ON $(n-1)$-DIMENSIONAL FACTORS OF I^n

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We recall that the cone $C(H)$ over a space H is defined as the product $H\times[0, 1]$ with each $(h, 0)$, $h\in H$, identified to a point v called the vertex. The part of $C(H)$ that corresponds to $H\times[0, 1]$ is the open cone, denoted by $OC(H)$, over H.

THEOREM 1. *Let K denote the cone over a compact Hausdorff space H. If the vertex of K has a neighborhood equivalent to E^n then $K\times I$ is a topological cube.*

As an immediate consequence, we obtain an affirmative answer to a question raised by Kwun and Raymond [7].

THEOREM 2. *For each $n\geqq 5$, there exists a space X such that $X\times I=I^n$ and* Bd X *contains no $(n-2)$-cell.*

Here, equality means topological equivalence. It may be noted that Theorem 2 does not hold for $n\leqq 4$ by a theorem due to Bing [1].

PROOF OF THEOREM 1. A proof that $OC(H)=E^n$ is available in various works such as [6]. Hence by [8, Proposition 3.1] Bd $K=H$ and Int $K=OC(H)$ as a generalized manifold with boundary, so we must have [11] $\text{Bd}(K\times I)=(\text{Bd } K\times I)\cup(K\times\text{Bd } I)$, and $\text{Int}(K\times I)=\text{Int } K\times\text{Int } I=OC(H)\times E^1=E^{n+1}$.

First, we observe $H\times E^1=OC(H)$ minus vertex$=S^{n-1}\times E^1$. By taking two point compactification, we see $\text{Bd}(K\times I)=$suspension over $H=S^n$. Note that uniqueness of this compactification follows from the uniqueness of one point compactification.

Each point of K has a neighborhood homeomorphic to $H\times I$ or E^n. To prove that $K\times I$ is a manifold with boundary we merely need to show that $H\times I^2$ is a manifold with boundary. But, this follows from the facts: (1) $H\times E^1=S^{n-1}\times E^1$, and (2) each point of I^2 has a neighborhood equivalent to $E^1\times I$.

Now, $\text{Bd}(K\times I)$ is collared [3]. Hence $K\times I$, being an $(n+1)$-cell plus an $(n+1)$-annulus around it, is a cell by the generalized Schoenflies theorem [2].

PROOF OF THEOREM 2. Let Y be the one point compactification of $(n-2)$-dimensional factor of E^{n-1}, $n\geqq 5$, described by Kwun [5] or Rosen [12]. Clearly, the boundary of cone X over Y contains no $(n-2)$-cell but suspension over Y is an S^{n-1}. Hence by Theorem 1 $X\times I=I^n$.

Received by the editors April 29, 1963.

REMARKS. It will not be difficult to verify that our most non-euclidean factor X is, in fact, the hyper space of an upper semi-continuous decomposition of I^{n-1} having densely distributed arcs in Bd I^{n-1} as nondegenerate elements. On the other hand, examples of Poénaru [**10**], Mazur [**9**] and Curtis [**4**] show that not all factors of cubes are conical neighborhoods. It will be interesting to know if $X \times I = I^n$ implies that Bd X is collared.

Finally, we wish to thank K. W. Kwun for his kind advice and information.

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