A SIMPLE PROOF OF A WELL-KNOWN OSCILLATION THEOREM

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THEOREM ([1], [2]). *The equation*

$$(ry')' + py = 0 \qquad (r > 0, \quad r \text{ and } p \text{ continuous}) \tag{1}$$

is oscillatory on $[0, +\infty)$ *provided*

$$\int^{\infty} \frac{1}{r} = \int^{\infty} p = +\infty. \tag{2}$$

PROOF. If (1) is nonoscillatory, the Riccati equation

$$z' + z^2/r + p = 0 \tag{3}$$

has a solution on some half-line $[a, \infty)$; thus, for large t,

$$z(t) + \int_a^t z^2/r = z(a) - \int_a^t p < 0. \tag{4}$$

Let $R(t) = \int_a^t z^2/r$; (4) says that

$$R^2 \leqq R' \cdot r \tag{5}$$

for $t \geqq b > a$ (b sufficiently large). Separation of variables and integration of (5) give

$$\int_b^t \frac{1}{r} \leqq R^{-1}(b) - R^{-1}(t) \leqq R^{-1}(b), \qquad t \geqq b,$$

which contradicts (2).

REFERENCES

1. W. Leighton, *On self-adjoint differential equations of second order*, J. London Math. Soc. **27** (1952), 37–47.

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