REMARK ON MY PAPER "ON A THEOREM OF J. L. WALSH"

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The theorem in the paper mentioned above, which appeared in these Proceedings (vol. 7 (1956) pp. 363–366), should be reworded to read as follows:

Let $f(x)$ and $f_n(x)$ $(n=1, 2, \cdots)$ be p times differentiable in the interval $a<x<b$ and let

$$\lim_{n=\infty} \operatorname*{Inf}_{x\in I, y\in I} \left| f_n(y) - f(x) \right| = 0 \tag{2}$$

for every open sub-interval I of (a, b). Then, given any $x_0\in(a, b)$ and any open sub-interval (α, β) of (a, b) containing x_0 the sequence $N=\{n\}$ can be written as a union of two (not necessarily both infinite) sequences $N_1=\{n_1\}$ and $N_2=\{n_2\}$ in such a way that for every n_1 there exists $x_{n_1}\in(\alpha, \beta)$ for which

$$f_{n_1}^{(p)}(x_{n_1}) = f^{(p)}(x_0) \tag{3}$$

while, if N_2 is infinite, we have

$$\limsup_{n_2=\infty} \int_{x_0-h}^{x_0+h} \left| f_{n_2}^{(p)}(x) - f^{(p)}(x) \right| dx = o(h) \tag{5}$$

as $0<h\to 0$. Moreover, if x_0 is not a local extremum point (in the wide sense) of $f^{(p)}(x)$ then the sequence N_2 may be taken as finite and the x_{n_1} satisfying (3) *may be taken so that we have*

$$\lim_{n_1=\infty} x_{n_1} = x_0. \tag{4}$$

The author is greatly indebted to Professor G. R. MacLane for calling his attention to the necessity of restating the theorem. An interesting supplement to the above is given in a note by G. R. MacLane (see this issue of Proceedings pp. 897–898).

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