KILLING KNOTS

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Let k be a tame knot in S^3, $N(k)$ a regular neighborhood of k and $M=S^3\setminus\operatorname{Int} N(k)$. M may be collapsed [1] to a 2-complex K. Let D be a disk in $N(k)$ with $D\cap\partial N(k)=\partial D$ and ∂D not contractible on $\partial N(k)$. $M=N(K)$ is the mapping cylinder C_f of a map $f\colon \partial N(K)\to K$, so $D\cup C_{f|\partial D}\cup K=L$ is a spine for $M\cup N(D)=U$. ∂U is a 2-sphere, so U is a cell and L is cellular. Then $S^3/L\cong S^3$. $k\cap L$ is a single point in the interior of D and $k\cap(S^3\setminus\operatorname{Int} U)$ is unknotted in $S^3\setminus\operatorname{Int} U$. It is easy to see that

THEOREM 1. *If k is a tame knot in S^3, there is a cellular 2-complex L in S^3 such that $k\cap L$ is a single point, and under the projection $p\colon S^3\to S^3/L\cong S^3$, $p(k)$ is a tame and unknotted simple closed curve.*

Note that if $k\cap L=\varnothing$, then $p(k)$ is unknotted if and only if k is unknotted.

THEOREM 2. *If $G=\pi_1(S^3\setminus k)$ is a knot group, there is a metric d on E^3 (inducing the standard topology) and a closed set P homeomorphic to E^1 such that P is "straight" (for any three points x, y, $z\in P$, $d(x, y)\pm d(y, z)=d(x, z)$) and $\pi_1(E^3\setminus P)=G$.*

PROOF. It may be assumed that $k\cap U=k\cap N(D)$ is a straight line segment. Let $P=k\cap\operatorname{Int} U$. Int. $U=E^3$ and inherits its metric from S^3. It is clear from the construction that $\pi_1(\operatorname{Int} U\setminus P)=\pi_1(S^3\setminus k)$.

REFERENCE

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