by x and y ; then if $V \cap L^2 = 0$ we are done by Theorem 1. If $z \in V \cap L^2$ and $z \neq 0$, then x and z still generate L . In this case $L = (x) + L^2$ so z has the form $z = \sum [a_i x + h_i, b_i x + h'_i]$ with a_i, b_i in K and h_i, h'_i in L^2 . This means that z is in L^3 . But any element of L^2 is a sum of words in x and z and each word must contain z i.e., $L^2 \subseteq L^3$ contrary to hypothesis.

HARVARD UNIVERSITY AND
WESTERN RESERVE UNIVERSITY

A REMARK ON SEMI-SIMPLE LIE ALGEBRAS

GORDON BROWN

The following observation seems to be lacking in the literature.

THEOREM. *The sum of the squares of the lengths of the roots of a semi-simple Lie algebra is equal to the dimension of a Cartan subalgebra.*

As usual the lengths of the roots are taken with respect to the non-degenerate Killing form $\text{Tr ad } x \text{ ad } y$.

PROOF. No generality is lost in assuming the ground field algebraically closed.

Consider the matrix M whose entries are the inner products (α, β) , where α, β range over all the roots with respect to the Cartan subalgebra H . We have

$$(\alpha, \beta) = \text{Tr ad } h_\alpha \text{ ad } h_\beta = \sum_\gamma \gamma(h_\alpha) \gamma(h_\beta) = \sum_\gamma (\alpha, \gamma) (\gamma, \beta)$$

where h_α denotes the dual of α ; i.e., $\alpha(h) = (h_\alpha, h)$ for all h in H . Thus $M^2 = M$ and $\text{Tr } M = \text{rank } M$. Hence $\sum_\alpha (\alpha, \alpha) = \dim H$.

UNIVERSITY OF ILLINOIS

Received by the editors March 25, 1963.