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A-GENUS AND INDECOMPOSABILITY OF DIFFERENTIABLE MANIFOLDS

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Introduction. In the previous paper [1] we have studied the conditions on which a differentiable manifold be indecomposable and cited many examples of indecomposable manifolds. In this paper we shall study the relations between A -genus and indecomposability of a differentiable manifold.

1. Hereafter we denote by X_n an n -dimensional compact orientable differentiable manifold. If $X_n = X_r \cdot X_s$, we say that X_n is *decomposable* and if not, we say that X_n is *indecomposable*. If $X_{4n} = X_n \cdot X_s$, we have

$$(1.1) \quad A(X_{4n}) = A(X_r)A(X_s);$$

where $A(X)$ denotes the A -genus of X and we define as follows:

$$(1.2) \quad A(X_n) = 0, \quad n \not\equiv 0 \pmod{4}.$$

If r and s are divisible by 4, the relation (1.1) follows from the general property of multiplicative series [2, p. 75]. According to the cobordism theory, the cobordism components of X_r ($r \not\equiv 0 \pmod{4}$) consist only of torsions. Hence the product $X_r \cdot X_s$ also consists only of torsions. Therefore $A(X_r \cdot X_s)$ is zero. Thus (1.1) holds in general. Meanwhile Atiyah and Hirzebruch have proved the following:

THEOREM 1 (ATIYAH AND HIRZEBRUCH [3]). *If X_{4n} is differentially imbedded in the $(8n-2q)$ -sphere, then $A(X_{4n})$ is divisible by 2^{q+1} . If moreover $q \equiv 2 \pmod{4}$, then $A(X_{4n})$ is divisible by 2^{q+2} .*

It is well known that an X_n is always differentially imbedded in the $2n$ -sphere. Hence we have from the above theorem

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$$(1.3) \quad A(X_{4n}) \equiv 0 \pmod{2}.$$

We have from (1.1) and (1.3):

THEOREM 2. *If $A(X_{4n}) \equiv 2 \pmod{4}$, such an X_{4n} is indecomposable.*

For example $P_{2^m}(c)$ is indecomposable, because

$$A(P_{2^m}(c)) = (-1)^m \binom{2m}{m}$$

(see [3]). We have proved the above theorem for the cases where $n=2, 3, 4$. We shall find another example of Theorem 2 in the next paragraph.

2. Let $P_{2m+1}(c)$ be the complex projective space of complex dimension $2m+1$. The total Pontryagin class of $P_{2m+1}(c)$ takes the form

$$(2.1) \quad p = (1 + g^2)^{2m+2}, \quad g \in H^2(p_{2m+1}(c), \mathbb{Z}).$$

Let X_{4m} be a compact orientable differentiable submanifold of $P_{2m+1}(c)$ and let λg be the cohomology class corresponding to the homology class represented by X_{4m} . The A -genus of X_{4m} is given by

$$(2.2) \quad A(X_{4m}) = \prod_i \frac{2\sqrt{\nu_i}}{\sinh 2\sqrt{\nu_i}}$$

where

$$(2.3) \quad p(X_{4m}) = \prod_i (1 + \nu_i).$$

Hence we have

$$(2.4) \quad A(X_{4m}) = \left[\frac{\sinh 2\lambda g}{2} \left(\frac{2g}{\sinh 2g} \right)^{2m+2} \right] [P_{2m+1}(c)] \quad [2, \text{p. 86}].$$

Taking

$$(2.5) \quad g^{2m+1}[P_{2m+1}(c)] = 1$$

into account, we have

$$(2.6) \quad A(X_{4m}) = \frac{1}{2\pi i} \int \frac{1}{z^{2m+2}} \left(\frac{\sinh 2\lambda z}{2} \right) \left(\frac{2z}{\sinh 2z} \right)^{2m+2} dz,$$

where the integrand should be integrated along a small circle around $z=0$ anti-clockwise. Changing the variable as follows:

$$(2.7) \quad \sinh 2z = t$$

we have

$$(2.8) \quad \begin{aligned} A(X_{4m}) &= \frac{2^{2m}}{2\pi i} \int \frac{\sinh 2\lambda z}{t^{2m+2}(1+t^2)^{1/2}} dt \\ &= \frac{2^{2m}}{2\pi i} \int \frac{((1+t^2)^{1/2}+t)^\lambda - ((1+t^2)^{1/2}-t)^\lambda}{2t^{2m+2}(1+t^2)^{1/2}} dt \end{aligned}$$

where the integrand should be integrated along the small circle around $t=0$ anti-clockwise. We have from (2.8):

$$(2.9) \quad \begin{aligned} A(X_{4m}) &= \frac{2^{2m}}{2\pi i} \int \frac{1}{t^{2m+2}} \left\{ \lambda t(1+t^2)^{(\lambda/2)-1} + \binom{\lambda}{3} t^3(1+t^2)^{(\lambda/2)-2} \right. \\ &\quad \left. + \binom{\lambda}{5} t^5(1+t^2)^{(\lambda/2)-3} + \dots \right\} dt \\ &= 2^{2m} \left\{ \lambda \binom{(\lambda/2)-1}{m} + \binom{\lambda}{3} \binom{(\lambda/2)-2}{m-1} \right. \\ &\quad \left. + \binom{\lambda}{5} \binom{(\lambda/2)-3}{m-2} + \dots + \binom{\lambda}{2m+1} \right\} \\ &= 2^{2m} \lambda(\lambda-2) \dots (\lambda-2m) \left\{ \frac{1}{2^m m!} + \frac{\lambda-1}{3! 2^{m-1} (m-1)!} \right. \\ &\quad \left. + \dots + \frac{(\lambda-1)(\lambda-3) \dots (\lambda-2m+1)}{(2m+1)!} \right\} \\ &= 2^{2m} \lambda f(\lambda). \end{aligned}$$

It is clear from (2.8) and (2.9) that

$$(2.10) \quad f(\lambda) = f(-\lambda)$$

and

$$(2.11) \quad \begin{aligned} f(2) &= f(4) = \dots = f(2m) = f(-2) \\ &= f(-4) = \dots = f(-2m) = 0. \end{aligned}$$

Therefore $f(\lambda)$ takes the form

$$(2.12) \quad f(\lambda) = c(\lambda-2)(\lambda-4) \dots (\lambda-2m)(\lambda+2)(\lambda+4) \dots (\lambda+2m).$$

Letting $\lambda=1$ in (2.9) we have

$$(2.13) \quad c = 1/2^m m! 3 \cdot 5 \dots (2m+1).$$

Hence we have from (2.9), (2.12) and (2.13)

$$(2.14) \quad A(X_{4m}) = \frac{2^m \lambda}{m!} \prod_{i=1}^m \left(\frac{\lambda^2 - 4i^2}{2i + 1} \right).$$

If $m = 2^s$ ($s \geq 1$) and λ is odd we have from (2.14)

$$(2.15) \quad A(X_{4m}) \equiv 2 \pmod{4}.$$

Hence we have from Theorem 2

THEOREM 3. *Let X_{4m} be a submanifold of $P_{2m+1}(c)$ corresponding to λg . Then X_{4m} is indecomposable provided that $m = 2^s$ ($s \geq 1$) and λ is odd.*

Next we consider the case where $w_2 = 0$. It is known that if $w_2 = 0$, then $\hat{A}(X_{4n}) = 2^{4n} A(X_{4n})$ is an integer, i.e., $A(X_{4n})$ is divisible by 2^{4n} [4]. We see from (2.14) that if $w_2 = 0$, then λ must be even. Thus we have

THEOREM 4. *If a submanifold X_{4n} of $P_{2m+1}(c)$ corresponding to λg has the vanishing w_2 , then λ is even.*

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