This proves $xy \in A$ and similarly $yx \in A$. Since I_f is obviously closed under addition, it is a two sided ideal. An application of Lemma 3 proves that A is commutative.

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A NOTE ON VALUED LINEAR SPACES

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Banaschewski [1] has given a simple and elegant proof of Hahn's embedding theorem for ordered abelian groups. His method can be used to prove the author's generalization of Hahn's theorem [2, p. 11]. In this note we make use of Banaschewski's method to prove a special case of the author's theorem (which is also a generalization of Hahn's theorem) that has been proven by Gravett [3].

Let (L, Δ, d) be a valued linear space [3]. That is, L is a vector space over a division ring K, Δ is a linearly ordered set with minimum element θ, and d is a mapping of L onto Δ such that for all $x, y \in L$, $d(x) = \theta$ if and only if $x = 0$, $d(x) = d(kx)$ for all $0 \neq k \in K$, and $d(x+y) \leqq \mathrm{Max} \ [d(x), d(y)]$. For each $\delta \in \Delta$, let $C^{\delta} = \{x \in L : d(x) \leqq \delta\}$ and let $C_{\delta} = \{x \in L : d(x) < \delta\}$. Let W be the vector space of all mappings f of Δ into the join of the spaces C^{δ}/C_{δ} for which $f(\delta) \in C^{\delta}/C_{\delta}$ and $R_f = \{\delta \in \Delta : f(\delta) \neq C_{\delta}\}$ is an inversely well ordered set. W is a subspace of the unrestricted direct sum V of the C^{δ}/C_{δ}. W is also a valued linear space (W, Δ, d'), with $d'(f)$ the largest $\delta \in R(f)$.

THEOREM. *There exists an isomorphism $x \to \bar{x}$ of L into W. Moreover, if $d(x) = \alpha$, then $\bar{x}(\alpha) = C_\alpha + x$ and $\bar{x}(\delta) = C_\delta$ for all $\alpha < \delta \in \Delta$. Thus this isomorphism is value-preserving.*

PROOF. Let $\mathfrak{S}$ be the set of all subspaces of L. There exists [1, Lemma 4, p. 431] a mapping π of $\mathfrak{S}$ into $\mathfrak{S}$ such that for all $C, D \in \mathfrak{S}$, $C \oplus \pi(C) = L$ and if $C \subseteq D$, then $\pi(C) \supseteq \pi(D)$. For $x \in L$ and $\delta \in \Delta$, let $\bar{x}(\delta) = C_\delta + x_\delta$, where $L = C^\delta \oplus \pi(C^\delta)$ and x_δ is the component of x in C^δ. Then $\bar{x} \in V$ and it follows easily that the mapping $x \to \bar{x}$ is an isomorphism of L into V. If $\alpha = d(x) < \delta$, then $x \in C^\alpha \subseteq C_\delta$, hence $\bar{x}(\alpha) = C_\alpha + x$ and $\bar{x}(\delta) = C_\delta$. To complete the proof it suffices to show that $R_{\bar{x}}$ is an inversely well ordered set (for each $0 \neq x$ in L). Let Γ be a nonempty subset of $R_{\bar{x}}$, and let C be the join of the C^γ for all γ in Γ. $L = \pi(C) \oplus C$, and $x = y + z$ where $y \in \pi(C)$ and $z \in C$. $d(z)$ is the greatest element in Γ. For if $d(z) < \gamma \in \Gamma$, then $\bar{z}(\gamma) = C_\gamma$, and since $\pi(C^\gamma) \supseteq \pi(C)$, $\bar{y}(\gamma) = C_\gamma$. But then $C_\gamma = \bar{y}(\gamma) + \bar{z}(\gamma) = \bar{x}(\gamma) \neq C_\gamma$.

If, as in [2], Δ is partially ordered and d is multiple valued, then practically the same proof gives the embedding theorem [2, p. 11].

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