that to every sequence $\alpha_1, \alpha_2, \cdots$ (with $\alpha_1 = 1$) of nonnegative integers there exists exactly one real number in the interval $[0, 1)$ to which the given sequence $\alpha_1, \alpha_2, \cdots$ corresponds.

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A SHORT PROOF OF AN INEQUALITY FOR THE PERMANENT FUNCTION

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Let A be a substochastic matrix, i.e., a square matrix of nonnegative numbers with each row sum no greater than 1. We have obtained a lower bound for the permanent of $I-A$.

THEOREM. *If A is a substochastic matrix, then*

$$\operatorname{per}(I - A) \geqq 0.$$

It was brought to our attention by Marcus and Minc [2] that Brualdi and Newman have proved this theorem. Indeed, two proofs of this theorem are contained in a paper that will appear in the Oxford Quarterly [1]. The proof that we shall give, shorter than and quite different from the Brualdi-Newman proofs, shows that this theorem is almost a corollary of the Ryser representation of the permanent.

Let B be an n-square matrix and let B_r denote a matrix obtained from B by replacing some r columns of B by zero columns. Let $S(B_r)$ be the product of the row sums of the matrix B_r. Ryser [3] has proved that the permanent of B is given by

$$\begin{aligned}\operatorname{per}(B) = S(B_0) &+ \sum (-1)S(B_1) + \sum (-1)^2 S(B_2) + \cdots \\ &+ \sum (-1)^{n-1} S(B_{n-1}),\end{aligned}$$

where $\sum(-1)^r S(B_r)$ denotes the sum over all $\binom{n}{r}$ replacements of r of the columns of B by zero columns.

Let $B = I - A$ where A is a substochastic matrix. The ith row sum of B_r is nonpositive or nonnegative according to whether the ith column of B_r is a zero or a nonzero column. Hence there are at least r row sums of B_r that are nonpositive and at least $n-r$ that are nonnegative. Therefore

Received by the editors September 2, 1965.

$$(-1)^r S(B_r) \geqq 0,$$
$$\text{per } (I - A) = \text{per } (B) \geqq 0.$$

We are indebted to Morris Newman for a preprint of [1].

Morris Newman informs me that essentially the same proof was communicated to him independently by Hazel Perfect.

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