COROLLARY. *If S is a finite groupoid which satisfies (1), (2) and (3), then S is an M -groupoid.*

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THE INDEX PROBLEM FOR INFINITE SYMMETRIC GROUPS

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Let M be an infinite set with cardinal X , $S(X, Y) = \{\sigma: \sigma \text{ is a permutation on } M \text{ such that } |\text{spt } \sigma| < Y\}$, where $\text{spt } \sigma = \{m \in M: \sigma(m) \neq m\}$. If X is a cardinal, denote its successor by X^* . Onofri [2] proved that $S(d, d^*)$ has no proper subgroups of finite index and $S(d, d)$ has precisely one, the alternating group $A(d)$. These results have been extended by Higman [1] and Scott [3]. Higman has shown that $S(X, d)$ has only one proper subgroup of index less than X , the alternating group $A(X)$, and $A(X)$ has no proper subgroups of index less than X . If Z is a cardinal such that $Z^Z < X$, Scott proved that $S(X, Y)$ for $Y > d$ has no subgroups of index less than or equal Z .

In this paper, the following generalization of these results is proven.

THEOREM. *If $d < Y \leq X^*$, $S(X, Y)$ has no proper subgroups of index less than X .*

LEMMA 1. *If $d \leq Z < Y$, $[S(X, Y): H] < X$, then H contains an element σ such that $|\text{spt } \sigma| = Z$ and $|M \setminus \text{spt } \sigma| = X$.*

PROOF. $H \cap S(X, Z^*)$ has index less than X in $S(X, Z^*)$ for any $Z < Y$. Since the index of $S(X, Z)$ in $S(X, Z^*)$ is greater than or equal X , there is $\sigma \in H$ such that $|\text{spt } \sigma| = Z$. If $Z < X$, then $|M \setminus \text{spt } \sigma| = X$. If $Z = X$, there are disjoint sets M_1 and M_2 such that $M = M_1 \cup M_2$ and $|M_1| = |M_2| = X$. Let $G = \{\sigma: \text{spt } \sigma \subset M_1\}$. Then $H \cap G$ has index less than X in G , hence there is $\sigma \in H \cap G$ such that $|\text{spt } \sigma| = X$ and since $\text{spt } \sigma \subset M_1$, $|M \setminus \text{spt } \sigma| = X$.

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LEMMA 2. Let Σ be a family of conjugate elements of $S(X, X^*)$ with disjoint supports and $|\Sigma| = X$. If H is a subgroup of $S(X, Y)$ with index less than X and H contains one element of Σ , then $\Sigma \subset H$.

PROOF. Let $G_0 = \{\sigma: \sigma\Sigma\sigma^{-1} = \Sigma\}$ and $H_0 = H \cap G_0$. There is a homomorphism ϕ from G_0 onto the group S of all permutations on Σ such that for each $\tau \in G_0$ and $\sigma \in \Sigma$, $\phi(\tau)$ maps σ onto $\tau\sigma\tau^{-1}$. The image of H_0 under ϕ has index less than X in S . If S' denotes the subgroup of S of permutations with finite support, then $\phi(H_0) \cap S'$ has index less than X in S' , hence is transitive on Σ . Thus if one member σ of Σ belongs to H , then all members of Σ are conjugate to σ by members of H_0 , hence $\Sigma \subset H$.

LEMMA 3. If $\sigma \in H$, $|M \setminus \text{spt } \sigma| = X$ and H is a subgroup of $S(X, Y)$ with index less than X , then H contains all conjugates of σ .

PROOF. Suppose τ is a conjugate of σ such that $\text{spt } \sigma \cap \text{spt } \tau = \emptyset$ and $|M \setminus \text{spt } \sigma \setminus \text{spt } \tau| = X$. Then by Lemma 2, $\tau \in H$. If ρ is any conjugate of σ , then ρ may be joined to σ by a chain of at most four elements such that consecutive elements satisfy the condition on τ and σ in the first statement of this paragraph. Thus $\rho \in H$.

Now suppose H is a subgroup of $S(X, Y)$ with index less than X , $d < Y \leq X^*$. For each cardinal Z , $d \leq Z < Y$, by Lemmas 1 and 3, H contains the conjugacy class of an element σ with $|\text{spt } \sigma| = Z$. Thus H contains $S(X, Z^*)$ for any $Z < Y$. Hence H contains $S(X, Y)$.

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