Consider now

$$\gamma = \phi(1) + \phi(\phi(1)) + \cdots .$$

Clearly $\gamma < \Omega_k$ since Ω_k is not cofinal with ω. But then from the definition of γ, $f(\gamma) \geqq \gamma$, an evident contradiction; this completes the proof.

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ON A PROBLEM OF G. BIRKHOFF

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In his book *Lattice theory*, G. Birkhoff proposed to prove that the representation of a finite partially ordered system as the product of indecomposable factors is unique within pairwise isomorphism of factors.[1] The present short note is to show that this is not the case in general. A simple counterexample, and indeed one of the simplest, perhaps, can be constructed as follows:

Let X be the lattice $\{0, 1\}$ of two elements 0, 1 $(0<1)$, for instance, and A be the partially ordered system

$$I + X + X^2 + X^3 + X^4 + X^5,$$

where I resp. X^i stands for the one-lattice resp. the direct product of i copies of X, and where $+$ means direct summation. The finite partially ordered system A may be expressed also by $f(X)$ with the polynomial $f(x) = 1+x+x^2+x^3+x^4+x^5$. Since every X^i has the up to isomorphism unique decomposition into indecomposable factors, $X^i = XX \cdots X$ (i factors), one sees easily that direct decompositions of A are, in the sense of isomorphism, in 1-1 correspondence with factorizations of polynomial $f(x) = 1+x+x^2+x^3+x^4+x^5$ into factors with non-negative rational integral coefficients. But our $f(x)$ has two distinct decompositions into factors which are irreducible in the prescribed sense, namely

$$f(x) = (1 + x)(1 + x^2 + x^4) = (1 + x^3)(1 + x + x^2).$$

Two direct decompositions

$$A = (I + X)(I + X^2 + X^4) = (I + X^3)(I + X + X^2)$$

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[1] G. Birkhoff, *Lattice theory*, Amer. Math. Soc. Colloquium Publications, vol. 25, New York, 1940, Problem 8.

of A give thus a counter-example to the conjecture.

Nevertheless the conjecture is true in the important special case of directly sum-indecomposable systems, as will be shown in a forthcoming paper by one of the writers.[2] Making use of the result we may formulate a criterion of unique decomposition as follows: First we decompose the given system into the direct sum of sum-indecomposable systems, then decompose the so-obtained summands into a product of (product-)indecomposable factors, construct a polynomial by taking a variable for each isomorphism type of such (product-)indecomposable factors, and finally examine whether the factorization of the polynomial (with several variables, perhaps) into factors irreducible within the domain of polynomials with non-negative rational integral coefficients is unique.[3]

We want also to suggest a generalization of the notion of partially ordered system by allowing negative coefficients, as in the case of algebraic complex in topology. This change of notion will make Birkhoff's conjecture true.

Added in proof: Our above example, communicated to Professor Birkhoff, has been incorporated into the revised second edition of *Lattice theory*, p. 26.

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[2] J. Hashimoto, *On the product decomposition of partially ordered set*, Mat. Japonicae vol. 1 (1949).

[3] This clarifies two extreme cases cited by Birkhoff, loc. cit.