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CERTAIN PROBLEMS OF DIFFERENTIABLE IMBEDDING

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1. The differentiable imbedding of the complex projective spaces has been studied by many authors [1; 3; 4; 5]. In this note we shall deal with the nonimbeddability of the submanifolds of a complex projective space. It was studied in [6] in a particular case.

We denote by $P_n(c)$ the complex projective space of complex dimension n. Let V_{2n-2} be a differentiable compact orientable submanifold of $P_n(c)$ corresponding to a cohomology class $v \in H^2(P_n(c), Z)$. Then the Pontrjagin class of V_{2n-2} is determined as follows [2]:

$$j\colon V_{2n-2} \to P_n(c), \tag{1.1}$$

$$\begin{aligned} &1 + p_1(V_{2n-2}) + p_2(V_{2n-2}) + \cdots \\ &\qquad = j^*[(1 + p_1(P_n(c)) + p_2(P_n(c)) + \cdots)(1 + v^2)^{-1}] \end{aligned} \tag{1.2}$$

where p_i denotes the Pontrjagin class of the dimension $4i$. We put as follows:

$$p = \sum_{k \geq 0} (-1)^k p_k = \prod_\alpha (1 - \gamma_\alpha), \tag{1.3}$$

$$\bar{p} = \sum_{k \geq 0} \bar{p}_k = \prod_\alpha (1 - \gamma_\alpha)^{-1}, \tag{1.4}$$

$$p \cdot \bar{p} = 1. \tag{1.5}$$

In the case of $P_n(c)$ we have

$$p = (1 - g_n^2)^{n+1}, \qquad g_n \in H^2(P_n(c), Z), \tag{1.6}$$

$$\bar{p} = (1 - g_n^2)^{-n-1}. \tag{1.7}$$

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Hence we have from (1.2), (1.4) and (1.7)

$$p(V_{2n-2}) = j^*[(1 - v^2)(1 - g_n^2)^{-n-1}]. \tag{1.8}$$

When $v=\lambda g$, where λ denotes some integer, we have from (1.8)

$$\begin{aligned} p(V_{2n-2}) &= j^*[(1 - \lambda^2 g_n^2)(1 - g_n^2)^{-n-1}] \\ &= j^*\Big[1 + (n + 1 - \lambda^2) g_n^2 + \cdots \\ &\quad + \left\{\frac{(n+1) \cdots (n+r)}{r!} - \frac{(n+1) \cdots (n+r-1)}{(r-1)!}\lambda^2\right\} g_n^{2r} \\ &\quad + \cdots\Big]. \end{aligned} \tag{1.10}$$

Meanwhile, if a compact orientable differentiable manifold V_m is differentiably imbedded in an $(m+q)$-dimensional euclidean space E_{m+q}, it must be that

$$p_k = 0, \qquad 2k \geqq q + 1. \tag{1.11}$$

Moreover, $p_k=0$ if $2k \geqq q$, since if $q=2k$, $p_k=E^2$, where E is the Euler class of the normal bundle, and $E=0$ in such a case.

Let us examine the term of the highest dimension in (1.10). When $n=2m+1$, it is

$$\left\{\frac{(2m + 2) \cdots (3m + 1)}{m!} - \frac{(2m + 2) \cdots (3m)}{(m - 1)!}\lambda^2\right\} g_{2m+1}^{2m}. \tag{1.12}$$

When $m>1$ the quantity (1.12) never vanishes. Hence we have from (1.11)

THEOREM 1. *Any compact orientable differentiable submanifold V_{4m} $(m>1)$ of $P_{2m+1}(c)$ cannot be differentiably imbedded in the E_{6m}.*

When $n=2m$, the term of the highest dimension in (1.10) is as follows:

$$\left\{\frac{(2m + 1) \cdots (3m - 1)}{(m - 1)!} - \frac{(2m + 1) \cdots (3m - 2)}{(m - 2)!}\lambda^2\right\} g_{2m}^{2m-2}. \tag{1.13}$$

If $m \neq 3$, the quantity (1.13) does not vanish. Hence we have

THEOREM 2. *Any compact orientable differentiable submanifold V_{4m-2} $(m \neq 3)$ of $P_{2m}(c)$ cannot be differentiably imbedded in the E_{6m-4}.*

The exceptional cases for above theorem are those where $m=3$ and

$\lambda = \pm 2$. Even those submanifolds cannot be imbedded in the E_{12}, because the coefficient of g_6^2 in (1.10) does not vanish.

2. The following theorem is available for our purpose:

THEOREM (ATIYAH-HIRZEBRUCH [4]). *Let X_{2n} be a differentiable manifold and its integral Stiefel-Whitney class w_3 be zero. If there exists a Chern character $z \in \mathrm{Ch}(X_{2n})$ whose $s(z)$ is odd, then such an X_{2n} cannot be imbedded in the sphere whose dimension is $4n-2\alpha(n)$, where $\alpha(n)$ denotes the number of* 1 *in the diadic expansion of n. In particular, if there exists a cohomology class $d \in H^2(X_{2n}, Z)$ whose $d^n(x_{2n})$ is odd, then X_{2n} cannot be imbedded in the sphere whose dimension is $4n-2\alpha(n)$.*

Let V_{2n-2} be a submanifold of $P_n(c)$, i.e.,

$$j\colon V_{2n-2} \to P_n(c) \tag{2.1}$$

and

$$v = \lambda g_n, \tag{2.2}$$

where v denotes the cohomology class corresponding to V_{2n-2}. Then we have

$$d = j^*(g_n), \tag{2.3}$$

$$d^{n-1}[V_{2n-2}] = j^*(g_n^{n-1})[V_{2n-2}] = (vg_n^{n-1})[P_n(c)] = \lambda(g_n^n)[P_n(c)] = \lambda. \tag{2.4}$$

Such a V_{2n-2} satisfies the condition $w_3=0$, because $w(P_n(c))$ lacks the terms of odd dimension and the normal bundle of V_{2n-2} has the same property. Hence we have from (2.4)

THEOREM 3. *If λ is odd, the submanifold of $P_n(c)$, corresponding to λg_n, cannot be differentiably imbedded in the $E_{4n-4-2\alpha(n-1)}$.*

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