## PROCEEDINGS

OF THE<br>\title{ American Mathematical Society }<br>EDITED BY<br>GUSTAV A. HEDLUND<br>G. P. HOCHSCHILD

A. C. SCHAEFFER

WITH THE ASSISTANCE OF
C. B. ALLENDOERFER
P. T. BATEMAN
R. H. BING
J. B. DIAZ
N. J. FINE
A. W. GOODMAN
I. M. SHEFEER

MAX SHIFFMAN
E. H. SPANIER

VOLUME 2, NUMBER 6
DECEMBER, 1951

PUBLISHED BY THE SOCIETY
MENASHA, WIS., AND PROVIDENCE, R. I.

## Journals Published by the

American Mathematical Society

## Bulletin of the American Mathematical Society

This journal is the official organ of the Society. It reports official acts of the Society and the details of its meetings, with abstracts of all research papers presented. It contains some of the officially invited addresses presented before the Society. It contains also announcements and notes on matters of current interest, and reviews of advanced mathematical books.

One volume of six numbers is planned for 1951. The subscription price is $\$ 7.00$. Sets of volumes of the BULLETIN are available as follows: vols. $1-36$ (1894-1930), $\$ 7.00$ each; vols. 37-53 (1931-1947), \$9.00 each; vols. 54-55 (1948-1949), \$11.00 each; vols. 56-57 (1950-1951), \$7.00 each.

## Proceedings of the American Mathematical Society

This journal is devoted entirely to research in pure and applied mathematics, and is devoted principally to the publication of original papers of moderate length. Volume 1 appeared in 1950.

One volume of six numbers is planned for 1951 . The subscription price is $\$ 11.00$. Vols. 1-2 (1950-1951) are available at the price of $\$ 11.00$ each.

## Transactions of the American Mathematical Society

This journal is devoted entirely to research in pure and applied mathematics, and includes in general longer memoirs than the Proceedings.

Normally, two volumes of three numbers each are published annually. The price per volume is $\$ 6.00$. Volumes $1-36(1900-1934)$ consist of four numbers each. Volumes 66-67 (1949) consist of two numbers each. Sets of volumes are available as follows: vols. 1-21 (1900-1920), \$7.00 each; vols. 22-32 (1921-1930), \$8.00 each; vols. 33-36 (1931-1934), $\$ 9.00$ each; vols. 37-71 (1935-1951), $\$ 6.00$ each.

## Mathematical Reviews

This journal contains abstracts and reviews of the current mathematical literature of the world. It is sponsored by thirteen mathematical organizations, located both in the United States and abroad.

The publication of Mathematical Reviews was begun in 1940. It appears monthly, excepting July, and the price per annual volume is $\$ 20.00$. Orders for complete volumes only are accepted. Volumes $1-12(1940-1951)$ are available at $\$ 20.00$ per volume.

The Proceedings of the American Mathematical Society is published bimonthly, in February April, June, August, October, and December. Price per annual volume is $\$ 11.00$. Subscriptions, orders for back numbers, and inquiries in regard to nondelivery of current numbers should be addressed to the American Mathematical Society, 450 Ahnaip Street, Menasha, Wis., or 80 Waterman Street, ProviAmerican Ma
dence 6, R. I.

Entered as second-class matter at the post office at Menasha, Wisconsin. Authorization is granted under the authority of the act of August 24, 1912, as amended by the act of August 4, 1947 (sec. 34.21, P. L. \& R.).

## INDEX OF VOLUME 2

Agnew, R. P. Mean values and Frullani integrals, 237.
Amitsur, S. A. Nil PI-rings, 538.
Amitsur, A. S., and Levitzki, J. Remarks on minimal identities for algebras, 320.
Apostol, T. M. Remark on the Hurwitz zeta function, 690.
Apostol, T. M., and Zuckerman, H. S. On magic squares constructed by the uniform step method, 557.
Arens, R. The adjoint of a bilinear operation, 839.
Aucoin, A. A. Systems of Diophantine equations, 760.
Aulbach, H. Some geometrical inequalities for sets in Hilbert space, 36.
Bang, T. A solution of the "plank problem," 990.
Bell, J. H. A note on the solution of the unilateral matrix equation, 553.
Blackwell, D. The range of certain vector integrals, 390.
Boas, R. P., and Chandrasekharan, K. Addendum: Derivatives of infinite order, 422.
Brafman, F. Generating functions of Jacobi and related polynomials, 942.
Breusch, R. On the distribution of the roots of a polynomial with integral coefficients, 939.

Brown, B. An extension of the Jacobson radical, 114.
Bruck, R. H. On a theorem of R. Moufang, 144.
Bruck, R. H., and Kleinfeld, E. The structure of alternative division rings, 878.
Buck, R. C. A factoring theorem for homomorphisms, 135.
Cairns, S. S. An elementary proof of the Jordan-Schoenflies theorem, 860.
Cameron, R. H. The first variation of an indefinite Wiener integral, 914.
Campbell, H. E. An extension of the "principal theorem" of Wedderburn, 581.
Carleson, L. On Bernstein's approximation problem, 953.
Carr, R. E. Pattern integration with improper Riemann integrals, 925.
Carr, R. E., and Hill, J. D. Pattern integration, 242.
Carruth, P. W. Sums and products of ordered systems, 896.
Chandrasekharan, K. See Boas, R. P.
Cheng, M. T. On a theorem of Nicolesco and generalized Laplace operators, 77.
Chernoff, H. An extension of a result of Liapounoff on the range of a vector measure, 722.
Chevalley, C. On a theorem of Gleason, 122.
Chevalley, C., and Kolchin, E. Two proofs of a theorem on algebraic groups, 126.
Civin, P. Approximation to conjugate functions, 207.
Clifford, A. H. A noncommutative ordinally simple linearly ordered group, 902.
Cohn, H. On finiteness conditions for a convex body, 544.

- On the finite determination of critical lattices, 547.

Court, L. M. A theorem on conditional extremes with an application to total differentials, 423.

Cowling, V. F. On the analytic continuation of Newton series, 28.

- On the distribution of the values of the partial sums of a Taylor series, 732.

Darling, D. A. Sums of symmetrical random variables, 511.
Davis, P., and Pollak, H. A theorem for kernel functions, 686.
Dieudonne, J. Algebraic homogeneous spaces over fields of characteristic two, 295.
Dinkines, F. Semi-automorphisms of symmetric and alternating groups, 478.
Edwards, R. E. On derivative and translational bases for periodic functions, 644.
Ellis, H. W. On the compatibility of the approximate Perron and the Cesdro-Perron integrals, 396.

Erdélyi, A. The general form of hypergeometric series of two variables, 374.
Erdös, P. On a theorem of Rådström, 205.
Feldman, C. The Wedderburn principal theorem in Banach algebras, 771.
Fine, N. J. Note on the Hurwitz zeta-function, 361.
Finkbeiner, D. T. A general dependence relation for lattices, 756.
Flanders, H. Elementary divisors of $A B$ and $B A, 871$.
Fort, M. K. A note on pointwise convergence, 34.

- A characterization of plane light open mappings, 175.

Freilich, G. On sets of constant width, 92.
Fulks, W. A generalization of Laplace's method, 613.
——On the solutions of the heat equation, 973.
Gardner, G. H. F. Canonical coordinates at a point for two skew-symmetric tensors, 328.
Gelbaum, B. A nonabsolute basis for Hilbert space, 720.
Gelbaum, G., Kalisch, G. K., and Olmsted, J. M. H. On the embedding of topological semigroups and integral domains, 807.
Gnanadoss, A. A. Linear difference equations with periodic coefficients, 699.
Goldstine, H. H., and von Neumann, J. Numerical inverting of matrices of high order. II, 188.
Goodman, A. W. Typically-real functions with assigned zeros, 349 .
Gottschalk, W. H. Choice functions and Tychonoff's theorem, 172.
Graue, L. C. A necessary and sufficient condition that a curve lie on a hyperquadric, 607.
Grosswald, E. On the integration scheme of Maréchal, 706.
Guggenheimer, H. A note on curvature and Betti numbers, 867.
Haimo, F. Some limits of Boolean algebras, 566.
Halperin, I. Non-measurable sets and the equation $f(x+y)=f(x)+f(y), 221$.
Hamilton, H. J. Vector subseries of maximum modulus, 87.
Hamilton, O. H. A fixed point theorem for pseudo-arcs and certain other metric continua, 173.

Hammer, P. C. The centroid of a convex body, 522.
-Convex bodies associated with a convex body, 781.
Hamstrom, M. E. Linear independence in abelian groups, 487.
Hanner, O., and Rådström, H. A generalization of a theorem of Fenchel, 589.
Hartman, P., and Wintner, A. On the behavior of Fourier sine transforms near the origin, 398.
Heins, M. A residue theorem for finite Blaschke products, 622.

- Interior mapping of an orientable surface into $S^{2}, 951$.

Herzog, F., and Piranian, G. On the univalence of functions whose derivative has a positive real part, 625.
Higman, D. G. Lattice homomorphisms induced by group homomorphisms, 467.
Hill, J. D. Note on a theorem in summability, 372.

- See Carr, R. E.

Hirschman, I. I., and Widder, D. V. On the products of functions represented as convolution transforms, 97.
Householder, A. S. Polynomial iterations to roots of algebraic equations, 718.
Hughes, N. J. S. The use of bilinear mappings in the classification of groups of class 2, 742.

Jacobson, N. Completely reducible Lie algebras of linear transformations, 105.
Johnson, R. E. The extended centralizer of a ring over a module, 891.
Jones, F. B. Certain homogeneous unicoherent indecomposable continua, 855.

Jonsson, B. A Boolean algebra without proper automorphisms, 766.
Kadison, R. V. Order properties of bounded self-adjoint operators, 505.
Kalisch, G. K. See Gelbaum, B.
Karush, W. Determination of the extreme values of the spectrum of a bounded self-adjoint operator, 980.
Kingsley, E. H. Bernstein polynomials for functions of two variables of class $C^{(k)}, 64$.
Kleinfeld, E. See Bruck, R. H.
Koehler, F. Note on a theorem of Gelfand and Šilov, 541.
Kolchin, E. See Chevalley, C.
Koopman, B. O. A probabilistic generalization of matric Banach algebras, 404.
Kuipers, L. On the representation of integers by sums of polynomials, 750.
Lelong, P. On a problem of M. A. Zorn, 12.
Leutert, W. On the convergence of approximate solutions of the heat equation to the exact solution, 433.
LeVeque, W. J. Note on the transcendence of certain series, 401.
Levine, J. Collineations in Weyl spaces of two dimensions, 264.
-Collineations in generalized spaces, 447, 993.

- Motions in linearly connected two-dimensional spaces, 932.

Levitzki, J. See Amitsur, A. S.
Lindgren, B. W. An integral on a space of continuous functions, 634.
Lorentz, G. G. Deferred Bernstein polynomials, 72.
Loud, W. S. Functions with prescribed Lipschitz condition, 358.
——The probability of a correct result with a certain rounding-off procedure, 440.
MacDuffee, C. C. Families of Lorentzian matrices, 794.
Makar, B. H. See Makar, R. H.
Makar, R. H., and Makar, B. H. On algebraic simple monic sets of polynomials, 526.
Mautner, F. I. On the decomposition of unitary representations of Lie groups, 490.
Mickle, E. J. Fréchet and Kerêkjartó equivalence, 518.
Miles, E. P. Certain properties of functions harmonic within a sphere, 213.
Miller, K. S. Self-adjoint factorizations of differential operators, 704.
Mitchell, J. A theorem in the geometry of matrices, 276.
Moise, E. E. A note of correction, 838.
Montgomery, D., and Zippin, L. Two-dimensional subgroups, 822.
Moore, E. F. Convexly generated $k$-dimensional measures, 597.
Moore, J. T. Division algebras over fields of formal power series, 874.
Moser, L. A theorem on quadratic residues, 503.
Mostow, G. D. A theorem on locally euclidean groups, 285.
Muhly, H. T. Integral bases and varieties multiply of the first species, 576.
Myers, S. B. Functional uniformities, 153.
Nakayama, T. Remark on the duality for noncommutative compact groups, 849.
von Neumann, J. See Goldstine, H. H.
Niven, I. Functions which represent prime numbers, 753.
Olmsted, J. M. H. See Gelbaum, B.
Ore, O. Some remarks on commutators, 307.
Papoulis, A. On the density theorem, 709.
Parker, E. T. On a question raised by Garrett Birkhoff, 901.
Peck, J. E. L. An ergodic theorem for a noncommutative semigroup of linear operators, 414.

Pettis, B. J. Remarks on a theorem of E. J. McShane, 166.
-On the continuity of parametric linear operations, 455.

Phillips, R. S. On one-parameter semi-groups of linear transformations, 234.

- A note on ergodic theory, 663.

Piranian, G. See Herzog, F.
du Plessis, N. A note about the derivatives of Legendre polynomials, 950.
Pollak, H. See Davis, P.
Pollard, H. The closure of translations in $L^{p}, 100$.
Price, G. B. Bounds for determinants with dominant principal diagonal, 497.
Rado, T. A remark on chain-homotopy, 458.
Rådström, H. See Hanner, 0.
Rajagopal, C. T. A note on generalized Tauberian theorems, 335.
Ramaswami, V. Number of integers in an assigned $A, P \leqq x$ and prime to primes greater than $x^{c}, 318$.
Reade, M. O. A characterization of minimal surfaces in isothermic representation, 47.
——On a mass distribution associated with a class of polynomials, 55.
Rechard, O. W. A note on the summability of infinite series by sequence to sequence and series to sequence transformations, 730.
Redheffer, R. M. Remarks on incompleteness of $\left\{e^{i \lambda_{n} x}\right\}$, nonaveraging sets, and entire functions, 365.
Riordan, J. Triangular permutation numbers, 429.
Robinson, R. M. Arithmetical definitions in the ring of integers, 279.
Rodeja F., E. G. Note on a lemma of A. W. Goodman, 314.
Ruderman, H. D. An extension of the nomogramic instrument, 262.
Rudin, W. A theorem on subharmonic functions, 209.
_- Positive infinities of potentials, 967.
——Green's second identity for generalized Laplacians, 970.
Ryser, H. J. A combinatorial theorem with an application to latin rectangles, 550.
Schafer, R. D. A theorem on the derivations of Jordan algebras, 290.
Schwartz, J. A note on the space $L_{p}^{*}, 270$.
Scott, W. R. Algebraically closed groups, 118.
Seth, B. R. Boundary conditions interpreted as conformal transformation, 1.
Shah, S. M. Some theorems on meromorphic functions, 694.
Shanks, D. A short proof of an identity of Euler, 747.
Sherman, S. Non-negative observables are squares, 31.
——Games and sub-games, 186.
Smiley, M. F. On the ideals and automorphisms of non-associative rings, 138.
Spitzbart, A. On the minimum of a certain integral, 246.
Spragens, W. H. On series of Walsh eigenfunctions, 202.
Stein, S. K. Convex maps, 464.
Straus, E. G. On the polynomials whose derivatives have integral values at the integers, 24.
Sunouchi, G. I. On the Walsh-Kaczmarz series, 5.
Sunouchi, G. I., and Yano, S. Notes on Fourier analysis. XXX. On the absolute convergence of certain series of functions, 380.
Swain, R. L. Approximate isometries in bounded spaces, 727.
Swingle, P. M. The closure of types of connected sets, 178.
Synge, J. L. On permanent vector-lines in $N$ dimensions, 370.
Thrall, R. M. On the projective structure of a modular lattice, 146.
Ullman, J. L. Two mapping properties of schlicht functions, 654.
Utz, W. R. Powers of a matrix over a lattice, 305.
Valentine, F. A. Arcwise convex sets, 159.

- A characterization of simply connected closed arcwise convex sets, 778.

Varma, R. S. On Appell polynomials, 593.
Verblunsky, S. On the shortest path through a number of points, 904.
Vinti, J. P. Note on a series of products of three Legendre polynomials, 19.
Walsh, J. L. On Rouchê's theorem and the integral-square measure of approximation, 671, 993.

Note on the location of the critical points of a real rational function, 682.
Warschawski, S. E. On conformal mapping of regions bounded by smooth curves, 254.
Widder, D. V. See Hirschman, I. I.
Wilansky, A. The bounded additive operation on Banach space, 46.

- Norms of matrix types for the spaces of convergent and bounded sequences, 738.

Williams, C. W. Recurrence and incompressibility, 798.
Wing, G. M. Averages of the coefficients of schlicht functions, 658.
Wintner, A. See Hartman, P.
Yano, S. On approximation by Walsh functions, 962.

- See Sunouchi, G. I.

Yood, B. On fixed points for semi-groups of linear operators, 225.
Young, G. S. A generalization of the Rutt-Roberts theorem, 586.
Zippin, L. See Montgomery, D.
Zuckerman, H. S. See Apostol, T. M.

# Books Published by the <br> American Mathematical Society 

Colloquium Publications
35. A. C. Schaeffer and D. C. Spencer, Coefficient regions for schlicht func- tions, 1950, xvi, 311 pp . $\$ 6.00$
34. J. L. Walsh, The location of critical points of analytic and harmonic functions, 1950, viii, 384 pp . 6.00
33. J. F. Ritt, Differential algebra, 1950, viii, 184 pp . ..... 4.40
32. R. L. Wilder, Topology of manifolds, 1949, x, 402 pp . ..... 7.0031. E. Hille, Functional analysis and semi-groups, 1948, xii, 528 pp.
7.50
30. T. Rad6, Length and area, 1948, vi, 572 pn . ..... 6.75
29. A. Weil, Foundations of algebraic geometry, 1946, xx, 288 pp. ..... 5.50
28. G. T. Whyburn, Analytic topology, 1942; reprinted, 1948, x, 280 pp . ..... 4.75
27. S. Lefshetz, Algebraic topology, 1942; reprinted, 1948, vi, 393 pp.
26. N. T.evinson, Gap and density theorems, 1940, viii, 246 pp .
6.004.00
25. G. Birkhoff, Lattice theory, 1940; enlarged and completely revised edi- tion, 1949, xiv, 283 pp . ..... 6.00
24. A. A. Albert, Structure of algebras, 1939, xii, 210 pp. ..... 4.00
23. G. Szegö, Orthogonal polynomials, 1939; reprinted, 1949, x, 403 pp. ..... 6.00
22. C. N. Moore, Summable series and convergence factors, 1938, vi, 105 pp. ..... 2.00
21. J. M. Thomas, Differential systems, 1937, x, 118 pp . ..... 2.00
20. J. L. Walsh, Interpolation and approximation by rational functions in the complex domain, 1935, x, 382 pp . ..... 6.00
19. R. E.A. C. Paley and N. Wiener, Fourier transforms in the complex domain, 1934; reprinted, 1949, viii, 184 pp . +portrait plate. ..... 3.75
18. M. Morse, The calculus of variations in the large, 1934; reprinted, 1948, $\mathrm{x}, 368 \mathrm{pp}$. ..... 5.75
17. J. H. M. Wedderburn, Lectures on matrices, 1934; reprinted, 1946; re- printed, 1949, x, 205 pp . ..... 3.35
16. G. A. Bliss, Algebraic functions, 1933; reprinted, 1948, x, 220 pp. ..... 3.75 ..... 3.75
15. M. H. Stone, Linear transformations in Hilbert space and their applica- tions to analysis, 1932 ; reprinted, 1951, viii, 622 pp . ..... 8.00
14. J. F. Ritt, Differential equations from the algebraic standpoint, 1932; reprinted, 1948, x, 172 pp . ..... 3.00
10. A. B. Coble, Algebraic geometry and theta functions, 1929; reprinted, 1948, viii, 282 pp. ..... 4.00
9. G. D. Birkhoff, Dynamical systems, 1927 ; reprinted, 1948, viii, 295 pp.
9. G. D. Birkhoff, Dynamical systems, 1927 ; reprinted, 1948, viii, 295 pp. ..... 4.25 ..... 4.25
8. L. P. Eisenhart, Non-Riemannian geometry, 1927; reprinted, 1949, viii, 184 pp. ..... 2.70
3. E. Kasner, Differential-geometric aspects of dynamics, 1913; reprinted, 1934; reprinted, 1948, iv, 117 pp. ..... 2.50
Mathematical Surveys
6. C. Chevalley, Introduction to the theory of algebraic functions of one variable, 1951, xii, 188 pp. ..... $\$ 4.00$
5. S. Bergman, The kernel function and conformal mapping, 1950, viii, 161 pp. ..... 4.00
4. O. F. G. Schilling, The theory of valuations, 1950, viii, 253 pp . ..... 6.00
3. M. Marden, The geometry of the zeros of a polynomial in a complex variable, 1949, x, 181 pp. ..... 5.00
2. N. Jacobson, The theory of rings, 1943; reprinted, 1949, vi, 151 pp . ..... 3.35

1. J. A. Shohat and J. D. Tamarkin, The problem of moments, 1943; re- printed, 1950, xiv, 144 pp . ..... 3.35
Semicentennial Publications
2. E. T. Bell, J. F. Ritt, N. Wiener, E. J. McShane, T. Y. Thomas, R. L. Wilder, G. C. Evans, J. L. Synge, and G. D. Birkhoff, Semicentennial addresses, 1938 , vi, 315 pp . ..... $\$ 4.50$
3. R. C. Archibald, A semicentennial history of the American Mathe- matical Society 1888-1938, with biographies and bibliographies of the past presidents, 1938 , xii, $261 \mathrm{pp} .+30$ plates. ..... 3.00
Proceedings of Symposia in Applied Mathematics
4. Electromagnetic theory, 1950, iv, 91 pp . ..... $\$ 3.00$ ..... $\$ 3.00$
Non-linear problems in mechanics of continua, 1949, iv, 219 pp. ..... 5.25

The price to members of the American Mathematical Society is $25 \%$ less than list. Volumes which are not listed above are out of print.

## CONTENTS

Vol. 2, No. 6 DECEMBER, 1951 Whole No. 12
The adjoint of a bilinear operation. By Richard Arens ..... Page
Remark on the duality for noncommutative compact groups. By Tadasi Nakayama ..... 849
Certain homogeneous unicoherent indecomposable continua. By F. B. Jones ..... 855
An elementary proof of the Jordan-Schoenflies theorem. By S. S. Cairns ..... 860
A note on curvature and Betti numbers. By H. GugGenheimer. ..... 867
Elementary divisors of $A B$ and $B A$. By Harley Flanders ..... 871
Division algebras over fields of formal power series. By J. T. Moore ..... 874
The structure of alternative division rings. By R. H. Bruck and Erwin Kleinfeld ..... 878
The extended centralizer of a ring over a module. By R. E. Johnson ..... 891
Sums and products of ordered systems. By P. W. Carruth ..... 896
On a question raised by Garrett Birkhoff. By E. T. Parker. ..... 901
A noncommutative ordinally simple linearly ordered group. By A. H. Clifford ..... 902
On the shortest path through a number of points. By S. Ver- BLUNSKY ..... 904
The first variation of an indefinite Wiener integral. By R. H. CAMERON ..... 914
Pattern integration with improper Riemann integrals. By R. E. Carr: ..... 925
Motions in linearly connected two-dimensional spaces. By Jack Levine ..... 932
On the distribution of the roots of a polynomial with integral coefficients. By Robert Breusch. ..... 939
Generating functions of Jacobi and related polynomials. By Fred Brafman ..... 942
A note about the derivatives of Legendre polynomials. By N. DU Plessis ..... 950
Interior mapping of an orientable surface into $S^{2}$. By Maurice Heins ..... 951
On Bernstein's approximation problem. By Lennart Carleson ..... 953
On approximation by Walsh functions. By Shigeki Yano ..... 962
Positive infinities of potentials. By Walter Rudin ..... 967
Green's second identity for generalized Laplacians. By Walter Rudin ..... 970
On the solutions of the heat equation. By W. Fulks ..... 973
Determination of the extreme values of the spectrum of a bounded self-adjoint operator. By William Karush ..... 980
A solution of the "plank problem." By Thøger Bang ..... 990
Errata, Volume 2. ..... 993
Index of Volume 2 ..... 994

[^0]
[^0]:    Articles for insertion in the Proceedings should be typewritten, double spaced, and the author should keep a complete copy. Papers in algebra and theory of numbers should be sent to G. P. Hochschild, Leet Oliver Memorial Hall, Yale University, New Haven 11, Conn.; in analysis to A. C. Schaeffer, North Hall, University of Wisconsin, Madison 6, Wis.; in other fields to G. A. Hedlund, Leet Oliver Memorial Hall, Yale University, New Haven 11, Conn. All other communications to the editors should be addressed to the Managing Editor, G. A. Hedlund, Leet Oliver Memorial Hall, Yale University, New Haven 11, Conn.

