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UNIVERSITY OF ILLINOIS

THE DISTANCE FROM $U(z)\cdot H^p$ TO 1

D. J. NEWMAN

If $U(z)$ is an inner function, then the set $U(z)\cdot H^p$ of all H^p multiples of $U(z)$ forms a closed subspace of H^p. In this note we compute the H^p distance between the constant function 1 and this closed subspace. It is of course well known that this distance is 0 if and only if $U(z)$ is a constant, i.e. if and only if $|U(0)|=1$. We will prove

THEOREM. dist $(1,\ U(z)\cdot H^p)=(1-|U(0)|^2)^{1/p}, \qquad p\geqq 1.$

PROOF. With

$$f_p(z) = \frac{1-(1-U(z)\overline{U}(0))^{2/p}}{U(z)}$$

we have

$$\begin{aligned}\|1-U(z)f_p(z)\| &= \left(\frac{1}{2\pi}\int_0^{2\pi}|1-U(z)\overline{U}(0)|^2 d\theta\right)^{1/p}\\ &= \left(\frac{1}{2\pi}\int_0^{2\pi}|U(z)-U(0)|^2 d\theta\right)^{1/p} = (1-|U(0)|^2)^{1/p},\end{aligned}$$

so that this distance is surely $\leqq(1-|U(0)|^2)^{1/p}$ and we need only show that $f_p(z)$ is the *closest* function to 1, i.e. that, for all $f(z)\in H^p$,

$$\|1-U(z)f(z)\|_p \geqq (1-|U(0)|^2)^{1/p}. \tag{1}$$

Consider

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$$I = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{U(z)} - f(z)\right)(U(z) - U(0))(1 - U(z)\overline{U}(0))^{1-2/p}d\theta$$
$$= \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{U(z)} - f_p(z)\right)(U(z) - U(0))(1 - U(z)\overline{U}(0))^{1-2/p}d\theta$$

(since $f(z) - f_p(z)$ is analytic).
Thus,

$$I = \frac{1}{2\pi}\int_0^{2\pi}\frac{(U(z) - U(0))}{U(z)}(1 - U(z)\overline{U}(0))d\theta$$
$$= \frac{1}{2\pi}\int_0^{2\pi}| U(z) - U(0) |^2 d\theta,$$

or

$$I = 1 - | U(0) |^2. \tag{2}$$

On the other hand, by Hölder's inequality, for $p > 1$,

$$I \leqq \left(\frac{1}{2\pi}\int_0^{2\pi}| 1 - U(z)f(z) |^p d\theta\right)^{1/p}.$$

$$\left(\frac{1}{2\pi}\int_0^{2\pi}(| U(z) - U(0) |^{2-2/p})^{p/(p-1)}d\theta\right)^{(p-1)/p}$$
$$= \|1 - U(z)f(z)\|_p\cdot\left(\frac{1}{2\pi}\int_0^{2\pi}| U(z) - U(0) |^2 d\theta\right)^{1-1/p}.$$

The same clearly holds for $p = 1$.

Hence

$$I \leqq \|1 - U(z)f(z)\|_p\cdot(1 - | U(0) |^2)^{1-1/p}. \tag{3}$$

Comparing (2) and (3) yields (1) immediately.

In the simple case of H^2 and a finite Blaschke product we obtain the

COROLLARY. *Let* $|z_i| < 1$, $i = 1, 2, \cdots, N$. *The minimum value of* $\sum | C_n |^2$ *subject to* $\sum_{n=0}^{\infty} C_n z_i^n = 1$, $i = 1, 2, \cdots, N$, *is* $1 - | z_1 z_2 \cdots z_N |^2$.

YESHIVA UNIVERSITY