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Notices

of the American Mathematical Society

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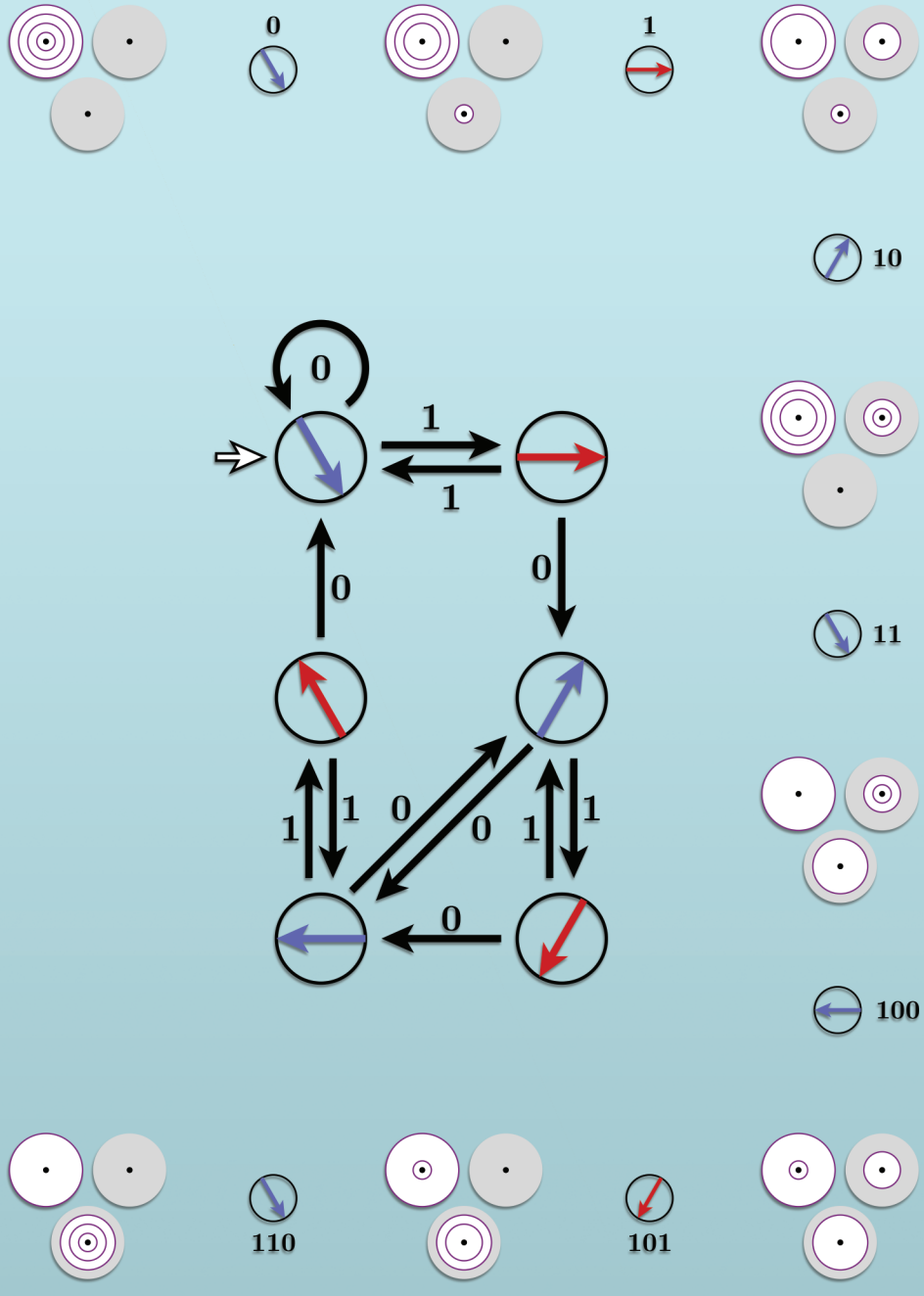

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American Mathematical Society

About the cover: Automatic sequences and the Tower of Hanoi (see page 258)

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Search for an Executive Director for the American Mathematical Society

Position

The Trustees of the American Mathematical Society seek candidates for the position of Executive Director of the Society to replace Dr. Donald McClure, who plans to retire in the summer of 2016. This position offers the appropriate candidate the opportunity to have a strong positive influence on all activities of the Society, as well as the responsibility of overseeing a large, complex, and diverse spectrum of people, publications, and budgets. The desired starting date is July 1, 2016.

Duties and terms of appointment

The American Mathematical Society, with headquarters in Providence, RI, is the oldest scientific organization of mathematicians in the U.S. The Society's activities are mainly directed toward the promotion and dissemination of mathematical research and scholarship, broadly defined; the improvement of mathematical education at all levels; increasing the appreciation and awareness by the general public of the role of mathematics in our society; and advancing the professional status of mathematicians. These aims are pursued mainly through an active program of publications, meetings, and conferences. The Society is a major publisher of mathematical books and journals, including MathSciNet, an organizer of numerous meetings and conferences each year, and a leading provider of electronic information in the mathematical sciences. The Society maintains a Washington office for purposes of advocacy and to improve interaction with federal agencies.

The Executive Director is the principal executive officer of the Society and is responsible for the execution and administration of the policies of the Society as approved by the Board of Trustees and by the Council. The Executive Director is a full-time employee of the Society appointed by the Trustees and is responsible for the operation of the Society's offices in Providence and Pawtucket, RI; Ann Arbor, MI; and Washington, DC. The Executive Director is an ex-officio member of the policy committees of the Society and is often called upon to represent the Society in its dealings with other scientific and scholarly bodies.
The Society employs a staff of about 200 in the four offices. The directors of the various divisions report directly to the Executive Director. A major part of the Society's budget is related to publications. Almost all operations (including the printing) of the publications program are done in-house. Information about the operations and finances of the Society can be found in its Annual Reports, available at www.ams.org/annual-reports.

The Executive Director serves at the pleasure of the Trustees. The terms of appointment, salary, and benefits will be consistent with the nature and responsibilities of the position and will be determined by mutual agreement between the Trustees and the prospective appointee.

Qualifications

Candidates for the office of Executive Director should have a Ph.D. (or equivalent) in mathematics, published research beyond the Ph.D., and significant administrative experience. The position calls for interaction with the staff, membership, and patrons of the Society as well as leaders of other scientific societies and publishing houses; thus leadership, communication skills, and diplomacy are prime requisites.

Applications

A search committee chaired by Robert Bryant (bryant@math.duke.edu) and Ruth Charney (charney@brandeis.edu) has been formed to seek and review applications. All communication with the committee will be held in confidence. Suggestions of suitable candidates are most welcome. Applicants can submit a CV and letter of interest to:

Executive Director Search Committee
c/o Carla D. Savage
Secretary, American Mathematical Society
Department of Computer Science
North Carolina State University
Raleigh, NC 27695-8206
ed-search@ams.org

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Notices

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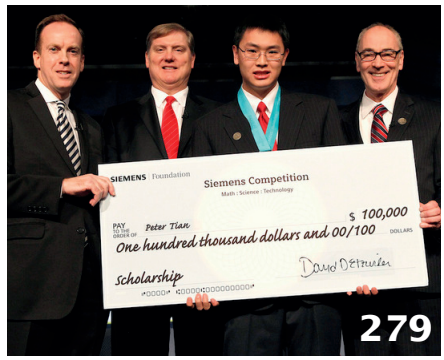
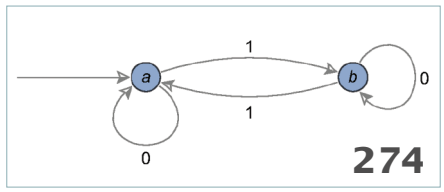
March 2015

Communications

Commentary





The March issue features the second installment of the Erdős centennial article and an interview with the new AMS President, Robert L. Bryant.

There is an announcement from MSRI about the first-ever National Math Festival taking place in Washington this year, a retrospective on the life and remarkable publishing career of Klaus Peters, and a memorial for founding father of mathematical physics Arthur Strong Wightman.

We wish you a pleasant Ides.—Steven G. Krantz, Editor

Features

Notices

of the American Mathematical Society

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Departments

From the AMS Secretary

CALL FOR NOMINATIONS

Chevalley Prize in Lie Theory

This prize was established by George Lusztig in 2014 to honor Claude Chevalley (1909–1984). Chevalley was a founding member of the Bourbaki group. He made fundamental contributions to class field theory, algebraic geometry, and group theory. His three-volume treatise on Lie groups served as standard reference for many decades. His classification of semisimple groups over an arbitrary algebraically closed field provides a link between Lie's theory of continuous groups and the theory of finite groups, to the enormous enrichment of both subjects.

This prize recognizes notable work in Lie Theory published during the preceding six years; a recipient should be at most twenty-five years past the Ph.D.

The US$8,000 prize will be awarded in even-numbered years without restriction on society membership, citizenship or venue of publication. The first award will be made in 2016.

Nominations with supporting information should be submitted using the online form available here:
www.ams.org/profession/prizes-awards/nominations

Include a short description of the work that is the basis of the nominations, including complete bibliographic citations. A brief curriculum vitae for the nominee should be included. Those who prefer to submit by postal mail may send nominations to AMS Secretary, Carla Savage, Box 8206, Computer Science Department, North Carolina State University, Raleigh, NC 27695-8206. The nominations will be forwarded by the secretary to the appropriate prize selection committee, which, in effect, will make final decisions on the award.

Deadline for nominations is June 30, 2015.

AMS
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A National Math Festival

Many organizations work to transform public perceptions of mathematics. The AMS's *Mathematical Moments, the Carriage House Lectures* of the Mathematical Association of America (MAA), the Math Awareness Month sponsored by the Joint Policy Board for Mathematics, the exhibits in the National Museum of Mathematics (MoMath), the films produced by the Mathematical Sciences Research Institute and the YouTube channel *Numberphile* it supports, the public events hosted by the math institutes, are all substantial contributions. But we're still approached, far too often, with the unabashed announcement: "I never liked/never was good at/always hated…math."

This is a symptom of a disease with pervasive effects, like the common cold. The condition has a pernicious influence in areas from elementary education to funding for basic research. Fear of mathematics reduces the fulfillment of human potential, increases inequality, and brakes economic growth by blocking the path to fulfilling and valuable careers. It prevents people from taking pleasure in math and from developing their capacity for this deep human endeavor.

In a major new effort, the Mathematical Sciences Research Institute (MSRI) and the Institute for Advanced Study (IAS) are collaborating on the first-ever National Math Festival, a series of events taking place April 16–18, 2015, in Washington, DC. The Festival will not cure the US of its disease, but we hope that it will be noticed widely and begin an influential tradition.

The Festival has so many activities and parts that it is difficult to describe! Here are some highlights:

Math Education and Math Teachers at the Festival: There will be separate events related to federal and state math education policy. On the federal level we will host a congressional briefing stressing the need for teachers' professionalism and (the right kinds of) content knowledge, and we will highlight programs that nurture some of our best math teachers, such as Math for America and the Institute for Advanced Study's Park City Mathematics Institute. On the state level, joining forces with the Council of Chief State School Officers, we will focus on the improving cooperation between the research and K–12 teaching communities, and the progress toward better "college- and career-ready standards." Every state has now adopted such standards, but much work remains.

Basic Research at the Festival: Basic research almost always involves mathematics. There will be a gala dinner, "Act from Thought: The Case for Basic Research" at the Great Hall of the Library of Congress. The chairs of the gala are the wonderful supporters of science and math, Jim and Marilyn Simons, with co-chairs Roger Strauch and Charles Simonyi, who head the Boards of MSRI and IAS. Among others, Eric Lander (former director of the Human Genome Project) will speak on the importance of mathematics in his work, and NOVA will be on hand to show off clips of its best math-related movies. The gala will bring together influential and powerful people from private and federal organizations that affect funding for research across the sciences.

Children's Books at the Festival: There will be a high-profile announcement of the winners of the first Mathical Book Prizes, "mathy" books for kids from tots to teens: five prizes plus a longer list that we'll distribute to parents, teachers, and librarians. This collaboration between MSRI and the Children's Book Council has a special feature: we are working with First Book, which distributes books at very low prices, to children in need, through a nationwide network (150,000+ nodes).

Come One, Come All! The events above are, by necessity, invitation-only—but a festival wouldn't be a festival without big public events, and there will be plenty! On Saturday, April 18, with the Smithsonian Museums we will offer mathematics applied and mathematics for fun! Hear about Sea Ice, Galaxies, Fireflies, Islamic Art, Drag Racing, Mathematical Magic, Minecraft, and NOVA labs! Ride MoMath's square-wheeled tricycle and visit the other twenty stations of the Math Midway filling the Enid A. Haupt Gardens next to the Smithsonian Castle! Follow the teams of Math Buskers of Guerilla Science! Enjoy Bridges Math+Art exhibition! Play the AMS's game show, "Who Wants to Be a Mathematician?" Compete in the MAA's mathematical scavenger hunt! And much more!

Outside the Beltway: To give the Festival truly national resonance, select science museums around the country will hold a special math event (with ZomeTools, we'll offer a geometric soap bubble kit) on the day or during the week of the Festival—which is also the middle of the JPBM's *Math Awareness Month*.

Such activity takes time to plan and costs money to publicize and execute. Far-sighted private foundations and individuals will provide resources (nothing is charged to the National Science Foundation). These leaders are the Alfred P. Sloan Foundation, the Carnegie Corporation of New York, the Charles and Lisa Simonyi Fund for Arts and Sciences, Google, the Gordon and Betty Moore Foundation, the Howard Hughes Medical Institute, the Kavli Foundation the Research Corporation, and the Simons Foundation. Our program partners—the Elwyn and Jenny Berlekamp Foundation, MoMath, and NOVA—have brought additional resources.

What if there were a festival and no one knew? Look at the new website, `mathfest.org`, for more information. Ask your local science museum to participate (information is on the website). Keep up the momentum with local public events. Bring kids to Washington on April 18. We won't cure the common cold—but together we can help spread and deepen the appreciation of math!

—David Eisenbud
Director, MSRI
de@msri.org

DOI: http://dx.doi.org/10.1090/noti1220

Letters to the Editor

Graduate School Recommendations

My thanks to Bernard Deconinck and Jan Medlock ["The Burden on Graduate School Recommenders," *Notices*, December 2014] for pointing out the burden imposed on recommenders by our current patchwork of application procedures for graduate school in mathematics, and for including the paragraph used by Robert Strichartz to address this issue in the letters he writes.

For many years, my own choice was to avoid completing online references whenever possible (`mathjobs.org` being a pleasant exception); until recently, most graduate programs would accept such letters upon request, often despite official posted policies to the contrary. However, the tide has clearly turned, and several programs now respond indicating that, while they are willing to accept hard copy, they cannot guarantee that their committee will read them.

Both the short- and long-term solutions proposed by the authors provide a better approach, with less risk of jeopardizing the recommendee's chances: Add a disclaimer to letters where necessary, and agitate for a uniform, mathjobs-style alternative. That latter step will succeed only if strongly supported by the AMS and its members.

Readers may enjoy the further discussion of this issue I found on an old (and apparently inactive) blog at `onlinereferences.blogspot.com`.

—Tevian Dray
Oregon State University
`tevian@math.oregonstate.edu`

(Received December 5, 2014)

Does the Fields Medal or Attracting Mates Ruin the Future of Geniuses?

While reading, with great interest, János Kollár's [4], I couldn't help thinking about another such study, of Satoshi Kanazawa [3], who finds that the decrease in productivity after major achievements is not specific to mathematics, but can be found in all creative fields, literature and painting included. In the fashionable style of "evolutionary psychology," in which human behavior and motivation is seen zoologically, Kanazawa finds that "genius" is the expression "of young men's proximate competitive desires, whose ultimate function in the ancestral environment would have been to increase reproductive success." In that view, mathematical achievement is a form of engaging in "cultural display in order to attract mates". Kanazawa attempts to show that this is so by amassing statistical evidence, for the fact that "marriage has a strong desistance effect on [...] genius," in the form of productivity graphs that show both women and unmarried men do not have the early peak and precipitous drop experienced by males who have achieved their goal of "attracting mates."

G. J. Borjas and K. B. Doran [1], however, completely ignore Kanazawa, and compare the Fields medalists with a very debatable list of "contenders" (picked among the "plenary speakers" at ICMs, all of them awardees of other prizes: Abel, Wolf, Cole, Bôcher, Veblen, Salem). That list conveniently misses Paul Erdős, whose independent elementary proof of the PNT (one of the results for which Atle Selberg received it (see [2], [5])) put him perhaps somewhat closer to "contendership" than the "plenary lecturers" chosen. It also misses Igor Shafarevich (although it lists his co-author Ilya Piatetski-Shapiro). If Erdős had been on that "top contender" list, he would have confirmed Borjas and Doran's finding that top contenders do (in this case *incredibly*) far better than medalists in terms of number of publications and their impact. Kanazawa would have said: I told you so, Erdős never cared about "attracting mates." Erdős, who lived to be eighty-three, would also have messed with the neat "conclusion" regarding the early demise of "top contenders" (whose "average age of death" is 60.5). Shafarevich, alive at ninety-one, would have also been of no help for the desired conclusion.

Whatever one thinks of the recurring fad of "evolutionary" approaches, an exclusionary focus on one distinction, as if fame and historical significance of results in mathematics depended only on it, seems to be an even narrower focus.

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[1] G. J. Borjas and K. B. Doran, Prizes and productivity: How winning the Fields Medal affects scientific output, *Journal of Human Resources* (2015).

[2] D. Goldfeld, The elementary proof of the prime number theorem: an historical perspective, `www.math.columbia.edu/~goldfeld/ErdosSelbergDispute.pdf`.

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[4] J. Kollár, Is there a curse of the Fields Medal?, *Notices of the AMS* **62**, no. 1 (2015), 21–25.

[5] J. Spencer and R. Graham, The elementary proof of the prime number theorem. With a note on the controversy by E. G. Straus and a postscript by Carl Pomerance, *Math. Intelligencer* **31** (2009), no. 3, 18–23.

—Victor Pambuccian
Arizona State University
`pamb@asu.edu`

(Received December 19, 2104)

Diversity Trumps Ability

Publishing Abigail Thompson's article "Does Diversity Trump Ability?" in the *Notices* was appropriate ["Does Diversity Trump Ability?", by Abigail Thompson, October 2014; see also Letters to the Editor, January 2015]. Her counterexample to the statement of Lu Hong and Scott Page, published in the *Proc. Nat. Acad. Sci. USA,* is correct. Claiming the opposite by referring to an additional **Condition 3** published three years later in Professor Page's book, but not included in the *Proceedings* paper, is disingenuous.

Diversity versus ability, particularly in regard to admitting students and hiring faculty, is a highly politicized constitutional issue. In their

paper, Professors Hong and Page attempted to support the diversity by an incorrect mathematical argument. There has been nothing uncivil or uncollegial in pointing out the error. It merely affirmed that "We live in the society and have eyes."

—Washek F. Pfeffer
University of California, Davis
washek@q.com

(Received December 31, 2014)

Reflections on Paul Erdős on His Birth Centenary, Part II

Krishnaswami Alladi and Steven G. Krantz, Coordinating Editors

This is Part II of the two-part feature on Paul Erdős following his centennial. Part I had contributions from Krishnaswami Alladi and Steven G. Krantz, Vera T. Sós and László Lovász, Ronald Graham and Joel Spencer, Jean-Pierre Kahane, and Mel Nathanson. Here in Part II we have six articles from contributors Noga Alon, Dan Goldston, András Sárközy, József Szabados, Gérald Tenenbaum, and Stephan Ramon Garcia and Amy L. Shoemaker.

Courtesy of Vera Sós.

Paul Erdős in a relaxed mood in Hungary.

Krishnaswami Alladi is professor of mathematics at the University of Florida. His email address is alladi@ufl.edu.

Steven G. Krantz is professor of mathematics at Washington University in St. Louis.

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Noga Alon

Paul Erdős and the Probabilistic Method

Probabilistic Beginnings

The probabilistic method is one of the most significant contributions of Paul Erdős. Indeed, Paul himself said, during his eightieth birthday conference in Keszthely, Hungary, that he believes the method will live long after him. This was the only time I heard him making any comment about the significance and impact of his work. He was always more interested in discussing new problems and results than in trying to assess their long-time expected merits.

The method is a powerful technique with numerous applications in combinatorics, graph theory, additive number theory and geometry.

Noga Alon is professor of mathematics at the Sackler School of Mathematics and Blavatnik School of Computer Science, Tel Aviv University, and Institute for Advanced Study, Princeton. His email address is nogaa@tau.ac.il.

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The basic idea is very simple: Trying to prove that a structure with certain desired properties exists, one defines an appropriate probability space of structures and then shows that the desired properties hold in this space with positive probability. The amazing fact is that this simple reasoning can lead to highly nontrivial results. The results and tools are far too numerous to cover in a few pages, and my aim here is to give only a glimpse of the topic by describing a few examples of questions and results that illustrate the method. All of these have been initiated by Erdős, motivated by his questions and results. The fact that there is still intensive ongoing work illustrates the influence and long-term impact of his work. More material on the subject can be found in the books [6], [8], [23], [26].

Ramsey Numbers

Ramsey theory is the study of the general phenomenon that every large structure, even if it looks totally chaotic, must contain a rather large well-organized substructure. This holds for many types of structures (though there are exceptions) and yields interesting applications in several mathematical areas. A detailed treatment of the subject can be found in [21].

Although several Ramsey-type theorems appeared earlier, the origin of Ramsey theory is usually credited to Frank Plumpton Ramsey, who proved in 1930 the fundamental theorem that edge colorings of finite or infinite graphs or hypergraphs satisfy such a theorem. The statement for finite graphs is as follows.

Let $H_1, H_2, \ldots, H_k$ be k finite, undirected, simple graphs. Then there is a finite number r such that in every edge coloring of the complete graph on r vertices by k colors, there is a monochromatic copy of H_i in color i for some $1 \le i \le k$. The smallest integer r that satisfies this property is called the (multicolor) *Ramsey number* of $H_1, \ldots, H_k$ and is denoted by $r(H_1, H_2, \ldots, H_k)$.

The determination or estimation of these numbers is usually a very difficult problem, one which fascinated Erdős since the '30s. When each graph H_i is a complete graph K_t with $t > 2$ vertices, the only values that are known precisely are those of $r(K_3, K_m)$ for $m \le 9$, $r(K_4, K_4)$, $r(K_4, K_5)$, and $r(K_3, K_3, K_3)$. The determination of the asymptotic behavior of Ramsey numbers up to a constant factor is also a very hard problem, and despite a lot of effort by various researchers there are only a few infinite families of graphs for which this behavior is known.

In one of the first applications of the probabilistic method in combinatorics, Erdős [12] proved that, if $\binom{n}{k} 2^{1-\binom{k}{2}} < 1$, then $R(K_k, K_k) > n$; that is, there exists a 2-coloring of the edges of the complete graph on n vertices containing no monochromatic clique of size k. This implies that $R(K_k, K_k) > 2^{k/2}$ for all $k \ge 3$. The proof is extremely short: the probability that a random two-edge coloring of K_n contains a monochromatic copy of K_k is at most $\binom{n}{k} 2^{1-\binom{k}{2}} < 1$, and hence there is a coloring with the required property.

It is worth noting that, although this argument seems trivial today, it was far from being obvious when published in 1947. In fact, several prominent researchers believed, before the publication of this short paper, that $R(K_k, K_k)$ may well be bounded by a polynomial in k. In particular, Paul Turán writes in [28] that he had conjectured for a while that $R(K_k, K_k)$ is roughly k^2 and that Erdős's result showing that this quantity behaves very differently than expected came to him as a big surprise.

My own first meeting with Paul Erdős took place when I was finishing high school in the early '70s in Haifa, Israel. Paul had a special visiting position at the Technion, and I met him during one of his visits. A few months before that I had read his probabilistic lower bound for the Ramsey numbers $R(K_k, K_k)$, formulated as a counting argument without any mention of probability, and noticed that the argument could be used to provide several similar results. I (proudly) told Erdős about my observations, and he encouraged me to keep thinking about these problems and gave me a book *The Art of Counting* [15], which had just been published at that time. This book contains selected publications of Erdős and is the first serious mathematical book I ever read. Reading it, and taking notes of much of its content, I quickly realized that it contained far more sophisticated extensions of the basic probabilistic lower bound proof of Erdős than the ones I observed. Paul, who surely knew well this fact, chose to suggest that I read the book and keep thinking about these problems, realizing that this is more stimulating than quickly pointing out the relevant references. Indeed, he always felt that young people interested in mathematics should be encouraged, and I am convinced that this approach was fruitful in many cases as it was in mine.

Returning to the asymptotics of Ramsey numbers, a particularly interesting example of an infinite family for which the behavior of the Ramsey number is known is the following result of Kim and of Ajtai, Komlós and Szemerédi.

Theorem 1 ([25], [1])**.** *There are two absolute positive constants c_1, c_2 such that*

$$c_1 m^2 / \log m \le r(K_3, K_m) \le c_2 m^2 / \log m$$

for all $m > 1$.

The upper bound, proved in [1], is probabilistic and applies a certain random greedy algorithm. There are several subsequent proofs, all based on probabilistic arguments. The lower bound is proved by a "semi-random" construction whose detailed analysis is subtle, relying on several large deviation inequalities. An alternative way of establishing the lower bound, which provides a better constant, appears in two recent papers, [20] and [7], that analyze the so-called "triangle-free process" suggested by Bollobás and Erdős. In this process one starts with a graph on n vertices with no edges and keeps adding uniformly chosen random edges among those that do not create a triangle. At the end, all these chosen edges are colored red and the nonchosen edges are colored blue. Clearly the resulting coloring contains no red triangle, and a careful analysis shows that with high probability there is no blue clique K_m for an appropriate choice of the initial size n.

It is worth noting that the question of obtaining a superlinear lower bound for $r(K_3, K_m)$ was mentioned already in [12], and Erdős established in [13], by an elegant probabilistic construction, an $\Omega(m^2/\log^2 m)$ lower bound.

Even less is known about the asymptotic behavior of multicolor Ramsey numbers, that is, Ramsey numbers with at least three colors. The asymptotic behavior of $r(K_3, K_3, K_m)$, for example, has been very poorly understood for quite some time, and Erdős and Sós conjectured in 1979 (cf., e.g., [10]) that

$$\lim_{m \to \infty} \frac{r(K_3, K_3, K_m)}{r(K_3, K_m)} = \infty.$$

This has been proved in [5], where it is shown that in fact $r(K_3, K_3, K_m)$ is equal, up to logarithmic factors, to m^3. A more complicated, related result proved in [5] that supplies the asymptotic behavior of infinitely many families of Ramsey numbers up to a constant factor is the following.

Theorem 2. *For every $t > 1$ and $s \geq (t-1)! + 1$ there are two positive constants c_1, c_2 such that, for every $m > 1$,*

$$c_1 \frac{m^t}{\log^t m} \leq r(K_{t,s}, K_{t,s}, K_{t,s}, K_m) \leq c_2 \frac{m^t}{\log^t m},$$

where $K_{t,s}$ is the complete bipartite graph with t vertices in one color class and s vertices in the other.

The proof of the lower bound is probabilistic: each of the first three color classes is a randomly shifted copy of an appropriate $K_{t,s}$-free graph that contains a relatively small number of large independent sets, as shown by combining spectral techniques with character sum estimates.

Sum-Free Subsets

A set A of integers is called *sum-free* if there is no solution to the equation $a + b = c$ with $a, b, c \in A$. Erdős [14] showed that any set A of n positive integers contains a sum-free subset of size at least $n/3$. The proof is a short and simple, yet intriguing, application of the probabilistic method. It proceeds by choosing a uniform random x in $(0, 1)$ and by observing that the set of all elements $a \in A$ satisfying $ax \mod 1 \in (1/3, 2/3)$ is sum-free and its expected size is $n/3$.

In [3] the authors showed that a similar proof gives a lower bound of $(n+1)/3$. Bourgain [9] has further improved this estimate to $(n+2)/3$. For quite some time it was not clear whether or not the constant $1/3$ could be replaced by a larger constant, until Eberhard, Green, and Manners proved in [11] that the constant $1/3$ is tight. Their proof is a sophisticated argument that contains a crucial probabilistic ingredient. The problem of deciding whether or not every set of n nonzero integers contains a sum-free subset of cardinality at least $n/3 + w(n)$, where $w(n)$ tends to infinity with n, remains open. It will be extremely surprising if there is no such $w(n)$.

List Coloring and Euclidean Ramsey Theory

The *list chromatic number* (or *choice number*) $\chi_\ell(G)$ of a graph $G = (V, E)$ is the minimum integer s such that for every assignment of a list of s colors to each vertex v of G, there is a proper vertex coloring of G in which the color of each vertex is in its list. This notion was introduced independently by Vizing in [29] and by Erdős, Rubin, and Taylor in [19]. In both papers the authors realized that this is a variant of usual coloring that exhibits several new interesting properties and that in general $\chi_\ell(G)$, which is always at least as large as the chromatic number of G, may be arbitrarily large even for graphs G of chromatic number 2.

For about ten years after the initial papers of Vizing and of Erdős, Rubin, and Taylor there was essentially no work on list coloring. Starting in the late '80s, the topic, motivated to a great extent by the many problems raised by Erdős and his collaborators in [19], received a considerable amount of attention. Paul Erdős himself told me in the early '90s that, when they wrote their paper, he thought that the topic was not very exciting and was pleasantly surprised to see that it eventually stimulated so much activity. I view this as a sign showing that Paul was essentially unable to ask any noninteresting questions. When he asked a question, even if at first sight it seemed artificial or nonappealing (even to Paul himself!), almost always it eventually turned out to be interesting.

It is natural to extend the notion of list coloring to hypergraphs. A hypergraph H is an ordered

pair (V, E), where V is a set of vertices and E is a collection of subsets of V, called edges. It is r-uniform if every edge contains exactly r vertices. Thus graphs are 2-uniform hypergraphs. The list chromatic number $\chi_\ell(H)$ of a hypergraph H is the minimum integer s such that, for every assignment of a list of s colors to each vertex of H, there is a vertex coloring of H assigning to each vertex a color from its list, with no monochromatic edges.

An intriguing property of list coloring of graphs which is not shared by ordinary vertex coloring is the fact that the list chromatic number of any (simple) graph with a large average degree is large. Indeed, it is shown in [2] that the list chromatic number of any graph with average degree d is at least $\Omega(\log d)$. For $r \geq 3$, simple examples show that there is no nontrivial lower bound on the list chromatic number of an r-uniform hypergraph in terms of its average degree. However, such a result does hold for simple hypergraphs. Recall that a hypergraph is *simple* if every two of its distinct edges share at most one vertex. The following result is proved in [4].

Theorem 3. *For every fixed $r \geq 2$ and $s \geq 2$, there is a $d = d(r, s)$ such that the list chromatic number of any simple r-uniform hypergraph with n vertices and at least nd edges is greater than s.*

A similar result for the special case of d-regular 3-uniform simple hypergraphs has been obtained independently in [22]. A subsequent proof with a much better upper estimate for $d(r, s)$ appears in a recent paper of Saxton and Thomason [27].

The proof of the theorem is probabilistic. For the simpler case of graphs it shows that if $G = (V, E)$ is a graph with average degree $d > 10^s$, then when we assign to each vertex of G a randomly chosen list consisting of s colors among the colors $\{1, 2, \ldots, 2s - 1\}$, then with high probability there is no proper coloring of G assigning to each vertex a color from its list. The precise argument requires some work, and the result suggests an interesting algorithmic question: given a graph $G = (V, E)$ with minimum degree $d > 10^s$, can we find, deterministically and efficiently, lists of size s for each $v \in V$ so that there is no proper coloring of G assigning to each vertex a color from its list? This problem is open, as is the simpler NP version of it, that is, that of exhibiting lists and providing a certificate that there is no proper coloring using them. Here the lists do not have to be found efficiently, and we require only that one will be able to check the certificate efficiently.

The last theorem has an interesting application in Euclidean Ramsey theory, yet another subject initiated by Erdős and his collaborators. A well-known problem of Hadwiger and Nelson is that of determining the minimum number of colors required to color the points of the Euclidean plane so that no two points at distance 1 have the same color. Hadwiger showed in 1945 that seven colors suffice, and Moser and Moser noted in 1961 that three colors do not suffice. These bounds have not been improved despite a considerable amount of effort by various researchers; see [24, pp. 150–152] and the references therein for more on the history of the problem.

A more general problem is considered in [16], [17], [18], where the main question is the investigation of finite point sets K in the Euclidean space for which any coloring of a Euclidean space of dimension d by r colors must contain a monochromatic isometric copy of K. There are lots of intriguing conjectures that appear in these papers. One of them asserts that, for any set K of three points which do not form an equilateral triangle, the minimum number of colors required for coloring the plane with no monochromatic isometric copy of K is three. The situation is very different for list coloring. A simple corollary of the theorem above is the following.

Theorem 4 ([4])**.** *For any finite set X in the Euclidean plane and for any positive integer s, there is an assignment of a list of size s to every point of the plane such that, whenever we color the points of the plane from their lists, there is a monochromatic isometric copy of X.*

The examples described in this brief survey include applications of the probabilistic method of Paul Erdős in graph theory, Ramsey theory, additive number theory, and combinatorial geometry. There have been recent results in the study of each of these examples, while the roots of all of them lie in the work and questions of Paul. There is no doubt that the study and application of probabilistic arguments will keep playing a crucial role in the development of many mathematical areas in the future, providing further evidence for the profound influence of Erdős. The comment he made at his eightieth birthday conference proved to be accurate: the probabilistic method does live and will stay alive long after him.

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D. A. Goldston

Erdős's Work on Primes

Prime numbers were an early and abiding interest of Erdős throughout his life. The second paper in his collected works [2] from 1932 gave a new proof of Tschebyschef's theorem on bounding the number of primes up to x. Many more papers on primes followed over more than sixty years. Of course Erdős published on topics in a large subset of mathematics, but from his talks later in life it was clear that the primes held a special attraction for him. In addition to his theorems, Erdős was constantly asking questions about primes and frequently offering money for solutions to his problems. These questions have led to surprising developments and new fields of study. Many remain unsolved, but a surprising number have been solved. In this short survey of some of Erdős's work on primes, I can mention only a selection of his results. I will also include a few of his lesser-known results that have interested me over many years.

The Elementary Proof of the Prime Number Theorem

By far the best known of Erdős's contributions to prime numbers is the elementary proof of the Prime Number Theorem (PNT) in 1949. This states, on letting $\pi(x)$ denote the number of primes $\le x$ that

$$\pi(x) \sim \frac{x}{\log x}, \quad \text{as } x \to \infty.$$

The PNT was first proved independently by Hadamard and de la Vallée-Poussin in 1896, but this and later proofs all depended on information on the zeros of the Riemann zeta-function $\zeta(s)$. It was eventually shown that the PNT follows from the fact that $\zeta(1+it) \neq 0$ for all real t and conversely that the PNT implies this result. Thus it was argued that no elementary proof could be obtained and if such a proof was found, then possibly this would be a step towards proving the Riemann Hypothesis. It was an electrifying moment in 1949 when an elementary proof was discovered. Much has been written about the circumstances and controversy surrounding how Atle Selberg and Erdős obtained their proofs, but putting aside

D. A. Goldston is professor of mathematics at San José State University. His email address is daniel.goldston@sjsu.edu.

the human story, both Erdős's and Selberg's papers, [5] and [27], describe well the mathematical contributions that each made to the proof. Both papers being elementary require as a prerequisite only a beginning knowledge of number theory and analysis and are well worth reading. In 1951 Erdős won the Cole Prize for his work in number theory with his paper [5] specifically mentioned, and in 1952 Selberg won the Fields Medal with his work on the elementary proof of PNT included as one of his major accomplishments. Some commentators have expressed disappointment over how little the elementary proof has influenced current work, but one can equally well argue that it is often our rarely used but prized possessions that save the day.

Photo courtesy of Krishnaswami Alladi.

Paul Erdős lecturing at the Institute of Mathematical Sciences, Madras, India, in January 1984. On the board is the inequality on large prime gaps alluded to in the Alladi-Krantz article and by Goldston. Also on the board is a US$500 problem of Erdős.

Large Gaps between Consecutive Primes

Erdős was very interested in the sequence of differences (or gaps) between consecutive primes and returned often to questions on this sequence. By the PNT there are $\sim \frac{x}{\log x}$ primes in the interval $[x, 2x]$, and therefore the average distance between primes in this interval is $\sim \log x$. If p_n denotes the n^{th} prime, then the n^{th} difference between consecutive primes $d_n = p_{n+1} - p_n$ is on average $\log p_n$. Much work has been done on irregularity in the distribution of these differences. For large gaps the best result known is that

$$p_{n+1} - p_n > c \log p_n \frac{\log\log p_n (\log\log\log\log p_n)}{(\log\log\log p_n)^2}$$

for infinitely many n. Erdős in 1935 [3] obtained the above result except for the quadruple log factor, which was first obtained by Rankin in 1938 [23]. The constant c has been improved many times over the last fifty years; the best result currently known is $c = 2e^{\gamma}$, where γ is Euler's constant, which is due to Pintz [22] in 1997. Erdős offered US$10,000 for anyone who could prove that this result is true for arbitrarily large c, which compared to other Erdős offers is a surprisingly large amount for what appears to be a very specialized result.

Erdős was always eager to examine patterns within patterns. In 1949 [6] he proposed showing that

$$D_k(n) = \min(d_n, d_{n+1}, \ldots, d_{n+k-1})$$

is sometimes large; i.e., there exist k consecutive extremely large gaps between consecutive primes. He succeeded in proving that $\limsup_{n\to\infty} D_2(n) = \infty$. The general problem was finally answered in 1981 by Helmut Maier[16], who proved the surprisingly strong result that

$$D_k(n) > c(k) \log p_n \frac{\log\log p_n (\log\log\log\log p_n)}{(\log\log\log p_n)^2}$$

for infinitely many n. This paper of Maier introduced the "Maier matrix method," which Maier in 1985 [17] used in a startling fashion to prove that the number of primes in intervals $[x, x + (\log x)^M]$ for any number M is distributed too irregularly to have an asymptotic formula. There have been many further applications of Maier's method, and recently it has been developed by Granville and Soundararajan [12] into a general uncertainty concept for sequences. This snowballing effect set off by a seemingly off-hand question of Erdős has occurred over and over again.

Small Gaps between Consecutive Primes

One expects that there will be infinitely many primes differing by any given even number and in particular that there will be infinitely many twin primes with difference 2. Up until 2013 it was widely believed to be an extremely hard problem to prove that any such even number exists, i.e., that the primes do not become isolated further and further apart when one examines larger and larger numbers. The year 2013 will go down in history as a spectacular year for prime numbers, because for the first time it was proved by Yitang Zhang that there actually exist even numbers which are the difference of infinitely many primes. Thus there always exist bounded gaps between primes. Later in 2013 James Maynard, by a different method, also obtained this result, finding infinitely often there are primes differing by a number less than 600 and also proving the even more spectacular result that there are bounded gaps between k primes for any given number k.

Progress on this problem prior to 2013 was painfully slow, and the results obtained concern

the sequence of normalized gaps between primes

$$\left\{\frac{p_{n+1}-p_n}{\log p_n}\right\}_{n=1}^{\infty}.$$

Since the average of this sequence is 1, the first goal is to prove that this sequence has a limit point less than 1 and thus establish the existence of infinitely many smaller-than-average gaps between primes. Letting

$$\Xi = \liminf_{n\to\infty} \frac{p_{n+1}-p_n}{\log p_n},$$

our goal is to prove $\Xi < 1$. The first contribution to this problem was due to Hardy and Littlewood in an unpublished paper from around 1922. They proved, using the circle method, that assuming the Generalized Riemann Hypothesis (GRH) one can prove that $\Xi \le \frac{2}{3}$; Rankin [25] improved this, still assuming GRH, to $\Xi \le \frac{3}{5}$. The first unconditional result was due to Erdős [4], who proved $\Xi \le 1 - A$ for an absolute positive constant A. Erdős first observed from the Brun sieve that the number of primes differing by k is bounded accurately, and then by adding this bound over k in an interval, one finds that it is not possible for all consecutive prime differences to be exactly located at the average spacing. This method of Erdős is frequently available as an add-on to other methods. For example, in the influential paper of Bombieri and Davenport [1] in 1965, they were able to make Hardy and Littlewood's method unconditional and prove $\Xi \le \frac{1}{2}$ and then use Erdős's method to improve this to $\Xi \le 0.46550\ldots$. When Maier in 1988 found a third method for finding small gaps between primes he combined it with both of the earlier methods to obtain $\Xi \le 0.2484\ldots$. This paper also answered for the first time the Erdős question of showing for

$$E_k(n) = \max\left(\frac{d_n}{\log p_n}, \frac{d_{n+1}}{\log p_{n+1}}, \ldots, \frac{d_{n+k-1}}{\log p_{n+k-1}}\right)$$

that $\liminf_{n\to\infty} E_2(n) < 1$. Because of the recent work of Maynard we now know $\liminf_{n\to\infty} E_k(n) = 0$ and much more. In 2005 Goldston, Pintz, and Yıldırım (see [11]) finally proved $\Xi = 0$, and their method was used as a starting point by both Zhang and Maynard.

With the recent advances on small gaps, it is reasonable to ask whether the earlier work should now be consigned to the dustbin of history. The answer is "no." The new methods are exquisitely adapted to examine primes and almost primes in various configurations, but often they do not say much about the number of primes in longer intervals with length around the average spacing between primes. The Erdős and Bombieri-Davenport methods are adapted and best applied to questions in this regime rather than very short gaps between primes.

At the end of his 1940 paper on small gaps between primes, Erdős made the conjecture that

$$\sum_{p_n \le x} (p_{n+1}-p_n)^2 = O(x\log x),$$

a result not likely to be proved anytime soon, since the Riemann Hypothesis only implies the bound $O(x\log^3 x)$ [26]. Erdős continued:

> This result if true must be very deep. I could not even prove the following very much more elementary conjecture : Let n be any integer and let $0 < a_1 < a_2 < \cdots < a_x < n$ be the $\phi(n)$ integers relatively prime to n; then
>
> $$\sum_{i=1}^{x-1} (a_{i+1}-a_i)^2 < c\frac{n^2}{\phi(n)}.$$

Progress was slow on this problem. Consider the generalized conjecture that

$$\sum_{i=1}^{\phi(n)} (a_{i+1}-a_i)^\gamma = O\left(\frac{n^\gamma}{\phi(n)^{\gamma-1}}\right).$$

Hooley [15] proved in 1963 that this is true for $1 \le \gamma < 2$. The breakthrough was in 1986 when Montgomery and Vaughan [27] proved not just Erdős's conjecture but that this bound holds for all fixed $\gamma \ge 1$. The techniques developed for this problem have subsequently been applied conditionally to higher moments for the primes in short intervals by Montgomery and Soundararajan [19].

Arithmetic Progressions of Primes

In 2004 Ben Green and Terry Tao proved a long-standing conjecture that there are arbitrarily long arithmetic progressions of primes [13]. Previously it was known only that there were 3 primes in arithmetic progression, and current computation has found only 26 primes in arithmetic progression. The Green-Tao theorem makes use of many difficult techniques, including Szemerédi's Theorem of 1975, which was conjectured by Erdős and Turàn in 1936. In 1973 Erdős made the following conjecture [8], which implies both Szemerédi's theorem and the Green-Tao theorem: For any set of positive integers where the sum of the reciprocal of the integers diverges, the set contains arbitrarily long arithmetic progressions. Erdős offered $3,000 for a proof, but no proof is yet in sight.

Limit Points for Normalized Prime Gaps and Jumping Champions of Primes

I will conclude by mentioning two amusing problems for which much can be conjectured but nothing of consequence proved. The first concerns the normalized consecutive prime gaps mentioned earlier. Ricci [24] and Erdős [7] independently

proved that the set of limit points for this sequence has positive Lebesgue measure. Erdős's proof is a model of clarity and cleverness. Erdős conjectured that the sequence is actually everywhere dense in $(0, \infty)$, although he could not find a single number which is a limit point. At the time only ∞ was a known limit point. Since 1955 there have been a few new results obtained. We now know from [11] that 0 is also a limit point, but no other limit point is known. In 1988 Hildebrand and Maier [14] proved, using Maier's method, that there are so many large limit points that the set of limit points has infinite measure, which answered a question of Erdős to show there is a finite limit point larger than 1. Finally, in 2013 Pintz used Zhang's result to show that there is an interval $[0, c]$ all of whose points are limit points. However, c is ineffective. Thus neither of these results helps answer Erdős's question of finding specified numbers which are limit points.

A second problem is concerned with the question of finding the most frequent difference between consecutive primes $\leq x$. This is referred to as the jumping champion up to x. For example, the prime differences up to $x = 12$ are $3 - 2 = 1$, $5 - 3 = 2$, $7 - 5 = 2$, and $11 - 7 = 4$, so the jumping champion up to 12 is 2. In 1980 Erdős and Straus [9] proved, assuming the Hardy-Littlewood conjectured asymptotic formula for the number of prime pairs, that the jumping champions must go to infinity. Numerically 6 is the jumping champion for $x \geq 947$ and continues to be so as far as has been computed, which is currently around 10^{15}. Odlyzko, Rubinstein, and Wolf [21] provided evidence for the conjecture that jumping champions will eventually transition from 6 to 30 and continue to increase through the sequence of primorials $2, 6, 30, 210, 2310, \ldots$, where the k^{th} primorial is formed from the product of the first k primes. In agreement with this conjecture, Ledoan and I [10] proved, assuming the same Hardy-Littlewood conjecture used by Erdős and Straus, that any given primorial will divide every sufficiently large jumping champion. Assuming stronger Hardy-Littlewood conjectures, we proved the Primorial Conjecture for sufficiently large x. Despite this, nothing unconditional beyond what computation reveals is known about jumping champions. We cannot prove or show through computation that there is any jumping champion larger than 6. We cannot even prove that 2 is not the jumping champion for all large x, although we would actually conjecture that 2 and powers of 2 are the biggest jumping losers.

Ernst Straus and Erdős first met in Princeton in 1944 and were close friends and coauthors of many papers together. One of Straus's sons, Dan Straus, is a chemistry professor and colleague of mine at San José State University. I asked him for one Erdős story for this article, and naturally he had many, many Erdős stories. I include here one he sent me which represents how I also remember Erdős.

> Paul Erdős visited UCSB sometime around 1975 when I was an undergraduate chemistry student, and my father arranged a meeting. Erdős took a college friend and me to lunch in Isla Vista and then we walked to the lagoon on campus near Storke Tower. Erdős was interested in chemistry and particularly in recent discoveries of complexes of the noble gasses, and it was fun talking with him, as always. We were standing at the top of a grassy slope scattered with ducks. Spontaneously, Erdős threw out his arms like a scarecrow and careened off after the ducks, which he chased tirelessly in wide arcs. He was a sight in his loose fitting pinstripe suit and sandals with dress socks, and he was very agile for a man of about sixty-three. It was a time of studied nonconformity, and students were everywhere in beads, sandals and such just watching Erdős. It was refreshing to watch one person we can all be sure never tried to be different or worried what people thought.

Note Added in Proof

2014 was another spectacular year for primes. The method introduced by Maynard, and independently obtained by Terry Tao, has been further developed by the Polymath 8b project `http://www.resmathsci.com/content/1/1/12`. It is now known (as of January 2015) that there are infinitely often two primes that differ by less than or equal to 246.

The US$10,000 Erdős problem on large gaps between primes was solved independently both by Maynard and by Ford, Green, Konyagin, and Tao. These five authors have now proven jointly that for the size of the large gaps in the Erdős problem the power of the triple logarithm in the denominator may be reduced from 2 to 1.

On the problem of limit points of normalized consecutive prime gaps, Banks, Freiberg, and Maynard have adapted the Maynard–Tao method for small gaps between primes with the method for large gaps to prove very strong results. In particular they have proved that at least 12.5 percent of all positive real numbers are limit points of this sequence. However we still do not know any specific positive real number that is a limit point.

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András Sárközy

Erdős and Sequences

Paul Erdős was one of the most prolific mathematicians in the history of mathematics. He was a leading personality in modern combinatorial mathematics. The large number of his disciples ("epsilons" as he called them) and coauthors also contributed to his great impact on twentieth-century mathematics.

I was nineteen years old, a second-year university student, when I received the following letter: "Dear Mr. Sárközy, I have heard about your nice results …from Paul Turán. Please, come and see me at the Mathematical Institute" (of the Hungarian Academy of Sciences). The letter was signed by Paul Erdős. I visited him soon after at the Mathematical Institute. I told him my results and I sketched the proofs. This was followed by a very fruitful and inspiring discussion, and at the end he asked a related question. As an answer to this question, I soon published my first paper based on an Erdős problem, and I was on the way to becoming one of his disciples. This first meeting of ours was followed by many others, and during one of them Erdős asked a further related problem. This I settled jointly with Endre Szemerédi, whom I introduced to Erdős, and from that point on the three of us worked jointly for several years. We usually met at Erdős's apartment, where he lived then with his mother. If one of our meetings was successful enough and we ended up with a nice result, Erdős asked his mother to give us a "few cubic centimeters of poison," which meant a few drops of *tokaji* (a rather sweet and heavy Hungarian dessert wine of good quality). Between 1966 and 1970 we published ten triple papers, all but one of

András Sárközy is professor of mathematics at Eötvös Loránd University. His email address is sarkozy@cs.elte.hu.

them written on divisibility properties of sequences of integers. Then Szemerédi switched to the Erdős-Turán problem on arithmetic progressions and later turned towards combinatorics, while I continued to work with Erdős in number theory (mostly on sequences of integers). I have written sixty-two joint papers with him, which puts me on the top in the list of his coauthors (András Hajnal is a close second with fifty-seven joint papers with him).

In classical number theory, one studies *special* sequences like primes, squares, etc. If in number theory we say just "sequence" (without any adjective), then we mean a general sequence; typically, we are looking for the connection between certain arithmetic properties and the density properties of sequences (or sets) of integers. The study of (general) sequences was started around 1930 by Schnirelmann's papers, and Erdős played a dominant role in the advance of this field. The first monograph written on sequences was the excellent book of Halberstam and Roth [19]. They write:

> Anyone who turns the pages of this book, will immediately notice the predominance of results due to Paul Erdős. In so far as the substance of this book may be said to define a distinct branch of number theory—and its wide range of topics in classical number theory appears to justify this claim—Erdős is certainly its founder. He was the first to recognize its true potential and has been the central figure in many of its developments.

In some of his papers written on *additive properties of sequences* Erdős studied Sidon sets. A set $\mathcal{A}$ of positive integers is said to be a Sidon set if all the sums $a + a'$ with $a \in \mathcal{A}$, $a' \in \mathcal{A}$, $a \le a'$ are distinct. Let $F(n)$ denote the maximal cardinality of a Sidon set $\mathcal{A}$ with $\mathcal{A} \subset \{1, 2, \dots, n\}$. Erdős and Turán [16] proved that $F(n) \le (1 + o(1))n^{1/2}$, and later Chowla and Erdős [3] independently also showed that $F(n) \ge (1 + o(1))n^{1/2}$. Erdős [22] also proved that, if $\mathcal{A}$ is an infinite Sidon set, then its counting function $A(n) = |\{a : a \in \mathcal{A},\ a \le n\}|$ must satisfy

$$\liminf_{n\to\infty} A(n) \left(\frac{\log n}{n}\right)^{1/2} < \infty.$$

As an answer to a question of Sidon (who was a Hungarian mathematician working in harmonic analysis), Erdős [4] proved that there is an infinite sequence $\mathcal{A}$ of positive integers such that, denoting the number of solutions of the equation

$$a + a' = n, \quad a \in \mathcal{A}, \quad a' \in \mathcal{A} \tag{1}$$

by $r(n)$, we have

$$c_1 \log n < r(n) < c_2 \log n$$

with some positive absolute constants c_1, c_2. He used a probabilistic method, and in [10] Erdős and Rényi gave further applications of this method. Later Erdős and Sárközy [11] generalized and extended this result of Erdős significantly.

Photo courtesy of Krishnaswami Alladi.

(From l to r, front row) Vera Sós, Andras Sárközy, and Paul Erdős at the Institute of Mathematical Sciences, Madras, India, in January 1984 just before the start of Erdős's lecture.

Write $R(x) = \sum_{n \le x} r(n)$, where $r(n)$ is the function defined above. Erdős and Fuchs [6] proved that, if $\mathcal{A}$ is an infinite sequence of positive integers, then

$$R(x) = cx + o\left(\frac{x^{1/4}}{(\log x)^{1/2}}\right) \tag{2}$$

cannot hold with a positive constant c. The significance of this result is based on the fact that it is closely related to the circle problem (which consists of the estimate of the number of lattice points in the circle $x^2 + y^2 \le r^2$). A further interesting feature of this result is that this is one of the first occasions when analytical tools were used to prove a theorem on general sequences. Jurkat (unpublished) and later Montgomery and Vaughan proved that the $o(\dots)$ term in (2) can be improved to $o(x^{1/4})$.

Erdős also studied *multiplicative properties* of sequences. A set $\mathcal{A}$ of positive integers is said to be primitive if there is no divisibility relation in it: if $a \in \mathcal{A}$, $a' \in \mathcal{A}$ and $a \ne a'$, then $a_i \nmid a_j$. Behrend proved that, if $\mathcal{A} \subset \{1, 2, \dots, n\}$ and $\mathcal{A}$ is primitive, then

$$\sum_{a\in\mathcal{A}} \frac{1}{a} < c\frac{\log n}{(\log\log n)^{1/2}} \tag{3}$$

for some absolute constant c. Erdős, Sárközy, and Szemerédi [12], [13] determined the smallest c with this property, and they also showed that, for infinite sets $\mathcal{A}$, the inequality (3) can be sharpened: if $\mathcal{A}$ is an infinite primitive set, then we have

$$\sum_{\substack{a\in\mathcal{A}\\ a\le x}} \frac{1}{a} = o\left(\frac{\log x}{(\log\log x)^{1/2}}\right).$$

Erdős [2] also proved another theorem on positive sequences: he proved that, if $\mathcal{A}$ is a primitive set with $\mathcal{A} \neq \{1\}$, then

$$\sum_{a\in\mathcal{A}} \frac{1}{a\log a} \leq c \tag{4}$$

with some absolute constant c. He conjectured that, for primitive sequences $\mathcal{A}$, the left-hand side of (4) is maximal if $\mathcal{A}$ is the set of the primes, and Erdős and Zhang [17] proved partial results in this direction.

Erdős also studied multiplicative Sidon sets, i.e., sets $\mathcal{A}$ of positive integers such that all the products aa' with $a \in \mathcal{A}$, $a' \in \mathcal{A}$, $a \leq a'$ are distinct. Let $G(n)$ denote the maximal cardinality of a multiplicative Sidon set $\mathcal{A}$ with $\mathcal{A} \subset \{1, 2, \dots, n\}$. He proved [5] that there are positive absolute constants c_1, c_2 such that, for $n > n_0$ we have

$$c_1 n^{3/4}(\log n)^{-3/2} < G(n) - \pi(n) < c_2 n^{3/4}(\log n)^{-3/2},$$

where $\pi(n)$ is the number of primes not exceeding n. This result is a beautiful application of combinatorics: the proof is based on estimates for the number of edges in bipartite graphs containing no cycles of length 4.

Many further results and also recent developments related to Erdős's works are surveyed in [21] (in the additive case) and [20] (in the multiplicative case).

Erdős and Szemerédi [14] studied a problem involving both sums and products. Let $a_1 < a_2 < \cdots < a_n$ be a sequence of positive integers. Consider the products and sums, i.e., all the integers of the form

$$a_i + a_j \text{ or } a_i a_j \text{ (with } 1 \leq i \leq j \leq n). \tag{5}$$

Denote by $f(n)$ the largest integer so that, for every $\{a_1, a_2, \dots, a_n\}$, there are at least $f(n)$ distinct integers of form (4). They proved that

$$n^{1+c_1} < f(n) < n^2 e^{-c_2 \log n/\log\log n} \text{ for } n > n_0(\varepsilon)$$

with some positive absolute constants c_1 and c_2. This result has been sharpened and extended in various directions. A survey of these results has been given by Bourgain [1].

Erdős also studied "hybrid" problems in which both general sequences and special sequences (e.g., the sequence of the primes) occur; in particular, he studied the number of prime factors of sums $a + b$ with $a \in \mathcal{A}$, $b \in \mathcal{B}$ (where $\mathcal{A}$, $\mathcal{B}$ are "large" or "dense" sets) jointly with Turán [15]. They proved in 1934 that, if $\mathcal{A}$ is a finite set of positive integers, then

$$\omega\Big(\prod_{a,a'\in\mathcal{A}} (a + a')\Big) > c \log |\mathcal{A}|,$$

where $\omega(n)$ denotes the number of distinct prime factors of n and c is a positive absolute constant. They also conjectured that this result can be extended to sums $a + b$ with $a \in \mathcal{A}$, $b \in \mathcal{B}$, where $\mathcal{A}$, $\mathcal{B}$ are finite sets of positive integers with $|\mathcal{A}| = |\mathcal{B}|$. This conjecture (in a slightly sharper form) was proved more than fifty years later by Győry, Stewart, and Tijdeman. Erdős, Maier, and Sárközy [8] studied the distribution of the numbers $\omega(a + b)$, and Erdős, Pomerance, Sárközy, and Stewart [9] the maximum of $\omega(a + b)$ with $a \in \mathcal{A}$, $b \in \mathcal{B}$ for "large" subsets $\mathcal{A}$, $\mathcal{B}$ of $\{1, 2, \dots, n\}$.

This survey would not be complete without mentioning some of Erdős's most famous problems. In 1936 Erdős and Turán conjectured that, if for any positive integers k and n we denote the size of the largest subset of $\{1, 2, \dots, n\}$ which does not contain an arithmetic progression of length k by $r_k(n)$, then for any fixed k we have $r_k(n) = o(n)$. First Roth in 1953 and then Szemerédi in 1969 settled the $k = 3$ and $k = 4$ special cases, respectively, and finally, in 1975, Szemerédi proved the conjecture in its general form. Szemerédi's theorem has been extended and sharpened in various directions (see [18] for a survey of these results). Erdős also conjectured that the following stronger statement is also true: if $\mathcal{A}$ is an infinite sequence of positive integers such that $\sum_{a\in\mathcal{A}} \frac{1}{a} = \infty$, then $\mathcal{A}$ contains arbitrarily long arithmetic progressions. A further related conjecture of his: if $e_1, e_2, \dots$ is any infinite sequence with $e_i \in \{-1, +1\}$ for $i = 1, 2, \dots$, then for an arbitrarily large real number K there exist positive integers n and d such that $|e_d + e_{2d} + \cdots + e_{nd}| > K$. These last two problems are still wide open.

Erdős and Turán conjectured that, if $\mathcal{A}$ is a set of positive integers such that $r(n) > 0$ for $n > n_0$ (where again $r(n)$ denotes the number of solutions of (1)), then $r(n)$ cannot be bounded: $\limsup_{n\to\infty} r(n) = \infty$. This conjecture is also still open.

There are many more problems in the Erdős–Graham book [7] and in his numerous problem papers; the majority of these problems remain unsolved.

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József Szabados

Erdős and Polynomial Interpolation

It is not easy to write about Paul Erdős. The number of stories on his way of life, on his method of work, on his interest in politics, is all well known. Nevertheless, it is important to remember him, to tell how we admire his devotion to mathematics.

My connection with him was sporadic and occasional. Although approximation theory was not his main field of interest, he contributed numerous significant results to this area of mathematics. In the early years of his career, mainly together with Paul Turán and Géza Grünwald, he was interested in interpolation and properties of polynomials. In these papers, sometimes his formulations and proofs were vague, and later he kept returning to his original problems by urging us to give more precise and simpler proofs or even to correct faulty arguments. In what follows we collect some examples of his outstanding achievements and loose statements which required further discussions. We restrict ourselves to Lagrange interpolation, although properties of polynomials would be a related area where he made significant progress as well.

Let $-1 \le x_{1n} < x_{2n} < \cdots < x_{nn} \le 1$ be an arbitrary system of nodes, and let

$$L_n(f,x) =: \sum_{k=1}^{n} f(x_{kn})\ell_{kn}(x)$$

be the uniquely determined *Lagrange interpolation polynomial* of degree at most $n-1$ of a continuous function $f(x)$ in the interval $[-1,1]$, where

$$\ell_{kn}(x) := \prod_{\substack{j=1 \\ j\neq k}}^{n} \frac{x - x_{jn}}{x_{kn} - x_{jn}}, \qquad k = 1, 2, \ldots, n,$$

are the *fundamental polynomials* of interpolation. The crucial quantity for the convergence of interpolation is the *Lebesgue constant* λ_n, which is the maximum over $[-1,1]$ of the *Lebesgue function*

$$\lambda_n(x) := \sum_{k=1}^{n} |\ell_{kn}(x)|.$$

G. Faber [11] proved that, no matter what the system of nodes of interpolation is, the Lebesgue constant λ_n is always at least $c_1 \log n - c_2$, with some constants $c_1, c_2 > 0$. Erdős [3] proved that the best constant here is $c_1 = 2/\pi$ (which can be attained for example for the roots of the Chebyshev polynomials $T_n(\cos(n \arccos x))$.

A direct consequence of this behavior of the Lebesgue constant is that Lagrange interpolation can never be uniformly convergent for all continuous functions. Far more is true: the celebrated theorem of Erdős and Vértesi [10] states that, for any system of nodes, one can always construct a continuous function whose interpolating polynomials diverge *almost everywhere.*

The phenomenon of divergence is in analogy with the behavior of the trigonometric Fourier series. However there, as is well known, the first arithmetic means (Fejér sums) converge uniformly

József Szabados is research professor emeritus of mathematics at the Alfréd Rényi Institute of Mathematics. His email address is szabados.jozsef@renyi.mta.hu.

to continuous functions. In a joint paper [4] with Géza Grünwald, Erdős claimed that this analogy does not hold for Lagrange interpolation: considering the arithmetic means

$$\frac{1}{n}\sum_{k=1}^{n} L_k(f,x) \tag{6}$$

of Lagrange interpolation based on the nodes of the Chebyshev polynomial $T_n(x) := \cos(n \arccos x)$, they diverge for some continuous function f. It turned out later that the proof was faulty: instead of (6), the proof works only for the arithmetic means of the absolute values of the interpolating polynomials. This error was corrected much later in the joint work [5] with G. Halász: in fact, they proved that (6) is at least $o(\log\log n)$ for some f_0, and this lower estimate is sharp.

Most of the time Erdős was correct in making conjectures for difficult problems, but not always. In [2] he conjectured that the minimum of the integral of the sum of squares of the fundamental polynomials of Lagrange interpolation, i.e.,

$$\int_{-1}^{1} \sum_{k=1}^{n} \ell_{kn}(x)^2\, dx,$$

is attained when the nodes are the roots of the integral of the Legendre polynomials. It was a reasonable conjecture, since these are the only nodes where the sum of the squares of the fundamental polynomials is bounded by 1 (in contrast to the Lebesgue constants mentioned above). However, I succeeded in proving that these nodes do not serve the minimum [12], and this was the occasion when Erdős started telling me problems. The solution of the mentioned problem of minimum is still unsolved, and it seems as hopeless as the exact construction of nodes of interpolation with minimal Lebesgue constant.

Our first joint work [6] is a kind of strengthening of the celebrated result of G. Faber mentioned above. We proved that the *integral* of the Lebesgue function over a fixed subinterval of interpolation

$$\int_a^b \lambda_n(x)\, dx, \qquad [a,b] \subset [-1,1]$$

is always at least $c(a,b)\log n$, where $c(a,b) > 0$ depends only on the length of the interval in question. This, of course, implies Faber's result. The proof is based on the simple observation that the sum of a positive number and its reciprocal is always at least 2. Then, in [9] we generalized this result for the case when the interval in question depends on n and proved also the sharpness of the result. The problem was further generalized in another direction by considering the *weighted* integral (with Jacobi weights) of the sum of *even* powers of the fundamental functions of interpolation (cf. [8]).

Of course, he was extremely fast in thinking and was rather annoyed when it turned out that somebody could not follow his train of thought. I have some sheets with his characteristic handwriting, but not much. Talking was more important for him than writing down occasionally long formulas. A typical situation was when he said, "Something is disturbing me," when we were stuck with a problem or something seemed to be too good to hold. Not for long, since he was always able to circumvent the difficulty.

In [1], Erdős (among others) considered the problem of loosening the strict condition of Lagrange interpolation when for n nodes we look for an interpolating polynomial of degree at most $n-1$. What if we allow polynomials of degree at most $n(1+\varepsilon)$, where ε is a fixed positive number? He claimed that, under some regularity conditions on the distribution of nodes (asymptotic equidistant distribution of the arccos of the nodes and a lower estimate on the distance of adjacent nodes), the interpolation polynomials of degree at most $n(1+\varepsilon)$ converge uniformly for all continuous functions. He did not prove this statement, but gave an indication how it can be deduced from another theorem of the same paper by a simple modification. We were unable to recover this "simple modification" and started working on the problem. Eventually, we came up with a rather long and sophisticated proof, gave an error estimate for the convergence, and proved the necessity of the mentioned regularity conditions (cf. [7]).

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Gérald Tenenbaum

Paul Erdős and the Divisors

Paul Erdős and I collaborated from March 1977 until his death (he called this a leave) in October 1996. During these nineteen years, most of our mathematical discussions were on the distribution of divisors of integers.

Analytic and probabilistic number theory is mainly concerned with understanding how the multiplicative and the additive structure of integers combine together or ignore each other. The twin primes conjecture and the Goldbach conjecture are enlightening examples: since nothing trivially forbids two primes from having difference two, this should happen with statistical frequency; similarly, if $2n$ is an arbitrary integer, the sequence $2n - p$, where p runs through all primes up to n, should contain a random quota of primes.

The famous *abc* conjecture of Masser and Oesterlé, dating from the early 1970s, is even more representative of this class of problems: addition should destroy multiplicative structure. Let us discuss this in more detail. The squarefree kernel, say $k(n)$, of an integer n is the product of all primes appearing in the canonical decomposition of n, ignoring exponents. A normal integer (in other words an integer belonging to a set that will almost surely show up when one picks an integer at random) has a "large" kernel (see [31] for recent progress on this question), whereas small kernels occur only for integers with a very special structure such as perfect powers or integers divisible by a large perfect power. In qualitative form, the *abc* conjecture states that when a and b are coprime and $c = a + b$, the three integers cannot be simultaneously abnormal. A thorough quantitative discussion of this question may be found in [30].

The distribution of divisors is a problem of similar type to those described above: a divisor has a very special multiplicative structure, since its prime factors, and even their exponents, are restricted in a drastic manner. Thus it is a basic number theoretic challenge to try to understand how the sequence of divisors is distributed by size, that is to say, with respect to additive structure.

Erdős was interested in all aspects of this question: usual behavior of the sequence of divisors of a random, or normal, integer; extremal properties involving divisors, that is, small and large values of arithmetic functions defined in terms of divisors; structure of sequences defined by constraints on the divisors of their elements; stochastic variations of divisor functions; etc.

It is clear that the divisors are made up from the prime factors. So the first step in the problem is to describe the growth of the sequence of prime factors. The step Erdős made was that of a giant. Before anyone else, he understood that basic results in probabilistic number theory, such as the Turán-Kubilius inequality,[1] yield a very strong and very surprising fact: in first approximation, the size of the jth prime factor of a normal integer n does not depend on n but only on j; more precisely, if we let $\{p_j(n)\}_{j=1}^{\omega(n)}$ denote the increasing sequence of the distinct prime factors of n, then we have

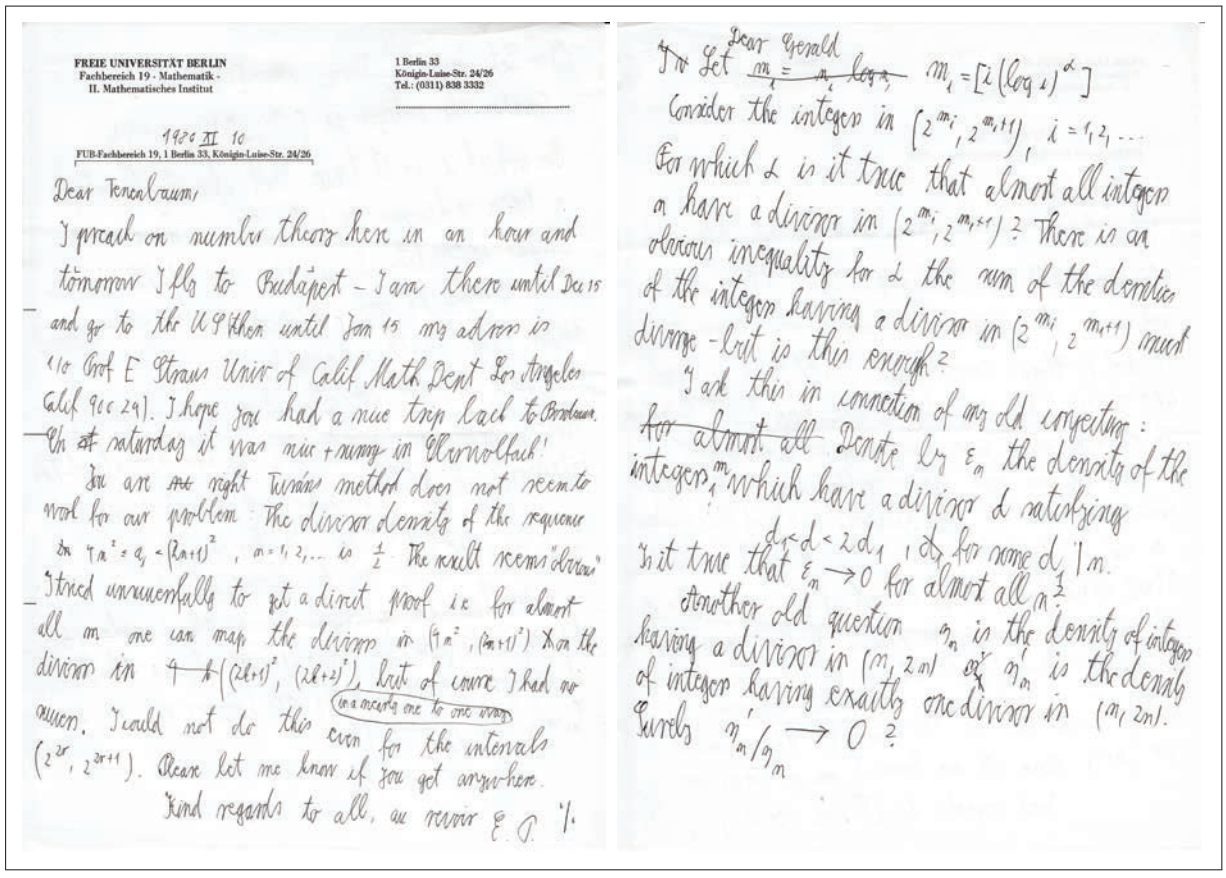
$$\log_2 p_j(n) \sim j \qquad (j \to \infty) \tag{1}$$

for almost all integers n.[2] The short proof may be retrieved in Erdős's paper [7]. It will not be reproduced here: we content ourselves with saying that the Turán-Kubilius inequality is the arithmetical analogue of the theorem from probability theory stating that the variance of a sum of independent random variables is the sum of the variances of its terms and that Erdős merely uses a Bienaymé-Chebyshev-type inequality to deduce his result. A much sharper result, actually exhibiting a Gaussian behavior of the prime factors around their means, is obtained in [5]. The reader may consult [18] (Chapter 1) for a detailed proof and [39] (Theorem III.3.10) for a simpler proof of a slightly weaker result.

From the above statement on prime factors, one could guess that the jth divisor of a normal integer is somewhat close to $\exp j^c$ with $c = 1/\log 2$. It was one of Erdős's outstanding qualities to devise a question that would precisely test how adequate the model is to the arithmetic nature of things. Since $c > 1$, we may deduce from the previous heuristics that the minimal ratio $E(n)$ between consecutive divisors, say d'/d, with $d < d'$, does not tend to 1 for almost all integers. However, as early as 1948 and probably much before, Erdős

Gérald Tenenbaum is professor of mathematics at the Université de Lorraine. His email address is gerald.tenenbaum@univ-lorraine.fr.

[1] *See, e.g., Elliott* [4] *or Tenenbaum* [39].
[2] *Here and in the sequel we denote by* $\log_k$ *the* k*-fold iterated logarithm.*



FREIE UNIVERSITÄT BERLIN
Fachbereich 19 - Mathematik -
II. Mathematisches Institut

1 Berlin 33
Königin-Luise-Str. 24/26
Tel.: (0311) 838 3332

1976 II 10
FUB-Fachbereich 19, 1 Berlin 33, Königin-Luise-Str. 24/26

Dear Tenenbaum,

I preach on number theory here in an hour and tomorrow I flg to Budapest - I am there until Dec 15 and go to the U of Ithica until Jan 15 my adress is c/o Prof E Straus Univ of Calif Math Dept Los Angeles Calif 90024. I hope you had a nice trip back to Canada. Ch saturday it was nice + sunny in Oberwolfach!

Yr are right Turáns method does not seem to work for our problem. The divisor density of the sequence $n^2 < a_i < (n+1)^2$, $i = 1, 2, \ldots$ is $\frac{1}{2}$. The result seems obvious. I tried unsuccessfully to get a direct proof i.e. for almost all n one can map the divisors in $(n^2, (n+1)^2)$ 1-1 on the divisors in $(2l+1)^2, (2l+2)^2$), but of course I had no success. I could not do this even for the intervals $(2^{2r}, 2^{2r+1})$. Please let me know if you get anywhere.

Kind regards to all, as ever P. E.

Dear Gerald

Let $m_1 = [2(\log 2)^\alpha]$

Consider the integers in $(2^{m_i}, 2^{m_i+1})$, $i = 1, 2, \ldots$

For which α is it true that almost all integers n have a divisor in $(2^{m_i}, 2^{m_i+1})$? There is an obvious inequality for α the sum of the densities of the integers having a divisor in $(2^{m_i}, 2^{m_i+1})$ must diverge - but is this enough?

I ask this in connection of my old conjecture: Denote by ε_n the density of the integers n which have a divisor d satisfying $d_1 < d < 2d_1$, $d_1 \mid n$.

Is it true that $\varepsilon_n \to 0$ for almost all n?

Another old question η_n is the density of integers having a divisor in $(n, 2n)$ or η'_n is the density of integers having exactly one divisor in $(n, 2n)$. Surely $\eta'_n/\eta_n \to 0$?

Letter from Erdős to Tenenbaum stating a problem on divisor density.

conjectured the exact opposite: for almost all n, one should have

$$(2)\quad E(n) := \min_{dd' \mid n,\, d<d'} d'/d = 1 + (\log n)^{1-\log 3+o(1)}.$$

The reason for this is precisely that the distribution of the prime factors fluctuates significantly around the mean and hence the quantities $\log d'/d$ should be fairly evenly distributed in the interval $[-\log n, \log n]$. Since these quantities are at least $3^{\omega(n)}$ in number and, by the Turán-Kubilius inequality or indeed from (1), we have $\omega(n) \sim \log_2 n$ for almost all n, we naturally arrive at (2). This conjecture was proved by Erdős-Hall [8] for the lower bound and by Maier-Tenenbaum [24] for the upper bound. A further refinement [29] comes even closer to the heuristics: for almost all n, we have

$$(3)\qquad E(n) = 1 + \frac{\log n}{3^{\omega(n)}}(\log_2 n)^{\vartheta_n}$$

where $-5 \leqslant \vartheta_n \leqslant 10$.

One of the most challenging problems remaining on the so-called subject of propinquity of divisors concerns the functions

$$E_r(n) := \min_{1\leqslant j\leqslant \tau(n)-r} \log\{d_{j+r}(n)/d_j(n)\} \qquad (r \geqslant 2),$$

where $\{d_j(n)\}_{j=1}^{\tau(n)}$ denotes the increasing sequence of the divisors of n. The precise normal behavior remains still unknown for all $r \geqslant 2$. Using techniques similar to that of the proof of Theorem 3 of [12], it can be shown that, on a sequence of natural density 1, we have

$$E_2(n) > (\log n)^{-\gamma_2+o(1)}$$

for some $\gamma_2 < \log 3 - 1$. Moreover, the methods and results of [25] yield, still normally,

$$E_r(n) \leqslant (\log n)^{-\beta_r+o(1)},$$

with

$$\beta_r := \frac{(\log 3 - 1)^m}{(3\log 3 - 1)^{m-1}}, \quad 2^{m-1} < r + 1 \leqslant 2^m.$$

Thus, we have

$$\beta_1 = \log 3 - 1 \approx 0.09861, \quad \beta_2 = \beta_3 \approx 0.00423,$$
$$\beta_r \approx 0.00018 \quad (4 \leqslant r \leqslant 7).$$

Also, it is proved in [25] (Thm. 1.1) that $E_r(n) > \tau(n)^{-1/r+o(1)}$ holds for almost all integers, uniformly in $r \geqslant 1$, and thus, on a sequence of density 1,

$$E_r(n) = 1/(\log n)^{o(1)} \qquad (r = r(n) \to \infty),$$

a result which might look surprising at first sight.

Maier and I conjecture the existence of a strictly decreasing sequence $\{\alpha_r\}_{r=1}^{\infty}$ such that we have

$$E_r(n) = (\log n)^{-\alpha_r + o(1)}$$

on a sequence of density 1. It is particularly irritating, for instance, to be unable to find a better normal upper bound for $E_2(n)$ than for $E_3(n)$.

I refer the reader to the recent survey [40] for a further account of Erdős's motivations for the conjecture (2), in particular related to the concept of set of multiples. The link is particularly apparent in Erdős's letter dated November 10, 1980, reproduced herein.

On page 1, Erdős mentions *divisor density*. The definition, due to R. R. Hall [15], is as follows. Let $\tau(n, \mathcal{A})$ designate the number of divisors of an integer n belonging to a sequence $\mathcal{A}$, and write $\tau(n) = \tau(n, \mathbb{Z}^+)$. We say that the integer sequence $\mathcal{A}$ has divisor density z, and we write $D\mathcal{A} = z$ if we have $\tau(n, \mathcal{A}) = \{z + o(1)\}\tau(n)$ as n tends to infinity on a sequence of natural density 1.[3]

Divisor density is a fruitful and surprising notion. For instance, Hall proved in [15] that, for any pair $(z, w) \in [0,1]^2$, there is an integer sequence $\mathcal{A}$ with divisor density z and logarithmic density w.[4] A criterion for $D\mathcal{A} = z$ is given in [33].

With the above definition, Erdős's question on page 1 may be stated as follows: define $\mathcal{A} := \bigcup_{m \geqslant 1} [4m^2, (2m+1)^2[\cap \mathbb{Z}^+$; is it true that $\mathcal{D}\mathcal{A} = \frac{1}{2}$? It needed a lot of work and the appeal to many deep results from analytic number theory, such as estimates of Karatsuba on exponential sums [22], to answer, positively, Erdős's question; see Hall-Tenenbaum [17] and Tenenbaum [38] (Theorem 11). To be slightly more precise, we observe that the sequence $\mathcal{A}$ may alternatively be defined by the condition $\left\langle \frac{1}{2}\sqrt{n} \right\rangle \leqslant \frac{1}{2}$ where $\langle x \rangle$ denotes the fractional part of the real number x. The two theorems above actually imply that, for any real number c, any $z \in [0,1]$, and any nonintegral positive number α, the sequence $\{n \geqslant 1 : \langle cn^\alpha \rangle \leqslant z\}$ has divisor density z.

On page 2 of the reproduced letter, Erdős asks a slightly different question: given a sequence $m_j := \lfloor j(\log j)^\alpha \rfloor$, and setting $\mathcal{A} := \bigcup_{j \geqslant 1} [2^{m_j}, 2^{m_j+1}[$, for which α do we have $\tau(n, \mathcal{A}) \geqslant 1$ on a sequence of asymptotic density 1? Here it is clear, and Erdős explicitly notes the fact that the problem is linked with conjecture (2): it deals with the distribution of divisors in dyadic intervals.

Once again, it took years of struggle to solve the problem: as proved in [37], the answer is positive for all α. As it turns out, one needs to take faster growing sequences to see a threshold: if we now define $m_j = j^\beta$, then Hall-Tenenbaum proved in 1992 [19] that the answer to Erdős's question is affirmative if, and only if, $\beta \leqslant 1/(1 - \log 2)$.

Aside from his interest in the normal behavior of the set of divisors, Erdős was also intrigued by extremal properties. Out of many of his problems, I extract two.

To describe the first, let us put, for $\alpha > 0$,

$$F_\alpha(n) := \sum_{1 \leqslant j < \tau(n)} \Big(\frac{d_{j+1}(n)}{d_j(n)} - 1 \Big)^\alpha.$$

The conjecture asserted that, for all $\alpha > 1$,

$$\liminf_{n \to \infty} F_\alpha(n) < \infty. \tag{4}$$

Since $F_1(n) \geqslant \log n$, it is clear that the condition $\alpha > 1$ cannot be weakened. This conjecture may be seen as dual to another conjecture of Erdős, related to the sequence $\{a_j\}_{j=1}^{\varphi(n)}$ of integers in $[1, n]$ and coprime to n. The problem here was to show that, for all $\gamma > 0$, we have

$$\limsup_{n \to \infty} \frac{\varphi(n)^{\gamma - 1}}{n^\gamma} \sum_{1 \leqslant j < \varphi(n)} (a_{j+1}(n) - a_j(n))^\gamma < \infty.$$

Improving a result of Hooley [20] disposing of the case $\gamma < 2$, Montgomery and Vaughan confirmed this conjecture in 1986 [27].

These two problems are specific to Erdős's particular way of thinking. He manufactured innocent-looking questions whose solution actually requires a deep understanding of the structure of integers defined by multiplicative constraints.

Conjecture (4) was solved by Vose in 1984. Since, from Holder's inequality, we have

$$(\log n)^\alpha \leqslant F_\alpha(n) \tau(n)^{\alpha - 1},$$

candidates to bounded values of $F_\alpha(n)$ must have a large number of divisors.[5] This led Erdős to the further conjecture that F_α should be bounded by natural sequences with many divisors, such as $n!$, l.c.m.$\{1, 2, \dots, n\}$, or $\prod_{p \leqslant n} p$. I could establish this in [35] as a consequence of a more general result proved by the saddle-point method.

The second extremal problem was asked by Erdős on numerous occasions and is referred to as Problem 23 in the appendix of Montgomery's book [28]. It states that, for suitable constant C, the inequality

$$\sum_{d \mid n,\, t \mid n,\, d < t} \frac{1}{t - d} \leqslant C\tau(n) \tag{5}$$

[3] *The natural density of an integer sequence is, when it exists, the limit, as $N \to \infty$, of the frequency of $\mathcal{A}$ among the N first integers.*

[4] *The logarithmic density of an integer sequence $\mathcal{A}$ is, when it exists, the value of the limit*

$$\delta(\mathcal{A}) := \lim_{N \to \infty} \frac{1}{\log N} \sum_{n \leqslant N,\, n \in \mathcal{A}} \frac{1}{n}.$$

[5] *Recall that a normal integer n has about $(\log n)^{\log 2 + o(1)}$ divisors.*

should hold for all positive integers n. Here again, the aim is to test the lacunarity of the sequence of the divisors of an integer: despite the fact that d and t in the above sum may get fairly close, we expect that this happens sufficiently rarely so that (5) remains true. Thus we are really facing a sieve problem of a delicate nature. I could establish (5) in a strong form, improved by La Bretèche [1].

Erdős's interest in divisors was so constant and so intense that a whole book would be necessary to describe his problems, attempts at solutions, and original methods on this topic—and the two already written books, [18] and [16], largely dominated by the work of Erdős, would only be a small part of the story. The references of this short survey constitute an incomplete and partial list of articles related to the subject, either by Erdős himself and his collaborators or inspired by his appealing way of thinking of mathematical problems.

Acknowledgments

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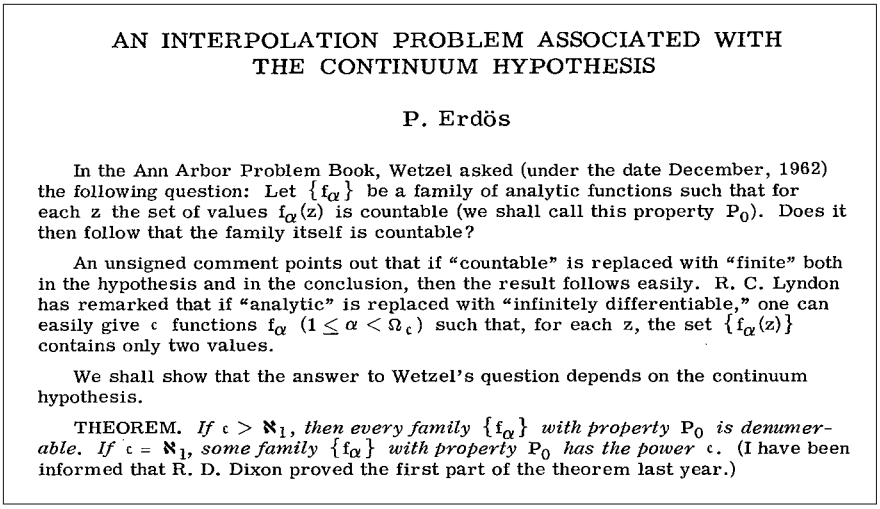

AN INTERPOLATION PROBLEM ASSOCIATED WITH THE CONTINUUM HYPOTHESIS

P. Erdös

In the Ann Arbor Problem Book, Wetzel asked (under the date December, 1962) the following question: Let $\{f_\alpha\}$ be a family of analytic functions such that for each z the set of values $f_\alpha(z)$ is countable (we shall call this property P_0). Does it then follow that the family itself is countable?

An unsigned comment points out that if "countable" is replaced with "finite" both in the hypothesis and in the conclusion, then the result follows easily. R. C. Lyndon has remarked that if "analytic" is replaced with "infinitely differentiable," one can easily give $\mathfrak{c}$ functions f_α $(1 \leq \alpha < \Omega_\mathfrak{c})$ such that, for each z, the set $\{f_\alpha(z)\}$ contains only two values.

We shall show that the answer to Wetzel's question depends on the continuum hypothesis.

THEOREM. *If* $\mathfrak{c} > \aleph_1$, *then every family* $\{f_\alpha\}$ *with property* P_0 *is denumerable. If* $\mathfrak{c} = \aleph_1$, *some family* $\{f_\alpha\}$ *with property* P_0 *has the power* $\mathfrak{c}$. (I have been informed that R. D. Dixon proved the first part of the theorem last year.)

Figure 1. The first few paragraphs of Erdős's paper [6].

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Stephan Ramon Garcia and Amy L. Shoemaker

Wetzel's Problem, Paul Erdős, and the Continuum Hypothesis: A Mathematical Mystery

We are concerned here with the curious history of *Wetzel's problem*: If $\{f_\alpha\}$ is a family of distinct analytic functions (on some fixed domain) such that for each z the set of values $\{f_\alpha(z)\}$ is countable, is the family itself countable?

In September 1963, Paul Erdős submitted to the *Michigan Mathematical Journal* a stunning solution to Wetzel's problem (Figure 1). He proved that an affirmative answer is equivalent to the negation of the continuum hypothesis. Erdős ends in an understated manner: "Paul Cohen's recent proof of the independence of the continuum hypothesis gives this problem some added interest." Together these results render Wetzel's problem undecidable in ZFC.

Erdős had a knack for solving "innocent-looking problems whose solutions shed light on the shape of the mathematical landscape" [7, p. 2]. In this case, the landscape he revealed was one of underground tunnels, surprising links, and glittering mysteries. However, our interest lies not with the solution itself but rather with the story of how Erdős encountered Wetzel's problem in the first place.

Our first exposure to Wetzel's problem was in *Proofs from The Book* by Aigner and Ziegler. "Paul Erdős liked to talk about The Book," they write, "in which God maintains the perfect proofs for mathematical theorems, following the dictum of G. H. Hardy that there is no permanent place for ugly mathematics. Erdős also said that you need not believe in God but, as a mathematician, you should believe in The Book" [1]. Erdős asked Aigner and Ziegler to assemble a moderate approximation of The Book; included in it was Erdős's answer to Wetzel's problem [1, pp. 102–6].

Regarding the origin of the problem, Erdős simply asserted that Wetzel posed the question in the Ann Arbor Problem Book in December 1962. Ziegler suggested several mathematicians who might be the Wetzel in question, eventually putting us in contact with John E. Wetzel (professor emeritus at the University of Illinois Urbana-Champaign), who confirmed that the problem was indeed his.

John Wetzel, born on March 6, 1932, in Hammond, Indiana, earned a B.S. in mathematics and physics from Purdue University in 1954 and went on to study mathematics at Stanford University (see Figure 2). While studying spaces of harmonic functions on Riemann surfaces under Halsey Royden, he posed the following question in his dissertation:

> Let V be a collection of harmonic functions on a Riemann surface R such that for each point p of R the set $V_p = \{v(p) : v \in V\}$ is countable. Must V then be countable? [9, p. 98]

Stephan Ramon Garcia is associate professor of mathematics at Pomona College. His email address is Stephan.Garcia@pomona.edu.

Amy L. Shoemaker is a graduate of Pomona College. Her email address is amyshoemaker2@gmail.com.

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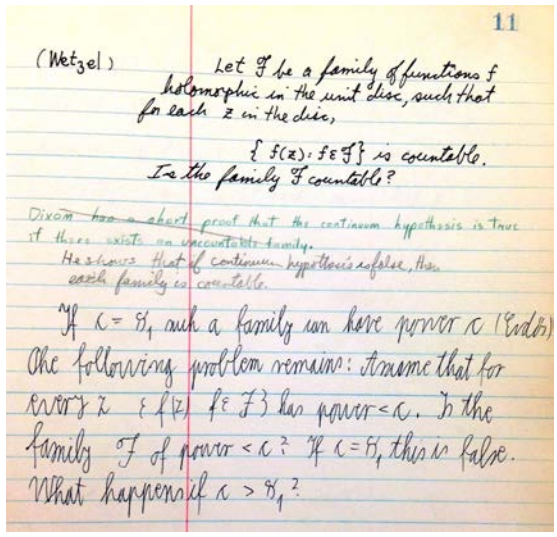
Photo courtesy of John Wetzel.

Figure 2. John E. Wetzel shortly after he left Stanford (L) and in recent years (R).



Courtesy of Margaret Lewis.

Figure 3. Page 11 of the Boneyard Book.

Regarding the origin of the question, Wetzel explained that Royden had asked him to investigate a specific conjecture:

> I thought about it for a while and eventually showed that what he conjectured was, in fact, not true. I remember reporting to him, thinking that all I had to do was write my work up in good form and I'd be finished with my dissertation; and I remember clearly Royden's reaction that my result would make up perhaps a third of an acceptable dissertation. The question might have had its genesis in the subsequent confusion. [8]

Wetzel left Stanford in 1961 to become an instructor at UIUC, having not yet finished his dissertation. He married Rebecca Sprunger in September 1962 and completed the writing of his dissertation in 1963. During this time, Paul Erdős visited the University of Illinois with his mother, who often accompanied him as he traveled from campus to campus [7, p. 7].

During one of his casual contacts with Erdős, Wetzel recounts,

> I mentioned the question to him, rather timidly, if memory serves. He thought about it briefly and said it was interesting—and that was the extent of my mathematical contact with him. I don't think he ever told me that he had settled the matter, but every time he visited the campus in the next few years he always asked me if I had any new interesting questions. [8]

Upon learning (probably not before 1966, Wetzel said) about Erdős's proof, Wetzel wrote to Royden that "Erdős has showed that the answer to a question I asked in my dissertation is closely tied to the continuum hypothesis! So once again a natural analysis question has grown horns" [8].

The Ann Arbor Problem Book that Erdős mentions seemed most likely to be the Math Club book, which has its own fiery history. In the words of Peter Duren (professor emeritus at the University of Michigan):

> When I came to Ann Arbor in 1962, I learned of a local tradition called "Math Club," an informal gathering of faculty and graduate students which met in the evening every month. A speaker was announced in advance, but the main attraction (in my view) was the series of 3-minute talks, unannounced and often spontaneous, that preceded the announced lecture. There people would tell their colleagues about neat mathematics they had come across, or raise questions, whatever they thought would be of interest. Afterwards each speaker was invited to write a short summary of his presentation in a book maintained for that purpose. The Secretary of the Math Club acted as guardian of the book, and both locals and visitors were invited to look through it. Unfortunately, the book was lost during the Christmas break of 1962–63, on the streets of Chicago. The man then serving as Secretary of the Math Club had carried the book (or books) with him when he drove to Chicago and had left it in his car overnight. Someone broke into the car and set it on fire, and the Math Club book was lost (among other items, including the car). The Math Club continued to meet, with a new book, but attendance gradually declined and the meetings were discontinued around 1990, as I recall. What Paul Erdős called the Ann Arbor Problem

> Book must have been the Math Club book. But his reference can't be checked, since the original entries for December 1962 no longer exist. [5]

However, Wetzel declared, "I have never visited the University of Michigan; I've never even been to Ann Arbor" [8]. This initially led us to conclude that Erdős erred in his citation. Given his unique manner of doing mathematics and his myriad colleagues, such a slipup is understandable. Duren wrote:

> I vividly recall him asking people (including me) at math conferences, "Where are you located?", which was the polite way of asking, "Who are you? I know I've met you somewhere." This was only natural, since he traveled so much and met so many mathematicians. It's easy to imagine that he didn't remember correctly where he had seen the problem. [5]

After much investigation, we are now able to trace the sequence of events from the original question sparked by Wetzel's dissertation to the problem treated by Erdős in his paper.

Within the UIUC mathematics library is a volume of particular importance for us. Wetzel explained:

> The library maintains a bound volume of blank pages called the Boneyard Book, in which faculty and visiting faculty are encouraged to write problems, including whatever supporting information or commentary they care to include, and faculty looking for interesting problems can browse in it for inspiration.[8]

Senior Library Specialist Margaret Lewis recovered the relevant page of the Boneyard Book (see Figure 3), despite the fact that the volume had been collecting dust in the UIUC archives for decades. Duren recounted that, during the early 1960s, Erdős traveled frequently between the University of Michigan and the University of Illinois Urbana-Champaign, so conflating the two schools' problem books would have been a natural mistake.

However, the Boneyard Book led to many more questions. There are clearly four scribes who contributed to the page, so our next task was to identify the mathematicians involved. While the final entry of the page certainly boasts Erdős's distinctive penmanship, the remaining three entries required further scrutiny.

Wetzel explained that the first entry, though attributed to him, was not written by him: "I haven't a clue who wrote the problem in our Boneyard Book and wrote my name next to it. I thought for a few moments that it might have been Ranga Rao, with whom I shared an office during my (and his) first year at Illinois, 1961–62. But his handwriting was recognizably 'Indian English penmanship,' and I think him unlikely" [8]. We therefore decided to investigate the second and third entries before tackling the first.

Jane Bergman, secretary of the chair of the UIUC math department, suggested that the Dixon mentioned in the Boneyard Book and Erdős's article (Figure 1) was Robert ("Bob") Dan Dixon:

> I was able to find out that Mr. Robert Dan Dixon received his PhD from Ohio State University in 1962 in the same year he was hired here as an instructor in Mathematics. In 1964 he was recommended for promotion to assistant professor, but there is no result in his file of that recommendation. I would guess that he moved on at that point, but there is nothing in his file to support my guess. He was born in 1936. I hope you find this helpful. [8]

A colleague of Wetzel's, George Robert ("Bob") Blakley, had more information about Dixon. Bob Blakley and Bob Dixon both arrived at UIUC in September 1962 as new PhD's and both left in 1964. Blakley, in personal correspondence with the authors and Wetzel, wrote:

> Bob [Dixon] went to the nascent Wright State University in the Dayton area as a founding father. First he founded the math department and headed it. Then he founded the computer science department and headed it…Late in the last century he retired from WSU, covered with glory…He still manages to fleece me regularly and disreputably in the most varied sorts of bets. But I think he has given up one hundred mile bike rides. [2]

Blakley provided us with three possible ways to contact the elusive Dixon: two email addresses which may or may not have been current, along with his home address. Wetzel sent a letter out to all three addresses, and luckily one route was successful. Dixon responded:

> I was there [UIUC] from the fall of 1962 until the spring of 1964. I remember Erdős visiting and may even have had some time with him but I don't remember discussing this problem. The handwriting in the book is puzzling. The entry describing your problem is not familiar to me. The first entry that mentioned me could be by Bob [Blakley] as he generally printed. The second entry refers to me in third person but I could have written it…Although I can't remember any details there is a bit of familiarity. I worked in complex analysis at the time and I had a very interesting course in graduate school that covered the relevant set theory.

> Lots of problems were thrown around in that group of young faculty. [4]

Dixon elaborated on "that group of young faculty" known as the SixtyTwo Illini Hall Group. In 1962 UIUC hired twenty new mathematics faculty to add to their faculty of one hundred; Illini Hall was located across from the mathematics building. According to Dixon:

> There was not a sense of privacy about the problems we were investigating. My own work was very specialized and detailed so I had no problems to share. Many others, like Jack Wetzel, did have problems that they proposed, or pointed us to, that they were curious about or needed to solve to get on to the problem they really wanted to work on. We fell then into three overlapping groups: proposers, solvers, and brokers. I was in the solvers group but not particularly successful, Jack may have been in all three. Bob Blakley was in all three but was effective as a broker…There were many others who participated in this interchange but my memory of names is bad. That said it was an experience that had more to do with my career than my own doctoral research. I suspect that was true of several of the other SixtyTwo Illini Hall Group. [4]

Upon receiving a copy of the Boneyard Book page, Blakley confirmed that he authored the entry in the Boneyard Book that reads, "Dixon has a short proof that the continuum hypothesis is true if there exists an uncountable family." Blakley also remarked that he feels "rather strongly…that Dixon is the third scribe." Given Erdős's parenthetical remark in his paper that "I have been informed that R. D. Dixon proved the first part of the theorem last year" [6], it is likely that Robert Dan Dixon was indeed the scribe of the third entry. Since Dixon expressed that he never spoke to Erdős about this problem during their overlapping time at UIUC, the information Erdős claimed was relayed to him almost certainly came from the Boneyard Book.

After immersion in the memories and details surrounding the problem, Wetzel recalled:

> I just remembered that I had given a faculty seminar and a departmental colloquium on the substance of my dissertation shortly after arriving at Illinois (even though the dissertation was not yet completely written), and that widened significantly the list of people who might have written the first entry in the Boneyard Book. [8]

A chance meeting between the first author and John P. D'Angelo (a professor at UIUC) finally revealed the most likely candidate for the first scribe. D'Angelo was convinced that the handwriting was that of Lee A. Rubel:

> Rubel, who died in 1995, often contributed to the Boneyard Book. Furthermore, his many interests included the interplay between logic and function theory…Rubel would have been quite interested in this problem, and the handwriting is remarkably similar to that of notes he wrote to me around 1979–80. [3]

Of the possible candidates, Wetzel remarks:

> Lee certainly strikes me as the more likely…I truly don't doubt that Rubel was the author, but I confess that I still find it a little surprising that he never mentioned it to me—admitting always the possibility that he did and I have forgotten. [8]

The final piece of evidence was a sample of Rubel's handwriting, obtained by D'Angelo from the UIUC archives, which appears to validate this conclusion. In fact, D'Angelo tells us, Rubel was the creator of the Boneyard Book.

Let us now return to Erdős's paper [6]. It was easy to jump to the conclusion that Erdős had erred in his citation, since the problem appears in the Boneyard Book at the University of Illinois, and Erdős saw it written there. However, there is another scenario that seems to fit the facts more closely. In the first few lines of his paper (see Figure 1), Erdős cites the "Ann Arbor Problem Book" as the source of Wetzel's problem and mentions "an unsigned comment" and a remark by Roger Lyndon (then a professor at Michigan). Neither of those comments appears on the relevant page of the Boneyard Book (Figure 3).

The only explanation is that someone had transported the problem to Michigan and had recorded it either in the Math Club book or in a separate book of open problems. Peter Duren conjectures that the problem was transferred by Lee Rubel himself, who was making frequent trips to Ann Arbor in those days to work with Allen Shields [5]. Erdős first saw the problem written there and came up with the beautiful result presented in [6]. If Erdős saw the problem in December 1962, it could well have been in the Math Club book which perished in Chicago. However, Duren reports having examined the Math Club book for 1962–91, which now resides in the Bentley Historical Library at UM. There he found a record that Paul Erdős gave a lecture (entitled "Some Unsolved Problems") at the Math Club on September 10, 1963, just a week before his paper [6] was received by the *Michigan Mathematical Journal* (on September 18). Thus it seems far more likely that Erdős saw the problem in Ann Arbor during his short visit of 1963, in which case the

Ann Arbor Problem Book was not lost in Chicago and may yet be found. However, Duren has no memory of an independent problem book, nor do his fellow retirees at Michigan. In any case, it seems clear that, after having submitted his paper [6], Erdős saw the problem in the Boneyard Book and learned that Dixon had obtained part of the result independently.

No story can ever have the entirety of its details pinned down. As Wetzel said, "It may require transfinite induction to bring this matter to a close." However, we have identified with a high degree of certainty the trajectory of Wetzel's question as it made its way to Erdős. It began in 1961, when Wetzel posed the original question (for harmonic functions on Riemann surfaces) in his evolving dissertation. When he arrived at UIUC in 1962, he gave a talk on his graduate research. Lee Rubel was almost certainly one of the attendees and likely transmitted the problem to Ann Arbor. Rubel wrote Wetzel's question in the Boneyard Book in 1962, and Bob Blakley responded with an entry claiming that Bob Dixon had a proof, assuming the truth of the continuum hypothesis. Dixon crossed out Blakley's entry and wrote (in third person), "He showed that if the continuum hypothesis is false, then each family is countable." Dixon's short proof was rediscovered and published by Erdős, who went on to prove that an affirmative answer to Wetzel's problem is equivalent to the negation of the Continuum Hypothesis. Erdős's Boneyard Book entry is likely from the fall of 1963, after he had submitted his proof to the *Michigan Mathematical Journal*.

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In Memory of Arthur Strong Wightman

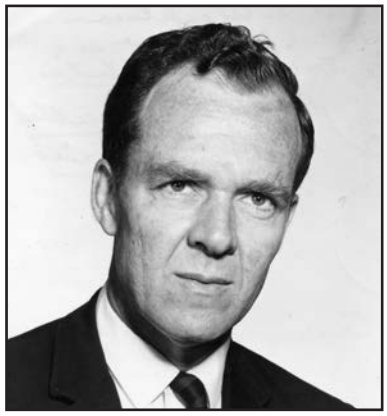
Barry Simon, Coordinating Editor

Arthur Strong Wightman, the Thomas D. Jones Professor of Mathematical Physics at Princeton University and a founding father of modern mathematical physics, passed away on January 13, 2013, at the age of ninety. He was a key figure in the development of a rigorous mathematical physics that uses ideas of functional analysis and operator theory. His influence went far beyond his published research, which mainly focused on clarifying the compatibility of special relativity with quantum theory in the framework of quantum field theory.

Wightman was born on March 30, 1922, in Rochester, NY. He served in the US Navy after getting an undergraduate degree at Yale in 1942. After the end of the Second World War, he came to Princeton intending to work with Eugene Wigner, who was spending most of his time at Oak Ridge. So Arthur wrote his 1949 PhD thesis "The moderation and absorption of negative pions in hydrogen" under John Wheeler.

Except for sabbaticals and other visits, he spent the remainder of his career in Princeton, initially in physics but eventually jointly in the two departments of mathematics and physics. His visits to Copenhagen in 1951–52 and 1956–57 and the interactions with Gunnar Källén and Lars Gårding in nearby Lund had a tremendous impact on him. He sometimes talked of his interactions there also with the previous generation, especially Wolfgang Pauli in Copenhagen and Marcel Riesz in Lund.

Gårding was a collaborator on parts of Wightman's most famous work, the development of what was called axiomatic field theory (and also on the classification of the representations of certain infinite-dimensional Heisenberg groups). The idea was that techniques of juggling the infinities of perturbation theoretic quantum field theory were remarkably successful but seemed to be mathematically shaky. Under these circumstances it was sensible to state precisely what one should mean by a relativistic quantum field. The result was a set of axioms, originally for vacuum expectations values in Wightman's 1956 paper and then in a variant of Gårding and Wightman in terms of operator-valued distributions. It is of course no coincidence that this use of distribution theory to revolutionize how mathematicians think about quantum field theory took place in the same place where Lars Hörmander was using distribution theory to revolutionize PDEs.



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Arthur Wightman, circa 1960.

The operator-valued distribution approach spread informally, and its success especially in the Haag–Ruelle scattering theory led to the 1964 publication of the Gårding-Wightman axioms and the celebrated book with Ray Streater, PCT, Spin and Statistics and All That (Princeton University Press, 1964).

Wightman's influence was magnified by an array of graduate students (see the list on the Math Genealogy website) and postdocs. His broad interests are illustrated by the fact that, while his most celebrated work is in axiomatic quantum field theory, he supervised PhD students in areas as wide as classical mechanics (Raphael de la Llave and Jerry Marsden), nonrelativistic quantum mechanics (John Dollard and Barry Simon), operator algebras (Huzihiro Araki and Robert Powers), Feynman diagrammatic perturbation theory (Vincent Rivasseau and Eugene Speer), statistical physics (Christian Gruber), and constructive quantum field theory (Arthur Jaffe and Oscar Lanford).

Barry Simon is the IBM Professor of Mathematics and Theoretical Physics at Caltech. His 1970 PhD thesis was supervised by Wightman. He and Wightman were colleagues on the Princeton faculty from 1969 to 1981. His email address is bsimon@caltech.edu.

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Picture taken by and courtesy of Arthur Jaffe.

Berlin dinner in 1981 for editors and advisors of the *Communications in Mathematical Physics,* including Jürgen Ehlers, Joel Lebowitz, Jüerg Fröhlich, Misha Polivanov, Elliott Lieb, James Glimm, Kurt Symanzik, Jean Ginibre, Thomas Spencer, Huzihiro Araki, Wightman, and Yakov Sinai.

Arthur had impact outside Princeton, his research, and his students. He helped set up the program in mathematical physics at IHES and encouraged the founding of Communications in Mathematical Physics. In 1973 Giorgio Velo and Arthur Wightman began their series of productive summer schools in Erice. He served as an associate editor of *Communications in Mathematical Physics*, as well as editing book series for Benjamin and for Princeton Press. As a member of the board of Princeton Press, he was a key early supporter in getting the Einstein Papers project under way. Arthur was the editor of Wigner's complete works published by Springer.

Arthur married Anna-Greta Larsson. She and their daughter, Robin, died from cancer at a young age. Arthur's second wife of thirty-five years, Ludmilla Popova Wightman, and his stepson, Todor Todorov, survive him.

For his work Wightman received the 1969 Dannie Heinemann Prize for Mathematical Physics from the American Physical Society and American Institute of Physics, and the inaugural Henri Poincaré Prize from the International Association of Mathematical Physics in 1997. He was the 1976 AMS Gibbs Lecturer. He was a member of the National Academy of Sciences; Fellow of the Royal Society of Arts, London; Fellow of the American Academy of Art and Sciences; a Doctor of Science of the Swiss Federal Institute of Technology (1968); and Doctor Honoris Causa of Göttingen University (1987).

We have collected some reminiscences from a few of the many scientists whose lives were impacted by Wightman. Princeton has a website (https://www.princeton.edu/physics/arthur-wightman/) with additional comments, and there is a charming blog post at quantumfrontiers.com/2013/03/13/remembering-arthur-wightman/ from a former undergraduate advisee. The reader may also want to consult two articles that appeared in the IAMP *News Bulletin*: from January 2013 (www.iamp.org/bulletins/old-bulletins/201301.pdf), an obituary by Arthur Jaffe and Barry Simon, and from April 2013 (www.iamp.org/bulletins/old-bulletins/201304.pdf), "Nine lessons of my teacher, Arthur Strong Wightman" by Arthur Jaffe.

Jüerg Fröhlich

In 1949 Arthur Strong Wightman got his PhD degree from Princeton for work entitled "The moderation and absorption of negative pions in hydrogen." Three years later, he co-authored a paper with Gian Carlo Wick and Eugene Wigner that bears the title "The intrinsic parity of elementary particles." In their seminal work the concept of a superselection rule was introduced. In the early fifties, in joint work with Lars Gårding, Arthur formulated the basic principles underlying local relativistic quantum field theory, and in the company of friends, colleagues, and students he then started to deduce physically relevant consequences from the so-called "Wightman axioms."

Arthur Wightman is not remembered for the work contained in his thesis. But he is and will be remembered for his studies of the fundamental principles underlying local relativistic quantum field theory and for having helped—and for having inspired his followers and students—to derive important results from those principles. The use of advanced mathematics was not an end in itself but was forced upon him and his peers because they wanted to *understand* relativistic quantum field theory, just as Born, Heisenberg, and Jordan did not consider the mathematics of infinite matrices to be an end in itself but a remedy in formulating and embellishing the new quantum mechanics.

Arthur Jaffe has called Wightman the "spiritual leader of (modern) mathematical physics," and he fears that "his death really marks the end of an era." I agree with the first statement, but am more optimistic than Jaffe concerning the second statement. We owe the twentieth-century revolutions in theoretical physics to people who were mathematical physicists in the very best sense of this term. In studying the deep problems of physics, they did not consider mathematics to be an end in itself but to be the natural tool for reading the *Book of Nature*—just as Arthur Wightman did thirty years later.

Jüerg Fröhlich is a professor emeritus at ETH Zürich. From 1974 till 1977 he was an assistant professor at Princeton and a mentee of Arthur Wightman. He frequently returned to Princeton for shorter or longer visits and always enjoyed the discussions with Wightman. His email address is juerg@phys.ethz.ch.

The clouds obfuscating the basic laws of the microcosm during the days of the old quantum theory and the clouds that concealed the theoretical foundations of particle physics in the early fifties dispersed through the work of people who had an intimate and nontrivial relationship with mathematics and used mathematics in a pragmatic but quite professional way. There are thick clouds lying over some of the deepest mysteries of the physics of our time. Theorists in the tradition of people like Arthur Wightman who have a taste for qualitative reasoning, do not despise mathematics, and attempt to do more than just solve one technical problem after another are likely to play an important role in helping to dispel those clouds in the future.

In my formative years I was fortunate to be under the spell of Arthur Wightman and to be exposed to his way of approaching problems in theoretical physics. I have profited from his vision and his patience with inexperienced youngsters. His exemplary intellectual integrity and his extraordinary generosity towards colleagues and students may be considered to be his main legacy. I remember him gratefully.

Picture taken by and courtesy of Arthur Jaffe.

Wightman and Rudolph Haag at the Boulder meeting of the International Association of Mathematical Physics, 1983.

Francesco Guerra

The first time I met Arthur S. Wightman was upon my arrival in Princeton in September 1970. I came from the University of Naples, where I had received my doctoral degree in 1964. However, I already had a quite complete idea about the scientific and human personality of Arthur. In fact, my mentors in theoretical physics, Eduardo R. Caianiello and Gianfausto Dell'Antonio, visited Princeton many times, were well acquainted with Arthur, and were his personal friends. During the sixties I was involved in research on renormalization theory, in the study of some simple models, and in the development of analytic regularization in configuration space. Moreover, I studied axiomatic field theory and was deeply impressed by the mathematically rigorous and physically relevant constructive program put forward by Arthur in his 1964 Cargese lectures. Hence my firm decision to spend some time in Princeton.

I remember with deep gratitude and affection the daily contacts with Arthur. I was impressed by his generous sharing of ideas and knowledge, by his participation in the unavoidable difficulties, the encouragement, and the help in developing research topics. An essential part of Arthur's style in personal relations was a typical use of irony and self-irony in their highest cultural meaning. I will give an example well fixed in my mind. At the beginning of 1972 I got some preliminary but quite strong results about the behavior of the vacuum in the infinite volume limit by using new methods of Euclidean quantum field theory recently developed by Edward Nelson. Quite naturally, I was hesitant to publish the results, because I was sure that much more could be easily obtained. One morning Arthur came to my office, asked about the status of the publication, and told me that I should hurry by giving the following image: "You are in the same situation as the young boy trying to stop the water from the dike by putting his small finger on the hole." It was good advice, promptly followed. In fact, Euclidean methods proved immediately to be very useful by producing a lot of new results in constructive quantum field theory. I was lucky to participate in the "flood" through a long and fruitful collaboration with Lon Rosen and Barry Simon, promoted by Arthur.

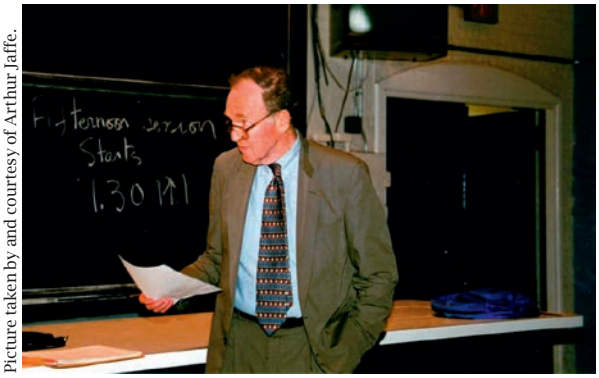
After leaving Princeton I was regularly in touch with Arthur through an enlightening correspondence.

The last time I saw him was during a visit to his clinic in 2008. The essential rich sprinkle of his mind was intact—in particular self-irony. He proudly showed us his last production: a wooden carriage train built for rehabilitation purposes.

Arthur will live in our minds and hearts forever.

Francesco Guerra is a professor of theoretical physics at the University of Rome "La Sapienza." He was invited to Princeton by Arthur Wightman as a research associate in the physics department in 1970–72. His email address is francesco.guerra@roma1.infn.it.

Klaus Hepp is a professor emeritus of theoretical physics at the Swiss Federal Institute Zürich (ETHZ). His 1962 PhD thesis was supervised by Res Jost, a close friend of Wightman. He was at the Institute for Advanced Study in Princeton 1964–66, interacting with Wightman as much as possible. His email address is khepp@phys.ethz.ch.



Picture taken by and courtesy of Arthur Jaffe.

Wightman lecturing at Harvard 1998.

Klaus Hepp

Remembering Arthur Wightman is for me, as for many of us, a return to the best time of my life. I "met" Arthur frequently before I came to Princeton. As a graduate student in Zürich, Res Jost asked me to explain in the proseminar Arthur's famous paper "Quantum field theory in terms of vacuum expectation values." At that time I imagined Arthur to be a very old man with a grim face. Later I saw a photo of him taken at the Varenna Summer School 1958, sitting on the ground with a boyish smile. In my master's thesis at ETH I struggled with Arthur's synthesis of group theory with holomorphic functions of several complex variables to simplify the proof of the theorem of Hall and Wightman. Now Arthur became for me a great conductor.

In the winter 1963–64 I first met Arthur personally in Paris. I remember a wonderful dinner at Le Procope, to which Arthur and Res Jost had invited Arthur Jaffe and me. Although I was from 1964 to 1966 in Princeton at the Institute for Advanced Study, I spent more time in Arthur's office and in his lectures than anywhere else. I enjoyed the dialogue he had with the audience, his struggles to present deep results, and I had the great chance to fill some gaps in the proof of the Bogoliubov Parasiuk theorem on renormalization, which Arthur had pointed out.

The Hepp family (Marie-Claude and I) were frequently Anna-Greta and Arthur's guests at their home. There we got impregnated by the writings of Barth, Bellow, Heller, Pynchon, Roth, Wolfe, and many others unknown to us illiterate Europeans: we became immersed in the Anglo-Saxon world, and we loved America. Once we went with Arthur to the famous football game, where Princeton lost to Dartmouth at the end of an undefeated season. At the Wightman party to celebrate the expected victory, the wine was seasoned with sadness.

In later times my interests drifted towards neurobiology, and I no longer interacted so much with Arthur and his school. The last time I saw Arthur was at the Rutgers meeting in honor of Elliott Lieb, watching the audience from a back seat and drifting slightly into his own world. I knew that I would never see him again, but I will never forget him.

Arthur Jaffe

First encounters not only shape our perceptions but can also influence one's destiny. Learning experimental physical chemistry as a Princeton undergraduate hardly prepared me for my eventual life's work. While realizing that I would not be happy following in my father's footsteps in medical research, Donald Spencer fortuitously encouraged my changing direction to study mathematics, and Charles Coulston Gillespie connected me with Clare College.

Only in Cambridge did I discover Arthur Wightman's wonderful papers. At the time I was struggling, with continual frustration, to learn mathematical physics. What a striking difference Arthur's clarity provided! Compared with papers where the physics ideas seemed to be hidden, intentionally or not, under layers of jargon, his were magic. Naturally I developed an intense desire to return to Princeton in spite of the temptation to remain in charming and stimulating Cambridge, England. Two years earlier a trajectory back there would have seemed unimaginable, but my long-distance exposure to Arthur Wightman made my motto "Princeton or bust."

The focal point for Princeton graduate students interested in physics and mathematics was the (Old) Fine Hall Common Room. Everyone needed to pass that space to go between the two departments, to read notices of seminars thumbtacked to the cork bulletin board in the hallway, to access the stairs to the famous library on the floor above, to ask a mathematical question, or to visit the local mailroom. One September afternoon I happened to be reading in a comfortable easy chair in the Common Room, opposite the door. Wightman was on leave at the IAS, and I had not yet met with him.

My perch had a clear view down the wood-paneled hallway lined by mathematics faculty offices. Suddenly the familiar, animated voice of Donald Spencer rang down the corridor. Shortly, he came into view, walking rapidly beside a tall, good-looking man who was casually dressed in a sports jacket, without a tie, and carrying a large,

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Arthur Jaffe is the Landon T. Clay Professor of Mathematics and Theoretical Science at Harvard University. He studied chemistry as a Princeton undergraduate, mathematics at Cambridge University, and later received his doctorate in physics with Arthur Wightman, submitting his thesis in 1965 titled "Dynamics of a cut-off $\lambda\phi^4$ field theory". He served as AMS president in 1997–98. His email address is arthur_jaffe@harvard.edu.

well-worn brown leather briefcase brimming with papers. The pair stopped in the hallway, not far from where I sat. Both men were compulsive talkers; they seemed completely engaged in their conversation, unaware that they were being observed.

Arthur Wightman appeared pleasant and even approachable, but formidable nonetheless. Spencer began to tell Wightman about a newly arrived student who planned to study mathematical physics, a student who sounded exactly like me. How extremely awkward to be the subject of a conversation you unintentionally overhear! Coming from an academic family, my father had often explained to me that professors have a human side too, which helped me understand that I had been derelict in my duty by not seeking out Arthur Wightman as soon as I arrived in Princeton. I soon did.

Eventually I became quite familiar with that omnipresent briefcase, with Arthur Wightman's modest Volkswagen Bug, and with Anna Greta's Mercedes. Two years later, at the IHES, someone coined the names "Big Arthur" and "Little Arthur" to distinguish us in conversation. Those adjectives not only described our relative ages and physiques, but also the way I felt at that moment in Fine Hall when I tried to disappear inconspicuously into the woodwork.

Chiara R. Nappi

I met Arthur in 1973 when I attended a summer school in constructive field theory that he organized at the Ettore Majorana Center in Erice, Sicily (Italy). I remember him sitting through all the lectures, many by his students and collaborators, very attentive and engaged. Arthur was an impressive guy, both intellectually and physically. He was tall and strong (Strong was his middle name).

He had attended Yale on an academic scholarship but had ended up playing basketball for Yale, something he was very proud of, although he readily acknowledged that the decisive element had been his size rather than his athletic skills. He and his wife, Anna-Greta, were quite a sight together. They went bird-watching through Sicily during the free weekend at the school. That was the first time I heard of bird-watching, and I was intrigued. When I arrived in Princeton in 1980, it was Arthur and his second wife, Ludmilla (by then Anna-Greta had lost her battle with cancer), who introduced me to birding.

Arthur was very proud of his association with both the math and physics departments at

Picture taken by and courtesy of Barry Simon.

Picture taken at mathematics department dinner in mid–1990s. Ludmilla Wightman is across from Arthur and from l to r are Lars Gårding, Elliott Lieb, and Christiane Fellbaum Lieb.

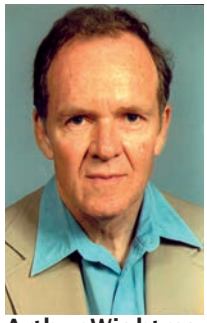
Princeton University and divided his loyalty evenly between them. One could always find him either in his office in Jadwin (physics department) or Fine Hall (math department). Nonetheless, he liked to joke that one of the advantages of being in two departments was like having both a wife and a lover. The wife thinks you are with the lover, the lover thinks you are with the wife, and you can hide somewhere and get some work done.

If you asked Arthur a question, he would say, "Do you have a minute?" To Arthur there was no such a thing as a quick answer. He knew so much, he had so much to say, so many details to reveal, so many connections to make. You sat there listening to all these facts that he remembered in exquisite detail, totally fascinated. Hours later you finally had the answer to the question you asked long ago, and in the process you had learned an awful lot about a lot of things you did not even know existed and enjoyed every moment of it.

Arthur's office was across the hall from mine. The walls were completely covered with books, and he had read them all. If you asked him where to find something, he would immediately pull out the right book and point you to the right page. Then there were the light blue notebooks, dozens of them, clearly labeled and filled with his neat handwriting. When the office was emptied a couple of years ago, it felt like the end of an era.

By then Arthur had already spent a few years in nursing homes. When his mind started failing him, he built an alternate reality around himself and continued to live in the world he loved. When I would go to visit him, he would ask, "Are you coming straight from your office? With all the construction going on, lately I have not been able to find the elevator from Fine Hall to Jadwin." Or he would say, "Princeton certainly is not what it used to be. Can you believe that there are people here who do not know who Dyson is?" But he never forgot things of the past. Even when he could not recognize the colleagues who visited him,

Chiara R. Nappi is a professor of physics at Princeton University. She and Arthur had been friends and colleagues for nearly four decades. Her email address is cnappi@princeton.edu


Picture courtesy of Ludmilla Wightman.

Arthur Wightman.

he nonetheless was extremely gracious and friendly, regaling them with memories of physics at Princeton decades earlier. He would say, "I know I should remember you, but I do not… I see I have a lot of remembering to do." Always good-humored, always dignified. A true giant, a great role model, and a gentleman to the very end.

Edward Nelson

I arrived in Princeton with a background in probability theory and functional analysis but not knowing a Hamiltonian from a Lagrangian. I was fascinated by Feynman's articles on path integrals and wanted to understand his application of them to quantum electrodynamics, so I attended Arthur Wightman's course on QED. But most of what I learned from Arthur was outside the classroom. For years he taught me physics by answering my questions with unfailing generosity and expertise.

Once, I was writing a paper in which I compared classical motion, as described in Whittaker's *Analytical Dynamics*, with quantum motion for a particle in a $1/r^2$ potential. In both cases there was a division: for angular momentum higher than a certain critical value, the motion was regular, and below the critical value, singular. But the critical values I obtained for the two cases differed by a factor of 2, which puzzled me greatly. I asked a well-known physicist about this. He gave me a lecture about the difference between classical and quantum mechanics, but with no mention of a factor of 2 and, unless I was paranoid, with an implicit suggestion that these matters were beyond the comprehension of a mere mathematician. Then I did what I should have done in the first place: I asked Arthur, telling him just what I have written above. He replied immediately, "Whittaker uses forces rather than potentials." (While a $1/r$ potential gives an attractive $1/r^2$ force, a $1/r^2$ potential gives an attractive $2/r^3$ force, and there was my missing factor of 2.) This answer was typical of Arthur's encyclopedic knowledge of the literature and his ready grasp of the essentials of a problem. But in another way the answer was quite untypical of him. In fact, it was unique in my experience and perhaps in the experience of everyone who knew him. The answer consisted of a single sentence!

Arthur was famous for his method of answering technical questions, with an impromptu but polished lecture that answered the question only after putting it in its scientific context, often revealing connections previously unknown to the questioner. Both Chiara Nappi and I expatiate on this at length in our articles on the Princeton physics department webpage `www.princeton.edu/physics/arthur-wightman/`, in memory of Arthur.

I am one of many who owe an immense debt of gratitude to Arthur for his enthusiastic, generous, and expert mentoring. This was not simply good-natured encouragement; it was informed by Arthur's clear vision of where mathematical physics should go and his uncompromising requirement of rigor. When I worked on stochastic mechanics, an unorthodox approach to quantum theory, not a word of encouragement came from Arthur. But he was not one of those scientists who attempt to suppress research directions of which they do not approve. I submitted a book on stochastic mechanics to the Princeton Series in Physics, of which he was one of the editors. After a long delay I asked Arthur what the situation was. He said he had no objection, but that I had better go talk with the other editor, which I did.

Arthur loved mathematical physics. He loved life, he loved people, he loved conversation. A man of great seriousness of purpose, he exuded fun.

David Ruelle

Wightman Field Theory is the scientific field where I started to do independent work (that was in Zurich, with Res Jost, in 1959). I had met Arthur Strong Wightman himself at physics summer schools and felt that his middle name was singularly appropriate. His vast knowledge of physics and mathematics had allowed him to choose in masterly fashion the axioms on which he based relativistic quantum field theory, and then he developed this Axiomatic or Wightman Field Theory in a beautiful way. He presented his ideas with force and clarity in articles, lectures, and a foundational set of notes (Bargman, Wightman, and Wigner), which unfortunately was left unfinished and seems now to be quite forgotten. One result which astounded me when I studied it was the fantastic calculation of the domain of holomorphy for the 3-point function, which he did with Gunnar Källén. For me these were the golden days, and thinking about them leaves me somewhat nostalgic. Arthur Wightman was the god of the part of science in which I worked, which was a paradise with a number of brilliant people: the Feldverein

Edward Nelson was professor emeritus of mathematics at Princeton University. He and Wightman were colleagues on the Princeton faculty beginning in 1959.

David Ruelle is an honorary professor at the Institut des Hautes Études Scientifiques (Bures-sur-Yvette) and a visiting professor at the Rutgers Math department. He has known Arthur Wightman since the late 1950s. His email address is `ruelle@ihes.fr`.

(Lehmann, Symanzik, Zimmermann), Jost, Haag, and many more. After a while I left the paradise, because I wanted something more constructive than axiomatic field theory, and started working in equilibrium statistical mechanics. But the interests of Arthur Wightman were vast, and I stayed in close scientific contact with him.

At a personal level, the image that comes to mind when I think of him is of Arthur Wightman at his home in Princeton among his books. Because of his role at the Princeton University Press, he had a vast number of books, knew them, and talked about them. His talk about books and other topics could, at times, be a bit overpowering. But listening to him could be quite rewarding. I remember for instance how he described the careful way in which J. Robert Oppenheimer expressed himself: writing down precisely what Oppie said, you had a text already fit to print.

Arthur Wightman was one of the founding fathers of modern mathematical physics at a time when there were only a few dozen people in this domain. His influence was strong, and science today keeps the mark of his great mind.

Barry Simon

Because of a wonderful high school physics teacher, I wound up as a physics major at Harvard, although I took lots of advanced math courses there. I realized early on that what I really wanted was to prove theorems about real physics. With Wally Gilbert, Paul Martin, and Julian Schwinger as my theoretical physics professors, I didn't see any of that at Harvard, so I cornered George Mackey in the middle of my junior year and asked if he knew anyone doing that sort of thing, and he told me about Arthur Wightman at Princeton. Princeton hadn't been on my radar, but I applied and went to graduate school there and, with Arthur and Ed Nelson as mentors, got a superb education.

When Arthur was starting out, he didn't have anyone like him! He went to Princeton intending to work with Wigner, who was at Oak Ridge, so Arthur did a more phenomenological thesis with Wheeler. His last year Wigner returned and Arthur set aside an hour a week to meet with him. As Arthur told me, "Sometimes we'd discuss my thesis, sometimes a point of physics or mathematics. But if need be, we'd discuss the weather, because darn it, he owed me that hour!"

Arthur told me this in the context of apologizing for going on leave to Europe during my second year, although he was conscientious about writing me long letters with useful advice. I remember one that suggested I use Padé summation on the divergent anharmonic oscillator perturbation series that began: "The specter of Padé is haunting Europe. S-matricists of the world unite. You have nothing to lose...."

Picture taken by Rudolph Haag and courtesy of Arthur Jaffe.

Wightman and Arthur Jaffe at the Boulder meeting of the IAMP, 1983.

The anecdote about Wigner illustrates Wightman's strong sense of right and wrong (in graduate school my friends and I used to joke that there was a reason his middle name was Strong). This carried over to his science. He used to complain to me about physicists who claimed there was no need for proofs from first principles because accurate prediction was a proof the arguments were correct. The phrases "intellectual coherence" and "intellectual honesty" were part of his response.

Combined with his sense of right and wrong was a phenomenal breadth of interest and knowledge (someone once remarked that the only thing more intimidating than the number of books in Arthur's office was that he knew what was in all of them) and an incredible generosity. The result was that ideas and research problems that most people would save for themselves were offered to students and postdocs, often not even resulting in joint publication.

One example of this involves dimensional renormalization of Feynman integrals. Arthur learned from Marcel Riesz the idea that one could understand the principal value integral by making the power one a complex variable, note there is a pole at $s=1$ in the analytic continuation, and view the principle value as the constant term in the Laurent series. Arthur had the idea that perhaps one could understand renormalization of Feynman integrals in the same way. He gave a simple low-order example to an undergrad for a senior thesis and then the full idea to Gene Speer as a PhD thesis problem. In Gene's hands it turned into dimensional renormalization, since a standard tool in both mathematical physics and theoretical physics. I could give many other examples of this sort.

Arthur Wightman was a giant figure who considerably impacted the field of mathematical physics in many ways. He was a wise and caring human being. I am fortunate to be among the many whose lives he changed.

Picture taken by and courtesy of Arthur Jaffe.

Wightman at his seminar in 301 Palmer Lab, 1965. The picture includes Gerhard Mack, Christian Gruber, Henri Epstein, Wightman, Rein Uritam, Rudolph Hwa, Anton Z. Capri, Dietrich Uhlenbrock, George Svetlichny, Jean Nuyts, Marcel Froissart, and several others.

Ray Streater

When I visited Princeton in 1963 to work with Wightman on a book, I shared his office. I noticed that he tutored students there, and I used to leave him when they arrived. One student, a young black man, was a particularly frequent visitor. Wightman treated him very kindly when he arrived, and his help went well beyond the physics. I realized after I returned to England in 1964 that segregation was still a problem in the USA: the huge demonstration against it, which took place in the summer of 1964, was indeed necessary.

Franco Strocchi

I met Arthur Wightman in the early sixties. During a one-semester visit to the physics department of Iowa State University, Fritz Rohrlich introduced me to the problem of a gauge independent formulation of quantum electrodynamics (QED), possibly improving Mandelstam's proposal. When back in Europe, I chose to spend a postdoctoral fellowship at Orsay, where Maurice Lévy had also worked on the same problem. In the process of finishing the paper with Rohrlich, I was struck by the fundamental paper by Wightman and Gårding on the mathematical formulation of quantum field theory, containing in particular a very neat section on the quantization of the electromagnetic potential. I then decided to spend my second year of fellowship at Princeton, where I arrived in January 1966. A new world was disclosed to me, and, in particular, I experienced the extraordinarily distinctive ability of Wightman to encourage and promote research work, according to his scientific style.

I went back to Princeton in the years 1972–73 with the aim of discussing with Wightman the implications of the Gauss law on the algebra of observables on the basis of a preprint with Ferrari and Picasso, which appeared only in 1974. The mathematical problem was the relation between the quantum number of the electric charge and the integral of the charge density: in the physical Coulomb gauge such a connection was plagued by the infinite renormalization constants due to vacuum polarization, as pointed out by Symanzik in his unpublished lectures of 1971. On the other hand, the Feynman-Gupta-Bleuler gauge offered a possible framework. Wightman reacted very favorably to this possibility and actually suggested an extensive control of the mathematical structure of that gauge. The result was a joint paper on the charge superselection rule and a proposal to write a book on what could be said on QED from a mathematical point of view.

We started exchanging letters and drafts. I still have a copy of Chapter I on classical electrodynamics and part of Chapter II on the Fock representation of the vector potential in a Krein-Hilbert space, both written by Wightman. We kept dreaming of our joint project during my visits to Princeton in 1976–77, 1980–81, and of Wightman to Pisa in 1993, but the project was slowly fading away. One of the reasons was that the subject was still in evolution, and it took time to fix some of the relevant issues. In fact, such a project turned out to be an endless source of interesting problems, such as the control of chiral symmetry breaking in the local quantization QED_{1+1}, with a mathematical analysis of the local quantization of the massless scalar field in two dimensions; the failure of the cluster property in local gauges; the nonregular representations of the field algebra defined by gauge invariant vacuum states; the construction and properties of the charged states; the proof of the charge superselection rule in the Coulomb gauge, the nonperturbative discussion of the Higgs mechanism; etc.

The influence of Wightman on my work has been so strong that I could hardly imagine what it would have been without him.

Ray Streater is a retired professor of mathematics at King's College London. He visited Wightman at Princeton University for six months, 1962–63, to help write up the account he had started of what is now known as "Wightman theory." It appeared in 1964 as a book entitled PCT, Spin and Statistics, and All That. *His email address is* raystreater@gmail.com.

Franco Strocchi is a retired professor of theoretical physics. On invitations by Wightman, he was in Princeton in the seventies and early eighties as research associate and as visiting professor. His email address is f.strocchi@sns.it.

Giorgio Velo

I became acquainted with Arthur Wightman at the beginning of 1964 when Arthur, on sabbatical in Europe, paid a short visit to Res Jost, then a professor at the ETH in Zurich. At that time I was passing a few months at the ETH in Zurich with an Italian fellowship to familiarize myself with rigorous quantum field theory. During the summer Arthur and I met again at the Cargese, Corsica (France) Summer School in Theoretical Physics. Arthur lectured extensively on two-dimensional solvable models in quantum field theory and stimulated my interest in the subject. As a speaker his style was simple and lucid. I started to understand why, besides being a leading scientist, he was also considered a gifted teacher.

In November of the same year I was in Princeton for one year--supported by a NATO fellowship, hosted by Arthur in his group. Arthur had a large number of students, postdocs, and others working on rigorous quantum theory under his leadership. He was generous in sharing his knowledge and experience with all of them and actually devoted an enormous amount of time to discussions. In connection with his Cargese lectures he suggested to me the problem of writing a rigorous solution of the two-dimensional Schwinger model. During that research I always received from Arthur both warm encouragement and help in order to overcome the technical difficulties. That year was crucial for my scientific growth. In Princeton I lived in an exciting atmosphere, and I had the strong impression of important scientific events taking place. I met colleagues and made friends for life: John Challifour, John Dollard, Marcel Guenin, Arthur Jaffe, Oscar Lanford, and others. During that year Arthur taught a course on quantum field theory for which I still have the notes. It was a remarkably wide presentation of various aspects of that discipline so inspiring that, as an immediate fallout, Klaus Hepp, who was in the audience, wrote an important paper on renormalization theory.

In subsequent years Arthur and I became independently interested in the theory of relativistically invariant wave equations, with particular attention to equations describing higher spin particles. We corresponded frequently on that subject. While I was working on specific examples, Arthur devoted himself mostly to general theory writing, thorough papers containing many suggestions for future developments.

Giorgio Velo was professor of mathematical methods for physics at the University of Bologna (Italy). Arthur Wightman and Giorgio Velo were co-directors of the International School of Mathematical Physics of the Ettore Majorana Center for Scientific Culture located in Erice, Sicily, Italy. His email address is Giorgio.Velo@bo.infn.it.

Wightman with Marston Morse and Anna-Greta Wightman at a social gathering in the Morse home, 1973.

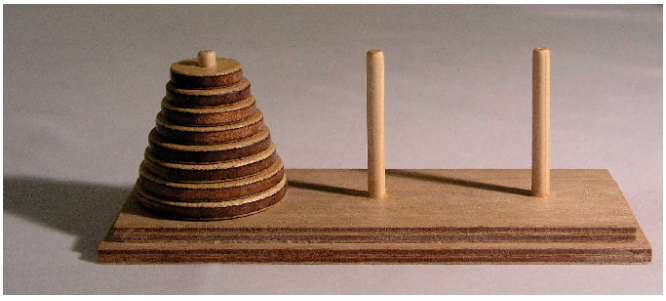
Starting in 1973, within the Ettore Majorana Center for Scientific Culture, located in Erice, Sicily (Italy), Arthur and I organized seven summer courses on various topics of mathematical physics: Constructive quantum field theory (1973), Renormalization theory (1975), Invariant wave equations (1977), Rigorous atomic and molecular physics (1980), Regular and chaotic motions in dynamical systems (1983), Fundamental problems of gauge field theory (1985), Constructive quantum field theory II (1988). They resulted in seven volumes of invited lectures, published variously by Springer, Reidel, and Plenum. I remember quite well Arthur's enthusiasm about this pedagogical activity.

The scientific community sorely misses a scholar and a gentleman, and I miss a true friend.

Automatic Sequences and the Tower of Hanoi

This month's cover was suggested by the article "WHAT IS an automatic sequence?" in this issue, written by Eric Rowland. One of the more curious examples of an automatic sequence is that which solves the well known and perhaps surprisingly popular problem of the Tower of Hanoi, first proposed by E. Lucas around 1884. The relation to automatic sequences can be found in §6.4 of the book by Allouche and Shallit referred to in Rowland's article.

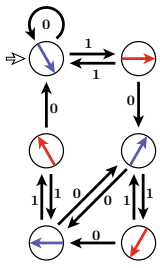
The problem is posed by a configuration of three pegs, on one of which are placed a number of rings of increasing radii, say from 1 at the top to some number N at the bottom.



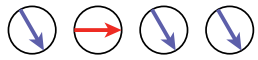
Tower of Hanoi | Plywood.

On the cover $N = 4$, but traditionally it is 8. The problem is to shift all rings from the initial peg to another, without ever placing a larger ring on top of a smaller one. As Allouche and Shallit explain, an optimal sequence of moves can be constructed as an automatic sequence. The sequence on the cover is one of symbols illustrating transfers to be made. For example,  marks a transfer of the disk on top of peg 1 to peg 3.

To figure out the nth move, first express n in bits. Then follow the path laid out by these bits, high order to low, in the following diagram:



The start is the node at upper left, and the move to be made is that indicated by the final node. For example, for $n = 6$ with bit expression 110 the path is



so one moves the small peg from peg 1 to peg 2.

The literature on this problem is enormous. Allouche and Shallit list many mathematical references, and the following web page is full of amusing information. `http://en.wikipedia.org/wiki/Tower_of_Hanoi`

The photograph of the Tower of Hanoi in plywood is taken from `http://commons.wikimedia.org/wiki/File: Tower_of_Hanoi.jpeg`

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—Bill Casselman
Graphics Editor
`notices-covers@ams.org`

Presidential Views: Interview with Robert L. Bryant

Allyn Jackson

Every other year, when a new AMS president takes office, the *Notices* publishes interviews with the outgoing and incoming presidents. What follows is an edited version of an interview with Robert L. Bryant, whose two-year term as president began on February 1, 2015. Bryant is the Phillip Griffiths Professor of Mathematics at Duke University and is an emeritus professor at the University of California at Berkeley. From 2007 until 2013 he was director of the Mathematical Sciences Research Institute (MSRI) in Berkeley. The interview was conducted in fall 2014 by *Notices* senior writer and deputy editor Allyn Jackson.
An interview with past-president David Vogan appeared in the February 2015 issue of the *Notices*.

Photo courtesy of Duke Photography; Lee Todd, Photographer.

Robert L. Bryant

***Notices*:** *While you were director of MSRI, you came into contact with a broad cross section of the mathematical community. Based on that experience, what would you say are the main strengths of the community today, and what are the main challenges it faces?*

Bryant: One strength is the vitality of the mathematical community. We are seeing incredible progress not only in traditional fields but in areas that reach out to other scientific disciplines. For example, in recent years compressed sensing, pattern recognition, and the challenges of dealing with large data sets have inspired a lot of developments in mathematics. Years ago we all thought topology was a fairly abstract subject and didn't realize that computational topology holds the key to understanding many aspects of data sets. The interactions with other disciplines have proved very fruitful and go far beyond the traditional interaction of mathematics and physics that has sustained and inspired us for a very long time. We are in many ways in a golden age in mathematics. In fact, one challenge is keeping up with all the things that are happening and serving the diverse community that is developing.

The mathematics institutes have a key role to play in this in sponsoring workshops and conferences within mathematics and in interdisciplinary areas as well. The AMS can also help, by having at its national conferences more speakers who highlight the advances that mathematics is making in real-world problems as well as the exciting theoretical developments that are making those advances possible.

***Notices*:** *What other venues are there for increasing these interactions?*

Bryant: We can work with people at the NSF [National Science Foundation] so that mathematicians become more aware of the funding opportunities that are available for interdisciplinary research. The NSF makes periodic announcements to the mathematics community about such initiatives. But those announcements don't necessarily inspire people or give them a clear idea of how they themselves can get involved. In addition to the announcements, there could be conferences that would give the folks at the NSF a chance to communicate directly with mathematicians who are interested in learning about these opportunities. It would be useful to sponsor speakers who would give colloquium talks in mathematics departments around the country and also to hold summer programs and research experiences, not only for undergraduates but also for graduate students who are interested in learning about these new areas. Those are all things we should explore.

***Notices*:** *Speaking of the NSF, how do you see the outlook for NSF funding for mathematics?*

Bryant: The people who watch Washington believe we shouldn't be planning at the moment for major increased funding for doing what we are

Allyn Jackson is senior writer and deputy editor of the Notices. *Her email address is* axj@ams.org.

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doing. But funding for interdisciplinary programs and initiatives often comes from outside the DMS [NSF's Division of Mathematical Sciences] or from other agencies, like the NIH [National Institutes of Health], the DoD [Department of Defense], and the DoE [Department of Energy].

Of course, we should take every opportunity to remind Congress and the American public of the benefits of mathematics. That is a more indirect but very important way for us to make the case for increasing mathematics funding.

***Notices*:** *What do you see as effective ways of raising public awareness about mathematics?*

Bryant: When we did public outreach events at MSRI, we found that there really is a significant public interested in the intellectual challenges in mathematics. We had a series of public lectures called "Math and…": "Math and Music" or "Math and Medicine" or "Math and Finance." We got a very good response from the public, from people who were interested in finding out what mathematicians are doing and what was exciting and important in the field. We could reach only people who were within a one-hour or at most two-hour drive from Berkeley. But with some additional investment in production values, we could make videos of the lectures suitable for, say, posting on YouTube. Or we could support one or more of these lecturers to speak at various places around the country. Those kinds of public event opportunities have not been fully utilized by either the mathematics or the scientific community.

If you look at sales of popular books on science or mathematics, they do pretty well. And there are movies, like the recent one on the life of Ramanujan. We could do more to raise public awareness by taking advantage of the human interest in people who devote their lives to science and mathematics.

There is a very interesting film by a French director, Olivier Peyon, called *How I Came to Hate Math* (*Comment j'ai détesté les Maths*). It starts out with interviews with people on the street, talking about their negative attitudes about math. There are also interviews with mathematicians, like Cédric Villani, who talk about what's exciting in mathematics and what their lives are like. Peyon came to MSRI while I was director and wanted to shoot some scenes. He wanted to talk with working mathematicians and find out: Why do you do this? What is so different about your life that you were attracted to mathematics and didn't succumb to the pervasive attitude that mathematics is frightening or boring? Full disclosure: I actually appear in the film, so I am not unbiased. But I've seen it, and I thought it was well done. A version of that film that featured interviews of everyday Americans being asked those questions would be quite interesting.

We were talking about challenges. One challenge is the need to reach out to underrepresented groups that have not traditionally been successful in mathematics. I don't know anyone who believes that the mathematics community should stay mostly male, mostly white, etc. But when you go to a mathematics department and see that most of the faculty are white males, that sends a message on its own. We need to do more to recruit underrepresented groups and make sure that they know that our community understands and appreciates them. That's a challenge for us as a workforce issue in the twenty-first century, but also for the health of our discipline. I would like to see the AMS take more of a role in meeting that challenge.

***Notices*:** *In 2014, for the first time, a woman, Maryam Mirzakhani, got a Fields Medal. What is the effect on women in mathematics?*

Bryant: Just after it happened, when I encountered women mathematicians I know, it was the first thing they talked about. That was evidence to me that it has a big psychological effect. Of course, we would like to believe that there really is a level playing field in mathematics, that you prove yourself by the results you get. But if you consider that until 2014 every single Fields Medalist was male, and you know that there are brilliant women mathematicians around the world who have done amazing things, it's hard not to feel that somehow their accomplishments haven't been as celebrated and recognized as those of their male counterparts. When Maryam Mirzakhani won the Fields Medal, people felt, "Ah!—'the glass ceiling' has been broken." People will be less likely to overlook the accomplishments of women in the future or less likely to think, "Oh, she's a good graduate student and will make a good colleague, but we don't expect her to be a leader." It does change people's attitudes. The sense of relief and pride I got from many of the women whom I talked with was palpable. I think we should not minimize that or make people feel that they shouldn't feel that way—as though we ought to be above noticing this. I think we should be proud that we are making progress in recognizing the many contributions of women in our field.

***Notices*:** *You got your PhD in 1979. Did you join the AMS right away?*

Bryant: If I remember correctly, I had a graduate student membership in the AMS. There was a gap of a few years, after I got my degree, when I was not a member; I re-joined the AMS in 1985.

***Notices*:** *That's how people thought then: After you get your PhD, you join the professional society. Now it's different. Young people are not joining in the same numbers as before. Two questions: How did the math community differ then from how it is now? And second, how can the AMS adapt to those differences?*

Bryant: Those are interesting questions. As soon as I got my first job, I very quickly got tenure. When you become tenured and you see yourself in a career that is going to be continuing probably for

the rest of your life, you realize that you are part of a community. Joining the professional organization just makes sense. You get greater access to networking, to talking to people who are making policy, to staying informed. For example, the reincarnation of the *Notices* as a monthly magazine has been enormously beneficial. When I get the *Notices* and the *Bulletin*, I spend a significant amount of time looking through them and picking out interesting articles to read. It gives me a feeling for what my colleagues are doing and what the challenges are that the community faces.

There are now many young mathematicians who are not on a tenure track and are not sure whether there's going to be a place for them in academia. This is speculation on my part, but maybe those young people feel that they are not really members of the professional community until they reach a more stable situation. This long gap between the PhD and a permanent position, which was much less common in the 1970s and 1980s, might also put young people at more distance from professional organizations. Of course, the AMS provides obvious benefits and support for young people, for example, through the Employment Register and MathJobs. But we should do our best to find out what we are *not* doing that would make young people feel that it is effective and useful for them to join the AMS. It's our mission, that's what the AMS does: it supports the profession, it supports its members.

Notices: *Do you think the younger generation is finding ways to stay connected electronically through things like MathOverFlow and blogs? Is that taking the place of the kind of interactions the AMS offers?*

Bryant: I hadn't thought of it in those terms. I confess that I was completely unaware of MathOverFlow in the first couple years that it existed. Maybe that's because I'm not of the generation that thinks that the first thing you do when you want to find out about something is to look at a blog or consult online interactive communities. But a friend noticed a question on MathOverFlow that he knew I could answer, and that's how I got involved. MathOverFlow is a very nice way to exchange information among research mathematicians and graduate students. It's a kind of electronic common room. If you go to many mathematics department common rooms today, you will find that, except during tea, they are mostly empty. People don't hang around common rooms anymore. In addition to MathOverFlow, there are widely read blogs, like those of Terry Tao and Tim Gowers that are really enlightening.

Journals that carry on the traditional role of highlighting important new developments and providing quality control, tools like MathSciNet that provide reviews and enable people to search the literature better—these are valuable things a professional organization can do. So I am all for those. I'm not sure there is a role for the AMS to insert itself into things like MathOverFlow, which seems to be working fine on its own. It is not necessary that the AMS become the sole source. But the AMS should do its best to help efforts like MathOverFlow to be as effective as possible by helping to spread the word.

Notices: *Is there anything else you wanted to say that we didn't touch on?*

Bryant: One thing I'd like to speak to is the role of the AMS in providing a forum for issues facing our community. I am thinking, for example, of the TPSE Math group [Transforming Post-Secondary Education in Mathematics]. With technology like MOOCs [Massive Open Online Courses] and shifting demographics changing the basic business model of many universities, there are challenges in STEM [science, technology, engineering, and mathematics] education and postsecondary training that we should address as a community. I would like to see the AMS become more involved in those efforts.

Our expertise in understanding what the issues are along with our experience in the classroom are both very valuable here. The AMS needs to be involved jointly with the MAA [Mathematical Association of America] and other professional organizations to make sure that we are doing the best job we can in helping develop effective educational tools and strategies. We also need to make sure that the successes and the difficulties are documented well enough so that people can clearly see what works and what doesn't. Then people will feel not lost but empowered and able to do things at their own institutions that will not only improve teaching and learning locally but will also better serve our community and society as a whole.

Special Projects Officer

Division of Meetings and Professional Services

Applications and nominations are invited for the position of Special Projects Officer in the Division of Meetings and Professional Services.

The division is the locus for programs and services that directly affect the entire mathematical community. Departments in the division work on meetings, surveys, professional development, research grants, employment services, educational outreach, public awareness, and membership development. The Special Projects Officer will work with the Associate Executive Director for Meetings and Professional Services in guiding and implementing these programs. The division includes approximately 20 staff in the Providence headquarters.

Responsibilities of the Special Projects Officer include:

- Providing mathematical expertise and support for such services as data collection, grant programs, and programs for early-career mathematicians;
- Serving as liaison to AMS officers and committees;
- Administering and overseeing grants and contracts;
- Writing grant proposals;
- Assisting in developing and implementing new programs for the Society.

Candidates should have an earned Ph.D. in one of the mathematical sciences and significant academic or administrative experience. Familiarity with the mathematical community and a strong interest in professional programs and services are essential. An interest in surveys and data analysis would be an asset.

This is a full-time position at the Society's headquarters in Providence, with a starting date of July 1, 2015. Salary will be commensurate with experience.

Applications must include a curriculum vitae and a letter describing the applicant's administrative and/or academic experience and his or her interest in the position.

Nominations and applications should be sent to:

Dr. T. Christine Stevens
Associate Executive Director
American Mathematical Society
201 Charles Street
Providence, RI 02904-2294 USA
email: aed-mps@ams.org
tel: 401-455-4139
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Applications received by May 4, 2015 will receive full consideration.

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Remembering Klaus Peters

Sigurdur Helgason and Dana Scott, Alice Peters and David Mumford

Sigurdur Helgason and Dana Scott

Klaus Peters as Mathematical Publisher

This piece is a supplement to a biographical note in the December 2014 issue of the *Notices* dealing with the highly respected publisher of scientific books, Dr. Klaus Peters. Here we intend to describe in more detail his remarkable career as a publisher of mathematical books.

After his doctorate in complex analysis in 1962 from the University of Erlangen, Klaus served as assistant professor at Erlangen for two years. Then he was invited by Springer Verlag to be its first in-house mathematics editor. That same year Springer opened its American office in New York. In 1972 Klaus was named one of Springer's directors, and he hired Alice Merker, who had earned degrees from Rochester and Chicago, to be a mathematics editor at Springer New York. They married that year, settled in Heidelberg, and worked there at Springer Verlag. This account is based on Alice Peters's recollection of their publishing activity during the last forty years.

The 1970s were a boom time for Springer. In addition to several new major volumes, Klaus started at that time at least two new book series with American editors: Applied Mathematical Sciences and Undergraduate Texts in Mathematics (edited by Halmos). At the same time Alice started a computer science program at Springer. This included textbooks, monographs, and the various Lecture Notes Series.

Klaus felt from the start that in order to build an enlarged mathematics publishing program he would need a full commitment where he would have responsibility for all aspects of the publishing process: acquisition, editing, production, pricing, promotion, etc. He realized that in order to build a world-class program, he would have to internationalize and involve American mathematicians. Richard Courant, who had maintained close ties with Springer even through the war, was his initial contact. They immediately forged a great relationship, and Courant introduced Klaus to the whole New York University/New Rochelle group, including Jürgen Moser, Fritz John, Kurt Friedrichs, Peter Lax, Cathleen Morawetz, among others, all of whom became trusted authors and advisors and also close personal friends. He also had close contact with Peter Hilton and Paul Halmos, both already editors of *Ergebnisse der Mathematik*. The connection with both Peter and Paul went beyond a professional relation, and Klaus considered both among his real friends.

Lecture Notes in Mathematics

Klaus was strongly involved with the beginning of Lecture Notes.

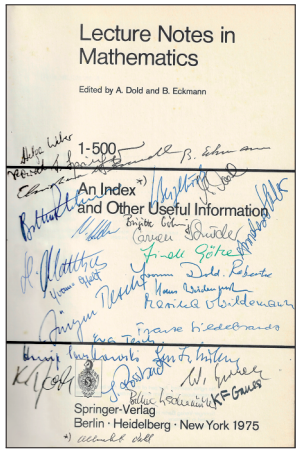


Photo courtesy of Sig Helgason.

A signed cover of Lecture Notes in Mathematics from a celebration of its first 500 volumes.

This may have come out of a discussion with Beno Eckmann (ETH Zurich) to find a way to distribute preprints or private communications that usually were sent only to a handful of closer colleagues to a wider audience, in particular, students. Klaus was convinced that this would be considered positively in the mathematics community and

Sigurdur Helgason is professor of mathematics at the Massachusetts Institute of Technology. His email address is helgason@mit.edu.

Dana Scott is Hillman University Professor of Computer Science, Philosophy, and Mathematical Logic, emeritus, at Carnegie Mellon University. His email address is dana.scott@cs.cmu.edu.

DOI: http://dx.doi.org/10.1090/noti1222

that it was timeliness rather than typesetting that would be the hallmark of the series. At the time, no one expected that the series would become so successful and respected. It now constitutes over two thousand volumes.

The Mathematical Intelligencer

The *Mathematical Intelligencer* was started by Alice, Klaus, and Walter Kaufmann-Buehler, their colleague in the mathematics editorial department at Springer. Walter had been working with Klaus, doing promotion for the math program, and he switched positions with Alice, taking her job at Springer NY after Klaus and Alice had met and decided to marry. The story of how and why the *Intelligencer* started is documented in an anniversary volume of the *Intelligencer*, Vol . 30 No. 1, 2008: pages 7–18 contain an interview with the founders.

In spite of successful publication activity, Klaus felt that financial considerations were gradually beginning to outweigh editorial decisions at Springer and thus would not preserve his ideal that the purpose of the company should be to do service to science. He failed to reach a satisfactory agreement with Springer owners on this issue and respectfully submitted his resignation.

From Springer to Birkhäuser to HBJ to A K Peters

In accordance with Klaus's point of view, Klaus and Alice decided to strike out on their own, intending to realize what he saw as a mathematical publisher's primary goal: service to the mathematical public. In his article [1] he outlines his philosophy of mathematical publishing, stressing particularly a publisher's obligation to the author to maximize the book's usefulness to the mathematical community. Most mathematicians know publishers for whom this obligation is secondary, even absent. A book, completely author prepared, may routinely be put out by a publisher as an eBook or printed only on demand, with a price of US$100.

Following his vision, Klaus contacted Chari Einsele, owner and publisher of Birkhäuser, after which Einsele immediately suggested that Klaus and Alice move to Basel and "take charge" of Birkhäuser. Klaus did not see this as a viable situation, because he felt that a modern-day scientific publisher needed to have an American office and program. So Klaus and Alice moved to Boston and started Birkhäuser Boston in 1979.

This was a courageous move: settling in a new country with three children under four and, with no prior experience, starting and running a company. Alice was the principal computer expert in the group and became mainly involved on the financial and

Photo courtesy of Alice Peters.

Alice and Klaus Peters.

production side. Klaus concentrated on the editorial work, starting the series Progress in Mathematics, which has now over one hundred fifty volumes. Birkhäuser grew and was thriving but then was put in jeopardy when the Birkhäuser printing company, owned also by the family, ran into difficulties. Through his contacts, Klaus arranged a sale of the publishing company to Harcourt Brace Jovanovich (Academic Press); however, this agreement fell apart because of a last-minute surprise takeover by Springer. Klaus then preferred to do something independent at that time, and, in fact, William Jovanovich quickly approached Klaus and Alice and asked them to start a new office for Harcourt Brace Jovanovich(HBJ)/Academic Press in Cambridge.

The Peters then built and managed the Boston office of HBJ, running Academic Press, and started the series Perspectives in Mathematics, which eventually included eighteen valuable books. They were responsible for the publishing program in mathematics, computer science, chemistry, and physics, with several other editors for the other fields. Unfortunately, HBJ ran into financial difficulties and was bought by General Cinema Corporation, which had little interest in the "idealistic" program envisioned by Klaus.

Klaus, with his passion and idealism, convinced Alice that they should start their own publishing company! With the help of Elwyn Berlekamp they then started A K Peters. The early years were hard for them and the company, both because of the long hours and financially. However, they held to their principles of publishing only high-quality books, even those whose sales would possibly be limited, as long as the books were of importance to the scientific community. Alice went to Babson College at night to complete work for her MBA in order to have a better knowledge of how to operate a business.

As the company grew, Klaus's interests also grew to include publishing in less-traditional

areas for a high-level scientific publisher. A new journal, *Experimental Mathematics*, was launched with the help of David Epstein and Silvio Levy and published by A K Peters. It was devoted to experimental aspects of mathematical research, including mathematical influence (even proofs) by computer. This grew out of discussions Klaus and Alice had with David Mumford. Among the notable books they published were *Hilbert* and *Courant*, both by Constance Reid, *The Honors Class* by Benjamin Yandell, *The Mathematical Experience* by Philip Davis and Reuben Hersh, *You Can Count on Monsters* by Richard Evan Schwartz, and *A Cultural History of Physics* by Károly Simonyi, as well as the popular book *Mathematical People* by Donald Albers and Gerald Alexanderson.

Klaus was extremely concerned about the state of education and sought ways to make mathematics more appealing. He saw the possibility to make a contribution by publishing stimulating books for young adults: a (fictionalized) biography of Gauss, a book on cryptography (*Cryptoclub*), and a whole program in recreational math, including origami, which he eventually worked on with the AMS. This really became a passion for him in later years.

He also made Herculean efforts to find ways to publish books that he felt should be made available, sometimes through clever sales of the books or through funding if the book could not support itself. With the last book that they published, Simonyi's *The Cultural History of Physics*, A K Peters sailed forth in full publishing glory. The book embodies Klaus's vision of the publication of a moderately priced masterpiece of very general interest, which in this case can be considered a work of art. Amazon.com contains some glowing endorsements. Sheldon Glashow's comments call it "fascinating for its wealth of data and for its anecdotal digressions," "marvelously filled with interesting and unfamiliar historical and technical explanations." In fact, Glashow, with Klaus's editorial help, was in the process of writing a supplementary volume.

A K Peters became what Klaus had envisioned: a company whose aim was to serve the scientific community. For over forty years, Alice and Klaus made a significant mark on mathematical publishing throughout the world. In the process, they published over one thousand books and earned universal goodwill in the mathematics community.

References

[1] KLAUS PETERS, Why publish mathematics? *Notices of the AMS*, August 1999.

[2] W. KAUFMANN-BÜHLER, K. PETERS, and A. PETERS, Mathematicians love books, *Mathematics Tomorrow*, Springer, 1981, pp. 121–126.

Alice Peters and David Mumford

Klaus Peters, Springer, and the Mission of Math Publishers

Klaus was a man with strong ideals who loved publishing and poured his heart into his work. His life and many contributions to publishing have been detailed in the previous article. Here we want to focus on a battle that he waged, one that concerns the math community as a whole. He fought fiercely for the ideal of publishing in service to the professional community and believed that *this required full editorial independence*. He was not afraid to put his career on the line for these ideals, and this led to his resignation from Springer after he had nearly single-handedly brought them to the forefront of mathematical publishing. How did this happen and what does this mean for our future?

Springer Verlag has worked closely with the mathematical community for over a hundred years. In 1923 Ferdinand Springer rescued the two preeminent mathematical journals of the time, *Mathematische Annalen* and *Mathematische Zeitschrift*, when they were in deep financial trouble. This resulted in a letter of appreciation signed by Richard Courant, Albert Einstein, and David Hilbert, among others. This collaboration resumed, of course, after World War II, and Klaus was welcomed into the firm by Ferdinand Springer with the word "service." He described the scene in an unfinished memoir as follows:

> One day my phone rang: "Springer here, please come to my office." Ferdinand Springer, the legendary publisher, did not usually deal with junior members of the staff nor had I been formally introduced to him. I went to his office unsure what this all meant. His personal secretary kindly advised that I should listen and quietly excuse myself when the "audience" was over. On entering his office I was greeted warmly as the new mathematics editor. Mathematics was one of Springer's favorite programs. He then proceeded to explain the raison d'être of a publisher: to facilitate the work of the authors by taking away the burdensome aspects of editing, producing, and most importantly distributing their work widely. He made it very clear that these added

Alice Peters is a publishing consultant and cofounder of the publishing company A K Peters. Her email address is alicepeters@gmail.com.

David Mumford is professor emeritus of mathematics at Brown University and Harvard University. His email address is David_Mumford@brown.edu.

values were the justification of a publisher's existence.

His fierce loyalty to authors and editors is confirmed by another story. When Ferdinand Springer sought to leave the occupied city of Berlin after World War II to rescue his family, he was stopped at a military control post. The commanding Russian officer demanded an explanation. Springer identified himself as a publisher of scientific books and journals (in his mind that was explanation enough) whereupon the officer commanded, "Tell me the names of the editors of such and such journal!" Springer had retained the names of Russian scientists and editors on the masthead of the journals they had served, despite the war. As he recited these names, the officer suddenly interrupted, "That's me, and I am honored to meet you." He provided Springer with free passage which allowed him to rejoin his family.

In 1971–72, Klaus formulated a plan to change the management of Springer from just the two active owners (Heinz Goetze and Konrad Springer) to a board of directors. He saw this as an important step in managing the growing company and as a necessary move for the future, as the owners were getting older. This plan was put in place with a board consisting of directors for editorial, production, marketing, and advertising (a bit of an anomaly, but that was an area that brought in a lot of money), each with equal votes. Klaus became the assistant to the editorial director and then later became the editorial director.

Some years later, a decision was made that the CFO should also become a director and that he would have the final say. Klaus very much disagreed with this decision and felt that it could alter his ideal that the purpose of the company should be serving science. He had many talks with Goetze about this and tried to convince him that there should be an editorial representative at the top (so to speak) on par with the financial director, and was always quite specific that it did not need to be him. When he was not able to sway Goetze's opinion, he told Goetze that he and Alice were going on vacation to France for one month, and if he did not hear from him otherwise, he would send in his letter of resignation on the last day of their vacation. Though he had just built a house in Heidelberg, had three children under the age of four and had "tenure" at Springer as a result of his joining the company from a university position, true to form he sent the letter of resignation. He was convinced that it would only be a matter of time until financial decisions outweighed editorial principles.

In fact, Klaus's beliefs were prophetic. Starting in 1999, Springer went through a succession of acquisitions. In 1999 the privately held publishing and mass media firm Bertelsmann acquired a majority share of Springer Verlag and formed the academic publishing firm BertelsmannSpringer. There were three subsequent acquisitions by private equity firms (in 2003, 2009, and 2013; the names of the firms are given in the Wikipedia entry for Springer Science+Business Media). In each case, the acquiring firm expected that Springer's return on capital could be increased under their management. For example, extending new distribution rights to scans enables a publisher to charge for online access to older articles that formerly generated no revenue. Early in the series of acquisitions, a group of six presidents of mathematical societies cooperated at the 2002 International Congress of Mathematicians on a proposal to the chief executive of BertelsmannSpringer to create a not-for-profit organization to take over the mathematics publishing part of the business. That was not to be. The private equity firms had a responsibility to place a priority on *return on investment* for their stakeholders, an incentive which often conflicts with the goal of *publishing as a service*, championed by Klaus,

Klaus understood this conflict of goals. In his own words [1]:

> The economic model of a scientific/technical/medical (STM) publisher plays a major role in its relationship with and impact on the scientific community. The major factors are pricing and distribution. Observing the publishing landscape from the point of view of the scientific community, one is struck by the enormous difference in pricing between journals and books at the large conglomerates and at the smaller houses. A closer look reveals that prices at large publishing houses are often higher by a factor of 2 or 3. An even more detailed analysis shows that the higher pricing is directly related to smaller unit sales. The "chicken and egg" question comes to mind, and we believe that the answer lies in the economic model on which that publishing philosophy is based. Selling fewer copies at higher prices yields a higher profit margin, keeps warehousing costs down, and generally requires less marketing effort and expense; the larger house can rely on its core market—the libraries—all the while ignoring the matter of expanding marginal sales. If one thinks of a publisher as an entity with an obligation to publicize and, therefore, to maximize the distribution of the ideas contained in its publications, such

> a publishing strategy certainly conflicts with the latter goal.

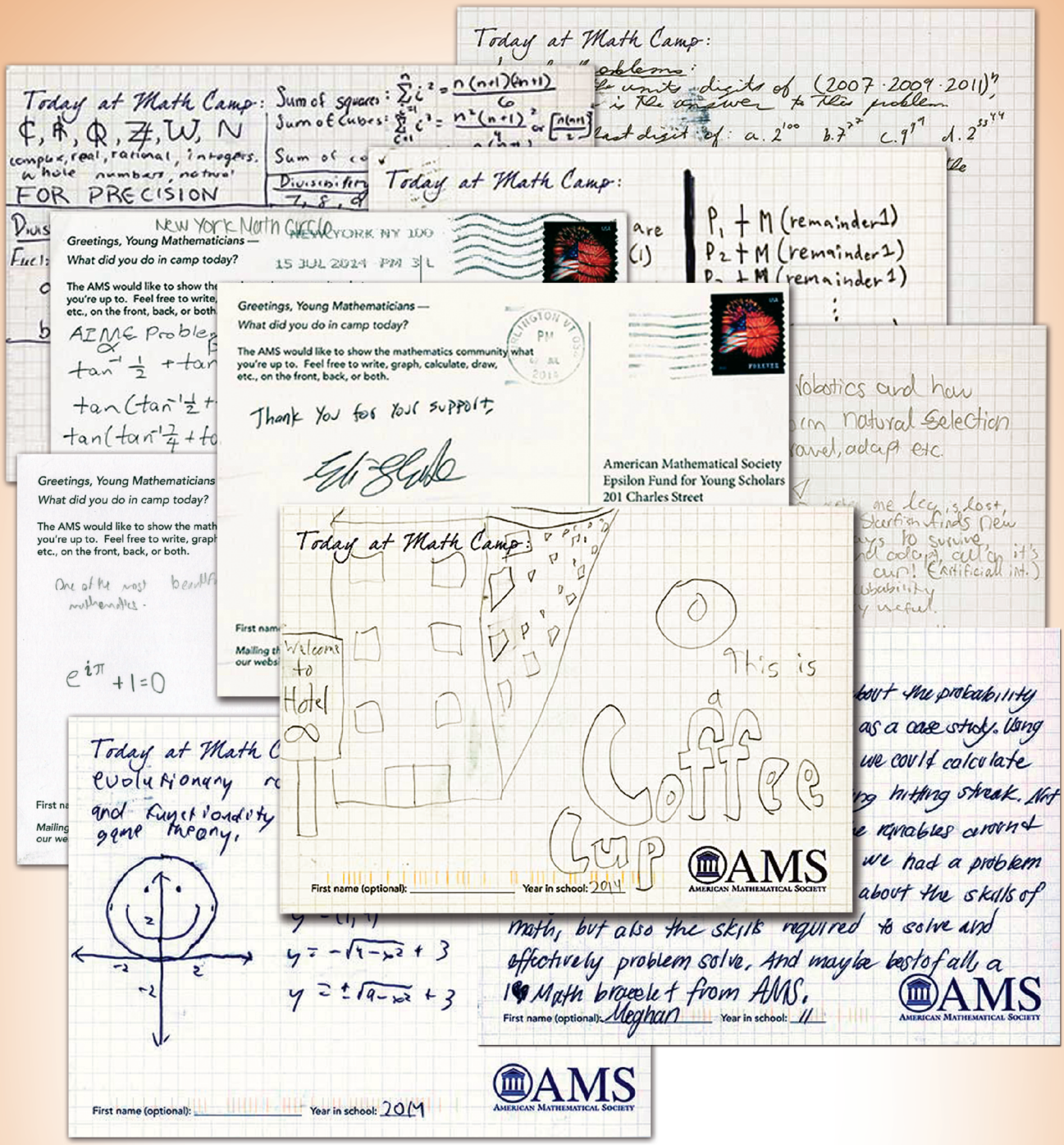
The idea of publishing as a service to the scientific community is something that was a core principle for Klaus throughout his life. In fact, just a few weeks before his death he wrote a short piece that he titled "A vanishing dream," articulating how essential this was to him—and to publishing in general.

> Alice and I feel that we have lived a dream to preserve and provide a service that was once considered worthwhile. I mean "publishing as a service." Some time ago I came across the speech that Ferdinand Springer gave in 1952 after he had become a member of the Rotary Club in Heidelberg. That speech represents the raison d'être and professional ethics of publishing that I learned from Springer in a personal conversation and from Heinz Goetze in my early years at Springer Verlag.
>
> That this concept (with few exceptions of small individual publishers) is widely lost is no secret but what bothers me intellectually is the fact that publishing companies can be run financially successfully without an intellectual mission and without thought to optimize sales (by numbers of copies) or to produce well-edited and designed books. They compensate these shortcomings by optimizing the bottom line through skimping on editorial and production costs and offsetting revenue loss from smaller per-title sales (by number) by inflating prices.

Mathematicians, by nature, want to concentrate on their work and resist worrying about the mechanics of communicating their results to their colleagues. But business models for publishing are changing rapidly in this digital age. Whether the new framework serves effectively the professional community or serves instead the financial firms that shift money from sector to sector following the scent of profit—this represents the major fork in the road that we are facing right now. Klaus's life and writings have a clear message: you need not be passive, and you can work to influence the outcome.

References

1. Klaus Peters, Why publish mathematics? *Notices of the AMS*, August 1999.



The **Epsilon Fund for Young Scholars** endowment supports summer camps for mathematically talented pre-college students.

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Mathematical Sciences Center (MSC) will hire a substantial statistics, number of post-doctorate fellows in the full spectrum of mathematical sciences. New and recent PhDs are encouraged for this position.

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The review process starts in December 2014, and closes by April 30, 2015. Applicants are encouraged to submit their applications before December15, 2014.

Tsinghua Sanya International Mathematics Forum (TSIMF)
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We invite proposals to organize workshops, conferences, research-in-team and other academic activities at the Tsinghua Sanya International Mathematics Forum (TSIMF).

TSIMF is an international conference center for mathematics. It is located in Sanya, a scenic city by the beach with excellent air quality. The facilities of TSIMF are built on a 140-acre land surrounded by pristine environment at Phoenix Hill of Phoenix Township. The total square footage of all the facilities is over 28,000 square meter that includes state-of-the-art conference facilities (over 9,000 square meter) to hold two international workshops simultaneously, a large library, a guesthouse (over 10,000 square meter) and the associated catering facilities, a large swimming pool, two tennis courts and other recreational facilities.

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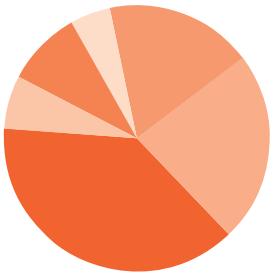
For information about TSIMF and proposal submission, please visit: http://msc.tsinghua.edu.cn/sanya/ or write to Ms. Yanyu Fang yyfang@math.tsinghua.edu.cn.



Preliminary Report on the 2013–2014 New Doctoral Recipients

William Yslas Vélez, James W. Maxwell, and Colleen Rose

This report presents a statistical profile of recipients of doctoral degrees awarded by departments in the mathematical sciences at universities in the United States during the period July 1, 2013, through June 30, 2014. The report includes a preliminary analysis of the fall 2014 employment plans of 2013–2014 doctoral recipients and a demographic profile summarizing characteristics of citizenship status and gender. The report on the 2013–2014 New Doctoral Recipients will include subsequent reports of additional 2013–2014 doctoral recipients from the departments that did not respond in time for this report, along with additional information provided by the doctoral recipients themselves. A list of the nonresponding departments is on page 273. This report uses the new groupings of doctoral-granting mathematics departments adopted by the Joint Data Committee starting with the 2012 AMS-ASA-IMS-MAA-SIAM Annual Survey of the Mathematical Sciences. Additional detail can be found on the AMS website at `www.ams.org/annual-survey/groups`.

Doctoral Degrees Awarded

The preliminary data shows that 1,728 new PhDs were awarded by 274 departments that responded in time for this report. These new PhDs consist of:

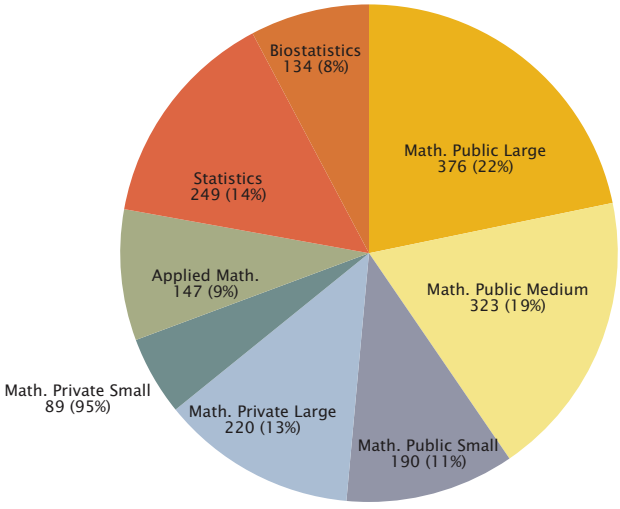
- 1,205 Males
- 523 Females
- 842 U.S. citizens
- 886 Non-U.S. citizens

Based on the data collected so far it is likely that the final count of PhDs awarded during 2013–2014 will exceed the record total of 1,843 reported for 2012–2013. The departments that have responded in both survey cycles reported 1,703 for 2013-2014 versus 1,658 for 2012–2013, a 2.7% increase.

Preliminary employment data on new PhDs shows that 9% are unemployed or not seeking employment and of those whose employment status is known (1,574):

- 78% are U.S. employed
- 13% are Non-U.S. employed
- 54% of those employed in the U.S. are U.S. citizens

Figure P.1: Number and Percentage of Degrees Awarded by Department Grouping*



Total Degrees Awarded: 1,728

*A description of the department groupings can be viewed at www.ams.org/annual-survey/groups.

William Yslas Vélez is a professor in the Department of Mathematics at University of Arizona. James W. Maxwell is AMS coordinator of special projects. Colleen A. Rose is AMS survey analyst.

Doctoral Degrees Awarded

The graphics below provide a snapshot of the preliminary data on new doctorates by gender, citizenship, employment status, employer type, and field of thesis. Watch for the release of the final report (Summer 2015) to see the full statistical profile along with comparative data on this cohort of new doctoral recipients.

Figure P.2: Gender of Doctoral Recipients by Department Grouping

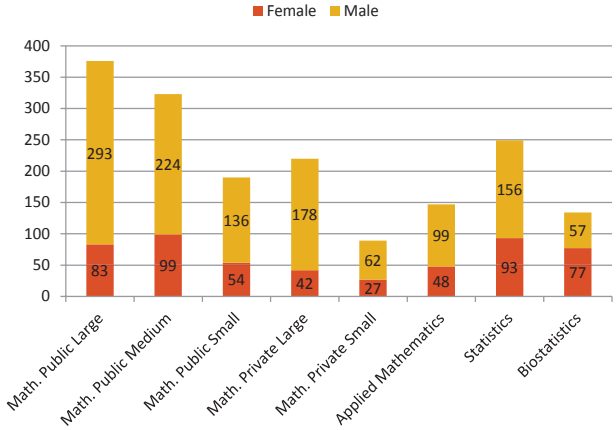


Figure P.3: Citizenship of Doctoral Recipients by Department Grouping

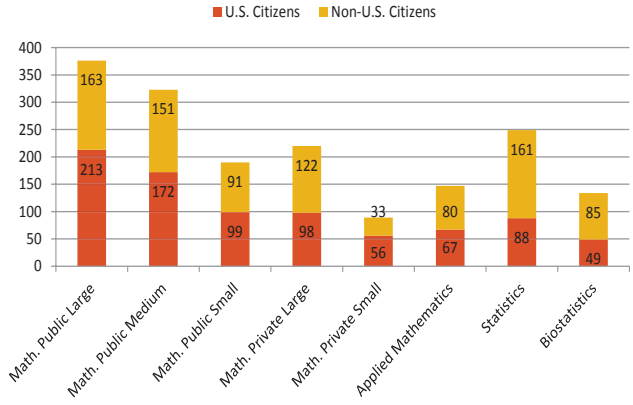
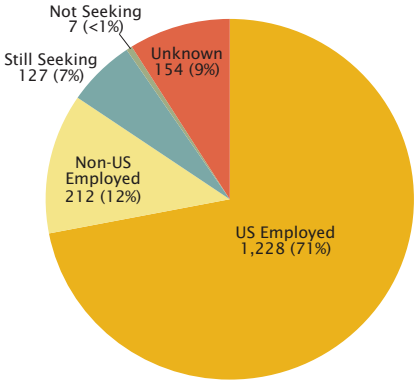
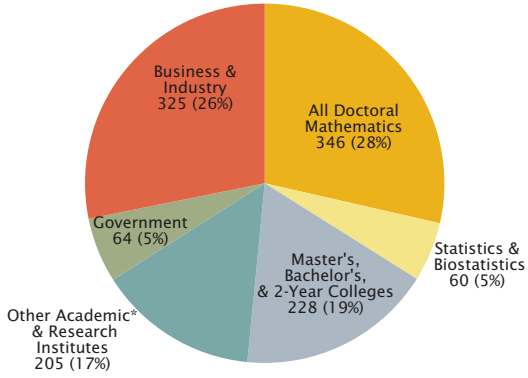


Figure P.4: Employment Status



Total Degrees Awarded: 1,728

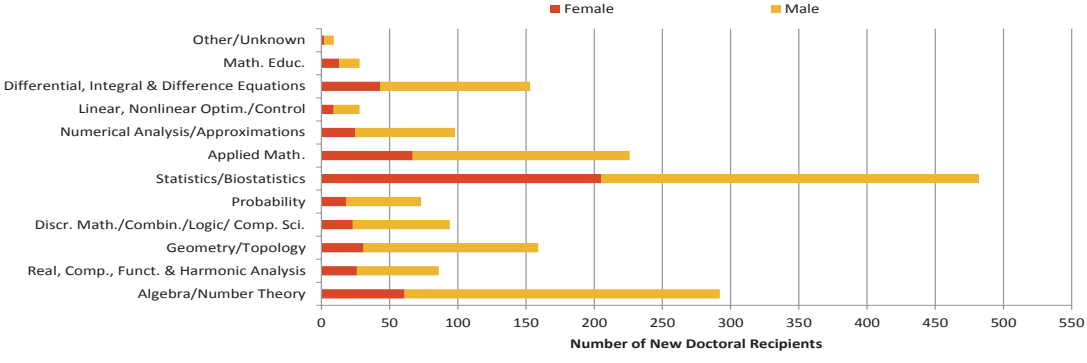
Figure P.5: U.S. Employed by Type of Employer



Total U.S. Employed: 1,228

*Other Academic consists of departments outside the mathematical sciences including numerous medical related units.

Figure P.6: New Doctoral Recipients by Field of Thesis and Gender



Departmental Response Rates

Survey Response Rates by New Groupings

Doctorates Granted
Departmental Response Rates

Math. Public Large	25 of 26 including	0 with no degrees
Math. Public Medium	40 of 40 including	0 with no degrees
Math. Public Small	57 of 64 including	6 with no degrees
Math. Private Large	24 of 24 including	0 with no degrees
Math. Private Small	26 of 28 including	4 with no degrees
Applied Math.	27 of 31 including	2 with no degrees
Statistics	45 of 58 including	1 with no degrees
Biostatistics	30 of 44 including	6 with no degrees
Total	274 of 315 including	19 with no degrees

Doctoral Degrees Not Yet Reported

The following mathematical sciences, applied mathematics, statistics, and biostatistics departments had not responded in time for their data to be included in this report. Those departments listed with an "*" have yet to respond as of January 24, 2015. Every effort will be made to collect the responses from these departments for inclusion in the New Doctoral Recipients Report which will be published in the August 2015 issue of *Notices of the AMS*.

Departments yet to respond can obtain copies of the Doctorates Granted survey forms on the AMS website at `www.ams.org/annual-survey/surveyforms`, by sending email to `ams-survey@ams.org`, or by calling 1-800-321-4267, ext. 4189.

Math. Public Large

University of Illinois at Chicago

Math. Public Medium

All departments responded.

Math. Public Small

Delaware State University
University of Missouri-St Louis
University of Nevada, Las Vegas
University of North Carolina at Charlotte
University of Southern Mississippi
University of Toledo
Utah State University

Math. Private Large

All departments responded.

Math. Private Small

Syracuse University
University of Denver

Applied Mathematics

Columbia University
University of Colorado, Boulder
University of Colorado, Denver
University of Pennsylvania

Statistics

Iowa State University
North Carolina State University
North Dakota State University
Oklahoma State University
Pennsylvania State University
University of California, Berkeley
University of California, Los Angeles
University of California, Santa Barbara
University of Georgia
University of Illinois- Urbana-Champaign
University of Kentucky
University of Virginia
Virginia Polytechnic Institute and State University

Biotatistics

Cornell University
Columbia University
Cornell University
Medical College of Wisconsin
St Louis University College for Public Health & Social Justice
University of Albany, SUNY
University of California, Los Angeles
University of Cincinnati
University of Illinois at Chicago
University of Kentucky
University of Louisville
University of Oklahoma, Health Science Center
University of South Carolina
University of South Florida

WHAT IS . . .

an Automatic Sequence?

Eric Rowland

A sequence $s(n)_{n\geq 0}$ is called *k-automatic* if $s(n)$ is a finite-memory function of the base-k digits of n. This means that some computer with only finitely many possible states can compute $s(n)$ for any n by reading the base-k digits of n one at a time (beginning with the least significant digit) and following a transition rule that specifies the next state of the computer as a function of both the current state and the current digit being read. Each possible state of the computer has an associated output value, and the result of the computation is the output value corresponding to the state of the computer after it has read the final digit. A computer of this kind is called an *automaton*, hence the name "automatic sequence."

For example, consider an automaton with only two states, q_1 and q_2, that reads binary representations of integers. Suppose the automaton starts in state q_1 and performs transitions according to the function $\delta : \{q_1, q_2\} \times \{0, 1\} \to \{q_1, q_2\}$ given by the following table.

δ	0	1
q_1	q_1	q_2
q_2	q_2	q_1

Let the output function $\tau : \{q_1, q_2\} \to \{a, b\}$ be given by $\tau(q_1) = a$ and $\tau(q_2) = b$. The first few terms of $s(n)_{n\geq 0}$ are as follows.

n	0	1	2	3	4	5	6	7
$s(n)$	a	b	b	a	b	a	a	b

Eric Rowland is a Marie Curie Actions COFUND postdoctoral fellow at the University of Liège. His email address is rowland@lacim.ca.

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For example, the standard binary representation of $n = 0$ is the empty word; to compute $s(0)$ the automaton starts in state q_1, performs no transitions, and outputs $\tau(q_1) = a$. When fed the binary digits of $n = 1 = 1_2$, the output of the automaton is

$$s(1) = \tau(\delta(q_1, 1)) = \tau(q_2) = b.$$

For $n = 2 = 10_2$, we get

$$s(2) = \tau(\delta(\delta(q_1, 0), 1)) = \tau(q_2) = b,$$

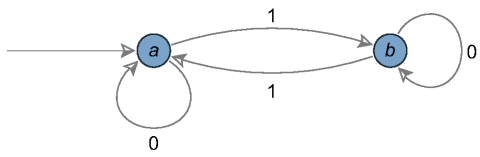
etc. To compute the value of $s(n)$, the automaton performs approximately $\log_2 n$ transitions.

The sequence

$$s(n)_{n\geq 0} = a, b, b, a, b, a, a, b, b, a, a, b, a, b, b, a, \ldots$$

is known as the Thue–Morse sequence. It is 2-automatic, since we are reading integers in base 2. Reading the least significant digit first is only a convention, since reading in the other direction turns out to give the same class of sequences.

It is illustrative to identify an automaton with a directed graph. We create a vertex for each state, and labeled edges encode the transition function δ. An unlabeled edge identifies the initial state, and each state is labeled with its output value. The automaton for the Thue–Morse sequence is the following:



We can compute $s(n)$ from the graph by starting in q_1 and following edges labeled with successive binary digits of n.

Since an automaton has only finitely many states, an automatic sequence is a sequence on a finite alphabet; and among sequences on finite alphabets, the class of automatic sequences is quite fundamental. There is no class known with more descriptive power and as many major properties. Automatic sequences generalize periodic sequences in the sense that each periodic sequence is k-automatic for all $k \geq 2$. In particular, this implies there is a "divisibility rule" for every integer in every base.

Automatic Sequences in Combinatorics

The structure of a k-automatic sequence reflects the recursive structure of the base-k digits of the integers, so it is not surprising that automatic sequences frequently arise from iteration. Suppose we iteratively replace the letters a and b in a word according to the morphism $a \to ab$ and $b \to ba$. Beginning with the word a, we obtain the sequence

$$\begin{aligned} a &\to ab \\ &\to abba \\ &\to abbabaab \\ &\to abbabaabbaababba \\ &\;\;\vdots \end{aligned}$$

which happens to consist of prefixes of the Thue–Morse sequence. In the limit, we obtain the Thue–Morse sequence itself, which is a fixed point of this morphism. Indeed, any morphism on a finite alphabet, where the image of each letter has length $k \geq 2$ and there is some letter a whose image begins with a, has an infinite fixed point. Cobham showed in 1972 that the letters of this fixed point form a k-automatic sequence.

Fixed points of morphisms are a central tool in combinatorics on words. A common question is whether a given pattern is avoidable in arbitrarily long words on a given alphabet. For instance, a *square* is a word of the form $w^2 = ww$, where w is a nonempty word. Are squares avoidable on a two-letter alphabet? Try to write down a long square-free word; it doesn't take long to determine whether this is possible. In 1912 Thue showed that cubes, i.e., words of the form w^3, are avoidable on a two-letter alphabet. His approach was to use properties of the morphism $a \to ab, b \to ba$ to conclude that the Thue–Morse sequence is cube-free.

In fact, Thue showed something stronger, namely, that the Thue–Morse sequence is overlap-free. An *overlap* is a word of the form wwc where w is a nonempty word whose first letter is c. Overlaps can be thought of as "$(2+\varepsilon)$-powers." Since overlaps are avoidable on a two-letter alphabet but squares are not, the exponent 2 is the *repetition threshold* for a two-letter alphabet.

Extensions of Thue's result have received much attention. In 1972 Dejean determined that the repetition threshold for a three-letter alphabet is $\frac{7}{4}$. A fractional power is a partial repetition; for example, *abbabbab* is the $\frac{8}{3}$-power $(abb)^{8/3}$. Dejean showed that it is not possible to avoid fractional $\frac{7}{4}$-powers on a three-letter alphabet, but it is possible to simultaneously avoid all $\frac{p}{q}$-powers with $\frac{p}{q} > \frac{7}{4}$. The morphism she used generates a 19-automatic sequence as opposed to Thue's 2-automatic sequence, but the broad idea of the proof is the same. For a general n-letter alphabet, Dejean's conjecture for the repetition threshold was finally confirmed through a number of additional papers by multiple authors. The last of these appeared only in 2011, nearly a century after Thue's results for a two-letter alphabet.

Automatic Sequences in Number Theory

Automatic sequences are also a useful tool in number theory, where they arise from an algebraic characterization of p-automatic sequences for prime p, which Christol discovered in 1979. Amazingly, p-automatic sequences correspond precisely to algebraic formal power series over finite fields $\mathbb{F}_q$ of characteristic p. That is, a sequence $s(n)_{n\geq 0}$ of elements in $\mathbb{F}_q$ is p-automatic if and only if $\sum_{n\geq 0} s(n)x^n$ is algebraic over $\mathbb{F}_q(x)$. For example, if we rename the letters in the alphabet for the Thue–Morse sequence, then the generating function $y = \sum_{n\geq 0} s(n)x^n$ of $s(n)_{n\geq 0} = 0,1,1,0,1,0,0,1,\ldots$ satisfies

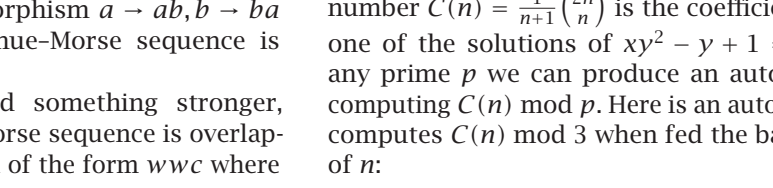
$$(1+x)^3y^2 + (1+x^2)y + x = 0$$

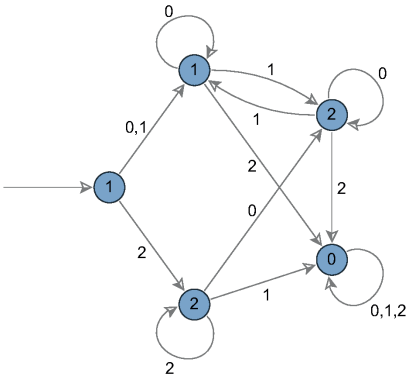
in $\mathbb{F}_2[\![x]\!]$.

An immediate application of this characterization is as follows. If $S(n)_{n\geq 0}$ is a sequence of integers such that $\sum_{n\geq 0} S(n)x^n$ is algebraic over $\mathbb{Q}(x)$, then projecting modulo p shows that

$$\sum_{n\geq 0} (S(n) \bmod p)x^n$$

is algebraic over $\mathbb{F}_p(x)$, and hence $(S(n) \bmod p)_{n\geq 0}$ is p-automatic. Many sequences that arise in enumeration settings have algebraic generating functions, so their behavior modulo primes can be assessed in this way. For example, the nth Catalan number $C(n) = \frac{1}{n+1}\binom{2n}{n}$ is the coefficient of x^n in one of the solutions of $xy^2 - y + 1 = 0$, so for any prime p we can produce an automaton for computing $C(n) \bmod p$. Here is an automaton that computes $C(n) \bmod 3$ when fed the base-3 digits of n:





Now consider the more general situation where we reduce an algebraic sequence modulo a prime power p^α. Since $\mathbb{Z}/(p^\alpha\mathbb{Z})$ is not a field for $\alpha \geq 2$, we can't use Christol's algebraic characterization to conclude that $(S(n) \bmod p^\alpha)_{n\geq 0}$ is p-automatic. However, Furstenberg showed that one can realize an algebraic sequence as the diagonal of a rational power series in two variables. The diagonal of a rational function lends itself to certain analyses, and it turns out to be good for computing an automaton for a sequence modulo p^α. Namely, there is an injection of the set of states of the automaton into the set of polynomials in $(\mathbb{Z}/(p^\alpha\mathbb{Z}))[x, y]$ with some bounded degree. Since this set is finite, the set of states is also finite and can be computed by polynomial arithmetic. The sequence of Catalan numbers forms the diagonal of

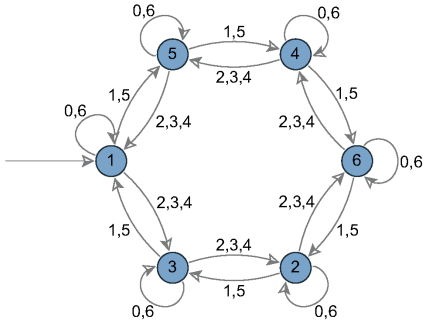
$$\frac{(y+1)\,(2xy^2+xy+x-1)}{xy^2+2xy+x-1}.$$

We can use this function to compute an automaton for $C(n) \bmod p^\alpha$ and obtain congruence information about the Catalan numbers. For example, $C(n) \not\equiv 3 \mod 4$ for all $n \geq 0$ simply because the automaton we compute does not output 3. Similar results for other sequences are completely routine to discover and prove [4].

Since this method works not just for algebraic sequences but more generally for diagonals of rational functions, it applies to many nonalgebraic combinatorial sequences as well as sequences that have arisen in number theoretic contexts. For example, the numbers $A(n) = \sum_{k=0}^{n} \binom{n}{k}^2 \binom{n+k}{k}^2$, which Apéry used to prove the irrationality of $\zeta(3)$, form the diagonal of

$$\frac{1}{(1-x_1-x_2)(1-x_3-x_4)-x_1x_2x_3x_4},$$

so $(A(n) \bmod p^\alpha)_{n\geq 0}$ is p-automatic. For $\alpha = 1$ this also follows from a result of Gessel, who proved that, if we write $n = n_l \cdots n_1 n_0$ in base p, then $A(n) \equiv \prod_{i=0}^{l} A(n_i) \mod p$. For $p^\alpha = 7$ we get the following particularly symmetric automaton which computes $A(n) \bmod 7$. The loops labeled $0, 6$ reflect that $A(0) \equiv A(6) \equiv 1 \mod 7$.



If $S(n)_{n\geq 0}$ is the diagonal of a rational power series, then $S(n)_{n\geq 0}$ is holonomic, meaning that it satisfies a linear recurrence with coefficients that are polynomials in n. Not every holonomic sequence is the diagonal of a rational power series; for example, $n!_{n\geq 0}$ grows too quickly. However, a conjecture of Christol [2] implies that every holonomic sequence of integers that grows at most exponentially is the diagonal of a rational function. If this conjecture is true, then essentially every sequence that arises in combinatorics is p-automatic when reduced modulo p^α.

Further Reading

[1] Jean-Paul Allouche and Jeffrey Shallit, *Automatic Sequences: Theory, Applications, Generalizations*, Cambridge University Press, Cambridge, 2003.

[2] Gilles Christol, Globally bounded solutions of differential equations, in *Analytic Number Theory (Tokyo, 1988)*, Lecture Notes in Mathematics, 1434, Springer-Berlin, 1990, pp. 45–64.

[3] James D. Currie, Pattern avoidance: themes and variations, *Theoretical Computer Science* **339** (2005), 7–18.

[4] Eric Rowland and Reem Yassawi, Automatic congruences for diagonals of rational functions, *Journal de Théorie des Nombres de Bordeaux*, to appear. `http://arxiv.org/abs/1310.8635` (2014).

Mathematics People

Behrend Awarded 2014 CRM-Fields-PIMS Prize

KAI BEHREND of the University of British Columbia has been awarded the CRM-Fields-PIMS prize for his work in algebraic geometry. According to the prize citation, "his contributions to the subject are noted both for their depth and scope. He has obtained fundamental results in the theory of algebraic stacks, Gromov–Witten theory and the study of Donaldson–Thomas invariants. In particular, his pioneering works on the construction of a 'virtual fundamental class' played a key role in laying the algebraic foundations of the Gromov–Witten theory. Later, he made a breakthrough in the study of the Donaldson–Thomas invariants by showing that, for certain spaces, the degree of the virtual fundamental class could be expressed as the topological Euler characteristic weighted by a natural constructible function, depending only on the intrinsic properties of the space. This function is now widely known as Behrend's function. It allowed the use of motivic methods to compute Donaldson–Thomas invariants and made it possible to obtain their categorified and motivic versions, which is currently among the hottest trends in the subject. In his earlier work, Professor Behrend obtained an important generalization of the Lefschetz trace formula for algebraic stacks, presently known as Behrend's trace formula. The ideas put forward by Kai Behrend have already proven to be immensely influential and will undoubtedly have a lasting impact on this area of mathematics."

The CRM-Fields-PIMS Prize recognizes exceptional achievement in the area of mathematical sciences. It is awarded by the Centre de Recherches Mathématiques (CRM), the Fields Institute, and the Pacific Institute for Mathematical Sciences (PIMS).

—From a CRM announcement

IEEE Control Systems Awards Given

The Institute of Electrical and Electronics Engineers awards the Control Systems Award annually. TAMER BASAR of the University of Illinois at Urbana-Champaign was awarded the 2014 award "for seminal contributions to dynamic games, stochastic and risk-sensitive control, control of networks, and hierarchical decision making". BRUCE FRANCIS of the University of Toronto received the 2015 award "for pioneering contributions to H-infinity, linear-multivariable, and digital control". The award recognizes an individual's "outstanding contributions to control systems engineering, science, or technology" and considers the seminal nature, depth, and breadth of contributions, as well as singular achievement and practical impact.

—From an IEEE announcement

Corwin Awarded Packard Fellowship

IVAN CORWIN of Columbia University has been awarded a Packard Fellowship by the David and Lucile Packard Foundation. Corwin's work attempts to unify algebraic structures within mathematics, to build bridges between these structures and domains of physics, and to discover universal phenomena within these domains. He has uncovered universal distributions (modern day parallels of the bell curve) in models of interface growth, traffic flow, mass transport, turbulence, and shock-fronts. The Packard Fellowships provide young scientists early in their careers with flexible funding and the freedom to take risks and explore new frontiers in their fields of study.

—From a Packard Foundation announcement

Leverhulme Prizes Awarded

Six mathematicians have been awarded Leverhulme Prizes by the Leverhulme Trust. ALEXANDROS BESKOS of University College London was recognized for his work in computational statistics and theory and applications of Monte Carlo methods. DANIEL KRAL of the University of Warwick was honored for his work in combinatorics. DAVID LOEFFLER of the University of Warwick and SARAH ZERBES of University College London were honored for their joint work in number theory, particularly construction of Euler system associated with convolution of modular forms. RICHARD SAMWORTH of the University

of Cambridge was recognized for his foundational and methodological contibutions to many areas of statistics. CORRINA ULCIGRAI of the University of Bristol was honored for her major contributions to ergodic theory and dynamical systems. The Leverhulme Trust awards Philip Leverhulme Prizes to outstanding researchers in a UK institution of higher education or research whose work has already attracted international recognition and whose future careers are exceptionally promising. Each year a number of disciplines are chosen for the awards; for 2014 the subject areas were mathematics and statistics. Prizes awarded are worth £100,000 (approximately US$156,000) and are to be used, over a two- to three-year period, to advance prizewinners' research.

—From a Leverhulme Trust announcement

Prizes of the Math Society of Japan

The Mathematical Society of Japan (MSJ) has awarded the following prizes for 2014.

The 2014 Spring Prize was awarded to YUKINOBU TODA of the University of Tokyo for his outstanding contributions to the study of derived categories of algebraic varieties. The Spring Prize is awarded to researchers under forty years of age who have obtained outstanding mathematical results.

The 2014 Autumn Prize was awarded to HIDEO KOZONO of Waseda University for his outstanding contributions to harmonic analytic research for stationary and nonstationary problems to the incompressible Navier-Stokes equation. The Autumn Prize is awarded without age restriction to people who have made exceptional contributions in their fields of research. The Spring Prize and the Autumn Prize are the most prestigious prizes awarded by the MSJ to its members.

The 2014 Algebra Prizes have been awarded to YUJI YOSHINO of Okayama University for the study of Cohen–Macaulay representation theory and to HIDEKAZU FURUSHO of Nagoya University for studies of Grothendieck–Teichmüller theory and multiple zeta values.

The 2014 Analysis Prizes were awarded to KAZUHIRO ISHIGE of Tohoku University for research on the qualitative analysis of solutions of linear and nonlinear heat equations; to HIROFUMI OSADA of Kyushu University for research on stochastic dynamics and geometry for infinite particle systems; and to HIDETAKA HAMADA of Kyushu Sangyo University for studies on the Loewner differential equation in several complex variables and holomorphic mappings on homogeneous unit balls.

The 2014 Geometry Prize was awarded to MASATAKE KURANISHI for a series of outstanding original works far beyond geometry, ranging from the Cartan–Kuranishi theory and CR-geometry to the Kuranishi family.

The 2014 Takebe Katahiro Prizes have been awarded to TOSHIMICHI USUBA of Kobe University for research in set theory with emphasis on $P_{\kappa\lambda}$ combinatorics and its applications; to MASAKI TSUKAMOTO of Kyoto University for work on the mean dimension of infinite dimensional moduli spaces; and NEAL BEZ of Saitama University for the study of various inequalities appearing in harmonic analysis and partial differential equations. The prize is given to young researchers who have obtained outstanding results.

The 2014 Takebe Katahiro Prizes for Encouragement of Young Researchers have been awarded to the following: KEIJI TAGAMI of the Tokyo Institute of Technology for the study of positivities of links and Khovanov type link invariants; TATSUYUKI HIKITA of Kyoto University for a new approach to the combinatorics of diagonal coinvariants; SHUN OHKUBO of the University of Tokyo for the study of p-adic Galois representations of a local field with imperfect residue field; NORISUKE IOKU of Ehime University for harmonic analytical research for partial differential equations with logarithmic singularities; and MAKOTO NAKASHIMA of the University of Tsukuba for the study of branching random walks in random environment. The prize is intended for young mathematicians who are deemed to have begun promising careers in research by obtaining significant results.

The *Journal of the Mathematical Society of Japan* Outstanding Paper Prizes for 2014 have been awarded to DAISUKE FUJIWARA of Gakushuin University for "An integration by parts formula for Feynman path integrals", **65**, No. 4, 2013, pp. 1273–1318; and to HISASHI OKAMOTO of Kyoto University for "Blow-up problems in the strained vorticity dynamics and critical exponents", **65**, No. 4, 2013, pp. 1079–1099.

—From MSJ announcements

Prizes of the New Zealand Mathematical Society

The New Zealand Mathematical Society (NZMS) has announced several awards for 2014.

DAVID VERE-JONES of Victoria University of Wellington was awarded the 2014 Jones Medal in recognition of "his lifetime achievement in statistics, both for his revolutionary research on modeling earthquakes and his teaching of statistics and mathematics in New Zealand".

MARSTON CONDER of the University of Auckland was awarded the Hector Medal for his "outstanding contributions to mathematics both internationally and locally, particularly in the construction and analysis of discrete objects with maximum symmetry under given conditions". DIMITRI LEEMANS of the University of Auckland received the 2014 NZMS Research Award "for his striking contributions to algebraic combinatorics that combine techniques from algebra, graph theory, combinatorics and number theory for the exploration and classification of highly symmetric geometric structures".

DAVID SIMPSON of Massey University was honored with the 2014 NZMS Early Career Award "for his contributions to the analysis of the effects of randomness and uncertainties in nonsmooth dynamical systems".

TIMM TRESKATIS of the University of Canterbury was awarded the 2014 Aitken Prize for the best contributed

talk by a student at the NZMS Colloquium for his talk, "Accelerated gradient vs. primal-dual methods in nonsmooth optimisation".

ANDREA BABYLON of Massey University received the 2014 Australia and New Zealand Industrial and Applied Mathematics (ANZIAM) poster prize for the best poster by an early career researcher at the NZMS Colloquium for her poster, "Modelling leptospirosis in livestock". Four mathematicians were chosen as Fellows of the New Zealand Mathematical Society: ASTRID AN HUEF, University of Otago; GAVEN MARTIN, Massey University; GRAHAM WEIR, Industrial Research Ltd., and SIR VAUGHAN JONES, University of California Berkeley.

—From an NZMS announcement

AAAS Fellows Chosen

The following mathematical scientists have been elected fellows of the Section on Mathematics of the American Association for the Advancement of Science (AAAS): JAMES M. CROWLEY, Society for Industrial and Applied Mathematics (SIAM); CHARLES L. EPSTEIN, University of Pennsylvania; NATAŠA JONOSKA, University of South Florida; KIRK E. JORDAN, IBM Research Division; YURI TSCHINKEL, New York University; and HOWARD (HOWIE) WEISS, Georgia Institute of Technology.

—From an AAAS announcement

2014 Siemens Competition

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Peter Tian, third from left.

Several high school students whose work involves the mathematical sciences have won prizes in the 2014 Siemens Competition in Math, Science, and Technology.

PETER TIAN, a senior at the Wellington School in Columbus, Ohio, won the grand prize of a US$100,000 scholarship in the individual category for his project, "Extremal Functions of Forbidden Multidimensional Matrices", which makes significant advances in the theory of pattern avoidance for higher dimensional matrices and may assist in computing the shortest rectilinear path among obstacles in space.

This in turn has potential applications to motion planning in space or circuit design, including implications for other 3D areas dealing with space obstacles, such as drone programming for obstacle avoidance and self-driving cars. His work also extends a number of known results and advances areas of pure mathematics. Within mathematics, his project has a direct application to hypergraphs, and it also has potential applications to other areas of combinatorics and computational geometry. Tian was cofounder of his school's math club and is a Research Science Institute scholar and Mathematical Olympiad Summer Program attendee. He was mentored by Jesse Geneson of the Massachusetts Institute of Technology.

JOSEPH ZURIER, a senior at Classical High School, Providence, Rhode Island, was awarded the second-place prize of a US$50,000 scholarship for his project, "Generalizations of the Joints Problem".

His research contributed to work on an open problem in

©The Siemens Foundation

Joseph Zurier

geometry. His project concerns the intersection of line and planes in three and four dimensions. He was also the top prizewinner in the Who Wants to Be a Mathematician competition in 2013. He was mentored by Ben Yang of the Massachusetts Institute of Technology. The team of JONATHAN CHAN, a senior at Bergen County Academies, Hackensack, New Jersey, and MICHAEL SEAMAN, homeschooled, of Short Hills, New Jersey, was awarded the US$40,000 team scholarship for their project, "On the Distribution of Discriminants over a Finite Field". They proved a theorem about the distribution of discriminants of monic polynomials in finite fields. They were mentored by Keith Conrad of the University of Connecticut. The team of SHAKTHI SHRIMA, homeschooled, of Austin, Texas; ADAM FORSYTH, a senior at Georgetown Day School, Washington, DC; and JACOB GUREV, a junior at Mira Loma High School, Sacramento, California, was awarded the US$20,000 team scholarship for their project, "Metacommutation of the Hurwitz Integers and the Projective Line over Fp". They characterized unique factorization in the Hurwitz integers by using methods taken from projective geometry. Their results have possible applications in cryptography and in quantum field theory. The team was mentored by Henry Cohn of the Massachusetts Institute of Technology.

—From a Siemens Competition announcement

Mathematics Opportunities

Call for Nominations for Prizes of the World Academy of Sciences

The World Academy of Sciences (TWAS) prizes are awarded to individual scientists in developing countries in recognition of outstanding contributions to knowledge in eight fields of science.

Eight awards are given each year in the fields of mathematics, medical sciences, biology, chemistry, physics, agricultural sciences, earth sciences, and engineering sciences. Each award consists of a prize of US$15,000 and a plaque. Candidates for the awards must be scientists who have been working and living in a developing country for at least ten years.

The deadline for nominations for the 2015 prizes is **February 28, 2015**. Nomination forms should be sent to: TWAS Prizes, International Centre for Theoretical Physics (ICTP) Campus, Strada Costiera 11, 1-34151 Trieste, Italy; phone: 39 040 2240 387; fax: 39 040 2240 7387; email: prizes@twas.org. Further information is available on the World Wide Web at twas.org/opportunity/twas-2015-prizes.

—From a TWAS announcement

Call for Nominations for Graham Wright Award

The Canadian Mathematical Society (CMS) is seeking nominations for the 2015 Graham Wright Award for Distinguished Service. This award recognizes individuals who have made sustained and significant contributions to the Canadian mathematical community and, in particular, to the Canadian Mathematical Society. Nominations should include a reasonably detailed rationale and be submitted by **March 31, 2015**, to gwaward@cms.math.ca. For more information see the website cms.math.ca/Prizes/dis-nom.

—From a CMS announcement

Project NExT 2015–2016

Project NExT (New Experiences in Teaching) is a professional development program for new and recent PhDs in the mathematical sciences (including pure and applied mathematics, statistics, operations research, and mathematics education). It addresses all aspects of an academic career: improving the teaching and learning of mathematics, engaging in research and scholarship, and participating in professional activities. It also provides the participants with a network of peers and mentors as they assume these responsibilities. For 2015 about eighty faculty members from colleges and universities throughout the country will be selected to participate in a Project NExT summer workshop and MathFest 2015, the Joint Mathematics Meetings in January 2016, and MathFest 2016. Faculty for whom the 2015–2016 academic year will be the first or second year of full-time teaching (post-PhD) at the college or university level are invited to apply to become Project NExT Fellows.

Applications are invited for the 2015–2016 fellowship year, the twenty-second year of Project NExT. The deadline for applications is **April 15, 2015**. For more information, see the Project NExT website, archives.math.utk.edu/projnext/, or contact David Kung, director, at projectnext@maa.org. Project NExT is a program of the Mathematical Association of America (MAA). It receives funding from the Mary P. Dolciani Halloran Foundation, the Educational Advancement Foundation, the American Mathematical Society, the American Statistical Association, the National Council of Teachers of Mathematics, the American Institute of Mathematics, the Association for Symbolic Logic, W. H. Freeman Publishers, and individuals and sections of the MAA.

—From a Mathematical Association of America announcement

For Your Information

Mathematics Awareness Month in 2015: Math Drives Careers

The Joint Policy Board for Mathematics (JPBM) has announced the theme for Mathematics Awareness Month in 2015: Math Drives Careers.

Innovation is an increasingly important factor in the growth of world economies. It is especially important in key economic sectors like manufacturing, materials, energy, biotechnology, healthcare, networks, and professional and business services. The advances in and applications of the mathematical sciences have become drivers of innovation as new systems and methodologies have become more complex. As mathematics drives innovation, it also drives careers.

Mathematics Awareness Month is sponsored each year by the JPBM (a collaborative effort of the American Mathematical Society, the American Statistical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics) to recognize the importance of mathematics through written materials and an accompanying poster that highlight mathematical developments and applications in a particular area.

—Joint Policy Board for Mathematics

Inside the AMS

Deaths of AMS Members

Richard F. Arenstorf, of Nashville, Tennessee, died on September 18, 2014. Born on November 7, 1929, he was a member of the Society for 57 years.

Serban A. Basarab, professor, Romanian Academy, Institute of Mathematics, Romania, died on July 14, 2014. Born on March 3, 1940, he was a member of the Society for 31 years.

Gilbert Baumslag, professor, City College (CUNY), died on October 20, 2014. Born on April 30, 1933, he was a member of the Society for 54 years.

Kathleen Baxter, of Seattle, Washington, died on November 5, 2012. Born on August 6, 1923, she was a member of the Society for 53 years.

David W. Bressler, of Davis, California, died on June 14, 2013. Born on September 7, 1923, he was a member of the Society for 57 years.

Tim D. Cochran, professor, Rice University, died on December 16, 2014. Born on April 7, 1955, he was a member of the Society for 35 years.

Richard M. Cohn, of New York, New York, died on June 17, 2014. Born in 1920, he was a member of the Society for 173 years.

Sergio Console, professor, University of Torino, died on November 4, 2013. Born on February 23, 1965, he was a member of the Society for 16 years.

Stephen H. Crandall, professor, Massachusetts Institute of Technology, died on October 29, 2013. Born on December 2, 1920, he was a member of the Society for 68 years.

Eugene Borisovich Dynkin, professor, Cornell University, died on November 14, 2014. Born on May 11, 1924, he was a member of the Society for 54 years.

Ralph Faudree, professor, University of Memphis, died on January 13, 2015. Born on August 23, 1939, he was a member of the Society for 52 years.

Peter J. A. Gaposchkin, of Berkeley, California, died on September 30, 2014. Born on April 5, 1940, he was a member of the Society for 51 years.

Samuel Gitler, professor, CINVESTAV del IPN, Mexico, died on September 9, 2014. Born on July 14, 1933, he was a member of the Society for 57 years.

David Gregory, of Ontario, Canada, died on July 12, 2013. Born on March 3, 1942, he was a member of the Society for 47 years.

Thomas W. Hungerford, of Chesterfield, Missouri, died on November 28, 2014. Born on March 21, 1936, he was a member of the Society for 52 years.

J. W. Jaworowski, professor, Indiana University, died on April 10, 2013. Born on March 2, 1928, he was a member of the Society for 52 years.

Donald W. Kahn, professor, University of Minnesota, died on January 16, 2015. Born on November 21, 1935, he was a member of the Society for 51 years.

Clifford A. Kottman, of Clinton, Virginia, died on March 26, 2014. Born on August 3, 1942, he was a member of the Society for 48 years.

Ray A. Kunze, of Laguna Woods, California, died on May 21, 2014. Born on March 7, 1928, he was a member of the Society for 58 years.

Sylvia Chin-Pi Lu, of San Jose, California, died on July 16, 2014. Born on March 11, 1928, she was a member of the Society for 52 years.

Margaret O. Marchand, of Minnetonka, Minnesota, died on June 4, 2014. Born on October 17, 1925, she was a member of the Society for 68 years.

John (Jim) Patterson Mayberry of Niagara Falls, New York, died on September 8, 2014. Born on July 17, 1929, he was a member of the Society for 63 years.

Robert F. McNaughton Jr., of Troy, New York, died on June 5, 2014. Born on March 13, 1924, he was a member of the Society for 45 years.

Jean Jacques Moreau, professor, University of Montpellier II, France, died on January 9, 2014. Born on July 31, 1923, he was a member of the Society for 52 years.

T. S. Bhanu Murthy, of Mumbai, India, died on July 31, 2014. Born on September 1, 1925, he was a member of the Society for 48 years.

Somashekhar A. Naimpally of Ontario, Canada, died on October 17, 2014. Born on August 31, 1931, he was a member of the Society for 51 years.

Edward Nelson, of Princeton, New Jersey, died on September 10, 2014. Born on May 4, 1932, he was a member of the Society for 61 years.

Mara D. Neusel, professor, Texas Tech University, died on September 5, 2014. Born on May 15, 1964, she was a member of the Society for 14 years.

Loren D. Olson, professor, University of Tromso, Norway, died on June 22, 2014. Born on December 9, 1942, he was a member of the Society for 48 years.

Mason Scott Osborne, professor, University of Washington, died on July 4, 2014. Born on September 2, 1946, he was a member of the Society for 43 years.

Harriet Ann Padberg, of Atherton, California, died on January 2, 2014. Born on November 13, 1922, she was a member of the Society for 61 years.

Bent E. Petersen, of Yachats, Oregon, died on April 3, 2014. Born on July 31, 1942, he was a member of the Society for 43 years.

Cary Baker Rader of Newark, Ohio, died on August 26, 2014. Born on January 20, 1941, he was a member of the Society for 23 years.

Irma M. Reiner, of Urbana, Illinois, died on April 4, 2014. Born on March 3, 1922, she was a member of the Society for 52 years.

Marcel K. Richter, of St. Paul, Minnesota, died on July 11, 2014. Born on June 26, 1932, he was a member of the Society for 52 years.

Helmut Rohrl, of La Jolla, California, died on January 30, 2014. Born on March 22, 1927, he was a member of the Society for 55 years.

Bernard D. Rudin, of Petaluma, California, died on December 21, 2013. Born on November 18, 1927, he was a member of the Society for 28 years.

Richard D. Schafer, of Lexington, Massachusetts, died on December 28, 2014. Born on February 25, 1918, he was a member of the Society for 73 years.

Hans Schneider, professor, University of Wisconsin, died on October 28, 2014. Born on January 24, 1927, he was a member of the Society for 58 years.

Augusta L. Schurrer, of Cedar Falls, Iowa, died on January 1, 2015. Born on October 11, 1925, she was a member of the Society for 69 years.

Peter H. Sellers, professor, Rockefeller University, died on November 15, 2014. Born on September 12, 1930, he was a member of the Society for 56 years.

Abdulalim A. Shabazz, of Ruston, Louisiana, died on June 25, 2014. Born on May 22, 1927, he was a member of the Society for 59 years.

Harold N. Shapiro, of Teaneck, New Jersey, died on December 12, 2013. Born on October 2, 1922, he was a member of the Society for 51 years.

Sankatha P. Singh, of Ontario, Canada, died on March 28, 2013. Born on January 27, 1937, he was a member of the Society for 48 years.

Mark R. Solomon, of Troy, Michigan, died on March 9, 2013. Born on August 23, 1945, he was a member of the Society for 5 years.

James D. Stewart, of Toronto, Canada, died on December 3, 2014. Born on March 29, 1941, he was a member of the Society for 49 years.

Russell A. Stokes, of Oxford, Mississippi, died on September 16, 2014. Born on May 1, 1922, he was a member of the Society for 55 years.

David A. Stone, of Highland, New Jersey, died on August 27, 2014. Born on January 27, 1945, he was a member of the Society for 36 years.

Dorothy Maharam Stone of Brookline, Massachusetts, died on September 27, 2014. Born on July 1, 1917, she was a member of the Society for 43 years.

Jan R. Strooker, professor, Utrecht University, died on August 16, 2014. Born on September 29, 1932, he was a member of the Society for 53 years.

Frank Sturm, professor, Auburn University, died on October 14, 2014. Born on October 8, 1983, he was a member of the Society for 7 years.

Peter A. Szego of San Jose, California, died on September 28, 2014. Born on July 18, 1925, he was a member of the Society for 46 years.

R. P. Tewarson of Stony Brook, New York, died on June 26, 2014. Born on November 17, 1930, he was a member of the Society for 52 years.

Garth H. M. Thomas, of Saskatoon, Canada, died on May 22, 2014. Born on August 4, 1926, he was a member of the Society for 64 years.

James R. Ward Jr., professor, University of Alabama at Birmingham, died on March 25, 2013. Born on August 2, 1944, he was a member of the Society for 38 years.

Oscar Wesler, of West Palm Beach, Florida, died on November 21, 2013. Born on July 13, 1921, he was a member of the Society for 52 years.

Neil L. White, professor, University of Florida, died on August 11, 2014. Born on January 25, 1945, he was a member of the Society for 48 years.

Ann Yasuhara, of Princeton, New Jersey, died on June 11, 2014. Born on March 8, 1932, she was a member of the Society for 51 years.

Paul J. Zwier of Grand Rapids, Michigan, died on August 21, 2014. Born on October 24, 1927, he was a member of the Society for 55 years.

AMERICAN MATHEMATICAL SOCIETY

Leroy P. Steele Prizes

Call for Nominations

The selection committee for these prizes requests nominations for consideration for the 2016 awards. Further information about the prizes can be found in the November 2013 *Notices*, pp. 1372–1377 (also available at http://www.ams.org/profession/prizes-awards/ams-prizes/steele-prize).

Three Leroy P. Steele Prizes are awarded each year in the following categories: (1) the Steele Prize for Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through PhD students; (2) the Steele Prize for Mathematical Exposition: for a book or substantial survey or expository-research paper; and (3) the Steele Prize for Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. In 2016 the prize for Seminal Contribution to Research will be awarded for a paper in applied mathematics.

Nomination with supporting information should be submitted to www.ams.org/profession/prizes-awards/nominations. Include a short description of the work that is the basis of the nomination, including complete bibliographic citations. A curriculum vitae should be included. Nominations for the Steele Prizes for Lifetime Achievement and for Mathematical Exposition will remain active and receive consideration for three consecutive years. Those who prefer to submit by regular mail may send nominations to the AMS Secretary, Carla Savage, 201 Charles Street, Providence, RI 02904. Those nominations will be forwarded by the secretary to the prize selection committee.

Deadline for nominations is March 31, 2015.

AMS
AMERICAN MATHEMATICAL SOCIETY
www.ams.org

2015 Class of the Fellows of the AMS

Sixty-three mathematical scientists from around the world have been named Fellows of the American Mathematical Society (AMS) for 2015, the program's second year.

The Fellows of the American Mathematical Society program recognizes members who have made outstanding contributions to the creation, exposition, advancement, communication, and utilization of mathematics. Among the goals of the program are to create an enlarged class of mathematicians recognized by their peers as distinguished for their contributions to the profession and to honor excellence.

The 2015 class of Fellows was honored at a dessert reception held during the Joint Mathematics Meetings in San Antonio, TX. Names of the individuals who are in this year's class, their institutions, and citations appear below.

The nomination period for Fellows is open each year from February 1 to March 31. For additional information about the Fellows program, as well as instructions for making nominations, visit the web page `www.ams.org/profession/ams-fellows`.

AMS Immediate Past President David Vogan addresses the Fellows at JMM 2015.

Prakash Belkale, University of North Carolina at Chapel Hill
For contributions to algebraic geometry and representation theory.

John J. Benedetto, University of Maryland
For contributions to theoretical and applied harmonic analysis as well as for editorial service, mentoring, and professional leadership.

Andreas R. Blass, University of Michigan
For contributions to logic, computer science, algebra, and geometry, as well as exposition and mentoring.

Jonathan Michael Borwein, University of Newcastle
For contributions to nonsmooth analysis and classical analysis as well as experimental mathematics and visualization of mathematics.

Maury Bramson, University of Minnesota-Twin Cities
For contributions to stochastic processes and their applications.

Martin Robert Bridson, University of Oxford
For contributions to geometric group theory as well as its exposition, and for service to the mathematical community.

Daniel Bump, Stanford University
For contributions to number theory, representation theory, combinatorics, and random matrix theory, as well as mathematical exposition.

Richard Canary, University of Michigan
For contributions to low-dimensional topology and hyperbolic geometry as well as for service and teaching in mathematics.

Alfonso Castro, Harvey Mudd College
For contributions to nonlinear analysis and elliptic partial differential equations as well as for service to individual departments and the larger community.

Xiuxiong Chen, Stony Brook University
For contributions to differential geometry, particularly the theory of extremal Kahler metrics.

Nikolai Chernov,* University of Alabama at Birmingham
For contributions to dynamical systems and statistical mechanics.

Henry Cohn, Microsoft Research
For contributions to discrete mathematics, including applications to computer science and physics.

Marc Culler, University of Illinois at Chicago
For contributions to geometry and topology of 3-manifolds, geometric group theory, and the development of software for mathematical research.

Michael W. Davis, Ohio State University, Columbus
For contributions to geometric group theory, transformation groups, and topology, particularly regarding aspherical manifolds.

Jesus A. DeLoera, University of California, Davis
For contributions to discrete geometry and combinatorial optimization as well as for service to the profession, including mentoring and diversity.

Paul Dupuis, Brown University
For contributions to the theoretical and numerical study of stochastic systems, as well as for editorial contributions and mentoring.

Alan Edelman, Massachusetts Institute of Technology
For contributions to random matrix theory, numerical linear algebra, high-performance algorithm, and applications.

Charles L. Epstein, University of Pennsylvania
For contributions to analysis, geometry, and applied mathematics including medical imaging, as well as for service to the profession.

James Allen Fill, Johns Hopkins University, Baltimore
For contributions to theory and applications of Markov chains and to probabilistic analysis of algorithms.

Donald A.S. Fraser, University of Toronto
For contributions to the theory and foundations of statistics, as well as for leadership and influence on the advancement of the statistical sciences.

Allan Greenleaf, University of Rochester
For contributions to inverse problems with applications to cloaking as well as for service to the AMS.

Continued on next page

Karsten Grove, University of Notre Dame
For contributions to Riemannian geometry.
Matthew J. Gursky, University of Notre Dame
For contributions to conformal geometry, nonlinear partial differential equations, and the geometry and topology of four-dimensional manifolds.
Robert M. Hardt, Rice University
For contributions to calculus of variations, partial differential equations, and analytic approaches to geometry and topology, as well as mentoring.
Aloysius G. Helminck, North Carolina State University
For contributions to the representations and applications of symmetric spaces as well as for leadership in developing programs that attract and retain mathematics students.
Mourad E. H. Ismail, King Saud University and the University of Central Florida
For contributions to classical analysis and special function theory, as well as for service to the community.
Carl G. Jockusch, University of Illinois, Urbana-Champaign
For contributions to logic, computability theory, and Turing structures.
Arthur J. Krener, University of California, Davis
For contributions to the geometric theory of nonlinear control and estimation.
Phil Kutzko, University of Iowa
For contributions to representations of p-adic groups and the local Langlands program, as well as for recruitment and mentoring of under-represented minority students.
David R. Larson, Texas A&M University
For contributions to operator theory and functional analysis as well as teaching of mathematics.
Irena Lasiecka, University of Memphis
For contributions to control theory of partial differential equations, mentorship, and service to professional societies.
Kristin E. Lauter, Microsoft Research
For contributions to arithmetic geometry and cryptography as well as service to the community.
Fanghua Lin, New York University, Courant Institute
For contributions to partial differential equations, geometric measure theory, calculus of variations, and applied analysis.
Hans Lindblad, Johns Hopkins University, Baltimore
For contributions to nonlinear partial differential equations and mathematical physics.
Mitchell Luskin, University of Minnesota, Twin Cities
For contributions to numerical analysis and applications to materials science.
Michael A. Mandell, Indiana University, Bloomington
For contributions to algebraic topology.
Donald A. Martin, University of California, Los Angeles
For contributions to mathematical logic, especially proofs and applications of determinacy, as well as service to the profession.
Gordana Matic, University of Georgia
For contributions to low-dimensional and contact topology.
Jonathan Christopher Mattingly, Duke University
For contributions to the analysis of stochastic systems.
Willard Miller Jr., University of Minnesota, Twin Cities
For contributions to applied mathematics, especially special function theory, and for service to the mathematical community.
Washington Mio, Florida State University
For contributions to topology as well as to the mathematics, statistics, and applications of shape analysis.


AMS Fellows Reception, JMM 2015.

Konstantin Mischaikow, Rutgers The State University of New Jersey New Brunswick
For contributions to dynamical systems as well as to applied and computational topology.
Irina Mitrea, Temple University
For contributions to partial differential equations and related fields as well as outreach to women and under-represented minorities at all educational levels.
Paul S. Muhly, University of Iowa
For contributions to operator theory as well as mentoring and service to the community.
Andrea R. Nahmod, University of Massachusetts, Amherst
For contributions to nonlinear Fourier analysis, harmonic analysis, and partial differential equations, as well as service to the mathematical community.
Gabriel Navarro, University of Valencia
For contributions to representation theory.
Alexander Olshanskii, Vanderbilt University
For contributions to combinatorial and geometric group theory.
Mihnea Popa, Northwestern University
For contributions to algebraic geometry.
Andrei Rapinchuk, University of Virginia
For contributions to the arithmetic theory of algebraic groups and geometry of locally symmetric spaces.
Charles Rezk, University of Illinois, Urbana-Champaign
For contributions to theoretical and computational aspects of algebraic topology and homotopy theory.
Igor Rivin, Temple University
For contributions to geometry and related fields, pure and applied.
Yongbin Ruan, University of Michigan
For contributions to geometry, theoretical physics, and their interconnections, particularly Gromov-Witten theory and quantum cohomology.
Wilhelm Schlag, University of Chicago
For contributions to harmonic analysis, mathematical physics, and nonlinear partial differential equations.
Rolf Schneider, Albert-Ludwigs-Universitat
For contributions to convex geometry.
Denis Serre, Ecole Normale Superieure de Lyon
For contributions to hyperbolic conservation laws and mathematical exposition.

AMS Associate Executive Director of Meetings and Professional Services, T. Christine Stevens with reception attendees Kenneth C. Millett, University of California, Santa Barbara, Robert D. Edwards, University of California, Los Angeles, and Janis Millett.

Brooke Shipley, University of Illinois at Chicago
For contributions to homotopy theory and homological algebra as well as for service to the mathematical community.
Cesar E. Silva, Williams College
For contributions to ergodic theory as well as exposition and mentoring undergraduates in research.
Christina Sormani, The City University of New York, Lehman College and The Graduate Center
For contributions to geometry, including the study of Ricci curvature, and for mentoring activities, especially for young mathematicians from underrepresented groups.
Michael Stillman, Cornell University
For contributions to implementation and algebra of symbolic computation.
Zhi-Qiang Wang, Utah State University
For contributions to nonlinear analysis and applications in nonlinear differential equations.
C. Eugene Wayne, Boston University
For contributions to dynamical systems and mathematical physics.
Charles A. Weibel, Rutgers The State University of New Jersey New Brunswick
For contributions to K-theory, motivic cohomology, and related fields.
Christopher Thomas Woodward, Rutgers The State University of New Jersey New Brunswick
For contributions to symplectic and algebraic geometry as well as for dedication to the teaching of mathematics.

* deceased

—See more at: `www.ams.org/profession/ams-fellows/new-fellows`

—Photos courtesy of Goen South.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The **editor** is the person to whom to send articles and letters for consideration. Articles include features, memorials, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The **production editor** is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar".

Permissions requests should be sent to: reprint-permission@ams.org.

Contact the **editor** at: notices@math.wustl.edu or by fax at 314-935-6839.

Contact the **production editor** at: notices@ams.org or by fax at 401-331-3842. Postal addresses for both may be found in the masthead.

Upcoming Deadlines

February 28, 2015: Nominations for the World Academy of Sciences (TWAS) Prizes. See "Mathematics Opportunities" in this issue.

February 28, 2015: Applications for the Third Heidelberg Laureate Forum. See www.heidelberg-laureate-forum.org.

March 2, 2015: Applications for EDGE for Women 2015 Summer Program. See the website edgeforwomen.org/.

March 15, 2015: Nominations for PIMS Education Prize. See www.pims.math.ca/pims-glance/prizes-awards.

March 31, 2015: Nominations for the Graham Wright Award for Distinguished Service of the Canadian Mathematical Society. See "Mathematics Opportunities" in this issue.

March 31, 2015: Applications for AMS-Simons Travel Grants program. See the website www.ams.org/programs/travel-grants/AMS-SimonsTG or contact Steven Ferrucci, email: ams-simons@ams.org , telephone: 800-321-4267, ext. 4113.

March 31, 2015: Applications for IPAM graduate summer school on Games and Contracts for Cyber-Physical Security. See the website www.ipam.ucla.edu.

April 15, 2015: Applications for Project NExT fellowships. See "Mathematics Opportunities" in this issue.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

AMS Bylaws—*November 2013, p. 1358*

AMS Email Addresses—*February 2015, p. 179*

AMS Governance 2014—*June/July 2014, p. 650*

AMS Officers and Committee Members—*October 2012, p. 1290*

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National Science Board—*March 2015, p. 290*

NRC Board on Mathematical Sciences and Their Applications—*March 2015, p. 290*

NSF Mathematical and Physical Sciences Advisory Committee—*February 2014, p. 202*

Program Officers for Federal Funding Agencies—*October 2013, p. 1188 (DoD, DoE); December 2014, p. 1369 (NSF Mathematics Education)*

Program Officers for NSF Division of Mathematical Sciences—*November 2014, p. 1264*

April 15, 2015: Nominations for 2015 Ostrowski Prize. See www.ostrowski.ch/index_e.php?ifile=preis.

April 15, 2015: Applications for fall 2015 semester of Math in Moscow. See www.mccme.ru/mathinmoscow, or contact: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: mim@mccme.ru. Information and application forms for the AMS scholarships are available on the AMS website at www.ams.org/programs/travel-grants/mimoscow, or contact: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence RI 02904-2294; email student-serv@ams.org.

May 1, 2015: Applications for May review for National Academies Research Associateship programs. See sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

May 1, 2015: Applications for AWM Travel Grants and Mathematics Education Research Travel Grants. See https://sites.google.com/site/awmmath/programs/travel-grants; telephone: 703-934-0163; or email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

August 1, 2015: Applications for August review for National Academies Research Associateship programs. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

October 1, 2015: Applications for AWM Travel Grants and Mathematics Education Research Travel Grants. See https://sites.google.com/site/awmmath/programs/travel-grants; telephone: 703-934-0163; or email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

November 1, 2015: Applications for November review for National Academies Research Associateship programs. See sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

Board on Mathematical Sciences and Their Applications, National Research Council

The Board on Mathematical Sciences and Their Applications (BMSA) was established in November 1984 to lead activities in the mathematical sciences at the National Research Council (NRC). The mission of BMSA is to support and promote the quality and health of the mathematical sciences and their benefits to the nation. Following are the current BMSA members.

Douglas N. Arnold, University of Minnesota

John B. Bell, Lawrence Berkeley National Laboratory

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Donald Saari, Chair, University of California Irvine

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Katepalli Sreenivasan, New York University

Elizabeth A. Thompson, University of Washington

The postal address for BMSA is: Board on Mathematical Sciences and Their Applications, National Academy of Sciences, Room K974, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2421; fax: 202-334-2422; email: bms@nas.edu; website: sites.nationalacademies.org/DEPS/BMSA/DEPS_047709 .

Book List

The Book List highlights recent books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. Suggestions for books to include on the list may be sent to notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

Alan M. Turing: Centenary Edition, by Sara Turing. Cambridge University Press, April 2012. ISBN-13: 978-11070-205-80. (Reviewed September 2014.)

Alan Turing: The Enigma, The Centenary Edition, by Andrew Hodges. Princeton University Press, May 2012. ISBN-13: 978-06911-556-47. (Reviewed September 2014.)

Alan Turing: His Work and Impact, edited by S. Barry Cooper and J. van Leeuwen. Elsevier, May 2013. ISBN-13: 978-01238-698-07. (Reviewed September 2014.)

Alan Turing's Electronic Brain: The Struggle to Build the ACE, the World's Fastest Computer, by B. Jack Copeland et al. Oxford University Press, May 2012. ISBN-13: 978-0-19-960915-4. (Reviewed September 2014.)

André-Louis Cholesky: Mathematician, Topographer and Army Officer, by Claude Brezinski and Dominique Tournès. Birkhäuser, August 2014. ISBN: 978-33190-813-42.

Arnold: Swimming Against the Tide, edited by Boris A. Khesin and

Serge L. Tabachnikov. AMS, September 2014. ISBN-13: 978-14704-169-97.

Baroque Science, by Ofer Gal and Raz Chen-Morris. University of Chicago Press, March 2013. ISBN-13: 978-02262-129-82.

Beating the Odds: The Life and Times of E. A. Milne, by Meg Weston. Imperial College Press, June 2013. ISBN-13: 978-18481-690-74.

Beautiful Geometry, by Eli Maor and Eugen Jost. Princeton University Press, January 2014. ISBN-13: 978-06911-509-94.

Beyond Banneker: Black Mathematicians and the Paths to Excellence, by Erica N. Walker. State University of New York Press, June 2014. ISBN-13: 978-14384-521-59.

Combinatorics: Ancient and Modern, by Robin Wilson and John J. Watkins. Oxford University Press, August 2013. ISBN-13: 978-01996-565-92.

A Curious History of Mathematics: The Big Ideas from Early Number Concepts to Chaos Theory, by Joel Levy. Andre Deutsch, February 2014. ISBN-13: 978-02330-038-56.

Doing Data Science: Straight Talk from the Frontline, by Rachel Schutt and Cathy O'Neil. O'Reilly Media, November 2013. ISBN: 978-1-449-35865-5. (Reviewed October 2014.)

Enlightening Symbols: A Short History of Mathematical Notation and Its Hidden Powers, by Joseph Mazur. Princeton University Press, March 2014. ISBN-13: 978-06911-546-33. (Reviewed in this issue.)

Experiencing Mathematics: What Do We Do, When We Do Mathematics?, by Reuben Hersh. AMS, February 2014. ISBN-13: 978-08218-942-00.

Fifty Visions of Mathematics, edited by Sam Parc. Oxford University Press, July 2014. ISBN-13: 978-01987-018-11.

The Formula: How Algorithms Solve All Our Problems—And Create More, by Luke Dormehl. Perigee Trade, November 2014. ISBN-13:978-03991-705-39.

Four Lives: A Celebration of Raymond Smullyan, edited by Jason Rosenhouse. Dover Publications, February 2014. ISBN-13: 978-04864-906-70.

Fractals: A Very Short Introduction, by Kenneth Falconer. Oxford University Press, December 2013. ISBN-13: 978-01996-759-82.

From Mathematics in Logic to Logic in Mathematics: Boole and Frege, by Aliou Tall. Docent Press, July 2014, ISBN-13: 978-0-9887449-7-4.

The Grapes of Math: How Life Reflects Numbers and Numbers Reflect Life, by Alex Bellos. Simon and Schuster, June 2014. ISBN: 978-14516-400-90.

Henri Poincaré: A Scientific Biography, by Jeremy Gray. Princeton University Press, November 2012. ISBN- 13: 978-06911-527-14. (Reviewed April 2014.)

A History in Sum: 150 Years of Mathematics at Harvard (1825–1975), by Steve Nadis and Shing-Tung Yau. Harvard University Press, October 2013. ISBN-13: 978-06747-250-03. (Reviewed June/July 2014.)

The Improbability Principle: Why Coincidences, Miracles, and Rare Events Happen Every Day, by David J. Hand. Scientific American/Farrar, Straus and Giroux, February 2014. ISBN-13: 978-03741-753-44. (Reviewed December 2014.)

Infinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World, by Amir Alexander. Scientific American/Farrar, Straus and Giroux, April 2014. ISBN-13: 978-03741-768-15.

James Clerk Maxwell: Perspective on his Life and Works, edited by Raymond Flood, Mark McCartney, and Andrew Whitaker. Oxford University Press, March 2014. ISBN-13: 978-01996-643-75.

Jane Austen, Game Theorist, by Michael Suk-Young Chwe. Princeton University Press, April 2013. ISBN-13: 978-06911-557-60.

L. E. J. Brouwer—Topologist, Intuitionist, Philosopher: How Mathematics Is Rooted in Life, by Dirk van Dalen. Springer (2013 edition), December 2012. ISBN-13: 978-14471-461-55. (Reviewed June/July 2014.)

Levels of Infinity: Selected Writings on Mathematics and Philosophy, by Hermann Weyl (edited and with an introduction by Peter Pesic). Dover Publications, January 2013. ISBN: 978-04864-890-32.

The Logic of Infinity, by Barnaby Sheppard. Cambridge University Press, May 2014. ISBN-13: 978-11076-786-68.

Love and Math: The Heart of Hidden Reality, by Edward Frenkel. Basic Books, October 2013. ISBN-13: 978-04650-507-41. (Reviewed October 2014.)

Math Bytes: Google Bombs, Chocolate-Covered Pi, and Other Cool Bits in Computing, by Tim Chartier. Princeton University Press, April 2014. ISBN-13: 978-06911-606-03.

Mathematical Expeditions: Exploring Word Problems Across the Ages, by Frank J. Swetz. Johns Hopkins University Press, June 2012. ISBN: 978-14214-043-87.

Mathematical Understanding of Nature: Essays on Amazing Physical-Phenomena and Their Understanding by Mathematicians, by V. I. Arnold. AMS, September 2014. ISBN-13: 978-14704-170-17.

The Mathematician's Shiva, by Stuart Rojstaczer. Penguin Books, September 2014. ISBN-13: 978-014312-631-7.

Mathematics and the Making of Modern Ireland: Trinity College Dublin from Cromwell to the Celtic Tiger, by David Attis. Docent Press, October 2014, ISBN-13:978-0-9887449-8-1.

Mathematics and the Real World: The Remarkable Role of Evolution in the Making of Mathematics, by Zvi Artstein. Prometheus Books, September 2014. ISBN-13: 978-16161-409-15.

The Mathematics Devotional: Celebrating the Wisdom and Beauty of Mathematics, by Clifford Pickover. Sterling, November 2014. ISBN-13: 978-14549-132-21.

Mathematics of the Transcendental, by Alain Badiou (translated by A. J. Bartlett and Alex Ling). Bloomsbury Academic, March 2014. ISBN-13: 978-14411-892-40.

Math in Minutes: 200 Key Concepts Explained in an Instant, by Paul Glendinning. Quercus, September 2013. ISBN-13: 978-16236-500-87.

Math in 100 Key Breakthroughs, by Richard Elwes. Quercus, December 2013. ISBN-13: 978-16236-505-44.

A Mind For Numbers: How to Excel at Math and Science (Even If You Flunked Algebra), by Barbara Oakley. Tarcher, July 2014. ISBN-13:*978-03991-652-45.*

The New York Times Book of Mathematics: More Than 100 Years of Writing by the Numbers, edited by Gina Kolata. Sterling, June 2013.

ISBN-13: 978-14027-932-26. (Reviewed May 2014.)

Numbers Are Forever, by Liz Strachan. Constable, March 2014. ISBN-13: 978-14721-110-43.

On Leibniz: Expanded Edition, by Nicholas Rescher. University of Pittsburgh Press, June 2013. ISBN-13: 978-08229-621-82.

Origins of Mathematical Words: A Comprehensive Dictionary of Latin, Greek, and Arabic Roots, by Anthony Lo Bello. Johns Hopkins University Press, November 2013. ISBN-13: 978-14214-109-82.

Parables, Parabolas and Catastrophes: Conversations on Mathematics, Science and Philosophy, by René Thom. Translated by Roy Lisker and edited by S. Peter Tsatsanis. Thombooks Press, November 2014 (distributed only by `amazon.ca` or `amazon.com`). ISBN-13:978-09939-269-07.

The Perfect Theory: A Century of Geniuses and the Battle over General Relativity, by Pedro G. Ferreira. Houghton Mifflin Harcourt, February 2014. ISBN-13: 978-05475-548-91.

Philosophy of Mathematics in the Twentieth Century, by Charles Parsons. Harvard University Press, March 2014. ISBN-13: 978-06747-280-66.

Pearls from a Lost City: The Lvov School of Mathematics, by Roman Duda (translated by Daniel Davies). AMS, July 2014. ISBN-13: 978-14704-107-66.

Probably Approximately Correct: Nature's Algorithms for Learning and Prospering in a Complex World, by Leslie Valiant. Basic Books, June 2013. ISBN-13: 978-04650-327-16. (Reviewed November 2014.)

Professor Stewart's Casebook of Mathematical Mysteries, by Ian Stewart. Basic Books, October 2014. ISBN-13: 978-04650-549-78.

Quantum Computing since Democritus, by Scott Aaronson. Cambridge University Press, March 2013. ISBN-13: 978-05211-995-68. (Reviewed November 2014.)

Ramanujan's Place in the World of Mathematics: Essays Providing a Comparative Study, by Krishnaswami Alladi. Springer, 2013. ISBN: 978-81322-076-65.

The Simpsons and Their Mathematical Secrets, by Simon Singh. Bloomsbury, October 2013. ISBN-13: 978-14088-353-02. (Reviewed January 2015.)

Struck by Genius: How a Brain Injury Made Me a Mathematical Marvel, by Jason Padgett and Maureen Ann Seaberg. Houghton Mifflin Harcourt, April 2014. ISBN-13: 978-05440-456-06.

Synthetic Philosophy of Contemporary Mathematics, by Fernando Zalamea. Urbanomic/Sequence Press, January 2013. ISBN: 978-09567-750-16.

The Tower of Hanoi: Myths and Maths, by Andreas M. Hinz, Sandi Klavzar, Uros Milutinovic, and Ciril Petr. Birkhäuser, January 2013. ISBN: 978-303-48023-69.

Turing: Pioneer of the Information Age, by Jack Copeland. Oxford University Press, January 2013. ISBN-13: 978-01996-397-93. (Reviewed September 2014.)

Turing's Cathedral: The Origins of the Digital Universe, by George Dyson. Pantheon/Vintage, December 2012. ISBN-13: 978-14000-759-97. (Reviewed August 2014.)

Undiluted Hocus-Pocus: The Autobiography of Martin Gardner. Princeton University Press, September 2013. ISBN-13: 978-06911-599-11. (Reviewed March 2014.)

The War of Guns and Mathematics: Mathematical Practices and Communities in France and Its Western Allies Around World War I, by David Aubin and Catherine Goldstein. AMS, October 2014. ISBN-13: 978-14704-146-96.

Why Is There Philosophy of Mathematics At All?, by Ian Hacking. Cambridge University Press, April 2014. ISBN-13: 978-11070-501-74. (Reviewed in this issue.)

Zombies and Calculus, by Colin Adams. Princeton University Press, September 2014. ISBN-13: 978-06911-619-07.

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at www.ams.org/cgi-bin/mathcal-submit.pl. The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at www.ams.org/mathcal/.

March 2015

* 2–3 **Workshop on spherical and hyperbolic geometry**, Institut de Recherche Mathématique Avancée, University of Strasbourg, France.
Organizers: A. Papadopoulos and D. Slutskiy (Strasbourg).
Invited speakers: N. A.Campo (Basel), V. Alberge (Strasbourg), N. Bergeron (Orsay), S. Dumitrescu (Nice), B. Ettaoui (Mulhouse), F. Fillastre (Cergy), E. Frenkel (Strasbourg), A. Papadopoulos (Strasbourg), D. Slutskiy (Strasbourg) and S. Yamada (Tokyo).
Language: The talks will be in English and they are intended for a general audience. Graduate students and young mathematicians are welcome.
Information: www-irma.u-strasbg.fr/article1481.html.

2–5 **Flow(ers) & Friends in Frankfurt (a workshop on Geometric Analysis)**, Goethe Universitaet Frankfurt, Frankfurt am Main, Germany. (Oct. 2014, p. 1106)

2–6 **Forty-Sixth Southeastern International Conference on Combinatorics, Graph Theory and Computing**, Florida Atlantic University, Boca Raton, Florida. (Feb. 2015. p 189)

2–13 **The interrelation between mathematical physics, number theory and non-commutative geometry**, Erwin Schrodinger International Institute for Mathematical Physics, Vienna, Austria. (Feb. 2015. p 189)

* 5–7 **40th University of Arkansas Spring Lecture Series**, University of Arkansas, Fayetteville, Arkansas.
Description: This year's spring lecture series conference is on representation stability. This conference will feature a series of five lectures by Benson Farb (University of Chicago), entitled: Cohomology, Polynomials, and Representations: an Eternal Golden Braid. We will have additional lectures by the following invited speakers: Tara Brendle (University of Glasgow), Allen Hatcher (Cornell University), Eriko Hironaka (Florida State University), Alexander Kupers (Stanford University), Dan Margalit (Georgia Institute of Technology), Jeremy Miller (Stanford University), Rohit Nagpal (University of Wisconsin), Andrew Putman (Rice University), Vic Reiner (University of Minnesota) and Melanie Wood (University of Wisconsin).
Information: math.uark.edu/3723.php.

* 6 **Columbia-Princeton Probability Day 2015**, Princeton University, Princeton, New Jersey.
Main Speakers: Davar Khoshnevisan (Utah), Fraydoun Rezakhanlou (Berkeley), Prasad Tetali (Georgia Tech), Balint Virag (Toronto). Junior Speakers: Alex Drewitz (Columbia), Leonid Petrov (Virginia).
Registration: Is free and will be open until February 26, 2015.
Information: The registration form and information can be found on the Probability Day website at orfe.princeton.edu/conferences/cp15/.

* 7 **3rd Annual Midwest Women in Mathematics Symposium**, Dominican University, River Forest, Illinois.
Description: We invite you to join us in strengthening the network of female mathematicians in the Midwest, encouraging collaborations and mentoring relationships at the Third Annual Midwest Women in Math Symposium, to be held at Dominican University on March 7, 2015. Based on the successful WIMS held in Southern California, the University of Illinois-Chicago hosted the first such event in the Midwest followed by a second meeting at the University of Notre Dame.
Plenary Talk: By Laura de Marco, Northwestern University.
Special Sessions: Algebra, algebraic and geometric topology, dynamical systems and ergodicity, mathematical logic, mathematical modeling, partial differential equations.
Information: www.dom.edu/wims.

9–13 **Hot Topics: Kadison-Singer, Interlacing Polynomials, and Beyond**, Mathematical Sciences Research Institute, Berkeley, California. (Jan. 2015, p. 74)

9–June 12 (NEW DATE) **Broad Perspectives and New Directions in Financial Mathematics**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Aug. 2014, p. 796)

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: www.ams.org/.

10–13 **Financial Mathematics Tutorials**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Dec. 2014, p. 1374)

* 13–15 **Meeting of the History and Pedagogy of Mathematics, Americas Section**, American University, Washington, DC.
Description: We seek a variety of talks on relations between the history and pedagogy of mathematics. Talks will be about 30 minutes long. Two special features of this meeting should be noted. American University has an extensive collection of old and rare mathematics texts, which we hope to visit on Friday afternoon. Furthermore, Washington's world renowned Phillips Collection, will be hosting a special exhibit, "Man Ray – Human Equations: A Journey from Mathematics to Shakespeare". This exhibit contains mathematical models from the Institut Henri Poincaré in Paris, alongside photographs of these models by Man Ray from the 1930s, and his paintings inspired by the models in the 1940s. We hope to tour this exhibit on Saturday afternoon. Abstracts are due by February 21, 2015. Abstracts and registrations may be submitted via the webpage.
Information: `www.hpm-americas.org`.

* 14–15 **Conference on Probability Theory and Combinatorial Optimization**, The Fuqua School of Business, Duke University, Durham, North Carolina.
Description: Specific areas of coverage include random structures and algorithms, random graphs, probabilistic combinatorial optimization, and concentration inequalities. The conference will provide an opportunity to honor J. Michael Steele's contributions to probability theory and combinatorial optimization on the occasion of his 65th birthday.
Registration: Is free.
Information: `sites.duke.edu/steele2015/`.

14–18 (UPDATED) **SIAM Conference on Computational Science and Engineering (CSE15)**, The Calvin L. Rampton Salt Palace Convention Center, Salt Lake City, Utah. (Jun/Jul 2014, p. 667)

15–20 **ALCOMA 15 - Algebraic Combinatorics and Applications**, Kloster Banz, Germany. (Dec. 2014, p. 1374)

16–20 **6th International Conference on High Performance Scientific Computing**, Hanoi, Vietnam. (Sept. 2014, p. 983)

16–20 **LMS Invited Lectures 2015: Cluster Algebras and Integrable Systems**, Durham University, Durham, United Kingdom. (Dec. 2014, p. 1375)

16–20 **Small Clusters, Polymer Vesicles and Unusual Minima**, Brown University, Providence, Rhode Island. (Mar. 2014, p. 317)

18–20 **Critical Issues in Mathematics Education 2015: Developmental Mathematics: For whom? Toward what ends?**, Mathematical Sciences Research Institute, Berkeley, California. (Oct. 2014, p. 1107)

18–20 **IAENG International Conference on Scientific Computing 2015**, Royal Garden Hotel, Hong Kong. (Aug. 2014, p. 796)

23–27 **AIM Workshop: Dynamical Algebraic Combinatorics**, American Institute of Mathematics, Palo Alto, California. (Aug. 2014, p. 796)

23–27 **MBI Workshop on Targeting Cancer Cell Proliferation and Metabolism Networks**, Mathematical Biosciences Institute, The Ohio State University, Jennings Hall 3rd Floor, 1735 Neil Ave., Columbus, Ohio. (Jun/Jul 2014, p. 667)

23–27 **Systemic Risk and Financial Networks**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Dec. 2014, p. 1375)

27 **Philosophy of Information and Information Processing**, Pembroke College, Oxford, United Kingdom. (Jun/Jul 2014, p. 667)

27–29 **Conference on Complex Analysis and Geometry on the Occasion of Sidney Webster's 70th birthday**, University of Wisconsin-Madison, Madison, Wisconsin. (Jan. 2015, p. 75)

30–31 **3rd IMA International Conference on Flood Risk**, Swansea University, Wales, United Kingdom. (Jun/Jul 2014, p. 667)

30–April 30 **Sets and Computations**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Oct. 2014, p. 1107)

April 2015

* 1–4 **Seminar on Stochastic Processes 2015**, University of Delaware, Newark, Delaware.
Plenary speakers: are Michel Ledoux (Kai Lai Chung Lecturer), Maria Gordina, Haya Kaspi, Lionel Levine, and Brian Rider. The main conference will be held on April 2–4, 2015. On April 1, there will be a special set of tutorial lectures given by Ivan Corwin aimed at new researchers. Participants are encouraged to present a poster and give a very short talk (complete a request on the registration form), or to present an open problem.
Local organizers: Nayantara Bhatnagar, Mokshay Madiman, Petr Plechac and Douglas Rizzolo.
Support deadline: Travel support for junior researchers is likely to be available. The request must be sent by January 28, 2015 (see website for details). Hotels listed on SSP Web page offer special rates until January 29, 2015.
Information: `www.mathsci.udel.edu/events/conferences/ssp/Pages/default.aspx`.

2–5 **The Second International Conference on Mathematics and Statistics, AUS-ICMS'15**, American University of Sharjah, Sharjah, United Arab Emirates. (Feb. 2015, p. 189)

7–10 **International Conference "Probability, Reliability and Stochastic Optimization" (PRESTO-2015)**, Taras Shevchenko National University of Kyiv, Kyiv, Ukraine. (Jan. 2015, p. 75)

9–11 **Latina/os in the Mathematical Sciences Conference**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Dec. 2014, p. 1375)

13–17 **Dynamics on Moduli Spaces**, Mathematical Sciences Research Institute, Berkeley, California. (Jun/Jul 2014, p. 667)

13–17 **Limit Shapes**, Brown University, Providence, Rhode Island. (Mar. 2014, p. 317)

13–17 **LMS-CMI Research School on Statistical Properties of Dynamical Systems**, Loughborough University, Loughborough, UK. (Feb. 2015, p. 190)

13–17 **MBI Workshop on Stem Cells, Development, and Cancer**, Mathematical Biosciences Institute, The Ohio State University, Jennings Hall 3rd Floor, 1735 Neil Ave., Columbus, Ohio. (Jun/Jul 2014, p. 667)

13–17 **The Mathematics of High Frequency Financial Markets: Limit Order Books, Frictions, Optimal Execution and Program Trading**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, CA. (Jan. 2015, p. 75)

14 **1st Cyber-Physical System Security Workshop (CPSS 2015)**, Singapore. (Jan. 2015, p. 75)

15–17 **BioDynamics 2015**, John McIntyre Conference Centre, Edinburgh, United Kingdom. (Feb. 2015, p. 190)

18 **Overview of Current Research and Applications in Financial Mathematics**, Farmingdale State College, Farmingdale, NY 11735. (Jan. 2015, p. 75)

18–19 **Underrepresented Students in Topology and Algebra Research Symposium (USTARS)**, Florida Gulf Coast University, Fort Myers, Florida. (Feb. 2015, p. 190)

19–22 **15th International Conference on Numerical Combustion**, Palais des Papes, Avignon, France. (Dec. 2014, p. 1375)

19–25 **Spring School on Variational Analysis, Paseky**, Paseky nad Jizerou, Czech Republic. (Nov. 2014, p. 1275)

24–25 **Finger Lakes Seminar**, University of Rochester, Rochester, New York. (Dec. 2014, p. 1375)

27–28 **ICMM 2015 International Conference on Mathematics and Mechanics**, Paris, France. (Feb. 2015, p. 190)

27–30 **The Cape Verde International Days on Mathematics**, Mindelo, Cape Verde. (Jan. 2015, p. 75)

May 2015

3–7 **Mal'tsev Meeting**, Sobolev Institute of Mathematics (SB RAS), Novosibirsk, Russia. (Nov. 2014, p. 1275)

* 4–8 **Advanced Numerical Methods in the Mathematical Sciences**, Institute for Scientific Computation, Texas A&M University, College Station, Texas.
Description: This workshop aims to bring together researchers working on different aspects of finite element discretization techniques and their applications to foster interaction among researchers from both industry and academia. Taking place at the university's main campus, this workshop will host a poster session and offer a number of invited presentations given by a mix of senior and junior researchers. We particularly encourage workshop participants to present posters. Additionally, preceding the workshop on May 3, 2015, will be a day of tutorials on various topics concerning finite element discretizations. Thanks to assistance from the National Science Foundation, we will offer young researchers and doctoral students financial support on a competitive basis.
Information: Those interested in this workshop should visit: `isc.tamu.edu/events/NumMethWkspTAMU2015/` or may contact `NumMethWkspTAMU2015@gmail.com` for additional information or assistance with registration, accommodations, etc.

* 4–8 **AIM Workshop: Integrated analysis for agricultural management strategies**, American Institute of Mathematics, San Jose, California.
Description: This workshop, sponsored by AIM and the NSF, will be devoted to the development of mathematical models to aid agricultural entities and water policy boards in the management of water resources.
Information: `aimath.org/workshops/upcoming/agmanagement`.

4–8 **Commodity Markets and their Financialization**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, CA. (Dec. 2014, p. 1375)

4–15 **Workshop on Stochastic Processes in Random Media**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Oct. 2014, p. 1107)

5–8 **The 11st Information Security Practice and Experience Conference (ISPEC 2015)**, Beijing, China. (Jan. 2015, p. 75)

6–10 **Arithmetic and Algebraic Differentiation: Witt Vectors, Number Theory and Differential Algebra**, University of California, Berkeley, California. (Dec. 2014, p. 1375)

6–10 **Representation Theory Workshop**, Uppsala University, Uppsala, Sweden. (Jun/Jul 2014, p. 668)

11–15 **Advances in Homogeneous Dynamics**, Mathematical Sciences Research Institute, Berkeley, California. (Jun/Jul 2014, p. 668)

13–16 **13th Viennese Workshop on Optimal Control and Dynamic Games**, Vienna University of Technology, Vienna, Austria. (Nov. 2014, p. 1275)

14–16 **49th Spring Topology and Dynamics Conference**, Bowling Green State University, Bowling Green, Ohio, USA. (Jan. 2015, p. 76)

14–16 **The 9th International Conference on Differential Equations and Dynamical Systems**, Dallas Campus (1910 Pacific Place, Dallas, TX 75201) and hosted by the Mathematics Department of Texas A&M University-Commerce, Dallas, TX. (Feb. 2015, p. 190)

14–17 **Modern Aspects of Complex Geometry: A Conference in Honor of Taft Professor David Minda**, University of Cincinnati Cincinnati, OH. (Jan. 2015, p. 76)

15–17 **Seymour Sherman Lecture and Conference: Probability and Statistical Physics**, Indiana University, Bloomington, Indiana. (Feb. 2015, p. 190)

15–19 **The 28th International Conference of The Jangjeon Mathematical Society**, Sherwood Club Kemer Hotel, Antalya, Turkey. (Jan. 2015, p. 76)

17–21 **SIAM Conference on Applications of Dynamical Systems (DS15)**, Snowbird Ski and Summer Resort, Snowbird, Utah. (May 2014, p. 556)

17–23 **Sixteenth International Conference on Functional Equations and Inequalities (16th ICFEI)**, Mathematical Research and Conference Center of the Institute of Mathematics of the Polish Academy of Sciences, Bedlewo, Poland. (Jan. 2015, p. 76)

18–22 **AIM Workshop: Carleson theorems and multilinear operators**, American Institute of Mathematics, Palo Alto, California. (Aug. 2014, p. 796)

18–22 **Forensic Analysis of Financial Data**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Dec. 2014, p. 1376)

18–22 **International Conference on Differential & Difference Equations and Applications 2015**, Military Academy, Amadora, Portugal. (Sept. 2014, p. 985)

18–23 **Recent Advances in Kähler Geometry**, Vanderbilt University, Nashville, Tennessee. (Feb. 2015, p. 190)

18–29 **Workshop on New Directions in Stein's Method**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Oct. 2014, p. 1107)

19–21 **The 2015 Midwest Combinatorics Conference**, University of Minnesota, Minneapolis, MN USA. (Jan. 2015, p. 76)

22–24 **Lehigh University Geometry and Topology Conference, Emphasizing Algebraic Topology**, Lehigh University, Bethlehem, Pennsylvania. (Feb. 2015, p. 190)

* 25–29 **International Conference on Numerical Partial Differential Equations and Their Applications**, Wuhan University, Wuhan, China.
Description: In the last few years we have observed many new and exciting directions, developments and applications in the area of numerical partial differential equations. This international conference aims to provide a forum for leading computational and applied mathematicians from around the world to present and exchange their latest research achievements on numerical PDEs and their applications. The conference encourages interactive research activities among speakers and participants, promotes collaborative research between China and the rest of the world, and provides an opportunity for young mathematicians to learn the current state of the art in the field and present their recent research results as well.
Information: `npdea.whu.edu.cn/`.

25–30 **Existence of Solutions for a Quasilinear Elliptic Equation with Nonlocal Boundary Conditions on Time Scales**, Prépartoiry School of Science and savings commertielles sciences and managements, Tlemcen, Algeria. (Feb. 2015, p. 191)

26–29 **Conference on Risk Analysis ICRA 6/RISK 2015**, Barcelona, Spain. (Feb. 2015, p. 191)

26–29 **8th Chaotic Modeling and Simulation International Conference**, Henri Poincaré Institute (IHP), Paris, France. (Jan. 2015, p. 76)

26–29 **Lebanese International Conference on Mathematics and Applications(LICMA.15)**, Lebanese University, Faculty of Sciences, Haddath, Beirut, Lebanon. (Feb. 2015, p. 191)

27–29 **6th International Conference on Modeling, Simulation and Applied Optimization**, Yildiz Technical University, Istanbul,Turkey. (Jan. 2015, p. 76)

27–30 **Seventh International Conference on Dynamic Systems and Applications & Fifth International Conference on Neural, Parallel, and Scientific Computations**, Department of Mathematics, Morehouse College, Atlanta, Georgia. (Aug. 2014, p. 797)

28–30 **Geometry, Arithmetic and Physics: Around Motives**, University of Strasbourg, Strasbourg, France.

28–31 **3rd International Conference on "Applied Mathematics & Approximation Theory-AMAT 2015"**, Ankara, Turkey. (May 2014, p. 557)

31–June 6 **Spring School on Analysis 2015: Function Spaces and Lineability IX**, Paseky nad Jizerou, Krkonose Mountains, Czech Republic. (Nov. 2014, p. 1275)

June 2015

1–5 **AIM Workshop: Stochastic methods for Non-Equilibrium Dynamical Systems**, American Institute of Mathematics, Palo Alto, California. (Sept. 2014, p. 985)

1–5 **Asymptotic Problems: Elliptic and Parabolic Issues**, Vilnius, Lithuania. (Feb. 2015, p. 191)

1–5 **Integrability in Mechanics and Geometry: Theory and Computations**, Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island. (Nov. 2014, p. 1275)

1–5 **MAMERN VI-2015: 6th International Conference on Approximation Methods and Numerical Modeling in Environment and Natural Resources**, University of Pau, Pau, France. (Jan. 2015, p. 76)

1–19 **Dynamics of Multi-Level Systems**, Max Planck Institute for the Physics of Complex Systems, Dresden, Germany. (Feb. 2015, p. 191)

1–July 31 **Networks in Biological Sciences**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Oct. 2014, p. 1107)

3–6 **International Conference on Recent Advances in Pure and Applied Mathematics (ICRAPAM 2015)**, Istanbul Commerce University, Sutluce, Istanbul, Turkey.

* 4–5 **French-Japanese Workshop on Teichmüller Spaces and Surface-Mapping Class Groups**, Institut de Recherche Mathématique Avancée, University of Strasbourg, France.
Organizers: A. Papadopoulos and V. Alberge (Strasbourg).
Invited speakers: N. A. Campo (Basel), J. Aramayona (Toulouse), N. Kawazumi (Tokyo), Y. Kuno (Tokyo), H. Miyachi (Osaka), K. Ohshika (Osaka), T. Sakasai (Tokyo), A. Sambarino (Orsay), J.-M. Schlenker (Luxembourg) and S. Yamada (Tokyo).
Language: The talks will be in English and they are intended for a general audience. Graduate students and young mathematicians are welcome.
Information: www-irma.u-strasbg.fr/article1487.html.

* 4–6 **International Conference on Recent Advances in Mathematical Biology, Analysis and Applications**, Department of Applied Mathematics, Aligarh Muslim University, Aligarh, India.
Description: The purpose of this event is to provide researchers, academicians and engineers with a platform that encourages the exchange of innovative ideas in mathematical analysis and its applications, and to provide an environment that encourages the formation of interdisciplinary collaborations. A focal theme will be the application of mathematics to the biological sciences.
Information: www.amu.ac.in/icmbaa.jsp.

5–10 **XVII-th International Conference on Geometry, Integrability and Quantization**, Sts. Constantine and Elena resort, Varna, Bulgaria. (Dec. 2014, p. 1377)

8–12 **AIM Workshop: Mathematical aspects of physics with non-self-adjoint operators**, American Institute of Mathematics, Palo Alto, California. (Aug. 2014, p. 797)*

8–12 **International Conference on Applied Analysis and Mathematical Modelling (ICAAMM2015)**, Yildiz Technical University, Davutpasa Campus, Istanbul, Turkey.
Description: Jointly organized by Yildiz Technical University, University Putra Malaysia and Malaysian Mathematical Sciences Soc. We would like to invite you to present a paper or a poster at the conference whose main theme is the topics presented below or general mathematics with applications. Please note that the conference language is English. Send us your abstracts through the online registration form for oral or poster presentations. The talks given by plenary speakers are to be 50 minutes long including a ten minute question and answer session while the talks from the other contributors are to be 20 minutes long including the question session. The aim of this conference is to bring together mathematicians working in the new trends of applications of math in Historical city, Istanbul.
Information: www.ntmsci.com/Conferences/ICAAMM2015.

11–15 **Tenth Panhellenic Logic Symposium**, University of Aegean, Samos, Greece. (Feb. 2015, p. 191)

14–18 **PDEs, Potential Theory and Function Spaces**, Linköping, Sweden. (Feb. 2015, p. 191)

14–19 **BIOMATH 2015: International Conference on Mathematical Methods and Models in Biosciences**, University Centre Bachinovo, South-West University, Blagoevgrad, Bulgaria. (Dec. 2014, p. 1377)

14–21 **Fifty-third International Symposium on Functional Equations (53rd ISFE)**, Hotel Pegaz, Krynica-Zdrój, Poland. (Dec. 2014, p. 1377)*

15–18 **Progress on Difference Equations 2015**, University of Beira Interior, Covilh, Portugal.
Description: This meeting, held under the auspicies of the International Society for Difference Equations, aims to be a forum where researchers can share their work and discuss the latest developments in the areas of difference equations, discrete dynamical systems and their applications. It is the nineth edition of PODE meetings and continues in the line of previous workshops held in Laufen (Germany) in 2007 and 2008, Bedlewo (Poland) in 2009, Xanthi (Greece) in 2010, Dublin (Ireland) in 2011, Richmond (USA) in 2012, Bialystok (Poland) in 2013, and Ismir (Turkey) in 2014.
Information: www.pode2015.ubi.pt/.

15–18 **Summer School, Mathematics in Savoie, MIS 2015 on "Evolution Equations: Long Time Behavior and Control"**, Departement Mathematics, Laboratoire de Mathématiques, Université de Savoie, Campus scientifique, Chambéry, France. (Jan. 2015, p. 76)

15–19 **Connections in Discrete Mathematics**, Simon Fraser University, Vancouver, Canada. (Jun/Jul 2014, p. 668)

15–19 **MEGA 2015: Effective Methods in Algebraic Geometry**, University of Trento, Povo (Trento), Italy. (Oct. 2014, p. 1107)

15–19 **Nordfjordeid Summer School 2015 "Lie Groups and Pseudogroups Actions: From Classical to Differential Invariants"**, Sophus Lie Conference Center, Norfjordeid, Norway. (Jan. 2015, p. 76)

15–26 **Geometric and Computational Spectral Theory–Séminaire de mathématiques supérieures**, Centre de recherches mathématiques, Montréal, Québec, Canada. (Feb. 2015, p. 191)

15–26 **Summer Graduate School — Geometric Group Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Nov. 2014, p. 1275)

17–19 **Espalia**, "Sapienza" Universita` di Roma, Rome, Italy. (Feb. 2015, p. 191)

* 18–19 **2nd IMA Conference on Mathematics in Finance**, University of Manchester, Manchester, United Kingdom.
Description: The purpose of the conference is to bring together academics and practitioners interested in mathematical modelling in finance. We invite researchers whose research work contains a strong mathematical element to attend. We have included a broad range of topics with the aim to expose participants to models, ideas and techniques that they may not be aware of, and to foster future collaborations. In recent years many of the underpinning

assumptions of classical Mathematical Finance have been under attack, and now efficient markets can no longer be assumed by default. The result is that many more complex real world interactions need to be taken into account, meaning that the techniques and models from other areas such as Real Options and Operations Research will be important for the future of Mathematics in Finance.
Information: ima.org.uk/conferences/conferences_calendar/maths_in_finance_2015.html.

20–27 **Physics and Mathematics of Nonlinear Phenomena**, Gallipoli, close to Lecce, South of Italy. (Feb. 2015, p. 191)

22–24 **3rd International Conference on "Graph Modelling in Engineering"**, University of Bielsko-Biala, Bielsko-Biala, Poland. (Jun/Jul 2014, p. 668)

* 22–25 **2015 Computer Aided Modeling, Simulation and Analysis (CAMSA)**, Banff, Canada.
Description: This workshop will provide a forum for scientists and engineers alike to present their latest findings on the subject of computer aided modeling, simulation and analysis.
Topics: Specific topics of interest include, but are not limited to: (1) Computer aided modeling and analysis (novel algorithms or applications in biology, physics, chemistry, mathematics, and mechanics) (2) Geometric modeling and processing (3) Numerical and design optimization (4) Finite Element, Boundary Element, or Meshless Simulation (5) Computer Vision and Image Processing.
Paper Submission Website: ess.iccsa.org.
Deadlines: Deadline for Full Paper Submission: February 1, 2015. Author Paper Review Acceptance or Revision Notification to Author: March 6, 2015. Submission of Final Paper: May 6, 2015.
Information: www.iccsa.org/workshops.

* 22–26 **Conference "Singular Landscapes", Celebrating Bernard Teissier's 70th birthday**, Centre Paul-Langevin, Aussois, France.
Description: This international conference is aimed at exploring various aspects of the geometry of analytic singular spaces. We shall especially emphasize the delicate interplay between commutative algebra and topology, and try to present new points of view on these problems that arose recently (valuations, non-archimedean geometry). This will be the opportunity to celebrate the accomplishments of B. Teissier in which all these aspects were always delicately interwoven. We shall also dedicate several talks to interactions between singularity theory and neuroscience.
Preliminary list of speakers: Comte, Georges; Cutkosky, Steven Dale; Dimca, Alexandru; Draisma, Jan; Gaffney, Terence; Huh, June; Khovanskii, Askold; Kurdyka, Krzysztof; Mclean, Mark; Mustata, Mircea; Nicaise, Johannes; Pe Pereira, Maria; Petitot, Jean; Temkin, Mikael; Ulrich, Bernd; Yin, Yimu; Yu, Tony Yue.
Contact: teissier2015@listes.math.cnrs.fr Registration is open till March 1, 2015.
Information: indico.math.cnrs.fr/conferenceDisplay.py?confId=202.

22–26 **International conference "Dynamical Systems and Their Applications"**, Institute of Mathematics of National Academy of Sciences of Ukraine, Kiyv, Ukraine. (Sept. 2014, p. 985)

25 **7th National Dyscalculia & MLD Conference**, The Cumberland Hotel, London. (Jan. 2015, p. 77)

* 25–27 **Thirty-Second Annual Workshop in Geometric Topology**, Texas Christian University, Fort Worth, Texas.
Description: Please mark your calendars for the upcoming Thirty-Second Annual Workshop in Geometric Topology.
Principal speaker: Wolfgang Lück. In addition, participants will have the opportunity to give 20-25 minute talks on their own work.
Funding: We anticipate funding that will allow us to cover a portion of the travel expenses for participants without other means of support. Graduate student, recent PhDs, and mathematicians in underrepresented groups working in geometric topology and geometric group theory are especially encouraged to apply for funding.
Information: The workshop web site and registration can be found at: faculty.tcu.edu/gfriedman/GTW2015/.

26–July 1 **The Eighth Congress of Romanian Mathematicians**, University A.I. Cuza, Iasi, Romania. (Dec. 2014, p. 1377)

28–July 4 **CNRS-PAN Mathematics Summer Institute in Krakow**, Krakow, Poland. (Jan. 2015, p. 77)

* 29–July 2 **9th Annual International Conference on Mathematics & Statistics: Education & Applications**, Athens, Greece.
Description: The Natural & Formal Sciences Research Division & Education Research Unit of the Athens Institute for Education and Research (ATINER), will hold their 9th Annual International Conference on Mathematics & Statistics: Education & Applications in Athens, Greece, from June 29-30 and July 1–2, 2015. The conference website is www.atiner.gr/edumatsta.htm. The conference is soliciting papers (in English only) from all areas of Mathematics & Statistics Education and other related fields. You may participate as panel organizer, presenter of one paper, chairperson of a session or observer. For programs of previous conferences and other information,
Fee structure: Information is available at www.atiner.gr/fees.htm.
Information: www.atiner.gr/edumatsta.htm.

29–July 3 **Geometry and Symmetry**, University of Pannonia, Veszprém, Hungary. (Jan. 2015, p. 77)

28–July 4 **The Fifth International Mathematical Conference on Quasigroups and Loops—LOOPS 2015**, Faculty of Computer Science and Engineering, UKIM University, Congress Center, Ohrid, R. Macedonia. (Jan. 2015, p. 77)

29–July 10 **Summer Graduate School — Mathematical Topics in Systems Biology**, Mathematical Sciences Research Institute, Berkeley, California. (Nov. 2014, p. 1276)

July 2015

1–3 **The 2015 International Conference of Applied and Engineering Mathematics**, Imperial College London, London, United Kingdom. (Feb. 2015, p. 191)

5–12 **24th International Conference on Nearrings, Nearfields and Related Topics**, Manipal Institute of Technology, Manipal University Manipal - 576 104, Karnataka, India. (Dec. 2014, p. 1377)

* 6–10 **29th Journées Arithmétiques**, University of Debrecen, Debrecen, Hungary.
Description: The "Journées Arithmétiques" is the most important European conference series in Number Theory. Although it originates in France already for many decades it is a real international conference, which is organized on a bi-annual basis. The French origin of this prestigious series of conference is emphasized by its name, and the fact that every second meeting is organized in France, and the intermediary conferences in other European countries.
Information: ja2015.math.unideb.hu/.

6–10 **Classical and Quantum Hyperbolic Geometry and Topology/Topologie et géométrie hyperbolique classique et Quantique**, Université Paris-Sud, Orsay, France. (Jun/Jul 2014, p. 668)

6–10 **Computational Geometric Topology in Arrangement Theory**, The Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, RI. (Feb. 2015, p. 192)

6–10 **Equadiff 2015**, Université Claude Bernard Lyon 1, Lyon, France. (Oct. 2014, p. 1107)

6–10 **GAP XIII Pohang "Derived Geometry"—Geometry and Physics**, IBS Center for Geometry and Physics, Pohang, South Korea. (Feb. 2015, p. 190)

6–10 **International Workshop on Operator Theory and Applications, IWOTA 2015**, Tbilisi, Georgia. (Jan. 2015, p. 77)

6–10 **Second International Conference New Trends in the Applications of Differential Equations in Sciences (NTADES2015)**, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria. (Feb. 2015, p. 192)

6–10 **10th IMACS Seminar on Monte Carlo Methods**, Johannes Kepler University Linz and Radon Institute for Computational and Applied Mathematics, Linz, Austria. (Oct. 2013, p. 1205)

8–10 **SIAM Conference on Control and Its Applications (CT15)**, Maison de la Mutualit, Paris, France. (Jun/Jul 2014, p. 668)

12–16 **VI Annual International Conference of the Georgian Mathematical Union**, Batumi, Georgia. (Jan. 2015, p. 77)

12–24 **Summer Graduate School — Gaps between Primes and Analytic Number Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Nov. 2014, p. 1276)

13–17 **Computational and Analytical Aspects of Image Reconstruction**, Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island. (Nov. 2014, p. 1276)

13–17 **12th International Conference on Finite Fields and Their Applications (Fq12)**, Skidmore College, Saratoga Springs, New York. (Feb. 2014, p. 214)

13–31 **AMS Summer Institute in Algebraic Geometry**, University of Utah, Salt Lake City, Utah. (Feb. 2015, p. 192)

13–December 18 **Coupling Geometric PDEs with Physics for Cell Morphology, Motility and Pattern Formation**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Mar. 2014, p. 318)

14–17 **International Conference on Nonlinear Operators, Differential Equations and Applications, ICNODEA-2015**, Babes-Bolyai University, Cluj-Napoca, Romania. (Aug. 2014, p. 797)

15–17 **International Conference on Applied Statistics 2015**, Pattaya, Thailand. (Feb. 2015, p. 192)

20–23 **ACA 2015: Applications of Computer Algebra**, Kalamata, Greece. (Jan. 2015, p. 77)

20–24 **9th International Symposium on Imprecise Probability: Theories and Applications**, Pescara, Italy. (Jan. 2015, p. 77)

20–24 **The 11th International Conference on Fixed Point Theory and its Applications**, Galatasaray University, Istanbul, Turkey. (Dec. 2013, p. 1497)

20–August 4 **XVIII Summer Diffiety School on Geometry of PDEs**, Piccolo Hotel Tanamalia, Lizzano in Belvedere (BO), Italy. (Feb. 2015, p. 192)

20–August 14 **Metric and Analytic Aspects of Moduli Spaces**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Mar. 2014, p. 318)

27–August 1 **XVIII International Congress on Mathematical Physics, ICMP 2015**, Pontificia Universidad Catolica de Chile, Santiago de Chile, RM, Chile. (Dec. 2014, p. 1378)

* 27–August 7 **Stochastic Analysis and Applications Mongolia 2015**, National University of Mongolia, Ulan Bator, Mongolia.
Invited mini-courses and special topic lecturers include: E. Baurdoux (London); L. Beck (Augsburg); J.Bertoin (Zurich); G. Borot (Bonn); M. Caballero (UNAM, Mexico); L. Doering (Mannheim); E. Eberlein (Freiburg); C. Goldschmidt (Oxford); A. Lambert (Paris); B. Oksendal (Oslo); C. Pardo (CIMAT, Mexico); L. Popovic (Montreal).
Additional talks: Will also be given by attendees.
Registration/Funding: There is no registration fee. Limited funding is available to (partially) cover travel expenses and/or accomodation costs. Preference is given to participants from Mongolia and developing countries. Please register (whether in need of funding or not) at our CIMPA page: `www.students.cimpa.info/login`. Please contact the organizers if you are interested in presenting at the meeting.
Information: `smcs.num.edu.mn/saam2015/`.

27–August 7 **Summer Graduate School — Incompressible Fluid Flows at High Reynolds Number**, Mathematical Sciences Research Institute, Berkeley, California. (Nov. 2014, p. 1276)

28–30 **Mathematics in Data Science- Exploring the Role of the Mathematical Sciences in an Evolving Discipline**, Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island. (Feb. 2015, p. 192)

August 2015

3–7 **AIM Workshop: First Passage Percolation and Related Models**, American Institute of Mathematics, Palo Alto, California. (Aug. 2014, p. 797)

3–7 **Differential and Combinatorial Aspects of Singularities**, TU Kaiserslautern, Germany. (Aug. 2014, p. 797)

3–7 **The 3rd Strathmore International Mathematics Conference Theme: Exploring Mathematics and its Applications**, Strathmore University, Nairobi, Kenya. (Oct. 2014, p. 1107)

* 3–8 **Logic Colloquium 2015**, Helsinki, Finland.
Description: The annual European Summer Meeting of the Association for Symbolic Logic, the Logic Colloquium 2015 (LC 2015), will be organized in Helsinki, Finland, 3-8 August 2015. Logic Colloquium 2015 is co-located with the 15th Conference of Logic, Methodology and Philosophy of Science, CLMPS 2015, and with the SLS Summer School in Logic.
Information: `www.helsinki.fi/lc2015/`.

4–6 **ICNHBAS [I] Current Trends in Mathematics & its Applications**, Hurghada, Egypt. (Feb. 2015, p. 193)

16–18 **Mathematics for Nonlinear Phenomena: Analysis and Computation - International Conference in Honor of Professor Yoshikazu Giga on his Sixtieth Birthday**, Sapporo Convention Center, Sapporo, Japan. (Jan. 2015, p. 77)

17–20 **The 8th International Conference on Lattice Path Combinatorics and Applications**, California State Polytechnic University, Pomona (Cal Poly Pomona), Pomona, California. (Feb. 2015, p. 193)

17–21 **20th International Summer School on Global Analysis and Applications "General Relativity: 100 Years After Hilbert"**, Stara Lesna, High Tatras, Slovakia.

* 18–21 **The 7th SEAMS-UGM 2015 International Conference on Mathematics and Its Applications**, Universitas Gadjah Mada, Yogyakarta, Indonesia.
Description: Department of Mathematics, Universitas Gadjah Mada (UGM), in cooperation with SEAMS (South East Asian Mathematical Society) organizes an International Conference on Mathematics and its Applications in Yogyakarta, Indonesia, once in every four years, as a commitment in mathematical activities to SEAMS. The first SEAMS-UGM Conference was held in 1991 and the last (sixth) conference was held in 2011. The scientific program will include invited lectures, contributed presentations and workshops.
Information: `seams2015.fmipa.ugm.ac.id/`.

19–December 18 **Mathematical, Foundational and Computational Aspects of the Higher Infinite**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Sept. 2014, p. 987)

24–27 **11th International Symposium on Geometric Function Theory and Applications**, Congress Centre of Ss. Cyril and Methodius University, Ohrid, Republic of Macedonia. (Feb. 2015, p. 193)

25–27 **7th International Conference on Research and Education in Mathematics (ICREM7)**, Kuala Lumpur, Malaysia. (Feb. 2015, p. 193)

27–29 **The 5th International Conference on Control and Optimization with Industrial Applications**, Baku, Azerbaijan. (Feb. 2015, p. 193)

* 31–September 3 **Finiteness Conditions in Topology and Algebra**, Queen's University, Belfast, United Kingdom.

Current list of speakers: Martin Bridson* (University of Oxford, UK); Dan Burghelea (Ohio State University, US); Ross Geoghegan (Binghamton University, US); Dessislava Kochloukova (University of Campinas, Brazil); Ian Leary (University of Southampton, UK); Alexandra Pettet* (University of British Columbia, Canada); Andrew Ranicki (University of Edinburgh, UK). (* to be confirmed)
Organizers: Thomas Huettemann and Alex Suciu.
Information: Further information will be posted in due course: www.northeastern.edu/suciu/Belfast2015/.

31–September 4 **Numerical Methods for Large-Scale Nonlinear Problems and Their Applications**, The Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, RI. (Feb. 2015, p. 193)

September 2015

1–August 31 **Call for Research Programmes 2015-2016**, Centre de Recerca Matematica, Bellaterra, Barcelona, Spain. (Sept. 2014, p. 987)

1–4 **IMA Conference on Numerical Methods for Simulation**, Mathematical Institute, University of Oxford, UK. (Aug. 2014, p. 797)

8–13 **International Conference on "Mathematical Analysis, Differential Equations and Their Applications" (MADEA 7)**, Baku, Azerbaijan.

8–December 11 **New Directions in Mathematical Approaches for Traffic Flow Management**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Dec. 2014, p. 1378)

9–11 **IMA Conference on Mathematics of Robotics**, St. Anne's College, Oxford, United Kingdom. (Sept. 2014, p. 987)

9–December 4 **ICERM Semester Program: Computational Aspects of the Langlands Program**, Brown University, Providence, Rhode Island. (Jun/Jul 2014, p. 669)

* 12–18 **International Conference "Harmonic Analysis and Approximations, VI"**, Tsaghkadzor, Armenia.
Description: The conference continues the series of international conferences on "Harmonic Analysis and Approximations" organized in Armenia. It is co-organized by the Institute of Mathematics of the Armenian National Academy of Sciences and Yerevan State University, and will be held at the Yerevan State University guesthouse, Tsaghkadzor, Armenia. The program of the conference will consist of invited 45 minutes plenary lectures and contributed 20 minutes talks.
The Programme Committee: Norair Arakelian (Armenia), Paul Gauthier (Canada), Boris Kashin Russia), Michael Lacey (USA), Wolfgang Luh (Germany), Alexander Olevskii (Israel), Alexandr Talalian (Armenia), Vladimir Temlyakov (USA), Przemyslaw Wojtaszczyk (Poland) The Organizing Commitee: Gegham Gevorkyan, Artur Sahakian, Aram Hakobyan, Michael Poghosyan The Tentative list of Speakers (accepted): Christoph Aistleitner (Austria), Sergei Bochkarev (Russia), Gegham Gevorkyan (Armenia), Ushangi Goginava (Georgia), Viktor Kolyada (Sweden), Sergei Konyagin (Russia), Alexander Olevskii (Israel), Konstantin Oskolkov (USA), Tino Ullrich (Germany), Przemyslaw Wojtaszczyk (Poland).
Information: mathconf.sci.am/haa2015/.

14–18 **The European Numerical Mathematics and Advanced Applications (ENUMATH) Conference**, Institute of Applied Mathematics, Middle East Technical University, Ankara, Turkey. (Dec. 2013, p. 1497)

14–18 **The Seventh Symposium on Nonlinear Analysis**, Faculty of Mathematics and Computer Sciences, Nicolaus Copernicus University, Toruń, Poland

16–21 **13th International Conference of The Mathematics Education for the Future Project: Mathematics Education in a Connected World**, Grand Hotel Baia Verde, Catania, Sicily, Italy. (Feb. 2015, p. 193)

17–19 **The 96th Encounter Between Mathematicians and Theoretical Physicists: Geometry and Biophysics**, University of Strasbourg, Strasbourg, France. (Dec. 2014, p. 1378)

21–26 **International Conference in Mathematics Education**, Catania, Sicily, Italy. (Aug. 2014, p. 797)

* 21–26 **Master-Class on Finsler Geometry and Applications to Low-Dimensional Geometry and Topology and Moduli Spaces**, University of Cagliari, Sardinia, Italy.
Description: The master-class on Finsler geometry and applications to low-dimensional geometry and topology and moduli spaces will take place at the University of Cagliari (Sardinia, Italy), on September 21-26, 2015. The focus of this Summer School will be on the following thematic areas: Finsler geometry, hyperbolic geometry, systolic geometry, Teichmüller theory.
Courses: The courses will be given by: Norbert A. Campo (Basel), Ivan Babenko (Montpellier), Ara Basmajian (CUNY), Ken'ichi Ohshika (Osaka), Hugo Parlier (Fribourg), Viktor Schroeder (Zurich), Sumio Yamada (Tokyo). Besides the courses, there will be a series of specialized lectures. The master-class is primarily intended for PhD students and young researchers.
Organizing Committee: R. Caddeo (Cagliari) and A. Papadopoulos (Strasbourg).
Registration: Is free of charge. PhD students and young mathematicians are particularly welcome. The arrival day is September 20, 2015 and the departure day is September 27, 2015.
Information: people.unica.it/renzoilariocaddeo/master-class/.

27–October 2 **Mathematical Foundations of Traffic**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, CA. (Jan. 2015, p. 78)

28–October 3 **Semester Workshop: Modular Forms and Curves of Low Genus: Computational Aspects**, Institute for Computational and Experimental Research in Mathematics (ICERM) at Brown University, Providence, Rhode Island.

October 2015

* 6–8 **Conference on Agricultural Statistics 2015**, Sarawak, Malaysia.
Description: The health and wealth of a nation and its potential to develop and grow,depends on its ability to feed its people. Accurate and timely statistics about the source and availability of basic agricultural supplies are essential. As nations develop and their economies grow, the need for immediate information will place increased emphasis on agricultural statistics. The rapidly growing world population will require increased productivity which will be enhanced by the use of statistical analysis resulting from the studies of agricultural sciences. Therefore, each nation should highly consider to provide a forum to foster a spirit of cooperation in the sharing of ideas and statistical methodology among the global nations to maintain continual improvement in the accuracy, timeliness and relevance of agricultural statistics.
Information: einspem.upm.edu.my/cas2015.

12–14 **SIAM Conference on Geometric and Physical Modeling (GDSPM15)**, Sheraton Salt Lake City Hotel, Salt Lake City, Utah. (Feb. 2015, p. 194)

12–16 **Workshop II: Traffic Estimation**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Dec. 2014, p. 1378)

* 18–24 **Workshop on Almost Hermitian and Contact Geometry**, International Mathematics Conference Center in Bedlewo near, Poznan, Poland.
Organizers: Thomas Friedrich (Berlin), Ilka Agricola (Marburg), Aleksy Tralle (Olsztyn).
Sponsors: Banach Center, Warsaw Center of Mathematics and Computer Science (WCNM).

Invited Speakers: Indranil Biswas, Gil Cavalcanti, Andrew Dancer, Marisa Fernandez, Hansjorg Geiges, Christina Toennesen-Friedman, Adriano Tomassini, Robert Wolak. The focus of the workshop is on the ineractions of almost hermitian geometry and metric aspects of contact geometry, as well as applicatons in mathematical physics.
Information: wmii.uwm.edu.pl/woahacg/index.php.

19–24 **Semester Workshop: Explicit Methods for Modularity of K3 Surfaces and Other Higher Weight Motives**, Institute for Computational and Experimental Research in Mathematics (ICERM) at Brown University, Providence, Rhode Island. (Feb. 2015, p. 194)

26–30 **Traffic Control**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, CA. (Dec. 2014, p. 1379)

November 2015

* 2–5 **International Conference on Coding and Cryptography**, USTHB, University of Algiers, Algiers, Algeria.
Description: The International Conference on Coding Theory and Cryptography will be organized by the Algebra and Number Theory laboratory of USTHB and will be held on the 2-5 November 2015 at USTHB. Mainly the topics of the conference are all aspects of theoretical and practical research in coding theory and cryptography. The conference is also open to all aspects of information theory. The purpose of this conference is to bring specialized researchers to present and discuss their research with a various wide of other specialists.
Information: www.latn.usthb.dz/spip.php?article35.

9–13 **Semester Workshop: Computational Aspects of L-functions**, Institute for Computational and Experimental Research in Mathematics (ICERM) at Brown University, Providence, Rhode Island. (Feb. 2015, p. 194)

16–20 **Decision Support for Traffic**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.

16–December 25 **Stochastic Methods in Game Theory**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Jan. 2015, p. 78)

December 2015

1–5 **BioInfoSummer 2014: Summer Symposium in Bioinformatics**, Monash University (Caufield Campus), Melbourne, Australia. (Jun/Jul 2014, p. 669)

* 16–20 **The 20th Asian Technology Conference in Mathematics (ATCM 2015)**, Leshan, China.
Description: The ATCM 2015 is an international conference to be held in Leshan China. To reach Leshan, you will land at Chengdu International Airport (CTU). Chengdu is known as a city of pandas. The ATCM 2015 will continue addressing technology-based issues in all Mathematical Sciences. Thanks to advanced technological tools such as computer algebra systems (CAS), interactive and dynamic geometry, and hand-held devices, the effectiveness of our teaching and learning, and the horizon of our research in mathematics and its applications continue to grow rapidly. The aim of this conference is to provide a forum for educators, researchers, teachers and experts in exchanging information regarding enhancing technology to enrich mathematics learning, teaching and research at all levels. English is the official language of the conference. ATCM averagely attracts 350 participants representing over 30 countries around the world. Be sure to submit your abstracts or full papers in time.
Information: atcm.mathandtech.org.

28–30 **Riemann Legacy Conference**, Sanya, Hainan, China.

31–January 4 **String Mathematics 2015**, Sanya, Hainan, China. (Dec. 2014, p. 1379)

January 2016

5–9 **Conference on General Relativity**, Sanya, Hainan, China. (Dec. 2014, p. 1379)

* 10–12 **ACM-SIAM Symposium on Discrete Algorithms (SODA16), being held with Analytic Algorithmics and Combinatorics (ANALCO16) and Algorithm Engineering and Experiments (ALENEX16)**, Crystal Gateway Marriott, Arlington, Virginia.
Description: Information on SODA, ALENEX and ANALCO will be available at www.siam.org/meetings/da16/ in June 2015.
Information: www.siam.org/meetings/da16/.

14–15 **Connections for Women: Differential Geometry**, Mathematical Sciences Research Institute, Berkeley, California. (Jun/Jul 2014, p. 669)

18–22 **Introductory Workshop: Modern Riemannian Geometry**, Mathematical Sciences Research Institute, Berkeley, California. (Jun/Jul 2014, p. 669)

* 26–28 **2nd International Conference on Mathematical Sciences and Statistics (ICMSS2016)**, Kuala Lumpur, Malaysia.
Description: The Department of Mathematics, Faculty of Science, Universiti Putra Malaysia(UPM) is proud to organise the 2nd International Conference on Mathematical Sciences and Statistics (ICMSS2016). This international conference aims to bring together academicians, researchers and scientists for knowledge sharing in Mathematics and Statistics areas. The ICMSS2016 serves as a good platform for the scientific community members to meet with each other and to exchange ideas. All accepted papers will be compiled and published in the American Institute of Physics proceedings (indexed by ISI Thomson). Selected papers will be published in a special issue of Malaysian Journal of Mathematical Sciences (indexed by SCOPUS), Springer Verlag (indexed by ISI Thomson) or Discovering Mathematics(Menemui Matematik) indexed by Zentralblatt MATH.
Information: math.upm.edu.my/icmss2016/.

February 2016

15–June 17 **Melt in the Mantle**, Isaac Newton Institute for Mathematical Sciences, Cambridge, United Kingdom. (Apr. 2014, p. 433)

April 2016

5–8 **SIAM Conference on Uncertainty Quantification (UQ16)**, SwissTech Convention Center, EPFL Campus, Lausanne, Switzerland.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

July 2016

* 25–29 **XXI Coloquio Latinoamericano de Álgebra**, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Buenos Aires, Argentina.
Description: The series "Coloquios Latinoamericanos de Algebra" started in 1981; until 1994 it was mainly a regional event that gathered mathematicians from Argentina and Chile and, occasionally, a few invited speakers from other countries. After a short interruption, these meetings were reinitiated in 2001, in Córdoba, Argentina, then continued in 2003 in Cocoyoc, México, in 2005 in Colonia, Uruguay, in 2007 in Medellín Colombia, in 2009 in São Pedro, Brasil, in 2012 in Pucón, Chile, and in 2014 in Lima, Peru.
Format: Following tradition, there will be plenary talks in the morning and thematic sessions in the afternoon. Wednesday afternoon will be free. Sessions: 1. Commutative Algebra and Algebraic Geometry. 2. Hopf Algebras. 3. Operator Algebras. 4. Rings and Algebras. 5. Algebraic Combinatorics. 6. Lie Groups and Representations. 7. Logic and Universal Algebra. 8. Homological Methods. 9. Representations of Algebras. 10. Group Theory.
Information: cms.dm.uba.ar/Members/gcorti/workgroup.2014-12-15.2886996475/index_html.

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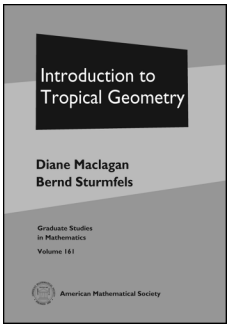
Algebra and Algebraic Geometry



Introduction to Tropical Geometry

Diane Maclagan, *University of Warwick, Coventry, United Kingdom*, and **Bernd Sturmfels**, *University of California, Berkeley, CA*

Tropical geometry is a combinatorial shadow of algebraic geometry, offering new polyhedral tools to compute invariants of algebraic varieties. It is based on tropical algebra, where the sum of two numbers is their minimum and the product is their sum. This turns polynomials into piecewise-linear functions, and their zero sets into polyhedral complexes. These tropical varieties retain a surprising amount of information about their classical counterparts.

Tropical geometry is a young subject that has undergone a rapid development since the beginning of the 21st century. While establishing itself as an area in its own right, deep connections have been made to many branches of pure and applied mathematics.

This book offers a self-contained introduction to tropical geometry, suitable as a course text for beginning graduate students. Proofs are provided for the main results, such as the Fundamental Theorem and the Structure Theorem. Numerous examples and explicit computations illustrate the main concepts. Each of the six chapters concludes with problems that will help the readers to practice their tropical skills, and to gain access to the research literature.

This wonderful book will appeal to students and researchers of all stripes: it begins at an undergraduate level and ends with deep connections to toric varieties, compactifications, and degenerations. In between, the authors provide the first complete proofs in book form of many fundamental results in the subject. The pages are sprinkled with illuminating examples, applications, and exercises, and the writing is lucid and meticulous throughout. It is that rare kind of book which will be used equally as an introductory text by students and as a reference for experts.

—Matt Baker, Georgia Institute of Technology

Tropical geometry is an exciting new field, which requires tools from various parts of mathematics and has connections with many areas. A short definition is given by Maclagan and Sturmfels: "Tropical geometry is a marriage between algebraic and polyhedral geometry". This wonderful book is a pleasant and rewarding journey through different landscapes, inviting the readers from a day at a beach to the hills of modern algebraic geometry. The authors present building blocks, examples and exercises as well as recent results in tropical geometry, with ingredients from algebra, combinatorics, symbolic computation, polyhedral geometry and algebraic geometry. The volume will appeal both to beginning graduate students willing to enter the field and to researchers, including experts.

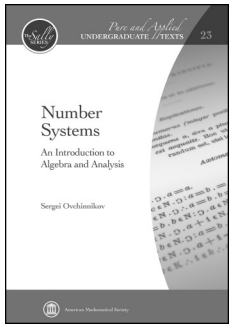
—Alicia Dickenstein, University of Buenos Aires, Argentina

Contents: Tropical islands; Building blocks; Tropical varieties; Tropical rain forest; Tropical garden; Toric connections; Bibliography; Index.

Graduate Studies in Mathematics, Volume 161

April 2015, approximately 364 pages, Hardcover, ISBN: 978-0-8218-5198-2, LC 2014036141, 2010 *Mathematics Subject Classification:* 14T05; 05B35, 12J25, 13P10, 14M25, 15A80, 52B20, **AMS members US$63.20**, List US$79, Order code GSM/161

Analysis



Number Systems

An Introduction to Algebra and Analysis

Sergei Ovchinnikov, *San Francisco State University, CA*

This book offers a rigorous and coherent introduction to the five basic number systems of mathematics, namely natural numbers, integers, rational numbers, real numbers, and complex numbers. It is a subject that many mathematicians believe should be learned by any student of mathematics, including future teachers.

The book starts with the development of Peano arithmetic in the first chapter, which includes mathematical induction and elements of recursion theory. It proceeds to an examination of integers that

also covers rings and ordered integral domains. The presentation of rational numbers includes material on ordered fields and convergence of sequences in these fields. Cauchy and Dedekind completeness properties of the field of real numbers are established, together with some properties of real continuous functions. An elementary proof of the Fundamental Theorem of Algebra is the highest point of the chapter on complex numbers. The great merit of the book lies in its extensive list of exercises following each chapter. These exercises are designed to assist the instructor and to enhance the learning experience of the students.

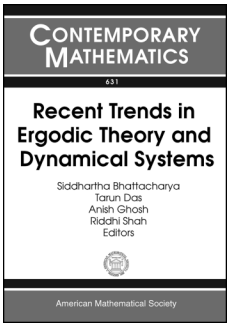
This item will also be of interest to those working in algebra and algebraic geometry.

Contents: Natural numbers; Integers; Rational numbers; Real numbers; Complex numbers; Sets, relations, functions; Bibliography; Index.

Pure and Applied Undergraduate Texts, Volume 23

April 2015, approximately 146 pages, Hardcover, ISBN: 978-1-4704-2018-5, LC 2014041492, 2010 *Mathematics Subject Classification:* 97F30, 97F40, 97F50, **AMS members US$48.80**, List US$61, Order code AMSTEXT/23

Differential Equations



Recent Trends in Ergodic Theory and Dynamical Systems

Siddhartha Bhattacharya, *Tata Institute of Fundamental Research, Mumbai, India*, **Tarun Das**, *University of New Delhi, India*, **Anish Ghosh**, *Tata Institute of Fundamental Research, Mumbai, India*, and **Riddhi Shah**, *Jawaharlal Nehru University, New Delhi, India*, Editors

This volume contains the proceedings of the International Conference on Recent Trends in Ergodic Theory and Dynamical Systems, in honor of S. G. Dani's 65th Birthday, held December 26–29, 2012, in Vadodara, India.

This volume covers many topics of ergodic theory, dynamical systems, number theory and probability measures on groups. Included are papers on Teichmüller dynamics, Diophantine approximation, iterated function systems, random walks and algebraic dynamical systems, as well as two surveys on the work of S. G. Dani.

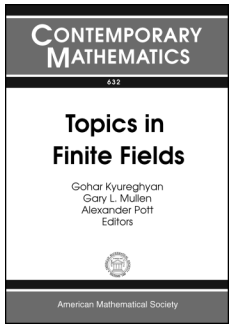
This item will also be of interest to those working in analysis.

Contents: M. S. Raghunathan, S. G. Dani as I have known him; **V. Bergelson** and **A. Leibman**, Cubic averages and large intersections; **C. R. E. Raja**, Liouville property on G-spaces; **V. Bernik** and **F. Götze**, A new connection between metric theory of Diophantine approximations and distribution of algebraic numbers; **J. S. Athreya**, **J. Chaika**, and **S. Lelièvre**, The gap distribution of slopes on the golden L; **E. Glasner** and **B. Weiss**, Uniformly recurrent subgroups; **D. Kleinbock** and **B. Weiss**, Values of binary quadratic forms at integer points and Schmidt games; **M. Denker** and **M. Yuri**, Conformal families of measures for general iterated function systems; **F. Ledrappier** and **R. Shah**, Dani's work on probability measures on groups; **Y. Guivarc'h** and **E. Le Page**, On the homogeneity at infinity of the stationary probability for an affine random walk; **D. W. Morris**, Dani's work on dynamical systems on homogeneous spaces; **e. H. el Abdalaoui** and **M. G. Nadkarni**, Calculus of generalized Riesz products; **A. Ghosh**, **A. Gorodnik**, and **A. Nevo**, Diophantine approximation exponents on homogeneous varieties; **H. Li**, **J. Peterson**, and **K. Schmidt**, Ergodicity of principal algebraic group actions; **V. Beresnevich** and **S. Velani**, A note on three problems in metric Diophantine approximation; **R. Miles**, **M. Staines**, and **T. Ward**, Dynamical invariants for group automorphisms.

Contemporary Mathematics, Volume 631

February 2015, 258 pages, Softcover, ISBN: 978-1-4704-0931-9, LC 2014021630, 2010 *Mathematics Subject Classification:* 22D40, 28D20, 37A15, 37A17, 37A20, 37A30, 37A35, 37B05, 37E35, 60B15, **AMS members US$84**, List US$105, Order code CONM/631

Number Theory



Topics in Finite Fields

Gohar Kyureghyan, *Otto-von-Guericke Universität, Magdeburg, Germany*, **Gary L. Mullen**, *Pennsylvania State University, University Park, PA*, and **Alexander Pott**, *Otto-von-Guericke Universität, Magdeburg, Germany*, Editors

This volume contains the proceedings of the 11th International Conference on Finite Fields and their Applications (Fq11), held July 22–26, 2013, in Magdeburg, Germany.

Finite Fields are fundamental structures in mathematics. They lead to interesting deep problems in number theory, play a major role in combinatorics and finite geometry, and have a vast amount of applications in computer science.

Papers in this volume cover these aspects of finite fields as well as applications in coding theory and cryptography.

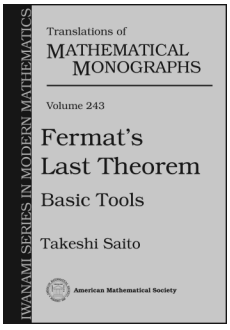
This item will also be of interest to those working in applications.

Contents: S. Abrahamyan and **M. Kyureghyan**, New recursive construction of normal polynomials over finite fields; **A. Aguglia** and **G. Korchmáros**, Collineation groups strongly irreducible on an oval in a projective plane of odd order; **I. N. Baoulina**, On the solvability of certain equations over finite fields; **M. Borello**, On automorphism groups of binary linear codes; **A. Canteaut** and **J. Roué**, Extended differential properties of cryptographic functions; **F. Caullery**, A divisibility criterion for exceptional APN functions; **A. Çeşmelioğlu** and **W. Meidl**, Non weakly regular bent polynomials from vectorial quadratic functions; **A. Cossidente**, **G. Korchmáros**, and **G. Marino**, Strongly regular graphs arising from Hermitian varieties; **J. Ducoat**, Generalized rank weights: A duality statement; **S. Fukasawa**, An upper bound for the number of Galois points for a plane curve; **R. Göttfert**, A generalization of the nonlinear combination generator; **Y. Hamahata**, Dedekind sums with a parameter in function fields; **M. Homma**, Numbers of points of hypersurfaces without lines over finite fields; **T. Honold**, **M. Kiermaier**, and **S. Kurz**, Optimal binary

subspace codes of length 6, constant dimension 3 and minimum subspace distance 7; **X.-D. Hou**, A survey of permutation binomials and trinomials over finite fields; **M.-D. Huang** and **A. K. Narayanan**, Computing class groups of function fields using stark units; **M.-D. Huang** and **A. K. Narayanan**, Finding primitive elements in finite fields of small characteristic; **R. Jurrius** and **R. Pellikaan**, The coset leader and list weight enumerator; **N. M. Katz**, Wieferich past and future; **M. Lavrauw** and **G. Van de Voorde**, Field reduction and linear sets in finite geometry; **S. Mesnager**, Bent functions from spreads; **A. Muratović-Ribić**, **A. Pott**, **D. Thomson**, and **Q. Wang**, On the characterization of a semi-multiplicative analogue of planar functions over finite fields; **M. Muzychuk**, A solution of an equivalence problem for semisimple cyclic codes; **J. Mykkeltveit** and **J. Szmidt**, On cross joining de Bruijn sequences; **D. Panario**, **A. Sakzad**, and **D. Thomson**, Ambiguity and deficiency of reversed Dickson permutations; **J. Wolfmann**, From near-bent to bent: A special case.

Contemporary Mathematics, Volume 632

February 2015, 371 pages, Softcover, ISBN: 978-0-8218-9860-4, LC 2014022869, 2010 *Mathematics Subject Classification:* 05Bxx, 11Txx, 11Gxx, 12Exx, 12Fxx, 12Yxx, 20Cxx, 51Exx, 94Axx, 94Bxx, **AMS members US$84**, List US$105, Order code CONM/632



Fermat's Last Theorem (2-Volume Set)

Takeshi Saito, *University of Tokyo, Japan*

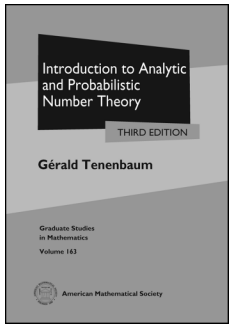
This 2-volume set (*Fermat's Last Theorem: Basic Tools* and *Fermat's Last Theorem: The Proof*) presents in full detail the proof of Fermat's Last Theorem given by Wiles and Taylor. With these two books, the reader will be able to see the whole picture of the proof to appreciate one of the deepest achievements in the history of mathematics.

Crucial arguments, including the so-called 3–5 trick, $R = T$ theorem, etc., are explained in depth. The proof relies on basic background materials in number theory and arithmetic geometry, such as elliptic curves, modular forms, Galois representations, deformation rings, modular curves over the integer rings, Galois cohomology, etc. The first four topics are crucial for the proof of Fermat's Last Theorem; they are also very important as tools in studying various other problems in modern algebraic number theory. In order to facilitate understanding the intricate proof, an outline of the whole argument is described in the first preliminary chapter of the first volume.

Contents: *Contents for Fermat's Last Theorem: Basic Tools:* Synopsis; Elliptic curves; Modular forms; Galois representations; The 3–5 trick; $R = T$; Commutative algebra; Deformation rings; Appendix A. Supplements to scheme theory; Bibliography; Symbol index; Subject index; *Contents for Fermat's Last Theorem: The Proof:* Modular curves over **Z**; Modular forms and Galois representations; Hecke modules; Selmer groups; Appendix B. Curves over discrete valuation rings; Appendix C. Finite commutative group scheme over $\mathbf{Z}_p$; Appendix D. Jacobian of a curve and its Néron model; Bibliography; Symbol index; Subject index.

Translations of Mathematical Monographs, Volume 243/245

December 2014, 434 pages, Softcover, ISBN: 978-1-4704-2216-5, LC 2013023932, 2010 *Mathematics Subject Classification:* 11D41; 11G05, 11F11, 11F80, 11G18, **AMS members US$68**, List US$85, Order code MMONO/243/245



Introduction to Analytic and Probabilistic Number Theory

Third Edition

Gérald Tenenbaum, *Institut Élie Cartan, Vandoeuvre-lès Nancy, France*

This book provides a self contained, thorough introduction to the analytic and probabilistic methods of number theory. The prerequisites being reduced to classical contents of undergraduate courses, it offers to students and young researchers a systematic and consistent account on the subject. It is also a convenient tool for professional mathematicians, who may use it for basic references concerning many fundamental topics.

Deliberately placing the methods before the results, the book will be of use beyond the particular material addressed directly. Each chapter is complemented with bibliographic notes, useful for descriptions of alternative viewpoints, and detailed exercises, often leading to research problems.

This third edition of a text that has become classical offers a renewed and considerably enhanced content, being expanded by more than 50 percent. Important new developments are included, along with original points of view on many essential branches of arithmetic and an accurate perspective on up-to-date bibliography.

The author has made important contributions to number theory and his mastery of the material is reflected in the exposition, which is lucid, elegant, and accurate.

—***Mathematical Reviews***

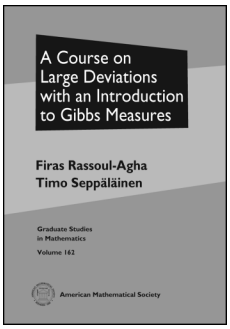
This item will also be of interest to those working in mathematical physics.

Contents: *Elementary methods:* Some tools from real analysis; Prime numbers; Arithmetic functions; Average orders; Sieve methods; Extremal orders; The method of van der Corput; Diophantine approximation; *Complex analysis methods:* The Euler gamma function; Generating functions: Dirichlet series; Summation formulae; The Riemann zeta function; The prime number theorem and the Riemann hypothesis; The Selberg-Delange method; Two arithmetic applications; Tauberian theorems; Primes in arithmetic progressions; *Probabilistic methods:* Densities; Limiting distributions of arithmetic functions; Normal order; Distribution of additive functions and mean values of multiplicative functions; Friable integers. The saddle-point method; Integers free of small factors; Bibliography; Index.

Graduate Studies in Mathematics, Volume 163

April 2015, 641 pages, Hardcover, ISBN: 978-0-8218-9854-3, LC 2014040135, 2010 *Mathematics Subject Classification:* 11-02; 11Axx, 11Jxx, 11Kxx, 11Lxx, 11Mxx, 11Nxx, **AMS members US$71.20**, List US$89, Order code GSM/163

Probability and Statistics



A Course on Large Deviations with an Introduction to Gibbs Measures

Firas Rassoul-Agha, *University of Utah, Salt Lake City, UT*, and **Timo Seppäläinen**, *University of Wisconsin–Madison, WI*

This is an introductory course on the methods of computing asymptotics of probabilities of rare events: the theory of large deviations. The book combines large deviation theory with basic statistical mechanics, namely Gibbs measures with their variational characterization and the phase transition of the Ising model, in a text intended for a one semester or quarter course.

The book begins with a straightforward approach to the key ideas and results of large deviation theory in the context of independent identically distributed random variables. This includes Cramér's theorem, relative entropy, Sanov's theorem, process level large deviations, convex duality, and change of measure arguments.

Dependence is introduced through the interactions potentials of equilibrium statistical mechanics. The phase transition of the Ising model is proved in two different ways: first in the classical way with the Peierls argument, Dobrushin's uniqueness condition, and correlation inequalities and then a second time through the percolation approach.

Beyond the large deviations of independent variables and Gibbs measures, later parts of the book treat large deviations of Markov chains, the Gärtner-Ellis theorem, and a large deviation theorem of Baxter and Jain that is then applied to a nonstationary process and a random walk in a dynamical random environment.

The book has been used with students from mathematics, statistics, engineering, and the sciences and has been written for a broad audience with advanced technical training. Appendixes review basic material from analysis and probability theory and also prove some of the technical results used in the text.

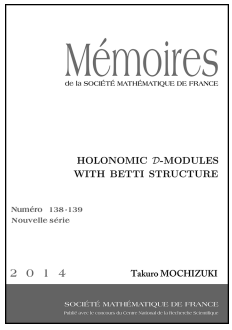
Contents: *Large deviations: General theory and i.i.d. processes:* Introductory discussion; The large deviation principle; Large deviations and asymptotics of integrals; Convex analysis in large deviation theory; Relative entropy and large deviations for empirical measures; Process level large deviations for i.i.d. fields; *Statistical mechanics:* Formalism for classical lattice systems; Large deviations and equilibrium statistical mechanics; Phase transition in the Ising model; Percolation approach to phase transition; *Additional large deviation topics:* Further asymptotics for i.i.d. random variables; Large deviations through the limiting generating function; Large deviations for Markov chains; Convexity criterion for large deviations; Nonstationary independent variables; Random walk in a dynamical random environment; *Appendixes:* Analysis; Probability; Inequalities from statistical mechanics; Nonnegative matrices; Bibliography; Notation index; Author index; General index.

Graduate Studies in Mathematics, Volume 162

April 2015, approximately 314 pages, Hardcover, ISBN: 978-0-8218-7578-0, LC 2014035275, 2010 *Mathematics Subject Classification:* 60-01, 60F10, 60J10, 60K35, 60K37, 82B05, 82B20, **AMS members US$63.20**, List US$79, Order code GSM/162

New AMS-Distributed Publications

Algebra and Algebraic Geometry



Holonomic $\mathcal{D}$-Modules with Betti Structure

Takuro Mochizuki, *Kyoto University, Japan*

The author defines the notion of Betti structure for holonomic $\mathcal{D}$-modules which are not necessarily regular singular. He establishes the fundamental functorial properties and also gives auxiliary analysis of holomorphic functions of various types on the real blow up.

This item will also be of interest to those working in analysis.

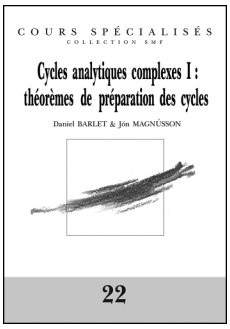
A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Preliminary; Good holonomic $\mathcal{D}$-modules and their de Rham complexes; Some sheaves on the real blow up; Complexes on the real blow up associated to good meromorphic flat bundles; Good K-structure; K-holonomic $\mathcal{D}$-modules; Functoriality properties; Derived category of algebraic K-holonomic $\mathcal{D}$-modules; Bibliography; Index.

Mémoires de la Société Mathématique de France, Number 138/139

December 2014, 205 pages, Softcover, ISBN: 978-2-85629-791-9, 2010 *Mathematics Subject Classification:* 14F10, 32C38, **AMS members US$48**, List US$60, Order code SMFMEM/138/139

Analysis



Cycles Analytiques Complexes I

Théorèmes de préparation des cycles

Daniel Barlet, *Université de Lorraine, France*, and **Jón Magnússon**, *Reykjavik University, Iceland*

A note to readers: This book is in French.

The aim of this book is, beginning with the standard knowledge of a master student in mathematics, to present some basic results in complex geometry from a firmly geometric point of view. The tools that are built are then used to describe analytic families of cycles of a given complex space, in order to show the first stability properties of this notion and to give some simple but fundamental applications of these results. This will be detailed and completed in the second volume.

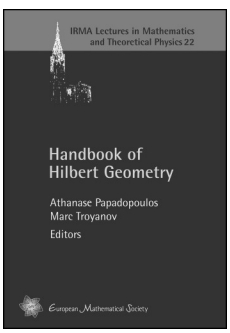
A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction générale; Préliminaires; Graphes multiformes et espaces complexes réduits; Analyse et géométrie sur un espace complexe réduit; Families de cycles en géométrie complexe; Appendix A. Complexification; Appendix B. Espaces vectoriels topologiques localement convexes; Bibliographie; Index.

Cours Spécialisés—Collection SMF, Number 22

December 2014, 533 pages, Softcover, ISBN: 978-2-85629-792-6, 2010 *Mathematics Subject Classification:* 32-01, 32-02, 32Cxx, 32Dxx, 32Exx, 32Fxx, 32Gxx, 32Hxx, **AMS members US$116**, List US$145, Order code COSP/22

General Interest



Handbook of Hilbert Geometry

Athanase Papadopoulos, *Université de Strasbourg*, and **Marc Troyanov**, *École Polytechnique Fédérale de Lausanne, Switzerland*, Editors

This volume presents surveys, written by experts in the field, on various classical and modern aspects of Hilbert geometry. They assume several points of view: Finsler geometry, calculus of variations, projective geometry, dynamical systems, and others. Some fruitful relations between Hilbert geometry and other subjects in mathematics are emphasized, including Teichmüller spaces, convexity theory, Perron–Frobenius theory, representation theory, partial differential equations, coarse geometry, ergodic theory, algebraic groups, Coxeter groups, geometric group theory, Lie groups and discrete group actions.

This book is addressed to both students who want to learn the theory and researchers in this area.

This item will also be of interest to those working in differential equations.

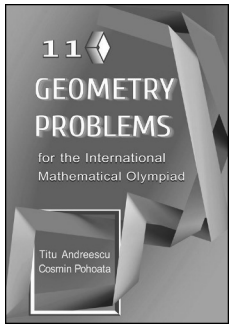
A publication of the European Mathematical Society. Distributed within the Americas by the American Mathematical Society.

Contents: Introducton; *Part I. Minkowski, Hilbert and Funk geometries:* **A. Papadopoulos** and **M. Troyanov**, Weak Minkowski spaces; **A. Papadopoulos** and **M. Troyanov**, From Funk to Hilbert geometry; **M. Troyanov**, Funk and Hilbert geometries from the Finslerian viewpoint; **C. Vernicos**, On the Hilbert geometry of convex polytopes; **C. Walsh**, The horofunction boundary and isometry group of the Hilbert geometry; **R. Guo**, Characterizations of hyperbolic geometry among Hilbert geometries; *Part II. Groups and dynamics in Hilbert geometry:* **M. Crampon**, The geodesic flow of Finsler and Hilbert geometries; **L. Marquis**, Around groups in Hilbert geometry; **A. Karlsson**, Dynamics of Hilbert nonexpansive maps; **B. Lemmens** and **R. Nussbaum**, Birkhoff's version of Hilbert's metric and and its applications in analysis; *Part III. Developments and applications:* **I. Kim** and **A. Papadopoulos**, Convex real projective structures and Hilbert metrics; **H. Miyachi**, **K. Ohshika**, and **S. Yamada**, Weil-Petersson Funk metric on Teichmüller space; **A. Papadopoulos** and **S. Yamada**, Funk and Hilbert geometries in spaces of constant curvature; *Part IV. History of the subject:* **M. Troyanov**, On the origin of Hilbert geometry; **A. Papadopoulos**, Hilbert's fourth problem; Open problems; List of contributors; Index.

IRMA Lectures in Mathematics and Theoretical Physics, Volume 22

December 2014, 460 pages, Hardcover, ISBN: 978-3-03719-147-7, 2010 *Mathematics Subject Classification:* 01A55, 35Q53, 37D25, 37D20, 37D40, 47H09, 51-00, 51-02, 51-03, 51A05, 51B20, 51F99, 51K05, 51K10, 51K99, 51M10, 52A07, 52A20, 52A99, 53A20, 53A35, 53B40, 53C22, 53C24, 53C60, 53C70, 54H20, 57S25, 58-00, 58-02, 58-03, 58B20, 58D05, **AMS members US$78.40**, List US$98, Order code EMSILMTP/22

Math Education



110 Geometry Problems for the International Mathematical Olympiad

Titu Andreescu, *University of Texas at Dallas, Richardson, TX*, and **Cosmin Pohoata**, *Columbia University, New York, NY*

This book represents a collection of carefully selected geometry problems designed for passionate geometers and students preparing for the IMO. Assuming the theory and the techniques presented in the first two geometry books published by XYZ Press, *106 Geometry Problems from the AwesomeMath Summer Program* and *107 Problems from*

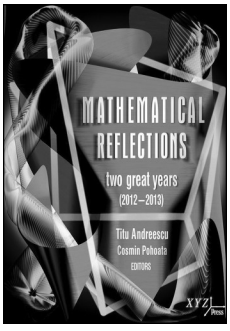
the AwesomeMath Year-Round Program, this book presents a multitude of beautiful synthetic solutions that are meant to give a sense of how one should think about difficult geometry problems. On average, each problem comes with at least two such solutions and with additional remarks about the underlying configuration.

A publication of XYZ Press. Distributed in North America by the American Mathematical Society.

Contents: Abbreviations and notation; Problems; Solutions; References and further reading.

XYZ Series, Volume 14

November 2014, 249 pages, Hardcover, ISBN: 978-0-9885622-2-6, 2010 *Mathematics Subject Classification:* 00A07, 97D50, 97U40, **AMS members US$47.96**, List US$59.95, Order code XYZ/14



Mathematical Reflections: Two Great Years (2012–2013)

Titu Andreescu, *University of Texas at Dallas, Richardson, TX*, and **Cosmin Pohoata**, *Columbia University, New York, NY*, Editors

This book is a compilation and revision of the 2012 and 2013 volumes from the online journal of the same name. This book is aimed at high school students, participants in math competitions, undergraduates, and anyone who has a fire for mathematics. Passionate readers submitted many of the problems, solutions, and articles and all require creativity, experience, and comprehensive mathematical knowledge. This book is a great resource for students training for advanced national and international mathematics competitions such as USAMO and IMO.

This item will also be of interest to those working in general interest.

A publication of XYZ Press. Distributed in North America by the American Mathematical Society.

Contents: Problems; Solutions; Articles; Problem author index; Article author index.

XYZ Series, Volume 15

November 2014, 499 pages, Hardcover, ISBN: 978-0-9885622-1-9, 2010 *Mathematics Subject Classification:* 00A07, 97D50, 97U40, **AMS members US$55.96**, List US$69.95, Order code XYZ/15

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There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: April 2015 issue–January 29, 2015; May 2015 issue–March 2, 2015; June/July 2015 issue–April 29, 2015; August 2015 issue–May 29, 2015; September 2015 issue–June 29, 2015; October 2015–July 29, 2015.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the US cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to US laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the US and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02904; or via fax: 401-331-3842; or send email to classads@ams.org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices.* However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Washington, District of Columbia

Georgetown University

March 7–8, 2015
Saturday – Sunday

Meeting #1107
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: January 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: March 2015
Issue of *Abstracts*: Volume 36, Issue 2

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Frederico Rodriguez Hertz, Pennsylvania State University, *Random dynamics and a formula for Furstenberg entropy*.

Nancy Hingston, The College of New Jersey, *Loop products, Poincare duality and dynamics*.

Simon Tavaré, Cambridge University, *Cancer by the numbers* (Einstein Public Lecture in Mathematics).

Yitang Zhang, University of New Hampshire, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Algebra and Representation Theory, **Ela Celikbas** and **Olgur Celikbas**, University of Connecticut, and **Frank Moore**, Wake Forest University.

Algebraic Structures Motivated by and Applied to Knot Theory, **Jozef H. Przytycki**, George Washington University, **Radmilla Sazdanovic**, North Carolina State University, and **Alexander N. Shumakovitch** and **Hao Wu**, George Washington University.

Asymptotic Problems for Stochastic Processes and PDEs, **Sandra Cerrai**, **Dmitry Dolgopyat**, **Mark Freidlin**, and **Leonid Koralov**, University of Maryland.

Bases and Frames in Hilbert Spaces and Applications, **Laura De Carli**, Florida International University.

Characterizing Uncertainty for Modeling Physical Processes, **Ali Arab**, Georgetown University.

Closure Operations in Commutative Algebra, **Neil Epstein**, George Mason University, and **Lance Edward Miller**, University of Arkansas.

Computable Structure Theory, **Rumen Dimitrov**, Western Illinois University, **Valentina Harizanov**, George

Washington University, and **Russell Miller**, Queens College and Graduate Center, City University of New York.

Conceptual Mathematical Models in Climate Science, **Hans Engler** and **Hans Kaper**, Georgetown University.

Convexity and Combinatorics, **Jim Lawrence** and **Valeriu Soltan**, George Mason University.

Crossing Numbers of Graphs, **Paul Kainen**, Georgetown University.

Data Assimilation: Recent Progress in Theory, Methods and Applications, **Evelyn M. Lunasin** and **Reza Malek-Madani**, United States Naval Academy.

Difference Equations and Applications, **Michael Radin**, Rochester Institute of Technology, and **Steven J. Miller**, Williams College.

Dynamical Systems Models of Physiological Processes, **Paula Grajdeanu**, Shenandoah University, and **Talitha Washington** and **Abdul-Aziz Yakubu**, Howard University.

Geometric Structures on Low-Dimensional Manifolds and their Invariants, **Cagatay Kutluhan**, University at Buffalo, and **Thomas E. Mark** and **Bulent Tosun**, University of Virginia.

History and Philosophy of Mathematics, **V. Frederick Rickey**, West Point Military Academy, and **James J. Tattersall**, Providence College.

Inverse Problems for Non-destructive Testing, **Nicolas Valdivia**, Naval Research Laboratory.

Iterated Integrals and Applications, **Ivan Horozov**, Washington University in St. Louis.

Mathematical Fluid Dynamics and Turbulence, **Zachary Bradshaw**, University of British Columbia, **Aseel Farhat**, Indiana University, and **Michele Coti Zelati**, University of Maryland.

Nonlinear Dispersive and Wave Equations with Applications to Fluids, **Pierre Germain** and **Zaher Hani**, New York University, and **Benoit Pausader**, Princeton University.

Nonlinear Partial Differential Equations in Sciences and Engineering, **Lorena Bociu**, North Carolina State University, **Ciprian Gal**, Florida International University, and **Daniel Toundykov**, University of Nebraska.

Number Theory in Ergodic Theory and Dynamical Systems, **Joe Herning**, Northern Virginia Community College, **Erblin Mehmetaj**, George Washington University and Georgetown University, **E. Arthur Robinson Jr.**, George Washington University, and **Tyler White**, Northern Virginia Community College.

Operator Theory on Analytic Function Spaces, **Robert F. Allen**, University of Wisconsin, La Cross, and **Flavia Colonna**, George Mason University.

Optimization Theory, Algorithms and Applications, **Olga Brezhneva**, Miami University, Oxford, OH, and **Igor Griva**, George Mason University.

Patterns in Permutations and Words, **Alexander Burstein**, Howard University.

Qualitative Behavior of Solutions of Partial Differential Equations, **Junping Shi**, College of William and Mary, and **Jiuyi Zhu**, John Hopkins University.

Quantum Algebras, Representations, and Categorifications, **Sean Clark** and **Weiqiang Wang**, University of Virginia.

Singularities: Algebraic and Analytic Aspects, **Claudia Miller**, Syracuse University, and **Sophia Vassiliadou**, Georgetown University.

Somos Sequences and Nonlinear Recurrences, **Andrew Vogt**, Georgetown University.

Spatial Evolutionary Models and Biological Invasions, **Judith Miller**, Georgetown University, and **Yuan Lou**, Ohio State University.

Stochastic Analysis and Stochastic PDEs, **Sandra Cerrai**, University of Maryland, and **Frederi Viens**, Purdue University.

Topology in Biology, **Paul Kainen**, Georgetown University.

Within-Host Disease Modeling, **Stanca Ciupe**, Virginia Polytechnic Institute, and **Sivan Leviyang**, Georgetown University.

East Lansing, Michigan

Michigan State University

March 14–15, 2015

Saturday – Sunday

Meeting #1108

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: January 2015
Program first available on AMS website: January 29, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 2

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Philippe R. Di Francesco, Departments of Mathematics and Physics, University of Illinois at Urbana-Champaign, *Integrable Combinatorics*.

Alexander Furman, University of Illinois at Chicago, *Hidden Symmetries of Some Groups*.

Vera Mikyoung Hur, University of Illinois at Urbana-Champaign, *Breaking the Waves*.

Mihnea Popa, Northwestern University, *Recent results on holomorphic one-forms*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Algebraic Combinatorics, **Carolina Benedetti**, **Peter Magyar**, and **Bruce Sagan**, Michigan State University.

Approximation Theory in Signal Processing and Computer Science, **Mark Iwen**, Michigan State University, **Rayan Saab**, University of California San Diego, and **Aditya Viswanathan**, Michigan State University.

Arithmetic of Hyperelliptic Curves, **Tony Shaska**, Oakland University.

Calculus of Variations, Nonlinear Partial Differential Equations, and Applications, **Moxun Tang** and **Baisheng Yan**, Michigan State University, and **Zhengfang Zhou**, MIchigan State University.

Combinatorics, Geometry, and Representation Theory of Homogeneous Spaces, **Mahir Bilen Can** and **Michael Joyce**, Tulane University, and **Miriam Logan**, Bowdoin College.

Complex Analysis in Several Variables and its Applications, **Debraj Chakrabarti**, Central Michigan University, and **Yunus Zeytuncu**, University of Michigan at Dearborn.

Conformal Geometry and Statistical Physics, **Ilia Binder**, University of Toronto, and **Dapeng Zhan**, Michigan State University.

Discrete Stochastic Models, **Michael Damron**, Indiana University, and **David Sivakoff**, The Ohio State University.

Extremal Graph Theory: Hypergraphs, Directed Graphs, and Other Generalizations, **Louis DeBiasio**, Miami University, and **Theodore Molla**, University of Illinois at Urbana-Champaign.

Floer Homology, Gauge Theory, and Symplectic Geometry, **David Duncan**, **Matt Hedden**, and **Tom Parker**, Michigan State University.

Fractals and Tilings, **Sze-Man Ngai**, Georgia Southern University, **Erin Pearse**, California Polytechnic State University, **Yang Wang**, Hong Kong University of Science and Technology, and **Yimin Xiao**, Michigan State University.

Fractional Calculus and Nonlocal Operators, **Mark M. Meerschaert** and **Russell Schwab**, Michigan State University.

Frames, Wavelets and Their Applications, **Palle Jorgensen**, University of Iowa, **Darrin Speegle**, St. Louis University, and **Yang Wang**, Hong Kong University of Science and Technology.

Geometry and Invariants of 3-Manifolds, **Oliver Dasbach**, Louisiana State University, and **Effie Kalfagianni**, Michigan State University.

Geometry of Manifolds, Singular Spaces, and Groups, **Benjamin Schmidt**, Michigan State University, and **Meera Mainkar**, Central Michigan University.

Groups and Representations, **Amanda Schaeffer Fry**, Metropolitan State University of Denver, **Jonathan Hall**, Michigan State University, and **Hung Nguyen**, University of Akron.

Harmonic Analysis and Applications, **Jarod Hart**, Wayne State University, **Nguyen Lam**, University of Pittsburgh, and **Guozhen Lu**, Wayne State University.

Harmonic Analysis and Partial Differential Equations, **Michael Goldberg**, University of Cincinnati, and **William Green**, Rose-Hulman Institute of Technology.

High-Frequency Problems, **Shlomo Levental** and **Mark Schroder**, Michigan State University.

Homotopy Continuation Methods and Their Applications to Science and Engineering, **Tianran Chen**, Michigan State University, and **Dhagash Mehta**, North Carolina State University.

Integrable Combinatorics, **Philippe Di Francesco** and **Rinat Kedem**, University of Illinois at Urbana-Champaign.

Interactions between Geometry, Group Theory, and Number Theory, **Benjamin Linowitz**, University of Michigan, and **D. B. Reynolds**, Purdue University.

Inverse Problems and Imaging, **Yulia Hristova**, University of Michigan-Dearborn, and **Linh Nguyen**, University of Idaho.

Knot Theory and Floer-Type Invariants, **Christopher Cornwell**, Université du Québec à Montréal, and **Faramarz Vafaee**, Caltech.

Mathematics in Industry and Industrial Problems with Mathematics Application, **Peiru Wu**, Michigan State University.

Modeling, Numerics, and Analysis of Electro-Diffusion Phenomena, **Peter W. Bates**, Michigan State University, **Weishi Liu**, University of Kansas, and **Mingji Zhang**, MIchigan State University.

New Developments in Actuarial Mathematics, **Emiliano A. Valdez**, Michigan State University.

New Developments in Stochastic Analysis, Stochastic Control and Related Fields, **Chao Zhu**, University of Wisconsin-Milwaukee.

Nonlinear Waves: Dynamics and Stability, **Keith Promislow** and **Qiliang Wu**, Michigan State University.

Phase Retrieval in Theory and Practice, **Matthew Fickus**, Air Force Institute of Technology, **Mark Iwen**, Michigan State University, and **Dustin Mixon**, Air Force Institute of Technology.

Random Fields and Long Range Dependence, **Mark M. Meerschaert** and **Yimin Xiao**, Michigan State University.

Random Matrices and Compressed Sensing, **Yang Liu**, Michigan State University.

Recent Advances in Finite Element and Discontinuous Galerkin Methods for Partial Differential Equations, **Aycil Cesmelioglu** and **Anna Maria Spagnuolo**, Oakland University.

Recent Advances in Mathematical Modeling of the Financial Markets, **Albert Cohen**, Michigan State University, and **Nick Costanzino**, University of Toronto.

Recent Advances in the Geometry of Submanifolds, Dedicated to the Memory of Franki Dillen (1963-2013), **Alfonso Carriazo Rubio**, University of Sevilla, **Yun Myung Oh**, Andrews University, **Bogdan D. Suceavă**, California State University, Fullerton, and **Joeri Van der Veken**, KU Leuven.

Smooth Dynamical Systems and Ergodic Theory, **Nicolai Haydn**, University of Southern California, and **Huyi Hu** and **Sheldon Newhouse**, Michigan State University.

Spectral Theory, Disorder, and Quantum Many Body Physics, **Peter D. Hislop**, University of Kentucky, and **Jeffrey Schenker**, Michigan State University.

Stochastic Partial Differential Equations and Applications, **Leszek Gawarecki**, Kettering University, and **Vidyadhar Mandrekar**, Michigan State University.

Survey of Biomathematics, **Hannah Callender**, University of Portland, **Peter Hinow**, University of Wisconsin,

Milwaukee, and **Deena Schmidt**, Case Western Reserve University.

The Geometry of Algebraic Varieties, **Kevin Tucker**, University of Illinois at Chicago, and **Brian Lehmann**, Boston College.

Topics in Noncommutative Algebra and Algebraic Geometry, **Jason Bell**, University of Waterloo, **Rajesh S. Kulkarni**, Michigan State University, and **Daniel Rogalski**, UC San Diego.

Huntsville, Alabama

University of Alabama in Huntsville

March 27–29, 2015

Friday – Sunday

Meeting #1109

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: January 2015
Program first available on AMS website: February 11, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 2

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Eva Bayer-Fluckiger, EPFL, *On the Euclidean Division*.

M. Gregory Forest, University of North Carolina at Chapel Hill, *Mathematics of Living Fluids*.

Dan Margalit, Georgia Institute of Technology, *Geometry, algebra, and dynamics of surfaces*.

Paul Pollack, University of Georgia, *Big doings with small gaps*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advances in the Theory and Applications of Dynamical Systems, **Shangbing Ai** and **Wenzhang Huang**, University of Alabama in Huntsville.

Analysis on Nonlinear Integral and Partial Differential Equations, **Tadele Mengesha** and **Tuoc Phan**, The University of Tennessee.

Analytic Methods in Elementary Number Theory, **Paul Pollack**, University of Georgia.

Fractal Geometry and Ergodic Theory, **Mrinal Kanti Roychowdhury**, University of Texas-Pan American.

Geometric Group Theory and Topology, **Tara Brendle**, University of Glasgow, **Christopher Leininger**, University of Illinois at Urbana-Champaign, and **Dan Margalit**, Georgia Institute of Technology.

Graph Theory, **Chris Stephens**, **Dong Ye**, and **Xiaoya Zha**, Middle Tennessee State University.

Mathematical Modeling in Ecology and Epidemiology, **Andrew Nevai** and **Zhisheng Shuai**, University of Central Florida.

New Developments in Population Dynamics and Epidemiology, **Jia Li**, University of Alabama in Huntsville, **Maia Martcheva**, University of Florida, and **Necibe Tuncer**, Florida Atlantic University.

Nonlinear Operator Theory and Partial Differential Equations, **Craig Cowan**, University of Manitoba, and **Claudio Morales**, University of Alabama in Huntsville.

Quadratic Forms in Arithmetic and Geometry, **Asher Auel**, Yale University, **Jorge Morales**, Louisiana State University, and **Anne Quéguiner-Mathieu**, Université Paris 13.

Recent Advances in Numerical Methods for Nonlinear Partial Differential Equations, **S. S. Ravindran**, University of Alabama in Huntsville.

Recent Progress in Differential Equations, **Mathew Gluck**, University of Alabama in Huntsville.

Recent Trends in Mathematical Biology, **Wandi Ding** and **Zachariah Sinkala**, Middle Tennessee State University.

Stochastic Analysis and Applications, **Parisa Fatheddin**, University of Alabama in Huntsville.

Stochastic Processes and Related Topics, **Paul Jung**, University of Alabama at Birmingham, **Erkan Nane**, Auburn University, and **Dongsheng Wu**, University of Alabama in Huntsville.

Topology and Topological Methods in Dynamical Systems, **John Mayer** and **Lex Oversteegen**, University of Alabama at Birmingham.

Las Vegas, Nevada

University of Nevada, Las Vegas

April 18–19, 2015

Saturday – Sunday

Meeting #1110

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: February 2015
Program first available on AMS website: March 5, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 2

Deadlines

For organizers: Expired
For abstracts: February 24, 2015

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Joel Hass, University of California, Davis, *Optimal diffeomorphisms of surfaces and some applications*.

Ko Honda, UCLA, *An invitation to Floer homology*.

Brendon Rhoades, University of California, San Diego, *Evaluating q-analogs in combinatorics and algebra*.

Bianca Viray, University of Washington, Seattle, *Reciprocity laws and rational points on varieties*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Algebraic Structures in Knot Theory (Code: SS 7A), **Sam Nelson**, Claremont McKenna College, and **Radmila Sazdanović**, North Carolina State University.

Algebraic and Enumerative Combinatorics (Code: SS 8A), **Drew Armstrong**, University of Miami, and **Brendon Rhoades**, University of California, San Diego.

Algebro-Geometric Methods in Graph Theory (Code: SS 24A), **Mohamed Omar**, Harvey Mudd College, and **Matthew T. Stamps**, KTH.

Arithmetic Geometry (Code: SS 18A), **Katherine E. Stange**, University of Colorado, Boulder, and **Bianca Viray**, University of Washington.

Cloaking and Metamaterials (Code: SS 9A), **Jichun Li**, University of Nevada, Las Vegas, and **Fernando Guevera Vasquez**, University of Utah.

Contact Geometry and Low-Dimensional Topology (Code: SS 25A), **Ko Honda** and **Erkao Bao**, University of California, Los Angeles, and **Lenhard Ng**, Duke University.

Data Analysis and Physical Processes (Code: SS 4A), **Hanna Makaruk**, Los Alamos National Laboratory, and **Eric Machorro**, National Security Technologies.

Developments of Numerical Methods and Computations for Fluid Flow Problems (Code: SS 11A), **Monika Neda**, University of Nevada, Las Vegas.

Evolution Problems at the Interface of Waves and Fluids (Code: SS 12A), **I. Bejenaru**, University of California, San Diego, and **B. Pausader** and **V. Vicol**, Princeton University.

Extremal and Structural Graph Theory (Code: SS 10A), **Bernard Lidický** and **Derrick Stolee**, Iowa State University.

Geometric Inequalities and Nonlinear Partial Differential Equations (Code: SS 19A), **Guozhen Lu**, Wayne State University, **Nguyen Lam**, University of Pittsburgh, and **Bernhard Ruf**, Università di Milano.

History Of Mathematics (Code: SS 23A), **Satish C. Bhatnagar**, University of Nevada, Las Vegas.

Inverse Problems and Related Mathematical Methods in Physics (Code: SS 1A), **Hanna Makaruk**, Los Alamos National Laboratory, and **Robert Owczarek**, University of New Mexico, Albuquerque.

Knots and 3-Manifolds (Code: SS 14A), **Abby Thompson** and **Anastasiia Tsvietkova**, University of California-Davis.

Mathematical and Numerical Aspects of Modeling Flows Through Porous Media (Code: SS 16A), **Aleksey S. Telyakovskiy** and **Stephen W. Wheatcraft**, University of Nevada, Reno.

Modeling and Numerical Studies for Coupled System of PDEs Arising From Interdisciplinary Problems (Code: SS 20A), **Pengtao Sun**, University of Nevada, Las Vegas.

New Developments in Noncommutative Algebra (Code: SS 22A), **Ellen Kirkman**, Wake Forest University, and **James Zhang**, University of Washington, Seattle.

Nonlinear Conservation Laws and Applications (Code: SS 6A), **Matthias Youngs**, Indiana University-Purdue University Columbus, **Cheng Yu**, University of Texas at Austin, and **Kun Zhao**, Tulane University.

Nonlinear Elliptic and Parabolic PDEs (Code: SS 17A), **Igor Kukavica**, University of Southern California, **Walter Rusin**, Oklahoma State University, and **Fei Wang**, University of Southern California.

Nonlinear PDEs and Variational Methods (Code: SS 5A), **David Costa**, **Zhonghai Ding**, and **Hossein Tehrani**, University of Nevada, Las Vegas.

Recent Advances in Finite Element Analysis and Applications (Code: SS 13A), **Jichun Li**, University of Las Vegas, and **Susanne Brenner**, Louisiana State University.

Recent Advances in Finite Element Analysis and Applications (Code: SS 21A), **Jichun Li**, University of Nevada, Las Vegas, and **Susanne Brenner**, Louisiana State University.

Set Theory (Code: SS 15A), **Derrick Dubose** and **Douglas Burke**, University of Nevada, Las Vegas.

Stochastic Analysis and Rough Paths (Code: SS 2A), **Fabrice Baudoin**, Purdue University, **David Nualart**, University of Kansas, and **Cheng Ouyang**, University of Illinois at Chicago.

Topics in Graph Theory: Structural and Extremal Problems (Code: SS 3A), **Jie Ma**, Carnegie Mellon University, **Hehui Wu**, Simon Fraser University, and **Gexin Yu**, College of William & Mary.

Porto, Portugal

University of Porto

June 10–13, 2015

Wednesday – Saturday

Meeting #1111

First Joint International Meeting involving the American Mathematical Society (AMS), the European Mathematical Society (EMS), and the Sociedade de Portuguesa Matematica (SPM).

Associate secretary: Georgia Benkart
Announcement issue of *Notices*: February 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses

Rui Loja Fernandes, University of Illinois at Urbana-Champaign, *Title to be announced*.

Irene Fonseca, Carnegie Mellon University, *Title to be announced*.

Annette Huber, Albert-Ludwigs-Universität, *Title to be announced*.

Mikhail Khovanov, Columbia University, *Title to be announced*.

André Neves, Imperial College London, *Title to be announced*.

Sylvia Serfaty, Université Pierre et Marie Curie Paris 6, *Title to be announced*.

Gigliola Staffilani, Massachusetts Institute of Technology, *Title to be announced*.

Marcelo Viana, Instituto de Matemática Pura e Aplicada, Brasil, *Title to be announced*.

Chicago, Illinois

Loyola University Chicago

October 3–4, 2015

Saturday – Sunday

Meeting #1112

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: June 2015
Program first available on AMS website: August 20, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 4

Deadlines

For organizers: March 10, 2015
For abstracts: August 11, 2015

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Julia Chuzhoy, Toyota Technological Institute at Chicago, *Title to be announced*.

Andrew Neitzke, The University of Texas at Austin, *Title to be announced*.

Sebastien Roch, University of Wisconsin-Madison, *Title to be announced*.

Peter Sarnak, Princeton University, *Title to be announced* (Erdős Memorial Lecture).

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebraic Methods Common to Association Schemes, Hopf Algebras, Tensor Categories, Finite Geometry, and Related Areas (Code: SS 1A), **Harvey Blau**, Northern Illinois University, **Sung Y. Song**, Iowa State University, and **Bangteng Xu**, Eastern Kentucky University.

Combinatorial and Computational Algebra (Code: SS 4A), **David Cook**, Eastern Illinois University, and **Sonja Mapes**, University of Notre Dame.

Groups, Rings, Group Rings, and Hopf Algebras – Celebrating the 75th Birthday of Professor Donald S. Passman (Code: SS 2A), **Jeffrey Bergen**, **Stefan Catoiu**, and **William Chin**, DePaul University.

The Mathematics of Evolution (Code: SS 3A), **Ruth Davidson** and **Ruriko Yoshida**, University of Illinois Urbana-Champaign.

Memphis, Tennessee

University of Memphis

October 17–18, 2015

Saturday – Sunday

Meeting #1113

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: August 2015
Program first available on AMS website: September 3, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 3

Deadlines

For organizers: March 17, 2015
For abstracts: August 25, 2015

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Mark van Hoeij, Florida State University, *Title to be announced*.

Vaughan Jones, Vanderbilt University, *Title to be announced*.

Mette Olufsen, North Carolina State University, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Operator Theory and Applications, in memory of James Jamison (Code: SS 5A), **Fernanda Botelho**,

University of Memphis, and **T.S.S.R.K. Rao**, Indian Statistical Institute Bangalore.

Banach Spaces and Applications (Code: SS 4A), **Anna Kaminska**, **Peikee Lin**, and **Bentuo Zheng**, University of Memphis.

Cahn-Hilliard and Related Equations and Applications. (Code: SS 11A), **Giséle Ruiz Goldstein**, University of Memphis, and **Alain Miranville**, Université de Poitiers.

Computational Analysis (Code: SS 1A), **George Anastassiou**, University of Memphis.

Control and Inverse Problems for Partial Differential Equations (Code: SS 6A), **Matthias Eller**, Georgetown University, **Shitao Liu**, Clemson University, and **Roberto Triggiani**, University of Memphis.

Difference Equations and Applications. (Code: SS 12A), **Michael A. Radin**, Rochester Institute of Technology, and **Youssef Raffoul**, University of Dayton.

Ergodic Theory (Code: SS 8A), **James T. Campbell** and **Mate Wierdl**, University of Memphis.

Extremal Graph Theory (Code: SS 3A), **Ralph Faudree**, University of Memphis.

Fractal Geometry and Dynamical Systems (Code: SS 2A), **Mrinal Kanti Roychowdhury**, University of Texas-Pan American.

Recent Advances in Commutative Algebra. (Code: SS 13A), **Sandra Spiroff**, University of Mississippi, and **Lance Miller**, University of Arkansas.

Recent Developments in the Statistical Analysis of Large Clustered Data (Code: SS 10A), **E. Olusegun George**, University of Memphis.

Spectra of Graphs and Hypergraphs. (Code: SS 9A), **Vladimir Nikiforov**, University of Memphis.

Stabilization, Control, and Analysis of Evolutionary Partial Differential Equations (Code: SS 7A), **George Avalos**, University of Nebraska Lincoln, **Scott Hansen**, Iowa State University, and **Justin Webster**, North Carolina State University & College of Charleston.

Fullerton, California

California State University, Fullerton

October 24–25, 2015

Saturday – Sunday

Meeting #1114

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2015
Program first available on AMS website: September 10, 2015
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 36, Issue 4

Deadlines

For organizers: March 27, 2015
For abstracts: September 1, 2015

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Mina Aganagic, University of California, Berkeley, *Title to be announced.*

John Lott, University of California, Berkeley, *Title to be announced.*

Eyal Lubetzky, Microsoft Research, Redmond, *Title to be announced.*

Zhiwei Yun, Stanford University, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Geometric Analysis (Code: SS 1A), **John Lott**, University of California, Berkeley, and **Aaron Naber**, Northwestern University.

Mathematicians and Outreach Programs (Code: SS 2A), **Olga Radko**, University of California Los Angeles, and **Bodgan D. Suceava**, California State University, Fullerton.

Spectral Asymptotics of Large Matrices (Code: SS 3A), **Alain Bourget** and **Tyler McMillen**, California State University, Fullerton.

New Brunswick, New Jersey

Rutgers University

November 14–15, 2015

Saturday – Sunday

Meeting #1115

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: September 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: November 2015
Issue of *Abstracts*: Volume 36, Issue 4

Deadlines

For organizers: April 14, 2015
For abstracts: September 22, 2015

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Lee Mosher, Rutgers University, *Title to be announced.*

Jill Pipher, Brown University, *Title to be announced.*

David Vogan, Massachusetts Institute of Technology, *Title to be announced*.

Wei Zhang, Columbia University, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advances in Valuation Theory (Code: SS 6A), **Samar El Hitti**, New York City College of Technology, City University of New York, **Franz-Viktor Kuhlmann**, University of Saskatchewan, and **Hans Schoutens**, New York City College of Technology, City University of New York.

Applications of CAT(0) Cube Complexes (Code: SS 1A), **Sean Cleary**, City College of New York and the City University of New York Graduate Center, and **Megan Owen**, Lehman College of the City University of New York.

Aspects of Minimal Surfaces in Riemannian Manifolds (Code: SS 4A), **Zheng Huang** and **Marcello Lucia**, City University of New York, Staten Island and Graduate Center.

Commutative Algebra (Code: SS 2A), **Laura Ghezzi**, New York City College of Technology, City University of New York, and **Jooyoun Hong**, Southern Connecticut State University.

Difference equations and applications (Code: SS 5A), **Manos Drymonis**, Providence College, **Evelina Lapierre**, Johnson and Wales University, and **Michael Radin**, Rochester institute of Technology.

On Geometric Topology: A Celebration of Jim West's 70th Birthday (Code: SS 3A), **Alexandre Dranishnikov**, University of Florida, **Steve Ferry**, Rutgers University, and **Boris Goldfarb**, State University of New York at Albany.

Representation Theory, Vertex Operator Algebras, and Related Topics (Code: SS 7A), **Corina Calinescu**, New York City College of Technology, City University of New York, **Andrew Douglas**, New York City College of Technology and Graduate Center, City University of New York, and **Joshua Sussan**, Medgar Evers College, City University of New York.

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday
Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2015
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2016
Issue of *Abstracts*: Volume 37, Issue 1

Deadlines

For organizers: April 1, 2015
For abstracts: To be announced

Athens, Georgia

University of Georgia

March 5–6, 2016

Saturday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 5, 2015
For abstracts: To be announced

Stony Brook, New York

State University of New York at Stony Brook

March 19–20, 2016

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 19, 2015
For abstracts: February 2, 2016

Salt Lake City, Utah

University of Utah

April 9–10, 2016

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Fargo, North Dakota

North Dakota State University

April 16–17, 2016

Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Convexity and Harmonic Analysis, **Maria Alfonseca-Cubero**, North Dakota State University, and **Dmitry Ryabogin**, Kent State University.

Ergodic Theory and Dynamical Systems, **Dogan Comez**, North Dakota State University, and **Mrinal Kanti Roychowdhury**, University of Texas-Pan American.

Mathematical Finance, **Indranil SenGupta**, North Dakota State University.

Brunswick, Maine

Bowdoin College

September 24–25, 2016

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: July 23, 2016

Denver, Colorado

University of Denver

October 8–9, 2016

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 8, 2016
For abstracts: August 16, 2016

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday
Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: October 2016
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2017
Issue of *Abstracts*: Volume 38, Issue 1

Deadlines

For organizers: April 1, 2016
For abstracts: To be announced

Charleston, South Carolina

College of Charleston

March 10–12, 2017

Friday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: November 10, 2016
For abstracts: To be announced

Bloomington, Indiana

Indiana University

April 1–2, 2017

Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Pullman, Washington

Washington State University

April 22–23, 2017

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday
Joint Mathematics Meetings, including the 124th Annual Meeting of the AMS, 101st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2017
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2017
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 16–19, 2019

Wednesday – Saturday
Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM)and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2018
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2018
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Central Section: **Georgia Benkart**, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: **Steven H. Weintraub**, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: **Brian D. Boe,** Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, e-mail: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated. Up-to-date meeting and conference information can be found at www.ams.org/meetings/.

Meetings:

Important Information regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 200 in the February 2015 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarly coded material (such as accent marks in text) must be typeset in LaTeX. Visit www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences in Cooperation with the AMS: (See www.ams.org/meetings/ for the most up-to-date information on these conferences.)

April 2–5, 2015: The Second International Conference on Mathematics and Statistics (AUS-ICMS '15). American University of Sharjah, United Arab Emirates.

July 13–31, 2015: 2015 Summer Research Institute on Algebraic Geometry, University of Utah, Salt Lake City, Utah.

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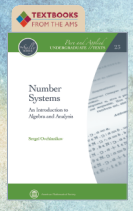
Recent Releases from the AMS



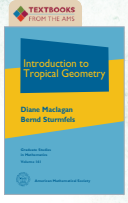
Number Systems

An Introduction to Algebra and Analysis

Sergei Ovchinnikov, *San Francisco State University, CA*

This book offers an introduction to the five basic number systems of mathematics, namely natural numbers, integers, rational numbers, real numbers, and complex numbers.

Pure and Applied Undergraduate Texts, Volume 23; 2015; 144 pages; Hardcover; ISBN: 978-1-4704-2018-5; List US$61; AMS members US$48.80; Order code AMSTEXT/23

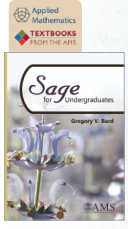


Introduction to Tropical Geometry

Diane Maclagan, *University of Warwick, Coventry, United Kingdom*, **and Bernd Sturmfels,** *University of California, Berkeley, CA*

This book offers a self-contained introduction to tropical geometry, suitable as a course text for beginning graduate students.

Graduate Studies in Mathematics, Volume 161; 2015; approximately 364 pages; Hardcover; ISBN: 978-0-8218-5198-2; List US$79; AMS members US$63.20; Order code GSM/161

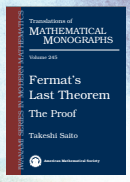


Sage for Undergraduates

Gregory V. Bard, *University of Wisconsin–Stout, Menomonie, WI*

A gentle introduction to Sage, the free and open-source mathematical software.

2015; 352 pages; Softcover; ISBN: 978-1-4704-1111-4; List US$29; All individuals US$21.75; Order code MBK/87



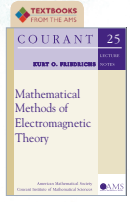
Fermat's Last Theorem

The Proof

Takeshi Saito, *University of Tokyo, Japan*

The second volume of the book on the proof of Fermat's last theorem by Wiles and Taylor.

Translations of Mathematical Monographs (*Iwanami Series in Modern Mathematics*), Volume 245; 2014; 234 pages; Softcover; ISBN: 978-0-8218-9849-9; List US$54; AMS members US$43.20; Order code MMONO/245



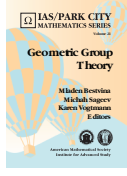
Mathematical Methods of Electromagnetic Theory

Kurt O. Friedrichs

An updated edition of Kurt Friedrichs's classic lecture notes on electromagnetic theory.

Titles in this series are co-published with the Courant Institute of Mathematical Sciences at New York University.

Courant Lecture Notes, Volume 25; 2014; 145 pages; Softcover; ISBN: 978-1-4704-1711-6; List US$34; AMS members US$27.20; Order code CLN/25



Geometric Group Theory

Mladen Bestvina, Michah Sageev, and Karen Vogtmann, Editors

An introduction to, and overview of, topics in geometric group theory.

Titles in this series are co-published with the Institute for Advanced Study/Park City Mathematics Institute. Members of the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) receive a 20% discount from list price.

IAS/Park City Mathematics Series, Volume 21; 2014; 399 pages; Hardcover; ISBN: 978-1-4704-1227-2; List US$90; AMS members US$72; Order code PCMS/21

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