

PROCEEDINGS  
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## Journals Published by the American Mathematical Society

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This journal is the official organ of the Society. It reports official acts of the Society and details of its meetings. It contains some of the officially invited addresses presented before the Society, research announcements and problems, and reviews of advanced mathematical books.

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Papers in algebra and number theory should be sent to ALEX ROSENBERG, Department of Mathematics, University of California, Berkeley 4, California; in probability, real variables, logic, and foundations to P. R. HALMOS, Eckhart Hall, University of Chicago, Chicago 37, Illinois; in abstract analysis to either P. R. HALMOS or ALEX ROSENBERG; in geometry and topology to E. H. SPANIER, Department of Mathematics, University of California, Berkeley 4, California; in other branches of analysis, applied mathematics, and all other fields to R. P. BOAS, Lunt Building, Northwestern University, Evanston, Illinois. All other communications to the editors should be addressed to the Managing Editor, P. R. HALMOS.

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This journal contains abstracts and reviews of the current mathematical literature of the world. It is sponsored by thirteen mathematical organizations, located both in the United States and abroad.

The publication of MATHEMATICAL REVIEWS was begun in 1940. Starting in 1961, it appears monthly, in two parts, and the price per annual volume is \$50.00. Prior to 1961 it appeared in eleven single issues. Orders for complete volumes only are accepted. Volumes 10 and 17 are not available. Volumes 1-9, 11-16, and 18-21 are available at the following prices: Vols. 1-14 (1940-1953), \$24.00 each; all other volumes \$50.00.

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### Notices of the American Mathematical Society

This journal announces the programs of the meetings of the Society. It carries the abstracts of all contributed papers presented at the meetings of the Society and publishes news items of interest to mathematical scientists.

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All communications should be addressed to the Editor, G. L. WALKER, 190 Hope Street, Providence 6, R. I. News items and insertions for each issue must be in the hands of the editor on or before the deadline for the abstracts of the papers to be presented in the meetings announced in that issue. These deadlines are published regularly on the back of the table of contents.

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