where $\tau = 1 - y$. Now as $y \to 1$, $\tau \to 0$, and it is known [2] that the last integral approaches $f(x)$ almost everywhere as $\tau \to 0$. This proves (10).

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ON HYPONORMAL OPERATORS

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A bounded linear operator T on a Hilbert space $\mathfrak{H}$ is said to be *hyponormal* in case $\|T^*x\| \leqq \|Tx\|$ for all $x \in \mathfrak{H}$. This short note gives a negative answer to the question raised in [1, p. 188]: "Does there exist a completely continuous hyponormal operator which is not normal?"

Theorem. *If T is hyponormal, $\|T^n\| = \|T\|^n$ for all n.*

Proof. It is sufficient to prove that $\|T\| = 1$ implies $\|T^n\| = 1$ for all n. Consider the following property:

(C_n) For every $\epsilon > 0$, there exists a unit vector x such that

$$\|T^n x\| \geqq 1 - \epsilon \text{ and } \|T^n x - T^* T^{n+1} x\| \leqq \epsilon.$$

(C_0) just says that 1 is an approximate proper value for the self-adjoint operator T^*T (see [1, p. 170]). (C_n) obviously implies $\|T^n\| = 1$. Now suppose that (C_n) is valid. For $\epsilon > 0$ and x (indicated in (C_n))

$$\begin{aligned}
\|T^{n+1}x - T^*T^{n+2}x\|^2 \\
&= \|T^{n+1}x\|^2 - 2\|T^{n+2}x\|^2 + \|T^*T^{n+2}x\|^2 \\
&\leqq \|T^n x\|^2 - \|T^{n+2}x\|^2 \text{ (because } \|T\| = \|T^*\| = 1) \\
&\leqq \|T^n x\|^2 - \|T^*T^{n+1}x\|^2 \text{ (because } T \text{ is hyponormal)} \\
&\leqq \|T^n x - T^*T^{n+1}x\| \{\|T^n x\| + \|T^*T^{n+1}x\|\} \leqq 2\epsilon \text{ by } (C_n).
\end{aligned}$$

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Also

$$\|T^{n+1}x\| \geqq \|T^{n+2}x\| \geqq \|T^*T^{n+1}x\|$$
$$\geqq \|T^n x\| - \epsilon \geqq 1 - 2\epsilon \text{ by } (\mathrm{C}_n).$$

Since $\epsilon > 0$ is arbitrary, (C_{n+1}) is valid.

COROLLARY 1. *Every nonzero hyponormal operator has a nonzero element in its spectrum.*

This follows from the above theorem via the known fact that the spectral radius of an operator T is equal to $\lim_{n\to\infty}\sqrt[n]{\|T^n\|}$.

COROLLARY 2. *Every completely continuous hyponormal operator is normal.*

PROOF. Let T be hyponormal and completely continuous. In view of a known property of a hyponormal operator (see [1, p. 168]) it is sufficient to prove that the set of all proper vectors for T is total, in other words, the set $\mathfrak{M}$ of all vectors orthogonal to every proper vector consists of only the null vector. Since $\mathfrak{M}$ reduces T (see [1, p. 168]), the restriction of T to $\mathfrak{M}$, denoted by $T|\mathfrak{M}$, is also hyponormal. The spectrum of $T|\mathfrak{M}$ consists of 0 only, for $T|\mathfrak{M}$ is completely continuous and has no proper value by the definition of $\mathfrak{M}$. By the above corollary this means that $T|\mathfrak{M}=0$ or $\mathfrak{M}=\{\theta\}$. The former is obviously excluded.

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