$$c_q(A_{qij} - A_{qji}) = 0 \tag{13}$$

is a necessary condition for the permanence of vector-lines. It is also sufficient, as we see from consideration of the order of the equations involved and the fact that if (13) holds, then $\Omega_{ij}=0$ implies $\partial\Omega_{ij}/\partial t=0$.

Hence we have this result: *A necessary and sufficient condition for the permanence of the vector-lines of c_i is* (13), *or equivalently, that the tensor c_qA_{qij}, that is,*

$$c_j(c_{i,k}v_k + \partial c_i/\partial t) + c_iv_{j,k}c_k,$$

shall be symmetric.

DUBLIN INSTITUTE FOR ADVANCED STUDIES

NOTE ON A THEOREM IN SUMMABILITY

J. D. HILL

Let T denote a regular matrix method of summability in the complex domain, that is to say, a transformation of the form

$$t_n = \sum_{k=1}^{\infty} a_{nk}s_k \qquad (n = 1, 2, 3, \cdots),$$

having the property that the convergence of $\{s_k\}$ to s always implies the existence of t_n for each n and the convergence of $\{t_n\}$ to s. It is well known that the following conditions of Silverman-Toeplitz are necessary and sufficient in order that T be regular: $a_{nk}=o(1)$ $(n\to\infty;$ $k=1, 2, 3, \cdots);$ $\sum_{k=1}^{\infty} a_{nk}=1+o(1)$ $(n\to\infty)$; and

$$\sum_{k=1}^{\infty} | a_{nk} | = O(1) \qquad (n \to \infty). \tag{1}$$

The following theorem was established recently by Henstock [2].[1]

THEOREM (HENSTOCK). *Let $y\equiv\{z_k\}$ be a given bounded sequence of complex numbers. Then there exist denumerably many sequences of*

Received by the editors July 5, 1950.

[1] Numbers in brackets refer to the references at the end of the paper.

zeros and ones, depending only on y, such that the summability of each of these by any regular matrix method T implies the T-summability of y.

The proof given by Henstock follows classical lines and is moderately complicated. The purpose of this note is to give a shorter and simpler proof based on the theory of linear operations [**1**]. To this end we consider the sequences $X(t) \equiv \{\alpha_k\}$ of zeros and ones, with infinitely many ones, where $t \equiv .\alpha_1\alpha_2\alpha_3 \cdots$ (radix 2). It is known [**3**] that the set of all sequences $X(t)$ constitutes a fundamental set in the Banach space (b) of all bounded complex sequences $x \equiv \{s_k\}$ with $\|x\| \equiv \sup_k |s_k|$. In other words, for $p = 1, 2, 3, \cdots$ and an arbitrarily given $y \in (b)$ there exist a finite number of complex constants $A_1^p, A_2^p, \cdots, A_{m_p}^p$ and corresponding sequences $X(t_1^p), X(t_2^p), \cdots, X(t_{m_p}^p)$ such that $\|y - [A_1^p X(t_1^p) + A_2^p X(t_2^p) + \cdots + A_{m_p}^p X(t_{m_p}^p)]\| < 1/p$. If y_p denotes the sequence in brackets we therefore have $y_p \to y$ in the norm of (b). Let T be a regular matrix method and write $f_n(x) = \sum_{k=1}^{\infty} a_{nk}s_k$ $(n = 1, 2, 3, \cdots)$ where $x = \{s_k\} \in (b)$. For each n, $f_n(x)$ is a linear operation on (b) to the space of complex numbers, and $\|f_n\| = \sum_{k=1}^{\infty} |a_{nk}|$ [**3**]. If each of the *denumerably many* sequences $X(t_i^p)$ $(i = 1, 2, \cdots, m_p; p = 1, 2, 3, \cdots)$ is summable-T, then the sequence y_p is summable-T, that is, the sequence $\{f_n(y_p)\}$ is convergent for each p. Since the sequence of norms $\{\|f_n\|\}$ is bounded by (1) and $y_p \to y$, it follows (as in [**1**, p. 79, proof of Theorem 3]) that the sequence $\{f_n(y)\}$ is convergent. This completes the proof.

References

1. S. Banach, *Théorie des opérations linéaires*, Warsaw, 1932.

2. R. Henstock, *The efficiency of matrices for bounded sequences*, J. London Math. Soc. vol. 25 (1950) pp. 27–33.

3. H. J. Hamilton and J. D. Hill, *Operation theory and multiple sequence transformations*, Duke Math. J. vol. 8 (1941) pp. 154–162.

MICHIGAN STATE COLLEGE