AN INEQUALITY FOR NONNEGATIVE ENTIRE FUNCTIONS[1]

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I give a simple proof of an inequality equivalent to one that was proved by S. Bernstein [1]. The proof also applies in higher dimensions, where Bernstein's method is not available.

THEOREM 1. *Let $f(z)$ be an entire function of exponential type τ, nonnegative and integrable on the real axis. Then*

$$f(x) \leqq (2\pi)^{-1}\tau \int_{-\infty}^{\infty} f(x)dx. \tag{1}$$

There is equality for $f(z) = z^{-2} \sin^2 \tau z/2$.
We have (see, e.g., [2, p. 103])

$$f(x) = \int_{-\tau}^{\tau} \phi(t)e^{ixt}dt. \tag{2}$$

A special case of Poisson's summation formula,[2] applied to (2), yields

$$\tau\phi(0) = \sum_{n=-\infty}^{\infty} f((2n\pi + x)/\tau). \tag{3}$$

Since the terms on the right are nonnegative, none of them can exceed $\tau\phi(0)$, which is the right-hand side of (1).

Now let $f(z, w)$ be an entire function of exponential type, absolutely integrable for real (z, w). We then have [5]

$$f(x, y) = \int_S \int \phi(t, u)e^{ixt+iyu}dtdu, \tag{4}$$

where S is a bounded convex set determined by the growth of $f(z, w)$. Consider lattices $\{a_{11}m+a_{21}n,\ a_{12}m+a_{22}n\}$ such that S is inside a lattice parallelogram that contains $(0, 0)$ and has area $4(a_{11}a_{22}-a_{12}a_{21}) = 4 \det[a]$; let $4D$ be the area of the smallest such parallelogram.

THEOREM 2. *If $f(x, y) \geqq 0$ then*

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[1] This was written while the author was President's Fellow at Northwestern University.

[2] The abstract form of Poisson's formula is given in [4, p. 153]; the special cases used here can be found in [3].

$$f(x, y) \leqq (2\pi)^{-2} D \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy. \tag{5}$$

If $[A]$ is the matrix $2\pi[a]^{-1}$ the analogue of (3) is

$$D\phi(0, 0) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(A_{11}m + A_{12}n + x, A_{21}m + A_{22}n + y),$$

and (5) follows in the same way as (1).

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2. R. P. Boas, Jr., *Entire functions*, Academic Press, New York, 1954.
3. S. Bochner, *Vorlesungen über Fouriersche Integrale*, Akademische Verlagsgesellschaft, Leipzig, 1932.
4. L. H. Loomis, *An introduction to abstract harmonic analysis*, Van Nostrand, New York, 1953.
5. M. Plancherel and G. Pólya, *Fonctions entières et intégrales de Fourier multiples* Comment. Math. Helv. **9** (1937), 224–248; **10** (1938), 110–163.

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REPRESENTATIONS OF BANACH SPACES[1]

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Banach and Mazur[2] proved that every separable Banach space B can be represented as the space $C(M)$ of continuous real functions on a compact metric space M. Since M is the continuous image of the Cantor set K, $C(M)$ can be imbedded in $C(K)$, and since functions in $C(K)$ can be extended preserving norm to functions over I^1, they conclude that B can be represented as a subspace of $C(I^1)$.

If B is not separable, it can be represented as $C(H)$, where H is compact Hausdorff. A compact Hausdorff space is the continuous image of some totally disconnected compact Hausdorff space T—for example, give the space the discrete topology, and let T be its Stone-Čech compactification. It follows that B is isomorphic to a subspace of $C(T)$. If T could be given a linear order inducing the same topology, we could fill in the missing intervals and obtain a compact con-

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[2] S. Banach, *Théorie des opérations linéaires*, Warsaw, 1932.