My Life and Functions

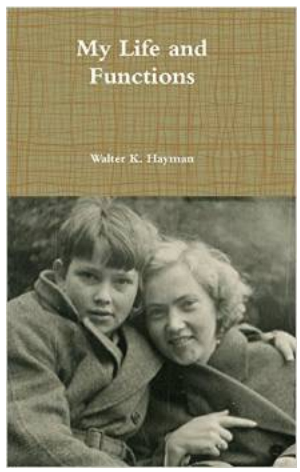
Reviewed by David Drasin

My Life and Functions
Walter K. Hayman
Logic Press, October 2014
Printed and distributed by lulu.com
152 pages
US$30.84 (hard cover)
ISBN-13: 978-1-326-03224-1
US$15.34 (paperback)
ISBN-13: 978-1-326-03020-9

Walter Kurt Hayman has led a remarkably full and interesting life, starting from accidents arising from his birth and ancestry to his later role as a leader of a major school of complex analysis. He was a prolific scientist, with extensive travels and interactions with major scientific and cultural leaders. As he nears his ninetieth birthday his memory remains strong. This autobiography is a relaxed review of his life, featuring abundant charming comments, anecdotes, insights and memories, with some illustrations. The author credits his current wife, Marie Jennings, with the impetus to write this book.

Less emphasis is given to mathematics, and the author provides little insight into the mechanics or vision behind his achievements. His impish sense of humor comes through consistently, and I could fill paragraphs with some of his short quotations and aphorisms. (One example is the thought that his "athletic prowess was probably inherited from [his] father. He claimed to be the best tennis teacher in the world, since all his students beat him after the first game.")

In recent years many of Walter's publications have been obituaries. Walter once mentioned



his frustration that, when preparing these, he invariably would recognize how little he knew of the subject's personal groundings, interests, or background. Efforts to compensate, including visits to family members and associates, were little compensation. (His obituary of Rolf Nevanlinna [MR 0671785] displays the seriousness and spirit with which he fulfilled these responsibilities.)

Some circumstances make having his own account especially notable. Walter was one of the large and talented German intellectual community which was displaced by the Nazi era; he emigrated to the UK at age twelve in 1938, only weeks before Kristallnacht dispelled any doubts of the future of Germans of Jewish ancestry (although by then the Haymans considered themselves Christian). One great-grandparent was the celebrated pianist Fanny Mendelssohn, sister of Felix and an important composer in her own right.[1] The author's middle name refers to his maternal grandfather, Kurt Hensel (who is credited with discovering p-adic numbers); Hensel's lemma is still part of the graduate algebra curriculum. Walter's final visit to his grandfather was shortly before leaving Germany,

David Drasin is professor emeritus of mathematics at Purdue University. His email address is drasin@purdue.edu.

DOI: http://dx.doi.org/10.1090/noti1250

[1] *Dover books still offers a selection of Fanny's piano music, and the recent very interesting documentary* Mendelssohn, the Nazis and Me, *directed by his daughter Sheila, includes testimony from Walter and fascinating insights from those times. Another Mendelssohn sister, Rebecka, married the mathematician P. G. L. Dirichlet.*

and his warm memories of their mathematical and musical interactions must have influenced his career choice.

The family's status and contacts allowed him and (when they arrived later) his parents to avoid the most severe hardships of a time of worldwide economic insecurity, and Walter was able to attend excellent schools and Cambridge University. Notable was Gourdonstown School, which had been founded in Scotland by his mother's cousin Kurt Hahn soon after Hahn's expulsion from Germany; enrollees would include members of the British Royal Family. The generosity and help which Walter and his family encountered in England is gratefully acknowledged throughout, and it was an important motivation for his lifelong service and charitable activities (another came from his joining the Society of Friends (Quakers) and his marriage to Margaret Crann, whom he met at a Friends meeting while both were at Cambridge). Walter was always conscious of the responsibilities which came from his prominence and successes.

It appears that Walter was reluctant to ask J. E. Littlewood to be his thesis supervisor, but he noticed that Mary L. Cartwright's work was one of the few directly cited in Littlewood's monograph *Lectures on the Theory of Functions* [MR 0012121], which made her a logical choice. He and Cartwright remained close until her death, and Walter's obituary of Dame Mary [MR 1866432] is a fascinating tribute.

His marriage to Margaret lasted forty-seven years, and their partnership also had consequences for the mathematical world. Margaret later was appointed Head of Maths (mathematics) at Putney High School and became well known in British education circles. In 1965 Walter and Margaret were invited by the Ministry of Education to a workshop on competitions. During the following summer, when the Hayman family made a motor trip to the Moscow International Congress, Walter was able to meet the Russian minister of education. The result was that, upon their return to the UK, the Haymans arranged that Britain participate in the International Mathematical Olympiad. The British team was the first non-Socialist entry, and today over one hundred nations compete. In her private life, Margaret was a respected violin player; Hayman's family life often included chamber music featuring the parents with their three daughters.

Upon Margaret's death, Walter married his former student Wafika al-Katifi and accepted her Moslem faith. This trajectory of religious affiliations would result in remarkable situations; for example, once when in Israel, "I stayed in a Presbyterian Guest House in Jerusalem…and I found myself on Sunday morning accompanying the hymns on the piano, a curious place and occupation for a Muslim Quaker." At present, he is married to Marie Jennings, whom he met a few years after Wafika's passing in 2001. The circumstances behind this courtship are very touching, and their partnership flourishes to this day.

Walter won major recognition from his British colleagues throughout his career: both Junior Berwick (now known as Berwick) and Senior Berwick Prizes (he was the first to receive both; the second double recipient only came decades later in 1994), culminating in the London Mathematical Society's deMorgan Medal (1995), considered the society's top recognition for mathematical achievement. He was elected Fellow of the Royal Society at age thirty. In spite of these accolades, he willingly recounts many doubts, weaknesses, and insecurities. As examples, his discomfort when first encountering the towering leaders of his research area, Rolf Nevanlinna and Lars Ahlfors, or his having the naiveté to directly ask A. S. Besicovitch to compare him to another student, Freeman Dyson: "He said quite rightly that Dyson was better."

The center of his professional career was his thirty years at Imperial College until the first of his retirements in 1985. The Hayman Monday morning seminar (at 11 sharp) featured work of students, colleagues, and recent literature, augmented by presentations from a steady stream of visitors. Walter was proud of his department's outstanding status in the British academic world. His pride in helping the department flourish is evident when discussing recruiting the complex dynamist I. N. Baker to the faculty well before the subject became fashionable. That the Fields Medalists Klaus Roth (now deceased) and Simon Donaldson were among his colleagues is of course mentioned. He also discusses the contributions and interesting personalities among his students and his clerical support staff (anyone who has encountered Walter's handwriting will appreciate the effort needed to bring his work to the scientific public).

Walter always thrived at mathematical conferences, where his direct, energetic, and unsnobbish behavior was atypical among leading mathematicians I have known. He considered himself duty-bound to ask questions, not being afraid to show that one need not be an expert to participate: "There's nothing worse than a dead silence at the end of a lecture," an inevitable result of "[m]athematicians are very used to listening to lectures which we do not understand." A familiar scene at meetings would be Walter with a younger attendee patiently addressing mathematical questions. These interactions often would lead to detailed answers at the meeting or by mail soon after (this process was important for my career), and he became an important research catalyst to the complex analysis community. In his phrasing, "…I found myself acting as a sort of Post Office."

His standing in British mathematics during the Cold War, along with this zeal for travel and scientific contact, had other consequences; however, many significant trips are not mentioned in the book. Visits to the USSR (1960) and China (1977) (both with Margaret) brought important recognition to mathematicians in both countries. In 1960 he was writing his book on meromorphic functions, where work of A. A. Goldberg would be highlighted. At the time, Goldberg was little known, since he was at Uzhgorod, a small provincial university. Moscow authorities were likely surprised that Goldberg's name was the first on a list Walter provided his hosts, and Goldberg was able to come to Moscow while Hayman was lecturing there. This imprimatur seems to have impressed the Soviet authorities, and Goldberg was then able to defend his (second) thesis and become a professor at Lviv University, where he developed a major research program and group.

Their 1977 China trip came soon after the end of China's Cultural Revolution. An American delegation, led by Saunders Mac Lane, had visited China shortly before, during the Cultural Revolution, but Walter's interest had already been piqued by a glowing report he received some years earlier from his colleague Cyril Offord. Offord had visited China in 1974, representing a British pro-China organization, and in Beijing had met the complex analysts Yang Lo and Zhang Guang Hou. Through Walter's repeated efforts Yang and Zhang visited London and Switzerland in 1978 in the first international purely scientific visit from China since 1966. This opened important opportunities for collaborative research and also helped give mathematics special internal prestige in China.

Unfortunately, Hayman's Moscow lecture (and the solutions to the problems he proposed there) were published in journals not covered by *Mathematical Reviews*; see `www.math.purdue.edu/~eremenko/dvi/as-curves.pdf`. On his return from China, Walter wrote to *Mathematical Reviews* urging full coverage of Chinese journals.

Although mathematics is not the main focus of the books under consideration or this review, some scientific contributions warrant mention.

a) The problem collections. For most of Walter's research career, there was no Internet, and political considerations made international contacts difficult. In 1962 the *Bulletin of the AMS* printed a list of twenty-five "Classical Function Theory Problems," produced at a meeting at Cornell. This was augmented five years later by Walter's small booklet *Research Problems in Function Theory*, based on a conference held three years earlier

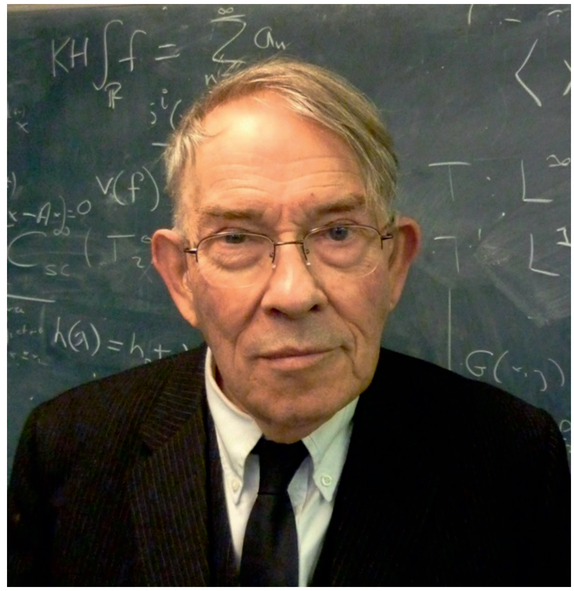


Photo courtesy of Christine Cockett, Department of Mathematics, University of York, UK.

Walter Hayman.

in London [MR 0217268]; it was praised not only for the quality and abundance of problems but for its coherent organization into research areas, accompanied by general background discussion. The price was minimal (this also holds for the volumes under review), and it became eagerly sought in Eastern Europe and Asia (i.e., China), where many older people still recount the heroics necessary to see it or well-worn photocopies of it. These collections were continued and expanded in four revisions and updates, usually with collaborators.

b) Books. Walter's research monographs remain in print and are valuable resources. Much as with his papers, they exhibit very careful style, and their scrupulous attention to detail also attracts elementary research students. Several of his results first appeared in these books.

The 1964 monograph *Meromorphic Functions* gave the first general exposition of the theory since Rolf Nevanlinna's monumental text from 1936 (revised in 1953).

The other two books are also significant. The impetus for the two-volume set *Subharmonic Functions* arose from plans of Walter's first research student, Paddy Kennedy, to write a major treatise on potential theory. However, Kennedy passed away with much of (what became) volume one unfinished. Walter completed that book as well as the more extensive companion, volume 2, which focused more on two-dimensional theory, including a full account of A. Baernstein's symmetrization theory.

Finally, his Cambridge tract, *Multivalent Functions*, deals with functions defined in the unit disk $D : \{|z| < 1\}$ which take on each value in the range (perhaps only in an average sense) at most $p < \infty$ times. The most important special case is $p = 1$, univalent functions. Much of the orientation of this book was also the framework in which some of Walter's first research work occurred. The revised edition has a self-contained treatment of L. de Branges's solution of the Bieberbach conjecture (see also below), as well as thorough accounts of symmetrization and the fundamental length-area principle.

c) Research. A few of Walter's famous results can be stated simply enough to be appreciated by many *Notices* readers.

i) Let $f(z)$ be a (nonconstant) entire function, and for $r > 0$ let

$$M(r) = \sup_{|z|=r} |f(z)|, \quad L(r) = \inf_{|z|=r} |f(z)|.$$

To reverse the obvious inequality $L(r) < M(r)$ requires additional information, the simplest being the order ρ of f:

$$\rho = \limsup_{r\to\infty} \frac{\log\log M(r)}{\log r};$$

for example, $\rho = k$ when $f(z) = \exp(z^k)$. Let us introduce $\sigma(\rho)$ as the best constant for which, given any $\varepsilon > 0$, we would have for any function f of order ρ:

$$\log L(r) > (\sigma(\rho) - \varepsilon) \log M(r),$$

at least on an unbounded sequence of r. About one-hundred years ago it was proved that, if $\rho < 1$, then $\sigma(\rho) = \cos \pi\rho$ is the best possible $\sigma(\rho)$, and soon after it was conjectured for $\rho > 1$ that $\sigma(\rho) = -1$. No further general result appeared in the next forty years, until Hayman [MR 0056083] showed the conjecture was false: indeed, for large ρ, $\sigma(\rho) > -A \log \rho$, with concrete estimates for A. (Only relatively recently, A. Fryntov [MR 1189746] disproved the conjecture for any $\rho > 1$; for $\rho = 1$ see [MR 1405053].)

ii) Picard's theorem asserts that a meromorphic function in the plane which omits three distinct values is constant (that this is sharp is seen by considering $w = f(z) = \tan z$, which omits $w = \pm i$). Walter established [MR 110807] the remarkable theorem that, if f is a (nonrational) meromorphic function, every derivative must assume every finite nonzero value infinitely often, with at most one exception. (The best previous result required that in addition f be entire, in which case there is automatically the additional omitted value $w = \infty$.) Refinements of these insights have led to the general principle known as "Hayman's Alternative," and this is Walter's most-cited paper.

iii) Until about thirty years ago, one of the two famous unsolved problems in complex analysis was Bieberbach's conjecture: If $f(z) = z + a_2 z^2 + a_3 z^3 \ldots$ is univalent in the unit disk, then $|a_n| \le n \; (n \ge 2)$, with equality only for the Koebe function $f(z) = z/(1-z)^2$. This was proved by L. de Branges in 1984, but two of Walter's contributions to the subject still endure (they also are valid for more general notions of univalence, where the Bieberbach conjecture is false). Thus if f is univalent (even in this general sense), the limit $L = \lim_n n^{-1}|a_n|$ exists, with $L < 1$ when f is not the Koebe function [MR 108586]. Finally, under the same hypothesis, he showed that

$$\big|\,|a_{n+1}| - |a_n|\,\big| \le A,$$

A an absolute constant. According to *Mathematical Reviews* [MR 148885] this was "hardly suspected to be true in the general case."

Acknowledgments

I am grateful to Steve Bell, Jennie Drasin, Alex Eremenko, Robert Kaufman, Dima Khavinson, Misha Sodin, Linda Sons, Allen Weitsman, Jang-Mei Wu, and Lo Yang for comments, corrections, and historical information.