delegation; he came from Jerusalem, where he had already worked with Aryeh Dvoretsky. I had just published my doctoral thesis, written under the supervision of Szolem Mandelbrojt, and it had very little to do with the mathematics of Erdős. Our common point was that my wife, who is of Hungarian origin, was there and spoke Hungarian. Since then he always asked me about her and her health and later about my children and grandchildren—epsilons and epsilon-squares—and he asked me about politics and about mathematical questions he had in mind: regularly the last few times about an exponent which describes how close to a constant the absolute value of a polynomial of degree n can be when the coefficients have absolute value 1. He remembered the exponent I had published and regularly forgot.

Shortly after 1954, I read the book of Paul Lévy on Brownian motion. Paul Lévy was impressed by a discovery of Dvoretsky, Erdős, and Kakutani on multiple points of the Brownian motion in the plane, namely, the existence of multiple points of nondenumerable order [DEK1], [DEK2]. It is a beautiful result indeed. Here is a reinforced version of the theorem, due to Jean-François Le Gall: Consider a compact set on the real line with empty interior, K, and a plane Brownian motion; then the plane Brownian motion has almost surely a K-multiple point z, meaning that the reciprocal image of z is homeomorphic to K through an increasing mapping [LG]. In turn this result impressed Wendelin Werner when he began to work on the plane Brownian motion with Le Gall as advisor. In this way Erdős (1954) is related to the Fields Medal received by Werner in 2006.

The Main Course: Bernoulli Convolutions

In 2000 Yuval Peres, William Schlag, and Boris Solomyak published a beautiful study on "Sixty years of Bernoulli convolutions" [PSS]. The starting point was the seminal papers of Erdős in 1939 and 1940. Since 2000 and quite recently, Bernoulli convolutions and self-similar measures were the subjects of bright contributions, including the brilliant lecture of Elon Lindenstrauss at the Erdős Centennial July 2013 [L]. Old conjectures remain and new ones appear. The quite recent paper of Pablo Shmerkin [Sh] gives new results and an excellent exposition of the whole subject. I shall try to describe the impulse given by Erdős and how things progressed with the notations in use today.

Given λ in the open interval $(0,1)$, we consider the random series

$$\sum_{n=0}^{\infty} \pm\lambda^n, \qquad (1)$$

where the signs + and − are chosen at random, independently each from the others, with probability 1/2. The sum is a random variable whose

Jean-Pierre Kahane is professor emeritus at the Université Paris-Sud à Orsay. His email address is jean-pierre.kahane@u-psud.fr.

distribution is the Bernoulli convolution under consideration,

$$\nu_\lambda = \mathop{*}_{n=0}^{\infty} \frac{1}{2}(\delta_{\lambda^n} + \delta_{-\lambda^n}), \tag{2}$$

and the characteristic function is the infinite product

$$\hat{\nu}_\lambda(\xi) = \prod_{n=0}^{\infty} \cos \lambda^n \xi \,. \tag{3}$$

When $\lambda = \frac{1}{2}$, ν_λ is nothing but $\frac{1}{4}\times$ Lebesgue measure on $(-2, 2)$. When $\lambda < \dfrac{1}{2}$, ν_λ is carried by a Cantor set whose ratio of dissection is λ. This set has Hausdorff dimension $\frac{\log 2}{\log 1/\lambda}$; it is self-similar, meaning that it is the union of two disjoint homothetic copies with ratio λ; and ν_λ is the natural measure thereon, giving equal masses to equal portions. When $\lambda > \frac{1}{2}$, (1) takes every value between $\frac{-1}{1-\lambda}$ and $\frac{1}{1-\lambda}$ and ν_λ is a self-similar measure, meaning the average of two copies obtained by similarities of ratio λ (affine functions with main coefficient λ). It has been known since 1935 [JW], [KW] that ν_λ is either absolutely continuous or purely singular with respect to the Lebesgue measure

$$\nu_\lambda \ll \text{Lebesgue} \qquad \text{or} \qquad \nu_\lambda \perp \text{Lebesgue}\,. \tag{4}$$

To decide between these two possibilities according to the value of λ is the first and the main question when $\lambda > \frac{1}{2}$.

This question was considered by Erdős in two papers published in 1939 and 1940 by the *American Journal of Mathematics* [E2], [E3]. The first deals with the question: when is

$$\limsup_{\xi\to\infty} \hat{\nu}_\lambda(\xi) > 0? \tag{5}$$

According to Riemann-Lebesgue, this can't happen when ν_λ is absolutely continuous; therefore (5) implies that ν_λ is singular.

The question makes sense for $\lambda > \frac{1}{2}$ as well as $\lambda < \frac{1}{2}$, but the implications are quite different: when $\lambda < \frac{1}{2}$, ν_λ is singular and (5) has a meaning in the Riemann-Cantor theory of trigonometric series [Z] ; when $\lambda > \frac{1}{2}$ it is a way—the only way we have up to now—to find λ for which $\nu_\lambda \perp$ Lebesgue. In any case it is convenient to introduce

$$\theta = 1/\lambda \tag{6}$$

and to use the formula

$$\hat{\nu}_\lambda(t\theta^N\pi) = \prod_{n=1}^{N} \cos(t\theta^n\pi) \prod_{n=0}^{\infty} \cos(t\lambda^n\pi) \tag{7}$$

where $1 \le t < \theta$. When θ is an integer > 2, choosing $t = 1$ gives (5). The same works whenever all θ^n are very close to integers. Erdős observed that it is the case indeed when θ is a Pisot-Vijayaraghavan or Pisot number, that is, an algebraic integer whose conjugates (other than θ itself) lie inside the unit circle of the complex plane (Erdős' condition).

It was the beginning of a long story where Raphaël Salem played the major role. First, including the integers larger than 2 among the Pisot numbers, the Erdős condition is necessary and sufficient for (5) to hold. Then, considering the case $\lambda < \frac{1}{2}$, the Erdős condition is necessary and sufficient for the support of ν_λ, the Cantor set described by (1), to be a set of uniqueness, or U-set, for trigonometric series; that is, the only trigonometric series that converges to 0 out of the set is the null series. This is one of the most striking relations between the theory of numbers and trigonometric series [S]. Finally, the set of Pisot numbers is closed [S]. Already $\frac{1+\sqrt{5}}{2}$ and the real root of $x^3 - x - 1 = 0$ had been recognized as Pisot numbers lying in the interval $(1, 2)$: it happens that the first is the smallest accumulation point of Pisot numbers, and the second, the smallest Pisot number [BDGPS].

The set of Pisot numbers, denoted by S, is now well understood [BDGPS], [M]. It is not the case for a companion introduced by Salem, the set T of algebraic integers τ whose conjugates other than τ lie in the closed disc $|z| \le 1$ with one at least (then all but two) on the boundary $|z| = 1$. Those numbers τ are now called Salem numbers. Every $\theta \in S$ is a limit in both directions of a sequence of $\tau \in T$, and no other accumulation point of T is known [S]. Is 1 an accumulation point for T? A section of [PSS], entitled "Bernoulli convolutions and Salem numbers," is devoted to this question, still unsolved.

The second paper of Erdős [E3] goes in the opposite direction: when is $\nu_\lambda \ll$ Lebesgue? His approach is another question: when is

$$\hat{\nu}_\lambda(\xi) = O(\xi^{-\alpha}) \qquad (\xi \to \infty) \tag{8}$$

for some $\alpha > 0$? His answer is that (8) holds for some $\alpha \simeq \alpha(\lambda) > 0$ for almost all λ. More exactly, given $\alpha > 0$, there exists $\delta > 0$ such that (8) holds for almost all $\lambda \in (1-\delta, 1)$. When (8) holds, we can make use of the identity

$$\hat{\nu}_{\sqrt{\lambda}}(\xi) = \hat{\nu}_\lambda(\xi)\hat{\nu}_\lambda(\xi\sqrt{\lambda}) \tag{9}$$

and its analogues to get

$$\hat{\nu}_{\sqrt{\lambda}}(\xi) = O(\xi^{-2\alpha})\,,\ \hat{\nu}_{\lambda^{1/k}}(\xi) = O(\xi^{-k\alpha}) \quad (\xi \to \infty)\,. \tag{10}$$

As soon as $k\alpha > 1/2$, the measure $\nu_{\lambda^{1/k}}$ is absolutely continuous with density in L^2; when $k\alpha > 1$ the density is continuous, and so on. That is a way to obtain information for λ near 1 from information for λ belonging to any interval. Anyway, $\nu_\lambda \ll$ Lebesgue for almost all λ belonging to some interval $(1-\delta, 1)$.

No progress was made for a long time. Adriano Garcia in 1962 gave a new method to find explicit values of λ for which $\nu_\lambda \ll$ Lebesgue, with bounded density. Here is a typical result of his: If θ is an algebraic integer with norm 2 and all conjugates **outside** the disc $|z| \le 1$, then $\nu_\lambda \ll$ Lebesgue with bounded density [G].

The short report that I published on these questions in 1971 contains an exposition of Erdős's proof in twelve lines (tersely written, according to [PSS]) with an improved conclusion: E_0, the λ-set such that (8) fails for every $\alpha > 0$, is 0-dimensional. In the statements of Erdős that I gave above, "almost all" can be replaced by "except on a 0–dimensional set" [K]. The combinatorial argument was later developed by Peres, Schlag, and Solomyak ([PSS], Proposition 6.1) and used by Shmerkin ([Sh], Propositions 2.2 and 2.3). However, the numerical consequence expressed in [PSS] is rather poor: if we want to have (8) with $\alpha = 0.6$ (therefore ν_λ is absolutely continuous with density in L^2) when $\lambda \in (1-\delta, 1)\backslash G$ with $\dim G < 1$, the combinatorial argument provides $1 - \delta = 2^{-2^{-10}}$, that is, $\delta \simeq 0.00066$.

The first breakthrough was realized by Solomyak in 1995: ν_λ is absolutely continuous with density in L^2 for almost all λ in $(\frac{1}{2}, 1)$ [So], [PS]. The second is quite recent: the exceptional set for which ν_λ is singular is not only of Lebesgue measure 0, but it is also 0-dimensional [Sh]. This last result, due to Pablo Shmerkin, echoes and uses another quite recent and deep discovery of Michael Hochman: Except for a λ-set of dimension 0, ν_λ has dimension 1, meaning that $\nu_\lambda(B) = 0$ when $\dim B < 1$ (we assume always $\frac{1}{2} < \lambda < 1$) [H].

Extensions and new developments on non-symmetric Bernoulli convolutions (+ and − in (1) having unequal probabilities) and on self-similar measures (not restricted to the real line) can be found in [PSS], [L], [Sh] and the references therein.

Going back to the questions: 1) when is $\nu_\lambda \ll$ Lebesgue? 2) when is (8) valid for some $\alpha > 0$ (power decay of $\hat{\nu}_\lambda$)? the answer is that they hold whenever $\lambda \in (\frac{1}{2}, 1)\backslash E$, E being a 0-dimensional set. E is not the same in question 1 and question 2; let us write E_1 and E_2 accordingly. We know that E_1 contains the inverses of the Pisot numbers < 2. It is easy to see that E_2 contains the inverses of the Salem as well as the Pisot numbers, and also θ^{-1} as soon as θ^n tends to 0 modulo 1 ($n \to \infty$). The study of those θ by Charles Pisot in 1938 should be mentioned here [P]. First, the result the set of those θ is denumerable. Then, the question does it contain anything other than Pisot numbers? And finally the proof: it is easy, but it is a paradigm for the combinatorial argument now known as Erdős-Kahane [PSS], [Sh].

Dreams and Goals

All investigations on the asymptotic behavior of the infinite product (3) rely on the formula (7):

$$\hat{\nu}_\lambda(t\theta^N\pi) = \prod_{n=1}^{N} \cos(t\theta^n\pi) \prod_{n=0}^{\infty} \cos(t\lambda^n\pi),$$

where $\theta = 1/\lambda$ and $1 \le t < \theta$. The second product is bounded, and the first involves the behavior of $t\theta^n$ modulo 1. Erdős in 1939 used the fact that θ^n modulo 1 tends to 0 rapidly when θ is a Pisot number. Let us look at the most classical result, going back to Hardy and Littlewood [HL] and to Hermann Weyl [W]: given t, the distribution of $t\theta^n$ modulo 1 is uniform on $\mathbb{R}/\mathbb{Z}$ for almost every θ. It implies that

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \log|\cos(t\theta^n\pi)| = \int_0^1 \log|\cos \pi x| dx = -\log 2^1;$$

therefore, given t, almost every λ in $(0,1)$ satisfies

$$(11) \qquad \hat{\nu}_\lambda(t\theta^N\pi) = O(2^{-N+\varepsilon N}) \qquad (N \to \infty)$$

and

$$(12) \qquad \hat{\nu}_\lambda(t\theta^N\pi) = \Omega(2^{-N-\varepsilon N}) \qquad (N \to \infty)$$

for all $\varepsilon > 0$. For almost every λ in $(0,1)$ (11) and (12) hold a.e. in t, but $O(\cdot)$ and $\Omega(\cdot)$ depend on λ and t. Let us write

$$(13) \quad g(\lambda) = \sup\{\alpha : \hat{\nu}_\lambda(\xi) = O(\xi^{-\alpha}) \ (\xi \to \infty)\}.$$

According to (12) we have a.e. in λ,

$$(14) \qquad g(\lambda) \le -\frac{\log 2}{\log \lambda}.$$

If we forget that $O(\cdot)$ in (11) depends on λ and t, we may dream and ask the question: Is it true that

$$(15) \qquad g(\lambda) = -\frac{\log 2}{\log \lambda}$$

for almost every λ in $(0,1)$? That is asked with a wrong factor $\frac{1}{2}$ in [K]; then a negative answer was given by Peres and Solomyak [PSo]. Here is what they proved. Writing

$$(16) \qquad \lambda_n = \binom{2n}{n} 2^{-2n}$$

$(\lambda_1 = \frac{1}{2}, \lambda_2 = \frac{3}{8}, \lambda_3 = \frac{5}{16}, \ldots, \lambda_n \sim \frac{1}{\sqrt{\pi n}} \ (n \to \infty))$, we have

$$(17) \qquad \begin{cases} \hat{\nu}_\lambda \in L^{2n}(\mathbb{R}) & \text{for almost all } \lambda \in (\lambda_n, 1), \\ \hat{\nu}_\lambda \notin L^{2n}(\mathbb{R}) & \text{for all } \lambda < \lambda_n. \end{cases}$$

We recover Solomyak's theorem when $n = 1$. The negative answer was obtained for $n = 2$ and $\lambda = \frac{1}{4}$.

[1] $\mathfrak{M}$ *denoting the mean value* $\mathfrak{M}(\log|\cos \pi x|) = \mathfrak{M}(\log|\sin \pi x|) = \mathfrak{M}(\log|\sin 2\pi x|) = \mathfrak{M}(\log|\cos \pi x|) + \mathfrak{M}(\log|\sin \pi x|) + \log 2$. *I am indebted to Mrs Anne Raoult for this footnote.*

A weak point of the beautiful result (17) is that all λ_n lie in $(0, \frac{1}{2})$. In order to have estimates on $(\frac{1}{2}, 1)$, Peres and Schlag considered weighted L^2 instead of L^{2n}. Here is a typical result: Suppose $J = [\lambda_0, \lambda_0'] \subset [\frac{1}{2}, 0.68]$ and $\lambda_0^{1+2\gamma} > \frac{1}{2}$; then

$$\int\int_{\mathbb{R}\times J} |\hat{\nu}_\lambda(\xi)|^2 \, |\xi|^{2\gamma} d\xi d\lambda < \infty; \tag{18}$$

hence

$$\hat{\nu}_\lambda(\xi) = o(\xi^{-\gamma}) \ (\xi \to \infty) \quad \text{for a.e. } \lambda \in J \text{ [PSS, section 7]}. \tag{19}$$

This is far from (15), but it is an important step in estimating $g(\lambda)$.

A general goal is the study of $g(\lambda)$ on $(0, 1)$. What happens "in general"? What happens in exceptional cases, or particular cases, has interesting relations with the theory of numbers. We already saw that $g(\lambda) = 0$ for Pisot and Salem numbers. When λ is a Garcia number, it was proved recently that $g(\lambda) > 0$ [DFW].

Let me express ancient problems [S] as dreams:

1) "θ^n modulo 1 tends to 0" implies that θ is Pisot.

2) A limit of Salem numbers is Pisot.

And here is the main dream on Bernoulli convolutions:

$\nu_\lambda \perp$ Lebesgue implies that λ is Pisot.

The behavior of powers modulo 1 is the matter of new investigations and problems [B, BM, K1].

References

[B] Yann Bugeaud, Distribution modulo 1 and Diophantine Approximation, *Cambridge Tracts in Mathematics* **193** (2012).

[BDGPS] Marie-José Bertin, Annette Decomps-Guilloux, M. Grandet-Hugot, Martine Pathiaux-Delefosse and Jean-Pierre Schreiber, *Pisot and Salem Numbers*, Birkhäuser, 1992.

[BM] Yann Bugeaud and Nikolay Moshchevitin, On fractional parts of powers of real numbers close to 1, *Math. Z.* , to appear.

[DEK1] Aryeh Dvoretsky, Paul Erdős and Shizuo Kakutani, Multiple points of paths of Brownian motion in the plane, *Bull. Res. Council Israël* **F3** (1954), 364–371.

[DEK2] ———, Points of multiplicity *c* of plane Brownian paths, *Bull. Res. Council Israël* **F7** (1958), 175–180.

[DFW] Xing-Rong Dai, De-Jun Feng, and Yang Wang, Refinable functions with noninteger dilations, *J. Funct. Anal.* **250** (2007), 1–20.

[E1] Paul Erdős, On highly composite numbers, *J. London Math. Soc.* **19** (1944), 130–133.

[E2] ———, On a family of symmetric Bernoulli convolutions, *Amer. J. Math.* **61** (1939), 974–975.

[E3] ———, On smoothness properties of Bernoulli convolutions, *Amer. J. Math.* **62** (1940), 180–186.

[EN] Paul Erdős and Jean-Louis Nicolas, Répartition des nombres super abondants, *Bulletin Soc. Math. France* **103** (1975), 65–90.

[G] Adriano Garsia, Arithmetic properties of Bernoulli convolutions, *Trans. Amer. Math. Soc.* **102** (1962), 409–432.

[H] Michael Hochman, On self-similar sets with overlap and inverse theorems for entropy, *Ann. of Math.* (2) **180** (2014), 773–822.

[HL] G. H. Hardy and J. E. Littlewood, Some problems of diophantine approximation, *Acta Math.* **37** (1914), 155–191; cf. p. 183.

[JW] Børge Jessen and Aurel Wintner, Distribution functions and the Riemann zeta function,*Trans. Amer. Math. Soc.* **38** (1935), 48–88.

[K] Jean-Pierre Kahane, Sur la distribution de certaines séries aléatoires, Colloque Th. Nombres [1969, Bordeaux], Bull. Soc. Math. France, *Mémoires* **25** (1971), 119–122.

[K1] Jean-Pierre Kahane, Sur la répartition des puissances modulo 1, C.R. *Acad. Sci. Paris*, série 1, **352** (2014) 383–385.

[KW] Richard Kershner and Aurel Wintner, On symmetric Bernoulli convolutions, *Amer. J. Math.* **57** (1935), 541–548.

[L] Elon Lindenstrauss, Bernoulli convolutions and self similar measures, Invited Plenary Lecture, Erdős Centennial, Budapest, July 2013.

[LG] Jean-François Le Gall, Le comportement du mouvement brownien entre deux instants où il passe par un point double, *J. Functional Analysis* **71** (1987), 246–262.

[M] Yves Meyer, *Algebraic Numbers and Harmonic Analysis,* North–Holland, 1972.

[N1] Jean-Louis Nicolas, *Paul Erdős 1913–1996,* Universalia 1997, la politique, les connaissances, la culture en 1996, Encyclopedia Universalis Paris, 1997, p. 471.

[N2] ———, Discours prononcé à l'occasion du doctorat honoris causa de l'Université de Limoges à Monsieur le Professeur Paul Erdős, *Gazette des mathématiciens* **33** (1987), 81–85.

[N3] ———, *Statistical Properties of Partitions, Paul Erdős and His Mathematics I*, Bolyai Math. Studies, 11, Budapest, 2002, pp. 537–547.

[N4] ———, *On Landau's function $g(n)$, The mathematics of Paul Erdős. I*, R. L. Graham and J. Nešetřil, eds., Algorithms and Combinatorics, 13, Springer, 1997, pp. 228–240; reedited by Springer, 2013, pp. 207–220.

[P] Charles Pisot, La répartition modulo 1 et les nombres algébriques, *Ann. Schola. Norm. Sup. Pisa* **2** (1938), 205–248.

[PS] Yuval Peres and Wilhelm Schlag, Smoothness of projections, Bernoulli convolutions, and the dimension of exceptions, *Duke Math. J.* **102** (2) (2000),193–251.

[PSo] Yuval Peres and Boris Solomyak, Self-similar measures and intersections of Cantor sets, *Trans. Amer. Math. Soc.* **350** (2007), 1–20.

[PSS] Yuval Peres, Wilhem Schlag, and Boris Solomyak, *Sixty Years of Bernoulli Convolutions*, Fractal geometry and stochastics, II (Greifswald/Koserow, 1998), vol. 46, Progr. Probab., Birkhäuser, Basel, 2000 pp. 39–65.

[S] Raphaël Salem, *Algebraic Numbers and Fourier Analysis,* Heath, 1963.

[Sh] Pablo Shmerkin, On the exceptional set for absolute continuity of Bernoulli convolutions, *Geom. Funct. Anal.* **24** (2014), 946–958.

[So] Boris Solomyak, On the random series $\sum \pm\lambda^n$ (an Erdős problem), *Ann. of Math. (2)*, **142** (3) (1955), 611–625.

[W] Hermann Weyl, Über die Gleichverteilung von Zahlen mod. Eins, *Math. Annalen* **77** (1916), 313–352, Satz 21.

[Z] Antoni Zygmund, *Trigonometric Series. I, II*, Cambridge Univ. Press, 1959.

Melvyn B. Nathanson

Paul Erdős and Additive Number Theory

Additive Bases

Paul Erdős, while he was still in his twenties, wrote a series of extraordinarily beautiful papers in additive and combinatorial number theory. The key concept is *additive basis*.

Let A be a set of nonnegative integers, let h be a positive integer, and let hA denote the set of integers that can be represented as the sum of exactly h elements of A, with repetitions allowed. A central problem in additive number theory is to describe the sumset hA. The set A is called an *additive basis of order h* if every nonnegative integer can be represented as the sum of exactly h elements of A. For example, the set of squares is a basis of order 4 (Lagrange's theorem), and the set of nonnegative cubes is a basis of order 9 (Wieferich's theorem).

The set A of nonnegative integers is an *asymptotic basis of order h* if hA contains every sufficiently large integer. For example, the set of squares is an asymptotic basis of order 4 but not of order 3. The set of nonnegative cubes is an asymptotic basis of order at most 7 (Linnik's theorem) and, by considering congruences modulo 9, an asymptotic basis of order at least 4. The Goldbach conjecture implies that the set of primes is an asymptotic basis of order 3. Helfgott [18] recently completed the proof of the ternary Goldbach conjecture: Every odd integer $n \geq 7$ is the sum of three primes.

The modern theory of additive number theory began with the work of Lev Genrikhovich Shnirel'man (1905–1938). In an extraordinary paper [38] published in Russian in 1930 and republished in an expanded form [39] in German in 1933, he proved that every sufficiently large integer is the sum of a bounded number of primes. Not only did Shnirel'man apply the Brun sieve, which Erdős subsequently developed into one of the most powerful tools in number theory, but he also introduced a new density for a set of integers that is exactly the right density for the investigation of additive bases. (For a survey of the classical bases in additive number theory, see Nathanson [28].)

Melvyn B. Nathanson is professor of mathematics at Lehman College (CUNY). His email address is melvyn.nathanson@lehman.cuny.edu.

Shnirel'man Density and Essential Components

The *counting function* $A(x)$ of a set A of nonnegative integers counts the number of positive integers in A that do not exceed x, that is,

$$A(x) = \sum_{\substack{a\in A \\ 1\leq a\leq x}} 1.$$

The *Shnirel'man density* of A is

$$\sigma(A) = \inf_{n=1,2,\ldots} \frac{A(n)}{n}.$$

The sum of the sets A and B is the set $A + B = \{a + b : a \in A \text{ and } b \in B\}$. Shnirel'man proved the fundamental sumset inequality:

$$\sigma(A+B) \geq \sigma(A) + \sigma(B) - \sigma(A)\sigma(B).$$

This implies that if $\sigma(A) > 0$, then A is a basis of order h for some h. This does not apply directly to the sets of kth powers and the set of primes, which have Shnirel'man density 0. However, it is straightforward that if $\sigma(A) = 0$ but $\sigma(h'A) > 0$ for some h', then A is a basis of order h for some h.

Landau conjectured the following strengthening of Shnirel'man's addition theorem, which was proved by Mann [23] in 1942:

$$\sigma(A+B) \geq \min(1, \sigma(A) + \sigma(B)).$$

Artin and Scherk [1] published a variant of Mann's proof, and Dyson [4], while an undergraduate at Cambridge, generalized Mann's inequality to h-fold sums. Nathanson [27] and Hegedüs, Piroska, and Ruzsa [17] have constructed examples to show that the Shnirel'man density theorems of Mann and Dyson are best possible.

We define the *lower asymptotic density* of a set A of nonnegative integers as follows:

$$d_L(A) = \liminf_{n=1,2,\ldots} \frac{A(n)}{n}.$$

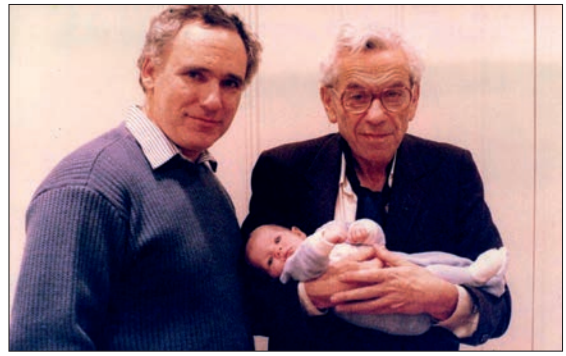
This is a more natural density than Shnirel'man density. A set A with asymptotic density $d_L(A) = 0$ has Shnirel'man density $\sigma(A) = 0$, but not conversely. A set A with asymptotic density $d_L(A) > 0$ is not necessarily an asymptotic basis of finite order, but A is an asymptotic basis if $d_L(A) > 0$ and $\gcd(A) = 1$ (cf. Nash and Nathanson [24]).

The set B of nonnegative integers is called an *essential component* if

$$\sigma(A+B) > \sigma(A)$$

for every set A such that $0 < \sigma(A) < 1$. Shnirel'man's inequality implies that every set of positive Shnirel'man density is an essential component. There exist sparse sets of zero asymptotic density that are not essential components. Khinchin [20] proved that the set of nonnegative squares is an essential component. Note that the set of squares is a basis of order 4. Using an extremely clever elementary argument, Erdős [6], at the age of twenty-two, proved the following considerable improvement: Every additive basis is an essential component. Greatly impressed, Landau celebrated this result in his 1937 Cambridge Tract *Über einige neuere Fortschritte der additiven Zahlentheorie* [22].

Plünnecke [33], [34], [35] and Ruzsa [37] have made important contributions to the study of essential components.



Mel Nathanson with Paul Erdős, who is holding Mel's infant son, Alex, in 1988.

The Erdős-Turán Conjecture

In another classic paper, published in 1941, Erdős and P. Turán [5] investigated Sidon sets. The set A of nonnegative integers is a *Sidon set* if every integer has at most one representation as the sum of two elements of A. They concluded their paper as follows:

> Let $f(n)$ denote the number of representations of n as $a_i + a_j, \ldots$. If $f(n) > 0$ for $n > n_0$, then $\limsup f(n) = \infty$. Here we may mention that the corresponding result for $g(n)$, the number of representations of n as $a_i a_j$, can be proved.

The additive statement is still a mystery. The Erdős-Turán conjecture, that the representation function of an asymptotic basis of order 2 is always unbounded, is a major unsolved problem in additive number theory.

Many years later, in 1964, Erdős [7] published the proof of the multiplicative statement. This proof was later simplified by Nešetřil and Rödl [32] and generalized by Nathanson [26].

Long ago, while a graduate student, I searched for a counterexample to the Erdős-Turán conjecture. Such a counterexample might be extremal in several ways. It might be "thin" in the sense that it contains few elements. Every asymptotic basis of order h has counting function $A(x) \geq cx^{1/h}$ for some $c > 0$ and all sufficiently large x. We call an additive basis of order h *thin* if $A(x) \leq c' x^{1/h}$ for some $c' > 0$ and all sufficiently large x. Thin bases exist. The first examples were constructed in the 1930s by Raikov [36] and by Stöhr [40], and Cassels [2], [29] later produced another important class of examples.

Alternatively, an asymptotic basis A of order h might be extremal in the sense that no proper subset of A is an asymptotic basis of order h. This means that removing *any* element of A destroys every representation of infinitely many integers. It is not obvious that minimal asymptotic bases exist, but I was able to construct asymptotic bases of order 2 that were both thin and minimal [25]. Of course, none was a counterexample to the Erdős-Turán conjecture.

Stöhr [41] gave the first definition of minimal asymptotic basis, and Härtter [16] gave a nonconstructive proof that there exist uncountably many minimal asymptotic bases of order h for every $h \geq 2$.

There is a natural dual to the concept of a minimal asymptotic basis. We call a set A an *asymptotic nonbasis of order* h if is not an asymptotic basis of order h, that is, if there are infinitely many positive integers not contained in the sumset hA. An asymptotic nonbasis of order h is *maximal* if $A \cup \{b\}$ is an asymptotic basis of order h for every nonnegative integer $b \notin A$. The set of even nonnegative integers is a trivial example of a maximal nonbasis of order h for every $h \geq 2$, and one can construct many other examples that are unions of the nonnegative parts of congruence classes. It is difficult to construct nontrivial examples.

I discussed this and other open problems in my first paper [25] in additive number theory. I did not know Erdős at the time, but I mailed him a preprint of the article. It still amazes me that he actually read this paper sent to him out of the blue by a completely obscure student, and he answered with a long letter in which he discussed his ideas about one of the problems. This led to correspondence, meetings, and joint work over several decades.

Extremal Properties of Bases

Here is a small sample of results on minimal bases and maximal nonbases.

Nathanson and Sarközy [31] proved that if A is a minimal asymptotic basis of order $h \geq 2$, then $d_L(A) \leq 1/h$. The proof uses Kneser's theorem [21] on the asymptotic density of sumsets, one of the most beautiful and most forgotten theorems in additive number theory. A well-known special case is Kneser's theorem for the sum of finite subsets of a finite abelian group.

Erdős and Nathanson [14] proved that for every $h \geq 2$, there exist minimal asymptotic bases of order h with asymptotic density $1/h$. Moreover, for every $\alpha \in (0, 1/(2h-2))$, there exist minimal asymptotic bases of order h with asymptotic density α. In particular, for every $\alpha \in (0, 1/2]$ there exist minimal asymptotic bases of order 2 with asymptotic density α.

Does every asymptotic basis A of order 2 contain a minimal asymptotic basis of order 2? Sometimes. Let $f(n)$ count the number of representations of n as the sum of two elements of A. If $f(n) > c \log n$ for some $c > (\log(4/3))^{-1}$ and all sufficiently large n, then A contains a minimal asymptotic basis of order 2 (Erdős-Nathanson [13]). This result is almost certainly not best possible.

Does every asymptotic basis of order 2 contain a minimal asymptotic basis of order 2? No. There exists an asymptotic basis A of order 2 with the following property: If $S \subseteq A$, then $A \setminus S$ is an asymptotic basis of order 2 if and only if S is finite (Erdős-Nathanson [12]).

There exist "trivial" maximal asymptotic nonbases of order h consisting of unions of arithmetic progressions [25]. However, for every $h \geq 2$ there also exist nontrivial maximal asymptotic nonbases of order h (Erdős-Nathanson [8], [11] and Deshouillers and Grekos [3]).

Is every asymptotic nonbasis of order h a subset of a maximal asymptotic nonbasis of order h? Sometimes. If $A \cup S$ is an asymptotic nonbasis of order 2 for every finite set $S \subseteq \mathbf{N} \setminus A$, then A contains a maximal asymptotic nonbasis of order 2.

Is every asymptotic basis of order h a subset of a maximal asymptotic nonbasis of order h? No. Hennefeld [19] proved that for every $h \geq 2$ there exists an asymptotic nonbasis A of order h such that if $S \subseteq \mathbf{N} \setminus A$, then $A \cup S$ is an asymptotic nonbasis A of order h if and only if the set $\mathbf{N} \setminus (A \cup S)$ is infinite.

Investigating extremal properties of additive bases is like exploring for new plant species in the Amazon rainforest. Much has been collected, but much more is unimagined. The following results about oscillations of bases and nonbases appear in [9], [10].

There exists a minimal asymptotic basis of order 2 such that $A \setminus \{x\}$ is a maximal asymptotic nonbasis of order 2 for every $x \in A$.

There exists a maximal asymptotic nonbasis of order 2 such that $A \cup \{y\}$ is a minimal asymptotic basis of order 2 for every $y \in \mathbf{N} \setminus A$.

There exists a partition of the nonnegative integers into disjoint sets A and B such that A is a minimal asymptotic basis of order 2 and B is a maximal asymptotic nonbasis of order 2.

There exists a partition of the nonnegative integers into disjoint sets A and B that oscillate in phase from minimal asymptotic basis of order 2 to maximal asymptotic nonbasis of order 2 as random elements are moved from the basis to the nonbasis infinitely often.

It is an open problem to extend these results to asymptotic bases of order $h \geq 3$. For a survey of extremal problems in additive number theory, see Nathanson [30].

References

[1] E. Artin and P. Scherk, On the sum of two sets of integers, *Annals Math.* **44** (1943), 138–142.

[2] J. W. S. Cassels, Über Basen der natürlichen Zahlenreihe, *Abhandlungen Math. Seminar Univ. Hamburg* **21** (1957), 247–257.

[3] J.-M. Deshouillers and G. Grekos, Propriétés extréales de bases additives, *Bull. Soc. Math. France* **107** (1979), 319–335.

[4] F. J. Dyson, A theorem on the densities of sets of integers, *J. London Math. Soc.* **20** (1945), 8–14.

[5] P. Erdős and P. Turán, On a problem of Sidon in additive number theory, and on some related problems, *J. London Math. Soc.* **16** (1941), 212–215.

[6] P. Erdős, On the arithmetical density of the sum of two sequences, one of which forms a basis for the integers, *Acta Arith.* **1** (1936), 197–200.

[7] ———, On the multiplicative representation of integers, *Israel J. Math.* **2** (1964), 251–261.

[8] P. Erdős and M. B. Nathanson, Maximal asymptotic nonbases, *Proc. Amer. Math. Soc.* **48** (1975), 57–60.

[9] ———, Oscillations of bases for the natural numbers, *Proc. Amer. Math. Soc.* **53** (1975), 253–358.

[10] ———, Partitions of the natural numbers into infinitely oscillating bases and nonbases, *Commentarii Math. Helvet.* **52** (1976), 171–182.

[11] ———, Nonbases of density zero not contained in maximal nonbases, *J. London Math. Soc. (2)* **15** (1977), no. 3, 403–405.

[12] ———, Sets of natural numbers with no minimal asymptotic bases, *Proc. Amer. Math. Soc.* **70** (1978), 100–102.

[13] ———, *Systems of Distinct Representatives and Minimal Bases in Additive Number Theory*, Number Theory, Carbondale, 1979 (Heidelberg) (M. B. Nathanson, ed.), Lecture Notes in Mathematics, vol. 751, Springer-Verlag, 1979, pp. 89–107.

[14] ———, Minimal asymptotic bases with prescribed densities, *Illinois J. Math.* **32** (1988), 562–574.

[15] H. Halberstam and K. F. Roth, *Sequences, Vol. 1*, Oxford University Press, Oxford, 1966; Reprinted by Springer-Verlag, Heidelberg, 1983.

[16] E. Härtter, Eine Bemerkung über periodische Minimalbasen für die Menge der nichtnegativen ganzen Zahlen, *J. Reine Angew. Math.* **214/215** (1964), 395–398.

[17] P. Hegedüs, G. Piroska, and I. Z. Ruzsa, On the Schnirelmann density of sumsets, *Publ. Math. Debrecen* **53** (1998), no. 3–4, 333–345.

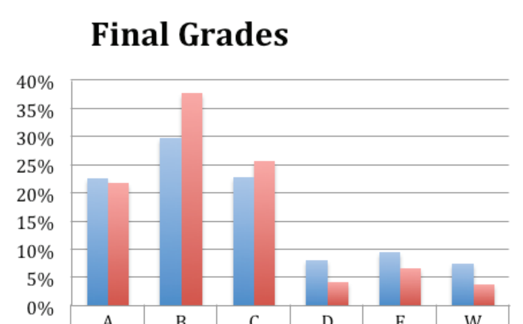
[18] H. A. Helfgott, *The ternary Goldbach conjecture is true*, arXiv:1312.7748 [math.NT], 2013.

[19] J. Hennefeld, Asymptotic nonbases which are not subsets of maximal asymptotic nonbases, *Proc. Amer. Math. Soc.* **62** (1977), 23–24.
[20] A. Ya. Khinchin, Über ein metrisches Problem der additiven Zahlentheorie, *Mat. Sbornik N.S.* **10** (1933), 180–189.
[21] M. Kneser, Abschätzungen der asymptotischen Dichte von Summenmengen, *Math. Z.* **58** (1953), 459–484.
[22] E. Landau, *Über einige neuere Fortschritte der additiven Zahlentheorie*, Cambridge University Press, Cambridge, 1937.
[23] H. B. Mann, A proof of the fundamental theorem on the density of sums of sets of positive integers, *Annals of Math.* **43** (1942), 523–527.
[24] J. C. M. Nash and M. B. Nathanson, Cofinite subsets of asymptotic bases for the positive integers, *J. Number Theory* **20** (1985), no. 3, 363–372.
[25] M. B. Nathanson, Minimal bases and maximal nonbases in additive number theory, *J. Number Theory* **6** (1974), 324–333.
[26] ———, Multiplicative representations of integers, *Israel J. Math.* **57** (1987), no. 2, 129–136.
[27] ———, Best possible results on the density of sumsets, *Analytic Number Theory (Allerton Park, IL, 1989)*, Progr. Math., vol. 85, Birkhäuser Boston, Boston, MA, 1990, pp. 395–403.
[28] ———, *Additive Number Theory: The Classical Bases*, Graduate Texts in Mathematics, vol. 164, Springer-Verlag, New York, 1996.
[29] ———, Cassels bases, *Additive Number Theory*, Springer, New York, 2010, pp. 259–285.
[30] ———, *Additive Number Theory: Extremal Problems and the Combinatorics of Sumsets*, Graduate Texts in Mathematics, Springer, New York, to appear.
[31] M. B. Nathanson and András Sárközy, On the maximum density of minimal asymptotic bases, *Proc. Amer. Math. Soc.* **105** (1989), 31–33.
[32] J. Nešetřil and V. Rödl, Two proofs in combinatorial number theory, *Proc. Amer. Math. Soc.* **93** (1985), 185–188.
[33] H. Plünnecke, Über ein metrisches Problem der additiven Zahlentheorie, *J. Reine Angew. Math.* **197** (1957), 97–103.
[34] ———, Über die Dichte der Summe zweier Mengen, deren eine die dichte null hat, *J. Reine Angew. Math.* **205** (1960), 1–20.
[35] ———, *Eigenschaften und Abschätzungen von Wirkungsfunktionen*, vol. 22, Berichte der Gesellschaft für Mathematik und Datenverarbeitung, Bonn, 1969.
[36] D. Raikov, Über die Basen der natürlichen Zahlentreihe, *Mat. Sbornik N.S. 2* **44** (1937), 595–597.
[37] I. Z. Ruzsa, Essential components, *Proc. London Math. Soc.* **54** (1987), 38–56.
[38] L. G. Shnirel'man, On the additive properties of integers, *Izv. Donskovo Politekh. Inst. Novocherkasske* **14** (1930), 3–27.
[39] ———, Über additive Eigenschaften von Zahlen, *Math. Annalen* **107** (1933), 649–690.
[40] A. Stöhr, Eine Basis h-Ordnung für die Menge aller natürlichen Zahlen, *Math. Zeit.* **42** (1937), 739–743.
[41] ———, Gelöste und ungelöste Fragen über Basen der natürlichen Zahlenreihe. I, II, *J. Reine Angew. Math.* **194** (1955), 40–65, 111–140.

Seven Characteristics of Successful Calculus Programs

David Bressoud and Chris Rasmussen

In these days of tight budgets and pressure to improve retention rates for science and engineering majors, many mathematics departments want to know what works, what are the most productive means of improving the effectiveness of calculus instruction. This was the impetus behind the study of Characteristics of Successful Programs in College Calculus[1] undertaken by the Mathematical Association of America. The study consisted of a national survey in fall 2010, followed by case study visits to seventeen institutions that were identified as "successful" because of their success in retention and the maintenance of "productive disposition," defined in [NRC 2001] as "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy."



	A	B	C	D	F	W
all PhD universities	22.7%	29.7%	22.9%	8.0%	9.4%	7.4%
5 case study sites	21.8%	37.8%	25.7%	4.2%	6.6%	3.8%

Figure 1. Instructor-reported final grades.

David Bressoud is professor of mathematics at Macalester College. His email address is `bressoud@macalester.edu`.

Chris Rasmussen is professor of mathematics education at San Diego State University. His email address is `crasmussen@mail.sdsu.edu`.

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Our survey revealed that Calculus I, as taught in our colleges and universities, is extremely efficient at lowering student confidence, enjoyment of mathematics, and desire to continue in a field that requires further mathematics. The institutions we selected bucked this trend. This report draws on our experiences at all seventeen colleges and universities but focuses on the insights drawn from those universities that offer a PhD in mathematics, the universities that both produce the largest numbers of science and engineering majors and that often struggle with how to balance the maintenance of high-quality research with attention to undergraduate education.

Case studies were conducted in the fall of 2012 at five of these universities: two large public research universities, one large private research university, one public technical university, and one private technical institute. We shall refer to these as:

PrTI: Private Technical Intitute. Private university. Data from nine sections of calculus with an average enrollment of 33.

PTU: Public Technical University. Public university. Data from seven sections with an average enrollment of 38 and one with an enrollment of 110.

LPU1: Large Public University. Data from forty-one sections with an average enrollment of 27.

LPU2: Large Public University. Data from four sections with an average enrollment of 200.

LPrU: Large Private University. Data from three sections with an average enrollment of 196 and one section with 32 students.

In addition to productive disposition and improved retention rates, the five also had noticeably higher grades (see Figure 1), cutting the D-F-W rate from 25 percent across all doctoral universities to only 15 percent at the case study sites. The difference was in B's and C's. The five case study universities actually gave out a slightly lower percentage of A's than the overall average.

We identified seven characteristics of the calculus programs at these five universities, characteristics that, as applicable, were also found at the other twelve case study sites.

1. **Regular use of local data to guide curricular and structural modifications.** In his description of the MAA study of *Models That Work* [Tucker, 1995], Alan Tucker wrote, "No matter how successful their current programs are, faculty members in the visited departments are not yet satisfied with the programs. Experimentation is continuous" [Tucker, 1996]. We found that not only was this true of the successful programs we studied, but these universities used the annual gathering and sharing of data on retention and grade distributions to guide this continuous experimentation. A bad semester was not dismissed as an anomaly but was viewed as an opportunity to understand what went wrong and what could be done to avoid a similar occurrence.

2. **Attention to the effectiveness of placement procedures**. Though this could be considered part of the first characteristic of successful programs, it received so much attention from all of the universities that we have elevated it to the level of a separate point. These universities evaluate and adjust their placement procedures on an annual basis. We also found a great deal of attention paid to those students near the cutoff, paying particular attention to programs in support of those allowed into Calculus I but most at risk and working with those who did not quite make the cut so that they were placed in programs that addressed their actual needs.

3. **Coordination of instruction, including the building of communities of practice**. As Tucker reported in 1996, "There is a great diversity of instructional and curricular approaches, varying from one visited department to another, and even varying within a single department." We found this, but we also found that those teaching calculus were in regular communication with the other instructors of this class. Of course, where classes were taught by graduate teaching assistants, there was much tighter coordination of instruction. In all cases, we found that common exams were used. The simple act of creation of such exams fostered communication among those teaching the course. In some cases, communication about teaching was much more intentional, sharing innovative pedagogies, assignments, and approaches to particular aspects of the curriculum. In all cases there was also a course coordinator, a position that was not rotating but a more or less permanent position with commensurate reduction in teaching load.

4. **Construction of challenging and engaging courses**. This is reflected in an observation that Tucker made in 1996: "Faculty members communicate explicitly and implicitly that the material studied by their students is important and that they expect their students to be successful in mathematical studies." It also is the first example of effective educational practice in *Student Success in College* [Kuh *et al.*, 2010, p. 11]: "Challenging intellectual and creative work is central to student learning and collegiate quality." None of the successful programs we studied believed that one could improve retention by making the course easier. Instructors used textbooks and selected problems that required students to delve into concepts and to work on modeling-type problems or even problems involving proofs. Interviews with students, most of whom had taken calculus in high school, revealed that they felt academically challenged in ways that went far beyond their high school courses.

5. **Use of student-centered pedagogies and active-learning strategies**. This is the second example of effective educational practice in [Kuh *et al.*, 2010], "Students learn more when they are intensely involved in their education and have opportunities to think about and apply what they are learning in different settings." As the first author learned twenty years ago when he surveyed Calculus I students at Penn State [Bressoud, 1994], few students know how to study or what it means to engage the mathematics, and most take a very passive role when attending a lecture. Active-learning strategies force students to engage the mathematical ideas and confront their own misconceptions. At LPU2, where class size makes active-learning difficult, we found it strongly encouraged and supported in the recitation sections.

6. **Effective training of graduate teaching assistants**. Graduate students play an important role in calculus instruction at all universities with doctoral programs, whether as teaching assistants in the break-out sections for large lectures or as the instructors of their own classes. The most successful universities have developed extensive programs for training, monitoring, and supporting these instructors. Running a successful training program is not a task that can be handed off to a single person. While there is always one coordinator, their effectiveness requires a core of faculty who are willing to participate in the graduate students' training that takes place before the start of the fall term and to assist in visiting classes and providing feedback.

7. **Proactive student support services, including the fostering of student academic and social integration**. This is a broad category that ranges from the building of a student-faculty community within the mathematics department to the specifics of support mechanisms for at-risk students. These are addressed in three of the effective practices identified in [Kuh *et al.*, 2010]: "Student Interactions with Faculty Members," "Enriching Educational Experiences," and "Supportive

Campus Environment." The first is mentioned in [Tucker, 1996]: "Extensive student-faculty interaction characterizes both the teaching and learning of mathematics, both inside and outside of the classroom." The universities we visited had rich programs of extracurricular activities within the mathematics department. They also had a variety of responses to supporting at-risk students. These included stretching Calculus I over two terms to

Table 1. Instructor Responses to 2010 Survey Questions on Practices in Calculus I.

Instructors	% who do this often or very often					
	Other doctoral universities	PrTI	PTU	LPU1	LPU2	LPrU
Lecture	84%	83%	71%	63%	100%	100%
Use online homework	61%	50%	100%	91%	0%	100%
Allow graphing calculators on exams	45%	0%	57%	97%	0%	0%
Ask students to explain their thinking in class	36%	33%	57%	82%	75%	33%
Have students work together in class	18%	17%	57%	97%	0%	33%
Hold whole class discussions	16%	17%	43%	55%	0%	33%
Have students give presentations in class	1%	17%	0%	36%	0%	0%

Key for Table 1: PrTI = Private Technical Institute, **PTU** = Public Technical University, **LPU1** = Large Public University, **LPU2** = Large Public University, **LPrU** = Large Private University

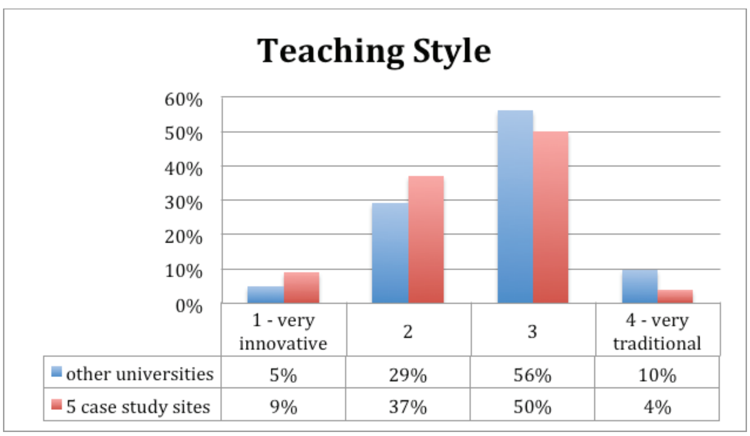


Figure 2: Instructor self-assessment of teaching style.

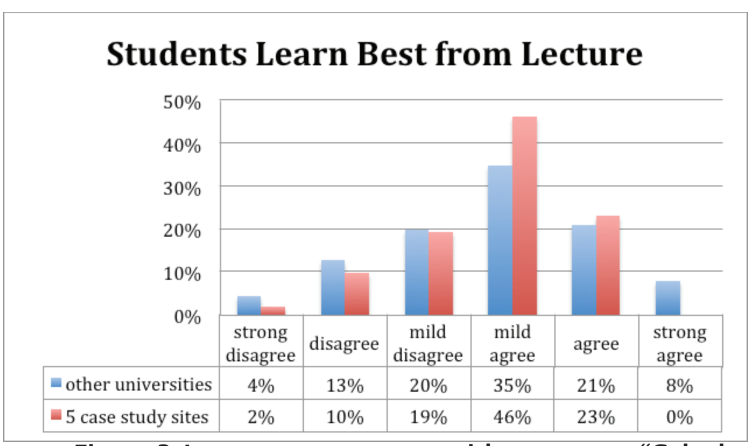


Figure 3: Instructor agreement with statement, "Calculus students learn best from lectures, provided they are clear and well organized."

allow for supplemental instruction in precalculus topics, providing "fall-back" courses for students who discovered after the first exam that they were in trouble in calculus, and working with student support services to ensure that students who were struggling got the help they needed. There also were heavily utilized learning centers that attracted all students as places to gather, work on assignments, and get help as needed. Often these were centers dedicated solely to helping students in calculus. What was common among all of the successful calculus programs was attention to the support of all students and a willingness to monitor and adjust the programs designed to help them.

There were some dramatic differences between instruction at the doctoral universities that were selected for the case study visits and instruction at all doctoral universities (see Table 1). Where the section size facilitated this—at PrTI, PTU, LPU1, and one section of LPrU—instructors made much less use of lecture and much more use of students working together, holding discussions, and making presentations. Three of the five have almost universal use of online homework, and a fourth uses it for half of the sections. Graphing calculators were allowed on exams in two of the five universities, though use was not consistent across sections. The most striking difference between these five universities and the overall survey was the number of instructors who ask students to explain their thinking.

Instructors at the case study sites still consider themselves to be fairly traditional (see Figure 2), though slightly less so than the national average. They also tend to agree with the statement, "Calculus students learn best from lectures, provided they are clear and well organized" (see Figure 3). Interestingly, not a single instructor at any of the case study sites strongly agreed with this statement. On the other hand, the instructors at the case study sites were slightly less likely to disagree with it. They clumped heavily toward mild agreement, suggesting an attitude of keeping an open mind and a willingness to try an approach that might be more productive.

References

[Bressoud, 1994] D. Bressoud, Student attitudes in first semester calculus, *MAA Focus*, vol. 14, (1994), 6–7.

[Kuh *et al.*, 2010] G. D. Kuh, J. Kinzie, J. H. Schuh, and E. J. Whitt, *Student Success in College: Creating Conditions That Matter*, Jossey-Bass, 2010.

[NRC 2001] National Research Council, *Adding It Up: Helping Children Learn Mathematics*, Kilpatrick, Swafford, and Findell (eds.), National Academy Press, 2001.

[Tucker, 1995] A. Tucker, *Models That Work: Case Studies in Effective Undergraduate Mathematics Programs*, MAA Notes #38, Mathematical Association of America, 1995.

[Tucker, 1996] ______, Models that work: Case studies in effective undergraduate mathematics programs, *Notices of the AMS*, vol. 43, (1996), 1356–1358.

About the Cover

x, y, and z

This month's cover was suggested by the review in this issue of the book *Enlightening Symbols* by Joseph Mazur. As Mazur mentions, albeit rather casually, it was Descartes who introduced into mathematics one of the most enduring conventions of our subject—the use of a, b, c, etc. as constants and x, y, z, etc. as variables. The cover image shows page 303 from the first edition of *Discours de la méthode*, in which this convention apparently first came fully to life.

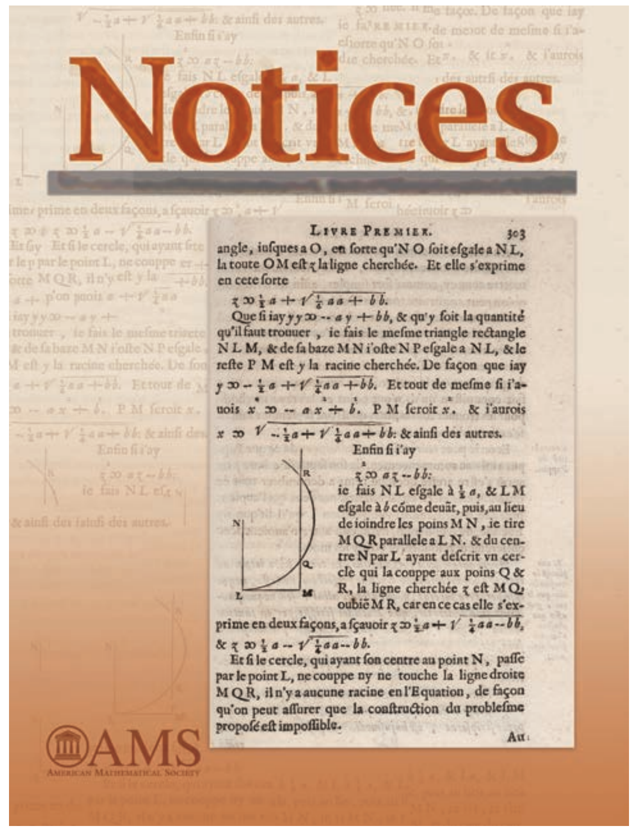
The *Discours* illustrates a curious feature of the history of mathematics—that randomness has played a more important role than one might expect. First of all, the circumstances of its publication are strange. It appeared in 1637, and in French, but not in France. Descartes had been having some trouble with French authorities, and the Netherlands was much more liberal in publishing policies. It was published in Leiden.

That it was in French seems quite reasonable to us, but in its time there were few scientific works in that language. Descartes himself asserts somewhere that he wrote in French because he wanted a wider readership. This is not a plausible claim. There is evidence that Descartes's Latin was rather weak, but he was a rather vain man … In fact it wasn't until René van Schooten the Younger published a Latin translation that it became widely read. It was the Latin edition, for example, that was Newton's textbook. What if van Schooten's father had not been a good friend of Descartes?

Descartes's choice of letters was one segment in the history of conventions regarding variables in algebra. Other major characters in the story are Viète and Harriot. Harriot himself was responsible for a few enduring conventions in algebra, among them the choice of n for integer variables. But it was Descartes who introduced exponents, so that (if we remember correctly), Harriot wrote *nnnnnnnnnnnnnnnnnnn*, where we follow Descartes in writing n^{19}. How did Harriot miss this?

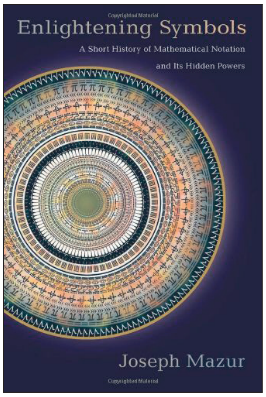
The question of enduring conventions is intriguing. What intuition led Descartes to his apparently canonical choice? Might history have taken some different path? Descartes's own conventions did not always pass the test. He is deservedly famous for devising the geometric interpretation of algebra, but it was Newton who rotated Descartes's axes so as to agree with our modern convention that the dependent variable is plotted vertically. Somehow, this seems to us, as to Newton's contemporaries, exactly right. Why?

The bilingual French/English edition of *La Géometrie*, which is part of the *Discours*, is available at archive.org/details/geometryofrene00desc.



The cover image was supplied from a copy of the *Discours* now in the Thomas Fisher Rare Book Library at the University of Toronto, to whom we are very grateful.

—*Bill Casselman*
Graphics Editor
notices-covers@ams.org



Enlightening Symbols: A Short History of Mathematical Notation and Its Hidden Powers

Reviewed by Robyn Arianrhod

Enlightening Symbols: A Short History of Mathematical Notation and Its Hidden Powers
Joseph Mazur
Princeton University Press, 2014
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It is easy to forget, with our four centuries of post-Cartesian hindsight, that for thousands of years arithmetic and algebra were done rhetorically or geometrically, without the benefit of our familiar x's and y's, our pluses, minuses, square root signs, indices, and other symbols. These symbols make algebraic processes so transparent, so universal, and so easily generalized that mathematical thought becomes simpler and more economical.

This simplicity is possible because mathematical symbols can "evoke subliminal, sharply focused perceptions and connections" [p. xiii], as Joseph Mazur puts it in his entertaining and insightful new book. "Just as with the symbolism in music and poetry, these mathematical symbols might also transfer metaphorical thoughts capable of conveying meaning through similarity, analogy, and resemblance, and hence are as capable of such transferences as words on a page. In reading an algebraic expression, the experienced mathematical mind leaps through an immense number of connections in relatively short neurotransmitter lag times" [p. xiii].

Thinking about the history of mathematics in terms of the history of symbolic mathematical notation offers an interesting perspective on the dramatic increase in mathematical progress from the seventeenth century onwards. It also makes us wonder how ancient mathematicians made as much progress as they did without an internalized, subconscious symbolic language to aid their thinking. *Enlightening Symbols* offers food for thought on both these themes: the history of mathematics and symbolic cognition. But Joe Mazur's skill lies in discussing deep ideas in an engaging and accessible style: this is a book aimed at the lay reader, although it will also be of interest to mathematicians and (perhaps especially) mathematics educators.

Some readers might wonder what exactly is the definition of "symbol" that applies to mathematics. In his introduction Mazur gives the etymology of the word: it comes from the Greek for "token" or "token of identity" and refers to "an ancient way of proving one's identity or one's relationship to another. A stick or bone would be broken in two, and each person in the relationship would be given one piece. To verify the relationship, the pieces would have to fit together perfectly" [p. xi].

On a deeper level, the word "suggests that when the familiar is thrown together with the unfamiliar, something new is created. Or, to put it another way, when an unconscious idea fits a conscious one, a new meaning emerges" [p. xi]. A classic example of the power of mathematical symbolism is Maxwell's use of (a component form of)differential vector calculus to describe the known facts of electromagnetism in terms of a *field* (his contemporaries had been using "action-at-a-distance" integral calculus); the bonus was that differentiating Maxwell's differential equations then produces mathematical wave equations, and mathematical wave equations suggest physical waves. Hertz was the first to experimentally produce wireless electromagnetic waves, nearly twenty-five years after Maxwell's theoretical prediction of their existence.

But Mazur points out that there is a difference between powerful, evocative symbolism and simple notation, the latter being simply a form of shorthand for words used in the rhetorical formulation of mathematical problems. For instance,

Robyn Arianrhod is an adjunct research fellow in the School of Mathematical Sciences at Monash University. Her email address is robyn.arianrhod@monash.edu.

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technically speaking the sign "+" comes into the category of notation, because + is shorthand for the letter "t" in the Latin word *et*, which means "and". Nevertheless, the meaning of "+" continued to evolve after Johannes Widmann introduced it in his 1489 book on mercantile calculation, where, according to Mazur, it did not refer to abstract operation of addition, but simply "meant 'excess', as in '+2 is two more than what was expected"' [p. 162]. Consequently, many fifteenth- and sixteenth-century mathematicians favored the notation "p" and "m" for "plus" and "minus", and there were many other signs used for these operations until "+" and "−" became universal in the eighteenth century.

The same was true for all our modern mathematical signs: abbreviations became ever more abbreviated until they became true symbols. So, while "a purist approach would be to distinguish symbolic representation from simple notation," Mazur favors the view that "numerals and all non-literal notation are different, but still considered symbols, for they represent things that they do not resemble" [pp. xi–xii]. With this definition in place, Part I of *Enlightening Symbols* is a brief history of numerals, and Part II is a history of symbolism in algebra. Part III is more speculative, a brief enquiry into the nature of symbolic cognition, including discussion of the similarities and differences between mathematical and other symbols.

Parts I and II each begin with a useful summary of key innovators; a quick glance at these lists shows just how long it took for mathematics to become the elegant and universal language it is today. The list for Part I focuses on the evolution of our Hindu-Arabic numeral system, beginning in the middle of the first millennium with the Bakhshâlî manuscript, followed by the work of Brahmagupta, whose *Brahmasphutasiddhanta* of 628 CE contains the first-known use of the concept of zero as a number. The list finishes with the thirteenth-century Europeans (including Fibonacci), who introduced the Indian numerals to Europe, so it took six centuries to accomplish the transition from India: first to the Middle East, whose Golden Age of translation and intellectual development included the work of the ninth-century Persian mathematician al-Khwarizmi, and then to Europe. It seems the Europeans knew about Indian numerals from the tenth century, but the concept of zero was so novel that it took another three centuries for the Indian numerical system to become fully accepted there: "The difficulty is in distinguishing placeholder from number. Accepting zero as a number representing the absence of quantity would have been a fantastically daring idea" [p. 64].

But Part I covers a much broader period than the rise of the Hindu-Arabic numerals: it includes the ancient number systems of the Babylonians, the Egyptians, Greeks, Hebrews, Chinese, Aztecs, and Mayans. To choose just one of the many interesting facts and points of discussion in this section, I was intrigued by Mazur's question, Why didn't the Greeks, with all their mathematical brilliance, "adopt the genius of the Babylonian system, such as its placeholders and relative ease of writing large numbers? The Babylonians had the right idea of positional notation, the clever idea of using the same digits to represent multiples of different powers of 60" [p. 21]. Alternatively, the ancient Chinese had come up with a clever decimal system that included symbols for powers of ten up to the fourth, so no placeholder symbol was needed. Mazur suggests that perhaps the Greeks used an abacus for calculations (later in Part I, he discusses the evolution of various abaci) or perhaps they were more interested in the "grand scope of mathematics itself" than in calculation. At any rate, it is astonishing to think how long it took before the Indian system—"the smartest system of all"—was developed.

To take another example from this section, Mazur discusses the widespread medieval art of finger-counting, as illustrated, for example, in Luca Pacioli's *Summa de Arithmetica* of 1494, a sample page of which is included as an illustration. But here as elsewhere in the book, Mazur does more than simply give an account of the art and its history: in this case, he also mentions intriguing research that suggests our brains may be hard-wired for counting, in the sense that both counting and finger movement are located in the left parietal lobe.

Part II begins with an anecdote about Mazur's visit to Oxford's Bodleian Library in order to peruse the oldest surviving copy of Euclid's *Elements*. Before being allowed to see this treasure, he had to take an oath to respect the library's property, and then he was asked to sign a special guestbook; to his amazement, he saw that just twelve lines above his own signature was that of Isaac Newton! As for Euclid, Mazur says, "I did not expect and could not find any symbols for addition, multiplication, or equality" [p. 86]. Euclid's work was entirely geometrical and rhetorical. The first known symbolic innovation did not occur until the third century, six hundred years after Euclid, when Diophantus (or his scribes) used abbreviations for unknowns, powers, and subtraction, although they were not the symbols we use today.

Mazur traces the long history of symbolic algebra, including the work of Diophantus, Brahmagupta and al-Khwarizmi, a host of fifteenth-, sixteenth-, and seventeenth-century Europeans (from Pacioli to Descartes to Newton and Leibniz), and finally, in the eighteenth and nineteenth centuries, Euler, William Jones (who introduced the Greek letter pi to denote the ratio of the circumference to the diameter of a circle), Dirichlet, and Hamilton. Along the way, he provides a fascinating tour of algebraic innovations, including the development of new number systems, notably complex numbers and quaternions, which, of course, are a world away from the ordinary numbers that developed from counting and measuring concrete "things." The square roots of negative numbers had been seen as "meaningless" right up to the seventeenth century, when, "with more general notation, more attention was paid to the 'meaningless' than had ever been paid before. So that attention called out the question: What is number?"[p. 148]. Key to this "more general notation" was Viète's decision, in 1591, to use vowels to represent unknowns and consonants to represent knowns and his "magnificent idea that those letters were also to be subject to algebraic reasoning and rules just as much as numbers." [p. 144]. Mazur's enthusiasm makes one thrill to the grand sweep of big ideas that too often we moderns mistake for small ones!

It wasn't until Descartes's *Geometria* of 1637 that almost all our modern algebraic notation was finally in place, so that "on page 69, for the first time, we find a perfectly readable account [of polynomial equations] that almost looks as if it is out of a twentieth-century textbook.…The symbol had finally arrived to liberate algebra from the informality of the word" [p. 156, xvii].

Fifty years later, Newton and Leibniz had systematized calculus, and Leibniz's brilliant notation is the one we use today. Newton's "pricked" letters, such as an x with a dot on top, also survive and are used to denote derivatives with respect to time. Mazur gives a brief but interesting comparison between Newton's and Leibniz's concepts of a derivative and notes the explosion of practical applications that followed in their wake.

He also notes, however, that amidst all this wondrous growth of symbolic mathematics, something was lost: mathematics became more specialized, less accessible to the public. Nowadays, he says, mathematics can seem like Lewis Carroll's nonsense rhyme *Jabberwocky* [p. 179]: "The Jabberwocky is what we get when we first encounter mathematics—or anything—we don't understand." Even applied mathematics "can be done without reference to any physically imaginable object other than a graphic symbol" [p. xviii], and this is why mathematics is more difficult to explain to the public than is, say, physics.

It's interesting to recall, in this context, that Newton famously chose to write *Principia* primarily in the language of geometry rather than that of his new symbolic calculus. And in developing the general theory of general relativity, Einstein, in contrast to Hilbert, chose to eschew a purely abstract mathematical formulation based on Lagrangian and Hamiltonian action principles; instead, he favored "psychologically natural" physical principles [1, p. 118]. Maxwell, too, was wary of banishing all physical content from our understanding of the symbolic equations of dynamics [2, p. 210], although he acknowledged that pure mathematics has given science many ideas whose discovery would not have been possible otherwise. His observation certainly applies to the bizarrely counterintuitive ideas of quantum theory!

To the uninitiated, this careful attention to balancing symbolic and psychological language is lost; even for the initiated, exploring the content of general relativity, for example, can require a lot of manipulation of graphic symbols whose physical content is buried deep within the layers of symbolic scaffolding that underlie the equations. Yet it is this very complexity that enables mathematical theories of nature to be expressed so economically.

To give readers some insight into why the rise of symbols helped mathematics become so abstract and so powerful, Mazur has included Part III, an absorbing, speculative excursion into the nature of thought, including mathematical cognition and the role of symbols in our thought processes [p. 207]. He tells us, for instance, that Jared Danker and John Anderson at Carnegie Mellon found that when subjects were asked to solve simple algebraic equations, "there was a strong interactional relationship between retrieval and representation in mathematical thinking." Research by Anthony Jansen, Kim Marriott, and Greg Yelland of Monash University found, in Mazur's words, "that experienced users of mathematics had an easier time identifying previously seen syntactically well-formed expressions than ill-formed ones. They found that the encoding of algebraic expressions is based primarily on processes that occur beyond the level of visual processing. For example, the well-formed string $7 - x$ is better recalled than ill-formed strings such as $7(x$" [p. 207].

They also found that "we 'read' algebraic expressions by their syntax, just as we do when processing sentences of natural language" [p. 208]. I found this fascinating in view of the fact that girls have long been considered better at language than at mathematics. Actually, current research [3], [4] suggests it is simply spatial ability that separates girls from boys in mathematical achievement and participation. This research also suggests that the spatial-ability stereotype itself could be contributing to the problem [4, p. 8], since there seems to be no genetic or hormonal basis for a gender difference in spatial skill [4, p. 7]. Indeed, it is easy for preconceived stereotypes to become reality: in some communities, and at different times in history, boys have been exposed to more spatially oriented toys and hobbies than girls, and girls have been expected to be inferior to boys in math and physics [see also 5, pp. 10, 19]. So I was intrigued that when describing his own thought processes on viewing the algebraic equation $x^2 - ab = 0$, Mazur commented, "I immediately know that $x = \pm\sqrt{ab}$. But I would also see a square and a rectangle that are aching to be compared" [p. 193]. I was intrigued because this is certainly an extremely simple visualization task, and yet I suspect that those of us who relate better to language than to spatial visualization would not automatically see geometrical interpretations of such equations.

Of course, not all women, or only women, are spatially challenged; in fact Mazur also discusses the earlier idea of "brain type" and quotes [p. 201] Henri Poincaré's words of a century ago (when most university students were male): "Among our students…some prefer to treat their problems 'by analysis', others 'by geometry'. The first are incapable of 'seeing in space'. The others are quickly tired of long calculations and become perplexed."

Fortunately, recent research [3] suggests that spatial skills can be learned; intriguingly, it also suggests [3, p. 369] that although these skills "strongly predict performance early in STEM [science, technology, engineering, and math] learning," it is less important for specialists, who "can rely on a great deal of *semantic* knowledge of the relevant spatial structures without having to perform classic mental spatial tasks[italics added]…." I have italicized the word "semantic" because this conclusion seems to fit nicely with the Monash and Carnegie Mellon research discussed above, and with the fact that a semantic geometrical connection, as well as a visual one, can be made between the square and rectangle implicit in $x^2 - ab = 0$. Mazur notes that because a and b "do not have specific values, the [geometric] exercise can only be one of symbolic manipulation. I would resort to the rules of algebra learned in school.…" [p. 193]. He goes on to give a subjective account of the algebraic thought processes involved in solving this little problem—an account that is necessarily subjective, because "we all think somewhat differently with brains that are exquisitely different, using richly assorted thinking styles that contribute to and account for the preciousness of being human" [p. 202].

I have touched only briefly on the content of Part III, and I have added my own digressions, but my point is that *Enlightening Symbols* is not only informative, it can also serve as a springboard for further thought or investigation, depending on the interests of the reader.

Of course, the book can also be read simply for its wide-ranging survey of mathematical history and its enjoyable, sometimes quirky, asides that make it more than just an accessible chronological history of mathematical symbolism. Let me take just one example at random. In 751 CE, during the battle of Talas between the Arabs and the Chinese over Kazakhstan, the Kazakhstani Arabs learned how to make paper from two Chinese prisoners-of-war. The availability of cheap paper helped foster the great period of Arab translation, which preserved ancient Greek texts that had been lost in the West.

At whatever depth one chooses to read it, *Enlightening Symbols* has something for everyone. It is entertaining and eclectic, and Mazur's personal and easy style helps connect us with those who led the long and winding search for the best ways to quantify and analyze our world. Their success has liberated us from "the shackles of our physical impressions of space"—and of the particular and the concrete—"enabling imagination to wander far beyond the tangible world we live in, and into the marvels of generality" [p. 154].

References

[1] A. Einstein, The foundation of the general theory of relativity, in *The Principle of Relativity: A collection of memoirs on the special and general theory of relativity*, by H. A. Lorentz, A. Einstein et al., translated from the German by W. Perrett and G. B. Jeffery, London, Methuen, 1923, and Dover (NY) reprint, 1952. Einstein's paper was originally published in German in *Annalen der Physik*, **49**, (1916).

[2] J. C. Maxwell, *A Treatise on Electricity and Magnetism*, Volume 2, reprinted in 1954 by Dover, NY (from the third edition of 1891).

[3] For a survey of research and a meta-analysis, see David H. Uttal, Nathaniel G. Meadow, Elizabeth Tipton, Linda L. Hand, Alison R. Alden, Christopher Warren, and Nora S. Newcombe, The malleability of spatial skills: A meta-analysis of training studies, *Psychological Bulletin* **139** (2013), no. 2, 352–402.

[4] For another review paper, see H. I. Scutt, S. K. Gilmartin, S. Sheppard, and S. Brunhaver, *Research-Informed Practices for Inclusive Science, Technology, Engineering and Math (STEM) Classrooms: Strategies for Educators to Close the Gender Gap*, American Society for Engineering Education, 2013 (paper presented at the 120th ASEE Annual Conference and Exposition, June 23–26, 2013).

[5] Jonathan M. Kane and Janet E. Mertz, Debunking myths about gender and mathematics performance, *Notices of the AMS* **59** (2012), no. 1, 10–20.

Interview with Yakov Sinai

Martin Raussen and Christian Skau

Yakov Sinai is the recipient of the 2014 Abel Prize of the Norwegian Academy of Science and Letters. This interview was conducted by Martin Raussen and Christian Skau in Oslo on May 19, 2014, in conjunction with the Abel Prize celebration. This article originally appeared in the September 2014 issue of the *Newsletter of the European Mathematical Society* and is reprinted here with permission of the EMS.

Professor Yakov Sinai

The Prize

Raussen and Skau: *Professor Sinai, first of all we would like to express our congratulations. You have been selected as the twelfth winner of the Abel Prize and you will receive the prize tomorrow. We are curious; did you have any expectations beforehand? How did you receive the information?*

Professor Sinai: In early March this year I got to know that the Abel Committee was interested in taking my photograph. A friend of mine told me this, and I thought this must mean something because this had never happened before. And then there was a telephone call from the Norwegian Academy of Science and Letters informing me about the prize.

Raussen and Skau: *And this was on the same day that the prize was announced here in Oslo?*

Professor Sinai: Yes. That happened on March 26.

Youth

Raussen and Skau: *You were born in Moscow in 1935 into a family of scientists. Your parents were both biologists, and your grandfather was well known in mathematics. We suppose this had important consequences for the development of your interests?*

Professor Sinai: Definitely, yes. How could I say no to this question? Everything was about mathematics and mathematical events. But, at that time, I preferred to play volleyball.

The influence of mathematics was not as direct as you may think. I participated in many olympiads in mathematics during my school years but never had any success and never won any awards. I say this to young people who have never won in olympiads; there may be compensation in the future.

At this time, my grandfather was of a great age and he did not have the energy to push me into mathematics. And I also have a half-brother, G. I. Barrenblatt, who worked at Moscow State University and who was convinced that I should pursue a career in mathematics.

Raussen and Skau: *Do you remember when you found out that you had an exceptional talent for mathematics?*

Professor Sinai: If at all, it happened very late. I was a graduate student when I brought my paper on entropy to my advisor, A. N. Kolmogorov and he said, "At last you can compete with my other students." But I am not sure that he was right and that I have an exceptional talent for mathematics.

Raussen and Skau: *You must have entered school at about the same time that Nazi Germany invaded Russia. How did the war influence your first years at school?*

Martin Raussen is associate professor of mathematics at Aalborg University, Denmark. His email address is `raussen@math.aau.dk`.

Christian Skau is professor of mathematics at the Norwegian University of Science and Technology, Trondheim, Norway. His email address is `csk@math.ntnu.no`.

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Professor Sinai: I entered school in 1943 after my family returned from the evacuation of Moscow. At that time boys and girls studied separately; at the end of each year, we had about ten exams. Before the evacuation, life was different. It was forbidden to leave windows open in the apartments in Moscow because it had to be dark. In 1943 windows were allowed to be open again. In Moscow there were no clear signs of war, but life was hard because of the time of Stalin. People had to behave in a special way.

Raussen and Skau: *And that also influenced life at school?*

Professor Sinai: It was everywhere; you could be expelled from school or even sent to prison for being controversial.

Raussen and Skau: *Were there teachers with a lot of influence on you, in particular in mathematics?*

Professor Sinai: We had a very good teacher in mathematics at our high school. His name was Vasily Alekseevich Efremov and he was a great old-style schoolteacher. He always brought us his problems in accurate handwriting on a piece of paper which he distributed among the students. Because of the well-organized and inspiring work, mathematics was very popular among us. We discussed and tried to solve his problems. At this time I was not among the best in the class. There were definitely other students who were much better than I.

Raussen and Skau: *What was your age at this point?*

Professor Sinai: This was still in high school just before I entered university. Thus, I was probably sixteen or seventeen years old.

Student at MSU-Mech-Mat

Raussen and Skau: *You entered the Faculty of Mechanics and Mathematics at Moscow State University in 1952 still a teenager. How was it to study at this famous institution as such a young student?*

Professor Sinai: We had a number of very good professors there. For example, the lecture course in analysis was given by M. A. Lavrentiev, who was a very famous scientist at the time. He was also involved in administration but was a great teacher and his lectures were very interesting. We also had a very good lecturer in classical mechanics, Chetaev. I was his student in the second year. Moreover, we had lectures in geometry given by Bachvalov, who was famous in Russia but not so much known in the West. There is a story about him: when we entered the university on 1 September, he came into the room and said, "Let's continue." And that was the beginning of his lecture course.

In algebra we had lectures by Dynkin, who was an excellent teacher for those who had started to study. These were lectures at a very high level. Dynkin used to hand out interesting problems for the enthusiastic students. Among such students in my year I could particularly mention I. Girsanov, who became a famous probabilist, and L. Seregin.

Raussen and Skau: *Was it Dynkin who inspired your first paper in mathematics?*

Professor Sinai: Yes. I was a student of Dynkin during the second and third years and I wrote the first paper under his supervision. I solved a problem that he formulated for me; this became my first published paper when I was a student in the third year. I loved the work I did and still do.

Dynkin wanted me to work on problems on Markov processes in the style of Feller. The papers by Feller became very popular in Moscow at that time, and Dynkin suggested that I should continue along this line. However, I was not very excited and interested in it.

Raussen and Skau: *To what extent were mathematics and mechanics integrated in the curriculum?*

Professor Sinai: These were independent parts of the curriculum. Everybody could attend lectures within each branch. I was attending lectures in mathematics and mechanics but also, to a minor degree, some lectures in physics. But on the whole it was mainly in the mathematics department.

Raussen and Skau: *We imagine that besides Dynkin, Kolmogorov and Gelfand must have been very important figures for you?*

Professor Sinai: Kolmogorov had many students, and I became one of them. His students had complete freedom to work on any problem. Kolmogorov loved to discuss their results. There were several cases when Kolmogorov wrote their papers in order to teach them how to write mathematical texts.

Kolmogorov organized a seminar, which was initially a seminar on random processes and later became a seminar on dynamical systems and ergodic theory. I began to attend, together with other mathematicians like Arnold, Alekseev, Tikhomirov, and others. Later I became a student of Kolmogorov. At that time he was also interested in problems of entropy in different compact and functional spaces. Questions of this type were very much discussed at that time.

Raussen and Skau: *But Gelfand tried to recruit you as a graduate student as well?*

Professor Sinai: Yes. Gelfand organized his famous seminar, which was attended by many mathematicians of different generations. I took part in it for many years. It happened, if I remember it correctly, in 1955 when Gelfand was writing a famous volume in his series of books on distributions. Gelfand was interested in probability theory and he wanted me to become his student. We had some discussions about it, and I told him that I wanted very much to work on problems related to probability theory but I had already written a paper under the supervision of Dynkin. He asked me, "Do you want to have probability theory as an appetizer or as a dessert?" I answered, "I want it as a main course." That was the end of the story....

This did not mean that our contact came to an end; we met many times, especially when he worked on problems in representation theory, which were connected with problems in ergodic theory, like the theory of horocycles and others. We discussed this many times. I attended Gelfand's seminars for many years, because Gelfand had the remarkable ability to explain difficult mathematical topics in a clear and simple way.

Dynamical Systems. Entropy and Chaos

Raussen and Skau: *Could you explain what a dynamical system is?*

Professor Sinai: We understand dynamical systems as objects that describe all types of evolution. The most interesting case is nonlinear dynamical systems, when the formulas for the dynamics of the evolution are nonlinear. There can be many different phenomena which require deep analysis.

Raussen and Skau: *And among these dynamical systems, what is an ergodic system?*

Professor Sinai: I have a very good example for an ergodic system which I always explain to my students. Suppose you want to buy a pair of shoes and you live in a house that has a shoe store. There are two different strategies: one is that you go to the store in your house every day to check out the shoes and eventually you find the best pair; another is to take your car and to spend a whole day searching for footwear all over town to find a place where they have the best shoes and you buy them immediately. The system is ergodic if the result of these two strategies is the same. The entropy characterizes the growth of the number of possibilities in dynamics. I heard the first explanation of this role of entropy from I. M. Gelfand.

Raussen and Skau: *Ergodic theory originally came from physics, in particular from the study of Hamiltonian equations. Can you explain in general terms what chaos is and how one can measure it?*

Professor Sinai: This is the subject of my lecture, which I will give the day after tomorrow, but I can summarize it briefly here. The main question concerns the difference between chaos theory and probability theory. In probability theory one deals with statistical experiments: say you toss a coin 100 times. One can have many different series resulting from this experiment and study the result.

If you consider the problem of chaos and, for example, want to measure the temperature at the same point you make the measurement during the year, you now have only one realization of the temperature. You cannot have a hundred realizations of the temperature at a given place and at a given time. So the theory of chaos studies the series when the results of measurements have a limit as time tends to infinity and how to describe this limit. The existence of the limit actually follows from some hypothesis about the equations of motion. This gives the existence of the distribution, which determines the value of all kinds of averages (or, it is better to say, the existence of the averages and also finding their values).

Then, the question is: What are the equations of motion which determine the distribution and these averages? The basic statement in chaos theory is that the dynamics must be unstable. Instability means that small perturbations of the initial conditions lead to large perturbations in the dynamics after some time.

Then there is a mathematical theory that says that if the system is unstable, the time averages exist and there is a possibility of calculating them. This is the general description of what is done in chaos. A more precise description requires more mathematics.

Raussen and Skau: *How do you measure chaos? Does entropy come into the picture here?*

Professor Sinai: If we understand chaos as mentioned already, i.e., as the existence of time averages and also properties related to mixing, then there is a natural description of chaos in terms of some special distributions. Entropy is used in the theory of unstable systems and it characterizes how many types of dynamics a given system can have. It is certainly a very useful notion, because the positivity of the entropy determines other properties of the systems that can be studied.

Physicists always expressed their hope that entropy would allow them to understand turbulence (see, for example, the paper by B. Chirikov and the books by A. Zaslavski, R. Sagdeev, and others). It is hard to say that this hypothesis is true. On the other hand, there are many situations in physics where systems have small entropy.

Definition of Entropy for Dynamical Systems

Raussen and Skau: *Kolmogorov had come up with the definition of entropy for Bernoulli shifts, but then he changed it to a definition that was not invariant. Then you came with the correct definition. What is now called the Kolmogorov-Sinai theorem gives an efficient way to compute the entropy.*

Professor Sinai: Kolmogorov started his seminar with von Neumann's theory of dynamical systems with pure point spectrum, which he explained in a purely probabilistic way. Later I found this approach in the book by Blanc-Lapierre and Fortet. Everything in Kolmogorov's seminars was very exciting. At that time we believed that the main problem in ergodic theory was to extend the theory of von Neumann to systems with continuous spectrum that can be constructed in terms of the second homology group of the spectrum with coefficients in the ring of bounded operators. It did not work, but the idea remained.

At that time, Kolmogorov spent his time primarily on problems in information theory and the concept of dimension of linear spaces. I do not know how it happened, but one day Kolmogorov came to his lecture and presented his definition of entropy. Using modern terminology, one can say that he gave the definition of entropy for Bernoulli shifts and thus proposed a new invariant for this class of dynamical systems. It was certainly a great result. Kolmogorov wrote his text. He submitted it for publication and left for Paris where he spent the whole semester. As is known, the text that was submitted for publication was different from what he explained in class. In his paper he introduced a new class of systems which he called quasi-regular. Later they were called K-systems (K for Kolmogorov). For this class of systems he introduced the notion of entropy. While Kolmogorov was away, I was thinking about a definition of entropy that could work for all dynamical systems. Later it appeared in my paper on entropy.

At that time, there was a clear feeling that for dynamical systems appearing in probability theory, the entropy is usually positive, while for dynamical systems generated by ODEs it should be zero. Thus, there seemed to be a possibility to distinguish dynamical systems in probability theory from dynamical systems in analysis.

Raussen and Skau: *How about your connection with Rokhlin?*

Professor Sinai: The story about my connection with Rokhlin, who later became a close friend of mine, started when Kolmogorov's paper on entropy appeared in 1958. At that time, Rokhlin lived in a small provincial town, Kolomna, not far from Moscow. He had a very good graduate student Leonid Abramov. There are several general theorems that were proven by Abramov, like the entropy of special flows and other things like Abramov's formula, etc. When Rokhlin heard about the paper by Kolmogorov, he sent Abramov to Moscow to find out what had really been done, what was the situation, and if possible to bring the text.

When Abramov came to Moscow, he found me; we talked a lot, and I taught him everything I knew. Abramov then invited me to Kolomna to talk to Rokhlin, and I accepted the invitation. I remember my first visit to Kolomna very well. Rokhlin had an apartment there, which was very orderly; everything was very accurate, and he was dressed very well. We began to talk, and he made a very strong impression on me.

Rokhlin had great experience in ergodic theory because he had already published several papers in this field. His doctoral thesis was also about this subject. Rokhlin formulated a number of interesting problems in ergodic theory. Some of them were connected with Rokhlin's theory of measurable partitions. This theory became very useful in ergodic theory, because through it one can understand conditional probabilities in probability theory much better.

One of the problems that I began to work on under the influence of Rokhlin was the calculation of entropy for group automorphisms of the two-dimensional torus. At that time it was not known that Kolmogorov's definition had to be modified; the analysis was rather difficult, and I could not achieve anything. Following the ideology of that time, I tried to prove that the entropy was zero, but all my attempts failed. Then I visited Kolmogorov and showed him my drawings. He said that it was clear in this case that the entropy must be positive. After that I proved the result.

At that time there was no question about publication of my paper, because Kolmogorov's paper on entropy had been published and it was not clear why another definition of entropy was needed. However, after some time, Rokhlin pointed out his result about the deficiencies in the definition by Kolmogorov. It became clear that I had to publish my paper with the definition and the calculation of the entropy for the automorphism which I had already done at that time.

This was the beginning of my contact with Rokhlin. After that he organized a seminar on ergodic theory in Moscow, which was attended by Arnold, Anosov, Alexeev, and others. In parallel, he had a seminar in topology where Novikov was the central figure.

Later Rokhlin moved to Leningrad (Saint Petersburg), and I used to go there to give talks at his seminar on later results.

Billiard Systems

Raussen and Skau: *You then came up with an extremely interesting example of an ergodic system, the so-called billiards. Can you explain what these are?*

Professor Sinai: A billiard, as people know, is the motion of a ball on the billiard table. An interesting mathematical theory arises if you allow the table to have a more or less arbitrary form. A natural question, which was actually raised by the Russian physicist Krylov long before the theory of entropy appeared, was: Which billiard systems have the same instability as the dynamics of particles moving in a space of negative curvature? Particles moving in a space with negative curvature yield the best example of unstable systems. The theory of billiards says that if the boundary of the table is concave, then the system is unstable in the sense we previously described. If we consider two initial conditions with different values of the velocities, then the corresponding trajectories diverge exponentially. If you consider a typical unstable billiard, namely the usual square billiard with a circle removed, then the difference between this billiard and the usual billiard is that for the unstable billiard the particles come to the holes much faster than for the usual billiard.

Raussen and Skau: *This may become a little technical now. You proved a very important result about systems with positive entropy. Given a system with positive entropy, you can find a Bernoulli shift, which is a so-called factor, with the same entropy. This implies that if you have two Bernoulli shifts with the same entropy, they are what is called weakly isomorphic. Ornstein proved later that entropy is a complete invariant for Bernoulli shifts. It follows then from the work of Ornstein that the billiard example is the most chaotic system and is actually a Bernoulli flow, right?*

Professor Sinai: From Ornstein's theorem it follows that if we have two ergodic billiard systems with the same value of entropy, then they are isomorphic. This is a remarkable and great result.

Raussen and Skau: *So coin tossing is, in a sense, similar to the deterministic billiard system—an amazing fact.*

Professor Sinai: My result says that if you have a system with positive entropy, there could be subsystems that move like Bernoulli shifts.

Raussen and Skau: *What about billiard systems in higher dimensions? Is anything known there?*

Professor Sinai: A lot of things are known. We have, for example, the result from the Hungarian mathematician Nåndor Simånyi, who is in Alabama now. He studied multidimensional dynamical systems that eventually become unstable and have positive entropy and are ergodic.

Raussen and Skau: *You introduced Markov partitions in your study of Anosov diffeomorphisms. This led to what later became known as the Sinai-Ruelle-Bowen measure, also referred to as the SRB-measure. Would you please explain?*

Professor Sinai: First of all, there was my paper where I constructed this measure for the case of the so-called Anosov systems, or just hyperbolic systems. Then there was a paper by Bowen and Ruelle where they extended this construction to systems considered by Smale, that is, Axiom A systems with hyperbolic behavior.

These measures are important if you study irreversible processes in these systems. Suppose you start with some nonequilibrium distribution and consider the evolution and you ask how a nonequilibrium distribution converges to the equilibrium one. The result of the theory says that the evolution is in a sense very nonuniform, along some directions the expansion is very small and all the time averages behave very well and converge to a limit. But along other directions this convergence is very erratic, and hence it can only be studied using probability theory. So the measures, which are called SRB-measures, are the ones which are smooth along some unstable directions and are very irregular along other directions. This is a class of measures that appears in the theory of evolution of distributions in the case of chaotic systems.

Raussen and Skau: *Are the SRB-measures related to Gibbs measures?*

Professor Sinai: Yes. These measures are examples of Gibbs measures, but the Gibbs measures are much more general objects.

Mathematics and Physics

Raussen and Skau: *Let's go back to more general questions, starting with the interplay between mathematics and physics. May we begin with the physicist Eugene Wigner, who in 1960 published the paper "The unreasonable effectiveness of mathematics in the natural sciences," in which he gave many examples showing how mathematical formalism advanced physical theory to an extent that was truly amazing? Do you have a similar experience?*

Professor Sinai: My impression is that this effectiveness of mathematics is no longer a surprise for people. There are so many cases, for instance, the fact that string theory is practically a mathematical theory for physics. Some time ago Joel Lebowitz organized a discussion about this phrase of Wigner, in particular, how it can be that mathematics is so effective. The conclusion was that this is just a well-established fact.

In my generation, there was a group of young mathematicians who decided to study physics seriously. However, there were different points of view of how to do mathematical physics. F. A. Berezin always stressed that mathematicians should prove only results that are interesting for physicists. R. L. Dobrushin and I always tried to find in physical results some possibilities for mathematical research.

Raussen and Skau: *On the other hand, there seems to be influence going in the opposite direction. Physicists have had a noteworthy impact on questions in quantum geometry and sometimes even in number theory. They have come up with formalisms that were not really developed in mathematics but nevertheless led to correct predictions which could be verified only after lengthy mathematical development.*

Professor Sinai: So mathematics is effective, but you can say that it is not effective enough.

Raussen and Skau: *You published in 2006 an article with the title "Mathematicians and Physicists = Cats and Dogs?" What is the main message of that paper?*

Professor Sinai: I wanted to show examples where mathematicians and physicists look at the same problems differently. One example for this is the following story: my student Pirogov and I worked on problems in the theory of phase transitions in statistical physics. We proved several theorems, and I went to meet the famous Russian physicist Ilya Lifshitz to show him our theory; Lifshitz replaced Lev Landau when Landau had his car accident, severely incapacitating him. When I presented the theory he stopped me and said, "It's very simple what you are talking about." He started to write formulas which eventually gave our results. I left him very much embarrassed and I started to think why this had happened. I realized that the final result of our theory was an obvious statement for him. He certainly did not know how to prove it, but he did not need the proof. He just used it as an obvious fact.

Raussen and Skau: *There is a famous quotation of the great Gauss: "Now I have the result. The only thing remaining is the proof." So intuition does play an important role in mathematics…*

Professor Sinai: I can also tell the following story, again connected to Gelfand. I explained to him a theorem, which we obtained together with Robert Minlos. And Gelfand said, "This is obvious. All physicists know this." So we asked him if it was so obvious, should we write a text of 200 pages with complete proofs? He looked at us and said, "Certainly, yes!"

A Jewish Mathematician in the Soviet Union

Raussen and Skau: *May we continue with a political question? You mentioned that being at school in the time of Stalin was not easy; life was still difficult for you when you entered university and started your career. You came from a Jewish family; in the Soviet Union, at least sometimes, a latent anti-Semitism prevailed.…*

Professor Sinai: I can mention two cases in my career when I encountered anti-Semitism. The first one was the entrance examination, which I failed. The influence of my grandfather, who was head of the Chair of Differential Geometry, and the help from the President of Moscow University, I.G. Petrovsky, were needed in order to give me the possibility of being admitted to the University. This was a clear sign that things were not simple.

The other case arose with my entrance examination to graduate school. This exam was about the history of the Communist Party; I was very bad in this topic and failed the exam (I don't want to discuss the details). But P. S. Alexandrov, who was head of the mathematical department at Moscow State University, together with Kolmogorov, visited the head of the Chair of the History of the Party and asked her to allow me to have another attempt. She gave permission, and I got a B on the second attempt, which was enough to enter the graduate school. The result was not clear a priori, and it could have gone either way.

Raussen and Skau: *In spite of these obstructions, it is quite obvious that many famous Russian mathematicians were and are of Jewish origin. This is quite amazing. Can you offer any explanation?*

Professor Sinai: First something trivial: Jews had more traditions in learning than other nations. They study the Bible, the Talmud, and other

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From left to right: Yakov Sinai, Martin Raussen, Christian Skau.

religious books and spend a lot of time doing this, which is conducive to learning. At that time, following the Jewish religion was strictly forbidden. People still did, however, but under very high pressure. If you do something under pressure, you work more. There is some kind of conservation law. This is my opinion of why Jews could succeed.

Raussen and Skau: *You had to be much better in order to get the same opportunities?*

Professor Sinai: I think it would be wrong to say that we had this feeling. We certainly tried to prepare for all exams and competitions. The result was not clear a priori, but there was always a hope that something could come out of it.

Raussen and Skau: *Perhaps another reason is (especially under Stalin but also later) that a lot of very intelligent people were attracted to the natural sciences because there were fewer restrictions than in, say, history or political science.…*

Professor Sinai: That is certainly true. I can give you one example: at the time, Mech-Mat, the Faculty of Mechanics and Mathematics, had many graduate students that came from other countries. The rule was that they could only have advisors who were members of the Party. But there were students who wanted to work with Arnold, with me, or perhaps with some other people. The way out of this situation was the following: there were a number of people in the Party who became the students' official advisors, but the students actually worked with professors and mathematicians who were not members of the Party.

East and West

Raussen and Skau: *You told us that you were not allowed to travel for many years, and this happened to a lot of Russian mathematicians at the time. Did these obstructions hamper or delay progress in science? Did it have the effect that Russian mathematics did not get recognition in the West that it deserved?*

Professor Sinai: It is very difficult to answer your question, because you are asking what would have happened if something didn't happen. It is just impossible to say. It certainly caused harm, but it is not clear how big it was.

Raussen and Skau: *Arnold was rather adamant about the lack of recognition. As a consequence of bad communication between East and West, results by Russian mathematicians during the isolation period were sometimes later rediscovered in the West. Therefore Russian mathematicians did not get the credit they deserved.*

Professor Sinai: I have perhaps a special point of view concerning this. The question is whether some results can be stolen or not. My point of view, to which many people probably won't agree, is that if a result can be stolen, it is not a very good result.

Raussen and Skau: *Tell us about the Landau Institute for Theoretical Physics in the Russian Academy of Sciences, your workplace for many years.*

Professor Sinai: For many years the Landau Institute was the best institution in Russia. It was organized after Lev Landau's untimely death as a result of a car accident. Its director, I. M. Khalatnikov, had a remarkable talent to find gifted people all over Russia and to invite them to the institute. After several years the Landau Institute had a very strong group of physicists like Abrikosov, Gorkov, Dzyaloshinski, A. B. Migdal, Larkin, Zakharov, Polyakov, A. A. Migdal, and many others. The group of mathematical physicists was headed by S. P. Novikov and was much smaller.

It turned out that there was a big area of theoretical physics in which mathematicians and physicists could understand each other very well. They could even work on similar problems. Among these mathematicians I can name Novikov, Krichever, Khanin, Shabat, and Bogoyavlenskiy. Sometimes we invited physicists to explain to us their results in our seminars. The tradition of discussing problems of mutual interest still prevails.

Raussen and Skau: *You moved in 1993 from the Landau Institute to Princeton University while still maintaining your position in Moscow. Why was it so attractive for you to go to the USA?*

Professor Sinai: That is an easy question. First of all, I had many friends at Princeton. When we met we always had many points for discussion and common interests. Another reason was that many people had escaped from Russia so the situation there was no longer what it was before. In previous times everybody was in Moscow and Saint Petersburg, and you could call everyone to ask questions or to have discussions. Now that became impossible. The working conditions were better in the West and in particular at Princeton.

Raussen and Skau: *You have now been in the USA for more than twenty years and you must know the American system almost as well as the Russian one. Could you tell us about how they compare from your perspective?*

Professor Sinai: Concerning academic lives, it seems to me that they are more or less similar. However, I must stress that I was never a member of any scientific committee at Mech-Mat at Moscow State University, and I was never invited to participate in any organizational meetings. Now I am chairman of the Scientific Council at the Institute of Transmission of Information.

Teaching and Collaboration

Raussen and Skau: *You have been teaching courses and seminars for almost all your career. Do you have a particular technique or philosophy?*

Professor Sinai: First of all, I like to teach undergraduate courses rather than graduate courses for the following reason: when you teach undergraduate courses you can easily see how your students become cleverer and more educated as they absorb new notions and connections and so on. When you teach graduate courses, the subject matter is usually a narrow piece of work, and students are mostly interested in some special issues that are needed for their theses. For me, that is less attractive.

My basic principle is as follows: if people do not understand my explanations, then this is my fault. I always ask students to ask questions as much as possible. Students who have asked me many questions during the lecture course have better chances for a good mark.

Raussen and Skau: *You have an impressive list of students that have done well after graduation under your supervision. Grigoriy Margulis, just to mention one name, won the Fields Medal in 1978, and he will give one of the Abel lectures related to your work later this week.…*

Professor Sinai: I think the reason for this is not because of me but because of the types of problems we worked on. We did very interesting mathematics and formulated interesting problems that students were attracted to. This is my explanation. Many students preferred to work independently, and I was never against this.

Raussen and Skau: *You are a very good example of the fact that mathematicians can flourish in late age as well. We came across a paper on number theory that you published this year together with two of your students. You have also published other papers related to number theory, so you must have kept an interest in that aspect of ergodic theory?*

Professor Sinai: Yes, definitely. In the field we are working in there are many problems that are more natural for ergodic theory than for number theory. I don't want to be specific, but we had a paper that was more natural for an ergodic theorist than for a number theorist, so we could get the results more easily.

Raussen and Skau: *Many joint papers appear on your list of publications. Apparently you like to have a lot of collaborators.*

Professor Sinai: Well, I would say that they like it! And I'm not against it. It has never happened that I have asked someone to be my coauthor. I can only talk about some problems and explain why they are interesting.

But you are right, I have had many coauthors. I very much liked collaborating with Dong Li, who is now a professor at the University of British Columbia. When we work on the same problem we call each other many times a day. There are many others of my students with whom I liked to work. It's different to work with different people. Certainly I can work with Russian mathematicians as well as with mathematicians from other countries. Sometimes I like to work alone, but with age I need coauthors.

Raussen and Skau: *You have only published one paper with Kolmogorov, but you have mentioned that you would have liked to publish more papers with him.*

Professor Sinai: At a certain time, Kolmogorov decided that the Soviet Union did not have enough applied statistics. He worked on theoretical statistics and found many beautiful and deep results, but he was not satisfied with the fact that the theorems in applied statistics were not used for practical purposes. He found a problem related to the motion of the rotational axis of the earth that could be studied with the help of mathematical statistics. French observatories published data about the axis of rotation every two weeks, and Kolmogorov wanted to construct statistical criteria that could predict this motion. He wanted us to work on this problem and invited a very good geophysicist, Yevgeny Fyodorov, who was one of the main experts in this field. We were sitting there; Kolmogorov and Fyodorov were present. Kolmogorov said, "Look at these people; they prefer to write a paper for *Doklady* instead of doing something useful" (*Doklady* was the leading Russian journal). In our joint paper (by M. Arató, A. Kolmogorov, and me) written on this occasion, practically everything was done and written by Kolmogorov. Later, M. Arató wrote a big monograph on that subject.

In other cases, I often tried to explain my latest results to Kolmogorov. Sometimes his reaction was unexpected: "Why did you work on that problem? You are already a grown-up." But usually his reaction was very friendly. I regret very much that we never worked together; perhaps the reason is a difference in style.

Raussen and Skau: *Wasn't it Kolmogorov who said that he spent a maximum of two weeks on a problem?*

Professor Sinai: Kolmogorov used to stress that he did not have papers on which he worked for a long time. He mostly prepared his papers, including the proof and the text, in just two weeks, and this was a major difference in our approaches. Kolmogorov was a person with a strong temperament and he could not do anything slowly. I worked on some of my papers for years.

Raussen and Skau: *He was a towering figure, not only in Russian mathematics but worldwide in the twentieth century.*

Professor Sinai: Yes, definitely. Can I tell you one more story about him? It was when Kolmogorov was close to eighty. I asked him how it happened that he was a pure mathematician, even though he worked on concrete physical problems like turbulence. He answered that he was studying the results of concrete experiments. He had a lot of papers with results from experiments lying on the floor. He was studying them, and in this way he came up with his hypotheses on turbulence.

Raussen and Skau: *So his intuition was motivated by physical considerations?*

Professor Sinai: Yes. He subscribed to physical journals, and one could say he was into physics in a big way.

Raussen and Skau: *Is that also true for you? Do you think mainly in terms of algebraic or analytic formulas? Or is it geometric intuition or even a mixture of all of that?*

Professor Sinai: It depends on the problem. I can come to the conclusion that some problem must have a specific answer. I just told a journalist the story about a problem in which I knew there should be a definite answer. I worked on this problem for two years, and at the end of that time I discovered that the answer was one-half!

In general, I probably prefer to develop theories, sometimes to find the right concepts, rather than [to solve] specific problems.

Raussen and Skau: *Have you had what we sometimes call a Poincaré moment, where all of sudden you see the proof?*

Professor Sinai: Ideas often come unexpectedly, sometimes like revelations. But it happens only after a long period, maybe years, of difficult work. It did not happen while trying to find a taxi or something similar. It was very hard work for a long time, but then suddenly there was a moment where it became clear how the problem could be solved.

Raussen and Skau: *If you yourself made a list of the results that you are most proud of, what would it look like?*

Professor Sinai: I like all of them.

Mountaineering

Raussen and Skau: *You mentioned Arnold, who died four years ago, an absolutely brilliant Russian mathematician. Arnold is, among many other things, known for his contributions to the so-called KAM theory. You both followed Kolmogorov's course and seminars in 1958. You told us that there was a close friendship already between your grandfathers.*

Both you and Arnold loved the outdoors and hiking. You once went to the Caucasus Mountains together, and you have to tell us the story about what happened when you stayed in the tents with the shepherds.

Professor Sinai: That is a very funny story. The weather was very bad; there was a lot of rain. We came to the shepherds' tent, and they let us in, and we could dry our clothes. We had lost our tent in the mountains, so we decided to go back to try to find it. We started to walk back, but these shepherds had some very big dogs—Caucasus dogs, a really big race. The shepherds weren't there any longer, and when the dogs found out that we were leaving, they surrounded us and started to bark ferociously. Arnold began to yell back with all the obscenities he knew, and the dogs did not touch him. But they attacked me. They didn't touch my skin, but they ripped my trousers apart. Finally, the shepherds came back and we were saved.

Raussen and Skau: *We would like to ask one final question that has nothing to do with mathematics. You have certainly focused on mathematics during your life, but surely you have developed other interests also?*

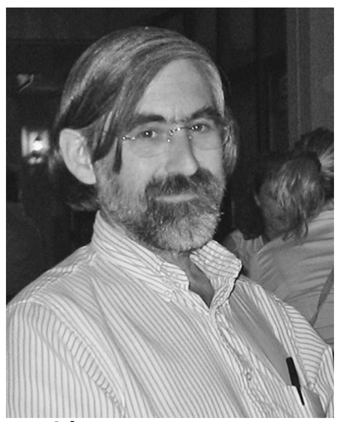
Professor Sinai: I was interested, especially in former years, in many different sports. I was a volleyball player and I liked to ski, both downhill and cross-country. I also liked mountaineering, but I cannot say I was a professional. I climbed often with a close friend of mine, Zakharov, who worked on integrable systems. We were climbing in the mountains together, and once we were on a very difficult 300-meter-long slope, which took us four hours to get down from! We had to use ropes and all sorts of gear. Nowadays, my possibilities are more limited.

Raussen and Skau: *Thank you very much for this most interesting conversation. We would like to thank you on behalf of the Norwegian, the Danish, and the European Mathematical Societies.*

Professor Sinai: Thank you very much.

Presidential Views: Interview with David Vogan

Every other year, when a new AMS president takes office, the *Notices* publishes interviews with the outgoing and incoming presidents. What follows is an edited version of an interview with David Vogan, whose two-year term as president ends on January 31, 2015. Vogan is the Norbert Wiener Professor of Mathematics at the Massachusetts Institute of Technology. The interview was conducted in fall 2014 by *Notices* senior writer and deputy editor Allyn Jackson.
An interview with president-elect Robert L. Bryant will appear in the March 2015 issue of the *Notices*.


David Vogan

Notices: *The first thing I would like to ask about is the PCAST report* Engage to Excel.[1] *Reactions to that report started bubbling up while you were still president-elect. You and several other mathematicians wrote a statement that appeared in the* Notices.[2] *Could you tell me your observations of the math community's reactions to the PCAST report?*

Vogan: The PCAST report was a very nice example of how strongly and wonderfully the mathematical community can react. I am probably going to be unfair to many other people if I say that my recollection is that Tara Holm, chair of the AMS Committee on Education, was really a driving force behind the statement that appeared in the *Notices*, which was a very reasonable reaction.

It seemed to a lot of mathematicians that the most important misunderstanding in the PCAST report was the idea that mathematicians didn't care about the obvious problems of students having difficulty with introductory math courses and that mathematicians had no idea about what to do about these problems. In fact, in some sense, almost every mathematics department is doing something to address these issues. So the first big reaction was just to try to point to the fact that there were a lot of good things going on already and then to try to make some of those good things available widely across the community.

Maybe one problem is that some of the best things that are going on are hard to learn about and hard to copy. Traditionally, people who were really good at teaching wrote a textbook. Then anybody could use, say, Nathan Jacobson's graduate algebra text, and Jacobson's understanding of how you should explain that kind of algebra became widespread. This was a wonderful system. A lot of the changes that are being developed now might have less to do with the content of what's being taught and more to do with the form and with the way things happen in a classroom. It is certainly more difficult to put those things in a textbook. The result is that we haven't understood well how to pass them around. So there is a lot of activity now trying to address that—to see how the people who have figured out great things about teaching at an elementary level can share what they do.

I should certainly mention the group that Eric Friedlander, Phillip Griffiths, and Mark Green started, called TPSE, Transforming Post Secondary Education in Mathematics. This also is aimed at highlighting new ideas about teaching mathematics and implementing them widely.

For many years the Committee on Education has had during its annual meeting an event for department chairs that offers interesting and useful information about math education. This year, Tara

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Allyn Jackson is senior writer and deputy editor of the Notices. *Her email address is* `axj@ams.org`.

[1] Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics, *a report by the President's Council of Advisors on Science and Technology, February 2012. See also "Presidential Report Draws Criticism from Mathematicians," by Allyn Jackson,* Notices, *October 2012.*

[2] *"Mathematicians' Central Role in Educating the STEM Workforce", by Eric M. Friedlander, Tara S. Holm, John Ewing, Rebecca Goldin, William H. Jaco, T. Christine Stevens, Abigail Thompson, and David A. Vogan Jr.,* Notices, *October 2012.*

DOI: http://dx.doi.org/10.1090/noti1215

Holm organized a meeting that had presentations by seven or eight speakers who talked about things that they were doing to address issues in beginning mathematics education. There was a huge amount of discussion: What can I do in my department? How do I go about taking advantage of these ideas? There have been panels at the Joint Math Meetings of a similar nature, also very well attended and inspiring a lot of discussion.

Notices: *During your presidency, the AMS started two new open access journals: the new series of the* Transactions *and the* Proceedings*. What are your observations about how that is going?*

Vogan: It went very smoothly. The only thing that has been disappointing about the journals is the level of response from authors. There is quite a bit of funding available in universities to support page charges in open access journals, because a lot of libraries see this as a reasonable thing for their budgets. The AMS hoped that with a new venue and the possibility of slightly improved publication times, people might be willing to make the extra effort to find out whether they had access to these resources. But not many have made this effort. The first time you try to figure out how to get your university to produce money for such page charges, it's a pain and takes time, but every other time you do it, it's easy. So I hope that the number of people who are able to take advantage of this will grow.

There was also a concern at the time the journals were launched that the UK was considering policies that could make open access publishing a requirement for UK-funded research. There was concern that such policies could be implemented in the US and elsewhere. If this kind of open access requirement goes into place anywhere, we are now at least in a position to continue publishing work from those places. The size of the journals is so far a bit of a disappointment, but I think that the reasons for setting up the option were sound and that it was done very well.

Notices: *Any other observations on the publishing front from your time as president?*

Vogan: I think the staff and the volunteer structure in the AMS are fairly united in thinking that the AMS should publish more mathematics, especially more research journal pages. There isn't a clear consensus about how to do this, but I think many people are coming to like the idea of having another large general research journal, with no restriction on field. That's not something that is going to happen next week or next year, but I think that the Committee on Publications is very amenable to such an idea and is starting to think about what would be involved in making it happen. People like the way the AMS does journal publishing. The prices are good. The backlogs are a pain, but the backlogs are there because there is more good mathematics to publish than we currently have the capacity to handle. Apart from the backlogs, people are quite happy with the way the AMS does publishing.

Notices: *The* Notices *has hosted a discussion of the revelations of Edward Snowden about the NSA [National Security Agency]. Can you tell me about how you see mathematicians' reactions to this issue? A reading on how the math community thinks about it?*

Vogan: That's a really tough question. What I have come to understand is that a very large fraction of the math community sees these questions very differently from how I do. I have tried to understand what the general views are, but it's been a very difficult process for me. What I think is that a lot of mathematicians see the work that's done by mathematicians at the NSA as building absolutely necessary tools; maybe sometimes these tools have been abused, as almost any tools can be, but that doesn't affect the need to build the tools. I think most mathematicians view the work of building those tools as honorable and important mathematical work.

Notices: *Not so many people seem interested in participating in the* Notices *discussion. I have the impression people are not too concerned.*

Vogan: I think they are not concerned about the mathematical aspects of the question. People may have issues with the way the executive branch [of the US government], for the past ten years at least, has been using these tools. But they don't have issues with the mathematical work underneath. I really liked Tom Hales's article in the *Notices.*[3] That article described mathematical work that should never have happened. He described work that was done using, not incredibly deep mathematics, but nevertheless using mathematics in a way that was just a mistake. There wasn't a justification for doing it. Hales made it clear that it has not been proven that NSA did what he described as a possibility. But what he described seemed to me to be using mathematics to lie and cheat, to make people believe things that were not true. And I don't see that as an acceptable way to use mathematics.

Notices: *One thing we talked about in the previous presidential interview is the future of the AMS membership. We talked about how the AMS can*

[3] *"The NSA Back Door to NIST," by Thomas C. Hales,* Notices, *February 2014.*

reach out to young people and encourage them to become members and participate in the Society. What are your thoughts on this two years later?

Vogan: It's still a serious issue. I am quite happy that it is one of the central themes of the strategic planning process that the AMS is now undertaking.

In getting ready for this interview, I looked at agendas for various committee meetings. One of the things in a bunch of the agendas is: How do we make the lectures at AMS meetings more accessible? And, more generally, how do we make mathematics more accessible? How do we make mathematics accessible to other mathematicians, to our students, to the world at large? This is something that is worked on constantly, and it is something that almost all mathematicians care about. Working on those issues in a good way, in a successful way, makes this an attractive organization.

If I open up MathSciNet now, next to any of the articles that are listed, there is a little button that says "Get this at MIT". I'm not completely sure how widespread these buttons are, but they are spreading, and they are making MathSciNet an even more powerful and useful tool. That's one big change. There have been many changes in MathSciNet. MathSciNet is a work in progress and gets better all the time. I think if we make tools like that, if we continue to make them more useful for all mathematicians, and in particular young people, then that's an aspect of making the organization more appealing.

Notices: *Is there anything else you wanted to talk about that we didn't cover or anything that happened during your presidency that you wanted to comment on?*

Vogan: I really liked a couple of transitions. The Membership and Professional Services Department at the AMS has in my mind been identified with Ellen Maycock for a very long time. She was what made that department a wonderful thing. Last summer, she began the process of retiring, and Chris Stevens stepped into that job. The department is going to be a little bit different, because Chris Stevens is different from Ellen Maycock, but it's going to continue to be wonderful. It's a great thing that this institution has continuity beyond the individuals that make it up. Also at *Math Reviews*, executive editor Graeme Fairweather retired at the beginning of June, and it was really unclear how we were going to find somebody to take over that enormous and really difficult job. And now Ed Dunne [formerly an AMS acquisitions editor] is doing it. There again, things are going to look different, and they are going to look great. Next February I am not going to be the president of the AMS; a wonderful, experienced new person is coming in. Things are going to be different, and they are going to be better.

Notices: *What was the most important thing you learned in your time as AMS president?*

Vogan: [laughs] Probably Sheila Rowland's email address [Sheila Rowland is a longtime AMS staffer in the Executive Director Department]. The staff of the AMS is just a fantastic thing. I admire the staff in a lot of math departments; they are fantastic and indispensable. But what goes on at the AMS is a whole different thing. From Don McClure and Sheila Rowland to Ed Dunne and Robin Marek [AMS development director] and Sandy Frost [now retired *Notices* managing editor]—every name that comes into my head makes me smile and makes me think about the way these people do the work of supporting mathematics. It's a fantastic thing.

The Mathematics Community and the NSA

POST-PUBLICATION EDITOR'S NOTE: This article is a part of the ongoing series "Mathematicians Discuss the Snowden Revelations". At the time of the writing of this piece Michael Wertheimer was the Director of Research at the NSA; he recently retired from that position. He can be reached at nsapao@nsa.gov.

This is the latest installment in the *Notices* discussion of the National Security Agency (NSA). Previous *Notices* pieces on this topic are:

"AMS Should Sever Ties with the NSA" (Letter to the Editor), by Alexander Beilinson (December 2013); "Dear NSA: Long-Term Security Depends on Freedom", by Stefan Forcey (January 2014); "The NSA Backdoor to NIST", by Thomas C. Hales (February 2014); "The NSA: A Betrayal of Trust", by Keith Devlin (June/July 2014); "The Mathematical Community and the National Security Agency", by Andrew Odlyzko (June/July 2014); "NSA and the Snowden Issues", by Richard George (August 2014); "The Danger of Success", by William Binney (September 2014); "Opposing an NSA Boycott" (Letter to the Editor), by Roger Schlafly (November 2014).

See also the Letters to the Editor in this issue.

Unsolicited submissions on this topic are welcome. Inquiries and submissions may be sent to notices-snowden@ams.org. Articles of 800 words or less are preferred. Those of 400 words or less can be considered as Letters to the Editor and should be sent to notices-letters@ams.org.

— *Allyn Jackson*
Notices *Deputy Editor*
axj@ams.org

Encryption and the NSA Role in International Standards

Michael Wertheimer

Michael Wertheimer is Director of Research at the National Security Agency. His email address is mawerth@nsa.gov.

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Over the past several months a discussion about the role of mathematics, mathematicians, and the activities of the National Security Agency has been hosted on the pages of the *Notices*. As an NSA mathematician I would like to provide some context to what has been reported in the press and share with the American Mathematical Society important facts and information. In particular I would like to address two hot-button issues shaping this conversation: "weakening" Internet encryption and impacts of data on privacy.

The US National Institute for Standards and Technology (NIST), the American National Standards Institute (ANSI), the Internet Engineering Task Force (IETF), and the International Standards Organization (ISO) are the four main bodies with which the NSA participates in developing standards for cryptography. NSA has worked with each of these for over twenty-five years. We value and are committed to the important work of these groups in producing secure cryptographic standards that protect global communications. NSA has a long and documented record of providing security enhancements to openly published international standards. However, recently our work has been questioned in several standards that are elliptic curve based, the most significant of which is an NIST-proposed random number generator that I discuss below.

NSA mathematicians remain steadfast in advocating secure international standards. Nevertheless, we are mindful that there has been considerable discussion regarding NIST publication SP 800-90A. This publication is entitled "Recommendation for Random Number Generation Using Deterministic Random Bit Generators" and contains specifications for four pseudorandom number generations for use in cryptographic applications. One of these describes a particular random number generator associated with NSA: the Dual Elliptic Curve Deterministic Random Bit Generator (Dual_EC_DRBG). The discussion centers on NSA's role in the design and advocacy for this algorithm despite a mathematical demonstration of the potential for a trapdoor.

A trapdoor, simply put, is information that allows the inverse of a seemingly one-way function to be computed easily. In other words, compute x from $f(x)$. In cryptographic applications, functions f are specifically designed to make the x to $f(x)$ computation very fast but the inverse computation intractable (hence, the term one-way). If an attacker knows "secret" information about f that allows an inverse to be calculated, the attacker might be able to decrypt messages or, in the case of the Dual_EC_DRBG, predict future outputs.

During the development of the ANSI standard based on the NIST publication, members of X9F1 (the ANSI-approved working group responsible for cryptographic tools) raised concerns about the potential that elliptic curve points used as parameters for the Dual_EC_DRBG could harbor a trapdoor secret known only to, and exploitable only by, the person who generated the points. As a result, the X9F1 committee expanded the standard to include verifiable random point generation. Since the NSA was using the algorithm at the time and had generated elliptic curve points for protecting Department of Defense users, the NSA-generated points were included in the standard. In other words, any implementation that used the NSA-generated points would be deemed compliant. Shortly thereafter, NIST negotiated with ANSI to use the ANSI Random Number Generation Standard as the basis for an NIST Random Number Generation Standard. ANSI also approved submitting a version of this standard to the ISO.

In 2007 several Microsoft researchers, including Microsoft's representative to the ANSI X9F1 committee that created the ANSI version of the standard, raised concerns in a talk at a cryptographic conference about the trapdoor potential in the Dual_EC_DRBG. These concerns were picked up by the media and widely disseminated. NIST and ANSI reviewed this information and elected to retain both the verifiable point generation scheme and the NSA-generated points.

In 2013 the same concerns were again raised and promulgated by the media. This time NSA's actions were portrayed as a subversion of standards. However, the facts remain:

- The Dual_EC_DRBG was one of four random number generators in the NIST standard; it is neither required nor the default.

- The NSA-generated elliptic curve points were necessary for accreditation of the Dual_EC_DRBG but only had to be implemented for actual use in certain DoD applications.

- The trapdoor concerns were openly studied by ANSI X9F1, NIST, and by the public in 2007.

With hindsight, NSA should have ceased supporting the dual EC_DRBG algorithm immediately after security researchers discovered the potential for a trapdoor. In truth, I can think of no better way to describe our failure to drop support for the Dual_EC_DRBG algorithm as anything other than regrettable. The costs to the Defense Department to deploy a new algorithm were not an adequate reason to sustain our support for a questionable algorithm. Indeed, we support NIST's April 2014 decision to remove the algorithm. Furthermore, we realize that our advocacy for the DUAL_EC_DRBG casts suspicion on the broader body of work NSA has done to promote secure standards. Indeed, some colleagues have extrapolated this single action to allege that NSA has a broader agenda to "undermine Internet encryption." A fair reading of our track record speaks otherwise. Nevertheless, we understand that NSA must be much more transparent in its standards work and act according to that transparency. That effort can begin with the AMS now.

NSA strongly endorses the NIST outline for cryptographic standards development, which can be found at `csrc.nist.gov/groups/ST/crypto-review/process.html`. One significant, and correct, change is that all NSA comments will be in writing and published for review. In other words, we will be open and transparent about our cryptographic contributions to standards. In addition, we will publish algorithms before they are considered for standardization to allow more time for public scrutiny (as we did recently with the new SIMON and SPECK algorithms, `eprint.iacr.org/2013/404.pdf`). With these measures in place, even those not disposed to trust NSA's motives can determine for themselves the appropriateness of our submissions, and we will continue to advocate for better security in open-source software, such as Security Enhancements for Linux and Security Enhancements for Android (`selinuxproject.org`).

We hope this open affirmation and our adherence to it will chart a course that all mathematicians will agree is appropriate and correct.

Data and Privacy

NSA mathematicians carry on a long and storied tradition of making and breaking codes and ciphers. Perhaps most celebrated are feats that our forebearers, American and Allied, made in breaking German and Japanese ciphers in World War II. Ironically, less than 5 percent of the encrypted material collected during that war was successfully decrypted, and of that amount only a scant fraction contributed to any sort of measurable action. Such is the nature of intelligence.

Today's communications environment makes 5 percent appear staggeringly large. The simple act of using a particular encryption algorithm no longer identifies the sender or receiver (as the

ENIGMA cipher did in World War II); the variety of protocols, products, and services for secure communications numbers in the thousands; and the ease and frequency of changing identifiable features is unprecedented. To achieve our foreign intelligence mission lawfully and effectively, NSA mathematicians lead efforts that determine how we "filter," "select," and "process" data while continuously verifying that our processes and procedures adhere to all legal and policy regulations.

Filtering algorithms decide what material is defeated, i.e., neither collected nor stored for analysis. Using aggregate numbers, of the exceedingly small proportion of the world's foreign communications we access, NSA algorithms filter out approximately 99.998 percent of the data it sees. The importance of these algorithms cannot be overstated: they form the bulwark of the legal and privacy protections in executing our mission. After the filtering process, surviving data must meet exacting criteria to be "selected" for subsequent processing and analysis. NSA mathematicians are at the forefront in designing the methods by which the selection criteria are expressed. The precision and accuracy of these methods are constantly improving and with those improvements come increased privacy protections.

I am reminded of an event shortly after the 9/11 attacks that may help to impress the importance of getting filtering and selection "right." Soon after allied operations launched in Afghanistan we came into possession of laptops left behind by retreating Taliban combatants. In one case we were able to retrieve an email listing in the customary to/from/subject/date format. There was only one English language email listed. The "to" and "from" addresses were nondescript (later confirmed to be combatants) and the subject line read: CONSOLIDATE YOUR DEBT. It is surely the case that the sender and receiver attempted to avoid allied collection of this operational message by triggering presumed "spam" filters. Indeed, this is exactly how intelligence and counterintelligence work: an escalating series of moves to discover and avoid discovery of information.

Adapting our filters and selectors to stay relevant while always operating within our legal and policy framework can never be perfect—but it is nearly so. Indeed, in a much-publicized account of 2,776 deviations from the rule set in 2012, a full 75 percent of these incidents occurred when an individual roamed from foreign soil to US soil and we failed to catch the fact in real time. The remaining 25 percent, about 700 in total, were human error (e.g., typing mistakes). Put into perspective, the average analyst at NSA makes a compliance mistake once every ten years.

The collection and analysis of data that lie between filtering (what we know we do not want) and selection (what we know we do want) is governed by a complex set of laws, policies, and implementing rules. This type of data, lawfully obtained and properly evaluated, helps us to avoid surprise. It is used to discover new threats, refine both our filters and selectors, and ultimately create a rising tide that lifts our intelligence insights and privacy protections. Mathematicians are leading the way to design and implement the algorithms that create this rising tide. Here we share many common interests with industry: e.g., big data analytics, cloud computing, machine learning, and advanced search. So-called metadata (intelligence information that can be ascertained without examining the actual content of a communication) plays a big role here, as our governing rules generally do not permit deep inspection when the aperture into our data set widens. Getting this right is paramount: the average NSA mathematician takes fourteen courses each year to be up-to-date on the procedures that govern these activities.

Some Parting Thoughts

I fondly recall the opportunity NSA gave me early in my career to return to the University of Pennsylvania and complete my PhD. During those formative years I had many opportunities to present results at AMS conferences, and I remember the warm embrace of colleagues who encouraged and supported my studies. I felt then, and I feel now, a connection to the mathematics community that goes beyond scholarship. That is why NSA Research is a major provider of grants for pure mathematical research, a participant in the National Physical Sciences Consortium, a sponsor of local high school teams for the American Regions Mathematics League, and sponsors of both undergraduate and graduate summer programs. Our research mathematicians serve on editorial boards, publish papers, teach at universities, and contribute time and energy to the AMS.

More broadly, NSA mathematicians are also fighters in the war on international terrorism, weapons of mass destruction proliferation, narcotics trafficking, and piracy. In fact, the overwhelming bulk of what we do is universally acknowledged as proper, measured, and important. We do so quietly and honorably.

It is my sincerest hope that the AMS will always see NSA mathematicians as an important part of its membership. I further hope that dialogue on important issues will always be respectful, informed, and focused on inclusivity.

State of the *Notices*

Steven G. Krantz

During my five years at the helm of the *Notices*, we have enjoyed a healthy flow of articles and a vigorous and constructive interaction with authors. At the beginning of my tenure, I expended considerable effort in finding and nurturing new articles, but I can now say that most of our articles originate through author initiative. We are similarly enjoying a modest but steady flow of Opinion pieces and Letters to the Editor. Clearly the *Notices* is filling an important role as a venue for top-quality mathematics exposition and for discussion of issues facing the mathematics profession.

The articles that we receive exhibit considerable diversity. In addition to articles on education and specialized areas of mathematics, we also receive articles about applications of mathematics to rather surprising areas of human endeavor, we receive articles about the philosophy of mathematics, and we even receive some satirical articles.

In the course of my five-year service, we have created two new columns for the *Notices*. The first of these, entitled *Doceamus* (Latin for "Let us teach"), is a column dedicated to teaching issues. Each column is limited to 1,200 words and is selected to focus on some didactic issue of current interest. So far, over forty *Doceamus* pieces have appeared. The second column that we created is called *Scripta Manent* (Latin for "Written words endure") and is dedicated to issues in publishing. We conceived the column as a place to discuss the rapid and dramatic changes that electronic media are wreaking on academic publishing. More than twenty pieces have appeared in *Scripta Manent*.

One of the dramatic events during my time as editor of the *Notices* was the publication of the article "Principles for implementing a potential solution to the Middle East conflict", by Thomas L. Saaty and H. L. Zoffer. This article suggested means by which mathematical ideas, such as game theory, could be used to help effect a peace in the Middle East conflict. The article struck reviewers as original, insightful, and certainly unusual. We published it in the November 2013 issue. The article garnered considerable attention and caused a nontrivial amount of tumult. People objected to both the substance and the slant of the article, and some objected even to the graphics. The upshot of this unrest is that AMS President David Vogan appointed a task force to study the matter and to recommend guidelines for handling such situations in the future.

Vogan treated this matter with considerable finesse, and the outcome of the deliberations of the task force was constructive and useful. As a result, it has now become standard procedure for the editor of the *Notices* to show every incoming Feature and Communication article to the entire *Notices* Editorial Board. No article can be published without at least two positive reactions from the Board. While the task force presented these procedures only as guidelines, the Board and I were happy to embrace them, and the resulting dialogue among all of the editors has proved to be heartening and helpful.

It must be said that the *Notices* could not consistently achieve such a high level of quality without decisive support from the AMS staff, especially managing editor Sandy Frost and deputy editor Allyn Jackson. I would particularly like to single out Sandy Frost, who worked on the *Notices* for two decades. She brought to the job a world of experience, decisive technical expertise, and considerable institutional memory. Sandy retired from the AMS last fall, and her professional contributions and good spirits have been sorely missed. She is replaced by Rachel Rossi, who is working hard to fill a big role. Rachel brings with her fifteen years of book and journal publishing experience, eight years in academic publishing.

Working as editor of the *Notices of the AMS* has proved to be a rewarding and fascinating task, and one that I view with pride and pleasure. It is delightful to interact with the authors, the readership, and the other editors. I look forward to my remaining time at the *Notices*.

Steven G. Krantz is editor of the Notices. His email address is `sk@math.wustl.edu`.

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Editing an Electronic Journal

Jeffrey Shallit

In 1998 Neil Sloane—sometimes known as the Father of Integer Sequences [1]—started a free electronic journal which he called the *Journal of Integer Sequences*. In the first four years of the journal's existence, fifty papers were published.

In 2002 Sloane decided to seek another editor, and I (influenced by Leonard Eugene Dickson's dictum [2])[1] volunteered to take over. I have edited the journal ever since, assisted by an editorial board of eight colleagues. Our journal is entirely free for both authors and readers and is hosted on computers at the University of Waterloo [3]. We get about 100–150 submissions per year, of which about 50–75 are published.

Editing the journal is rather time-consuming, and I estimate that I spend roughly one day a week on it. Here I reflect on some of the things I've learned while editing the journal for twelve years.

Referees

At the *Journal of Integer Sequences*, to avoid burdening referees unduly, we try not to ask the same referee twice within a year. Unless the paper is exceptionally long or there are other extenuating circumstances, we ask for referee reports to be completed within two months. After two months we send a reminder and, if necessary, another reminder after three months. In the rare case when no report is produced after four months, we give the referee a one-week ultimatum. If there is still no report, we look for a different referee. The result is that our mean time from submission to a decision is under one hundred days.

The delay between submission and decision is mostly due to waiting for referee reports. Good referees tell you right away if they are able to write a report and deliver their report on time. Bad referees don't answer your initial request quickly, don't deliver a report on time, and don't respond to repeated requests for the report. The worst referees of all, however, are those who agree to write the report and then string you along with an endless series of "I'm almost done with the report, and I should have it to you next week" messages. I once had a referee do this to me for almost a year until I finally gave up. This particular referee was very convincing, I have to say.

Potential referees offer many reasons for not agreeing to read a paper. One explained that he no longer refereed any papers at all because he objected, on philosophical grounds, to the imposition of any time limit on the production of the report. I notice that he continues to publish and presumably get his own papers refereed by others.

Finally, one referee who we've asked at least four times kept refusing because the papers we sent him were "not in his area of competence," even though they evidently were. Pretending incompetence certainly succeeds as a strategy to avoid more work, although I imagine it doesn't improve one's reputation.

Jeffrey Shallit is professor of mathematics at the University of Waterloo. His email address is `shallit@cs.uwaterloo.ca`.

[1] *"...every person should aim to perform at some time in his life some serious useful work for which it is highly improbable that there will be any reward whatever other than his satisfaction therefrom."*

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Two Paradoxes of Refereeing

The first paradox of refereeing is that good referees are not compensated for doing a good job on time. Instead, they are effectively penalized by additional requests for refereeing, while bad referees (or people who don't agree to referee) are rewarded by having less work to do. I have no easy solution to this paradox, although when good referees submit papers to our journal, we do try to match them with other good referees.

Matching referees with a paper leads to the second paradox of refereeing: very good papers are typically sent to very good mathematicians as referees. Not-so-good papers don't usually get sent to the mathematicians of the highest reputation for two reasons:

1. There are many more mediocre papers than truly excellent mathematicians.
2. You want to conserve the resources of great mathematicians by not asking them very often, and by sending them papers that are deserving of their time.

The result is that very good papers, which get sent to good mathematicians with high standards, might very well have a lower acceptance rate than much weaker papers. I suppose it is the role of the editor to try to adjust for this paradox.

Referee Reports

In twelve years as editor-in-chief, I've seen more than a thousand referee reports, both good ones and bad ones. To paraphrase Tolstoy, good referee reports are all alike: they evaluate the quality and correctness of the paper; give specific, detailed suggestions about how it could be improved; mention relevant papers missing from the bibliography; and end with an explicit recommendation to accept, revise, or reject.

Bad referee reports, however, are each bad in their own way. For example, here is one report I got: "I had a brief look at the paper, not worth my or anyone else's time." I'm not sure what I am supposed to do with this, even if it's true. I can hardly send it verbatim to the author.

One risk of electronic referee reports that does not seem very widely known is the risk of revealing your identity (name, institution, and so forth) in the "metadata" that accompanies your report. In my experience, this is particularly true of reports that are prepared using Microsoft Word (and one reason that we ask referees not to prepare their reports in this fashion).

Authors

Although most authors appreciate the work done by referees, a few do not. One author, a rather well-known mathematician, fell in this latter class. After I forwarded a report with some reasonable suggestions for improvements, he sent a reply as follows:

> I feel bad for you and JIS for messing up so badly. You are hereby granted twenty-four hours to accept our paper subject to the few minor revisions that we agree with, without going back to that stupid referee.

Needless to say, we did not give in to this kind of threat, and the paper was not published.

Another author, upon receiving a negative report from the world's expert on the subject, complained that our journal is "a forum taken to belong to an elite that think to hold [*sic*] the absolute truth delivering a decision based on an incredibly deprecatory pseudo review" and attributed the rejection to his belonging to the group of "people that are obviously not in the social network of the journal." Luckily, responses like this are the exception, not the rule.

Perhaps because of the content of our journal, we also get a nontrivial number of submissions from amateurs claiming to solve major open problems, such as Goldbach's conjecture. Lately we have resorted to requiring that these kinds of papers be accompanied by a physical (nonelectronic) signed letter from a PhD mathematician at a university, stating that he or she has read the submission and agrees it is correct. This requirement has cut down on our time commitment in handling low-quality submissions.

Plagiarism

Another problem that we've had to deal with is plagiarism. I once had a paper submitted that plagiarized almost word-for-word parts of a paper listed in the submission's bibliography. I learned this from the referee report, written by…the author of that paper in the bibliography! Needless to say, this was not exactly the smartest move on the part of the author.

Most cases of plagiarism, however, have not concerned mathematical content but arise because the authors of the papers feel their English is not up to the standards of a scientific paper. They "borrow" sections of the introduction of other papers.These authors often don't seem to understand that what they have done is not legitimate.

Preparing Papers

At our journal, we have no editorial staff to prepare papers, so the job falls on the shoulders of the authors and the editor-in-chief. It's hard to teach people how to write good papers, but Steven Krantz's book [4] is a good start.

Our journal relies on LaTeX as the source code for manuscripts. We have a certain journal style, which we distribute to authors [5] and ask them to follow. Most of the recommendations there are along the lines of "brush your teeth," but it's startling how few authors follow them.

For example, despite the fact that mathematicians are intimately familiar with the notion of using a variable name to denote a numerical quantity, for some inexplicable reason this familiarity does not extend to giving names to theorems, lemmas, and so forth using labels in LaTeX. Many authors insist on "hard-coding" these references. This choice makes it very hard to revise the paper, since inserting a new theorem requires renumbering throughout.

There is a relatively short list of latex errors that are made again and again by authors [6]. And the American Mathematical Society has a set of packages (amsmath, amscd, amsthm, amsfonts, amssymb) that make life much easier for a mathematician. Use them!

Conclusions

Running an electronic journal has been a great experience for me. I've learned a lot about manuscript preparation. I get to see interesting papers before they appear and contribute to improving their presentation. Sometimes I even get inspired to write my own papers following up on submissions. I think I underestimated, however, how much time editorial work would take and the kinds of challenges I would face. Perhaps this brief note lets you know what you're in for if you decide to do the same.

References

[1] *The On-Line Encyclopedia of Integer Sequences*, `oeis.org`.
[2] L. E. Dickson, *History of the Theory of Numbers*, Vol. II, Chelsea Publishing Co., New York, 1996, p. xxi.
[3] `cs.uwaterloo.ca/journals/JIS/`
[4] Steven G. Krantz, *A Primer of Mathematical Writing*, American Mathematical Society, 1996.
[5] `cs.uwaterloo.ca/journals/JIS/texrecs.pdf`
[6] `recursed.blogspot.ca/2013/05/ten-common-latex-errors.html`

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History of Mathematics: Seeking Truth and Inspiring Students

Alex M. McAllister and Diana White

Our doctoral programs helped us master the research methodologies and disciplinary norms of our respective fields in pure mathematics. However, as professors, we both developed a strong desire to deepen our knowledge of the history of mathematics and its research methodologies. In this column, we provide a brief overview, anchored in concrete examples and based on our experiences, of how learning more about the history of mathematics as well as the basics of historiography can benefit working mathematicians professionally.

Rigorous Research

As mathematicians, we base a great deal of our work on deductive reasoning from definitions and axioms. As a result, we may adopt a skeptical view toward disciplines that we perceive as lacking the same "rigor." We might also view historical research as simply creating lists of people, papers, accomplishments, and inventions. Yet the history of mathematics is so much more than this!

In addition to describing the development of mathematics, historians of mathematics seek to understand the influence of ambient culture, social conventions and norms, and mathematicians as individuals and in relationships with others. They care about the extramathematical work and interests of mathematicians, because these often provide the broader context of and influences on their work. A robust understanding of history is developed through a dynamic process of "zooming in" on specific details and "zooming out" to a broader perspective of contemporary cultural, social, political, and scientific influences. Historical research produces reliable, accurate historical knowledge via methodologies that, while different, are every bit as sophisticated and well developed as the practices of mathematicians and scientists.

Historical Accuracy and Contextual Understanding

As students and later as professors, we typically become familiar with a variety of historical anecdotes related to mathematics, which we may in turn repeat to our students or others. However, because of our lack of expertise in the history of mathematics, we are usually unable to evaluate whether these anecdotes are true or mere apocryphal tales.

The tales of mathematics involve more than just the mathematics itself. Mathematical anecdotes are perhaps akin to family gossip, capitalizing on humans' innate love of sharing stories. Students often

Alex M. McAllister is professor of mathematics at Centre College. His email address is `alex.mcallister@centre.edu`.

Diana White is associate professor of mathematics at the University of Colorado Denver. Her email address is `Diana.White@ucdenver.edu`.

The authors first learned about many of these ideas from the two-day MAA Short Course: Reading, Writing, Doing the History of Mathematics: Learning the Methods of Historical Research, just before the 2014 Joint Mathematics Meetings.

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benefit from this humanizing of mathematics and can come to perceive mathematics as a dynamic evolving field of study and inquiry rather than just some static, staid collection of facts. At the same time, though, each story we tell should be true rather than hagiography or a sort of false fishing tale about "the one that got away." As mathematicians we aspire to share mathematical truths with our students, and this goal should also motivate our fidelity to accurately representing the history of mathematics.

Some scholars, especially novices, have a tendency to read modern results and perspectives into the writings and results of preceding generations of mathematicians. For example, the geometric results found in Euclid's *Elements* can be interpreted and explained in terms of algebra. A modern mathematician might take this algebraic interpretation based on current techniques as evidence that the Greeks knew algebra. This observation might lead them to mistakenly posit that *Elements* is a geometric garland covering the underlying algebra rather than acknowledge its true nature as a synthesis of the state-of-the-art in geometry.

Historians strive to adopt the unique perspectives of the mathematicians from each time, place, and social context that is being studied. Babylonian, Greek, Chinese, Indian, Islamic, European, and modern mathematicians all explored different questions, developed distinct approaches for grappling with these questions, and varied in their perceived need for and rigor of mathematical arguments. In short, historians must be careful to investigate mathematical developments in their appropriate context to avoid creating a "whig" history of mathematics which presents the past as an inevitable progression toward the more enlightened present while ignoring mathematical pathways that are not part of our current methodologies.

Effective Teaching Using the History of Mathematics

The history of mathematics can inform our approach to teaching entire fields of mathematics or specific topics within a particular field. For the former, a prime example is Otto Toeplitz's *Genetic Approach to Calculus*, which presents the ideas of calculus from a historical perspective of ideas evolving over centuries by examining the particular results of Archimedes, Kepler, Galileo, Fermat, and culminating with Newton and Leibniz. For the latter, we discuss an example shared by Fred Rickey at one of last year's Joint Mathematics Meetings' minicourses (1). He presents his Real Analysis class with Cauchy's "proof" that if a series of continuous functions converges, then the series converges to a continuous function. For homework, his students explore counterexamples to this supposed "theorem." During the next class period, after his students express their frustration with the "false proof," Rickey helps them understand the error in Cauchy's proof and then develops the notion of uniform convergence.

Studying the history of mathematics can also help students understand and embrace mathematical traditions. For example, the notions of proof and rigor have had different meanings in various times and cultures. In the fourth century BCE, Euclid provided a deductive proof of the Pythagorean Theorem (in *Elements*, see also Liu Hui's annotated version of *Jiu zhang suan shu* Nine Chapters on the Mathematical Art) from 263 CE provided a "proof by picture" of this result that was held in similarly high regard by his peers. Approaching the modern era, in 1899 CE, David Hilbert introduced a "more rigorous" development of Euclidean geometry in his *Grundlagen der Geometrie*. A comparison of these various proofs provides a wonderful, focused opportunity for students to discuss the evolving nature of proof itself.

We regularly create and teach proofs by induction. Yet many prior generations of mathematicians were content with using some variation of the phrase "and so on" when it appeared clear that the arguments would continue indefinitely in the same fashion. For example, Euclid's original proof that there exist infinitely many primes considers only the case of there being exactly three primes. We now explain "his" proof with the supposition that there exists some arbitrary, finite number of primes, but in Euclid's time and for centuries afterwards, his original proof was regarded as sufficient and rigorous. Our students can better understand the contemporary notion of rigorous proof when we contrast modern proofs with the arguments of our mathematical ancestors.

Resources for Deepening Knowledge

Just as in mathematical research, the quest for new historical knowledge can require great persistence. Some good starting places include the *Dictionary of Scientific Biography* (2) and the *MacTutor History of Mathematics* archive (3), both of which contain references to original sources for further study. A historian must go deeper, though, and examine extant, original manuscripts housed in archives. Fortunately, libraries continue scanning their archival materials and providing access through the Web. In parallel, translators are making an increasing number of manuscripts available in diverse languages, although translations can present their own challenges in their accuracy and reliability.

The Mathematical Association of America's History of Mathematics Special Interest Group (HOMSIGMAA) provides many resources on their website (4). These links include references for mathematicians interested in the basics of reading, writing, and doing the history of mathematics. Another starting place might be attending a talk on the history of mathematics at the next Joint Math

Meetings or at MathFest. Exploring the history of mathematics through these various resources enables us to grow professionally and to come to think differently about our profession and our mathematical ancestors.

Final Thoughts

Our doctoral programs provide solid training for mathematical research, and increasingly our profession augments this learning with preparation for teaching. We propose an additional step in our ongoing professional development both as teachers and scholars: learning how our discipline evolved into our contemporary study of mathematics as well as how historical research is conducted.

The history of mathematics can inform our understanding of basic topics, strengthen our teaching, and help us better understand our place in the overall human story of mathematics. Not only will we grow as mathematicians, we will also build our capacity to help our students understand mathematics as a human endeavor, developing in response to and driving various aspects of society. Some of us may even decide to go beyond being consumers of the history of mathematics and to learn the in-depth research procedures of historiography, making original contributions to this field.

References

[1] Fred Rickey at `fredrickey.info/` shared this example at the MAA Short Course: Reading, Writing, Doing the History of Mathematics: Learning the Methods of Historical Research, just before the 2014 Joint Mathematics Meetings.

[2] *Dictionary of Scientific Biography* at `historyofmathematics.org`.

[3] *MacTutor History of Mathematics* archive at `www-history.mcs.st-andrews.ac.uk/`.

[4] HOMSIGMAA at `historyofmathematics.org/`.

Mathematics Opportunities

Call for Nominations for the 2015 Ostrowski Prize

The Ostrowski Foundation is seeking nominations for the 2015 Ostrowski Prize. The prize is awarded every two years for recent outstanding achievements in pure mathematics and in the foundation of numerical mathematics. The prize carries a cash award of 100,000 Swiss francs (approximately US$103,000).

Nominations should include a CV of the candidate, a letter of nomination, and two or three letters of reference. The chair of the 2015 jury is Christian Berg of the University of Copenhagen, Denmark. Nominations should be sent to `berg@math.ku.dk` by **April 15, 2015**. For more information, see the website `www.ostrowski.ch/index_e.php?ifile=preis`.

—From an Ostrowski Foundation announcement

Call for Applications for the Third Heidelberg Laureate Forum

The Third Heidelberg Laureate Forum (HLF) will be held August 23–28, 2015. It will bring together winners of the Abel Prize and the Fields Medal, both in mathematics, as well as the Turing Award and the Nevanlinna Prize, both in computer science, in Heidelberg, Germany.

The Heidelberg Laureate Forum Foundation (HLFF) is looking for outstanding young mathematicians and computer scientists from all over the world who would like to get the chance to personally meet distinguished experts from both disciplines and find out how to become leading scientists in their fields. The deadline for applications is **February 28, 2015**. Applications must be submitted online at: `application.heidelberg-laureate-forum.org`. The Forum is organized by the HLFF in cooperation with Klaus Tschira Stiftung and the Heidelberg Institute for Theoretical Studies (the Forum's founders), as well as the Association for Computing Machinery, the International Mathematical Union, and the Norwegian Academy of Science and Letters. For more information, see `www.heidelberg-laureate-forum.org`.

—From an HLF announcement

Mathematics People

2015 ICIAM Prizes Awarded

The International Council for Industrial and Applied Mathematics (ICIAM) has announced the awarding of five major prizes for 2015.

ANNALISA BUFFA, director of the Institute for Applied Mathematics and Information Technologies (Pavia-Genoa-Milan section), has been awarded the Collatz Prize "in recognition of her spectacular use of deep and sophisticated mathematical concepts to obtain outstanding contributions to the development of computer simulations in science and industry." The prize is awarded to researchers under the age of forty-two for outstanding work in industrial and applied mathematics.

ANDREW J. MAJDA of the Courant Institute of Mathematical Sciences, New York University, has been awarded the Lagrange Prize "in recognition of his ground-breaking, original, fundamental, and pioneering contributions to applied mathematics and, in particular, to wave front propagation and combustion, scattering theory, fluid dynamics and atmosphere climate science". The prize recognizes researchers who have made exceptional contributions to applied mathematics throughout their careers.

JEAN-MICHEL CORON of Université Pierre et Marie Curie has been awarded the Maxwell Prize "for his fundamental and original contributions to the study of variational methods for partial differential equations and the nonlinear control of nonlinear partial differential equations". The prize is awarded to a mathematician "who has demonstrated originality in applied mathematics".

BJÖRN ENGQUIST of the University of Texas at Austin has been named the recipient of the Pioneer Prize "for fundamental contributions in the field of applied mathematics, numerical analysis and scientific computing which have had long-lasting impact in the field as well as successful applications in science, engineering and industry". The prize is awarded for pioneering work introducing applied mathematical methods and scientific computing techniques to an industrial problem area or a new scientific field of applications.

LI TATSIEN of Fudan University, Shanghai, China, has been awarded the Su Buchin Prize "in recognition of his outstanding contributions to applied mathematics and to the dissemination of mathematical sciences in developing countries". The prize recognizes outstanding contributions "in the application of mathematics to emerging economies and human development, in particular at the economic and cultural level in developing countries".

All prizes carry a cash award of US$5,000 and will be presented at the ICIAM Congress to be held in Beijing, China, August 10–14, 2015.

—From an ICIAM announcement

Yuan Awarded TWAS Prize

YA-XIANG YUAN of the Chinese Academy of Sciences has been awarded the 2014 TWAS Prize in Mathematics of the Academy of Sciences for the Developing World (TWAS). He was recognized for his contribution to numerical methods for nonlinear optimization, particularly to nonlinear conjugate gradient methods, trust region algorithms, quasi-Newton methods, and subspace methods.

Yuan received his PhD from the University of Cambridge in 1986. He has been affiliated with the Chinese Academy of Sciences in various positions since 1988. He has held a number of visiting positions, including at the University of Colorado Boulder and Northwestern University. He was elected a Fellow of the Society for Industrial and Applied Mathematics (SIAM) in 2011 and received the Shiing S. Chern Mathematics Award in 2011. He became a Fellow of the AMS in 2012 and served on the editorial board of *Mathematics of Computation*.

The TWAS Prize carries a cash award of US$15,000. Yuan will deliver a lecture at the TWAS general meeting in 2015.

—From a TWAS announcement

Tserunyan Awarded Emil Artin Junior Prize

Anush Tserunyan of the University of Illinois at Urbana-Champaign has been awarded the 2015 Emil Artin Junior Prize in Mathematics for her paper "Finite generators for countable group actions in the Borel and Baire category settings", *Advances in Mathematics*, **269**, (2015), 585–646. Established in 2001, the Emil Artin Junior Prize in Mathematics carries a cash award of US$1,000 and is presented usually every year to a student or former student of an Armenian educational institution who is under the age of thirty-five for outstanding contributions to algebra, geometry, topology, and number theory—the fields in which Emil Artin made major contributions. The prize committee consisted of A. Basmajian, Y. Movsisyan, and V. Pambuccian.

—Victor Pambuccian, New College, Arizona State University

Poggio Awarded Swartz Prize

Tomaso Poggio of the Massachusetts Institute of Technology has been named the recipient of the Swartz Prize for Theoretical and Computational Neuroscience of the Society of Neuroscience (SfN). He was honored for his contributions to the development of computational and theoretical models of brain functions, particularly the human visual system and how the brain accomplishes visual recognition. The prize carries a cash award of US$25,000 and recognizes significant cumulative contributions to theoretical models or computational methods in neuroscience.

—From an NSF announcement

2014 CMS Robinson Awards Announced

The Canadian Mathematical Society (CMS) has awarded the 2014 G. de B. Robinson Award jointly to Jonathan M. Borwein and Wadim Zudilin), both of the University of Newcastle, New South Wales; James Wan of Singapore University; and Armin Straub of the University of Illinois for their paper, "Densities of short uniform random walks" (with an appendix by Don Zagier), *Canadian Journal of Mathematics* **64**, no. 5 (2012), 961–990. Jan Nekovář of Université Pierre et Marie Curie was also honored with a Robinson Award for his paper "Level raising and anticyclotomic Selmer groups for Hilbert modular forms of weight two", *Canadian Journal of Mathematics* **64**, no. 3 (2012), 588–649. The award is given in recognition of outstanding contributions to the *Canadian Journal of Mathematics* or the *Canadian Mathematical Bulletin*.

—From a CMS announcement

Cautis Awarded Aisenstadt Prize

Sabin Cautis of the University of British Columbia has been awarded the 2014 André Aisenstadt Prize in Mathematics of the Centre de Recherches Mathématiques (CRM). The prize citation reads in part: "Sabin Cautis works at the crossroads of algebraic geometry, representation theory, and low-dimensional topology. In his earlier work (joint with the 2011 André Aisenstadt Prize recipient, Joel Kamnitzer), he developed a new approach to Khovanov's knot invariants, which uses algebraic geometry and is inspired by mirror symmetry. Dr. Cautis is a world leader in the area of categorification. His results are expected to have a lasting impact on the field and lead to important developments in low-dimensional topology, the geometric Langlands program, and the mathematical aspects of quantum physics. In particular, his recent work with Anthony Licata on categorification of Heisenberg algebras and vertex operators is a major step in the direction outlined by Igor Frenkel towards categorification of conformal field theory." Cautis received his PhD from Harvard University in 2006 and has held positions at the University of Southern California, Columbia University, the Mathematical Sciences Research Institute, and Rice University. He was an Alfred P. Sloan Foundation Fellow in 2011–2013. The prize recognizes outstanding research achievement by a young Canadian mathematician.

—From a CRM announcement

Rhodes Scholars Announced

The Rhodes Trust has named its scholars for 2015. Among them are four students whose work involves the mathematical sciences.

Noam Angrist of Brookline, Massachusetts, graduated from the Massachusetts Institute of Technology in 2013 with majors in economics and mathematics. While at MIT, he did economic research for the World Bank, the White House, and on the Affordable Care Act, and also founded an enrichment program combining athletics and academics for low-income youth that achieved considerable success measured by achievement and college matriculation. As a Fulbright Scholar in Botswana, he founded an NGO for HIV education designed to discourage intergenerational sex ("sugar daddy awareness"). Its success led him to raise the money to extend the program to 340 schools, and he now plans to launch it in four other southern African countries. He hopes to continue to apply his economics acumen to assess and develop poverty-alleviation projects that work. He will do the MSc in Evidence-Based Social Intervention and Policy Evaluation at Oxford.

Ruth C. Fong of Somerset, New Jersey, is a senior at Harvard University majoring in computer science. Her senior thesis focuses on how computers can intuitively identify and perceive objects in a way that more closely mimics the human brain. She was chosen to teach three undergraduate computer science courses and one for graduate students as well. She won a highly competitive scholarship from Apple for women in technology and a Tech in the World Fellowship to work on infectious disease data in Tanzania. Ruth is also extremely active as an advocate for autism-related causes and was a director of Big Sibs program in Boston's Chinatown. She is a member of a dance troupe and enjoys both hip-hop and traditional Chinese dance. She plans to do the MSc in Mathematics and Foundations in Computer Science and the MSc in Computer Science at Oxford.

Sai P. Gourisankar of Atlanta, Georgia, is a senior at the University of Texas, where he will graduate in May with a BS in chemical engineering and a BA in liberal arts. He also has a minor in German. He is a Goldwater Scholar and a Churchill Scholar with a 4.0 across multiple disciplines. He has several publications relating to his work in nanotechnology, particularly relating to nanoclusters. He is also president of an organization that fosters discussion between the humanities and the sciences. At Oxford, he plans to do the MSc in mathematical modeling and scientific computing and the MSc in mathematical and theoretical physics.

Peter N. Kalugin of Albuquerque, New Mexico, is a senior at Johns Hopkins University, from which he will receive a BS in molecular and cellular biology and a BA in mathematics with a minor in physics. A Goldwater Scholar, and elected to Phi Beta Kappa as a junior, he has worked in basic science labs on cell signaling and immunobiology and completed a thesis on brain tumor biology. He has also worked on statistical analysis of medical data sets at the University of New Mexico. Peter plays the alto saxophone, speaks five languages, has taught English to new immigrant students, volunteered at orphanages in Mongolia and Nepal, and founded an organization to connect other university students with local NGOs abroad. He is passionate about pediatric oncology and research in cancer cell growth. He plans to do the MSc by research in oncology at Oxford.

—From a Rhodes Trust announcement

Verma Awarded Bhatnagar Prize

Kaushal Kumar Verma of the Indian Institute of Technology has been awarded the 2014 Shanti Swarup Bhatnagar Prize for Science and Technology in the mathematical sciences. He works in complex analysis. The prize is awarded by the Council of Scientific Research and Industrial Development to recognize outstanding Indian work in science and technology. Shanti Swarup Bhatnagar was the founding director of the Council. It is the highest award for science in India. The prize carries a cash award of 500,000 rupees (approximately US$8,100).

—Council of Scientific Research and Industrial Development, India

2014 Infosys Prize Awarded


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Madu Sudan

Madhu Sudan of Microsoft Research and the Massachusetts Institute of Technology has been awarded the 2014 Infosys Prize in mathematical sciences by the Infosys Science Foundation. He was recognized "for his seminal contributions to probabilistically checkable proofs and error-correcting codes". The prizewinners are selected based on significant progress showcased in their chosen spheres, as well as for the impact their research will have on the specific field.

—From an Infosys Science Foundation announcement

Van Raamsdonk Awarded CAP/CRM Prize

Mark Van Raamsdonk of the University of British Columbia has been awarded the 2014 CAP/CRM Prize in Theoretical and Mathematical Physics by the Canadian Association of Physicists (CAP) and the Centre de Recherches Mathématiques (CRM) "for his highly original, influential contributions to several areas of theoretical physics, including string theory, quantum field theory, and quantum gravity". The award recognizes research excellence in the fields of theoretical and mathematical physics in Canada.

—From a CAP/CRM announcement

2014 Prix la Recherche

Valeria Banica of the University of Evry-Val-d'Essone has been awarded the Prix la Recherche for her work on stable vortices in fluids. The research prizes, given by the French magazine *La Recherche*, honor the best scientific work conducted in France during the preceding year.

—From a La Recherche *announcement*

Inside the AMS

From the Public Awareness Office

2014 AMS Einstein Public Lecture in Mathematics. JAMES SIMONS, chairman of the Simons Foundation, gave the lecture "Mathematics, Common Sense, and Good Luck" at the AMS Fall Western Sectional Meeting at San Francisco State University. He ruminated on the three intertwining subjects of his careers—mathematics, finance, and philanthropy—and afterward met some AMS-Simons Travel Grant recipients. See the website `www.ams.org/meetings/lectures/einstein-2014`. The 2015 Einstein Public Lecture will take place on March 7 at the Spring Eastern Sectional Meeting at Georgetown University.

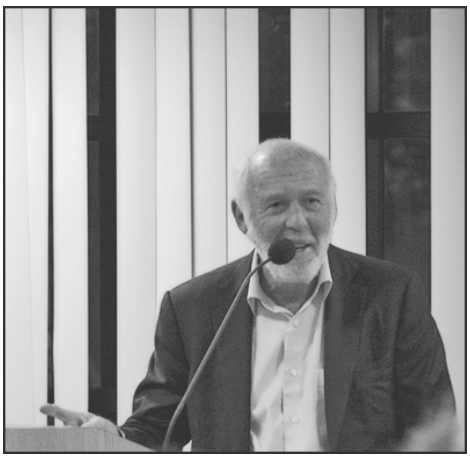


Photo by Lisa Sze

—*Annette Emerson and Mike Breen*
AMS Public Awareness Officers
`paoffice@ams.org`

AMS Email Support for Frequently Asked Questions

A number of email addresses have been established for contacting the AMS staff regarding frequently asked questions. The following is a list of those addresses together with a description of the types of inquiries that should be made through each address.

abs-coord@ams.org for questions regarding a particular abstract or abstracts questions in general.

acquisitions@ams.org to contact the AMS Acquisitions Department.

ams@ams.org to contact AMS Headquarters in Providence, Rhode Island.

amsdc@ams.org to contact the Society's office in Washington, D.C.

amsfellows@ams.org to inquire about the Fellows of the AMS.

amsmem@ams.org to request information about membership in the AMS and about dues payments or to ask any general membership questions; may also be used to submit address changes.

ams-simons@ams.org for information about the AMS Simons Travel Grants Program.

ams-survey@ams.org for information or questions about the Annual Survey of the Mathematical Sciences or to request reprints of survey reports.

bookstore@ams.org for inquiries related to the online AMS Bookstore.

classads@ams.org to submit classified advertising for the *Notices*.

cust-serv@ams.org for general information about AMS products (including electronic products), to send address changes, or to conduct any general correspondence with the Society's Sales, Marketing & Member Services Department.

development@ams.org for information about charitable giving to the AMS.

eims-info@ams.org to request information about Employment Information in the Mathematical Sciences (EIMS). For ad rates and to submit ads go to `eims.ams.org`.

emp-info@ams.org for information regarding AMS employment and career services.

eprod-support@ams.org for technical questions regarding AMS electronic products and services.

gradprg-ad@ams.org to inquire about a listing or ad in the Find Graduate Programs online service.

mathcal@ams.org to send information to be included in the "Mathematics Calendar" section of the *Notices*.

mathjobs@ams.org for questions about the online job application service `Mathjobs.org`.

mathprograms@ams.org for questions about the online program application service `Mathprograms.org`.

mathrev@ams.org to submit reviews to *Mathematical Reviews* and to send correspondence related to reviews or other editorial questions.

meet@ams.org to request general information about Society meetings and conferences.

mmsb@ams.org for information or questions about registration and housing for the Joint Mathematics Meetings (Mathematics Meetings Service Bureau).

msn-support@ams.org for technical questions regarding MathSciNet.

notices@ams.org to send correspondence to the production editor of the *Notices*, including items for the news columns.

EMS Monograph Award

The EMS Monograph Award is assigned every two years to the author(s) of a monograph in any area of mathematics that is judged by the selection committee to be an outstanding contribution to its field. The prize is endowed with 10,000 Euro and the winning monograph will be published by the EMS Publishing House in the series "EMS Tracts in Mathematics".

The first award was announced in the June 2014 issue of the Newsletter of the EMS.
The second award will be announced in 2016, the deadline for submissions is 30 June 2015.

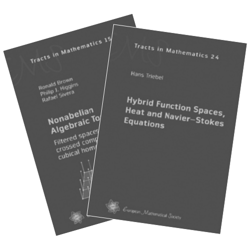
Submission of manuscripts
The monograph must be original and unpublished, written in English and should not be submitted elsewhere until an editorial decision is rendered on the submission. Monographs should preferably be typeset in TeX. Authors should send a pdf file of the manuscript by email to:

awards@ems-ph.org

Scientific Committee
John Coates, Pierre Degond, Carlos Kenig, Jaroslav Nešetřil, Michael Roeckner, Vladimir Turaev

EMS Tracts in Mathematics

Editorial Board:
Carlos E. Kenig (University of Chicago, USA); Andrew Ranicki (University of Edinburgh, UK); Michael Röckner (Universität Bielefeld, Germany, and Purdue University, USA); Vladimir Turaev (Indiana University, Bloomington, USA); Alexander Varchenko (University of North Carolina at Chapel Hill, USA)



This series includes advanced texts and monographs covering all fields in pure and applied mathematics. Tracts will give a reliable introduction and reference to special fields of current research. The books in the series will in most cases be authored monographs, although edited volumes may be published if appropriate. They are addressed to graduate students seeking access to research topics as well as to the experts in the field working at the frontier of research.

To appear:
Vol. 24 Hans Triebel: *Hybrid Function Spaces, Heat and Navier–Stokes Equations;* 978-3-03719-150-7. 2015. 196 pages. Approx. US$ 64

and the Winners of the EMS Monograph Award 2014:
Vol. 22 Patrick Dehornoy with François Digne, Eddy Godelle, Daan Krammer and Jean Michel: *Foundations of Garside Theory* 978-3-03719-139-2. 2015. Approx. 700 pages. Approx. $ 128
Vol. 23 Augusto C. Ponce: *Elliptic PDEs, Measures and Capacities. From the Poisson Equation to Nonlinear Thomas–Fermi Problems* 978-3-03719-140-8. 2015. Approx. 350 pages. Approx. $ 78

European Mathematical Society Publishing House
Seminar for Applied Mathematics, ETH-Zentrum SEW A27
CH-8092 Zürich, Switzerland
orders@ems-ph.org, www.ems-ph.org

notices-ads@ams.org to submit electronically paid display ads for the *Notices*.

notices-booklist@ams.org to submit suggestions for books to be included in the "Book List" in the *Notices*.

notices-letters@ams.org to submit letters and opinion pieces to the *Notices*.

notices-whatis@ams.org to comment on or send suggestions for topics for the "WHAT IS...?" column in the *Notices*.

nsagrants@ams.org for information about the NSA-AMS Mathematical Sciences Program.

paoffice@ams.org to contact the AMS Public Awareness Office.

president@ams.org to contact the AMS president.

prof-serv@ams.org to send correspondence about AMS professional programs and services.

promorequests@ams.org to request AMS giveaway materials such as posters, brochures, and catalogs for use at mathematical conferences, exhibits, and workshops.

publications@ams.org to send correspondence to the AMS Publication Division.

pub-submit@ams.org to submit accepted electronic manuscripts to AMS publications (other than Abstracts). See `www.ams.org/submit-book-journal` to electronically submit accepted manuscripts to the AMS book and journal programs.

reprint-permission@ams.org to request permission to reprint material from Society publications.

royalties@ams.org for AMS authors to direct questions about royalty payments.

sales@ams.org to inquire about reselling or distributing AMS publications or to send correspondence to the AMS Sales, Marketing & Member Services Department.

secretary@ams.org to contact the AMS secretary.

student-serv@ams.org for questions about AMS programs and services for students.

tech-support@ams.org to contact the Society's typesetting Technical Support Group.

textbooks@ams.org to request examination copies or inquire about using AMS publications as course texts.

webmaster@ams.org for general information or for assistance in accessing and using the AMS website.

Deaths of AMS Members

Antal Bege, professor, Babes-Bolyai University, died on March 22, 2012. Born on November 26, 1962, he was a member of the Society for 10 years.

Michael Charles Richard Butler, professor, University of Liverpool, died on December 18, 2012. Born on January 6, 1929, he was a member of the Society for 10 years.

Egbert Harzheim, of Cologne, Germany, died on December 14, 2012. Born on February 11, 1932, he was a member of the Society for 48 years.

Stephen Vagi of Rockport, Maine, died on June 8, 2012. Born on October 19, 1930, he was a member of the Society for 26 years.

2014 Election Results

In the elections of 2014 the Society elected a vice president, a trustee, five members at large of the Council, three members of the Nominating Committee, and two members of the Editorial Boards Committee.

Vice President

Elected as the new vice president is **Carlos E. Kenig** from the University of Chicago. Term is three years (1 February 2015–31 January 2018).

Trustee

Elected as trustee is **Joseph H. Silverman** from Brown University. Term is five years (1 February 2015–31 January 2020).

Members at Large of the Council

Elected as new members at large of the Council are:
Matthew Baker from Georgia Institute of Technology
Edward Frenkel from University of California, Berkeley
Pamela Gorkin from Bucknell University
Wen-Ching Winnie Li from Pennsylvania State University
Mary Pugh from the University of Toronto
Terms are three years (1 February 2015–31 January 2018).

Nominating Committee

Elected as new members of the Nominating Committee are:
Douglas N. Arnold from the University of Minnesota
Christine Guenther from Pacific University
Kavita Ramanan from Brown University
Terms are three years (1 January 2015–31 December 2017).

Editorial Boards Committee

Elected as new members of the Editorial Boards Committee are:
Danny Calegari from the University of Chicago
Hee Oh from Yale University
Terms are three years (1 February 2015–31 January 2018).

2015 AMS Election

Nominations by Petition

Vice President or Member at Large

One position of vice president and member of the Council *ex officio* for a term of three years is to be filled in the election of 2015. The Council intends to nominate at least two candidates, among whom may be candidates nominated by petition as described in the rules and procedures.

Five positions of member at large of the Council for a term of three years are to be filled in the same election. The Council intends to nominate at least ten candidates, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

Petitions are presented to the Council, which, according to Section 2 of Article VII of the bylaws, makes the nominations. The Council of 23 January 1979 stated the intent of the Council of nominating all persons on whose behalf there were valid petitions.

Prior to presentation to the Council, petitions in support of a candidate for the position of vice president or of member at large of the Council must have at least fifty valid signatures and must conform to several rules and operational considerations, which are described below.

Editorial Boards Committee

Two places on the Editorial Boards Committee will be filled by election. There will be four continuing members of the Editorial Boards Committee.

The President will name at least four candidates for these two places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and operational considerations, described below, should be followed.

Nominating Committee

Three places on the Nominating Committee will be filled by election. There will be six continuing members of the Nominating Committee.

The President will name at least six candidates for these three places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and operational considerations, described below, should be followed.

Rules and Procedures

Use separate copies of the form for each candidate for vice president, member at large, or member of the Nominating and Editorial Boards Committees.

1. To be considered, petitions must be addressed to Carla D. Savage, Secretary, American Mathematical Society, 201 Charles Street, Providence, RI 02904–2294 USA, and must arrive by 24 February 2015.
2. The name of the candidate must be given as it appears in the *Combined Membership List* (www.ams.org/cml). If the name does not appear in the list, as in the case of a new member or by error, it must be as it appears in the mailing lists, for example on the mailing label of the *Notices*. If the name does not identify the candidate uniquely, append the member code, which may be obtained from the candidate's mailing label or by the candidate contacting the AMS headquarters in Providence (amsmem@ams.org).
3. The petition for a single candidate may consist of several sheets each bearing the statement of the petition, including the name of the position, and signatures. The name of the candidate must be exactly the same on all sheets.
4. On the next page is a sample form for petitions. Petitioners may make and use photocopies or reasonable facsimiles.
5. A signature is valid when it is clearly that of the member whose name and address is given in the left-hand column.
6. The signature may be in the style chosen by the signer. However, the printed name and address will be checked against the *Combined Membership List* and the mailing lists. No attempt will be made to match variants of names with the form of name in the *CML*. A name neither in the *CML* nor on the mailing lists is not that of a member. (Example: The name Carla D. Savage is that of a member. The name C. Savage appears not to be.)
7. When a petition meeting these various requirements appears, the secretary will ask the candidate to indicate willingness to be included on the ballot. Petitioners can facilitate the procedure by accompanying the petitions with a signed statement from the candidate giving consent.

Nomination Petition

for 2015 Election

The undersigned members of the American Mathematical Society propose the name of

____

as a candidate for the position of (check one):

- ☐ **Vice President** (term beginning 02/01/2016)
- ☐ **Member at Large of the Council** (term beginning 02/01/2016)
- ☐ **Member of the Nominating Committee** (term beginning 01/01/2016)
- ☐ **Member of the Editorial Boards Committee** (term beginning 02/01/2016)

of the American Mathematical Society.

Return petitions by 24 February 2015 to:
Carla D. Savage, AMS Secretary, 201 Charles Street, Providence, RI 02904-2294 USA

Name and address (printed or typed)	
	Signature
	Signature
	Signature
	Signature
	Signature
	Signature

AMERICAN MATHEMATICAL SOCIETY

Leroy P. Steele Prizes

Call for Nominations

The selection committee for these prizes requests nominations for consideration for the 2016 awards. Further information about the prizes can be found in the April 2014 *Notices*, pp. 393–397 (also available at http://www.ams.org/profession/prizes-awards/ams-prizes/steele-prize).

Three Leroy P. Steele Prizes are awarded each year in the following categories: (1) the Steele Prize for Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through PhD students; (2) the Steele Prize for Mathematical Exposition: for a book or substantial survey or expository-research paper; and (3) the Steele Prize for Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. In 2016 the prize for Seminal Contribution to Research will be awarded for a paper in applied mathematics.

Nomination with supporting information should be submitted to www.ams.org/profession/prizes-awards/nominations. Include a short description of the work that is the basis of the nomination, including complete bibliographic citations. A curriculum vitae should be included. Nominations for the Steele Prizes for Lifetime Achievement and for Mathematical Exposition will remain active and receive consideration for three consecutive years. Those who prefer to submit by regular mail may send nominations to the AMS Secretary, Carla Savage, 201 Charles Street, Providence, RI 02904. Those nominations will be forwarded by the secretary to the prize selection committee.

Deadline for nominations is March 31, 2015.

AMS
AMERICAN MATHEMATICAL SOCIETY
www.ams.org

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The **editor** is the person to whom to send articles and letters for consideration. Articles include features, memorials, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The **production editor** is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar".

Permissions requests should be sent to: reprint-permission@ams.org.

Contact the **editor** at: notices@math.wustl.edu or by fax at 314-935-6839.

Contact the **production editor** at: notices@ams.org or by fax at 401-331-3842. Postal addresses for both may be found in the masthead.

Upcoming Deadlines

January 15, 2015: Applications for AMS-AAAS Mass Media Summer Fellowships. See the website at www.aaas.org/program/aaas-mass-mediascience-engineering-fellows-program . Applicants may contact Dione Rossiter, project director, AAAS Mass Media Science & Engineering Fellows Program, 1200 New York Avenue, NW, Washington, DC 20005; telephone: 202-326-6645; email: drossite@aaas.org. Further information is also available at www.ams.org/programs/ams-fellowships/media-fellow/massmediafellow .

January 22, 2015: Full proposals for NSF Major Research Implementation (MRI) program. See the website www.nsf.gov/pubs/2013/nsf13517/nsf13517.htm.

January 31, 2015: Nominations for Early Career Award of the Canadian Applied and Industrial Mathematics Society (CAIMS) and the Pacific Institute for Mathematical Sciences (PIMS). See www.pims.math.ca/pims-glance/prizes-awards.

January 31, 2015: Entries for AWM Essay Contest. Contact Heather Lewis at hlewis5@naz.edu or see the website https://sites.google.com/site/awmmath/home.

February 1, 2015: Applications for February review for National Academies Research Associateship programs. See the website sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2760; fax: 202-334-2759; email: rap@nas.edu.

February 1, 2015: Applications for AWM Travel Grants, Mathematics Education Research Travel Grants, Mathematics Mentoring Travel Grants, and Mathematics Education Research Mentoring Travel Grants. See the website https://sites.google.com/site/awmmath/programs/travel-grants ; telephone: 703-934-0163; or email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

February 9, 2015: Applications for travel support for IPAM Latinas in Mathematical Sciences Conference. See the website www.ipam.ucla.edu.

February 12, 2015: Applications for IPAM Research in Industrial Projects for Students (RIPS) programs. See the website www.ipam.ucla.edu.

February 28, 2015: Applications for the Third Heidelberg Laureate Forum. See "Mathematics Opportunities" in this issue.

March 2, 2015: Applications for EDGE for Women 2015 Summer Program. See the website www.edgeforwomen.org/.

March 15, 2015: Nominations for PIMS Education Prize. See www.pims.math.ca/pims-glance/prizes-awards.

March 31, 2015: Applications for AMS-Simons Travel Grants program. See the website www.ams.org/programs/travel-grants/AMS-SimonsTG or contact Steven Ferrucci, email: ams-simons@ams.org, telephone: 800-321-4267, ext. 4113.

March 31, 2015: Applications for IPAM graduate summer school on Games and Contracts for Cyber-Physical Security. See the website www.ipam.ucla.edu.

April 15, 2015: Nominations for 2015 Ostrowski Prize. See "Mathematics Opportunities" in this issue.

April 15, 2015: Applications for fall 2015 semester of Math in Moscow. See www.mccme.ru/mathinmoscow, or contact: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: mim@mccme.ru. Information and application forms for the AMS scholarships are available on the AMS website at www.ams.org/programs/travel-grants/mimoscow, or contact: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence RI 02904-2294; email: student-serv@ams.org .

May 1, 2015: Applications for May review for National Academies Research Associateship programs. See sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2760; fax: 202-334-2759; email: rap@nas.edu.

May 1, 2015: Applications for AWM Travel Grants and Mathematics Education Research Travel Grants. See https://sites.google.com/site/awmmath/programs/travel-grants ; telephone: 703-934-0163; or email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

August 1, 2015: Applications for August review for National Academies Research Associateship programs. See sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2760; fax: 202-334-2759; email: rap@nas.edu.

October 1, 2015: Applications for AWM Travel Grants and Mathematics Education Research Travel Grants. See https://sites.google.com/site/awmmath/programs/travel-grants ; telephone: 703-934-0163; or email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

November 1, 2015: Applications for November review for National Academies Research Associateship programs. See sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2760; fax: 202-334-2759; email: rap@nas.edu.

Book List

The Book List highlights recent books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. Suggestions for books to include on the list may be sent to notices-booklist@ams.org*.*

*Added to "Book List" since the list's last appearance.

Alan M. Turing: Centenary Edition, by Sara Turing. Cambridge University Press, April 2012. ISBN-13: 978-11070-205-80. (Reviewed September 2014.)

Alan Turing: The Enigma, The Centenary Edition, by Andrew Hodges. Princeton University Press, May 2012. ISBN-13: 978-06911-556-47. (Reviewed September 2014.)

Alan Turing: His Work and Impact, edited by S. Barry Cooper and J. van Leeuwen. Elsevier, May 2013. ISBN-13: 978-01238-698-07. (Reviewed September 2014.)

Alan Turing's Electronic Brain: The Struggle to Build the ACE, the World's Fastest Computer, by B. Jack Copeland et al. Oxford University Press, May 2012. ISBN-13: 978-0-19-960915-4. (Reviewed September 2014.)

André-Louis Cholesky: Mathematician, Topographer and Army Officer, by Claude Brezinski and Dominique Tournès. Birkhäuser, August 2014. ISBN: 978-33190-813-42.

**Arnold: Swimming Against the Tide,* edited by Boris A. Khesin and Serge L. Tabachnikov. AMS, September 2014. ISBN-13: 978-14704-169-97.

**Baroque Science,* by Ofer Gal and Raz Chen-Morris. University of Chicago Press, March 2013. ISBN-13: 978-02262-129-82.

**Beating the Odds: The Life and Times of E. A. Milne,* by Meg Weston. Imperial College Press, June 2013. ISBN-13: 978-18481-690-74.

**Beautiful Geometry,* by Eli Maor and Eugen Jost. Princeton University Press, January 2014. ISBN-13: 978-06911-509-94.

Beyond Banneker: Black Mathematicians and the Paths to Excellence, by Erica N. Walker. State University of New York Press, June 2014. ISBN-13: 978-14384-521-59.

**Combinatorics: Ancient and Modern,* by Robin Wilson and John J. Watkins. Oxford University Press, August 2013. ISBN-13: 978-01996-565-92.

A Curious History of Mathematics: The Big Ideas from Early Number Concepts to Chaos Theory, by Joel Levy. Andre Deutsch, February 2014. ISBN-13: 978-02330-038-56.

Doing Data Science: Straight Talk from the Frontline, by Rachel Schutt and Cathy O'Neil. O'Reilly Media, November 2013. ISBN: 978-1-449-35865-5. (Reviewed October 2014.)

Enlightening Symbols: A Short History of Mathematical Notation and Its Hidden Powers, by Joseph Mazur. Princeton University Press, March 2014. ISBN-13: 978-06911-546-33. (Reviewed in this issue.)

**Experiencing Mathematics: What Do We Do, When We Do Mathematics?,* by Reuben Hersh. AMS, February 2014. ISBN-13: 978-08218-942-00.

**Fifty Visions of Mathematics,* edited by Sam Parc. Oxford University Press, July 2014. ISBN-13: 978-01987-018-11.

**The Formula: How Algorithms Solve All Our Problems—And Create More,* by Luke Dormehl. Perigee Trade, November 2014. ISBN-13:978-03991-705-39.

Four Lives: A Celebration of Raymond Smullyan, edited by Jason Rosenhouse. Dover Publications, February 2014. ISBN-13: 978-04864-906-70.

Fractals: A Very Short Introduction, by Kenneth Falconer. Oxford University Press, December 2013. ISBN-13: 978-01996-759-82.

From Mathematics in Logic to Logic in Mathematics: Boole and Frege, by Aliou Tall. Docent Press, July 2014, ISBN-13: 978-0-9887449-7-4.

The Grapes of Math: How Life Reflects Numbers and Numbers Reflect Life, by Alex Bellos. Simon and Schuster, June 2014. ISBN: 978-14516-400-90.

Henri Poincaré: A Scientific Biography, by Jeremy Gray. Princeton University Press, November 2012. ISBN- 13: 978-06911-527-14. (Reviewed April 2014.)

A History in Sum: 150 Years of Mathematics at Harvard (1825–1975), by Steve Nadis and Shing-Tung Yau. Harvard University Press, October 2013. ISBN-13: 978-06747-250-03. (Reviewed June/July 2014.)

The Improbability Principle: Why Coincidences, Miracles, and Rare Events Happen Every Day, by David J. Hand. Scientific American/Farrar, Straus and Giroux, February 2014. ISBN-13: 978-03741-753-44. (Reviewed December 2014.)

Infinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World, by Amir Alexander. Scientific American/Farrar, Straus and Giroux, April 2014. ISBN-13: 978-03741-768-15.

**James Clerk Maxwell: Perspective on his Life and Works,* edited by Raymond Flood Mark McCartney, and Andrew Whitaker. Oxford University Press, March 2014. ISBN-13: 978-01996-643-75.

Jane Austen, Game Theorist, by Michael Suk-Young Chwe. Princeton University Press, April 2013. ISBN-13: 978-06911-557-60.

L. E. J. Brouwer—Topologist, Intuitionist, Philosopher: How Mathematics Is Rooted in Life, by Dirk van Dalen. Springer (2013 edition), December 2012. ISBN-13: 978-14471-461-55. (Reviewed June/July 2014.)

Levels of Infinity: Selected Writings on Mathematics and Philosophy, by Hermann Weyl (edited and with an introduction by Peter Pesic). Dover Publications, January 2013. ISBN: 978-04864-890-32.

The Logic of Infinity, by Barnaby Sheppard. Cambridge University Press, May 2014. ISBN-13: 978-11076-786-68.

Love and Math: The Heart of Hidden Reality, by Edward Frenkel. Basic Books, October 2013. ISBN-13: 978-04650-507-41. (Reviewed October 2014.)

Math Bytes: Google Bombs, Chocolate-Covered Pi, and Other Cool Bits in Computing, by Tim Chartier. Princeton University Press, April 2014. ISBN-13: 978-06911-606-03.

Mathematical Expeditions: Exploring Word Problems Across the Ages, by Frank J. Swetz. Johns Hopkins University Press, June 2012. ISBN: 978-14214-043-87.

**Mathematical Understanding of Nature: Essays on Amazing Physical-Phenomena and Their Understanding by Mathematicians, by V. I. Arnold. AMS, September 2014. ISBN-13: 978-14704-170-17.*

The Mathematician's Shiva, by Stuart Rojstaczer. Penguin Books, September 2014. ISBN-13: 978-014312-631-7.

Mathematics and the Making of Modern Ireland: Trinity College Dublin from Cromwell to the Celtic Tiger, by David Attis. Docent Press, October 2014, ISBN-13:978-0-9887449-8-1.

Mathematics and the Real World: The Remarkable Role of Evolution in the Making of Mathematics, by Zvi Artstein. Prometheus Books, September 2014. ISBN-13: 978-16161-409-15.

The Mathematics Devotional: Celebrating the Wisdom and Beauty of Mathematics, by Clifford Pickover. Sterling, November 2014. ISBN-13: 978-14549-132-21.

Mathematics of the Transcendental, by Alain Badiou (translated by A. J. Bartlett and Alex Ling). Bloomsbury Academic, March 2014. ISBN-13: 978-14411-892-40.

Math in Minutes: 200 Key Concepts Explained in an Instant, by Paul Glendinning. Quercus, September 2013. ISBN-13: 978-16236-500-87.

Math in 100 Key Breakthroughs, by Richard Elwes. Quercus, December 2013. ISBN-13: 978-16236-505-44.

A Mind For Numbers: How to Excel at Math and Science (Even If You Flunked Algebra), by Barbara Oakley. Tarcher, July 2014. ISBN-13:*978-03991-652-45.*

The New York Times Book of Mathematics: More Than 100 Years of Writing by the Numbers, edited by Gina Kolata. Sterling, June 2013. ISBN-13: 978-14027-932-26. (Reviewed May 2014.)

Numbers Are Forever, by Liz Strachan. Constable, March 2014. ISBN-13: 978-14721-110-43.

**On Leibniz: Expanded Edition,* by Nicholas Rescher. University of Pittsburgh Press, June 2013. ISBN-13: 978-08229-621-82.

**Origins of Mathematical Words: A Comprehensive Dictionary of Latin, Greek, and Arabic Roots,* by Anthony Lo Bello. Johns Hopkins University Press, November 2013. ISBN-13: 978-14214-109-82.

Parables, Parabolas and Catastrophes: Conversations on Mathematics, Science and Philosophy, by

René Thom. Translated by Roy Lisker and edited by S. Peter Tsatsanis. Thombooks Press, November 2014 (distributed only by amazon.ca or amazon.com). ISBN-13:978-09939-269-07.

The Perfect Theory: A Century of Geniuses and the Battle over General Relativity, by Pedro G. Ferreira. Houghton Mifflin Harcourt, February 2014. ISBN-13: 978-05475-548-91.

Philosophy of Mathematics in the Twentieth Century, by Charles Parsons. Harvard University Press, March 2014. ISBN-13: 978-06747-280-66.

Pearls from a Lost City: The Lvov School of Mathematics, by Roman Duda (translated by Daniel Davies). AMS, July 2014. ISBN-13: 978-14704-107-66.

Probably Approximately Correct: Nature's Algorithms for Learning and Prospering in a Complex World, by Leslie Valiant. Basic Books, June 2013. ISBN-13: 978-04650-327-16. (Reviewed November 2014.)

Professor Stewart's Casebook of Mathematical Mysteries, by Ian Stewart. Basic Books, October 2014. ISBN-13: 978-04650-549-78.

Quantum Computing since Democritus, by Scott Aaronson. Cambridge University Press, March 2013. ISBN-13: 978-05211-995-68. (Reviewed November 2014.)

Ramanujan's Place in the World of Mathematics: Essays Providing a Comparative Study, by Krishnaswami Alladi. Springer, 2013. ISBN: 978-81322-076-65.

The Simpsons and Their Mathematical Secrets, by Simon Singh. Bloomsbury, October 2013. ISBN-13: 978-14088-353-02. (Reviewed January 2015.)

Struck by Genius: How a Brain Injury Made Me a Mathematical Marvel, by Jason Padgett and Maureen Ann Seaberg. Houghton Mifflin Harcourt, April 2014. ISBN-13: 978-05440-456-06.

Synthetic Philosophy of Contemporary Mathematics, by Fernando Zalamea. Urbanomic/Sequence Press, January 2013. ISBN: 978-09567-750-16.

The Tower of Hanoi: Myths and Maths, by Andreas M. Hinz, Sandi Klavzar, Uros Milutinovic, and Ciril Petr. Birkhäuser, January 2013. ISBN: 978-303-48023-69.

Turing: Pioneer of the Information Age, by Jack Copeland. Oxford University Press, January 2013. ISBN-13: 978-01996-397-93. (Reviewed September 2014.)

Turing's Cathedral: The Origins of the Digital Universe, by George Dyson. Pantheon/Vintage, December 2012. ISBN-13: 978-14000-759-97. (Reviewed August 2014.)

Undiluted Hocus-Pocus: The Autobiography of Martin Gardner. Princeton University Press, September 2013. ISBN-13: 978-06911-599-11. (Reviewed March 2014.)

**The War of Guns and Mathematics: Mathematical Practices and Communities in France and Its Western Allies Around World War I,* by David Aubin and Catherine Goldstein. AMS, October 2014. ISBN-13: 978-14704-146-96.

Why Is There Philosophy of Mathematics At All?, by Ian Hacking. Cambridge University Press, April 2014. ISBN-13: 978-11070-501-74. (Reviewed in this issue.)

Zombies and Calculus, by Colin Adams. Princeton University Press, September 2014. ISBN-13: 978-06911-619-07.

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at www.ams.org/cgi-bin/mathcal-submit.pl. The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at www.ams.org/mathcal/.

March 2015

* 2–6 **Forty-Sixth Southeastern International Confereence on Combinatorics, Graph Theory and Computing**, Florida Atlantic University, Boca Raton, Florida

Description: Celebrating its 46th year, the Conference brings together mathematicians and others interested in combinatorics, graph theory and compuuting, and their interactions. The conference lectures and contributed papers, as well as the opportunities for informal conversations, have proved to be of great interest to participants. The Conference is an excellent event for graduate students. The 46th Conference will feature distinguished invited plenary lecturers: sarah-marie belcastro, Smith College; Ezra Brown, Virginia Tech; Jo Ellis-Monaghan, St. Michael's College of Vermont; Gary Gordon, Lafayette College; Heiko Harborth, TU Braunschweig. There will be contributed papers, as well as invited special sessions of papers on: Combinatorial Designs and Applications (organized by Spyros Magliveras); Graph Polynomials and their Applications (Jo Ellis-Monaghan and Iain Moffat); Matroids (Gary Gordon).

Location: math.fau.edu/cgtc/cgtc46/

* 2–13 **The interrelation between mathematical physics, number theory and non-commutative geometry**, Erwin Schrodinger International Institute for Mathematical Physics, Vienna, Austria.

Description: Over the past few years, the newly discovered relation between Feynman integrals in quantum field theory and Grothendieck's theory of motives of algebraic varieties has become a topic of growing importance in theoretical and mathematical physics. In particular, the field has developed along different parallel lines, and there is a pressing need to create more interaction between researchers involved in the various aspects of this area of research. In particular, we single out four different directions among which we hope to stimulate new interactions: (1) Motives and periods in perturbative quantum field theory; (2) amplitudes and super Yang-Mills theory; (3) algebraic structures in renormalization; (4) quantum field theory on noncommutative spacetimes. There will be a one week Master Class followed by a one week Research Meeting in which these topics will be the focus.

Information: www.esi.ac.at/activities/events/2015/the-interrelation-between-mathematical-physics-number-theory-and-non-commutative-geometry.

* 16–20 **LMS Invited Lectures 2015: Cluster algebras and integrable systems**, Durham University, Durham, United Kingdom.

Description: The minicourse consisting of ten lectures will be devoted to the fast growing area on the intersection of cluster algebras theory and integrable systems, and interactions of these with other areas of mathematics and theoretical physics. The course will be fully accessible to postgraduate students and non-specialists interested in the topic. There will also be supplementary lectures by Robert Marsh (Leeds), Andy Hone (Kent), and Sebastian Franco (CCNY).

Information: www.maths.dur.ac.uk/users/pavel.tumarkin/LMS2015/.

April 2015

* 2–5 **The Second International Conference on Mathematics and Statistics, AUS-ICMS'15**, American University of Sharjah, Sharjah, United Arab Emirates.

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: www.ams.org/.

Description: In cooperation with the American Mathematical Society, AUS-ICMS'15 goal is to offer a forum for researchers and scientists working in all fields of mathematics and statistics from both academia and industry to come together, exchange research ideas and discuss recent developments in mathematical research. The conference promotes interaction between AUS faculty and other researchers from the region and worldwide. The technical program of the AUS-ICMS'15 includes keynote lectures delivered by renowned mathematicians, special sessions and contributed papers. Special sessions are organized in the following areas: Algebras and coalgebras; commutative algebra and its applications; designs, codes and graphs; machine learning and data mining; mathematical and computational methods in biology and medicine; topology, geometry and applications and mathematical finance and probability.
Information: `www.aus.edu/ICMS15`.

* 13–17 **LMS-CMI Research School on Statistical Properties of Dynamical Systems**, Loughborough University, Loughborough, UK
Description: The aim of the school is to provide research students with knowledge and training on basic tools needed to study statistical properties of dynamical systems. Students are not assumed to have any knowledge in the subject. In the first two days the lectures will aim at general background and tools needed in the area. More advanced topics related to current advances in the area will start to appear on the third day of the school. The lecturers will be Mark Demers (Fairfield, USA), Carlangelo Liverani (Rome) and Ian Melbourne (Warwick). Tutorial support is also provided. The deadline for registration is January 30, 2015.
Location: `www.claymath.org/events/statistical-properties-dynamical-systems`

* 15–17 **BioDynamics 2015**, John McIntyre Conference Centre, Edinburgh, United Kingdom.
Description: This will be the second in a series of international workshops designed to bring together biologists, mathematicians, clinicians, physicists, and computer scientists who are interested in dynamical systems in the biological and medical sciences. They provide a unique and exciting forum for multidisciplinary interactions, which we hope will lead to rewarding collaborations between theoretical, experimental, and clinical scientists. There will be a number of keynote presentations delivered by world-leading scientists, who will talk about their cutting-edge current research and highlight important future challenges within their field.
Information: `www.bio-dynamics.org/events/biodynamics-2015-0`.

* 18–19 **Underrepresented Students in Topology and Algebra Research Symposium (USTARS)**, Florida Gulf Coast University, Fort Myers, Florida.
Description: USTARS creates a venue where Algebra and Topology graduate students from underrepresented groups present their work and form research and social support networks with other mathematicians with related research interests. This event is a two-day research symposium which will consist of underrepresented speakers giving 30-minute research talks. These presentations will run in parallel sessions and will be divided by topics based on a broad definition of Algebra and Topology. In addition, two distinguished graduate students and one invited faculty mentor will each give one-hour talks and a research poster session featuring invited undergraduate students will also be included. The event will close with a panel discussion addressing critical transitions of undergraduate and graduate students.
Information: `www.ustars.org`.

* 27–28 **ICMM 2015 International Conference on Mathematics and Mechanics**, Paris, France.
Description: The aim of this conference is to promote research in the field of Mathematics and Mechanics. Another goal is to facilitate exchange of new ideas in these fields and to create a dialogue between scientists and practitioners.
Information: `mathematics.conference-site.com`.

May 2015

* 14–16 **The 9th International Conference on Differential Equations and Dynamical Systems**, Dallas Campus (1910 Pacific Place, Dallas, TX 75201) and hosted by the Mathematics Department of Texas A&M University-Commerce, Dallas, TX
Description: The 9th International Conference on Differential Equations and Dynamical Systems will be held in Dallas, Texas, USA, May 14–16, 2015. The objective of this conference is to share the recent progress and advances in the theory and applications of differential equations and dynamical systems. Researchers and educators in all fields of differential equations, functional equations, difference equations, applied analysis, dynamics and applications including modeling, control and computations are invited to attend and share their research results. This conference provides a unique international forum where exciting interactions and communications take place among researchers and it brings fruitful direction and collaborations to the world community. There will be several invited lectures covering recent developments in problems of current interest and important applications in various disciplines.
Location: `www.watsci.org/deds2015/`

* 15–17 **Seymour Sherman Lecture and Conference: Probability and Statistical Physics**, Indiana University, Bloomington, Indiana.
Description: This conference will feature 16 young researchers at the forefront of different subfields of probability relating to Sherman's work, mainly, statistical physics and probability. The Sherman Lecturer will be Andrea Montanari (Stanford).
Support: Full support is available for graduate students, who should apply by March 15, 2015.
Information: `pages.iu.edu/~rdlyons/sherman/2015.html`.

* 18–23 **Recent Advances in Kähler Geometry**, Vanderbilt University, Nashville, Tennessee.
Description: Kähler geometry is an area with deep roots and very rapid and diverse advances. Remarkable progress has been made recently in different directions such as the study of Kähler-Einstein metrics on Fano manifolds, complete Calabi-Yau manifolds, as well as extremal Kähler metrics and their link with algebraic-geometric notions of stability on polarized manifolds. The main objective of the conference is to bring together leading experts in the separate but related fields of differential geometry, geometric analysis and algebraic geometry, to present recent results and future directions of research in the field. The conference is organized in conjunction with the Shanks Lecture Series. The 30th Shanks Lecturer is going to be Professor Shing-Tung Yau, Harvard University.
Information: `www.math.vanderbilt.edu/~kahlergeometry`.

* 22–24 **Lehigh University Geometry and Topology Conference, Emphasizing Algebraic Topology**, Lehigh University, Bethlehem, Pennsylvania.
Description: This year the conference will emphasize algebraic topology, in celebration of Don Davis's 70th birthday.
Principal speakers: Gunnar Carlsson (Stanford University and Ayasdi Corp.), Michael Farber (Queen Mary University of London), Paul Goerss (Northwestern University), Kathryn Hess (École Polytechnique Fédérale de Lausanne), Mike Hopkins (Harvard University), Doug Ravenel (University of Rochester), Dennis Sullivan (Stony Brook University and City University of New York). Also contributed talks.
Supporter: The conference is supported by Journal of Differential Geometry, Lehigh University, and NSF. Limited travel support is available, and the priority will be given to recent PhDs, current graduate students and members of underrepresented groups.
Information: `www.lehigh.edu/~dlj0/geotop.html`.

* 25–30 **Existence of Solutions for a Quasilinear Elliptic Equation with Nonlocal Boundary Conditions on Time Scales**, Prépartoiry School of Science and savings commertielles sciences and managements, Tlemcen, Algeria.
Description: Abstract: The purpose of this work is the construction upper and lower solutions for a class of second order quasilinear elliptic equation subject to nonlocal boundary conditions.

* 26–29 **Lebanese International Conference on Mathematics and Applications(LICMA.15)**, Lebenese University, Faculty of sciences, Haddath, Beirut, Lebanon.
Description: LICMA '15 will witness a large scope of mathematical areas and applications. It aims to promote research interests in different fields of pure and applied mathematics and their diverse applications. In addition, it provides a forum for the exchange of ideas and the latest results within a multidisciplinary setting.
Information: www.licma.net.

June 2015

* 1–5 **Asymptotic Problems: Elliptic and Parabolic Issues**, Vilnius, Lithuania.
Description: Besides asymptotic analysis of elliptic and parabolic problems, the topics of the conference include fluid mechanics, free boundary problems, evolution problems in general, calculus of variations, homogenization, modeling and numerical analysis.
Information: secure.estravel.eu/asymptotic_problems_2015/.

* 1–19 **Dynamics of Multi-Level systems**, Max Planck Institute for the Physics of Complex Systems, Dresden, Germany.
Description: The emergence of levels is a key feature in a variety of complex systems. Experts from different disciplines will discuss current trends in the theoretical understanding of multi-level and multiscale systems and present applications from physics and chemistry, sociology and economics, ecology and neuroscience. Workshop topics include: multiscale methods; hierarchies in dynamical systems; physics of information; self-organization and level identification; multilevel networks; coarse graining; aggregation methods The school will provide the opportunity to learn key techniques and theories from mathematics and physics for dealing with multiscale and multilevel systems.
Information: Applications for the school and/or workshop have to be submitted via the school and workshop webpage: www.pks.mpg.de/~dymult15/ until February 28, 2015.

* 3–6 **International Conference on Recent Advances in Pure and Applied Mathematics (ICRAPAM 2015)**, Istanbul Commerce University, Sutluce, Istanbul, Turkey.
Description: International Conference on Recent Advances in Pure and Applied Mathematics (ICRAPAM 2015) is aimed to bring researchers and professionals to discuss recent developments in both theoretical and applied mathematics and to create a professional knowledge exchange platform between mathematicians. Prospective authors are invited to submit their one-page abstracts on the related, but not limited, following topics of interest: Numerical analysis, ordinary and partial differential equations, scientific computing, boundary value problems, approximation theory, sequence spaces and summability, real analysis, functional analysis, fixed point theory, optimization, geometry, computational geometry, differential geometry, applied algebra, combinatorics, complex analysis, flow dynamics, control, mathematical modelling in scientific disciplines, computing theory, numerical and semi-numerical algorithms, game theory, operations research, optimization techniques, symbolic computation.
Information: www.icrapam.org.

* 11–15 **Tenth Panhellenic Logic Symposium**, University of Aegean, Samos, Greece.
Description: The Panhellenic Logic Symposium, a biennial scientific event established in 1997, is open to researchers worldwide who work in logic broadly conceived. PLS 10 is hosted by the Department of Mathematics of the University of the Aegean, Greece, and will take place at the island of Samos. Please see call for papers at the webpage. Special attention will be given to work by young researchers and PhD students.
Information: samosweb.aegean.gr/pls10/.

* 14–18 **PDEs, Potential Theory and Function Spaces**, Linköping, Sweden.
Description: This conference is organized in honour of Lars Inge Hedberg (1935–2005). The aim of the conference is to present the state-of-the-art in these three closely connected fields, which have a long tradition at Linköping University. Our goal is to bring together experts as well as young researchers interested in these topics, and to foster applications and collaboration between these subjects, as well as within them. The conference will consist of a number of plenary lectures (45 min.) and of short talks (20–25 min.) in parallel sessions in the above three topics. There will be a possibility for the participants to submit abstracts to be considered for the short talks. We will try to accommodate as many talks as we can but of course the number of slots is limited. There will be no conference fee.
Information: conferences.mai.liu.se/PPF-2015/.

* 15–26 **Geometric and Computational Spectral Theory–Séminaire de mathématiques supérieures**, Centre de recherches mathématiques, Montréal, Québec, Canada.
Description: The 2015 Séminaire de Mathématiques Supérieures will feature about a dozen minicourses on geometry of eigenvalues, geometry of eigenfunctions, spectral theory on manifolds with singularities, and computational spectral theory. There has been a number of remarkable recent developments in these closely related fields. The goal of the summer school is to shed light on different facets of modern spectral theory and to provide a unique opportunity for graduate students and young researchers to get a "big picture" of this rapidly evolving area of mathematics. The lectures will be given by the leading experts in the subject. The minicourses will be complemented by guided exercises sessions, as well as by several invited talks by the junior participants who have already made important contributions to the field. A particularly novel aspect of the school is the emphasis on the interactions between spectral geometry and computational spectral theory.
Information: www.crm.umontreal.ca/sms/2015/index_e.php.

* 17–19 **Espalia**, "Sapienza" Universitá di Roma, Rome, Italy.
Description: Three Italo-spanish days in PDEs, calculus of variations, and applications.
Information: www.sbai.uniroma1.it/conferenze/espalia2015/homepage/Home.html.

* 20–27 **Physics and Mathematics of Nonlinear Phenomena**, Gallipoli, close to Lecce, South of Italy.
Description: The conference follows the long established tradition of the Nonlinear Physics meetings organized by the research group on Integrable Systems of Universita' del Salento and held in Gallipoli (Lecce, Italy).
Information: More information on previous editions and on the current conference can be found at our website: pmnp.unisalento.it.

July 2015

* 1–3 **The 2015 International Conference of Applied and Engineering Mathematics**, Imperial College London, London, United Kingdom.
Description: The conference ICAEM'15 is held under the World Congress on Engineering 2015.

Organizer: The WCE 2015 is organized by the International Association of Engineers (IAENG), and serves as good platforms for the engineering community members to meet with each other and to exchange ideas. The last IAENG conferences attracted more than one thousand participants from over 30 countries. All submitted papers will be under peer review and accepted papers will be published in the conference proceeding (ISBN: 978-988-19253-4-3). The abstracts will be indexed and available at major academic databases. The accepted papers will also be considered for publication in the special issues of the journal Engineering Letters, in IAENG journals and in edited books.

Deadlines: Calls for Manuscript Submissions Draft Paper Submission: March 6, 2015. Camera-Ready Papers Due & Registration: March 31, 2015. WCE 2015: July 1–3, 2015.

Information: `www.iaeng.org/WCE2015/ICAEM2015.html`.

* 6–10 **Computational Geometric Topology in Arrangement Theory**, The Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, RI

Description: This workshop will bring together mathematicians working on combinatorial, geometric and topological properties of arrangements. In addition to fundamental open problems in the area, we will emphasize connections to tropical geometry, configuration spaces, and applications (coding theory, statistical eco-nomics, topological robotics), building bridges between those working on different aspects of the area. The main aim of the workshop is to discuss computational issues that arise in studying topological and combinatorial invariants of arrangements. The workshop will be comprised of two main activities: A series of short courses by leading experts and research or expository talks.

Location: `icerm.brown.edu/topical_workshops/tw15-4-cgtat/`

* 6–10 **GAP XIII Pohang “Derived Geometry”—Geometry and Physics**, IBS Center for Geometry and Physics, Pohang, South Korea

Description: GAP (Geometry and Physics—Séminaire itinérant) is a series of conferences and summer schools held annually since 2003 in various countries around the world. GAP XIII Pohang is the thirteenth edition, focused on Derived Geometry.

Mini-Course Speakers: Kenji Fukaya, Mikhail Kapranov, Erik Verlinde Plenary

Speakers: Aleksel Bondal, Andrei Caldararu, Ionut Ciocan-Fontanine, Andrea D’Agnolo, Vladimir Hinich, Ludmil Katzarkov, Si Li, Marco Manetti, David Nadler*, Ryszard Nest, Alexander Polishchuk*, Bertrand Toën, Éric Vasserot, Gabriele Vezzosi (*: to be confirmed)

Location: `cgp.ibs.re.kr/conferences/gapxiii/`

* 6–10 **Second International Conference New Trends in the Applications of Differential Equations in Sciences (NTADES2015)**, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria

Description: Second International Conference on New Trends of Differential Equations in Sciences is organized by the Department of Differential Equations and Mathematical Physics at Institute of Mathematics and Informatics, Bulgarian Academy of Sciences. The conference consists of invited and contributed papers. Prospective authors are invited to submit their papers on topics included but not limited to applications of differential equations in: Mathematical Physics; Mathematical Finance; Mathematical Biology; Nonlinear waves; Fractional Analysis; Neuroscience.

Deadline for Submission of Abstracts: April 30th, 2015.

Chair of the Program Committee: Prof. Angela Slavova

Location: `www.math.bas.bg/ntades`

* 13–31 **AMS Summer Institute in Algebraic Geometry**, University of Utah, Salt Lake City, Utah.

Description: The forty-fifth Summer Research Institute, sponsored by the American Mathematical Society and organized in collaboration with the Clay Mathematics Institute, will be devoted to algebraic geometry. The goal of the three-week institute is to review major achievements in and around algebraic geometry in the past decade, and bring the attendants to the forefront of the relevant subjects. The three weeks will be roughly focused respectively on 1) Analytic methods, birational geometry and classification, commutative algebra and computational geometry, Hodge theory, singularities, and characteristic p methods. 2) Derived algebraic geometry, derived categories, geometric representation theory, Gromov-Witten and Donaldson-Thomas theories, mirror symmetry, tropical geometry. 3) Rational points and diophantine problems, p-adic Hodge theory, arithmetic fundamental groups, topology of algebraic varieties, cycles, and cohomology theories.

Deadline: January 15, 2015.

Information:
`sites.google.com/site/2015summerinstitute`.

* 15–17 **International Conference on Applied Statistics 2015**, Pattaya, Thailand

Description: The ICAS 2015: International Conference on Applied Statistics aims to bring together the national and international statisticians, other scientists, educators and students from academia, industries, government, and research institutes to exchange and share their experiences and research results about all aspects of statistics and applied statistics. It also provides a platform for them to mutually interact and share their thoughts on the recent innovations, practical challenges encountered and the solutions adopted in the difference areas of the subject. All accepted full papers which are presented at the conference will be published in our conference proceedings which will be available for download a couple of days after the last day of conference. The selected papers will be published in Thailand Statistician (ISSN: 1985-9057), indexed in Zentralblatt fur Mathematik and TCI.

Location: `icas2015.kmutt.ac.th/`

* 20–August 4 **XVIII Summer Diffiety School on Geometry of PDEs**, Piccolo Hotel Tanamalia, Lizzano in Belvedere (BO), Italy.

Description: The aim of this permanent School is to introduce undergraduate and PhD. students in Mathematics and Physics as well as post-doctoral researchers in a specific area of Mathematics and Teoretical Physics: Secondary Calculus. A diffiety is a geometrical object that properly formalizes the concept of the solution space of a given system of (nonlinear) PDEs, much as an algebraic variety does with respect to solutions of a given system of algebraic equations. Secondary Calculus is a natural diffiety analogue of the standard calculus on smooth manifolds, and as such leads to a very rich general theory of nonlinear PDEs. Moreover, it appears to be a natural language for quantum physics, just as the standard calculus is the natural language for classical physics.

Information: `www.levi-civita.org/Activities/DiffietySchools/xviii-summer-diffiety-school`.

* 28–30 **Mathematics in Data Science- Exploring the Role of the Mathematical Sciences in an Evolving Discipline**, Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island.

Description: The goal of this workshop is to bring together mathematicians and data scientists to participate in a discussion of current methods and outstanding problems in data science. The workshop is particularly aimed at mathematicians interested in pursuing research or a career in data science who wish to gain an understanding of this rapidly evolving field and the ways in which mathematics can contribute. Researchers currently working in data science are also encouraged to attend, to share ideas about mathematical methodologies and challenges. A number of experienced data scientists with a variety of backgrounds from academics, national laboratories, and industry (including startups) will be invited. The program will include overview and technical talks, several panels

consisting of practitioners with different experience levels, and one or more poster sessions.
Information: icerm.brown.edu/topical_workshops/tw15-6-mds/.

August 2015

* 4–6 **ICNHBAS [I] Current Trends in Mathematics & its Applications**, Hurghada, Egypt.
Description: ICNHBAS 2015 will be devoted to new challenges and innovations in Fundamental and Applied Sciences and to encourage and promote collaborations in the wide topics. Mathematics Session [I] in this conference will primarily focus on revolutionary approaches and results developed newly in pure and applied mathematics. The Session will include recent hot achievements in the following topics: www.nhbas.com/Mathematics.aspx. The Mathematics Session will consist of plenary lectures, invited and contributed oral and poster presentations, which will be organized to bring new visions and horizons for all the participants
Information: www.nhbas.com/Mathematics.aspx.

* 17–20 **The 8th International Conference on Lattice Path Combinatorics and Applications**, California State Polytechnic University, Pomona (Cal Poly Pomona), Pomona, California.
Description: Topics to be covered (but not limited to): Lattice path enumeration ; plane partitions; Young tableaux; q-calculus; orthogonal polynomials; random walks; nonparametric statistical inference; discrete distributions and urn models; queueing theory; analysis of algorithms; graph theory and applications; self-dual codes and unimodular lattices; bijections between paths and other combinatoric structures.
Information: www.csupomona.edu/~math/CONFERENCE/index.html.

* 24–27 **11th International Symposium on Geometric Function Theory and Applications**, Congress Centre of Ss. Cyril and Methodius University, Ohrid, Republic of Macedonia.
Description: GFTA 2015 is a continuation of series of annual symposia: 2005: Canada, University of Victoria 2006: Romania, Transilvania University of Brasov 2007: Turkey, Istanbul Kultur University 2008: Malaysia, University Kebangsaan Malaysia 2009: Romania, Lucian Blaga University of Sibiu 2010: Bulgaria, Bulgarian Academy of Science 2011: Romania, Babes-Bolyai University of Cluj-Napoca 2012: R. Macedonia, Ss. Cyril and Methodius University in Skopje 2013: Turkey, Isik University 2014: Romania, University of Oradea The aim of the symposium is to bring together leading experts and young researchers working on topics mainly related to Univalent and Geometric Function Theory and to present their recent work to the mathematical community. The Symposium is open to other fields of mathematics.
Information: www.research-publication.com/index.php/gfta-2015.

* 27–29 **The 5th International Conference on Control and Optimization with Industrial Applications**, Baku, Azerbaijan.
Description: The 1st Conference on Control and Optimization with Industrial Applications-COIA.05 was held in 2005 in Botanic Park, Baku, Azerbaijan. As it was noted by specialists this event was the most representative conference on these topics. A number of specialists from over the world participated in COIA.05. The 2nd conference of this series COIA.08 was held in 2008 also in Baku and was also attractive for the specialists from over the world. Considering the interest of the scientists and to attract new countries and institutions, it was decided to organize the 3rd one in the abroad. COIA.2011 was held in Bilkent University, Ankara, Turkey in 2011. There the purpose of the group of mathematicians from Bulgaria. We invite all specialists from over the world to come and participate in the work of this Conference.
Information: www.coia-conf.org.

* 31–September 4 **Numerical Methods for Large-Scale Nonlinear Problems and Their Applications**, The Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, RI
Description: Over the last 20 years or so, Newton-Krylov methods have developed to maturity, allowing effective fully-coupled treatment of a broad range of large-scale nonlinear problems. This development has set the stage for addressing more difficult problems with more challenging features. Additionally, applications for which state-of-the-art Newton-Krylov approaches are inapplicable have recently exposed several basic research questions. At the same time, there remain many problem-specific methods and legacy codes that are still useful and can be regarded as a resource for further development. This workshop will include mathematicians and computer scientists who work on algorithm design, implementation, and analysis, together with disciplinary scientists and engineers who use the algorithms in applications and have a working knowledge of their capabilities, weaknesses, and limitations.
Information: icerm.brown.edu/topical_workshops/tw15-5-nmln/p

September 2015

* 8–13 **International Conference on "Mathematical Analysis, Differential Equations and Their Applications" (MADEA 7)**, Baku, Azerbaijan.
Description: This is Azerbaijan-Turkish-Ukrainian scientific conference in the field of mathematical analysis, differential equations and their applications and organized by Azerbaijan National Aviation Academy, Institute of Mathematics and Mekhanics of NAS of Azerbaijan, Mersin University (Turkey), Institute of Mathematics of NAS of Ukraine, Kyiv Taras Shevchenko National University (Ukraine) and Yu. A. Mitropolskiy International Mathematical Center of NAS of Ukraine.
Scientific Fields: Applied analysis, approximation theory, extremal problems, functional analysis, functional-differential and stochastic equations, functions of real and complex variables, harmonic analysis, integral transformations, interpolation theory, partial differential equations, qualitative and asymptotic methods in the theory of differential equations, summability theory.
Information: madea2015.imm.az.

* 16–21 **13th International Conference of The Mathematics Education for the Future Project: Mathematics Education in a Connected World**, Grand Hotel Baia Verde, Catania, Sicily, Italy.
Description: Our 12th International Conference of the Mathematics Education for the Future Project in 2014 in Montenegro was attended by 174 people from 29 countries. Our conferences bring together many innovative movers and shakers from around the world, and are renowned for their friendly and productive atmosphere. The conference title, Mathematics Education in a Connected World, continues our search for innovative ways in which mathematics, science, computing and statistics education can succeed in our increasingly connected world. We now call for papers and workshops (which can be peer reviewed) with the possibility of future publication in a book or journal. Please email Alan at alan@cdnalma.poznan.pl for further details.
Information: directorymathsed.net/montenegro/AAAFACatania3.pdf.

* 28–October 3 **Semester Workshop: Modular Forms and Curves of Low Genus: Computational Aspects**, Institute for Computational and Experimental Research in Mathematics (ICERM) at Brown University, Providence, Rhode Island.
Description: One of the crowning achievements of number theory in the 20th century is the construction of the modularity correspondence between elliptic curves with rational coefficients and modular forms of weight 2. The consequences of this result resound throughout number theory; for instance, it enables the resolution of certain problems of diophantine equations (e.g., Fermat's last theo-

rem) as well as the systematic tabulation of elliptic curves, which in turn provides the basis for many new conjectures and results. The aim of this workshop is to lay the groundwork for extending this correspondence to curves of small genus over number fields. The general framework for this correspondence is predicted by the Langlands program, but much remains to be made explicit. We will explore theoretical, algorithmic, computational, and experimental questions on both sides of the correspondence, with an eye towards tabulation of numerical data and formulation of precise conjectures.
Information: icerm.brown.edu/sp-f15-w1/.

October 2015

* 12–14 **SIAM Conference on Geometric and Physical Modeling (GDSPM15)**, Sheraton Salt Lake City Hotel, Salt Lake City, Utah.
Description: This biennial joint conference (started in 2009) represents a historic union of these communities, their rich academic and industrial histories, as well as the common intellectual themes that continue to move them forward. Over the past twenty years the meetings of the SIAM Special Interest Activity Group on Geometric Design have been one of the main general international conferences on geometric modeling and related areas, and have been well attended by mathematicians and engineers from academia, industry, and government. Since its inception in 1991, the ACM Symposium on Solid and Physical Modeling has been the primary international forum for disseminating research results and exchanging new ideas in relevant mathematical theory, solid modeling, physical modeling, geometric design, analysis, simulation and processing, shape computing and visualization, and various applications.
Information: www.siam.org/meetings/gdspm15/.

* 19–24 **Semester Workshop: Explicit Methods for Modularity of K3 Surfaces and Other Higher Weight Motives**, Institute for Computational and Experimental Research in Mathematics (ICERM) at Brown University, Providence, Rhode Island.
Description: Only recently has it become feasible to do large scale verification of the predictions of the Langlands program in higher rank cases and to present the results in a way that is accessible widely to mathematicians. Moving from the understanding of Galois representations attached to elliptic curves to those attached to surfaces and other higher-dimensional varieties poses interesting problems in both arithmetic, algebra, geometry, and analysis. In this workshop, we will consider computational and other explicit aspects of modular forms in higher rank. Topics covered will include: K3 surfaces and their connections to modular forms on orthogonal groups, algebraic modular forms associated to classical groups and their computation, and motives arising from general Calabi-Yau varieties accessible to explicit methods, including hypergeometric motives.
Information: icerm.brown.edu/sp-f15-w2/.

November 2015

* 9–13 **Semester Workshop: Computational Aspects of L-functions**, Institute for Computational and Experimental Research in Mathematics (ICERM) at Brown University, Providence, Rhode Island.
Description: This conference will revolve around several themes: the computational complexity of L-functions; statistical problems concerning L-functions, such as the distribution of their values, and zeros, moments of L-functions, statistics and size of ranks in families of elliptic curves; practical implementations of algorithms and their applications to testing various conjectures about L-functions; rigorous and certifiable computations of L-functions. One goal is to stimulate dialogue between theoreticians and computationally minded researchers regarding problems to which computation might provide insight or important confirmation of conjectures. In the other direction, we hope that discussions will lead to new ideas concerning algorithms for L-functions.
Information: icerm.brown.edu/sp-f15-w3/.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

August 2016

* 1–5 **XVI International Conference on Hyperbolic Problems: Theory, Numerics, Applications**, RWTH Aachen University, Aachen, Germany.
Description: The objective of the conference is to bring together scientists with interests in the theoretical, applied, and computational aspects of hyperbolic partial differential equations and of related mathematical models appearing in the area of applied sciences.
Information: www.hyp2016.de.

* 17–19 **Connections for Women: Geometric Group Theory**, Mathematical Sciences Research Institute, Berkeley, California.
Description: This three-day workshop will feature talks by six prominent female mathematicians on a wide range of topics in geometric group theory. Each speaker will give two lectures, separated by a break-out session during which participants will meet in small groups to discuss ideas presented in the first lecture. The workshop is open to all mathematicians.
Information: www.msri.org/workshops/768.

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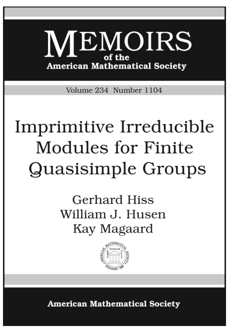
To subscribe to email notification of new AMS publications, please go to www.ams.org/bookstore-email*.*

Algebra and Algebraic Geometry



Imprimitive Irreducible Modules for Finite Quasisimple Groups

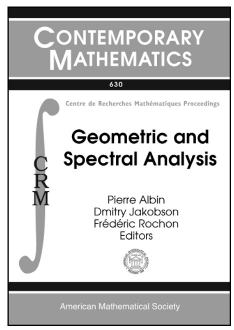
Gerhard Hiss, *Lehrstuhl D für Mathematik, RWTH Aachen University, Germany*, **William J. Husen**, *Ohio State University, Columbus, OH*, and **Kay Magaard**, *University of Birmingham, UK*

Contents: Acknowledgements; Introduction; Generalities; Sporadic groups and the Tits group; Alternating groups; Exceptional Schur multipliers and exceptional isomorphisms; Groups of Lie type: Induction from non-parabolic subgroups; Groups of Lie type: Induction from parabolic subgroups; Groups of Lie type: char$(K) = 0$; Classical groups: char$(K) = 0$; Exceptional groups; Bibliography.

Memoirs of the American Mathematical Society, Volume 234, Number 1104

March 2015, 114 pages, Softcover, ISBN: 978-1-4704-0960-9, LC 2014042350, 2010 *Mathematics Subject Classification:* 20B15, 20C33, 20C34, 20E28; 20B25, 20C15, 20C20, **Individual member US$48**, List US$80, Institutional member US$64, Order code MEMO/234/1104

Analysis



Geometric and Spectral Analysis

Pierre Albin, *University of Illinois at Urbana-Champaign, IL*, **Dmitry Jakobson**, *McGill University, Montreal, Quebec, Canada*, and **Frédéric Rochon**, *Université du Québec à Montréal, Québec, Canada*, Editors

In 2012, the Centre de Recherches Mathématiques was at the center of many interesting developments in geometric and spectral analysis, with a thematic program on Geometric Analysis and Spectral Theory followed by a thematic year on Moduli Spaces, Extremality and Global Invariants.

This volume contains original contributions as well as useful survey articles of recent developments by participants from three of the workshops organized during these programs: Geometry of Eigenvalues and Eigenfunctions, held from June 4–8, 2012; Manifolds of Metrics and Probabilistic Methods in Geometry and Analysis, held from July 2–6, 2012; and Spectral Invariants on Non-compact and Singular Spaces, held from July 23–27, 2012.

The topics covered in this volume include Fourier integral operators, eigenfunctions, probability and analysis on singular spaces, complex geometry, Kähler-Einstein metrics, analytic torsion, and Strichartz estimates.

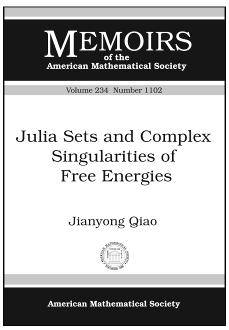
This item will also be of interest to those working in geometry and topology.

This book is co-published with the Centre de Recherches Mathématiques.

Contents: **M. Braverman** and **B. Vertman**, A new proof of a Bismut-Zhang formula for some class of representations; **R. J. Berman**, Tunneling, the Quillen metric and analytic torsion for high powers of a holomorphic line bundle; **V. Guillemin**, **E. Legendre**, and **R. Sena-Dias**, Simple spectrum and Rayleigh quotients; **Y. A. Rubinstein**, Smooth and singular Kähler-Einstein metrics; **G. A. Mendoza**, Complex b-manifolds; **B.-W. Schulze**, Iterative structures on singular manifolds; **Z. Lu** and **J. Rowlett**, The fundamental gap and one-dimensional collapse; **R. Ponge**, The logarithmic singularities of the Green functions of the conformal powers of the Laplacian; **Y. Safarov**, A symbolic calculus for Fourier integral operators; **D. Baskin**, **J. L. Marzuola**, and **J. Wunsch**, Strichartz estimates on exterior polygonal domains; **N. Burq** and **G. Lebeau**, Probabilistic Sobolev embeddings, applications to eigenfunctions estimates; **L. I. Nicolaescu**, Random smooth functions on compact Riemannian manifolds; **B. T. Nguyen**, **D. S. Grebenkov**, and **A. L. Delytsin**, On the exponential decay of Laplacian eigenfunctions in planar domains with branches; **G. Poliquin**, Bounds on the principal frequency of the p-Laplacian.

Contemporary Mathematics, Volume 630

December 2014, 366 pages, Softcover, ISBN: 978-1-4704-1043-8, LC 2014021478, 2010 *Mathematics Subject Classification:* 58J05, 58J40, 58J50, 58J52, 53C55, 60D05, 49R05, 46E35, 53A30, 35Q41, **AMS members US$100.80**, List US$126, Order code CONM/630



Julia Sets and Complex Singularities of Free Energies

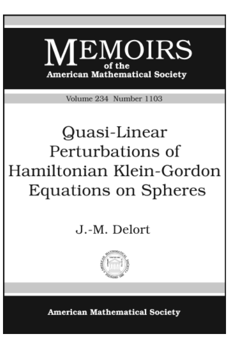
Jianyong Qiao, *School of Science, Beijing University of Posts and Telecommunications, People's Republic of China*

Contents: Introduction; Complex dynamics and Potts models; Dynamical complexity of renormalization transformations; Connectivity of Julia sets; Jordan domains and Fatou components; Critical exponent of free energy; Bibliography.

Memoirs of the American Mathematical Society, Volume 234, Number 1102

March 2015, 89 pages, Softcover, ISBN: 978-1-4704-0982-1, LC 2014041891, 2010 *Mathematics Subject Classification:* 37F10, 37F45; 82B20, 82B28, **Individual member US$45.60**, List US$76, Institutional member US$60.80, Order code MEMO/234/1102

Differential Equations



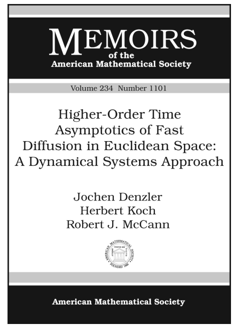
Quasi-Linear Perturbations of Hamiltonian Klein-Gordon Equations on Spheres

J.-M. Delort, *Université Paris-Nord, Villetaneuse, France*

Contents: Introduction; Statement of the main theorem; Symbolic calculus; Quasi-linear Birkhoff normal forms method; Proof of the main theorem; Appendix; Bibliography.

Memoirs of the American Mathematical Society, Volume 234, Number 1103

March 2015, 80 pages, Softcover, ISBN: 978-1-4704-0983-8, LC 2014041958, 2010 *Mathematics Subject Classification:* 35L72, 35S50, 37K45, **Individual member US$40.80**, List US$68, Institutional member US$54.40, Order code MEMO/234/1103



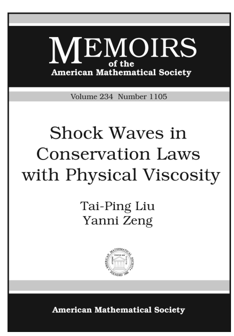
Higher-Order Time Asymptotics of Fast Diffusion in Euclidean Space: A Dynamical Systems Approach

Jochen Denzler, *University of Tennessee, Knoxville, TN*, **Herbert Koch**, *Mathematisches Institut der Universität Bonn, Germany*, and **Robert J. McCann**, *University of Toronto, Ontario, Canada*

Contents: Introduction; Overview of obstructions and strategies, and notation; The nonlinear and linear equations in cigar coordinates; The cigar as a Riemannian manifold; Uniform manifolds and Hölder spaces; Schauder estimates for the heat equation; Quantitative global well-posedness of the linear and nonlinear equations in Hölder spaces; The spectrum of the linearized equation; Proof of Theorem 1.1; Asymptotic estimates in weighted spaces: The case $m < \frac{n}{n+2}$; Higher asymptotics in weighted spaces: The case $m > \frac{n}{n+2}$. Proof of Theorem 1.2 and its corollaries; Appendix A. Pedestrian derivation of all Schauder estimates; Bibliography.

Memoirs of the American Mathematical Society, Volume 234, Number 1101

March 2015, 81 pages, Softcover, ISBN: 978-1-4704-1408-5, LC 2014041890, 2010 *Mathematics Subject Classification:* 35B40, 35K61, 37L10, 58J50, 76S05, **Individual member US$42**, List US$70, Institutional member US$56, Order code MEMO/234/1101



Shock Waves in Conservation Laws with Physical Viscosity

Tai-Ping Liu, *Institute of Mathematics, Academia Sinica, Taipei, Taiwan, and Stanford University, CA*, and **Yanni Zeng**, *University of Alabama at Birmingham, AL*

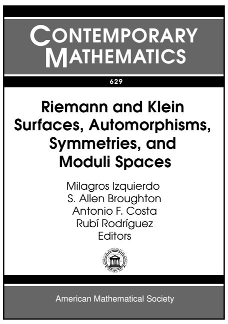
This item will also be of interest to those working in mathematical physics.

Contents: Introduction; Preliminaries; Green's functions for systems with constant coefficients; Green's function for systems linearized along shock profiles; Estimates on Green's function; Estimates on crossing of initial layer; Estimates on truncation error; Energy type estimates; Wave interaction; Stability analysis; Application to magnetohydrodynamics; Bibliography.

Memoirs of the American Mathematical Society, Volume 234, Number 1105

March 2015, 168 pages, Softcover, ISBN: 978-1-4704-1016-2, LC 2014041959, 2010 *Mathematics Subject Classification:* 35K59, 35L67; 35L65, 35Q35, 35A08, 35B40, 35B35, 76W05, 76N15, **Individual member US$53.40**, List US$89, Institutional member US$71.20, Order code MEMO/234/1105

Geometry and Topology



Riemann and Klein Surfaces, Automorphisms, Symmetries and Moduli Spaces

Milagros Izquierdo, *Linköping University, Sweden*, **S. Allen Broughton**, *Rose-Hulman Institute of Technology, Terre Haute, IN*, **Antonio F. Costa**, *Universidad Nacional de Educación a Distancia, Madrid, Spain*, and **Rubí E. Rodríguez**, *Pontificia Universidad Católica de Chile, Santiago, Chile*, Editors

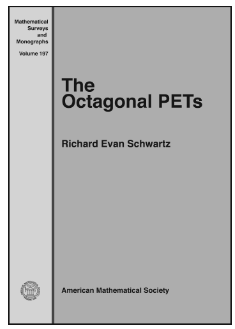
This volume contains the proceedings of the conference on Riemann and Klein Surfaces, Symmetries and Moduli Spaces, in honor of Emilio Bujalance, held from June 24–28, 2013, at Linköping University.

The conference and this volume are devoted to the mathematics that Emilio Bujalance has worked with in the following areas, all with a computational flavor: Riemann and Klein surfaces, automorphisms of real and complex surfaces, group actions on surfaces and topological properties of moduli spaces of complex curves and Abelian varieties.

Contents: **L. Beshaj**, **T. Shaska**, and **C. Shor**, On Jacobians of curves with superelliptic components; **S. A. Broughton**, Superelliptic surfaces as p-gonal surfaces; **S. A. Broughton** and **A. Wootton**, Exceptional automorphisms of (generalized) super elliptic surfaces; **A. Carocca**, **R. E. Rodríguez**, and **A. M. Rojas**, Symmetric group actions on Jacobian varieties; **F.-J. Cirre** and **R. A. Hidalgo**, Normal coverings of hyperelliptic real Riemann surfaces; **M. D. E. Conder**, Large group actions on surfaces; **M. D. E. Conder** and **B. P. Zimmermann**, Maximal bordered surface groups versus maximal handlebody groups; **A. F. Costa**, **M. Izquierdo**, and **A. M. Porto**, Maximal and non-maximal NEC and Fuchsian groups uniformizing Klein and Riemann surfaces; **J. J. Etayo** and **E. Martínez**, On the minimum genus problem on bordered Klein surfaces for automorphisms of even order; **J. Gilman**, Computing adapted bases of conformal automorphism groups of Riemann surfaces; **E. Girondo**, **D. Torres-Teigell**, and **J. Wolfart**, Fields of definition of uniform dessins on quasiplatonic surfaces; **V. G. Gutiérrez** and **S. López de Medrano**, Surfaces as complete intersections; **G. Gromadzki** and **R. A. Hidalgo**, Conjugacy classes of symmetries of compact Kleinian 3-manifolds; **G. Gromadzki**, **A. Weaver**, and **A. Wootton**, Connectivity and dimension of the p-locus in moduli space; **G. Gromadzki** and **X. Zhao**, Free degree of periodic self-homeomorphisms of compact bordered orientable surfaces; **F. Herrlich**, p-adic origamis; **G. A. Jones**, Regular dessins with a given automorphism group; **M. Kreuzer** and **G. Rosenberger**, Growth in Hecke groups; **J. Rodríguez**, Some results on abelian groups of automorphisms of compact Riemann surfaces; **R. E. Rodríguez**, Abelian varieties and group actions; **D. Singerman**, The remarkable Accola-Maclachlan surfaces; **K. Stokes** and **M. Bras-Amorós**, Patterns in semigroups associated with combinatorial configurations; **P. Turbek**, Computing equations, automorphisms and symmetries of Riemann surfaces.

Contemporary Mathematics, Volume 629

November 2014, 348 pages, Softcover, ISBN: 978-1-4704-1093-3, LC 2014012547, 2010 *Mathematics Subject Classification:* 30Fxx, 14Hxx, 20Hxx, 57Mxx, 14Gxx, 14Kxx, 14Jxx, **AMS members US$91.20**, List US$114, Order code CONM/629



The Octagonal PETs

Richard Evan Schwartz, *Brown University, Providence, RI*

A polytope exchange transformation is a (discontinuous) map from a polytope to itself that is a translation wherever it is defined. The 1-dimensional examples, interval exchange transformations, have been studied fruitfully for many years and have deep connections to other areas of mathematics, such as Teichmüller theory. This book introduces a general method for constructing polytope exchange transformations in higher dimensions and then studies the simplest example of the construction in detail. The simplest case is a 1-parameter family of polygon exchange transformations that turns out to be closely related to outer billiards on semi-regular octagons. The 1-parameter family admits a complete renormalization scheme, and this structure allows for a fairly complete analysis both of the system and of outer billiards on semi-regular octagons. The material in this book was discovered through computer experimentation. On the other hand, the proofs are traditional, except for a few rigorous computer-assisted calculations.

This item will also be of interest to those working in differential equations.

Contents: Introduction; *Friends of the octagonal PETs:* Background; Multigraph PETs; The alternating grid system; Outer billiards on semiregular octagons; Quarter turn compositions; *Renormalization and symmetry:* Elementary properties; Orbit stability and combinatorics; Bilateral symmetry; Proof of the main theorem; The renormalization map; Properties of the tiling; *Metric properties:* The filling lemma; The covering lemma; Further geometric results; Properties of the limit set; Hausdorff convergence; Recurrence relations; Hausdorff dimension bounds; *Topological properties:* Controlling the limit set; The arc case; Further symmetries of the tiling; The forest case; The Cantor set case; Dynamics in the arc case; *Computational details:* Computational methods; The calculations; The raw data; Bibliography.

Mathematical Surveys and Monographs, Volume 197

July 2014, 212 pages, Hardcover, ISBN: 978-1-4704-1522-8, LC 2014006823, 2010 *Mathematics Subject Classification:* 37E20, 37E05, 37E15, **AMS members US$72**, List US$90, Order code SURV/197

Classified Advertisements

Positions available, items for sale, services available, and more

GEORGIA

GEORGIA
Georgia State University
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The Department of Mathematics and Statistics, of Georgia State University invites applications for Limited Term Faculty, Lecturer respectively. Positions require a PhD in Mathematics, Statistics or related field. Submit applications to www.mathjobs.org. An offer of employment will be conditional upon background verification. Georgia State University is one of four Research Universities of the University System of Georgia and an EEO/AA institution.

000010

OHIO

THE OHIO STATE UNIVERSITY
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Columbus, OH

Description: The College of Arts and Sciences at The Ohio State University seeks applications to fill an Eminent Scholar position in Scientific Computation within the Department of Mathematics. The ideal candidate will be someone who will be a leader in the research field of Scientific Computation and will be able to carry out interdisciplinary research forging new connections with other departments at The Ohio State University and other institutions. It is expected that this faculty member will be significantly engaged with the graduate and undergraduate programs in the Mathematics Department e.g. directing PhD, master's thesis and/or directing undergraduate research. This position is partially funded by Ohio State's Discovery Themes Initiative, a significant faculty hiring investment in key thematic areas in which the university can build on its culture of academic collaboration to make a global impact.

Requirements: A doctoral degree in Mathematics is required at the time of application. He or she will have an excellent research record in the area of Scientific Computation and evidence of strong teaching ability.

Application Instructions Applications should be submitted online at www.mathjobs.org. If you cannot apply online, please contact facultysearch@math.ohio-state.edu or write to: Hiring Committee, Department of Mathematics, The Ohio State University, 231 W. 18th Avenue, Columbus, OH 43210. Application Deadline: 01/31/2015.

The Ohio State University is an Equal Opportunity Employer. All qualified applicants will receive consideration for employment without regard to race, color, religion, sex, sexual orientation or identity, national origin, disability status, or protected veteran status.

000011

HONG KONG

HONG KONG
The University of Hong Kong
Department of Mathematics
Associate Professor
Tenure-track Associate Professor/
Assistant Professor (3 posts)
(Ref.: 201401325)

Applications are invited for tenure-track appointment as Associate Professor/Assistant Professor (3 posts) in the Department of Mathematics, to commence from September 1, 2015 or as soon as possible thereafter. The appointment will initially be made on a three-year term basis, with the possibility of renewal and with consideration for tenure during the second three-year contract.

The Department of Mathematics provides a solid general undergraduate education in mathematics, offers supervision in graduate study for students with a strong interest in and a capacity for mathematics, and engages in teaching and research aiming at a high international standing. Information about the Department can be obtained at www.hku.hk/math/.

Candidates in all areas of Applied Mathematics and Mathematical Sciences will be considered, with preference given to those working in the areas of Scientific Computing, Computational Mathematics, Financial Mathematics, Operations Research, and Optimization. The appointees are

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2015 rate is $3.50 per word with a minimum two-line headline. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: March 2015 issue–January 2, 2015; April 2015 issue–January 29, 2015; May 2015 issue–March 2, 2015; June/July 2015 issue–April 29, 2015; August 2015 issue–May 29, 2015; September 2015 issue–June 29, 2015.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02904; or via fax: 401-331-3842; or send email to classads@ams.org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

expected to actively engage in outreach and service.

A globally competitive remuneration package commensurate with qualifications and experience will be offered. At current rates, salaries tax does not exceed 15% of gross income. The appointment will attract a contract-end gratuity and University contribution to a retirement benefits scheme, totalling up to 15 as well as annual leave, and medical benefits. Housing benefits will be provided as applicable.

Applicants should send a completed application form, together with an up-to-date C.V. containing information on educational and professional experience, a complete list of publications, a survey of past research and teaching experience, a research plan for the next few years, and a statement on teaching philosophy by e-mail to `scmath@hku.hk`. They should also arrange for submission, to the same e-mail address as stated above, three reference letters from senior academics. One of these senior academics should be asked to comment on the applicant's ability in teaching, or the applicant should arrange to have an additional reference letter on his/her teaching sent to the same e-mail address as stated above. Please indicate clearly which level they wish to be considered for and the reference number in the subject of the e-mail. Application forms (341/1111) can be downloaded at `www.hku.hk/apptunit/form-ext.doc`. Further particulars can be obtained at `jobs.hku.hk/`. Review of applications will start from February 1, 2015 and continue until all the posts are filled.

The University thanks applicants for their interest, but advises that only shortlisted applicants will be notified of the application result.

The University is an Equal Opportunities Employer and is committed to a no-smoking policy.

000009

INDIA

THE INFOSYS CHAIR VISITING PROFESSORSHIPS
Indian Institute of Science, Bangalore, India

The Infosys Foundation has endowed Visiting Professorships in Mathematics and Physics at the Indian Institute of Science (IISc). The purpose of these Professorships is to bring distinguished scientists in the broad areas of Physical and Mathematical Sciences to the IISc campus for collaborative research interactions. Visiting Professors are expected to spend a total of one year at IISc; visits can be spread out over a period of 4 years. The remuneration package for these Visiting Professorships is internationally competitive. If you are interested in this Professorship, please contact a faculty member at IISc who will then take the process forward. If you would like to nominate a distinguished Physicist or Mathematician for this Professorship, please send a nomination to the Chairman, Division of Physical and Mathematical Sciences, Indian Institute of Science by email: `ichair@admin.iisc.ernet.in`.

000012

ASSISTANCE SOUGHT

In Preparation of Paper Setting Forth Solution to the $3x+1$ Problem.

I believe I have solved the very difficult $3x + 1$ Problem. Reasons I believe this are given below. But I am not an academic mathematician (my degree is in computer science, and I have worked as a researcher in the computer industry). Therefore I feel it is essential that the version of the paper that I submit to a journal is prepared with the assistance of an academic mathematician – preferably a number theorist – who has published at least, say, five papers. In return, I will offer any reasonable consulting fee, and/or generous mention of the mathematician in the Acknowledgements. (This will mean considerable prestige for the mathematician, considering the reputation of the Problem.) If, during the preparation, the mathematician makes a significant contribution to the paper, I will also offer him or her shared authorship.

The paper is "A Solution to the $3x + 1$ Problem" on `occampress.com`.

Reasons I believe I have solved the Problem are: (1) my own repeated checking; (2) the fact that there were well over $2,700$ visits to the paper in 2014, many of them certainly in response to four announcements of the solution that I placed in the AMS "Notices" and SIAM "News" and in response to my Letter to the Editor in the Nov. issue of "Notices", which gave the online address of the paper. In all these visits I received only four claims of errors. Each of these turned out to be a result of misunderstandings of a fundamental proof. Refutations of these claims are now a part of the paper.

Peter Schorer, `peteschorer@gmail.com`.

000008

PUBLICATIONS FOR SALE

Books for sale on amazon.com.

A. J. Jerri: 1. *Intro to Integral Equations with Applications*, 2. *Introduction to Wavelets* (Both with Students Solutions Manuals), and 3. *Advances in the Gibbs Phenomenon.*

000074

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000001

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000049

General Information Regarding Meetings & Conferences of the AMS

Speakers and Organizers: The Council has decreed that no paper, whether invited or contributed, may be listed in the program of a meeting of the Society unless an abstract of the paper has been received in Providence prior to the deadline.

Special Sessions: The number of Special Sessions at an Annual Meeting is limited. Special Sessions at annual meetings are held under the supervision of the Program Committee for National Meetings and, for sectional meetings, under the supervision of each Section Program Committee. They are administered by the associate secretary in charge of that meeting with staff assistance from the Meetings and Conferences Department in Providence. (See the list of associate secretaries on page 216 of this issue.)

Each person selected to give an Invited Address is also invited to generate a Special Session, either by personally organizing one or by having it organized by others. Proposals to organize a Special Session are sometimes solicited either by a program committee or by the associate secretary. Other proposals should be submitted to the associate secretary in charge of that meeting (who is an ex officio member of the program committee) at the address listed on page 216. These proposals must be in the hands of the associate secretary at least seven months (for sectional meetings) or nine months (for national meetings) prior to the meeting at which the Special Session is to be held in order that the committee may consider all the proposals for Special Sessions simultaneously. Special Sessions must be announced in the *Notices* in a timely fashion so that any Society member who so wishes may submit an abstract for consideration for presentation in the Special Session.

Talks in Special Sessions are usually limited to twenty minutes; however, organizers who wish to allocate more time to individual speakers may do so within certain limits. A great many of the papers presented in Special Sessions at meetings of the Society are invited papers, but any member of the Society who wishes to do so may submit an abstract for consideration for presentation in a Special Session, provided it is submitted to the AMS prior to the special early deadline for consideration. Contributors should know that there is a limit to the size of a single Special Session, so sometimes all places are filled by invitation. An author *may* speak by invitation in more than one Special Session at the same meeting. Papers submitted for consideration for inclusion in Special Sessions but not accepted will receive consideration for a contributed paper session, unless specific instructions to the contrary are given.

The Society reserves the right of first refusal for the publication of proceedings of any Special Session. If published by the AMS, these proceedings appear in the book series *Contemporary Mathematics*. For more detailed information on organizing a Special Session, see `www.ams.org/meetings/specialsessionmanual.html`.

Contributed Papers: The Society also accepts abstracts for ten-minute contributed papers. These abstracts will be grouped by related *Mathematical Reviews* subject classifications into sessions to the extent possible. The title and author of each paper accepted and the time of presentation will be listed in the program of the meeting. Although an individual may present only one ten-minute contributed paper at a meeting, any combination of joint authorship may be accepted, provided no individual speaks more than once.

Other Sessions: In accordance with policy established by the AMS Committee on Meetings and Conferences, mathematicians interested in organizing a session (for either an annual or a sectional meeting) on employment opportunities inside or outside academia for young mathematicians should contact the associate secretary for the meeting with a proposal by the stated deadline. Also, potential organizers for poster sessions on a topic of choice should contact the associate secretary before the deadline.

Abstracts: Abstracts for all papers must be received by the meeting coordinator in Providence by the stated deadline. Unfortunately, late papers cannot be accommodated.

Submission Procedures: Visit the Meetings and Conferences homepage on the Web at `www.ams.org/meetings` and select "Submit an abstract".

Site Selection for Sectional Meetings

Sectional meeting sites are recommended by the associate secretary for the section and approved by the Secretariat. Recommendations are usually made eighteen to twenty-four months in advance. Host departments supply local information, ten to fifteen rooms with overhead projectors and a laptop projector for contributed paper sessions and Special Sessions, an auditorium with twin overhead projectors and a laptop projector for Invited Addresses, space for registration activities and an AMS book exhibit, and registration clerks. The Society partially reimburses for the rental of facilities and equipment and for staffing the registration desk. Most host departments volunteer; to do so, or for more information, contact the associate secretary for the section.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Washington, District of Columbia

Georgetown University

March 7–8, 2015

Saturday – Sunday

Meeting #1107

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: January 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: March 2015
Issue of *Abstracts*: Volume 36, Issue 2

Deadlines

For organizers: Expired
For abstracts: January 20, 2015

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Frederico Rodriguez Hertz, Pennsylvania State University, *Random dynamics and a formula for Furstenberg entropy*.

Nancy Hingston, The College of New Jersey, *Loop products, Poincare duality and dynamics*.

Simon Tavaré, Cambridge University, *Cancer by the numbers* (Einstein Public Lecture in Mathematics).

Yitang Zhang, University of New Hampshire, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Algebra and Representation Theory (Code: SS 13A), **Ela Celikbas** and **Olgur Celikbas**, University of Connecticut, and **Frank Moore**, Wake Forest University.

Algebraic Structures Motivated by and Applied to Knot Theory (Code: SS 18A), **Jozef H. Przytycki**, George Washington University, and **Radmilla Sazdanovic**, North Carolina State University.

Asymptotic Problems for Stochastic Processes and PDEs (Code: SS 19A), **Sandra Cerrai**, **Dmitry Dolgopyat**, **Mark Freidlin**, and **Leonid Koralov**, University of Maryland.

Bases and Frames in Hilbert Spaces and Applications (Code: SS 26A), **Laura De Carli**, Florida International University.

Characterizing Uncertainty for Modeling Physical Processes (Code: SS 21A), **Ali Arab**, Georgetown University.

Closure Operations in Commutative Algebra (Code: SS 28A), **Neil Epstein**, George Mason University, and **Lance Edward Miller**, University of Arkansas.

Computable Structure Theory (Code: SS 8A), **Rumen Dimitrov**, Western Illinois University, **Valentina Harizanov**, George Washington University, and **Russell Miller**, Queens College and Graduate Center, City University of New York.

Conceptual Mathematical Models in Climate Science (Code: SS 5A), **Hans Engler** and **Hans Kaper**, Georgetown University.

Convexity and Combinatorics (Code: SS 9A), **Jim Lawrence** and **Valeriu Soltan**, George Mason University.

Crossing Numbers of Graphs (Code: SS 3A), **Paul Kainen**, Georgetown University.

Data Assimilation: Recent Progress in Theory, Methods and Applications (Code: SS 23A), **Evelyn M. Lunasin** and **Reza Malek-Madani**, United States Naval Academy.

Difference Equations and Applications (Code: SS 32A), **Michael Radin**, Rochester Institute of Technology, and **Steven J. Miller**, Williams College.

Dynamical Systems Models of Physiological Processes (Code: SS 27A), **Paula Grajdeanu**, Shenandoah University, and **Talitha Washington** and **Abdul-Aziz Yakubu**, Howard University.

Geometric Structures on Low-Dimensional Manifolds and their Invariants (Code: SS 24A), **Cagatay Kutluhan**, University at Buffalo, and **Thomas E. Mark** and **Bulent Tosun**, University of Virginia.

History and Philosophy of Mathematics (Code: SS 15A), **V. Frederick Rickey**, West Point Military Academy, and **James J. Tattersall**, Providence College.

Inverse Problems for Non-destructive Testing (Code: SS 22A), **Nicolas Valdivia**, Naval Research Laboratory.

Iterated Integrals and Applications (Code: SS 12A), **Ivan Horozov**, Washington University in St. Louis.

Mathematical Fluid Dynamics and Turbulence (Code: SS 17A), **Zachary Bradshaw**, University of British Columbia, **Aseel Farhat**, Indiana University, and **Michele Coti Zelati**, University of Maryland.

Nonlinear Dispersive and Wave Equations with Applications to Fluids (Code: SS 14A), **Pierre Germain** and **Zaher Hani**, New York University, and **Benoit Pausader**, Princeton University.

Nonlinear Partial Differential Equations in Sciences and Engineering (Code: SS 16A), **Lorena Bociu**, North Carolina State University, **Ciprian Gal**, Florida International University, and **Daniel Toundykov**, University of Nebraska.

Number Theory in Ergodic Theory and Dynamical Systems (Code: SS 29A), **Joe Herning**, Northern Virginia Community College, **Erblin Mehmetaj**, George Washington University and Georgetown University, **E. Arthur Robinson Jr.**, George Washington University, and **Tyler White**, Northern Virginia Community College.

Operator Theory on Analytic Function Spaces (Code: SS 11A), **Robert F. Allen**, University of Wisconsin, La Cross, and **Flavia Colonna**, George Mason University.

Optimization Theory, Algorithms and Applications (Code: SS 20A), **Olga Brezhneva**, Miami University, Oxford, OH, and **Igor Griva**, George Mason University.

Patterns in Permutations and Words (Code: SS 30A), **Alexander Burstein**, Howard University.

Qualitative Behavior of Solutions of Partial Differential Equations (Code: SS 7A), **Junping Shi**, College of William and Mary, and **Jiuyi Zhu**, John Hopkins University.

Quantum Algebras, Representations, and Categorifications (Code: SS 2A), **Sean Clark** and **Weiqiang Wang**, University of Virginia.

Singularities: Algebraic and Analytic Aspects (Code: SS 31A), **Claudia Miller**, Syracuse University, and **Sophia Vassiliadou**, Georgetown University.

Somos Sequences and Nonlinear Recurrences (Code: SS 10A), **Andrew Vogt**, Georgetown University.

Spatial Evolutionary Models and Biological Invasions (Code: SS 6A), **Judith Miller**, Georgetown University, and **Yuan Lou**, Ohio State University.

Stochastic Analysis and Stochastic PDEs (Code: SS 25A), **Sandra Cerrai**, University of Maryland, and **Frederi Viens**, Purdue University.

Topology in Biology (Code: SS 4A), **Paul Kainen**, Georgetown University.

Within-Host Disease Modeling (Code: SS 1A), **Stanca Ciupe**, Virginia Polytechnic Institute, and **Sivan Leviyang**, Georgetown University.

East Lansing, Michigan

Michigan State University

March 14–15, 2015
Saturday – Sunday

Meeting #1108
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: January 2015

Program first available on AMS website: January 29, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 2

Deadlines

For organizers: Expired
For abstracts: January 20, 2015

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Philippe Di Francesco, University of Illinois, *Integrable Combinatorics*.

Alexander Furman, University of Illinois at Chicago, *Hidden Symmetries of Some Groups*.

Vera Mikyoung Hur, University of Illinois at Urbana-Champaign, *Breaking the Waves*.

Mihnea Popa, Northwestern University, *Recent Results on Holomorphic One-forms*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Combinatorics (Code: SS 19A), **Carolina Benedetti**, **Peter Magyar**, and **Bruce Sagan**, Michigan State University.

Approximation Theory in Signal Processing and Computer Science (Code: SS 5A), **Mark Iwen**, Michigan State University, **Rayan Saab**, University of California San Diego, and **Aditya Viswanathan**, Michigan State University.

Arithmetic of Hyperelliptic Curves (Code: SS 3A), **Tony Shaska**, Oakland University.

Calculus of Variations, Nonlinear Partial Differential Equations, and Applications (Code: SS 31A), **Moxun Tang** and **Baisheng Yan**, Michigan State University, and **Zhengfang Zhou**, MIchigan State University.

Combinatorics, Geometry, and Representation Theory of Homogeneous Spaces (Code: SS 36A), **Mahir Bilen Can** and **Michael Joyce**, Tulane University, and **Miriam Logan**, Bowdoin College.

Complex Analysis in Several Variables and its Applications (Code: SS 11A), **Debraj Chakrabarti**, Central Michigan University, and **Yunus Zeytuncu**, University of Michigan at Dearborn.

Conformal Geometry and Statistical Physics (Code: SS 20A), **Ilia Binder**, University of Toronto, and **Dapeng Zhan**, Michigan State University.

Discrete Stochastic Models (Code: SS 29A), **Michael Damron**, Indiana University, and **David Sivakoff**, The Ohio State University.

Extremal Graph Theory: Hypergraphs, Directed Graphs, and Other Generalizations (Code: SS 25A), **Louis DeBiasio**, Miami University, and **Theodore Molla**, University of Illinois at Urbana-Champaign.

Floer Homology, Gauge Theory, and Symplectic Geometry (Code: SS 37A), **David Duncan**, **Matt Hedden**, and **Tom Parker**, Michigan State University.

Fractals and Tilings (Code: SS 10A), **Sze-Man Ngai**, Georgia Southern University, **Erin Pearse**, California Polytechnic State University, **Yang Wang**, Hong Kong University of Science and Technology, and **Yimin Xiao**, Michigan State University.

Fractional Calculus and Nonlocal Operators (Code: SS 1A), **Mark M. Meerschaert** and **Russell Schwab**, Michigan State University.

Frames, Wavelets and Their Applications (Code: SS 16A), **Palle Jorgensen**, University of Iowa, **Darrin Speegle**, St. Louis University, and **Yang Wang**, Hong Kong University of Science and Technology.

Geometry and Invariants of 3-Manifolds (Code: SS 22A), **Oliver Dasbach**, Louisiana State University, and **Effie Kalfagianni**, Michigan State University.

Geometry of Manifolds, Singular Spaces, and Groups (Code: SS 18A), **Benjamin Schmidt**, Michigan State University, and **Meera Mainkar**, Central Michigan University.

Groups and Representations (Code: SS 9A), **Amanda Schaeffer Fry**, Metropolitan State University of Denver, **Jonathan Hall**, Michigan State University, and **Hung Nguyen**, University of Akron.

Harmonic Analysis and Applications (Code: SS 27A), **Jarod Hart**, Wayne State University, **Nguyen Lam**, University of Pittsburgh, and **Guozhen Lu**, Wayne State University.

Harmonic Analysis and Partial Differential Equations (Code: SS 26A), **Michael Goldberg**, University of Cincinnati, and **William Green**, Rose-Hulman Institute of Technology.

High-Frequency Problems (Code: SS 14A), **Shlomo Levental** and **Mark Schroder**, Michigan State University.

Homotopy Continuation Methods and Their Applications to Science and Engineering (Code: SS 6A), **Tianran Chen**, Michigan State University, and **Dhagash Mehta**, North Carolina State University.

Integrable Combinatorics (Code: SS 28A), **Philippe Di Francesco** and **Rinat Kedem**, University of Illinois at Urbana-Champaign.

Interactions between Geometry, Group Theory, and Number Theory (Code: SS 24A), **Benjamin Linowitz**, University of Michigan, and **D. B. Reynolds**, Purdue University.

Inverse Problems and Imaging (Code: SS 21A), **Yulia Hristova**, University of Michigan-Dearborn, and **Linh Nguyen**, University of Idaho.

Knot Theory and Floer-Type Invariants (Code: SS 34A), **Christopher Cornwell**, Université du Québec à Montréal, and **Faramarz Vafaee**, Caltech.

Mathematics in Industry and Industrial Problems with Mathematics Application (Code: SS 30A), **Peiru Wu**, Michigan State University.

Modeling, Numerics, and Analysis of Electro-Diffusion Phenomena (Code: SS 17A), **Peter W. Bates**, Michigan State University, **Weishi Liu**, University of Kansas, and **Mingji Zhang**, MIchigan State University.

New Developments in Actuarial Mathematics (Code: SS 15A), **Emiliano A. Valdez**, Michigan State University.

New Developments in Stochastic Analysis, Stochastic Control and Related Fields (Code: SS 7A), **Chao Zhu**, University of Wisconsin-Milwaukee.

Nonlinear Waves: Dynamics and Stability (Code: SS 23A), **Keith Promislow** and **Qiliang Wu**, Michigan State University.

Phase Retrieval in Theory and Practice (Code: SS 8A), **Matthew Fickus**, Air Force Institute of Technology, **Mark Iwen**, Michigan State University, and **Dustin Mixon**, Air Force Institute of Technology.

Random Fields and Long Range Dependence (Code: SS 2A), **Mark M. Meerschaert** and **Yimin Xiao**, Michigan State University.

Random Matrices and Compressed Sensing (Code: SS 40A), **Yang Liu**, Michigan State University.

Recent Advances in Finite Element and Discontinuous Galerkin Methods for Partial Differential Equations (Code: SS 33A), **Aycil Cesmelioglu** and **Anna Maria Spagnuolo**, Oakland University.

Recent Advances in Mathematical Modeling of the Financial Markets (Code: SS 32A), **Albert Cohen**, Michigan State University, and **Nick Costanzino**, University of Toronto.

Recent Advances in the Geometry of Submanifolds, Dedicated to the Memory of Franki Dillen (1963-2013) (Code: SS 12A), **Alfonso Carriazo Rubio**, University of Sevilla, **Yun Myung Oh**, Andrews University, **Bogdan D. Suceavă**, California State University, Fullerton, and **Joeri Van der Veken**, KU Leuven.

Smooth Dynamical Systems and Ergodic Theory (Code: SS 35A), **Nicolai Haydn**, University of Southern California, and **Huyi Hu** and **Sheldon Newhouse**, Michigan State University.

Spectral Theory, Disorder, and Quantum Many Body Physics (Code: SS 38A), **Peter D. Hislop**, University of Kentucky, and **Jeffrey Schenker**, Michigan State University.

Stochastic Partial Differential Equations and Applications (Code: SS 4A), **Leszek Gawarecki**, Kettering University, and **Vidyadhar Mandrekar**, Michigan State University.

Survey of Biomathematics (Code: SS 13A), **Hannah Callender**, University of Portland, **Peter Hinow**, University of Wisconsin, Milwaukee, and **Deena Schmidt**, Case Western Reserve University.

The Geometry of Algebraic Varieties (Code: SS 41A), **Kevin Tucker**, University of Illinois at Chicago, and **Brian Lehmann**, Boston College.

Topics in Noncommutative Algebra and Algebraic Geometry (Code: SS 39A), **Jason Bell**, University of Waterloo, **Rajesh S. Kulkarni**, Michigan State University, and **Daniel Rogalski**, UC San Diego.

Huntsville, Alabama

University of Alabama in Huntsville

March 27–29, 2015

Friday – Sunday

Meeting #1109

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: January 2015
Program first available on AMS website: February 11, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 2

Deadlines

For organizers: Expired
For abstracts: February 4, 2015

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Eva Bayer-Fluckiger, EPFL, *On the Euclidean Division.*

M. Gregory Forest, University of North Carolina at Chapel Hill, *Mathematics of Living Fluids.*

Dan Margalit, Georgia Institute of Technology, *Geometry, algebra, and dynamics of surfaces.*

Paul Pollack, University of Georgia, *Big doings with small gaps.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advances in the Theory and Applications of Dynamical Systems (Code: SS 6A), **Shangbing Ai** and **Wenzhang Huang**, University of Alabama in Huntsville.

Analysis on Nonlinear Integral and Partial Differential Equations (Code: SS 14A), **Tadele Mengesha** and **Tuoc Phan**, The University of Tennessee.

Analytic Methods in Elementary Number Theory (Code: SS 9A), **Paul Pollack**, University of Georgia.

Fractal Geometry and Ergodic Theory (Code: SS 1A), **Mrinal Kanti Roychowdhury**, University of Texas-Pan American.

Geometric Group Theory and Topology (Code: SS 15A), **Tara Brendle**, University of Glasgow, **Christopher Leininger**, University of Illinois at Urbana-Champaign, and **Dan Margalit**, Georgia Institute of Technology.

Graph Theory (Code: SS 11A), **Chris Stephens**, **Dong Ye**, and **Xiaoya Zha**, Middle Tennessee State University.

Mathematical Modeling in Ecology and Epidemiology (Code: SS 16A), **Andrew Nevai** and **Zhisheng Shuai**, University of Central Florida.

New Developments in Population Dynamics and Epidemiology (Code: SS 4A), **Jia Li**, University of Alabama in Huntsville, **Maia Martcheva**, University of Florida, and **Necibe Tuncer**, Florida Atlantic University.

Nonlinear Operator Theory and Partial Differential Equations (Code: SS 7A), **Craig Cowan**, University of Manitoba, and **Claudio Morales**, University of Alabama in Huntsville.

Quadratic Forms in Arithmetic and Geometry (Code: SS 12A), **Asher Auel**, Yale University, **Jorge Morales**, Louisiana State University, and **Anne Quéguiner-Mathieu**, Université Paris 13.

Recent Advances in Numerical Methods for Nonlinear Partial Differential Equations (Code: SS 10A), **S. S. Ravindran**, University of Alabama in Huntsville.

Recent Progress in Differential Equations (Code: SS 8A), **Mathew Gluck**, University of Alabama in Huntsville.

Recent Trends in Mathematical Biology (Code: SS 3A), **Wandi Ding** and **Zachariah Sinkala**, Middle Tennessee State University.

Stochastic Analysis and Applications (Code: SS 13A), **Parisa Fatheddin**, University of Alabama in Huntsville.

Stochastic Processes and Related Topics (Code: SS 2A), **Paul Jung**, University of Alabama at Birmingham, **Erkan Nane**, Auburn University, and **Dongsheng Wu**, University of Alabama in Huntsville.

Topology and Topological Methods in Dynamical Systems (Code: SS 5A), **John Mayer** and **Lex Oversteegen**, University of Alabama at Birmingham.

Las Vegas, Nevada

University of Nevada, Las Vegas

April 18–19, 2015

Saturday – Sunday

Meeting #1110

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: February 2015
Program first available on AMS website: March 5, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 2

Deadlines

For organizers: Expired
For abstracts: February 24, 2015

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Joel Hass, University of California, Davis, *Optimal diffeomorphisms of surfaces and some applications.*

Ko Honda, UCLA, *An invitation to Floer homology.*

Brendon Rhoades, University of California, San Diego, *Evaluating q-analogs in combinatorics and algebra.*

Bianca Viray, University of Washington, Seattle, *Reciprocity laws and rational points on varieties.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Structures in Knot Theory (Code: SS 7A), **Sam Nelson**, Claremont McKenna College, and **Radmila Sazdanovic**, North Carolina State University.

Algebraic and Enumerative Combinatorics (Code: SS 8A), **Drew Armstrong**, University of Miami, and **Brendon Rhoades**, University of California San Diego.

Algebraic-Geometric Methods in Graph Theory (Code: SS 24A), **Mohamed Omar**, Harvey Mudd College, and **Matthew T. Stamps**, KTH.

Arithmetic Geometry (Code: SS 18A), **Katherine E. Stange**, University of Colorado, Boulder, and **Bianca Viray**, University of Washington.

Cloaking and Metamaterials (Code: SS 9A), **Jichun Li**, University of Nevada, Las Vegas, and **Fernando Guevera Vasquez**, University of Utah.

Contact Geometry and Low-Dimensional Topology (Code: SS 25A), **Ko Honda, Erkao Bao**, University of California, Los Angeles, and **Lenhard Ng**, Duke University.

Data Analysis and Physical Processes (Code: SS 4A), **Hanna Makaruk**, Los Alamos National Laboratory, and **Eric Machorro**, National Security Technologies.

Developments of Numerical Methods and Computations for Fluid Flow Problems (Code: SS 11A), **Monika Neda**, University of Nevada, Las Vegas.

*Evolution Problems at the Interface of Waves and Fluids (*Code 12A), **I. Bejenaru**, University of California, San Diego, **B. Pausader** and **V. Vicol**, Princeton University.

Extremal and Structural Graph Theory (Code: SS 10A), **Bernard Lidický** and **Derrick Stolee**, Iowa State University

Geometric Inequalities and Nonlinear Partial Differential Equations (Code: SS 19A), **Guozhen Lu**, Wayne State University, **Nguyen Lam**, University of Pittsburgh, and **Bernhard Ruf**, Università di Milano.

History Of Mathematics (Code: SS 23A), **Satish C. Bhatnagar**, University of Nevada, Las Vegas.

Inverse Problems and Related Mathematical Methods in Physics (Code: SS 1A), **Hanna Makaruk**, Los Alamos National Laboratory, and **Robert Owczarek**, University of New Mexico, Albuquerque.

Knots and 3-Manifolds (Code: SS 14A), **Abby Thompson** and **Anastasiia Tsvietkova**, University of California-Davis.

Mathematical and Numerical Aspects of Modeling Flows Through Porous Media (Code: SS 16A), **Aleksey S. Telyakovskiy** and **Stephen W. Wheatcraft,** University of Nevada, Reno.

Modeling and Numerical Studies for Coupled System of PDEs Arising From Interdisciplinary Problems (Code: SS 20A), **Pengtao Sun**, University of Nevada, Las Vegas.

New Developments in Noncommutative Algebra (Code: SS 22A), **Ellen Kirkman**, Wake Forest University, and **James Zhang**, University of Washington, Seattle.

Nonlinear Conservation Laws and Applications (Code: SS 6A), **Matthias Youngs**, Indiana University-Purdue University Columbus, **Cheng Yu**, University of Texas at Austin, and **Kun Zhao**, Tulane University.

Nonlinear Elliptic and Parabolic PDEs (Code: SS 17A), **Igor Kukavica**, University of Southern California, **Walter Rusin,** Oklahoma State University, and **Fei Wang**, University of Southern California.

Nonlinear PDEs and Variational Methods (Code: SS 5A), **David Costa, Zhonghai Ding,** and **Hossein Tehrani**, University of Nevada, Las Vegas.

Recent Advances in Finite Element Analysis and Applications (Code: SS 21A), **Jichun Li,** University of Nevada, Las Vegas, and **Susanne Brenner**, Louisiana State University.

Set Theory (Code: SS 15A), **Derrick Dubose** and **Douglas Burke**, University of Nevada, Las Vegas.

Stochastic Analysis and Rough Paths (Code: SS 2A), **Fabrice Baudoin**, Purdue University, **David Nualart**, University of Kansas, and **Cheng Ouyang**, University of Illinois at Chicago.

Topics in Graph Theory (Code: SS 3A), **Jie Ma**, Carnegie Mellon University, **Hehui Wu**, Simon Fraser University, and **Gexin Yu**, College of William & Mary.

Session for Contributed Talks

There also will be a session for 10-minute contributed talks. Please see the abstracts submission form at www.ams.org/cgi-bin/abstracts/abstract.pl. **The deadline for all abstracts submissions is February 24, 2015.**

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. **Las Vegas is a popular destination in the spring, so please make your reservations early!** Special discounted rates were negotiated with the hotels listed below. Other hotels for this meeting may be added to the AMS website; check www.ams.org/meetings/sectional/sectional.html for more information. **Rates quoted do not include hotel tax of 12 percent.** Participants must state that they are with the **American Mathematical Society (AMS) Math Meeting at the University of Nevada, Las Vegas** to receive the discounted rates. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details when you make your reservation.**

Alexis Park All Suite Resort, 375 E. Harmon Ave., Las Vegas, NV 89169; 800-582-2228, 702-796-3300 (phone), 702-796-4334 (fax). It is about a .6 mile walk to the meeting site. This all-suite hotel is a nonsmoking property. Rooms are US$79 per night for a room with a king bed, coffee maker, refrigerator, and a desk. Wired and wireless internet are available for an extra charge. There is a restaurant and a bar on site. **When you make your reservation you will be charged a deposit equal to one night's lodging; this is fully refundable provided that you cancel at least 48 hours prior to your arrival. The deadline for reservations is April 1, 2015.**

Bluegreen Club 36, 372 E. Tropicana Ave., Las Vegas, NV 89169; 702-656-2900 (phone) 702-856-2912 (fax). It is about a 1.5 mile walk to the meeting site (30 minutes). Rates are US$82 per room, per night; there is an additional charge if you need houskeeping services. Rooms include one king bed and a queen sleeper sofa, coffee maker, microwave oven, and refrigerator. Amenities include Wi-Fi, free local calls, free use of a copy machine, free parking, and an indoor pool. Smoking and nonsmoking rooms are available. Also on the property are a Subway sandwich shop and a convenience store. Check-in time is 4:00 p.m. and check out time is 10:00 a.m. Please note that this is a time-share property, and you may be asked to view a presentation or take a tour to purchase a unit; you are under no obligation to do so. **You must pay for one night's accommodation when you make your reservation. Your deposit is fully refundable if you cancel at least 72 hours prior to arrival. The deadline for reservations at the above rate is March 17, 2015.**

Candlewood Suites Hotel, 4034 Paradise Rd., Las Vegas, NV 89169; 702-836-3660; about a mile from the meeting site. Rates are US$89 for a queen-bedded studio with fully equipped kitchen, including free wireless Internet, and free parking. You will also find a convenience store/market on the premises which is open 24 hours, a swimming pool and Jacuzzi, a workout room, and a snack shop. The Candlewood Gazebo Grill is usually open 24 hours . **The deadline for reservations is March 17, 2015. Please see the AMS website at** `www.ams.org/meetings/sectional/2218_other.html` **for a direct link to make your reservation at this hotel.**

Hyatt Place Las Vegas, 4520 Paradise Rd., Las Vegas, NV, 89169; 800-491-6126 or 702-369-3366 (phone) 701-369-1689 (fax); US$119/king bed or two double beds plus sofa sleeper, wet bar, and refrigerator, and coffee maker. The rate also includes complimentary hot breakfast buffet, free shuttle to/from the airport, free wireless Internet throughout the hotel, outdoor pool, fitness room, and 24-hour business center. It is about a .6 mile walk to the meeting site on campus.

Platinum Hotel, 211 E. Flamingo Rd., Las Vegas, NV 89169; 702-365-5000; for reservations go to `https://bookings.ihotelier.com/Platinum-Hotel-Las-Vegas/bookings.isp?hotelid=15019&identifier=UNLV`. The cost is US$119 for a king-bed room with parlor area including a pull-out couch, full kitchen with microwave and coffee maker, and high speed internet; suites with two queen beds are also available for the same price. This full service hotel is nongaming and nonsmoking, and features the Kil@Wat Restaurant (open for breakfast and lunch) and the contemporary Stir and Bar Lounge. Room service is available for breakfast, lunch, and dinner. **You must pay for one night's accommodation when making your reservation; this is fully refundable if you cancel at least 72 hours prior to your arrival. The deadline for reservations is March 17, 2015.**

Food Services

Information will be available on site.

Registration and Meeting Information

Advance Registration

Advance registration for this meeting will open on **January 19, 2015.** Fees will be US$56 for AMS members, US$78 for nonmembers; and US$5 for students, unemployed mathematicians, and emeritus members.

Onsite Information and Registration

Registration, the book exhibit, Special Sessions, and Invited Addresses will be located in the Classroom Building Complex located just behind the Thomas and Mack Center. The registration desk will be open on Saturday, April 18, 7:30 a.m.–4:00 p.m. and Sunday, April 19, 8:00 a.m.–noon. Fees are the same as advance registration and will be payable on site via cash, check, or credit card.

Special Needs

It is the goal of the AMS to ensure that its conferences are accessible to all, regardless of disability. The AMS shall strive, unless it is not practicable, to choose venues that are fully accessible to the physically handicapped.

If special needs accommodations are necessary in order for you to participate in an AMS Sectional Meeting, please communicate your needs in advance to the AMS Meetings Department by:

- Registering early for the meeting
- Checking the appropriate box on the registration form
- Sending an email request to the AMS Meetings Department at `mmsb@ams.org` or `meet@ams.org`.

Other Activities

Book Sales: Stop by the on-site AMS bookstore to review our newest publications and take advantage of exhibit discounts! AMS members receive 40 percent off list price. Nonmembers receive a 25 percent discount. Not a member? Ask about the benefits of AMS membership. Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

Parking

Free parking on Saturday and Sunday is available in the Black Lot (with parking garage) next to the Thomas and Mack Center and is accessible from Swenson St., and in the N lot nearby which can be accessed from Harmon Avenue, off of Swenson St. If you need to park on campus on Friday, you may use the metered lots or obtain a parking pass. Please see the details at the Parking and Transportation Services website at `parking.unlv.edu`.

Travel

Airport: You should plan to fly into McCarran International Airport (LAS); see `www.mccarran.com` for details. Taxis are available outside the baggage claim area. Some hotels provide free shuttle services; please check when making your reservations.

Car Rental: Hertz is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at `www.hertz.com`, click on the box "I have a discount", and type in our convention number (CV): 04N30005. You can also call Hertz directly at 800-654-2240 (U.S. and Canada) or 1-405-749-4434 (other countries). At the time of reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available.

Driving to UNLV: Please use your favorite travel website for the best advice on driving to campus. The main address of the campus is 4505 South Maryland Parkway, Las Vegas, NV 89154. Please see `www.unlv.edu/maps` for avariety of maps. On Saturday and Sunday you should plan to park in the Black Lot near the Thomas and Mack Center or the N lot near the Classroom Building Complex. These are clearly marked on campus maps.

Social Networking

Participants and speakers are encouraged to tweet about the meeting using the hashtag #AMSmtg.

Weather

During the month of April the average high temperature is in the high 70s, the average low temperature is in the low 50s and there is very little rainfall.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at `sites.nationalacademies.org/pga/biso/visas/` and `travel.state.gov/visa/visa_1750.html`. If you need a preliminary conference invitation in order to secure a visa, please send your request to `aba@ams.org`.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of "binding" or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence

- property ownership

- bank accounts

- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor's home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

Porto, Portugal

University of Porto

June 10–13, 2015

Wednesday – Saturday

Meeting #1111

First Joint International Meeting involving the American Mathematical Society (AMS), the European Mathematical Society (EMS), and the Sociedade de Portuguesa Matematica (SPM).

Associate secretary: Georgia Benkart
Announcement issue of *Notices*: February 2015
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Rui Loja Fernandes, University of Illinois at Urbana-Champaign, *Title to be announced.*

Irene Fonseca, Carnegie Mellon University, *Title to be announced.*

Annette Huber, Albert-Ludwigs-Universität, *Title to be announced.*

Mikhail Khovanov, Columbia University, *Title to be announced.*

André Neves, Imperial College London, *Title to be announced.*

Sylvia Serfaty, Université Pierre et Marie Curie Paris 6, *Title to be announced.*

Gigliola Staffilani, Massachusetts Institute of Technology, *Title to be announced.*

Marcelo Viana, Instituto de Matemática Pura e Aplicada, Brasil, *Title to be announced.*

This announcement was composed with information taken from the website maintained by the local organizers at http://aep-math2015.spm.pt/. Please watch this website for the most up-to-date information.

Abstract Submissions

Talks in Special Sessions are generally by invitation of the organizers. The abstract submission deadline was June 1, 2014.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. These hotels were reserved in their entirety for participants of the meeting, for the dates of the meeting. The organizing committee has selected a number of conveniently located hotels and has secured special rates for conference participants and their travel companions, provided the reservations are made by **April 30, 2015**. Breakfast is included in the quoted prices below. Additional information on each hotel is available through the hotel's webpage. To book a room at the special rate, at any of the hotels below, please visit https://www.hfhotels.com/gb/?b2b . The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

Hotel HF Ipanema Porto (⋆⋆⋆⋆)

Rates are Single (Standard): EUR€57, US$70.60; Twin (Standard): EUR€63, US$78.03; Extra bed: EUR€26, US$32.20 per night. Free Wi-Fi Internet in the entire hotel with public transports in front of the hotel. Cancellation and early check-out policies vary; be sure to check when you make your reservation.

Hotel HF Tuela Porto (⋆⋆⋆)

Rates are Single (Standard): EUR€49, US$60.69; Twin (Standard): EUR€56, US$69.36; Extra bed: EUR€20, US$ 24.77 per night. Free Wi-Fi Internet in the entire hotel, one minute away from public transports. Cancellation and early check-out policies vary; be sure to check when you make your reservation.

Hotel HF Fénix Porto (⋆⋆⋆⋆)

Rates are Single (Standard): EUR€57, US$70.60; Twin (Standard): EUR€63, US$78.03; Single (Privilege): EUR€60, US$ 74.32; Twin (Privilege): EUR€67, US$82.99; Extra bed: EUR€26, US$32.20 per night. Free Wi-Fi Internet in the entire hotel, 1 minute away from public transports. Cancellation and early check-out policies vary; be sure to check when you make your reservation.

Hotel HF Ipanema Park (⋆⋆⋆⋆⋆)

Rates are Single (Standard): EUR€60, US$74.32; Twin (Standard): EUR€69, US$85.46; Single/Twin (Executive): EUR€71, US$87.94; Extra bed: EUR€30, US$ 37.16 per night. Free Wi-Fi Internet in the entire hotel and outdoor swimming pool, 1 minute away from public transports Cancellation and early check-out policies vary; be sure to check when you make your reservation.

BessaHotel (⋆⋆⋆⋆)

Rates are Single (Standard): EUR€69, US$85.46; Twin (Standard): EUR€79, US$ 97.85 per night. Prices include breakfast, WiFi and access to the gym. Cancellation and early check-out policies vary; be sure to check when you make your reservation.

Additional accommodation options can be found at http://aep-math2015.spm.pt/accommodation.

Portugal's electrical current is 220–240 V; and sockets take the standard continental European dual round-pronged plugs.

Registration and Meeting Information

The online registration is available at aep-math2015.spm.pt/registration_fees. Registration includes all conference materials, full access to sessions and lectures, refreshments at coffee breaks and Welcome Reception.

Registration fees

To take advantage of registration fees at the reduced rate, participants must register by April 30, 2015.

Chicago, Illinois

Loyola University Chicago

October 3–4, 2015

Saturday – Sunday

Meeting #1112

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: June 2015
Program first available on AMS website: August 20, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 4

Deadlines

For organizers: March 10, 2015
For abstracts: August 11, 2015

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Julia Chuzhoy, Toyota Technological Institute at Chicago, *Title to be announced*.

Andrew Neitzke, The University of Texas at Austin, *Title to be announced*.

Sebastien Roch, University of Wisconsin-Madison, *Title to be announced*.

Peter Sarnak, Princeton University, *Title to be announced* (Erdős Memorial Lecture).

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Algebraic Methods Common to Association Schemes, Hopf Algebras, Tensor Categories, Finite Geometry, and Related Areas (Code: SS 1A), **Harvey Blau**, Northern Illinois University, **Sung Y. Song**, Iowa State University, and **Bangteng Xu**, Eastern Kentucky University.

Groups, Rings, Group Rings, and Hopf Algebras—Celebrating the 75th Birthday of Professor Donald S. Passman (Code: SS 2A), **Jeffrey Bergen**, **Stefan Catoiu**, and **William Chin**, DePaul University.

The Mathematics of Evolution (Code: SS 3A), **Ruth Davidson** and **Ruriko Yoshida**, University of Illinois Urbana-Champaign.

Memphis, Tennessee

University of Memphis

October 17–18, 2015

Saturday – Sunday

Meeting #1113

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: August 2015
Program first available on AMS website: September 3, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 3

Deadlines

For organizers: March 17, 2015
For abstracts: August 25, 2015
The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Mark van Hoeij, Florida State University, *Title to be announced*.

Vaughan Jones, Vanderbilt University, *Title to be announced*.

Mette Olufsen, North Carolina State University, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `http://www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advances in Operator Theory and Applications. (Code: SS 5A), **Fernanda Botelho**, University of Memphis.

Banach Spaces and Applications (Code: SS 4A), **Anna Kaminska**, **Peikee Lin**, and **Bentuo Zheng**, University of Memphis.

Cahn-Hilliard and Related Equations and Applications. (Code: SS 11A), **Giséle Ruiz Goldstein**, University of Memphis, and **Alain Miranville**, Université de Poitiers.

Computational Analysis (Code: SS 1A), **George Anastassiou**, University of Memphis.

Control and Inverse Problems for Partial Differential Equations (Code: SS 6A), **Matthias Eller**, Georgetown University, **Shitao Liu**, Clemson University, and **Roberto Triggiani**, University of Memphis.

Difference Equations and Applications. (Code: SS 12A), **Michael A. Radin**, Rochester Institute of Technology, and **Youssef Raffoul**, University of Dayton.

Ergodic Theory (Code: SS 8A), **James T. Campbell** and **Mate Wierdl**, University of Memphis.

Extremal Graph Theory (Code: SS 3A), **Ralph Faudree**, University of Memphis.

Fractal Geometry and Dynamical Systems (Code: SS 2A), **Mrinal Kanti Roychowdhury**, University of Texas-Pan American.

Recent Advances in Commutative Algebra. (Code: SS 13A), **Sandra Spiroff**, University of Mississippi, and **Lance Miller**, University of Arkansas.

Recent Developments in the Statistical Analysis of Large Clustered Data (Code: SS 10A), **E. Olusegun George**, University of Memphis.

Spectra of Graphs and Hypergraphs. (Code: SS 9A), **Vladimir Nikiforov**, University of Memphis.

Stabilization, Control, and Analysis of Evolutionary Partial Differential Equations (Code: SS 7A), **George Avalos**, University of Nebraska Lincoln, **Scott Hansen**, Iowa State University, and **Justin Webster**, North Carolina State University & College of Charleston.

Fullerton, California

California State University, Fullerton

October 24–25, 2015

Saturday – Sunday

Meeting #1114

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2015
Program first available on AMS website: September 10, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 4

Deadlines

For organizers: March 27, 2015
For abstracts: September 1, 2015

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Mina Aganagic, University of California, Berkeley, *Title to be announced.*

John Lott, University of California, Berkeley, *Title to be announced.*

Eyal Lubetzky, Microsoft Research, Redmond, *Title to be announced.*

Zhiwei Yun, Stanford University, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Geometric Analysis (Code: SS 1A), **John Lott**, University of California, Berkeley, and **Aaron Naber**, Northwestern University.

Mathematicians and Outreach Programs (Code: SS 2A), **Olga Radko**, University of California Los Angeles, and **Bodgan D. Suceava**, California State University, Fullerton.

New Brunswick, New Jersey

Rutgers University

November 14–15, 2015

Saturday – Sunday

Meeting #1115

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: September 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: November 2015
Issue of *Abstracts*: Volume 36, Issue 4

Deadlines

For organizers: April 14, 2015
For abstracts: September 22, 2015

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Lee Mosher, Rutgers University, *Title to be announced.*

Jill Pipher, Brown University, *Title to be announced.*

David Vogan, Massachusetts Institute of Technology, *Title to be announced.*

Wei Zhang, Columbia University, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at http://www.ams.org/cgi-bin/abstracts/abstract.pl.

Applications of CAT(0) Cube Complexes (Code: SS 1A), **Sean Cleary**, City College of New York and the City University of New York Graduate Center, and **Megan Owen**, Lehman College of the City University of New York.

Aspects of Minimal Surfaces in Riemannian Manifolds (Code: SS 4A), **Zheng Huang** and **Marcello Lucia**, City University of New York, Staten Island and Graduate Center.

Commutative Algebra (Code: SS 2A), **Laura Ghezzi**, New York City College of Technology, City University of New York, and **Jooyoun Hong**, Southern Connecticut State University.

Difference equations and applications (Code: SS 5A), **Manos Drymonis**, Providence College, **Evelina Lapierre**, Johnson and Wales University, and **Michael Radin**, Rochester institute of Technology.

On Geometric Topology: A Celebration of Jim West's 70th Birthday (Code: SS 3A), **Boris Goldfarb**, State University of New York at Albany.

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday

Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For abstracts: To be announced

Athens, Georgia

University of Georgia

March 5–6, 2016

Saturday – Sunday

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 5, 2015
For abstracts: To be announced

Stony Brook, New York

State University of New York at Stony Brook

March 19–20, 2016

Saturday – Sunday

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 19, 2015
For abstracts: February 2, 2016

Salt Lake City, Utah

University of Utah

April 9–10, 2016

Saturday – Sunday

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Fargo, North Dakota

North Dakota State University

April 16–17, 2016

Saturday – Sunday

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

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Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Convexity and Harmonic Analysis (Code: SS 2A), **Maria Alfonseca-Cubero**, North Dakota State University, and **Dmitry Ryabogin**, Kent State University.

Ergodic Theory and Dynamical Systems (Code: SS 1A), **Dogan Comez**, North Dakota State University, and **Mrinal Kanti Roychowdhury**, University of Texas-Pan American.

Mathematical Finance (Code: SS 3A), **Indranil SenGupta**, North Dakota State University.

Brunswick, Maine

Bowdoin College

September 24–25, 2016

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: July 23, 2016

Denver, Colorado

University of Denver

October 8–9, 2016

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 8, 2016
For abstracts: August 16, 2016

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday
Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For abstracts: To be announced

Charleston, South Carolina

College of Charleston

March 10–12, 2017

Friday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: November 10, 2016
For abstracts: To be announced

Bloomington, Indiana

Indiana University

April 1–2, 2017

Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Pullman, Washington

Washington State University

April 22–23, 2017

Saturday – Sunday

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday

Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: October 2017

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2017

For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 16–19, 2019

Wednesday – Saturday

Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM)and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: October 2018

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2018

For abstracts: To be announced

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