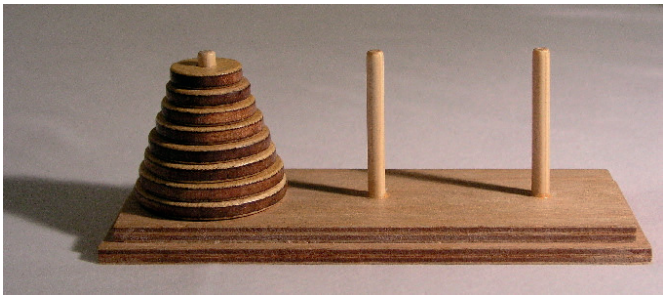
Automatic Sequences and the Tower of Hanoi

This month's cover was suggested by the article "WHAT IS an automatic sequence?" in this issue, written by Eric Rowland. One of the more curious examples of an automatic sequence is that which solves the well known and perhaps surprisingly popular problem of the Tower of Hanoi, first proposed by E. Lucas around 1884. The relation to automatic sequences can be found in §6.4 of the book by Allouche and Shallit referred to in Rowland's article.

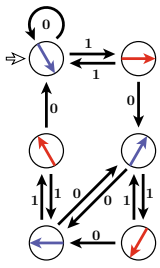
The problem is posed by a configuration of three pegs, on one of which are placed a number of rings of increasing radii, say from 1 at the top to some number N at the bottom.



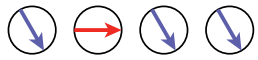
Tower of Hanoi | Plywood.

On the cover $N = 4$, but traditionally it is 8. The problem is to shift all rings from the initial peg to another, without ever placing a larger ring on top of a smaller one. As Allouche and Shallit explain, an optimal sequence of moves can be constructed as an automatic sequence. The sequence on the cover is one of symbols illustrating transfers to be made. For example,  marks a transfer of the disk on top of peg 1 to peg 3.

To figure out the nth move, first express n in bits. Then follow the path laid out by these bits, high order to low, in the following diagram:



The start is the node at upper left, and the move to be made is that indicated by the final node. For example, for $n = 6$ with bit expression 110 the path is



so one moves the small peg from peg 1 to peg 2.

The literature on this problem is enormous. Allouche and Shallit list many mathematical references, and the following web page is full of amusing information. `http://en.wikipedia.org/wiki/Tower_of_Hanoi`

The photograph of the Tower of Hanoi in plywood is taken from `http://commons.wikimedia.org/wiki/File: Tower_of_Hanoi.jpeg`

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—Bill Casselman
Graphics Editor
`notices-covers@ams.org`