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A NOTE ON UNSTABLE HOMEOMORPHISMS[1]

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In [1] W. R. Utz introduced the concept of an *unstable*[2] *homeomorphism* and raised the question of whether there exists an unstable homeomorphism of a compact continuum onto itself. In this note an example of such an homeomorphism will be given.

Let C denote the complex unit circle and for each $z\in C$, let $g(z)=z^2$. Then $g:C$ onto C determines an inverse limit space $\Sigma_2 = \{(a_0, a_1, a_2, \cdots) \mid$ for each non-negative integer i, $a_i\in C$ and $g(a_{i+1})=a_i\}$. For $a, b\in\Sigma_2$, the function $\rho(a, b)=\sum_{i=0}^{\infty} |a_i-b_i|/2^i$ is a metric for Σ_2; Σ_2 is familiar as the "two-solenoid," and is a compact, indecomposable continuum. Define $f:\Sigma_2$ onto Σ_2 as follows: for each $a=(a_0, a_1, \cdots)\in\Sigma_2$, let $f(a)=[g(a_0), g(a_1), \cdots]$. Then $f(a) =(a_0^2, a_1^2, \cdots)=(a_0^2, a_0, a_1, \cdots)$, $f^{-1}(a)=(a_1, a_2, a_3, \cdots)$, and f is a homeomorphism of Σ_2 onto Σ_2.

To show that f is unstable, suppose that $a=(a_0, a_1, \cdots)$ and $b=(b_0, b_1, \cdots)$ are distinct points of Σ_2. Consider, as Case 1, that $a_0\neq b_0$. Let $e^{i\theta}=a_0$, $e^{i\phi}=b_0$, where $0\leqq\theta, \phi<2\pi$. Then there exists a non-negative integer n such that the angle between the terminal rays of $2^n\theta$ and $2^n\phi$ is greater than $\pi/2$. Then $\rho[f^n(a), f^n(b)]\geqq |a_0^{2^n}-b_0^{2^n}| =|e^{i2^n\theta}-e^{i2^n\phi}|>1$.

Case 2: for some integer $n>0$, $a_n\neq b_n$, but $a_i=b_i$, for $0\leqq i<n$. Then $f^{-n}(a)=(a_n, a_{n+1}, a_{n+2}, \cdots)$, $f^{-n}(b)=(b_n, b_{n+1}, b_{n+2}, \cdots)$, and there-

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[2] A homeomorphism f of a compact metric space X onto X is said to be *unstable* provided there exists a fixed positive number δ, such that if x and y are distinct points of X, then there exists an integer n, such that $\rho[f^n(x), f^n(y)]$ is greater than δ.

fore $\rho[f^{-n}(a), f^{-n}(b)] \geqq |a_n - b_n|$, which is equal to 2, because $a_n^2 = b_n^2$. Therefore f is unstable.

Other examples. If, instead, we take $C = [0, 1]$, and $g: C$ onto C defined by

$$g(x) = \begin{cases} 2x, & \text{for} \quad 0 \leqq x \leqq 1/2 \\ 2 - 2x & \text{for} \quad 1/2 < x \leqq 1, \end{cases}$$

then the inverse limit space is a well known indecomposable continuum that can be embedded in the plane. The homeomorphism f, defined just as above, is unstable relative to the metric defined as above, and thus would be so relative to the metric in the plane. Furthermore, as in both examples, f leaves a point fixed, two such continua could be joined at their fixed point, yielding an example in which the space is a decomposable continuum.

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