SHORTER NOTES

The purpose of this department is to publish very short papers of an unusually elegant and polished character, for which there is normally no other outlet.

A PROOF OF THE NONRETRACTIBILITY OF A CELL ONTO ITS BOUNDARY

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By appealing to the simplicial approximation theorem [2, p. 64] it suffices to prove that there is no simplicial retraction of a subdivision of a closed n-simplex E onto its boundary ∂E.

Suppose $f: E \to \partial E$ is a simplicial retraction. Let a be the barycenter of an $(n-1)$-simplex $A \subset \partial E$. The point is this: *$f^{-1}(a)$ is a compact one-dimensional manifold whose boundary is contained in ∂E. The component of $f^{-1}(a)$ containing a is thus a broken line segment with one endpoint at a; but the other endpoint cannot exist.* It would have to be a point of ∂E different from a which maps onto a under f, contradicting the assumption that $f|\partial E$ is the identity.

The proof that $f^{-1}(a)$ has the stated properties is simple and classical (cf. [3]). Any n-simplex B mapping onto A has precisely two faces mapping onto A, so that $B \cap f^{-1}(a)$ is the line segment in B joining the barycenters of the two faces. These line segments fit together to form a manifold whose boundary is in ∂E because every $(n-1)$-simplex C of E is incident to either one or two n-simplices, according to whether $C \subset \partial E$ or not.

The same proof applies if E is a compact triangulated manifold with boundary ∂E. More generally, the proof works if E is a finite n-dimensional complex such that each $(n-1)$-simplex is a face of not more than two n-simplices, and ∂E is the union of those $(n-1)$-simplices incident to at most one n-simplex.

In the case where E is a compact differentiable manifold, one may use the differentiable approximation theorem in place of the simplicial one, and take a to be a regular value. This of course requires a theorem such as Brown's [1, Theorem 3.III], Dubovitski's [4, Theorem 4], or Sard's [5] on the existence of regular values.

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