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AMERICAN MATHEMATICAL SOCIETY

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Bulletin of the American Mathematical Society

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Mathematical Reviews

This journal is devoted to abstracts and reviews of the current mathematical literature of the world. Two volumes will be published in 1963, Volume 25 and Volume 26. Each volume will consist of 6 regular issues plus an index issue. In each regular issue the abstracts and reviews are grouped under subject headings. Publication began in 1940.

Subscription price per volume of 7 issues is $50.00; 2 volumes per year $100.00. Back issues: volumes 1–16 (1940–1955) $42.00 per volume; volumes 17–24 (1956–1962) $50.00 per volume.

Soviet Mathematics—Doklady

This journal contains the entire pure mathematics section of the DOKLADY AKADEMII NAUK SSSR in translation. It appears six times a year, each bimonthly issue corresponding to one volume of the Soviet DOKLADY. (The DOKLADY AKADEMII NAUK SSSR is issued three times a month, six issues constituting a volume.)

Current subscriptions are as follows: Domestic, $17.50; foreign, $20.00. Single issues are $5.00. Volumes 1, 2 and 3 (1960–1962) are available at $30.00 per volume or $5.00 per issue.

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This is a cover to cover translation into English of Acta Mathematica Sinica, published by Academia Sinica Peking, People's Republic of China. Acta Mathematica Sinica contains current research in all fields of pure mathematics. Two volumes, Vols. 2 and 3, are planned for 1963. Vol. 1 (1962) $30.00. Subscription price per volume is $20.00.

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This journal announces the programs of the meetings of the Society. It carries the abstracts of all contributed papers presented at the meetings of the Society and publishes news items of interest to mathematical scientists.

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