## About the Cover

# Leibniz Inventing the Calculus

This issue contains a review by Eberhard Knobloch of a collection of articles about Leibniz. The articles under review are mostly concerned with Leibniz's philosophical writings, but of course to us he is best known for his mathematics, primarily his role in the invention of differential and integral calculus. Indeed, the term "calculus," signifying an algorithm, is his own.

The manuscript page on the cover shows the moment that Leibniz introduced the integral sign $\int$. It is from one of several memoranda he wrote to himself during his remarkable stay in Paris during the years 1672–1676. These were discovered in Hannover and transcribed by Carl Immanuel Gerhardt in the mid-nineteenth century. The particular one at hand is dated October 29, 1675.

Leibniz's scribblings are certainly hard to read, and at some point the manuscript seems to have suffered water damage. In Gerhardt's transcription (with a minor correction): *Utile erit scribi* $\int$ *pro omn., ut* $\int \ell$ *pro omn.* $\ell$*, id est summa ipsorum* $\ell$*. Itaque fiet* $\frac{\overline{\int \ell}^{1}}{2a} \sqcap \int \overline{\overline{\int \ell}\, \frac{\ell}{a}}$ *et* $\int \overline{x\ell} \sqcap x\overline{\int \ell} - \int \overline{\overline{\int \ell}}$*. Et ita apparebit semper observari legem homogeneorum, quod utile est ut calculi errores vitentur. Nota: si analytice detur* $\int \ell$*, dabitur etiam* $\ell$*. Ergo si detur* $\int\int \ell$*, dabitur etiam* $\ell$*, sed non si datur* $\ell$*, dabitur et* $\int \ell$*. Semper* $\int x \sqcap \frac{x^2}{2}$.

In the English of J. M. Child, this starts out
*It will be useful to write* $\int$ *for omn., so that*

$$\int \ell \sqcap \operatorname{omn} \ell, \text{ or the sum of the } \ell\text{'s}.$$

Here omn is an abbreviation of *omnia*, which for Leibniz means an infinite sum of all of something. He writes $\sqcap$ for $=$. Instead of parentheses he writes lines over expressions. He hasn't got around yet to writing $\int \ldots dx$, or to using functional notation. These all come later. He uses a kind of "dummy" variable $a$ because he says that homogeneous expressions make errors easier to spot. Curiously, one of these $a$'s is missed in Gerhardt's transcription, and subsequent authors continue this error because the manuscript is not available to them.

Leibniz then goes on to say that

$$\frac{(\int \ell)^2}{2} \sqcap \int \left( \int \ell \right),$$

or in our only slightly different terminology

$$\frac{x^2}{2} \sqcap \int x\, dx .$$

He also exhibits a version of what we know as integration by parts that eventually gives him:

$$\int x^2\, dx \sqcap \frac{x^3}{3} .$$

References for this material include:

- Carl Immanuel Gerhardt, **Der Briefwechsel von Gottfried Wilhelm Leibniz mit Mathematikern**, Berlin, 1899.

  This is a transcription of Leibniz's original Latin. The passage we are concerned with is to be found on p. 154. As you can see from the image above, Gerhardt's task cannot have been easy. It ought not to be very surprising that an $a$ was left out of one denominator.
- J. M. Child, **The Early Mathematical Manuscripts of Leibniz**, Open Court, 1920.

  The passage at hand is on p. 80.
- André Weil, "A review of 'Leibniz in Paris 1672–1676' by Joseph Hofmann," *Bulletin of the AMS*, 1975.

  This an extremely enjoyable account.
- C. H. Edwards Jr., **The Historical Development of the Calculus**, Springer, 1979.

  Pages 252–254 are about this memorandum.

The first three of these can be found on the Internet for free download.

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